DATA 621 - Homework 3

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Introduction

Your objective is to build a binary logistic regression model on the training data set to predict whether the neighbourhood will be at risk for high crime levels. You will provide classifications and probabilities for the evaluation data set using your binary logistic regression model. You can only use the variables given to you (or, variables that you derive from the variables provided).

1. Data Exploration

1.1 Load Libraries

1.2 Read in data

1.2.1 Create data dictionary

Variable Name	Definition	NA
zn	proportion of residential land zoned for large lots (over 25000 square feet)	Outcome variable
indus	proportion of non-retail business acres per suburb	Outcome variable
chas	a dummy var. for whether the suburb borders the Charles River (1) or not (0)	Outcome variable
nox	nitrogen oxides concentration (parts per 10 million)	Outcome variable
rm	average number of rooms per dwelling	Outcome variable
age	proportion of owner-occupied units built prior to 1940	Outcome variable
dis	weighted mean of distances to five Boston employment centers	Outcome variable
rad	index of accessibility to radial highways	Outcome variable
tax	full-value property-tax rate per $10,000	Outcome variable
ptratio	pupil-teacher ratio by town	Outcome variable
lstat	lower status of the population (percent)	Outcome variable
medv	median value of owner-occupied homes in $1000s	Outcome variable

1.3 Basic dataset statistics

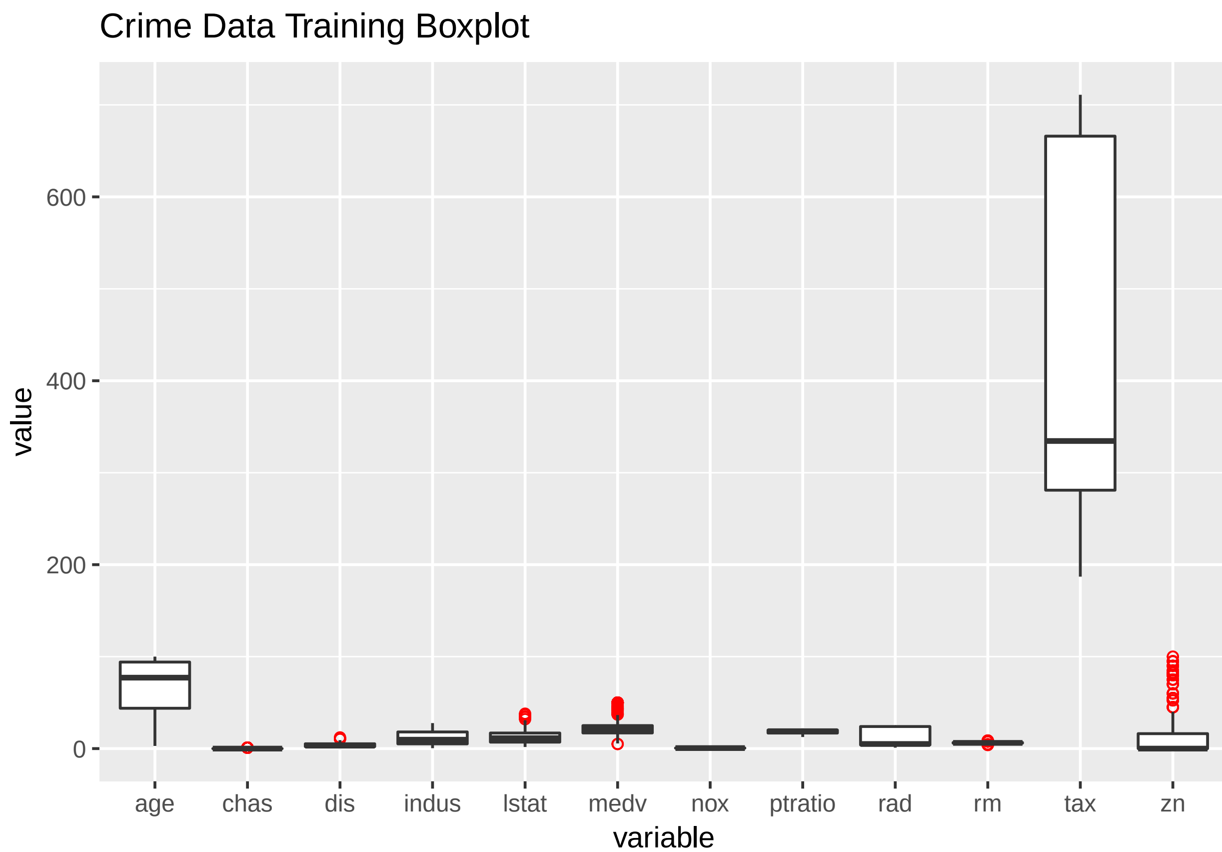
	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew	kurtosis	se
zn	1	466	11.5772532	23.3646511	0.00000	5.3542781	0.0000000	0.0000	100.0000	100.0000	2.1768152	3.8135765	1.0823466
