# MATH1023-01: Lecture Notes

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# Author's Note

These lecture notes are a compilation of material from the course Analytic Geometry and Calculus I (MATH1032) at La Roche University for the Fall 2024 Semester, supplemented with personal notes and reflections on the subject matter. My notes may be accessed at https://github.com/JoshuaWKelly/MATH1032-01-Lecture_Notes/tree/main. The formatting and style of these notes are inspired by the Feynman Lectures on Physics (https://www.feynmanlectures.caltech.edu/), and aim to present the concepts of calculus and analytic geometry in an engaging and accessible manner – similar to how Richard Feynman conveyed complex physics topics.

The content primarily draws from *Calculus Volume 1* by Gilbert Strang et al.[2], a foundational text that provides a thorough introduction to calculus. Problems, examples, and exercises referenced in these notes are sourced directly from this textbook unless otherwise noted. The intention is to provide students with a resource that not only follows the course curriculum but also adds depth and clarity to the material covered in lectures.

I hope these notes serve as a helpful guide for anyone studying calculus and encourage further exploration and understanding of the subject.

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# Contents

# Chapter 1

# Important Formulas

## 1.1 Linear Functions

### 1.1.1 Slope-Intercept Form

$$f(x) = mx + b \tag{1.1}$$

### 1.1.2 Point-Slope Form

$$y - y_1 = m(x - x_1) \tag{1.2}$$

### 1.1.3 Standard Form

$$ax + by = c, \tag{1.3}$$

$$a + b \neq 0 \tag{1.4}$$

### 1.1.4 Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{1.5}$$

## 1.2 Quadratic Functions

### 1.2.1 Vertex Form

$$f(x) = a(x - h)^2 + k \tag{1.6}$$

### 1.2.2 Standard Form

$$f(x) = ax^2 + bx + c \tag{1.7}$$

### 1.2.3 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1.8}$$

## 1.3 Exponential Functions

### 1.3.1 Exponential Growth

$$f(x) = ab^x \tag{1.9}$$

### 1.3.2 Exponential Decay

$$f(x) = ab^{-x} \tag{1.10}$$

## 1.4 Logarithmic Functions

### 1.4.1 Common Logarithm

$$f(x) = \log_b(x) \tag{1.11}$$

### 1.4.2 Natural Logarithm

$$f(x) = \ln(x) \tag{1.12}$$

## 1.5 Trigonometric Functions

### 1.5.1 Sine Function

$$f(x) = \sin(x) \tag{1.13}$$

### 1.5.2 Cosine Function

$$f(x) = \cos(x) \tag{1.14}$$

### 1.5.3 Tangent Function

$$f(x) = \tan(x) \tag{1.15}$$

## 1.6 Limits

### 1.6.1 Definition of a Limit

$$\lim_{x \to a} f(x) = L \tag{1.16}$$

### 1.6.2 Limit Laws

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) \tag{1.17}$$

$$\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) \tag{1.18}$$

$$\lim_{x \to a}[cf(x)] = c \lim_{x \to a} f(x) \tag{1.19}$$

$$\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x) \tag{1.20}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \tag{1.21}$$

$$\lim_{x \to a}[f(x)]^n = [\lim_{x \to a} f(x)]^n \tag{1.22}$$

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} \tag{1.23}$$

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} \tag{1.24}$$

$$\lim_{x \to a} f(x)^{g(x)} = \left[\lim_{x \to a} f(x)\right]^{\lim_{x \to a} g(x)} \tag{1.25}$$

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)} \tag{1.26}$$

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)} \tag{1.27}$$

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)} \tag{1.28}$$

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)} \tag{1.29}$$

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{\lim_{x \to a} f(x)} \tag{1.30}$$

