Lab Investigation: The motion of a Cart rolling down a ramp

Joshua Liu

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1 Hypothesis

State the type of motion you think a cart rolling down a ramp will experience as well as justify **why** you think it will experience this type of motion.

I think the type of motion this cart will experience will be uniform acceleration as the cart is moving down a ramp. I think this will happen as the cart will convert gravitational energy into kinetic energy, meaning there will be constant acceleration.

2 Procedure

1. Set up an inclined plane as shown in the given Figure. Clamp the timer near the top of the board. Set up a fixed stop at the bottom of the board.

2. Obtain a length of ticker tape slightly shorter than the incline ramp. Feed the tape through the timer and attach the end of the tape to the back of the cart.

3. Turn on the timer and allow the cart to roll down the incline ramp. Have your partner stop the cart at the end of the ramp.

4. Choose a clear dot at the beginning to be the reference point at the time of $0.00s$. The timer likely makes dots at a frequency of $60Hz$. The time interval between two points is $\frac{1}{60}s$. For convenience, mark off

intervals every six dots along the tape so that each interval represents $0.10s$ up to $2.0s$ 20 dots.

5. Measure the position of the cart with respect to the reference point after $0.10s$, and record the data in a position-time table.

Trial 1		Trial 2	
Time	Position	Time	Position
0.0	0.000	0.0	0.000
0.1	0.005	0.1	0.004
0.2	0.012	0.2	0.010
0.3	0.024	0.3	0.019
0.4	0.042	0.4	0.030
0.5	0.063	0.5	0.043
0.6	0.089	0.6	0.059
0.7	0.113	0.7	0.076
0.8	0.150	0.8	0.099
0.9	0.187	0.9	0.116
1.0	0.226	1.0	0.138
1.1	0.270	1.1	0.161
1.2	0.318	1.2	0.184
1.3	0.369	1.3	0.209
1.4	0.423	1.4	0.234
1.5	0.481	1.5	0.260
1.6	0.543	1.6	0.287
1.7	0.608	1.7	0.314
1.8	0.676	1.8	0.343
1.9	0.750	1.9	0.372
2.0	0.825	2.0	0.401

6. Before you dismantle the ramp, measure the height of textbooks (H) and the length of the ramp (L) to measure the angle of inclination of the ramp.

   - Trial 1, 5 Textbooks: $H = 0.118m, L = 2.27m$
   - Trial 2, 3 Textbooks: $H = 0.071m, L = 2.27m$

7. Use the formula $\theta = sin^{-1}(\frac{H}{L})$ to find the angle of inclination of the ramp.

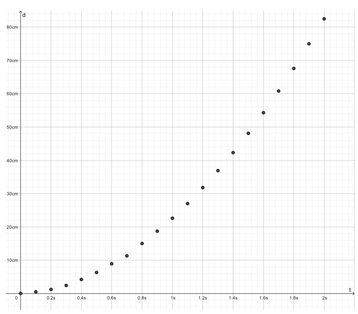
- Trial 1, $\theta = sin^{-1}(\frac{0.118}{2.27}) = 2.99°$
- Trial 2, $\theta = sin^{-1}(\frac{0.071}{2.27}) = 1.79°$

8. Repeat the previous steps for one more different angle to have a total of two trials.

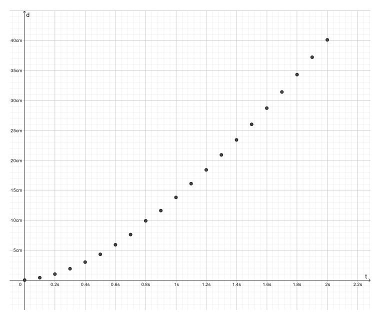
3 Analysis

3.1 Graphical Method

1. Use the position and time data that you collected in the procedure to plot a position-time graph for the motion of the cart down the ramp for every trial.

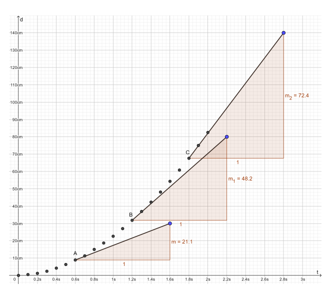


Trial one

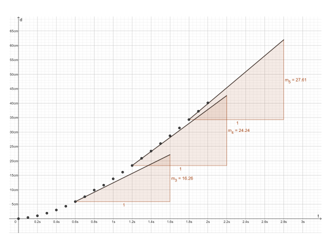


Trial two

2. Draw tangents of the cart every 0.60 s at three specific times.



Trial one



Trial two

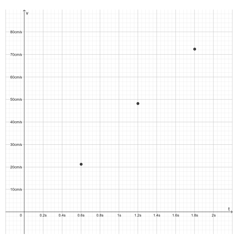
Figure 1: Tangents of both trials

3. Calculate the slops of the tangents which equal the instantaneous velocities at these chosen specific times.

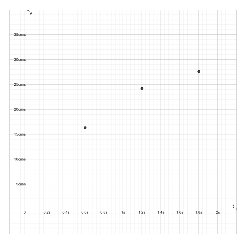
    - The slopes are calculated in Figure **??**, as the Δx for each tangent "triangle" is 1, meaning the Δy is the slope of the tangent.