indus	2	466	11.1050215	6.8458549	9.69000	10.9082353	9.3403800	0.4600	27.7400	27.2800	0.2885450	-1.2432132	0.3171281
chas	3	466	0.0708155	0.2567920	0.00000	0.0000000	0.0000000	0.0000	1.0000	1.0000	3.3354899	9.1451313	0.0118957
nox	4	466	0.5543105	0.1166667	0.53800	0.5442684	0.1334340	0.3890	0.8710	0.4820	0.7463281	-0.0357736	0.0054045
rm	5	466	6.2906738	0.7048513	6.21000	6.2570615	0.5166861	3.8630	8.7800	4.9170	0.4793202	1.5424378	0.0326516
age	6	466	68.3675966	28.3213784	77.15000	70.9553476	30.0226500	2.9000	100.0000	97.1000	-0.5777075	-1.0098814	1.3119625
dis	7	466	3.7956929	2.1069496	3.19095	3.5443647	1.9144814	1.1296	12.1265	10.9969	0.9988926	0.4719679	0.0976026
rad	8	466	9.5300429	8.6859272	5.00000	8.6978610	1.4826000	1.0000	24.0000	23.0000	1.0102788	-0.8619110	0.4023678
tax	9	466	409.5021459	167.9000887	334.50000	401.5080214	104.5233000	187.0000	711.0000	524.0000	0.6593136	-1.1480456	7.7778214
ptratio	10	466	18.3984979	2.1968447	18.90000	18.5970588	1.9273800	12.6000	22.0000	9.4000	-0.7542681	-0.4003627	0.1017669
lstat	11	466	12.6314592	7.1018907	11.35000	11.8809626	7.0720020	1.7300	37.9700	36.2400	0.9055864	0.5033688	0.3289887
medv	12	466	22.5892704	9.2396814	21.20000	21.6304813	6.0045300	5.0000	50.0000	45.0000	1.0766920	1.3737825	0.4280200
target	13	466	0.4914163	0.5004636	0.00000	0.4893048	0.0000000	0.0000	1.0000	1.0000	0.0342293	-2.0031131	0.0231835

The training data has 466 cases, with 13 predictor variables. Each case represents a neighbourhood in Boston. Our large sample size satisfies one of the requirements to fit our data to a logistic model.

Amazingly, there is not a single `NA` in the entire dataset, which will make our data cleaning job much easier!