# Chapter 2

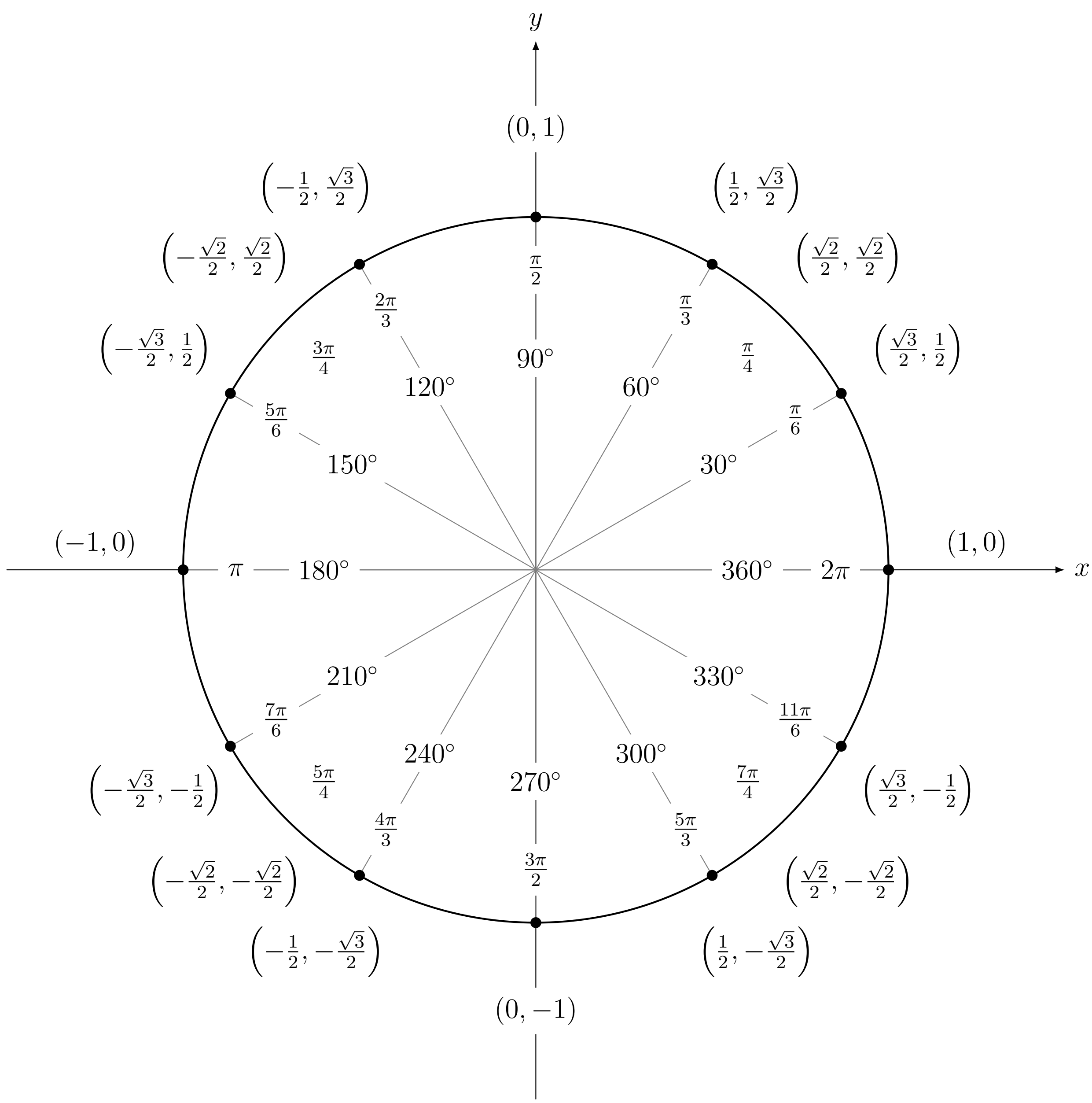
# Definitions

## 2.1 Linear Functions

One of the most important functions in mathematics is the linear function. A linear function is a function that can be written in the form $f(x) = mx+b$, where $m$ is the slope of the line and $b$ is the $y$-intercept.

