Analysis

To begin the tables below show the angular frequency of the pendulums. The height from 1-4 corresponds to ℓ where the spring is attached where 1 is the closest mounting location to the pivot and 4 is the furthest.

Pendulum 1				
Height	Spring 1($\frac{1}{s}$)	Spring 2($\frac{1}{s}$)	Spring 3($\frac{1}{s}$)	Spring 4($\frac{1}{s}$)
1 (Even)	4.40 $\pm$0.1	4.19$\pm$0.3	4.40$\pm$0.7	4.40$\pm$0.7
1 (Odd)	4.19$\pm$0.1	4.10$\pm$0.7	4.08$\pm$0.7	5.01$\pm$0.7
2 (Even)	5.03$\pm$0.3	4.40$\pm$0.1	4.71$\pm$0.1	4.71$\pm$0.5
2 (Odd)	4.19$\pm$0.3	4.40$\pm$0.1	4.61$\pm$0.1	5.39$\pm$0.1
3 (Even)	5.24$\pm$0.3	4.49$\pm$0.3	5.03$\pm$0.3	7.12$\pm$0.3
3 (Odd)	5.39$\pm$0.3	4.32$\pm$0.5	5.03$\pm$0.7	6.28$\pm$0.7
4 (Even)	7.54$\pm$1	4.40$\pm$1	5.86$\pm$1	6.28$\pm$0.3
4 (Odd)	5.03$\pm$0.7	4.69$\pm$0.7	6.28$\pm$1	7.54$\pm$1

Pendulum 2				
Height	Spring 1($\frac{1}{s}$)	Spring 2($\frac{1}{s}$)	Spring 3($\frac{1}{s}$)	Spring 4($\frac{1}{s}$)
1 (Even)	4.42$\pm$0.3	4.19$\pm$1	4.44$\pm$0.7	4.89$\pm$0.7
1 (Odd)	4.44$\pm$0.3	4.40$\pm$0.7	4.19$\pm$0.3	5.03$\pm$0.7
2 (Even)	5.03$\pm$0.1	4.43$\pm$0.3	4.71$\pm$0.7	4.89$\pm$0.3
2 (Odd)	5.03$\pm$0.3	4.57$\pm$0.3	4.95$\pm$0.7	5.45$\pm$0.7
3 (Even)	5.03$\pm$0.3	4.61$\pm$0.1	4.73$\pm$0.7	6.28$\pm$0.3
3 (Odd)	4.89$\pm$0.7	4.19$\pm$0.7	5.01$\pm$0.1	6.70$\pm$1
4 (Even)	6.28$\pm$1	4.40$\pm$1	5.71$\pm$1	6.91$\pm$0.1
4 (Odd)	6.28$\pm$0.7	4.89$\pm$0.1	7.10$\pm$1	7.96$\pm$1

The value for the angular frequency was calculated by simply finding the frequency of the pendulums and then using FFT package from scipy to compute the frequency of the waves. From there multiply the frequency by 2π gives the angular frequency. The uncertainty of the angular frequency was decided on how good the data was. For example below is two figure where the pendulum displayed a bad and a good set. The figure on the left had random oscillator motion while the data set on the left both pendulum showed very similar amplitude and period. The reason why we had these bad data was dependant on the spring and where the mounting location was. The left figure data was taking from the third spring on the third mounting location. During the trial the spring kept shifting up and down on the hook of the mounting location as a result it produced this non-uniform oscillatory motion.

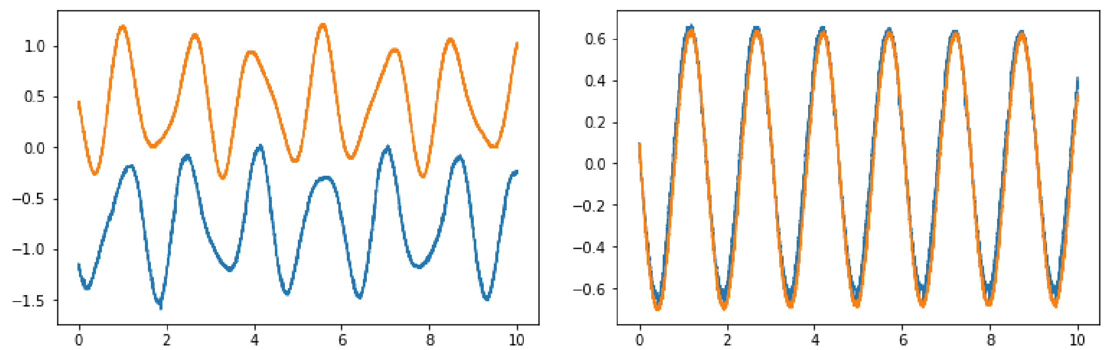
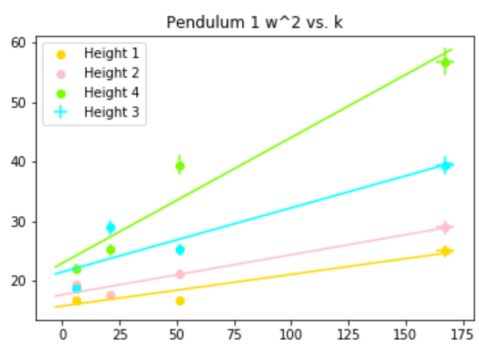
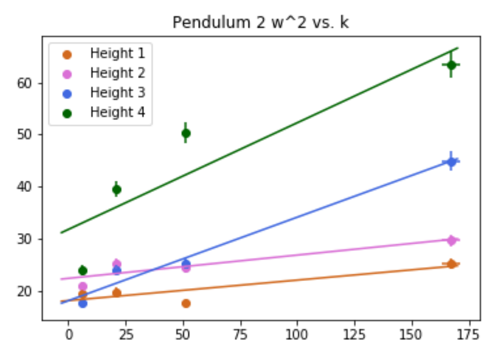


