Math Notes

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1 Introduction

Thanks to google, Advanced Function Textbook

- 30% Exam
- 55% Tests
- 10% Quizes
- 5% Assignments

2 Chapter 1

2.1 Domain and range

$$y = 3x - 5$$

Domain=$\{x \in R\}$ Range=$\{y \in R\}$
$y = x^2$ is a function because it passes the vertical line text
$x = y^2$ is not a function because it does not pass vertical line test.

2.2 Absolute Value

$$|-22| = 22$$

$$|10 - 8| = |-8| = |8|$$

Suppose $|x| < 8$ is true which means $-8 < x < 8$

2.3 Transformations

$$y = \frac{1}{2} f(\frac{1}{4}(x - 5)) + 6$$

Vertical compression by a factor of $\frac{1}{2}$
Horizontal stretch by a factor of 4
Horizontal translation 5 units to the right
Vertical translation 6 units up
$y = \sqrt{x}$ to $y = -3\sqrt{x - 5}$ makes a mapping of $(x, y) \rightarrow (x + 5, -3y)$
Domain=$\{x \in R | x \geq 5\}$ Range=$\{y \in R | x \leq 0\}$

x	y
0	0
1	1
4	2
9	3

x	y
5	0
6	-3
9	-6
14	-9

Transformations follow the format $y = a \times f(b(x - c)) + d$ Noticed the $-c$ and the $+d$, they are set that way so when you list the transformations they follow a patter

Vertical stretch by a factor of a

Horizontal stretch by a factor of $\frac{1}{b}$

Horizontal translation c units to the right

Vertical translation d units up

Make sure the translations are listed as if you're reading the function from the left to right, it is the correct way.

Transformations are the way you describe the transformation say $y = x - 5$ you say *Horizontal translation 5 units right* the mapping should follow the statement so $(x, y) \to (x + 5, y)$

2.4 Inverse Function

$$(2,5) \to (5,2)$$
$$(4,-8) \to (-8,4)$$
$$f(1) = 2$$
$$f(2)^{-1} = 1$$

$$f(x) = 3x + 4$$
$$y = 3x + 4$$
$$x = 3y + 4$$
$$x - 4 = 3y$$
$$\frac{x-4}{3} = y$$
$$f(x)^{-1} = \frac{x-4}{3}$$

$$g(x) = 4(x-3)^2 + 1$$
$$x = 4(y-3)^2 + 1$$
$$\frac{x-1}{4}$$
$$\pm\sqrt{\frac{x-1}{4}} + 3 = y$$
$$f(x)^{-1} = 3 \pm \sqrt{\frac{x-1}{4}}$$

The idea of inverse functions is to think of the function as $y = mx + b$ or $y = ax^2 + bx + c$, where the y is on the left and the rest is on the right, calculating the inverse is just isolating for x then replacing y with $f(x)^{-1}$ and x with x, the process shown above is a more formal way to show it.

2.5 Piecewise Functions

I do not want to draw graphs...

This chapter is about putting together functions, knowing how to graph them, how to get the function from the graph, and how to adjust so that functions are continuous

2.6 Operations with Functions

When it gives you a two sets and tell you to add, subtract, multiply, or whatever you take the y values and you perform the operation on those values. Make sure your x values are the same

$f = \{(0, 1)\}$ $g = \{(3, 2)\}$ $f + g$ This does not work as the x values are different.

$f = \{(0, 1)\}$ $g = \{(0, 2)\}$ $f - g$ This would equal to $\{(0, -1)\}$

The rest is just adding functions

3 Chapter 3

3.1 Polynomials

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$

3.2 Characteristics of Polynomials

Polynomials of an even degree have both end points going in the same direction Polynomials of an odd degree have both end points going in different directions

3.3 Polynomials

Say we have a factored polynomial which follows $y = a(x - x_1)(x - x_2)(x - x_3)^3$ it could also be $y = a(x - x_1)(x - x_2)^2$ We can use certain characteristics of the factored form to draw a graph, looking at each factor $((x - x_n))$ if it has a power (order) of 1 it passes through that x axis
If it is order 2 it touched the point at the x axis and "bounces off of it"
Order 3 it changes "concavity" (You will learn this later)

say $y = 2(x + 1)^2(x - 3)$, at $(0, -1)$ it will bounce off the axis, and at $(x - 3)$ it will go through that point

View the graph on Desmos.

