Math Exam Review: Answers

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1 Introduction

2 Exam review questions

1. For what values of the variable(s) is each rational expression not defined?

   a)
   $$\frac{y-2}{5x(y-2)}$$
   x and y are not defined, they're variables

   b)
   $$\frac{x^2-7x+12}{x^2-x-12}$$
   x is not defined, it's a variable

2. Simplify the following and state restrictions.

   a)
   $$\begin{aligned} &\frac{1}{x-1}-\frac{1}{1-x} && x\neq 1\\ &=\frac{1}{x-1}+\frac{1}{-(1-x)} && x\neq 1\\ &=\frac{1}{x-1}+\frac{1}{x-1} && x\neq 1\\ &=\frac{2}{x-1} && x\neq 1 \end{aligned}$$

b)

$$\frac{a - a^3 + \frac{2}{a}}{\frac{8}{a} - 2a^3} \qquad a \neq 0, \frac{8}{a} - 2a^3 \neq 0$$

$$= \frac{\frac{a^2 - a^4 + 2}{a}}{\frac{8 - 2a^4}{a}} \qquad a \neq 0, 8 - 2a^4 \neq 0$$

$$= \frac{a^2 - a^4 + 2}{8 - 2a^4} \qquad a \neq 0, a^4 \neq 4$$

$$= \frac{(2 - a^2)(a^2 + 1)}{2(4 - a^4)} \qquad a \neq 0, \pm\sqrt{2}$$

$$= \frac{(2 - a^2)(a^2 + 1)}{2(2 - a^2)(2 + a^2)} \qquad a \neq 0, \pm\sqrt{2}$$

$$= \frac{a^2 + 1}{2(2 + a^2)} \qquad a \neq 0, \pm\sqrt{2}$$

$$= \frac{a^2 + 1}{4 + 2a^2} \qquad a \neq 0, \pm\sqrt{2}$$

c)

$$\frac{x^2 + 4x - 5}{2x^2 + 11x + 5} \div \frac{x^2 - 3x + 2}{2x^2 + 5x + 2}$$

$$= \frac{(x + 5)(x - 1)}{(2x + 1)(x + 5)} \div \frac{(x - 2)(x - 1)}{(2x + 1)(x + 1)} \qquad x \neq -\frac{1}{2}, -5, -1$$

$$= \frac{(x + 5)(x - 1)}{(2x + 1)(x + 5)} \cdot \frac{(2x + 1)(x + 1)}{(x - 2)(x - 1)} \qquad x \neq -\frac{1}{2}, -5, -1, 2, 1$$

$$= \frac{\cancel{(x + 5)}\cancel{(x - 1)}}{\cancel{(2x + 1)}\cancel{(x + 5)}} \cdot \frac{\cancel{(2x + 1)}(x + 1)}{(x - 2)\cancel{(x - 1)}} \qquad x \neq -\frac{1}{2}, -5, -1, 2, 1$$

$$= \frac{x + 1}{x - 2} \qquad x \neq -\frac{1}{2}, -5, -1, 2, 1$$

d)

$$\frac{x^2 - 4x + 3}{x^2 - x}$$

$$= \frac{(x - 3)(x - 1)}{x(x - 1)} \qquad x \neq 0, 1$$

$$= \frac{(x - 3)\cancel{(x - 1)}}{x\cancel{(x - 1)}} \qquad x \neq 0, 1$$

$$= \frac{x - 3}{x} \qquad x \neq 0, 1$$

e)

$$\begin{aligned}
&\frac{x^2+x-6}{x^2+5x+4} \div \frac{x^2+4x+3}{x^2+6x+8} \\
&= \frac{(x+3)(x-2)}{(x+4)(x+1)} \div \frac{(x+3)(x+1)}{(x+4)(x+2)} && x \neq -4, -1, -2 \\
&= \frac{(x+3)(x-2)}{(x+4)(x+1)} \cdot \frac{(x+4)(x+2)}{(x+3)(x+1)} && x \neq -4, -1, -2, -3, -1 \\
&= \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+4)}(x+1)} \cdot \frac{\cancel{(x+4)}(x+2)}{\cancel{(x+3)}(x+1)} && x \neq -4, -1, -2, -3, -1 \\
&= \frac{(x-2)(x+1)}{(x+1)^2} && x \neq -4, -1, -2, -3, -1 \\
&= \frac{x^2-x-2}{x^2+2x+1} && x \neq -4, -1, -2, -3, -1
\end{aligned}$$

