Nuclear Counting

Josh Lau (2630767), Riley Delaney (20605226)

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Abstract

In this lab we determine characteristic curve and dead time of Geiger tubes, test the expected relationship with sample size and Gaussian distribution, and obtained the decay energy of Cl-36. We found that the dead time for our Geiger tube was $1400 \pm 200\mu sec$ and it]s characteristic curve has long plateau but relatively steeper slope. Next we found that the as the sample size increased the expected relationship correlated better with out data. Lastly we obtained a beta decay of $1.06 \pm 0.08 MeV$ with a 7% error with the expected result.

Introduction

In this experiment, we are counting nuclear particles through various methods with a Geiger Counter. Geiger Counters are tool that can measure the radioactive of a particle by counting the number of beta, alpha and gamma particles that are given off. This is useful since radiation is dangerous, so being able to detect high level of radiation is very important in fields such as nuclear physics and is useful for nuclear radiation protection.

One of the first attempts at measuring nuclear particles was using a microscope that could rotate around golden foil. This, however, was not the best method since it took time for individuals eyes to adjust to the darkness as well as there was a limited time that an individual could look and get an accurate count.(APS Physics 2012)

Hans Geiger thought that he could make an apparatus that was more accurate however. The first Geiger Counter which was a Crookes tube with electrodes that would measure the alpha radiation that moved through the system. This could only measure alpha radiation, until he worked with one of his students, Walther Muller where they improved it to be more portable, efficient and even able to detect particles other then alpha particles. (APS Physics 2012)

In this experiment, we tested a few different effects. The first part, we adjusted voltage to determine the effect and rate that increasing voltage had on the detection and the releasing of particles. We also determined the deadtime, the time that the detector cannot detect particles, for the system. Lastly, we used aluminium of varies thickness to determine the shielding of the aluminium as well as determine the beta decay of Cl-36.

Theoretical Background

Characteristics of Geiger Tube

An essential information about a Geiger tube is it's voltage characteristic curve, something that is unique between each counter tube. To find the the curve a radioactive source must first be placed at a fixed distance from the counter (Experimental Nucleonics). Then the count rate can be graphed as a function of the applied voltages. By doing so a curve below can be graphed.

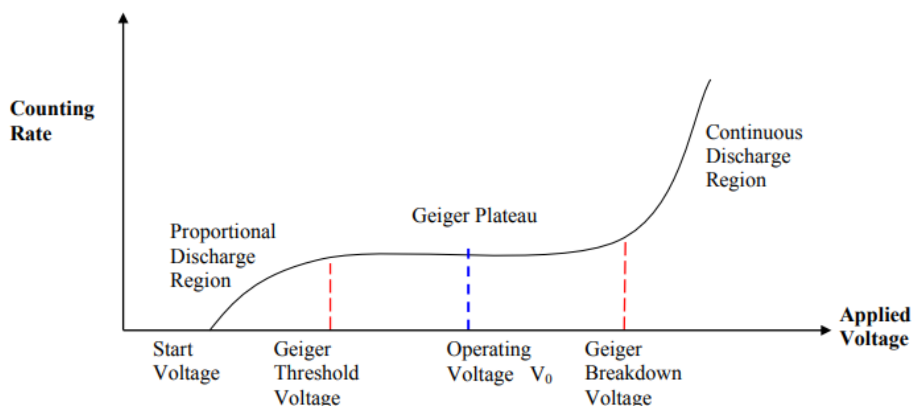


Figure 1: This image was directly taken from (OPERATING CHARACTERISTICS...[Date Unknown])

Initially the potential difference is too small for the counter to detect and until it reaches the threshold voltage (Experimental Nucleonics). Once the threshold is reached the counter rate will become constant even as the voltage increases. The optimal voltage to work in will be in the middle of the Geiger plateau since the count rate will be almost independent from the voltage. Beyond the plateau at the breakdown voltage where continuous discharge occurs and the count rate becomes unreliable. In general the center of the plateau should be 200v long and should not have a slope greater than 0.05%. A plateau that is as flat and long as possible is desirable however as the counter ages the curve tends to increase in slope and the length shortens.

