Advanced Applied Statistics Homework 3 Solutions

Joshua D. Ingram

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1.

We define our 4 basis functions for the smoothed linear regression with splines model as follows,

$$h_1(x) = 1$$

$$h_2(x) = x$$

$$h_3(x) = \begin{cases} (x - \eta_1) & \text{if } x > \eta_1 \\ 0 & \text{otherwise} \end{cases}$$

$$h_4(x) = \begin{cases} (x - \eta_2) & \text{if } x > \eta_2 \\ 0 & \text{otherwise,} \end{cases}$$

Now,

$$f(x) = \sum_{j=1}^{4} \beta_j h_j(x).$$

Check that for the basis functions h_1, h_2, h_3 and h_4 from the end of the class today (Monday):

a.

for any $x \leq \eta_1$, we get

$$f(x) = \beta_1 + \beta_2 x$$

Answer: Given the definitions of the basis functions above, for any $x \leq \eta_1$,

$$f(x) = \beta_1 + \beta_2 x + \beta_3 * 0 + \beta_4 * 0 = \beta_1 + \beta_2 x.$$

b.

For any x such that $\eta_1 < x < \eta_2$, we get

$$f(x) = \beta_1 - \beta_3 \eta_1 + (\beta_2 + \beta_3) x$$

Answer: Given the definitions of the basis functions above, for any $\eta_1 < x < \eta_2$,

$$f(x) = \beta_1 + \beta_2 x + \beta_3(x - \eta_1) = \beta_1 + \beta_2 x + \beta_3 x - \beta_3 \eta_1$$

$$f(x) = \beta_1 - \beta_3 \eta_1 + (\beta_2 + \beta_3)x.$$

c.

When plugging in $x = \eta_1$ in part b above, we get the expression in part a. (i.e., there is a continuity between the two linear pieces.)

Answer: Given the definitions of the basis functions above, for $x = \eta_1$, we may plug in to the equation in part b,

$$f(\eta_1) = \beta_1 - \beta_3 \eta_1 + (\beta_2 + \beta_3)\eta_1 = \beta_1 - \beta_3 \eta_1 + \beta_2 \eta_1 + \beta_3 \eta_1$$

$$= \beta_1 + \beta_2 \eta_1.$$

This is the same as if we plugged in $x = \eta_1$ into the expression in part a.