4. Record these velocities in a table indicating the time and the velocity at that particular time to plot a velocity-time graph for every trial.

Trial 1		Trial 2	
Time	Position	Time	Position
0.6	0.212	0.6	0.163
1.2	0.482	1.2	0.242
1.8	0.724	1.8	0.276

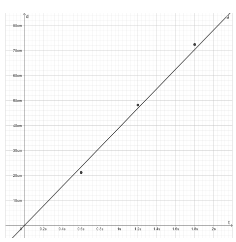


Trial one

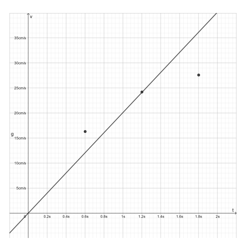


Trial two

5. Draw a line of best fit for every graph that best approximates the relationship.

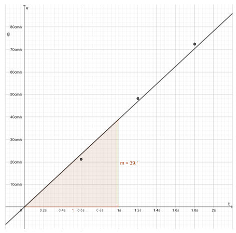


Trial one

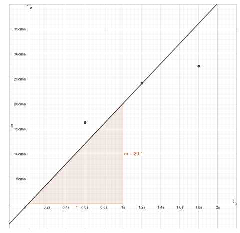


Trial two

6. Calculate the slope of the velocity-time graph to determine the average acceleration ($\vec{a}$) for every graph.

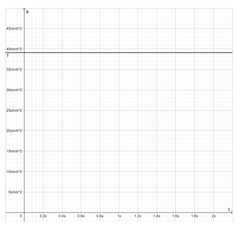


Trial one: $\Delta x = 1$
$\Delta y = 0.391$ $\vec{a} = slope = 0.391$

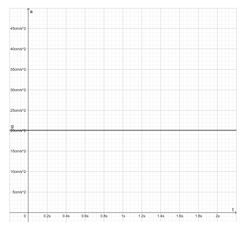


Trial two: $\Delta x = 1$
$\Delta y = 0.201$ $\vec{a} = slope = 0.201$

7. Draw a-t graphs for the two trials.



Trial one: $\vec{a} = 0.391$



Trial two: $\vec{a} = 0.201$

8. The acceleration down the ramp ($\vec{a}$) is defined by the formula $\vec{a} = \vec{g}sin\theta$, therefore $g = \frac{a}{sin\theta}$ where g is the acceleration due to gravity and (a) is the acceleration along the ramp.

   - Trial One:

$$\begin{aligned} g &= \frac{a}{sin\theta} \\ &= \frac{0.391}{sin(2.99°)} \\ &= 7.50 \end{aligned}$$

   - Trial Two:

$$\begin{aligned} g &= \frac{a}{sin\theta} \\ &= \frac{0.201}{sin(1.79°)} \\ &= 6.43 \end{aligned}$$

9. Compare the experimental value of g to the theoretical value of g =9.81 m/s^2 using the percentage of error calculation and the percentage of difference for the two values you got.

   - Trial One:

$$\begin{aligned}\%ofError &= \frac{|Approx - Exact|}{|Exact|}\\ &= \frac{|7.5 - 9.81|}{9.81}\\ &= 23.5\%\end{aligned}$$

$$\begin{aligned}\%ofDifference &= |\frac{Approx - Exact}{\frac{Approx+Exact}{2}}|\\ &= |\frac{7.5 - 9.81}{\frac{7.5+9.81}{2}}|\\ &= 26.7\%\end{aligned}$$

   - Trial Two:

$$\begin{aligned}\%ofError &= \frac{|Approx - Exact|}{|Exact|}\\ &= \frac{|6.43 - 9.81|}{9.81}\\ &= 34.5\%\end{aligned}$$

$$\begin{aligned}\%ofDifference &= |\frac{Approx - Exact}{\frac{Approx+Exact}{2}}|\\ &= |\frac{6.43 - 9.81}{\frac{6.43+9.81}{2}}|\\ &= 41.6\%\end{aligned}$$

10. Compare the experimental graphical value of g to the theoretical value of g =9.81 m/s^2 using the percentage of error calculation and the percentage of difference for the two values you got.

