16.4 Green's Theorem

Pages: 1096 - 1103

Joshua D. Ingram

Green's Theorem

Green's Theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

- The convention of **positive orientation** is used in Green's Theorem, refering to a single *counterclockwise* traversal of C. In other words, the region is always to the left as a function $\mathbf{r}(t)$ traverses C.

Theorem 16.4.1 - Green's Theorem

Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C. If P and Q have continuous partial derivatives on an open region that contains D, then

$$\oint_C Pdx + Qdy = \oint_C \mathbf{F} \cdot d\mathrm{r} = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA$$

We can also use Green's Theorem to find areas. Since the area of D is $\iint_D 1dA$, we wish to choose P and Q such that

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$$

Given this, Green's theorem gives the following formulas for the area of D:

$$A = \oint_C xdy = -\oint_C ydx = \frac{1}{2}\oint_C xdy - ydx$$