Analysis

Method (A) of determining speed of light.

To begin at a qualitative standpoint looking through the travelling microscope at low hertz an image of two red dots are formed. The image is blurring most likely due to the air and diffraction patterns can also be made out from each dot. As the hertz increases the dots start to move from left to right on the lens. At a certain frequency they merge into one. Once the mirror is rotating at maximum speed there is only one red dot on the lens. The effects of the dots moving can really be observed once the voltage goes from max to 20v quickly. This causes the mirror to rotate slower and the dots can be observe to move from right to left quickly and separating back into the two.

Next given the three measurements of the distance from the beam splitter to the rotating mirror the average distance was taken. The reason behind measuring the distance using the 10cm, 20cm, and 30cm mark on the tape measure was to limit any manufacturing error in the tape measure. Taking the average gives $r = 7.108 \pm 0.0009m$. This was calculated by taking all 3 measurements adding them and dividing by 3. The example calculation below shows how the uncertainty was determined:

$$\frac{\sqrt{(0.008)^2+0.008)^2+0.008)^2}}{3} = \frac{\sqrt{3(0.008)^2}}{3} = \frac{0.01386}{3} = 0.00046188 = 0.0005$$

Using the r value, d can be calculating using equation (PUT NUMBER) which gives that $d = 12.892 \pm 0.0005m$. With r and d the only numbers need to calculate the speed of light is finding the slope between the travelling microscope position versus frequency of the mirror.

From the data obtained in the appendix, graphs were produced to plot the travelling microscope position in cm to the frequency in the oscilloscope in hertz. Figure 1 contains all 3 trials done on recording the change in position of the microscope as the frequency increased. Without looking at the numbers the graphs suggest a linear relationship between the two which agrees with our theory. Trial 1 and Trial 3 will seem to have a good fit since most points lie on a straight line however Trial 2's data won't seem to have an as good linear fit due to the points being either above or below the predicted line, this will be clearly seen once the the graphs are linearised. Next Figure 2 shows the same graphs but linearised. The linearisation was done in python using polyfit from numpy. The program uses least square method of determining the line of best fit. As expected the graphs of Trial 1 and Trial 3 provided a line of best fit that correlated well with the data points. On the other hand Trial 2's points hovered above and below the predicted line. Out of all three trials it seemed that Trial 3 had best points since all expect for 1 is touching the line of best fit.

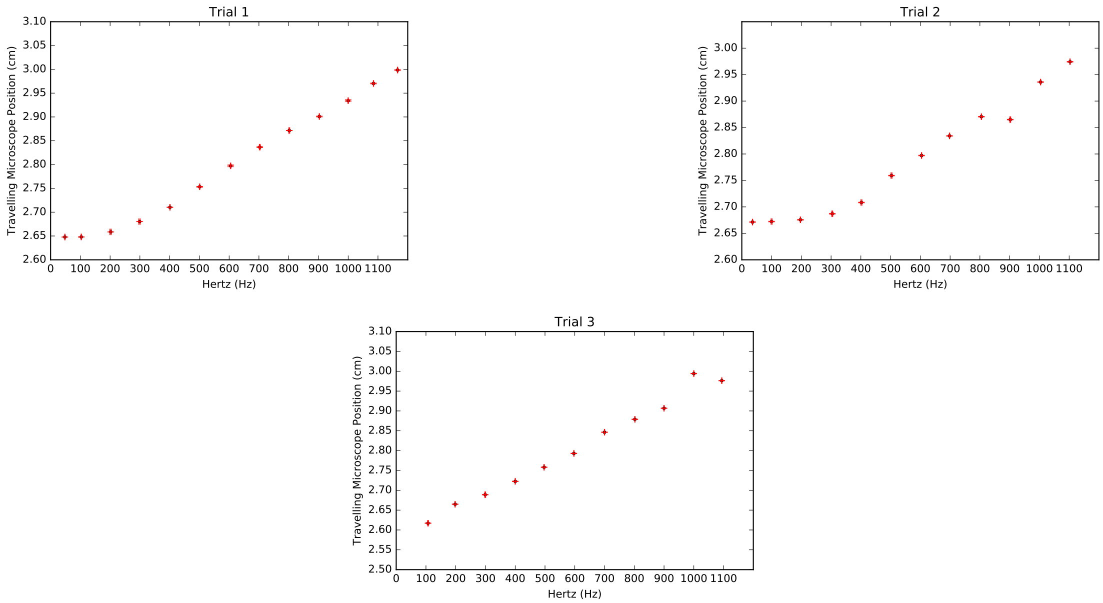

Figure 1: Graphs of the change in position with respect to the frequency.