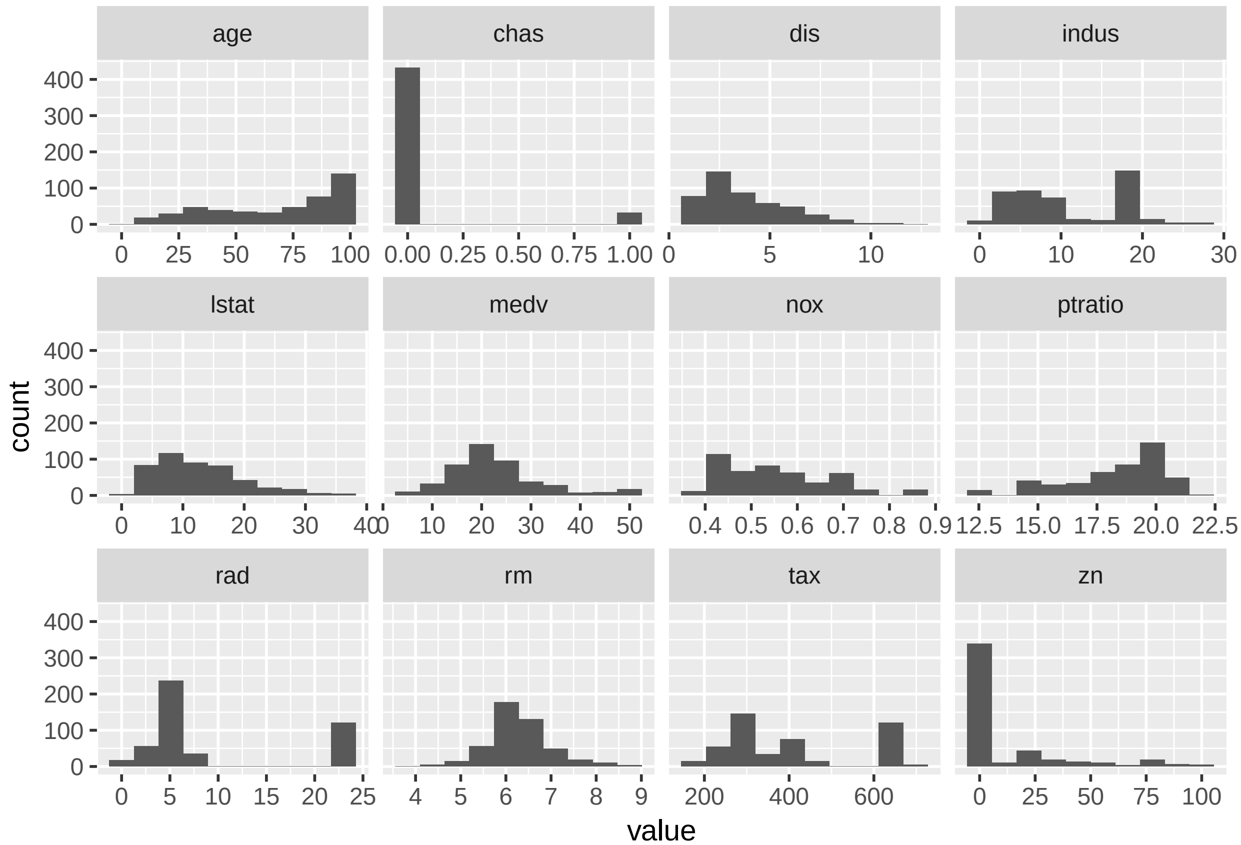
1.4 Summary Graphs

1.4.1 Boxplot



Aside from `zn`, this dataset doesn't have too many outliers.