f)

$$\begin{aligned}
&\frac{x-3}{x+2} - \frac{6}{x-4} && x \neq 4, -2 \\
&= \frac{(x-3)(x-4)-6(x+2)}{(x+2)(x-4)} && x \neq 4, -2 \\
&= \frac{x^2-7x+12-6x-12}{(x+2)(x-4)} && x \neq 4, -2 \\
&= \frac{x^2-13x}{(x+2)(x-4)} && x \neq 4, -2 \\
&= \frac{x^2-13x}{x^2-2x-8} && x \neq 4, -2
\end{aligned}$$

g)

$$\begin{aligned}
&\frac{2x^2-x-1}{3x^2+x-2} \div \frac{2x^2-5x-3}{3x^2-11x+6} \\
&= \frac{(2x-1)(x+1)}{(3x-2)(x+1)} \div \frac{(2x+1)(x-3)}{(3x-2)(x-3)} && x \neq \frac{2}{3}, -1, 3 \\
&= \frac{(2x-1)(x+1)}{(3x-2)(x+1)} \cdot \frac{(3x-2)(x-3)}{(2x+1)(x-3)} && x \neq \frac{2}{3}, -1, 3, -\frac{1}{2} \\
&= \frac{(2x-1)\cancel{(x+1)}}{\cancel{(3x-2)}\cancel{(x+1)}} \cdot \frac{\cancel{(3x-2)}\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}} && x \neq \frac{2}{3}, -1, 3, -\frac{1}{2} \\
&= \frac{2x-1}{2x+1} && x \neq \frac{2}{3}, -1, 3, -\frac{1}{2}
\end{aligned}$$

h)

$$
\begin{aligned}
&\frac{3}{x^2-3x-4}-\frac{2}{x^2+5x+4} \\
&=\frac{3}{(x-4)(x+1)}-\frac{2}{(x+4)(x+1)} && x\neq 4,-1,-4 \\
&=\frac{3(x+4)-2(x-4)}{(x-4)(x+1)(x+4)} && x\neq 4,-1,-4 \\
&=\frac{3x+12-2x+8}{(x-4)(x+1)(x+4)} && x\neq 4,-1,-4 \\
&=\frac{x+20}{(x-4)(x+1)(x+4)} && x\neq 4,-1,-4
\end{aligned}
$$

3. Factor Completely.

a)

$$
\begin{aligned}
&4ax^5-\frac{axy^4}{4} \\
&=ax(4x^4-\frac{y^4}{4}) \\
&=\frac{ax(16x^4-y^4)}{4} \\
&=\frac{ax(4x^2-y^2)(4x^2+y^2)}{4} \\
&=\frac{ax(2x-y)(2x+y)(4x^2+y^2)}{4}
\end{aligned}
$$

4. Solve by factoring

a)

$$
\begin{aligned}
2x^2-9x+10&=0 \\
(2x+5)(x+2)&=0 \\
2x+5&=0 && -x+2=0 \\
x&=-\frac{5}{2} && x=-2 \\
x&=-\frac{5}{2},-2
\end{aligned}
$$

b)

$$
\begin{aligned}
2x^2-7x&=4 \\
2x^2-7x-4&=0 \\
(2x+1)(x-4)&=0 \\
2x+1&=0 && x-4=0 \\
x&=-\frac{1}{2} && x=4 \\
x&=-\frac{1}{2},4
\end{aligned}
$$

5. Solve $5x^2 + 1 = 8x$ using the quadratic formula. Express the answer in simplest radical form.

$$5x^2 + 1 = 8x \qquad ax^2 + bc + c = 0$$

$$5x^2 - 8x + 1 = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 5 \cdot 1}}{2 \cdot 5}$$