## 2.2 Unit Circle

The unit circle is a circle with a radius of 1. It is centered at the origin of the coordinate plane and is used to define the trigonometric functions.




Credit: Supreme Aryal unit

## 2.3 Transformations of Functions

| Transformation of $f (c > 0)$ | Effect on the graph of $f$ |
|---|---|
| $f(x) + c$ | Vertical shift up $c$ units |
| $f(x) - c$ | Vertical shift down $c$ units |
| $f(x + c)$ | Shift left by $c$ units |
| $f(x - c)$ | Shift right by $c$ units |
| $cf(x)$ | Vertical Stretch if $c > 1$; vertical compression if $0 < c < 1$ |
| $f(cx)$ | Horizontal stretch if $0 < c < 1$; horizontal compression if $c > 1$ |
| $-f(x)$ | Reflection about the $x$-axis |
| $f(-x)$ | Reflection about the $y$-axis |

Table 1.7 Transformations of Functions

## 2.4 Common Angles

| Degrees | Radians | Degrees | Radians |
|---|---|---|---|
| 0 | 0 | 120 | $\frac{2\pi}{3}$ |
| 30 | $\frac{\pi}{6}$ | 135 | $\frac{3\pi}{4}$ |
| 45 | $\frac{\pi}{4}$ | 150 | $\frac{5\pi}{6}$ |
| 60 | $\frac{\pi}{3}$ | 180 | $\pi$ |
| 90 | $\frac{\pi}{2}$ | ... | ... |

Table 1.8 Common Angles Expressed in Degrees and Radians

## 2.5 Trigonometric Functions

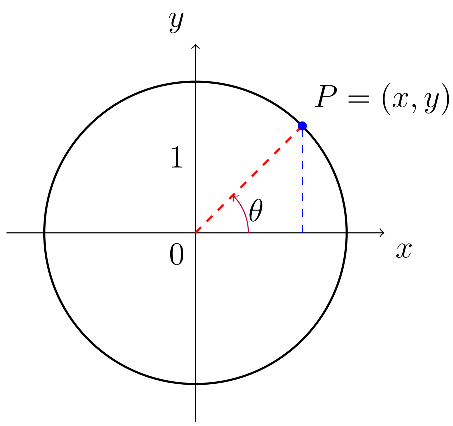



**Figure 1.31** The angle 0 is in standard position. The values of the trigonometric functions for 0 are defined in terms of the coordinates $x$ and $y$.

Let $P = (x, y)$ be a point on the unit circle centered at the origin $O$. Let $\theta$ be an angle with an initial side along the positive $x$-axis and a terminal side given by the segment $OP$. The **trigonometric functions** are then defined as

$$\sin\theta = y$$

$$\cos\theta = x$$

$$\tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{1}{y}$$

$$\sec\theta = \frac{1}{x}$$

$$\cot\theta = \frac{x}{y}$$

If $x = 0$, $\sec\theta$, and $\tan\theta$ are defined. If $y = 0$, then $\cot\theta$ and $\csc\theta$ are undefined.

### 2.5.1 Trigonometric Identities

**Reciprocal Identities**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{2.1}$$

$$\csc\theta = \frac{1}{\sin\theta} \tag{2.2}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \tag{2.3}$$

$$\sec\theta = \frac{1}{\cos\theta} \tag{2.4}$$

**Pythagorean identities**

$$\sin^2\theta + \cos^2\theta = 1 \tag{2.5}$$

$$1 + \tan^2\theta + 1 = \sec^2\theta \tag{2.6}$$

$$1 + \cot^2\theta = \csc^2\theta \tag{2.7}$$

**Addition and subtraction formulas**

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{2.8}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \tag{2.9}$$

**Double-angle formulas**

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{2.10}$$

$$\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta \tag{2.11}$$

# Chapter 3

# Review of Functions

## 3.1 Introduction

## 3.2 Linear Functions

A linear function is a function that can be written in the form $f(x) = mx + b$, where $m$ is the slope of the line and $b$ is the $y$-intercept.