Figure 1: Bad and Good Data

Depending on the how good the data set was the associated uncertainty would be higher for data set that showed non-uniform motion like the left figure and low uncertainty for those that were similar to the right figure.

Next below is the figure for ω^2 versus k and along with the table of the intercept and slope





	Pendulum 1		Pendulum 2	
Height	Slope ($\frac{1}{kg}$)	Intercept ($\frac{1}{s}$)	Slope ($\frac{1}{kg}$)	Intercept($\frac{1}{s}$)
1	0.053±0.002	15.78±0.5	0.039±0.002	18.06±0.5
2	0.067±0.002	17.67±0.5	0.045±0.002	22.34±0.5
3	0.11±0.002	21.51±0.5	0.16±0.002	18.05±0.5
4	0.21±0.002	22.99±0.6	0.20±0.002	31.75±0.8

The two pendulums were separated in to figures for the sake of clarity, when combined the graph becomes too busy with error bars and polyfitted lines. The error bars for the k value were calculate using the values from the analysis of part a and the ω^2 derived from the uncertainty of ω in the previous table. The uncertainty calculation of ω^2 were carried out using the method from Phys 360B error note(Treatment of Errors):

$$e_a\% = \sqrt[2]{(e_b\%)^2 + (e_c\%)^2} \tag{1}$$

Next the uncertainty for the slope were calculated using (Treatment of Errors) equation where:

$$\frac{\Delta S}{S} \approx \frac{S_{max} - S_{min}}{2S} \tag{2}$$

The maximum and minimum slope were calculated for each data set and the difference divided by 2 were found to be the uncertainty. Likewise the same method was used to find the intercept's uncertainty. Looking at equation 8a from (COUPLED OSCILLATORS):

$$\omega^2 = {\omega_0}^2 + \frac{2k\ell^2}{mL^2} \tag{3}$$

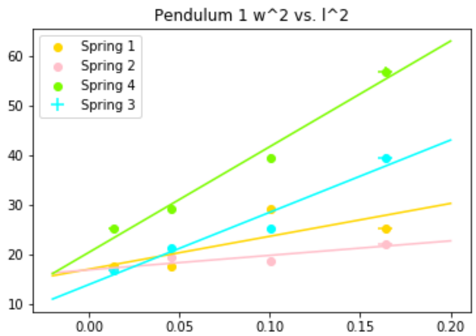
The intercept from the graph should be equal to the first term on the right hand side ${\omega_0}^2$ and the slope should be equal to $\frac{2\ell^2}{mL^2}$. Using the length of the pendulum as $0.594 \pm 0.002m$ that makes ${\omega_0}^2 = 16.49 \pm 0.06\frac{1}{s^2}$. Next for the slope: using the height from 1 to 4 as $0.117 \pm 0.003m$, $0.214 \pm 0.003m$, $0.317 \pm 0.002m$, and $0.405 \pm 0.002m$ and the mass for pendulum being $2.931 \pm 0.00005kg$ and pendulum 2 being $2.941 \pm 0.00005kg$. Plugging these values in gives the slope of pendulum 1 from height 1 to 4: $0.0264 \pm 0.00007\frac{1}{kg}$, $0.0882 \pm 0.0001\frac{1}{kg}$, $0.194 \pm 0.0001\frac{1}{kg}$, and $0.316 \pm 0.0001\frac{1}{kg}$. Likewise for pendulum 2 the slopes were: $0.0265 \pm 0.00007\frac{1}{kg}$, $0.0885 \pm 0.0001\frac{1}{kg}$, $0.195 \pm 0.0001\frac{1}{kg}$ and $0.317 \pm 0.0001\frac{1}{kg}$.

