Probability: Chapter 1 - Combinatorial Analysis

Joshua Ingram

1.2 The Basic Principles of Counting

If r experiments are to be performed where experiment 1 has n_1 possible outcomes, and for each outcome of experiment 1 there are n_2 possible outcomes of experiment 2, etc...

Then there is a total of

$$\boxed{\mathbf{n_1 \cdot n_2 \cdot ... \cdot n_r}}$$

possible outcomes from the r experiments

1.3 Permutations

Permutation - an arrangement of objects where order matters *(e.g. abc is distinct from acb)*

Calculate Number of Permutations

For n objects, there are

$$\boxed{n \cdot (n-1) \cdot (n-2) \cdot ... \cdot 3 \cdot 2 \cdot 1 = n!}$$

permutations

- There is *no replacement*

Permutations with Identical Objects

For n objects where n_1 are alike, n_2 are alike, $\ldots$, n_r are alike, there are

$$\boxed{\frac{n!}{n_1!n_2!...n_r!}}$$

different permutations

1.4 Combinations

Combination - arrangement of objects where order does not matter *(e.g. abc = bca, abc $\neq$ abd)*

Combinations of size r from n Objects

For n objects, there are

$$\boxed{\binom{n}{r} = \frac{n!}{(n-r)!r!}}$$

combinations of size r

Pascal's Identity

$$\boxed{\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}} \quad 1 \leq r \leq n$$

The Binomial Theorem

$$\boxed{(x+y)^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}}$$

1.5 Multinomial Coefficients

Finding all Possible Divisons of Groups

For n distinct items that are to be divided into r groups of size $n_1, ..., n_r$, such that $\sum_{i=1}^{r} n_i = n$, there are

$$\boxed{\binom{n}{n_1, n_2, ..., n_r} = \frac{n!}{n_1! n_2! ... n_r!}}$$

different divisions

The Multinomial Theorem

$$\boxed{(x_1 + x_2 + ... + x_r)^k = \sum_{n_1,...n_r : n_1+...n_r=n} \binom{n}{n_1, n_2, ...n_r} x_1^{n_1} x_2^{n_2} ... x_r^{n_r}}$$

1.6 The Number of Integer Solutions of Equations

Positive Integer Solutions

There are

$$\boxed{\binom{n-1}{r-1}}$$

distinct positive-valued vectors satisfying: $x_1 + x_2 + ... + x_r = n$

Nonnegative Integer Solutions

There are

$$\boxed{\binom{n+r-1}{r-1}}$$

distinct nonnegative integer-valued vectors satisfying: $x_1 + x_2 + ... + x_r = n$