$$x = \frac{8 \pm \sqrt{44}}{10}$$

$$x = \frac{8 \pm 4\sqrt{11}}{10}$$

$$x = \frac{8 + 4\sqrt{11}}{10} \qquad x = \frac{8 - 4\sqrt{11}}{10}$$

$$x = \frac{4 + 2\sqrt{11}}{5} \qquad x = \frac{4 - 2\sqrt{11}}{5}$$

$$x = \frac{4 + 2\sqrt{11}}{5}, \frac{4 - 2\sqrt{11}}{5}$$

6. Evaluate the expression $E(x) = \frac{x^2+5x+6}{x^2+7x+10}$ for $x = -11$

$$E(x) = \frac{x^2 + 5x + 6}{x^2 + 7x + 10}$$

$$E(x) = \frac{(x+3)(x+2)}{(x+5)(x+2)}$$

$$E(x) = \frac{x+3}{x+2}$$

$$E(-11) = \frac{-11+3}{-11+2}$$

$$E(-11) = \frac{8}{9}$$

7. Solve for x (leave in answers in the radical form if necessary)

a)

$$-2x^2 - 3x + 1 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot -2 \cdot 1}}{2 \cdot -2} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{17}}{-4}$$

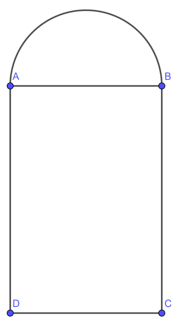
$$x = \frac{-3 \pm \sqrt{17}}{4}$$

$$x = \frac{-3 + \sqrt{17}}{4}, \frac{-3 - \sqrt{17}}{4}$$

b)

$$
\begin{aligned}
6^{x-5} &= \frac{1}{36^{2x+5}}\\
6^{x-5} &= 36^{-(2x+5)}\\
6^{x-5} &= 6^{2(-2x-5)}\\
6^{x-5} &= 6^{-4x-10}\\
x-5 &= -4x-10\\
5x &= -5\\
x &= -1
\end{aligned}
$$

8. A rectangular window is continued at the top side with a semi-circle. The perimeter of the window is 10m. Find the dimensions of the window that will maximize the area of the window.



$$
\begin{aligned}
2(\overline{AB}+\overline{BC}) &= 10\\
\overline{AB}+\overline{BC} &= 5
\end{aligned}
$$

Let x represent $\overline{AB}$

$$
\begin{aligned}
Area_{ABCD} &= x(10-x) \quad & Area_{\widehat{AB}} &= (\frac{x}{2})^2\pi\\
&= 10x-x^2 & &= \frac{x^2\pi}{4}
\end{aligned}
$$

$$
\begin{aligned}
Area &= 10x-x^2+\frac{x^2\pi}{4}\\
0 &= x(10-x+\frac{x\pi}{4})\\
0 &= x(x(\frac{-4+\pi}{4})-10)\\
x &= 0, \frac{40}{\pi-4}
\end{aligned}
$$

Maximum area is the average of the x_1, x_2, meaning maximum area is where $x = \frac{20}{\pi-4}$. No need to really worry about this, I was told it doesn't matter.

9. Determine the vertex of $y = -4x^2 - 16x + 5$ and state if it is a maximum or minimum

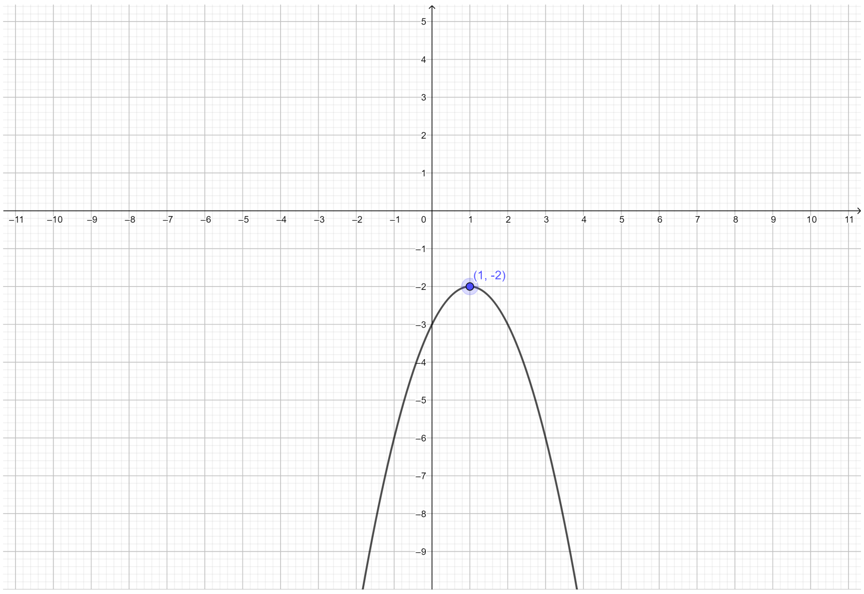
   Remember that looking of the first term (the $-4x^2$) if it's facing up (Is positive), then it is a minimum, if facing down (Is negative), the it is a maximum. In this case, $-4x^2$ is negative, meaning it was a **maximum**