Measuring the Dead Time

The dead time is caused by the fact that negative ions are collected quicker than than the positive ions mainly due to the difference in mass. The counter discharge occurs very close to the central wire (Experimental Nucleonics). As the electrons are collected first a sheath of positive ions move towards the outer surface of the cylinder (Experimental Nucleonics). This reduces the field to a point below the threshold value and the tube stops counting. Once the positive ion's sheath radius is large enough the field in the wire is restored (Experimental Nucleonics). This process is called the dead time where particles entering the tube during this time will not be detected. In most cases the dead time is on the order of hundreds of microseconds. The following derivation was taken from)Experimental Nucleonics) :

Let N be the count/sec for a counter with zero dead time. Let n be count/sec of a counter with dead time T sec. Then nT will be how long the tube is insensitive for in one second. As a result

$$NnT = N - n \tag{1}$$

where Nnt is the number of particles that weren't counted. Then if there were two separate sources n_1 and n_2 then

$$N_1 n_1 T = N_1 - n_1 \tag{2}$$

$$N_2 n_2 T = N_2 - n_2 \tag{3}$$

Adding equation (2) and (3) together

$$Tn_3(N_1 + N_2) = (N_1 + N_2) - n_3 \tag{4}$$

where n_3 is when both sources are counting together. Then rearranging equation (4) gives

$$T = \frac{(N_1 + N_2) - n_3}{n_3(N_1 + N_2)} \tag{5}$$

From this approximately the equation is

$$T = \frac{n_1 + n_2 - n_3}{2n_1 n_2} \tag{6}$$

Nature of Radioactive Decay

Due to the fact that radioactive decay occurs randomly no true count rate can be achieved. However an overall mean rate count can be calculated (Experimental Nucleonics). As number of counts increases meaning a higher sample size the count rate approaches towards the mean. Radioactive decay follows the Poisson distribution which expresses a probability curve of given events to occur over a fixed interval. A approximate graph of the Poisson is the Gaussian distribution.

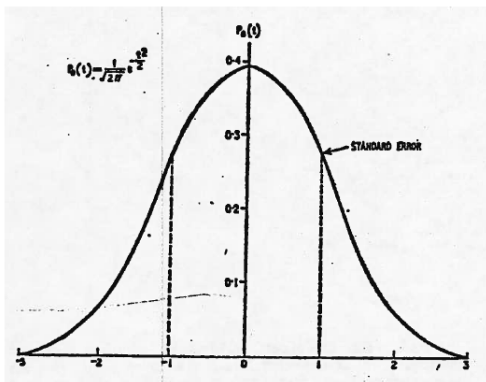


Figure 2: Diagram Directly Taken from (Experimental Nucleonics)

The width of the graph is known as the variance v. The square root of the variance is the standard deviation σ. The standard deviation expresses how spread out all the numbers are. With a small standard deviation the graph will look thinner and with a large standard deviation the graph will look thicker. The relative error is given by (Experimental Nucleonics)

$$t = \frac{n - m}{\sigma} \tag{7}$$

where n is sample size or observed count and m is the mean count. Then the probability of relative error can between interval t and t + dt is given by (Experimental Nucleonics)

$$P(t)dt = (2\pi)^{\frac{-1}{2}} e^{\frac{-t^2}{2}} dt \tag{8}$$

The standard deviation can be calculated (Experimental Nucleonics)

$$\sigma = \sqrt{n} \tag{9}$$

Then the true count will be $n \pm \sqrt{n}$ (Experimental Nucleonics)

Range of Beta Particles in Aluminium

For the most part the factor that effects distance travelled by particle in a material is the density-thickness ($\frac{mg}{cm^3}$). Even if two material have different densities as long as their density thickness values are the same the amount absorption will be the same. The figure below is of the absorption of aluminium with P^{32}. The tail part of the graph is due to Bremsstrahlung radiation and is linear. The dotted line represents the backward extrapolation of the Bremsstrahlung radiation. The curve of the beta radiation can be extrapolated to the x-axis which is given by the other dotted line. The place where both extrapolated lines intersect gives the maximum energy of the beta particles.