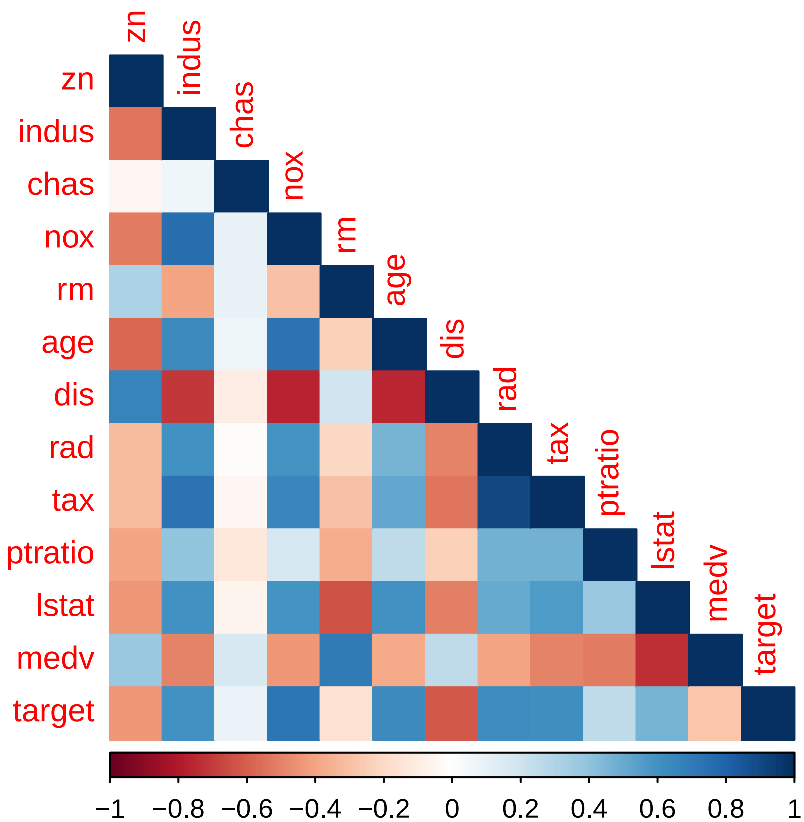
1.4.2 Histogram



We can see some variables, `age`, `chas`, `rad`, `zn`, in particular, are strongly skewed.

1.4.3 Correlation

1.4.3.1 Correlation Heatmap



1.4.3.2 Correlation (with response) table

	P-value	Correlation with response
zn	1.41560261828797e-22	-0.431681757278317
indus	7.84565111436042e-48	0.604850736249927
chas	0.0843481072424482	0.0800418718031737
nox	1.68013085953093e-77	0.726106218470473
rm	0.00095422493976375	-0.152553344896326
age	6.24573590524504e-53	0.63010624884034
dis	1.4501758603908e-50	-0.618673121688883
rad	1.64759680001221e-52	0.628104918805408
tax	4.70956558462997e-49	0.611113314858614
ptratio	4.05273029910627e-08	0.250848917529503
lstat	7.07143116645425e-27	0.469127015198214
medv	2.92493073681285e-09	-0.270550708927679

From the above correlation analysis, it appears that `chas` is not correlated with neither the response variable, nor any of the other predictor variables. This is important to note, since we may consider removing it from the final model.

Another concern is the high correlation between `rad` and `tax` - a staggering 0.9064632! We may want to remove one of these predictors from our model to prevent muddying it with collinearity.

2. Data Preparation

2.1 Missing Data

As noted earlier, the dataset is remarkably whole, so we may proceed without worrying about having to imputate any data.

2.2 Normality of Predictor Variables

As can be seen in the distribution plots in section 1.4.2, many of the predictor variables are not normally distributed. However, since logistic regression makes no assumptions, including the normality of the variables, we can safely skip this step, and keep the variables as they are.

2.3 Add or Remove Variables

As mentioned before, we'll consider removing two variables for one of our models. `chas`, due to it's low correlation with any of the other variables, and either `rad` or `tax`, due to high collinearity between the two.

Other than the variables mentioned above, I don't see any reason to remove any variables. Furthermore, there isn't enough implicit information from which we could possibly derive new variables.

2.4 Variable Transformation

339 of 466 cases in the `zn` variable have a value of 0, or roughly 72.75%. We may want to convert this to a binary variable, where

$$zn = \begin{cases} 0 & zn = 0 \\ 1 & zn \neq 0 \end{cases}$$

Additionally, we'll convert both the `target` variable, as well as `chas`, from integers to factors.

2.5 Outliers

I believe that once we recode the variable `zn` as outlined in section 2.4, we will no longer have the outlier issue that is currently affecting the predictor.

3. Build Models

Note that I will not be using any sort of 'automatic' model selection, e.g. stepwise regression. After reading this article, I've decided to forego any automated choosing, and build (and test) the models myself.