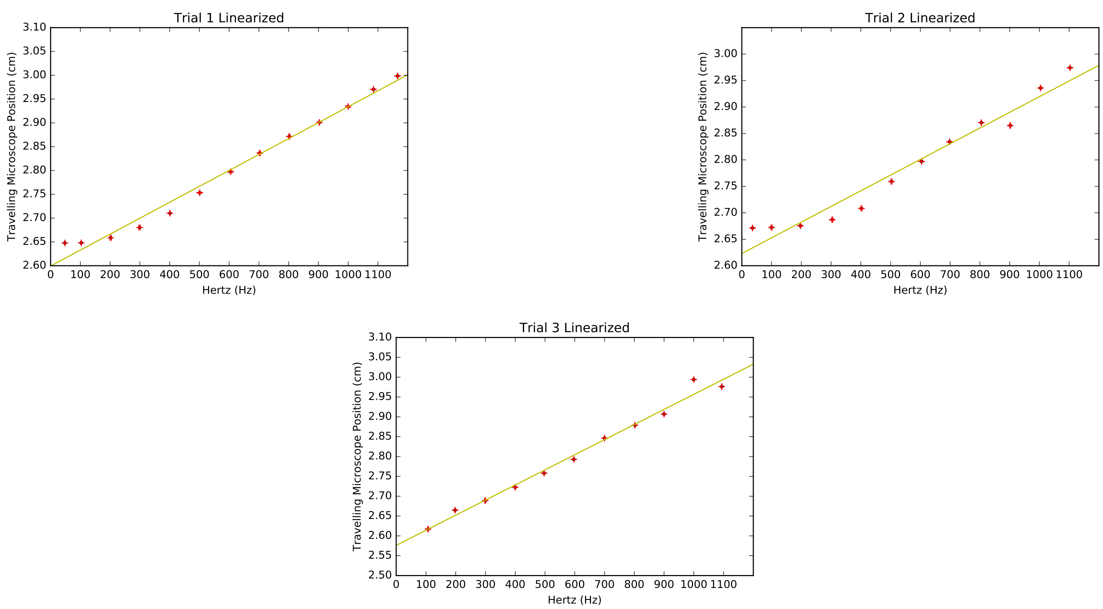

Figure 2: Graphs of the change in position with respect to the frequency linearised

Next a table with the slope and uncertainty of the 3 trials appears below:[1]

Trial	Slope ($\frac{cm}{s}$)	Intercept(cm)
1	$(3.34 \pm 0.01)x10^-4$	2.60 ± 0.0003
2	$(2.96 \pm 0.02)x10^-4$	2.62 ± 0.0002
3	$(3.81 \pm 0.003)x10^-4$	2.57 ± 0.00008

Table 1: Table of slope and intercept of the line of best fit.

First to calculate the errors of the slope and intercept an example calculation of trial 1 is done below: To find the upper and lower bound of the slope we took the addition or difference of the hertz and position uncertainty. For example if the hertz was 48 $\pm 1hz$ then the upper limit for that number will be 49 hz and lower will be 47. This was done for all 12 different and position. As an example the first 2 and last position and hertz were calculated in the following table:[2]

Trial 1		Trial 1 Upper		Trial 1 Lower	
Hertz	Position	Hertz	Position	Hertz	Position
48 $\pm 1hz$	2.6478 $\pm 0.0003cm$	49hz	2.6481cm	47hz	2.6475cm
103 $\pm 1hz$	2.648 $\pm 0.0005cm$	104hz	2.6485	102hz	2.6475
.	.	.	.	.	.
.	.	.	.	.	.
1085 $\pm 1hz$	2.9703 $\pm 0.001cm$	1086hz	2.9713cm	1084hz	2.9693cm

Table 2: Trial 1 Upper and Lower Position and Hertz

Doing this we found the slope for the upper and lower bound by graphing the two data sets and poly fitting a linear relationship. As a result the slope for the upper bound gave a value of 0.000380875 and lower bound 0.000380323. With these two upper and lower slopes the difference divided by 2 was taken.
$(m_{\text{up}} - m_{\text{low}})/2 = (0.000380875 - 0.000380323)/2 = 1.082x10^-6 = 1x10^-6$ The exact same procedure was done with the upper and lower bound intercept which resulted a value of 0.000262 which is equal to 0.0003. These method of finding the uncertainty using the upper and lower bound were based on the methods outlined in ”ASSESSMENT OF EXPERIMENTAL ERRORS” of the courses notes.