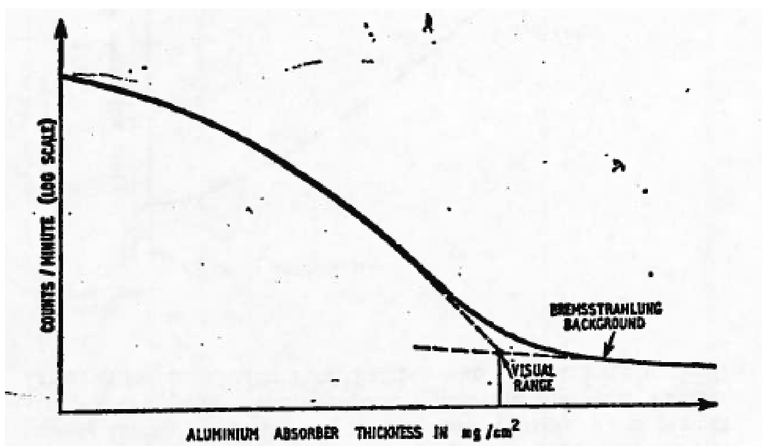


Figure 3: Diagram Directly Taken from (Experimental Nucleonics)

Procedure

For all four experiments the program LabView will be used however different programs within LaView will be used in four sections. First thing when arriving at the lab is to turn on the Power supple to warm it up (Phys 360AB/460AB..). Set the voltage to 0 and turn on the rest of the equipment(Phys 360AB/460AB..). Throughout the experiment the source is always on the fourth slot from the detector. Whenever the voltage is adjusted the sources should not be under the detector. The data should be saved in the c drive under exp14, then make a folder with the name of the group members, this will be where the data is saved. Important note for the majority of this procedure our time interval was set to 40 seconds instead of 60 seconds like the manual have instructed us to do. The main driving force behind this deviation is just for the interest of saving time.

Characteristics of Geiger Tube
Equipments:
(1) Computer
(2) Circuit Box for Geiger Tube
(3) HV Power Supply
(4) Chlorine 36 Source
(5) LND 712 Tube
(6) LND 72314 Tube
(7) Scaler
(8) Lead Shield

First open tubchar on Labview, make sure it is the latest version. Next make sure the voltage is set to 0. Then hook up one of the Geiger Tube to the control box. Once the Geiger is hooked onto the circuit box lift the top part of cylindrical lead shield off and put the counter inside. There should be a slot where the tube fits in perfectly in the shield. Once that is done close the top and make sure that when closing the top the detector is still in the slot. For LND 712 measure the tube from 300-650 volts and LND 72314 between 300-700 volts (Phys 360AB/460AB..). Use voltage step of 25v and on the program use a sampling time of 40s. Make sure that every time the voltage is increased the source is taken out of the detector. Also keep an eye on the scaler to make sure that the detector is counting. Keep repeating this process until the range is done and then do everything again for the other tube. After finishing both tubes decide which tube is better and that will be the one used for the next sections.

Measuring the Dead Time
Equipments: Same as in Last Section except now it is the double beta Chlorine 36.

First open deadtime on LabView again it should be the latest version. Then install the Geiger tube that was decided from previous section. The voltage used now should be the voltage at the center of the plateau found from the previous section. This voltage will remain the same throughout this section. Next take out the Cl 36 from the source plate and measure the background for 40 seconds. Then put either of the half of the source into the source slot, this will be considered your source ”1” then remove source 1 and put the other half and it will be considered source ”2”. Lastly put both sources on the source plate then measure the count. This process will done for 5 times.

Nature of Radioactive Decay
Equipment: Same as in section on ”Characteristics of Geiger Tube”

First open stasti in LabView again always pick the latest version. Next determine how long it would take to measure 20 counts with the Cl 36 at the chosen voltage (Phys 360AB/460AB..). Once the time is obtained input it into the program and set the size to 10,100,300,500 trials . For each different size take a screen shot of the histogram and Gaussian distribution displayed .

