MATH1023-01: Lecture Notes

Joshua W. Kelly

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Author's Note

These lecture notes are a compilation of material from the course Analytic Geometry and Calculus I (MATH1032) at La Roche University for the Fall 2024 Semester, supplemented with personal notes and reflections on the subject matter. My notes may be accessed at https://github.com/JoshuaWKelly/MATH1032-01-Lecture_Notes/tree/main. The formatting and style of these notes are inspired by the Feynman Lectures on Physics (https://www.feynmanlectures.caltech.edu/), and aim to present the concepts of calculus and analytic geometry in an engaging and accessible manner – similar to how Richard Feynman conveyed complex physics topics.

The content primarily draws from *Calculus Volume 1* by Gilbert Strang et al.[1], a foundational text that provides a thorough introduction to calculus. Problems, examples, and exercises referenced in these notes are sourced directly from this textbook unless otherwise noted. The intention is to provide students with a resource that not only follows the course curriculum but also adds depth and clarity to the material covered in lectures.

I hope these notes serve as a helpful guide for anyone studying calculus and encourage further exploration and understanding of the subject.

Joshua W. Kelly
La Roche University

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Chapter 1

Important Formulas

1.1 Linear Functions

1.1.1 Slope-Intercept Form

$$f(x) = mx + b \tag{1.1}$$

1.1.2 Point-Slope Form

$$y - y_1 = m(x - x_1) \tag{1.2}$$

1.1.3 Standard Form

$$ax + by = c, \tag{1.3}$$

$$a + b \neq 0 \tag{1.4}$$

1.1.4 Slope Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{1.5}$$

1.2 Quadratic Functions

1.2.1 Vertex Form

$$f(x) = a(x - h)^2 + k \tag{1.6}$$

1.2.2 Standard Form

$$f(x) = ax^2 + bx + c \tag{1.7}$$

1.2.3 Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1.8}$$

1.3 Exponential Functions

1.3.1 Exponential Growth

$$f(x) = ab^x \tag{1.9}$$

1.3.2 Exponential Decay

$$f(x) = ab^{-x} \tag{1.10}$$

1.4 Logarithmic Functions

1.4.1 Common Logarithm

$$f(x) = \log_b(x) \tag{1.11}$$

1.4.2 Natural Logarithm

$$f(x) = \ln(x) \tag{1.12}$$

1.5 Trigonometric Functions

1.5.1 Sine Function

$$f(x) = \sin(x) \tag{1.13}$$

1.5.2 Cosine Function

$$f(x) = \cos(x) \tag{1.14}$$

1.5.3 Tangent Function

$$f(x) = \tan(x) \tag{1.15}$$

1.6 Limits

1.6.1 Definition of a Limit

$$\lim_{x \to a} f(x) = L \tag{1.16}$$

1.6.2 Limit Laws

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) \tag{1.17}$$

$$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) \tag{1.18}$$

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x) \tag{1.19}$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x) \tag{1.20}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \tag{1.21}$$

$$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n \tag{1.22}$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} \tag{1.23}$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} \tag{1.24}$$

$$\lim_{x\to a} f(x)^{g(x)} = \left[\lim_{x\to a} f(x)\right]^{\lim_{x\to a} g(x)} \tag{1.25}$$

$$\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{\lim_{x\to a} f(x)} \tag{1.26}$$

$$\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{\lim_{x\to a} f(x)} \tag{1.27}$$

$$\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{\lim_{x\to a} f(x)} \tag{1.28}$$

$$\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{\lim_{x\to a} f(x)} \tag{1.29}$$