You can also take this idea to find the rough functions of graphs.

Turning points are points in the polynomial which "turn" the way it's facing, better known as maximum and minimum, think of it as the vertices of the polynomial

3.4 Transformations of cubic and quartic functions

TLDR: Just transformations of function cubic and quartic functions are involved, (Really just transformation of functions)

3.5 Dividing Polynomials

Remember long division is middle school? $3\overline{)253}$ dividing polynomials has the same idea,

$(x^3 - 2x + 2)/(x - 4)$

$$\begin{array}{rrrrr}
 & x^2 & +4x & +14 & \\
\cline{2-5}
x-4) & x^3+0x^2 & -2x & +2 & \\
 & -x^3+4x^2 & & & \\
\cline{2-2}
 & 4x^2 & -2x & & \\
 & -4x^2 & +16x & & \\
\cline{2-3}
 & & 14x & +2 & \\
 & & -14x & +56 & \\
\cline{3-4}
 & & & 58 &
\end{array}$$

Dividing by this we get $x^2 + 4x + 14$ remainder 58, written out is $x^2 + 4x + 14 + \frac{58}{x-4}$

There is also synthetic division it's takes less space, and only works in cases where you have $x - x_1$. I suggest long division, it's like a tool which does it all while synthetic only has one use case, but a common one.

Remainder theorem

say you have a polynomial divided by a linear, to find the remainder you solve for x in the linear function and plug that into the polynomial

$$(x^3 - 2x + 2)/(x - 4)$$

$$x - 4 = 0 \rightarrow x = 4$$

$$4^3 - 2 * 4 + 2 = 58$$

This only works when dividing by linear function

You can use this theorem to solve some problems

$8x^3 + 10x^2 - px - 5$ is divisible by $2x + 1$

You can use the remainder theorem

$$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$$

$$P(x) = 8x^3 + 10x^2 - px - 5$$

$$P(-\frac{1}{2}) = \frac{1}{2}p - 3 = 0$$

$$p = 6$$

You can also use long division to find it but it will take long (I'd still do it tho :¿)

3.6 Factoring Polynomials

So you got a formula for factoring quadratic polynomials, what about cubic? quartic? there is a formula for cubic but it's scary.

So what we can do is try to find a factor in a polynomial by brute forcing for a factor.

Say factor $x^3 + 2x^2 - 5x - 6$, using the factor theorem before try plugging in values and see if you can find one which equals 0. Through brute force we find $P(-1) = 0$ meaning $x + 1$ is a factor of the polynomial

$$
\begin{array}{r}
x^2 \quad + x - 6 \\
x+1 \overline{\smash{)}\; x^3 + 2x^2 - 5x - 6} \\
\underline{- x^3 \quad - x^2} \qquad\qquad \\
x^2 - 5x \quad\;\; \\
\underline{- x^2 \quad - x} \quad\;\; \\
- 6x - 6 \\
\underline{6x + 6} \\
0
\end{array}
$$

Knowing the factor and the quadratic part we can easily factor the quadratic to get the factored polynomial $(x + 1)(x + 3)(x - 2)$

3.7 Difference Of Cubes

Difference of cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Sum of cubes $a^3 + b^3 = (a^2 - ab + b^2)$

3.8 Solving Polynomials

$$y = 2x(x - 1)(x + 2)(x - 2)$$

$$x = 0, 1, -2, 2$$

I mean, this is grade 11 polynomials.

4 Chapter 4

4.1 Polynomial Inequalities

$(x + 2)(x - 3)(x + 1) \geq 0$ Graphing this function you get two intervals which satisfy this condition, $-2 \leq x \leq -1$ or $x \geq 3$

4.2 Rate Of Change

Rate of change $m = \frac{y_2 - y_1}{x_2 - x_1}$ you should already know this from middle school. Use to find slopes of functions at a and b

We can use this to estimate the tangent at any point in a function by finding the slope with two points that are very close together.