Range of Beta Particles in Aluminium

First open betarang in LabView again use the latest version. The shielding of aluminium will be placed on the slot above the Cl-36. Use 40 seconds for the time interval on LabView. Measure the number of counts from thin to thick aluminium sheets. Each time the shield is changed make sure to take out both the plate for the Cl-36 and shield together. Never expose the detector with the source without shielding (Phys 360AB/460AB..). Keep doing this process of changing aluminium thickness until 3 points on the tail of the plot is achieved. This is one of those times where too many data can be bad once the 3 points on the tail is achieved the experiment should stop since in this case too many data will make the analysis more difficult.

Analysis

Characteristics of Geiger Tube

To begin the total count is a function of voltage is graphed for the Geiger tubes LND 712 and LND 72314. This results in these two graphs:

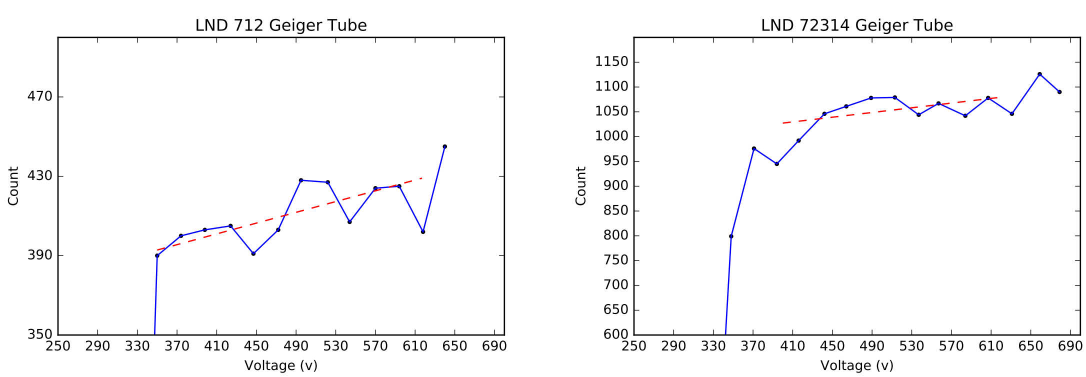


Figure 4: Plot of Count versus Voltage for LND 712 and LND 72314 Geiger Tube.

In the graph the solid blue line is a plot of the counts from the respective voltage ranges. Within the graph there is also the red dotted line which we estimated to be where the plateau is. This estimation is mainly based on looking at where it seems likely for the plateau to start and begin and also looking count that they are still within about 50 counts from one another. The main argument behind estimating the plateau is to approximate the slope of the plateau. This way we can compare the slope of each estimated plateau and also the length to decide which tube to use. First the slope of LND 712 and LND 72314 are 0.136 and 0.245 respectively. Having a flatter slope is more desirable since that means the count rate is less dependant upon the voltage applied. Another important aspect is for the plateau to be long this way when estimating the center of the plateau it gives more room for error. With all this in mind we decided that LND 72314 was the one we wanted to go with. However both tubes have their pros and cons. For LND 72314 it's pro is that the graph looks more like the desirable curve for a Geiger tube where there is a sharp jump pass the threshold voltage and long plateau. The only downside compared to LND 314 is that the slope is greater by about a factor of 1.8. Another reason for picking LND 72314 is that the points don't oscillate about the estimated plateau line. Meaning the points are on average closer to the line then that of the LND 712. As for the pros and cons of LND 712 it is exactly the opposite with LND 72314 where there is a flatter slope but the points oscillate more about the estimated plateau line and the length is shorter. With all these in mind we decided to use LND 72314 and estimated that the optimal voltage for the center of the plateau is at 500v.

Measuring Dead Time

There was not much to observe qualitatively, however, there was a difference in two sources. The first source had a silver coating where as the first one was more clear and white. There was also four different counts that were taken, background, source 1, source 2 and both sources.

First, to determine the deadtime, the averages of the four counts and the standard deviation where taken and are recorded in the table below:

	Background	Source 1	Source 2	Both Sources
Mean	126	3720	990	4450
Standard Deviation	8	90	40	30

Using equation 6 for deadtime was given to be:

$$T = \frac{(n_1 + n_2 - n_3)}{2n_1n_2} \tag{10}$$

Using the values that were calculated the deadtime is:

$$T = \frac{3720 + 990 + 4450}{2x3720x990} = 3.53x10^{-5}(40)sec = 1400sec$$

Multiplying by 40 seconds since there trials run for 40 seconds.