10. Complete the square and represent graphically the following quadratic equation $y = -x^2 + 2x - 3$

$$
y = -(x^2 - 2x + 3)
$$

$$y = -((x-1)^2 + 3 - 1)$$
$$y = -(x-1)^2 - 2$$

Here is the graphical representation, vertex is $(1, -2)$, facing down.



11. Given that θ is an angle in standard position, $0° \le \theta \le 360°$, with $\cos\theta = -\frac{\sqrt{3}}{2}$. Find all possible values of θ. Remember CAST rule

$$\cos\beta = \frac{\sqrt{3}}{2}$$
$$\beta = 30°$$

$$\cos(180 - \beta) = \cos(180 + \beta) = -\cos\beta$$

$$180 - \beta = \theta \qquad 180 + \beta = \theta$$
$$180 - 30 = \theta \qquad 180 + 30 = \theta$$
$$150 = \theta \qquad 210 = \theta$$
$$\theta = 150°, 210°$$

12. In $\triangle ABC, \angle B = 107°$, $a = 18.7$, $c = 10.5$. Solve the triangle. as you're given two sides and one angle (SAS) you can use the cosine law. $c^2 = a^2 + b^2 - 2ab\cos C$ to find the side,

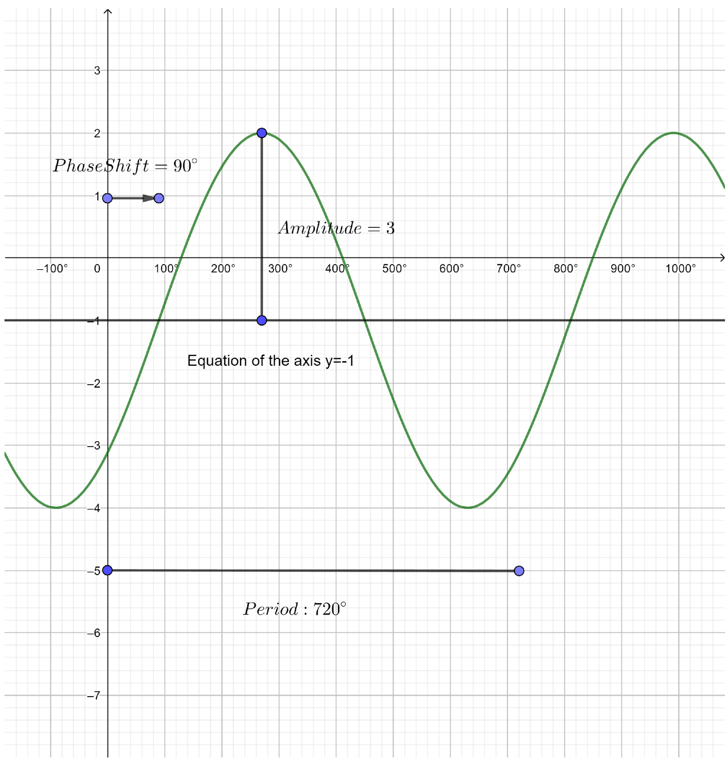
the sine law $\frac{\sin A}{a} = \frac{\sin B}{B} = \frac{\sin C}{C}$ to find the angles

$$\begin{aligned}
b^2 &= a^2 + c^2 - 2ac\cos B \\
b^2 &= 18.7^2 + 10.5^2 - 2 \cdot 18.7 \cdot 10.5 \cdot \cos 107^\circ \\
b &= 23.974035297475865 \\
\frac{\sin B}{b} &= \frac{\sin C}{c} \\
\frac{\sin 107^\circ}{23.984} &= \frac{\sin C}{10.5} \\
0.419 &= \sin C \\
24.8^\circ &= \angle C \\
\angle A &= 180 - 24.8 - 107 \\
\angle A &= 48.2
\end{aligned}$$