With the three slopes the average of the three can be taken. This gives a $\bar{m} = (3.37 \pm 0.0008)x10^-4$ the uncertainty in the slope was calculated using the exact same method the average value of r. Where the square of the uncertainties were summed and a square root was taken and finally divided by 3 because there were 3 trials. This slope will then be used to calculate the speed of light. Using the equation (INPUT NUMBER)

$\bar{m} = \frac{4\pi rd}{c}$, where r = 7.108m, d = 12.892m and converting $\bar{m}$ to meters which gives $\bar{m} = 3.37x10^-6$

Solving for c gives
$c = \frac{4\pi rd}{\bar{m}} = \frac{4\pi rd}{\bar{m}} = \frac{4\pi(7.108)(12.892)}{3.37x10^-6} = (3.41 \pm 0.0009)x10^8 \frac{m}{s}$

The error was calculated by converting the uncertainty of r,d, and $\bar{m}$ into percentages. This gives $r = 7.108 \pm 0.007\%$, $d = 12.892 \pm 0.007\%$, and $\bar{m} = 3.37 \pm 0.023\%$. Using the rule for uncertainty when multiplying gives the following equation:

$\sqrt{(0.007\%)^2 + (0.007\%)^2 + (0.023\%)^2} = 0.025\% = 0.03\%$
Finally 0.03% is equivalent to $9x10^4 \frac{m}{s}$

Comparing this to the accepted value of $2.99x10^8$ our percent error is about 14% which is significant higher then our uncertainty can account for. This most likely means that the average slope calculated from the graphs were off by some degree. Going reverse the average slope should have been about $3.85x10^-6$ using the accepted speed of light value. Interesting trial 3's slope was $3.81x10^-6)$ if the number is divided by 100 on table 1 since that is in cm per second. The most likely outlier is trial 2 since it resulted in such a small slope it brings the average down significantly. Even looking at the graph trial 2's numbers don't seem to correlate very well with the linear fit. It's difficult to pin point which exact point on trial 2 is causing problems since all of the data hover above or below the line. But if we take trial 2 out of consideration and only take average of trial 1 and 3 then that would result in a slope of $3.575x10^-6$ which will still give us a percent error of about 7% but it drops down by a factor of 2.

Looking at the reason behind this significant percent error it is most likely due to the slope and not the measured r and d values. The reason is as follows, given the equation (INPUT SPEED OF LIGHT EQUATION), c depends on r,d, and $\bar{m}$. If the measuring r was off by 10cm for example then that means that d will also be off by 10 since they are related. But the errors counter balances each other in a way. If r increases or decreases then d will do the opposite. So when the two are multiplied the effect of the measurement error is reduced. As a result a change in $\bar{m}$ will greatly effect the value of c since there isn't another variable to counter that error like in the case of r and d. With that being said some instrumental error may have occurred when measuring r. When it comes to measuring long distance the tape measure tend to dip in the middle. Even when my partner and I pulled tried to add tension it still dips a significant amount. As a result the r value we calculated will be larger since length is increased due to the dip. As for errors caused by experimenters the most significant one will be when using the travelling microscope. At the beginning it's not too difficult to find the center point of whichever dot was chosen to observe. However as the frequency increases the image of the dot begins to get blurry and elongate. Especially once the two dots merge into one it's a lot more difficult determine where the center is. At the same time the shape is no longer circular it has more of a football shape where the long side is on the y axis. A combination of diffraction pattern and the air causing the laser to blur also contributes to the lack of clarity where the center is. As a result it gets increasingly difficult to accurately measure the center as the frequency increases. Taking everything into consideration most likely the reason behind this large discrepancy is due to sources of error caused by the experimenter given the fact that $\bar{c}$ depends more heavily on $\bar{m}$ which is itself depends more on experimenter errors as oppose to instrumental errors. More specifically the most significant errors are more likely to be cause the experimenter due to the fact that trying to center the position of the microscope to laser get increasingly difficult.

Method (B) of determining speed of light.