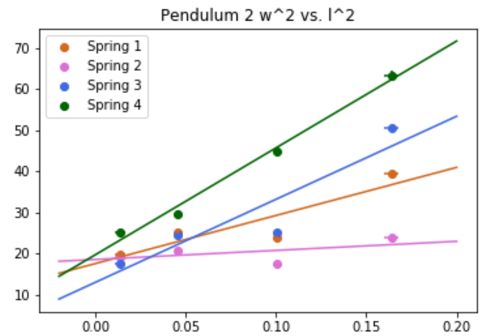
Overall the results were quite different to the theoretical value. Larger difference can be seen on the slope as oppose to the intercept. Looking at the slope first interestingly the slope were under the theoretical value for all the data sets of pendulum 1 and 2. Reason for the slope being under the theoretical value might be the result of either the spring constant being too high or the ω^2 was calculated to be too low. Next looking at the intercept this time the results varied between either above or below the theoretical value of 16.49. For the most part the values were quite different with the closest intercept being 15.78 with % difference being approximately 4.3% while the largest difference being the intercept of 31.75 with a 192.5% difference.

One possible reason behind these largest discrepancy between observation and theoretical value is first the human factor in this experiment. It sometimes is difficult to try to displace the exact same angle for both pendulums as a result this would lead to pendulum 1 and 2 having difference ω^2. Secondly during the experiment we noticed that the pendulum on the right had significantly more friction than the one on the left. As a result even when the two pendulums were displaced the same the pendulum on the right slowed down significantly from the friction.

Lastly ω^2 versus ℓ^2 would be look at. Similarly in the previous analysis the graph were broken into pendulum 1 and 2, and the table below consist of the slope and intercept values.

The error for the slope and intercept were calculated using the same method from ω^2 vs k graphs.





	Pendulum 1		Pendulum 2	
Spring	Slope($\frac{1}{s^2m^2}$)	Intercept($\frac{1}{s}$)	Slope($\frac{1}{s^2m^2}$)	Intercept($\frac{1}{s}$)
1	66.29$\pm$2	16.97$\pm$0.5	116.96$\pm$2	17.62$\pm$0.5
2	29.25$\pm$2	16.83$\pm$0.5	22.05$\pm$2	18.63$\pm$0.5
3	145.74$\pm$4	13.86$\pm$0.5	201.98$\pm$8	13.04$\pm$0.5
4	213.14$\pm$8	20.34$\pm$0.5	259.90$\pm$8	19.75$\pm$0.5

Using the same procedure from before the intercept would be the first term on the right and same as the previous analysis ${\omega_0}^2 = 16.49 \pm 0.06\frac{1}{s^2}$. Next the slope is calculated using $\frac{2k}{mL^2}$. As a result the slope for pendulum 1 from spring 1 to 4 is: $47.4 \pm 0.7\frac{1}{s^2m^2}$, $13.2 \pm 0.08\frac{1}{s^2m^2}$, $114.8 \pm 3\frac{1}{s^2m^2}$, and $376.8 \pm 9\frac{1}{s^2m^2}$. Likewise for pendulum 2 the slopes are $47.6 \pm 0.7\frac{1}{s^2m^2}$, $13.3 \pm 0.08\frac{1}{s^2m^2}$, $115.2 \pm 3\frac{1}{s^2m^2}$, and $378.1 \pm 9\frac{1}{s^2m^2}$.

Similarly to the previous analysis the slopes were slopes were quite different to that of the theoretical value. Tthis time the results for spring 1 to 3 were higher than the predicted value however the fourth spring was significantly under the predicted value. During the experiment the fourth spring also gave us some trouble since it was so stiff. As a result any displacement too large causes the pendulum swing too fast. To try to minimize this the angular displacement was made as small as possible but at one point we couldn't go any smaller since it was hard to judge whether the two pendulum had equal displacements. Additionally the effect of the right pendulum's higher friction causes the non-uniform oscillatory motion. Next the intercept for this data set were much closer to the predicted value of 16.49. The data are a lot more clustered to the predicted value with the closest having a 2.1% difference and the furthest being 123.3% difference. The furthest point still has a significant difference however it was much better than the previous analysis of 192.5%.

To summarize the results for both ω^2 vs k and ω^2 vs ℓ^2 graphs deviated significantly from the predicted results. Generally the slopes were quite quite different however the intercepts of the graphs showed better results. The main factors of these deviation were most likely caused by human factors and also one of the pendulums having significantly more friction than the other.

References

Gardiner, Jeff. n.d. Treatment of Errors. Waterloo (On): University of Waterloo.

Gardiner, Jeff. n.d. COUPLED OSCILLATORS. Waterloo (On): University of Waterloo.