16.8 - Stoke's Theorem

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Stokes' Theorem

Theorem 16.8.1 - Stokes' Theorem

Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let **F** be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains S. Then

$$\boxed{\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S curl\, \mathbf{F} \cdot d\mathbf{S}}$$

Since

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T}\, ds \quad \text{and} \quad \iint_S curl\, \mathbf{F} \cdot d\mathbf{S} = \iint_S curl\, \mathbf{F} \cdot \mathbf{n}\, dS$$

Stokes' Theorem say that the line integral around the boundary curve of S of the tagential component of **F** is equal to the surface integral over S of the normal component of the curl of **F**.

This positively oriented boundary curve of the oriented surface S is often written as ∂S, so the theorem can be rewritten as

$$\iint_S curl\, \mathbf{F} \cdot d\mathbf{S} = \int_{\partial S} \mathbf{F} \cdot d\mathbf{r}$$

If S_1 and S_2 are oriented surfaces with the same oriented boundary curve C and both satisfy the hypothesis of Stokes' Theorem, then

$$\iint_{S_1} curl\, \mathbf{F} \cdot \; d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r} = \iint_{S_2} curl\, \mathbf{F} \cdot \; d\mathbf{S}$$