Similarly as the previous part qualitative comments will discussed first. Once the set-up is complete and the oscilloscope detects the laser, there are two large dibs on the screen while there a many smaller dips on the sides. In the perfect world there should be only 2 large dips while every else is flat however due to factors such as wire, air interference, and also room lighting we get small signals along with the ones we are interested in.

Following the procedure in the first test we used only 2 plane mirrors and a corner cube. In the first test we made the distance from the beam splitter to lens to be 5 $\pm 0.005m$ and from the lens to M_1 was measured to be 4.422 $\pm 0.005m$. From there M_1 to M_2 was 10.20 $\pm 0.01m$ and finally M_2 to Corner Cube was 10.10 $\pm 0.01m$. So the total distance light would travel would be the sum of all these distance multiplied by 2 since it travels both ways. A mistake of forgetting to measure the distance between to beam splitter and detector was made. This small distance might contribute to the difference between actual speed and our calculated one. Below will be a sample calculation of finding c_1:

Total path length of light (c_1) -
$2(5 + 4.422 + 10.20 + 10.10) = 59.444 \pm 0.02m$

The uncertainty was calculated by summing the squares and taking the square root -
$e_1 = \sqrt{(0.005)^2 + (0.005)^2 + (0.01)^2 + (0.01)^2} = 0.0158 = 0.02$

Time $t_1 = 197.480 \pm 2ns$

Using the formula (Gardiner 2015)-

$$c_1 = \frac{l_1 \pm e_1}{t_1 \pm e_t} = \frac{l_1}{t_1} \pm \sqrt{(\%e_1)^2 + (\%e_t)^2} = c_1 \pm e_1 \tag{1}$$

Putting the numbers in

$$\frac{59.444m}{197.48x10{-}9s} \pm \sqrt{(0.0336\%)^2 + (1.01\%)^2} = (3.01 \pm 0.03)x10^8 \frac{m}{s}$$

Using the exact same procedure c_2 and c_3 were obtained:
$c_2 = (3.01 \pm 0.01)x10^8 \frac{m}{s}$ and $c_3 = (3.04 \pm 0.03)x10^8 \frac{m}{s}$
The values for the distance can be referenced in the appendix section Part B.

To calculate mean speed of light and error the following formula will be used (Gardiner 2015) -

$$\bar{c} = \frac{\sum_{n=1}^{3} \frac{c_n}{(e_n)^2}}{\sum_{n=1}^{3} \frac{1}{(e_n)^2}} \text{ and } e_{\text{mean}} = \sqrt{\frac{1}{\sum_{n=1}^{3} \frac{1}{(e_n)^2}}}$$

plugging in the values

$$\bar{c} = \frac{\frac{3.01}{0.03^2} + \frac{3.01}{0.01^2} + \frac{3.04}{0.03^2}}{\frac{1}{0.03^2} + \frac{1}{0.01^2} + \frac{1}{0.03^2}} = 3.01x10^8 \frac{m}{s} \text{ and}$$

$$e_{\text{mean}} = \sqrt{\frac{1}{\frac{1}{0.03^2} + \frac{1}{0.01^2} + \frac{1}{0.03^2}}} \quad 0.009x10^8 \frac{m}{s}$$

Therefore calculated speed of light $\bar{c} = (3.01 \pm 0.009)x10^8 \frac{m}{s}$

Comparing this with the accepted value of $2.99x10^8$ we have a percent error of 0.67% but the error bound of 0.009 is just outside of the accepted value. This means the uncertainty for some of the measurement were most likely smaller than it should be. For example when it comes to distance exceeding 10 meter the uncertainty should certainty be higher than 1 cm just because of the fact at 10 m the tape measure starts to dip significantly. As a result if this experiment was conducted again the error in measuring distances should be significantly higher. The results turned out consistent with theory, when it comes to part b the equation used was much simpler than in part a since it is just using the definition of velocity, change in distance over change in time. It was expected that this experiment would fall within 1% of the expected value and we obtained at percent error of 0.67%. Overall the results were consistent with the accepted value. It's also important to again keep in mind that this experiment was not done in vacuum so air plays role in reducing the accuracy of this experiment. To reduce the uncertainty in the time measurement of the oscilloscope we tried to use different positions to see which one would gives us the best result. For example instead of using the dips inflection point of the waves could have been used. Ultimately we decided the best position would be using the peak of the waves.