13. On graph paper provided, sketch one cycle of $y = 3\sin\frac{1}{2}(\theta - 90^\circ) - 1$

$$\begin{aligned}
&\text{Amplitude} = 3 \\
&\text{Equation of the axis} : y = -1 \\
&\text{Phase Shift} = 90^\circ right \\
&\text{Period} : \frac{360}{\frac{1}{2}} = 720^\circ
\end{aligned}$$

The graph is drawn as shown



14. Given the function $y = -4\cos 2(\theta + \frac{180}{3}) + 5$, state the amplitude, period, phase shift, and

range.

$$\begin{aligned}
&\text{Amplitude} = 4\\
&\text{Equation of the axis} : y = 5\\
&\text{Phase Shift} = \frac{180}{3} = 60° left\\
&\text{Period} : \frac{360}{2} = 180°
\end{aligned}$$

15. Solve the equation for $0 \leq \theta \leq 360°$. $\sin\theta(3 - 4\cos^2\theta) = 0$

$$\sin\theta = 0 \qquad\qquad 3 - 4\cos^2\theta = 0$$

$$\theta = 0°, 180°, 360° \qquad\qquad \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = 0°, 180°, 360° \qquad\qquad \theta = 180 - 30, 180 + 30$$

$$\theta = 0°, 180°, 360° \qquad\qquad \theta = 150°, 210°$$

$$\theta = 0°, 150°, 180°, 210°, 360°$$

16. Solve the equation $2\sin^2\theta - \sin\theta - 1 = 0$ for $0 \leq \theta \leq 360°$.

Let x represent $\sin\theta$

$$\begin{aligned}
2x^2 - x - 1 &= 0\\
(2x + 1)(x - 1) &= 0\\
x &= -\frac{1}{2}, 1
\end{aligned}$$

$$\sin\theta = -\frac{1}{2} \qquad\qquad \sin\theta = 1$$

$$\theta = 180 + 30, 360 - 30 \qquad\qquad \theta = 90°$$

$$\theta = 210°, 330° \qquad\qquad \theta = 90°$$

$$\theta = 90°, 210°, 330°$$

17. Question Missing there's just nothing in the paper

18. Question Missing there's just nothing in the paper

19. Prove the identity:

   a) $\frac{\sin^2\theta - 1}{\cos^2\theta - 1} = \frac{1}{\sin^2\theta} - 1$

   Remember that $\sin^2\theta + \cos^2\theta = 1$

$$\begin{aligned}
LS &= \frac{\sin^2\theta - 1}{\cos^2\theta - 1}\\
&= \frac{-\cos^2\theta}{-\sin^2\theta}\\
&= \frac{\cos^2\theta}{\sin^2\theta}
\end{aligned}
\qquad\qquad
\begin{aligned}
RS &= \frac{1}{\sin^2\theta} - 1\\
&= \frac{1 - \sin^2\theta}{\sin^2\theta}\\
&= \frac{\cos^2\theta}{\sin^2\theta}
\end{aligned}$$

$$\because LS = RS$$

$$\therefore \frac{\sin^2\theta - 1}{\cos^2\theta - 1} = \frac{1}{\sin^2\theta} - 1$$

b) $\tan\theta + \frac{\cos\theta}{1+\sin\theta} = \frac{1}{\cos\theta}$

Remember that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$
\begin{aligned}
LS &= \tan\theta + \frac{\cos\theta}{1+\sin\theta} \\
&= \frac{\sin\theta(1+\sin\theta)+\cos\theta}{\cos\theta(1+\sin\theta)} \\
&= \frac{sin^2\theta + \cos^2\theta + \sin\theta}{\cos\theta(1+\sin\theta)} \\
&= \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)} \\
&= \frac{1}{\cos\theta}
\end{aligned}
\qquad\qquad RS = \frac{1}{\cos\theta}
$$

$$\because LS = RS$$
$$\therefore \tan\theta + \frac{\cos\theta}{1+\sin\theta} = \frac{1}{\cos\theta}$$

c) $\frac{\sin x + \tan x}{\cos x + 1} = \tan x$

$$
\begin{aligned}
LS &= \frac{\sin x + \tan x}{\cos x + 1} \\
&= \frac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1} \\
&= \frac{\frac{\sin x \cos x + \sin x}{\cos x}}{\cos x + 1} \\
&= \frac{\sin x \cos x + \sin x}{\cos x(\cos x + 1)} \\
&= \frac{\sin x(\cos x + 1)}{\cos x(\cos + 1)} \\
&= \frac{\sin x}{\cos x} \\
&= \tan x
\end{aligned}
\qquad\qquad RS = \tan x
$$