3.1 Model 1

My first model will use the original dataset as is, without any variable changes. This will serve as a sort of benchmark with which to gauge the effectiveness of our changes.

```
##
## Call:
## glm(formula = target ~ ., family = binomial(link = "logit"),
##     data = crime.training)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.8464  -0.1445  -0.0017   0.0029   3.4665
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -40.822934   6.632913  -6.155 7.53e-10 ***
## zn           -0.065946   0.034656  -1.903  0.05706 .
## indus        -0.064614   0.047622  -1.357  0.17485
## chas1         0.910765   0.755546   1.205  0.22803
## nox          49.122297   7.931706   6.193 5.90e-10 ***
## rm           -0.587488   0.722847  -0.813  0.41637
## age           0.034189   0.013814   2.475  0.01333 *
## dis           0.738660   0.230275   3.208  0.00134 **
## rad           0.666366   0.163152   4.084 4.42e-05 ***
## tax          -0.006171   0.002955  -2.089  0.03674 *
## ptratio       0.402566   0.126627   3.179  0.00148 **
## lstat         0.045869   0.054049   0.849  0.39608
## medv          0.180824   0.068294   2.648  0.00810 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 645.88  on 465  degrees of freedom
## Residual deviance: 192.05  on 453  degrees of freedom
## AIC: 218.05
##
## Number of Fisher Scoring iterations: 9
```

3.1.1 Model 1 Interpretation

There are several variables that are not significant to the model (i.e. $P > 0.05$), including `indus`, `chas`, `rm`, `lstat`, with `zn` right on the border of 0.05.

`zn`, `indus`, `rm`, and `tax` are all negatively correlated to the `target` variable, meaning an increase in any of these is correlated with a lower occurence of crime.

	Coefficient	Possible Reasoning
zn	-0.0659	More large homes would indicate a wealthier neighbourhood (unless zn is referring to apartment buildings)
indus	-0.0646	More likely to be a suburban (rather than urban) neighbourhood
chas1	0.9108	I'm not familiar with the Boston area
nox	49.1223	Higher pollution could be due to industry or a poorly-funded area, both of which attract crime
rm	-0.5875	More rooms means a larger home, which would mean a wealthier neighbourhood
age	0.0342	Older units are more likely to be occupied by lower-income residents, and lower-income neighbourhoods are more likely to have crime
dis	0.7387	Neighbourhoods farther away from employment centers have higher crime, possibly due to unemployment
rad	0.6664	Access to highways might indicate a more urban neighbourhood, which tend to have higher crime
tax	-0.0062	This one is unclear. Higher tax rate could be due to size of unit, or overall high tax rate for that area
ptratio	0.4026	Higher ratio is more likely in poorly-funded districts, which tend to have higher crime
lstat	0.0459	Lower income neighbourhoods tend to have more crime
medv	0.1808	Surprising that neighbourhoods with higher-valued homes had more crime

The model has an AIC (Akaike information criterion) of 218.05, and a BIC (Bayesian information criterion) of 271.92.

With a Null deviance of 645.88, and a Residual deviance of 192.05, we get a difference of 453.83.

Lastly, let's run an ANOVA Chi-Square test to view the effect each predictor variable is having on the response variable.

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: target
##
## Terms added sequentially (first to last)
##
##
##         Df Deviance Resid. Df Resid. Dev  Pr(>Chi)
## NULL                      465     645.88
## zn       1  127.411       464     518.46 < 2.2e-16 ***
## indus    1   86.433       463     432.03 < 2.2e-16 ***
## chas     1    1.274       462     430.76  0.258981
## nox      1  150.804       461     279.95 < 2.2e-16 ***
## rm       1    6.755       460     273.20  0.009349 **
## age      1    0.217       459     272.98  0.641515
## dis      1    7.981       458     265.00  0.004727 **
## rad      1   53.018       457     211.98 3.305e-13 ***
## tax      1    5.562       456     206.42  0.018355 *
## ptratio  1    5.657       455     200.76  0.017388 *
## lstat    1    0.814       454     199.95  0.366872
## medv     1    7.904       453     192.05  0.004933 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