5 Chapter 5

5.1 Rational Functions

Say we want to graph $y = \frac{1}{x-5}$ there are two "lines" we can draw that describes the function will never pass through, where they go to and from infinity at that point, we call those "asymptotes", for example, that function has two asymptotes, $x = 5$ and $y = 0$

Say we have a quadratic ration function $y = \frac{1}{x^2-x-6}$ when we graph $x^2 - x - 6$ we know the x intercepts are -2 and 3 and we know $\frac{1}{0}$ is undefined so these two points are asymptotes $x = -2$ and $x = 3$ there is also a asymptote at $y = 0$. Domain $= \{x \in R | x \neq -2, 3\}$, Range $= \{y \in R | y < -\frac{1}{2}$ or $y > 0\}$

5.2 Graph Rational Functions

So how do we graph something like $y = \frac{ax+b}{cx+d}$

$y = \frac{x+4}{2x+5}$ We have a horizontal asymptote at $y = \frac{a}{c}$ or $y = \frac{1}{2}$ and a vertical asymptote $2x + 5 = 0 \to x = -\frac{5}{2}$

These functions can also have a function which can be a "hole" say $y = \frac{x+2}{2x+4}$ if we were to calculate it's "vertical asymptote" $2x + 4 = 0 \to x = -2$ if we also calculate the numerator at this point we also get 0 meaning we get $\frac{0}{0}$, this means it is a hole instead of a vertical asymptotes. hole at $(-2, \frac{1}{2})$ and a horizontal asymptote at $y = -\frac{1}{2}$

Solve the inequality $x(x-5)^2(x+6)^2 > 0$

Zeros are $x = 0, 5, -6$. We calculate the values between the intervals of $-6, 0, 5$ and noticed $x < -6$ we get a positive number, $-6 < x < 0$ negative $0 < x < 5$ positive $5 < x$ positive.

Meaning at $x < -6, 0 < x < 5, x > 5$ is rings true.

Find a rational function with a hole at $x = 1$ vertical asymptotes at $x = 5$ and horizontal asymptote at $y = -8$

$$\frac{-8x(x-1)}{(x-1)(x-5)}$$

$x - 5$ means vertical asymptote, both $x - 1$ represent the hole, and $-8x$ and $x - 5$ represent vertical asymptote.

To find the end behavior (when x approaches negative and positive infinity, usually) of a rational function, you take a look at it's horizontal asymptote.

$f(x) = \frac{6x-5}{3x+12}$

H.A. $y = 2$. V.A. $x = -4$

1. As $x \to -\infty$, $y \to 2$ from above
2. As $x \to +\infty$, $y \to 2$ from below
3. As $x \to -4^-$, $y \to +\infty$
4. As $x \to -4^+$, $y \to -\infty$

5.3 Solving Rational Inequalities

Simplify and factor, check the zeros of both the numerator and denominator. Check the intervals between zeros

6 Chapter 6

6.1 Exponential and Logarithmic Functions

Properties of logarithms

1. $a^b = c \to \log_a c = b$
2. $\log_a 1 = 0$
3. $\log_a a = 1$
4. $\log_a a^b = b$
5. $\log_{a^b} a^c = \frac{c}{b}$
6. $\log_a b^c = c \log_a b$
7. $\log_a b + \log_a c = \log_a bc$
8. $\log_a b - \log_a c = \log_a \frac{b}{c}$
9. $\log ab = \frac{\log b}{\log a}$
10. $a^{\log_a b} = b$

Logarithms are the inverse of exponents
Really just understand the rules and use a calculator when possible
$y = a\sin(k(x-d)) + c$

- amplitude $a = \frac{\text{max}-\text{min}}{2}$
- $k = \frac{2\pi}{\text{period}}$
- period $= \frac{2\pi}{k}$
- $d =$ phase shift
- axis $= c = \frac{\text{max}+\text{min}}{2}$