16.1 Vector Fields

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Vector Fields

Definition - 2D Vecor Field

Let D be a set in $\mathbb{R}^2$. A **vector field on** $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns to each point (x,y) in D a two-dimensional vector $\mathbf{F}$(x,y)

$\mathbf{F}$ can be written in terms of its *component functions* P and Q:

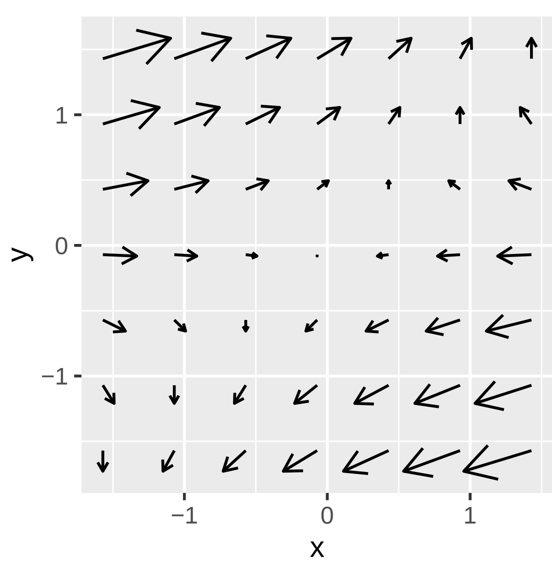
$$\boxed{\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j} = \langle P(x,y), Q(x,y) \rangle}$$

or in a shorter form,

$$\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$$

Example

Vector field such that $\mathbf{F}(x,y) = \langle y - x, sin(y) \rangle$:



Definition - 3D Vector Field

Let E be a subset of $\mathbb{R}^3$. A **vector field on** $\mathbb{R}^3$ is a function $\mathbf{F}$ that assigns to each point (x,y,z) in E a three-dimensional vector $\mathbf{F}$(x,y,z)

$$\boxed{\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k} = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle}$$

Gradient Fields

Definition - Gradient Vector

If f is a scalar function of three variables x, y, and z, then the **gradient** of f is the vector function $\nabla f(x, y, z)$ defined by

$$\boxed{\nabla f(x, y, z) = f_x(x, y, z)\mathbf{i} + f_y(x, y, z)\mathbf{j} + f_z(x, y, z)\mathbf{k}}$$

Definition - Conservative Vector Field

If there exists a function f such that $\mathbf{F} = \nabla f$, then $\mathbf{F}$ is a **conservative vector field**.

Side Notes

- For a vector field function $\mathbf{F}$, $\mathbf{F}(x, y, z)$ can also be written as $\mathbf{F}(\mathbf{x})$, such that $\mathbf{x} = (x, y, z)$
- $\nabla f(x, y, z)$ is called a **gradient vector field**
- For $\mathbf{F} = \nabla f$, f is called the **potential function** of $\mathbf{F}$