The uncertainty formula is:

$$e_T = T\sqrt{(\frac{\sqrt{e_1^2 + e_2^2 + e_3^2}}{n_1 + n_2 - n_b})^2 + (\%e_1^2 + \%e_2^2)^2} \tag{11}$$

$$e_T = T\sqrt{(\frac{\sqrt{90^2 + 40^2 + 30^2}}{3720 + 990 - 4450})^2 + ((\frac{90}{3720})^2 + (\frac{40}{990})^2)^2} = 5x10^{-6}sec$$

The deadtime of the system was $1400 \pm 200\mu sec$. The expected range is a few hundred microseconds. The actual value was higher then a few hundred microseconds,however, these could be due to the age of the Geiger Counter.

Nature of Radioactive Decay
To begin a table for the mean, standard deviation, and mean deviation is created below for the 4 difference sample sizes:

Sample Size	Mean	Standard Deviation	Mean Deviation
10	20.90	4.09	3.50
100	19.13	3.87	3.04
300	19.66	4.11	3.18
500	19.36	4.18	3.38

Table 1: Table of Mean, Standard Deviation, and Mean Deviation for 4 Sample Sizes

The 4 relationship to test are as follows:
(1) $\sigma = \sqrt{m}$, where m is the mean count
(2) mean deviation = $\frac{4}{5}\sigma$
(3) Approx $\frac{1}{3}$ of the deviation exceed the standard deviation
(4) Approx $\frac{1}{20}$ of the deviation exceed twice the standard deviation

As an example calculation sample size of 500 will be done step by step. First to answer the question simply take the square root of the mean count which is 19.36 from the table above. This gives a value of 4.4, compare this with the standard deviation of 4.18 this is off by about 8% from the theory. Next taking $\frac{4}{5}\sigma$ gives 3.34 which compared to the mean deviation of 3.38 it has only a percent error of 1%. Next we must first calculate the n value for one deviation from the mean. using equation and plugging t = 1, m =19.36, $\sigma = 4.18$ gives

$$1 = \frac{n - 19.36}{4.18} \tag{12}$$

rearranging gives it in terms of n

$$n = 4.18(1) + 19.36 = 23.54 \tag{13}$$

However this is only the positive side of the tail, the left side of the mean must also be calculated where t = -1, doing so gives a n = 15.18. So now we just have to calculate of those 500 data set how many are less than 15.18 and greater than 23.54. The result is 180 points fall within two limits. Then calculating $\frac{180}{500}$ gives 36% compared to the theoretical result of 33% we were off by less than 3%. Lastly doing similar procedure for 2 σ. Using equation (7) again:

$$2 = \frac{n - 19.36}{4.18} \tag{14}$$

$$n = 4.18(2) + 19.36 = 27.72 \tag{15}$$

We once again have to calculate the left end of the tail where t = -2 this gives n = 11. Now same procedure as one sigma we find of the 500 data points how many fall less than 11 and greater than 27.72. This gives 20 data points that are in the limit. This means that 4% of the deviation exceed twice the standard deviation. Comparing this to the expected value of 5% we were only off by 1%.

Next the table below summarizes the result of the 4 questions for all 4 trials:

Sample Size	σ vs. $\sqrt{m}$	mean deviation vs. $\frac{4}{5}\sigma$	1σ	2σ
10	4.09 vs. 4.57	3.50 vs. 3.272	0.30	0
100	3.87 vs. 4.37	3.04 vs. 3.096	0.31	0.07
300	4.11 vs. 4.43	3.18 vs. 3.29	$\frac{1}{3}$	$\frac{1}{20}$
500	4.18 vs. 4.4	3.38 vs. 3.34	0.36	0.04