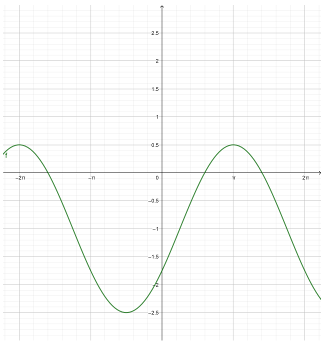
$$\because LS = RS$$
$$\therefore \frac{\sin x + \tan x}{\cos x + 1} = \tan x$$

20. Complete the table

Sine Function	$y = -3\sin(2x - 60) + 1$	
Amplitude		2
Period		270°
Phase Shift		30° left
Vertical Shift		None
Domain		
Range		

Sine Function	$y=-3\sin(2x-60)+1$	$\underline{2\sin(\frac{4}{3}(x+30))}$
Amplitude	3	2
Period	180°	270°
Phase Shift	30° right	30° left
Vertical Shift	1 up	None
Domain	D=$\{x \in \mathbb{R}\}$	D=$\{x \in \mathbb{R}\}$
Range	R=$\{y \in \mathbb{R} \mid -2 \leq y \leq 4\}$	R=$\{y \in \mathbb{R} \mid -2 \leq y \leq 2\}$

21. State a possible equation for the cosine function shown.



$$Min = -2.5 \qquad Max = 0.5$$

$$\text{Amplitude} = \frac{Max - Min}{2} \qquad \text{Period} = \frac{2\pi}{3\pi}$$

$$\text{Amplitude} = \frac{0.5 + 2.5}{2} \qquad \text{Period} = \frac{2}{3}$$

$$\text{Amplitude} = 1.5$$

$$\text{Equation of the axis: } y = \frac{Max + Min}{2} \qquad \text{Phase Shift} = \pi \text{ right}$$

$$\text{Equation of the axis: } y = \frac{-2.5 + 0.5}{2}$$

$$\text{Equation of the axis: } y = -1$$

$$y = 1.5\cos(\frac{2}{3}(x - \pi)) - 1$$

22. Consider the function $y = 3\sin 2(x + 90) + 5$.
    State the Phase Shift, Period, Vertical displacement, Amplitude, Domain, Range
    a) Graph the function over two complete graphs

Phase Shift	90° left
Period	$\frac{360}{2} = 180°$
Vertical Displacement	5
Amplitude	3
Domain	$D = \{x \in \mathbb{R}\}$
Range	$R = \{y \in \mathbb{R} \mid 2 \leq y \leq 8\}$

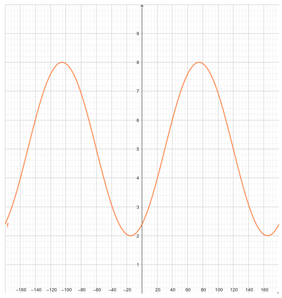
    I will most likely not do the graph, if you really want me to add this, ask me on discord.

23. Graph the function $y = -3\sin(2x - 240)$ for $-180° \leq x \leq 180°$

$$y = -3\sin(2x - 240)$$

$$y = -3\sin(2(x - 120))$$

Phase Shift	$120°$ right
Period	$\frac{360}{2} = 180°$
Vertical Displacement	0
Amplitude	3
Domain	$D = \{x \in \mathbb{R}\}$
Range	$R = \{y \in \mathbb{R} \mid -3 \leq y \leq 3\}$



24. On a typical day at an ocean port, the water has a maximum depth of $18m$ at 6:00 a.m. The minimum depth of 9 m occurs 6.8 h later. Sketch one complete cycle and write an equation of the form: $h = a\cos b(t-d)+c$ to describe the relationship between the depth h of the water and the time t. *I'm assuming t is in terms of hours as before it says 6.8 h later.*

$$\text{Max} = 18 \qquad \text{Min} = 9$$

$$\text{Vertical Displacement} = \frac{Max + Min}{2} \qquad \text{Amplitude} = \frac{Max - Min}{2}$$

$$\text{Vertical Displacement} = \frac{18+9}{2} \qquad \text{Amplitude} = \frac{18-9}{2}$$

$$\text{Vertical Displacement/c} = \frac{27}{2} \qquad \text{Amplitude/a} = \frac{9}{2}$$

$$\text{Phase Shift} = 6.8 \text{ right} \qquad \text{Period} = 2 * 6.8$$

$$\text{Phase Shift/d} = 6.8 \text{ right} \qquad \text{Period/b} = 13.6$$

$$h = 13.5\cos 13.6(t - 6.8) + 4.5$$

Again, graphs are annoying, if you want me to explain ask me on discord.