Reference

Gardiner J. 2015. PHYS 360A Modern Physics Laboratory 1. Waterloo(ON): University of Waterloo

Appendix

Part A
Distance from Beam Splitting Glass "S" to Rotating Mirror "M_1":
1) 7.103 $\pm 0.008m$ - Used 10cm mark
2) 7.108 $\pm 0.008m$ - Used 20cm mark
3) 7.112 $\pm 0.008m$ - Used 30cm mark

Distance from M_1 to Lens:
2.89 $\pm 0.005m$ - Used 10cm and 20cm mark

Distance from Lens to M_2:
1.48 $\pm 0.005m$

Distance from M_2 to M_3:
8.52 $\pm 0.01m$

Trial 1		Trial 2		Trial 3	
Hertz	Position	Hertz	Position	Hertz	Position
48 $\pm 1hz$	2.6478 $\pm 0.0003cm$	36 $\pm 1hz$	2.6713 $\pm 0.0007cm$	n/a	n/a
103 $\pm 1hz$	2.648 $\pm 0.0005cm$	100 $\pm 2hz$	2.6723 $\pm 0.0007cm$	107 $\pm 2hz$	2.6171 $\pm 0.0007cm$
202 $\pm 3hz$	2.6586 $\pm 0.0003cm$	197 $\pm 1hz$	2.6758 $\pm 0.0007cm$	198 $\pm 1hz$	2.6649 $\pm 0.0007cm$
299 $\pm 3hz$	2.6802 $\pm 0.0003cm$	304 $\pm 2hz$	2.6870 $\pm 0.0007cm$	299 $\pm 2hz$	2.6889 $\pm 0.0007cm$
401 $\pm 1hz$	2.7103 $\pm 0.0005cm$	402 $\pm 2hz$	2.7083 $\pm 0.0007cm$	400 $\pm 1hz$	2.7225 $\pm 0.0007cm$
501 $\pm 1hz$	2.7533 $\pm 0.001cm$	503 $\pm 2hz$	2.7542 $\pm 0.0007cm$	497 $\pm 1hz$	2.7531 $\pm 0.0007cm$
605 $\pm 1hz$	2.7973 $\pm 0.002cm$	604 $\pm 1hz$	2.7971 $\pm 0.0007cm$	597 $\pm 1hz$	2.7929 $\pm 0.0007cm$
703 $\pm 2hz$	2.8366 $\pm 0.001cm$	698 $\pm 1hz$	2.8342 $\pm 0.0007cm$	700 $\pm 1hz$	2.8463 $\pm 0.0007cm$
802 $\pm 2hz$	2.8715 $\pm 0.0005cm$	805 $\pm 1hz$	2.8705 $\pm 0.0007cm$	802 $\pm 1hz$	2.8789 $\pm 0.0007cm$
903 $\pm 1hz$	2.9009 $\pm 0.001cm$	902 $\pm 2hz$	2.8950 $\pm 0.0007cm$	900 $\pm 1hz$	2.9068 $\pm 0.0007cm$
1000 $\pm 1hz$	2.9342 $\pm 0.002cm$	1004 $\pm 1hz$	2.9359 $\pm 0.0007cm$	1000 $\pm 1hz$	2.9941 $\pm 0.0007cm$
1085 $\pm 1hz$	2.9703 $\pm 0.001cm$	1103 $\pm 1hz$	2.9743 $\pm 0.0007cm$	1094 $\pm 1hz$	2.9763 $\pm 0.0007cm$

Table 3: Part A Data Set

Part B
Second Test -

Laser to Lens:
5 $\pm 0.005m$

Distance from lens to M_1:
4.422 $\pm 0.005m$

Distance from M_1 to M_2:
$10.20 \pm 0.01 m$

Distance from M_2 to M_3:
$10.16 \pm 0.01 m$

Distance from M_3 to Collimator:
$10.12 \pm 0.01 m$

Oscilloscope - Using both valleys
$\Delta t = 264.80 \pm 1 ns$

Third Test -

Laser to Lens:
$5 \pm 0.005 m$

Distance from lens to M_1:
$4.422 \pm 0.005 m$

Distance from M_1 to M_2:
$10.20 \pm 0.01 m$

Distance from M_2 to M_3:
$12.28 \pm 0.03 m$

Distance from M_3 to Collimator:
$10.22 \pm 0.01 m$

Oscilloscope - Using both valleys
$\Delta t = 276.80 \pm 3 ns$