Table 2: Table of The Expected Relationships from Brown

The overall trend does match with what is expected as the sample size goes up. As the same size rises the expected relationship correlate better, for example looking at the 1σ and 2σ column the values gets increasingly closer to $\frac{1}{3}$ and $\frac{1}{20}$ as the sample increases. An interesting not is that for our trial of 300 the values of 1 and 2 σ got exactly the approximate values. This agrees with the fact that as the sample size is increased the distribution approaches more Gaussian. This is also clearly evident by looking at histograms on Figure 5. As the number of sample size increases a clearly define mean peak and Gaussian shape starts to form. The histogram for 100,300, and 500 peak also agrees with what was expected, since we calculated the necessary time for 20 counts on average all the trials in each size should be close to the mean of 20. This is clearly evident in the 100 trial where majority of the occurrences happened at 20 counts. To conclude as the sample sizes increases the expected relationship from Brown correlates with the data.

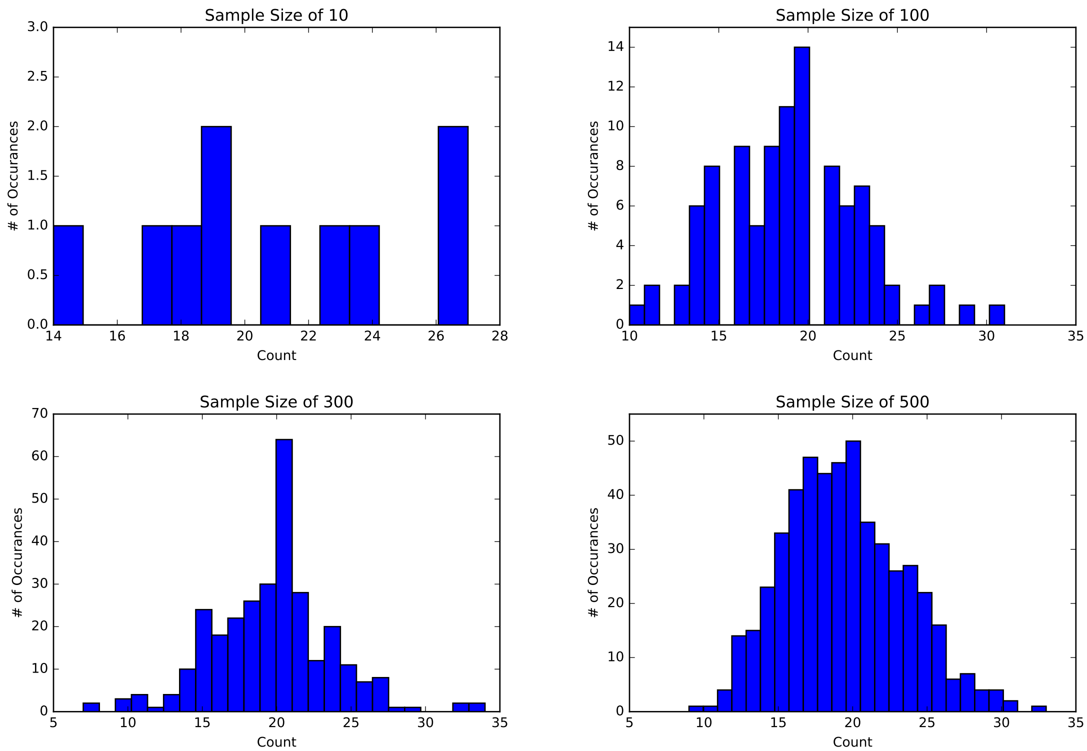


Figure 5: Histogram of sample sizes 10,100,300, and 500.