25. Given the function $f(x) = \frac{x+1}{x^2}$, determine and simplify $f(\frac{1}{x})$

$$f(\frac{1}{x}) = \frac{\frac{1}{x}+1}{(\frac{1}{x})^2}$$

$$f(\frac{1}{x}) = \frac{\frac{1+x}{x}}{\frac{1}{x^2}}$$

$$f(\frac{1}{x}) = \frac{x^2(x+1)}{x}$$

$$f(\frac{1}{x}) = x^2 + x$$

26. Solve the triangle $\triangle ABC$ in which $AB = 23.3cm$ and $BC = 26.8cm$ and $\angle ABC = 113°$. Round the angles to the nearest degree and lengths to one decimal place. Include diagram.

Use the cosine law to find the missing side$c^2 = a^2 + b^2 - 2ab\cos C$. Then use sine law to find the missing angles $\frac{\sin A}{a} = \frac{\sin B}{B} = \frac{\sin C}{C}$.

$$c = 23.3 \qquad a = 26.8 \qquad B = 113°$$

$$\begin{aligned}
b^2 &= a^2 + c^2 - 2 \times a \times c \cos B \\
b^2 &= 26.8^2 + 23.3^2 - 2 \times 23.3 \times 26.8 \cos 113° \\
b^2 &= 1749.11 \\
b &= 41.8 \\
\frac{\sin A}{a} &= \frac{\sin B}{B} \\
\frac{\sin A}{26.8} &= \frac{\sin 113°}{41.8} \\
\sin A &= \frac{26.8 \times \sin 113°}{41.8} \\
A &= \arcsin(\frac{26.8 \times \sin 113°}{41.8}) \\
A &= 36° \\
C &= 180 - A - B \\
C &= 180 - 36 - 113 \\
C &= 31°
\end{aligned}$$

For diagram refer to this link.

27. Determine how much money is needed today in order to have \$6000 in three years at 6% compounded quarterly.

    Compound interest formula: $A = P(1 + \frac{r}{n})^{nt}$. (If you want to understand these variables search online)

$$\begin{aligned}
P &= 6000 & r &= 0.06 \\
n &= 4 & t &= 3
\end{aligned}$$

$$\begin{aligned}
A &= 6000(1 + \frac{0.06}{4})^{3*4} \\
A &= 7173.71
\end{aligned}$$

28. Suppose you begin a saving program to have \$10 000 after 10 years. You plan to make regular deposits every month into an investment account that pays 6% compounded monthly, calculate each regular deposit.

    This uses the future value annuity formula $FV = P\frac{(1+\frac{r}{n})^{nt}-1}{\frac{r}{n}}$

$$\begin{aligned}
n &= 12 & t &= 10 \\
r &= 0.06 & FV &= 10000
\end{aligned}$$

$$\begin{aligned}
FV &= P\frac{(1+\frac{r}{n})^{nt}-1}{\frac{r}{n}} \\
10000 &= P\frac{(1+\frac{0.06}{12})^{12\times 10}-1}{\frac{0.06}{12}} \\
\frac{10000}{\frac{(1+\frac{0.06}{12})^{12\times 10}-1}{\frac{0.06}{12}}} &= P \\
61.02 &= P
\end{aligned}$$

29. A penny is tossed into the air from a bridge and falls to the water below. The height of the penny h metres, relative to the water t seconds after being thrown is given by $h = -5t^2 + 10t + 20$.

   a) Determine the max height of the penny above the water.

   I am *pretty* sure that this involves completing the square. *Maybe I got the name wrong*

$$\begin{aligned}
h &= -5t^2 + 10t + 20 \\
h &= -5(t^2 - 2t - 4) \\
h &= -5((t-1)^2 - 5) \\
h &= -5(t-1)^2 + 25\text{Vertex: } (1, 25)
\end{aligned}$$

   Maximum height is 25 metres

   b) How long does it take the penny to reach its max height?

   it takes 1 second.