$$\lim_{x\to a}\frac{1}{f(x)} = \frac{1}{\lim_{x\to a} f(x)} \tag{1.30}$$

Chapter 2

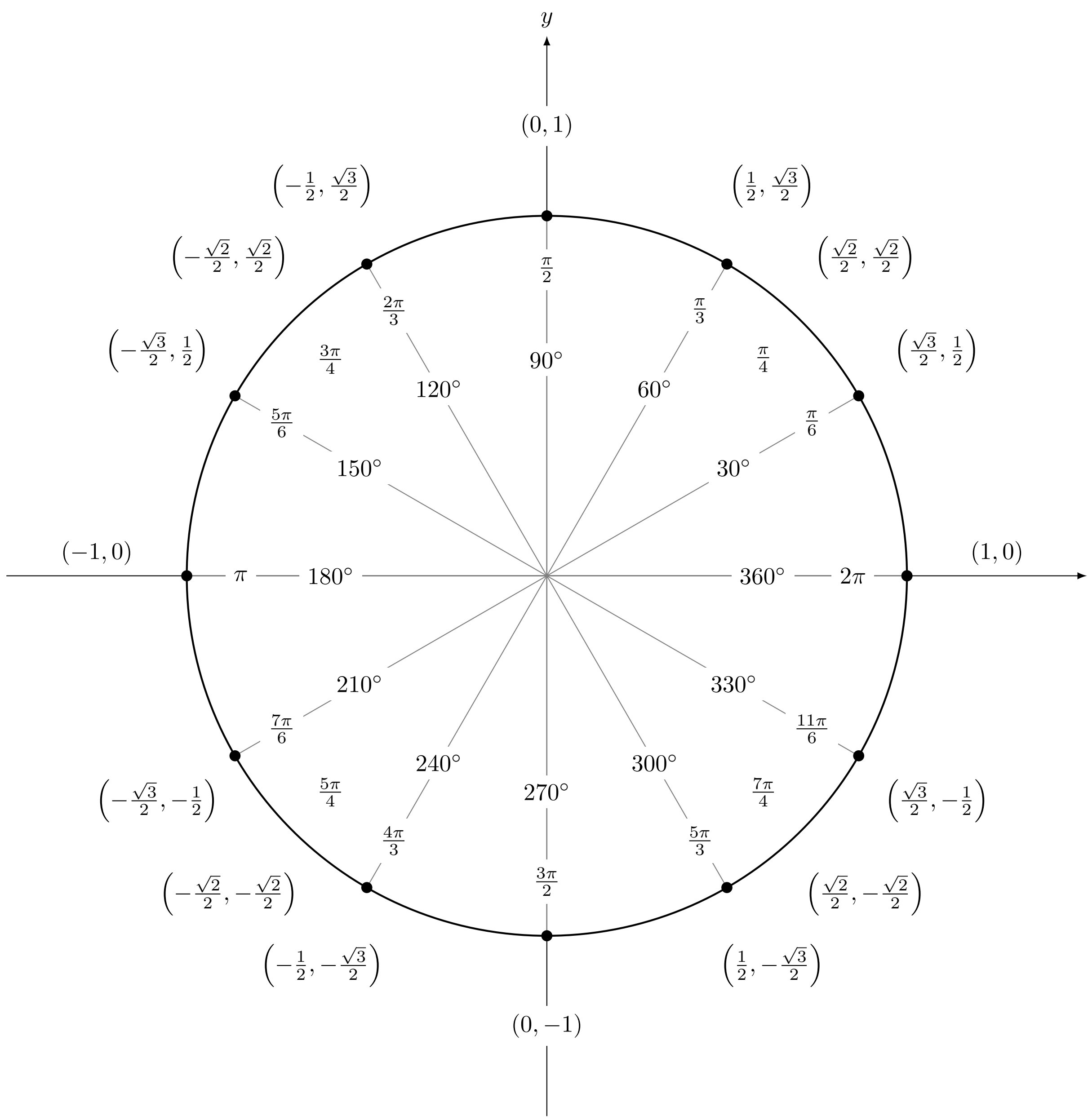
Definitions

2.1 Linear Functions

One of the most important functions in mathematics is the linear function. A linear function is a function that can be written in the form $f(x) = mx + b$, where m is the slope of the line and b is the y-intercept.

2.2 Unit Circle

The unit circle is a circle with a radius of 1. It is centered at the origin of the coordinate plane and is used to define the trigonometric functions.



y
$(0,1)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{2}$
90°
$\frac{2\pi}{3}$
120°
$\frac{\pi}{3}$
60°
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
$\frac{5\pi}{6}$
150°
$\frac{\pi}{6}$
30°
$(-1,0)$
π
180°
360°
2π
$(1,0)$
210°
$\frac{7\pi}{6}$
330°
$\frac{11\pi}{6}$
240°
$\frac{5\pi}{4}$
$\frac{4\pi}{3}$
270°
$\frac{3\pi}{2}$
300°
$\frac{7\pi}{4}$
$\frac{5\pi}{3}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$(0,-1)$

2.3 Transformations of Functions

Transformation of $f (c > 0)$	Effect on the graph of f
$f(x) + c$	Vertical shift up c units
$f(x) - c$	Vertical shift down c units
$f(x + c)$	Shift left by c units
$f(x - c)$	Shift right by c units
$cf(x)$	Vertical Stretch if $c > 1$; vertical compression if $0 < c < 1$
$f(cx)$	Horizontal stretch if $0 < c < 1$; horizontal compression if $c > 1$
$-f(x)$	Reflection about the x-axis
$f(-x)$	Reflection about the y-axis

Table 1.7 Transformations of Functions

2.4 Common Angles

Degrees	Radians	Degrees	Radians
0	0	120	$\frac{2\pi}{3}$
30	$\frac{\pi}{6}$	135	$\frac{3\pi}{4}$
45	$\frac{\pi}{4}$	150	$\frac{5\pi}{6}$
60	$\frac{\pi}{3}$	180	π
90	$\frac{\pi}{2}$	...	...