   - Trial One:

$$\begin{aligned}\%ofError &= \frac{|Approx - Exact|}{|Exact|}\\ &= \frac{|7.5 - 9.81|}{9.81}\\ &= 23.5\%\end{aligned}$$

$$\begin{aligned}\%ofDifference &= |\frac{Approx - Exact}{\frac{Approx+Exact}{2}}|\\ &= |\frac{7.5 - 9.81}{\frac{7.5+9.81}{2}}|\\ &= 26.7\%\end{aligned}$$

- Trial Two:

$$\begin{aligned}\%ofError &= \frac{|Approx - Exact|}{|Exact|}\\ &= \frac{|6.43 - 9.81|}{9.81}\\ &= 34.5\%\end{aligned}$$

$$\begin{aligned}\%ofDifference &= |\frac{Approx - Exact}{\frac{Approx+Exact}{2}}|\\ &= |\frac{6.43 - 9.81}{\frac{6.43+9.81}{2}}|\\ &= 41.6\%\end{aligned}$$

1. Determine an equation for the position time graph. The graph is a parabola and its vertex is the origin (0,0), therefore the equation is $\Delta\vec{d} = k\Delta\vec{t}^2$. Where Δd is the displacement in (m), Δt is the time in (s), and k is a constant.

2. Determine the value of the constant k for from the coordinate of three points that you drew tangents at them before in the graphical method. Be sure to determine and include the units for k.

Trial 1			Trial 2		
Δt	$\Delta\vec{d}$	$\frac{\Delta\vec{d}}{\Delta t^2} = k$	Δt	$\Delta\vec{d}$	$\frac{\Delta\vec{d}}{\Delta t^2} = k$
0.6	0.089	0.247	0.6	0.059	0.164
1.2	0.318	0.221	1.2	0.184	0.128
1.8	0.676	0.209	1.8	0.343	0.106

3. If you have different values for k in the same graph, use the average.

- Trial One:

$$\begin{aligned}k &= \frac{0.247 + 0.221 + 0.209}{3}\\ &= 0.226\end{aligned}$$

- Trial Two:

$$\begin{aligned}k &= \frac{0.164 + 0.128 + 0.106}{3}\\ &= 0.133\end{aligned}$$

4. If the cart has a constant acceleration, it motion should follow the equation $\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$. The initial velocity of the cart is zero so the equation becomes $\Delta\vec{d} = \vec{v_1}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. It looks similar to the equation you derived using mathematical modelling therefore $k = \frac{1}{2}a$, where a is the acceleration along the ramp.

   - Trial One:

$$\begin{aligned} a &= 2k \\ &= 2 * 0.226 \\ &= 0.452 \end{aligned}$$

   - Trial Two:

$$\begin{aligned} a &= 2k \\ &= 2 * 0.133 \\ &= 0.266 \end{aligned}$$

5. Theoretically $a = g\sin\theta$, where (θ) is the angle of inclination of the ramp and (g) is the acceleration due to gravity. Therefore $k = \frac{1}{2}g\sin\theta$ and $g = \frac{2k}{\sin\theta}$.

   - Trial One:

$$\begin{aligned} g &= \frac{a}{sin\theta} \\ &= \frac{0.452}{sin(2.99^\circ)} \\ &= 8.67 \end{aligned}$$

   - Trial Two:

$$\begin{aligned} g &= \frac{a}{sin\theta} \\ &= \frac{0.226}{sin(1.79^\circ)} \\ &= 7.24 \end{aligned}$$

6. Compare the experimental value of g to the theoretical value of g =9.81 m/s^2 using the percentage of error calculation and the percentage of difference for the two values you got.

   - Trial One:

$$\%ofError = \frac{|Approx - Exact|}{|Exact|}$$
$$= \frac{|8.76 - 9.81|}{9.81}$$
$$= 10.7\%$$

$$\%ofDifference = |\frac{Approx - Exact}{\frac{Approx+Exact}{2}}|$$
$$= |\frac{8.76 - 9.81}{\frac{8.76+9.81}{2}}|$$
$$= 11.3\%$$

- Trial Two:

$$\%ofError = \frac{|Approx - Exact|}{|Exact|}$$
$$= \frac{|7.24 - 9.81|}{9.81}$$
$$= 26.2\%$$

$$\%ofDifference = |\frac{Approx - Exact}{\frac{Approx+Exact}{2}}|$$
$$= |\frac{7.24 - 9.81}{\frac{7.24+9.81}{2}}|$$
$$= 30.1\%$$

4 Conclusion

Did the cart experience uniform motion, uniform acceleration or non-uniform acceleration while moving down the ramp? Explain how you know it experienced this type of motion. Use your velocity-time graph to justify graph to justify your explanation.

The cart experienced uniform acceleration. If the object is uniform acceleration, it's acceleration-time graph would be a horizontal line, while it's velocity-time graph would be a linear line. Looking at the velocity-time graph for both trials, we see that they are linear. I also know this cart experienced uniform acceleration as the position-time graph shows a parabolic shape.

5 Source of Error

State all the human and equipment sources of error, within the lab, that may have affected your results.

- Lubrication of the cart
- Debris along the track
- Friction along the track
- Rough measurements of textbooks
- Miscounting the dots