   Note: The x value of the maximum is the average of the two x intercepts $\frac{x_1+x_2}{2}$. Replacing x_1, x_2 with the quadratic formula you get the x value of vertex is $-\frac{b}{a}$.

30. What transformations have been performed on $y = f(x)$ to attain $y = 3f(-x+1) - 2$?

$$y = 3f(-(x-1)) - 2$$

Vertical stretch by a factor of 3
Reflect along the y axis
Horizontal shift 1 unit to the right
Vertical shift 2 units down

31. Describe the transformations applied to $y = \sqrt{49 - x^2}$ to produce $y = -2\sqrt{49 - (x+5)^2} - 6$.

Reflect along the x axis
Vertical stretch by a factor of 2
Horizontal shift 5 units to the left
Vertical shift 6 units down

32. Given $f(x) = 4x^2 - 5$, determine

   a) $f(2)$

$$\begin{aligned}
f(2) &= 4 \times 2^2 - 5 \\
f(2) &= 11
\end{aligned}$$

b) $f(x) = 31$

$$\begin{aligned} 31 &= 4x^2 - 5 \\ 36 &= 4x^2 \\ x &= \pm 3 \end{aligned}$$

33. Determine the inverse of $f(x) = \frac{2}{5+x} - 1$, and state its domain.

$$\begin{aligned} f(x) &= \frac{2}{5+x} - 1 \\ x &= \frac{2}{5+f'(x)} - 1 \\ x+1 &= \frac{2}{5+f'(x)} \\ 5+f'(x) &= \frac{2}{x+1} \\ f'(x) &= \frac{2}{x+1} - 5 \\ R &= \{f(x) \in \mathbb{R} \mid f(x) \neq -1\} \\ D' &= \{x \in \mathbb{R} \mid x \neq -1\} \end{aligned}$$

34. For each case,
    - find the inverse
    - state if the inverse is a function or not

a) $f(x) = \frac{1}{2}x - 3$

$$\begin{aligned} x &= \frac{1}{2}f'(x) - 3 \\ x+3 &= \frac{1}{2}f'(x) \\ f'(x) &= 2x + 6 \\ &\text{This inverse is a function} \end{aligned}$$

b) $f(x) = x^2 + 2$

$$\begin{aligned} x &= f'(x)^2 + 2 \\ x-2 &= f'(x)^2 \\ \pm\sqrt{x-2} &= f'(x) \\ &\text{This inverse is not a function} \end{aligned}$$

c) $f(x) = \frac{2x}{x+1}$

$$\begin{aligned} x &= \frac{2f'(x)}{f'(x)+1} \\ xf'(x) + x &= 2f'(x) \\ f'(x)(x-2) &= -x \\ f'(x) &= \frac{x}{2-x} \\ &\text{This inverse is a function} \end{aligned}$$

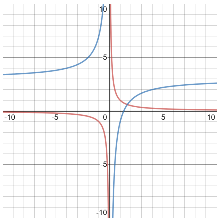
d) $f(x) = 2(x+2)^2 + 2$

$$
\begin{aligned}
x &= 2(f'(x)+2)^2 + 2 \\
x - 2 &= 2(f'(x)+2)^2 \\
\frac{x-2}{2} &= (f'(x)+2)^2 \\
\pm\frac{x-2}{2} - 2 &= f'(x)
\end{aligned}
$$

This inverse is not a function

35. Given $f(x) = \frac{1}{x}$

a) Sketch the image of $y = 2f(-\frac{1}{2}x) + 3$



b) Write and simplify the equation of the image in (a).

$$
\begin{aligned}
y &= 2f(-\frac{1}{2}x) + 3 \\
y &= 2 \times \frac{1}{-\frac{1}{2}x} + 3 \\
y &= 2 \times \frac{2}{x} + 3 \\
y &= \frac{4}{x} + 3
\end{aligned}
$$

c) State the domain and range of the image in (a).

$$
\begin{aligned}
D &= \{x \in \mathbb{R} \mid x \neq 0\} \\
R &= \{y \in \mathbb{R} \mid y \neq 3\}
\end{aligned}
$$

Equations Used