### 3.2.1 Hyperbolic Functions

Hyperbolic cosine

$$\cosh x = \frac{e^x + e^{-x}}{2} \tag{3.1}$$

# Chapter 4

# Limits

## 4.1 Introduction

This is the introduction section of my document.

## 4.2 Intutive Definition of a Limit

Imagine you're standing on the shore, looking out at the ocean. You see a boat far away, and it's moving towards you. As the boat gets closer, it appears larger and clearer, but if it were to keep coming closer indefinitely, it would eventually reach you, right at your feet. Now, we might say that the boat "approaches" you as it moves closer and closer.

In mathematics, a limit is a way of describing what happens when we look at how something changes as we move closer to a certain point. Think of it as focusing on what happens in the "long run" as we approach a specific point, rather than what happens exactly at that point.

Let's use an example with numbers: Imagine you have a sequence of numbers that gets closer and closer to 10, like 9, 9.9, 9.99, 9.999, and so on. Even though none of these numbers are exactly 10, we can say that "in the limit," these numbers are approaching 10. The limit of this sequence is 10 because, as we go further along in the sequence, the numbers get arbitrarily close to 10.

In this sense, a limit is about getting closer and closer to something without necessarily ever reaching it. It's about the behavior of a function or a sequence as we move toward a certain point or as the input grows indefinitely.

In mathematical terms, if we say the limit of $f(x)$ as $x$ approaches $a$ is $L$, we mean that we can get $f(x)$ as close as we want to $L$ by taking x sufficiently close to $a$, but not necessarily equal to $a$.

## 4.3 Preview of Calculus

**Formula.**

$$m_{sec} = \frac{f(x) - f(a)}{x - a} \tag{4.1}$$

## 4.4 The Limit of A Function

## 4.5 The Limit Laws

## 4.6 Continuity

## 4.7 The Precise Definition of a Limit

**Precise Definition of a Limit.**

$$\lim_{x \to a} f(x) = L \tag{4.2}$$

**Definition of a Limit.** The limit of a function $f(x)$ as $x$ approaches $a$ is $L$ if for every $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

**Example 1.** Enter an example here.

**Important.** It is important that...

**Formula.** Enter a formula here.

**Proof.** This is a proof

$$s(t) = \text{position of the object at time } t \tag{4.3}$$

**Example 2.2.**

$$s(t) = 16t^2 + 64$$

a) $[0.49, 0.50]$

$$\frac{s(0.5) - s(0.49)}{0.5 - 0.49} = -15.84 \tag{4.4}$$

b) $[0.50, 0.51]$

$$\frac{s(0.51) - s(0.5)}{0.51 - 0.5} = 16.16u \tag{4.5}$$

# Chapter 5

# Derivatives

One of the most important concepts in calculus is the derivative. The derivative of a function is a measure of how the function changes as its input changes. It tells us how the function is changing at any given point, and it is used to solve a wide variety of problems in mathematics, science, and engineering.

# Bibliography


[1] Supreme Aryal. *Example: Unit circle*. EN. Mar. 2010. URL: https://texample.net/tikz/examples/unit-circle/.

[2] Gilbert Strang et al. *Calculus Volume 1*. EN. OpenStax, Mar. 2016. ISBN: 978-1-947172-13-5. URL: https://openstax.org/details/books/calculus-volume-1/.

# Appendix A

# Appendix

One

Let $f(x)$ be a $\mathbb{R}$. Then $f(x)$ is a function of $x$ if for each $x$ in the domain of $f(x)$, there is exactly one value of $f(x)$.

Let, $f(x) = 9 + 5$ where $9 \neq 0$