Table 1.8 Common Angles Expressed in Degrees and Radians

2.5 Trigonometric Functions

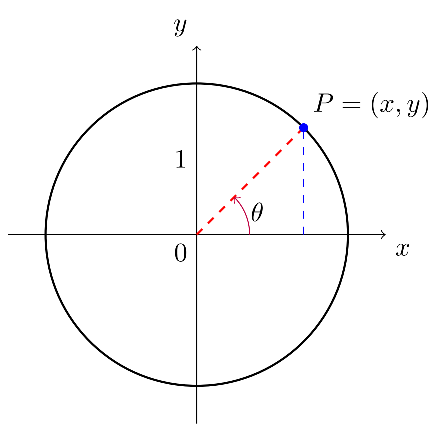


Figure 1.31 The angle 0 is in standard position. The values of the trigonometric functions for 0 are defined in terms of the coordinates x and y.

Let $P = (x, y)$ be a point on the unit circle centered at the origin O. Let θ be an angle with an initial side along the positive x-axis and a terminal side given by the segment OP. The **trigonometric functions** are then defined as

$$\begin{aligned}
\sin\theta &= y \\
\cos\theta &= x \\
\tan\theta &= \frac{y}{x} \\
\csc\theta &= \frac{1}{y} \\
\sec\theta &= \frac{1}{x} \\
\cot\theta &= \frac{x}{y}
\end{aligned}$$

If $x = 0$, $\sec\theta$, and $\tan\theta$ are defined. If $y = 0$, then $\cot\theta$ and $\csc\theta$ are undefined.

2.5.1 Trigonometric Identities

Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{2.1}$$

$$\csc\theta = \frac{1}{\sin\theta} \tag{2.2}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \tag{2.3}$$

$$\sec\theta = \frac{1}{\cos\theta} \tag{2.4}$$

Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1 \tag{2.5}$$

$$1 + \tan^2\theta + 1 = \sec^2\theta \tag{2.6}$$

$$1 + \cot^2\theta = \csc^2\theta \tag{2.7}$$

Addition and subtraction formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \tag{2.8}$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \tag{2.9}$$

Double-angle formulas

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{2.10}$$

$$\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta \tag{2.11}$$

Chapter 3

Review of Functions

3.1 Introduction

This is the introduction section of my document.

3.2 Linear Functions

A linear function is a function that can be written in the form $f(x) = mx + b$, where m is the slope of the line and b is the y-intercept.

3.2.1 Hyperbolic Functions

Hyperbolic cosine

$$\cosh x = \frac{e^x + e^{-x}}{2} \tag{3.1}$$

Chapter 4

Limits

4.1 Introduction

This is the introduction section of my document.

4.2 Intutive Definition of a Limit

4.3 Preview of Calculus

Formula.

$$m_{sec} = \frac{f(x) - f(a)}{x - a} \tag{4.1}$$

4.4 The Limit of A Function

4.5 The Limit Laws

4.6 Continuity

4.7 The Precise Definition of a Limit

Precise Definition of a Limit.

$$\lim_{x \to a} f(x) = L \tag{4.2}$$

Definition of a Limit. The limit of a function $f(x)$ as x approaches a is L if for every $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

Example 1. Enter an example here.

Important. It is important that...

Formula. Enter a formula here.

Proof. This is a proof

$$s(t) = \text{position of the object at time } t \tag{4.3}$$

Example 2.2.

$$s(t) = 16t^2 + 64$$

a) $[0.49, 0.50]$

$$\frac{s(0.5) - s(0.49)}{0.5 - 0.49} = -15.84 \tag{4.4}$$

b) $[0.50, 0.51]$

$$\frac{s(0.51) - s(0.5)}{0.51 - 0.5} = 16.16u \tag{4.5}$$

Chapter 5

Derivatives

Bibliography

[1] Gilbert Strang et al. *Calculus Volume 1*. EN. OpenStax, Mar. 2016. ISBN: 978-1-947172-13-5. URL: https://openstax.org/details/books/calculus-volume-1/.

Appendix A

Appendix

One