3.2 Model 2

For our second model, we'll remove the variables deemed insignificant in model 1.

```
##
## Call:
## glm(formula = target ~ . - indus - chas - rm - lstat, family = binomial(link = "logit"),
##     data = crime.training)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.8295  -0.1752  -0.0021   0.0032   3.4191
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -37.415922   6.035013  -6.200 5.65e-10 ***
## zn           -0.068648   0.032019  -2.144  0.03203 *
## nox          42.807768   6.678692   6.410 1.46e-10 ***
## age           0.032950   0.010951   3.009  0.00262 **
## dis           0.654896   0.214050   3.060  0.00222 **
## rad           0.725109   0.149788   4.841 1.29e-06 ***
## tax          -0.007756   0.002653  -2.924  0.00346 **
## ptratio       0.323628   0.111390   2.905  0.00367 **
## medv          0.110472   0.035445   3.117  0.00183 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 645.88  on 465  degrees of freedom
## Residual deviance: 197.32  on 457  degrees of freedom
## AIC: 215.32
##
## Number of Fisher Scoring iterations: 9
```

3.2.1 Model 2 Interpretation

	Coefficient	Possible Reasoning
zn	-0.0686	More large homes would indicate a wealthier neighbourhood (unless zn is referring to apartment buildings)
nox	42.8078	Higher pollution could be due to industry or a poorly-funded area, both of which attract crime
age	0.033	Older units are more likely to be occupied by lower-income residents, and lower-income neighbourhoods are more likely to have crime
dis	0.6549	Neighbourhoods farther away from employment centers have higher crime, possibly due to unemployment
rad	0.7251	Access to highways might indicate a more urban neighbourhood, which tend to have higher crime
tax	-0.0078	This one is unclear. Higher tax rate could be due to size of unit, or overall high tax rate for that area
ptratio	0.3236	Higher ratio is more likely in poorly-funded districts, which tend to have higher crime
medv	0.1105	Surprising that neighbourhoods with higher-valued homes had more crime

This model has an AIC of 215.32, and a BIC of 252.62.

With a Null deviance of 645.88, and a Residual deviance of 197.32, we get a difference of 448.55.

Once again, we'll run an ANOVA Chi-Square test on this model.

```
## Analysis of Deviance Table
##
## Model: binomial, link: logit
##
## Response: target
##
## Terms added sequentially (first to last)
##
##
##         Df Deviance Resid. Df Resid. Dev  Pr(>Chi)
## NULL                      465     645.88
## zn       1  127.411       464     518.46 < 2.2e-16 ***
## nox      1  230.177       463     288.29 < 2.2e-16 ***
## age      1    0.767       462     287.52 0.3810001
## dis      1    4.296       461     283.22 0.0382133 *
## rad      1   55.953       460     227.27 7.423e-14 ***
## tax      1   15.916       459     211.35 6.620e-05 ***
## ptratio  1    2.706       458     208.65 0.0999454 .
## medv     1   11.326       457     197.32 0.0007644 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This model has a smaller difference between deviances, but has a slightly lower AIC.

3.3 Model 3

For the last model, we'll transform `zn` to a binary variable.

```
##
## Call:
## glm(formula = target ~ ., family = binomial(link = "logit"),
##     data = crime.training.copy)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.8321  -0.1891  -0.0112   0.0030   3.5349
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -37.511394   5.986257  -6.266 3.70e-10 ***
## zn1          -1.718763   0.734165  -2.341  0.01923 *
## nox          43.985459   6.683768   6.581 4.67e-11 ***
## age           0.034869   0.011108   3.139  0.00169 **
## dis           0.704363   0.215670   3.266  0.00109 **
## rad           0.719840   0.146091   4.927 8.34e-07 ***
## tax          -0.007773   0.002607  -2.981  0.00287 **
## ptratio       0.283659   0.117259   2.419  0.01556 *
## medv          0.108308   0.035554   3.046  0.00232 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 645.88  on 465  degrees of freedom
## Residual deviance: 197.45  on 457  degrees of freedom
## AIC: 215.45
##
## Number of Fisher Scoring iterations: 9
```

3.4 Comparison of Models

References

- http://userwww.sfsu.edu/efc/classes/biol710/logistic/logisticreg.htm
- https://www.hackerearth.com/practice/machine-learning/machine-learning-algorithms/logistic-regression-analysis-r/tutorial/
- https://stats.stackexchange.com/questions/59879/logistic-regression-anova-chi-square-test-vs-significance-of-coefficients-