Range of Beta Particles in Aluminium

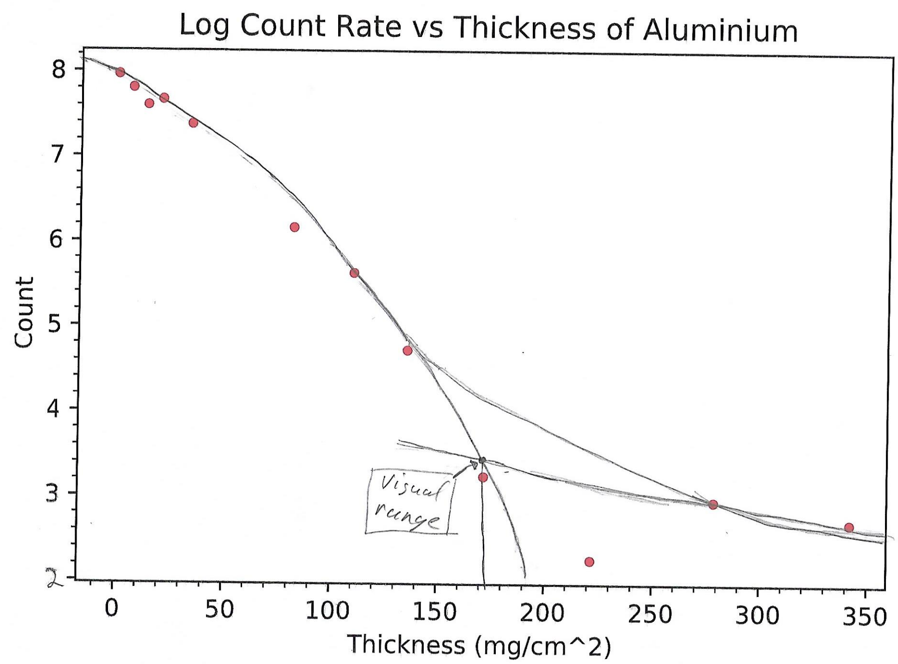
The count of no aluminium or source is from the Geiger Counter was 8. The ratio for $(n_1 + n_2)/(n_b)$, from part 2, can be used to determine the counts lost due to deadtime. Combining these two ideas, the counts can be adjusted for deadtime and background radiation:

$$n_{new} = (n_{old} - n_{bkg})\frac{n_1 + n_2}{n_b} \tag{16}$$

A sample calculation is:

$$n_{new} = (2714 - 8)\frac{4710}{4450} = 2877 counts$$

The table with all of these values can be located in the appendix. With these values, the graph of the log of the counts vs the thickness can be plotted for the visual range:



From this graph, the visual range is around $172mg/cm^2$, however, due to the having to hand draw the functions instead of using a more accurate computer regression function, the value with uncertainty s $172 \pm 7mg/cm^2$.

From the curve in Halliday(Experimental Nucleonics), a visual range of $172 \pm 7mg/cm^2$ has a beta decay of $1.06 \pm 0.08MeV$. The actual beta decay of Cl-36 is 1.142MeV(Nucleide 2012c). The actual values is slightly outside of the uncertainty. This could mean that the rough sketch had more uncertainty then initial thought. The percent error is low with a percent error of 7%.

Conclusion

For the first part, LND 72314 was chosen because it had a a better plateau leading to a better estimate for the average count. Both had pros and cons though, such as LND 314 having a flatter slope. However, it was decided that LND 72314 had the most pros.

The deadtime of the system was $(1400 \pm 200\mu sec)$. The expected range is around a hundred or a few hundred. The actual value is a degree higher then the expected deadtime. This could be due to precise of the instruments or could be the age of the Geiger Counter.

Fr the third part, the trend does match expectations as the sample size increases. For σ_1 the value approaches $\frac{1}{3}$ and σ_2 approaches a value of $\frac{1}{20}$ for increasing sample sizes. This agrees that increasing sample size makes the curve more Gaussian. The expected average for the curve is 20 counts, which the histogram peaks are around for the different sample sizes.

The visual range is $172 \pm 7mg/cm^2$ for the aluminium. This corresponds to a beta decay rate of $1.06 \pm 0.08MeV$ and the actual value is 1.152MeV (Nucleide 2012c). The percent error was 7%, therefore the experiment value is relatively close.

Appendix

Background	Source 1	Source 2	Both
128	3567	1041	4458
127	3655	949	4434
137	3779	934	4394
111	3790	1031	4472
126	3785	992	4476

Absorber	Count	Adjusted Count
0	2714	2877
6.8	2318	2468
13.8	1892	2006
20.7	2024	2145
34.4	1512	1603
82.2	448	475
110.3	263	279
135.7	106	112
171.6	24	25
221.4	9	10
278.6	18	19
342.2	14	15

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