Theoretical Considerations

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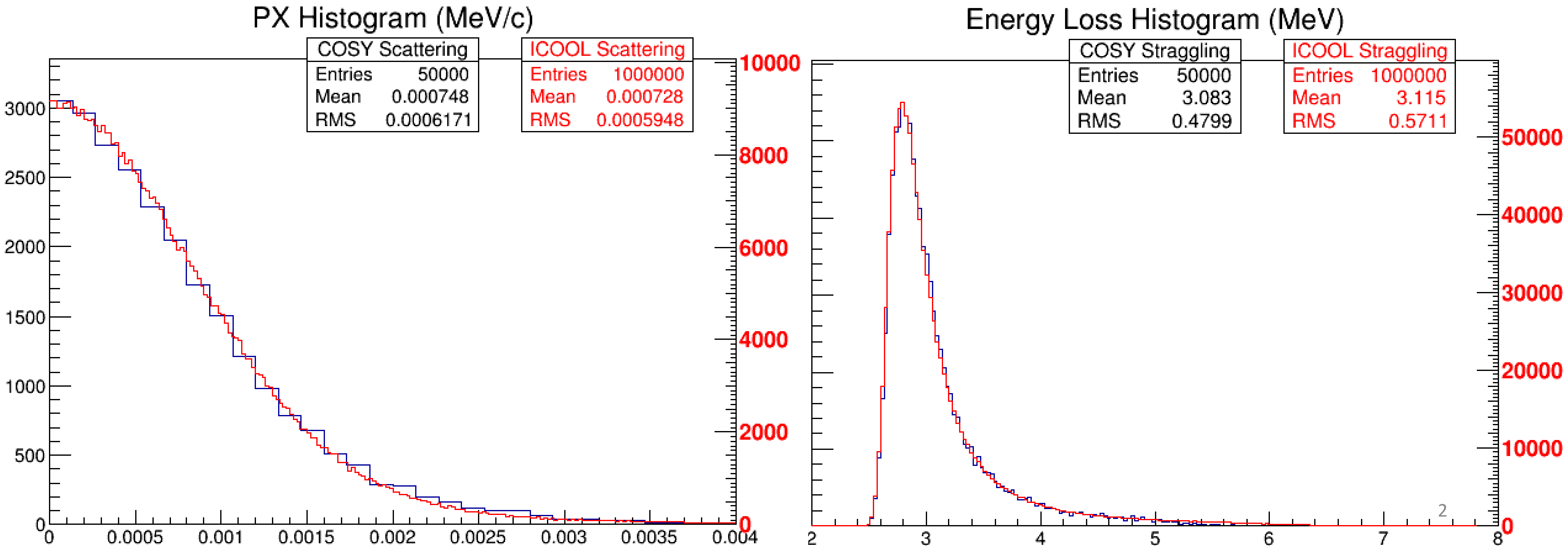
07.14.14

10 cm Results Revisited

- Transverse looks okay, longitudinal looks great!
- Transverse was done using a Gaussian-Enge distribution –need more particles to say anything conclusive.



Transverse Momentum: Which Model Is Best?

- Theory (3 parameters): $\chi^2 = 1665^*$
- Theory (5 parameters):

$$f(u) = \begin{cases} A_0 e^{-a(1-u)}, & u_0 \leq u \\ A_1 \dfrac{1}{(b-u)^d}, & u \leq u_0 \end{cases}$$

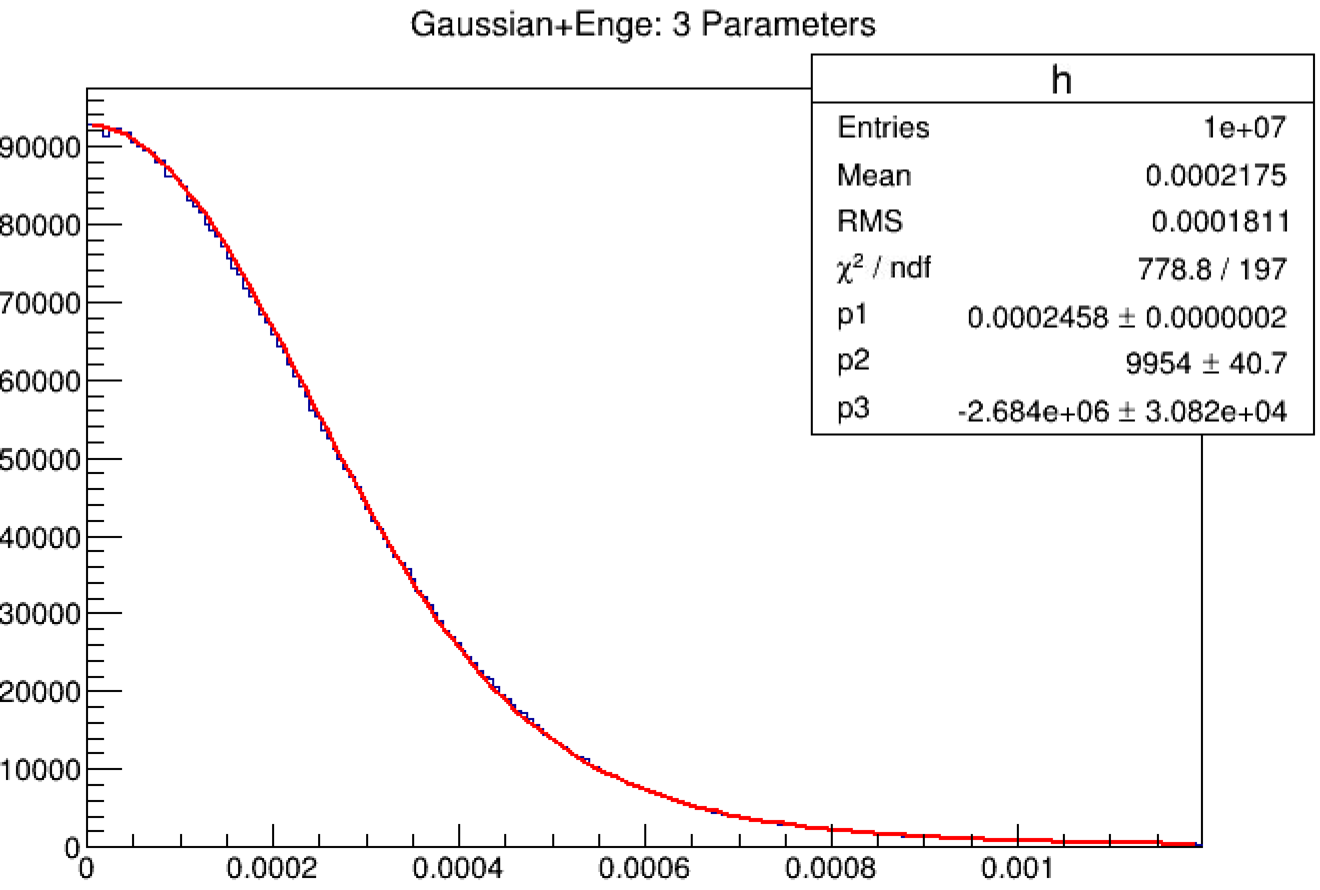
$$u = \cos\theta = \frac{P_z}{\sqrt{P_z^2 + P_x^2}}$$

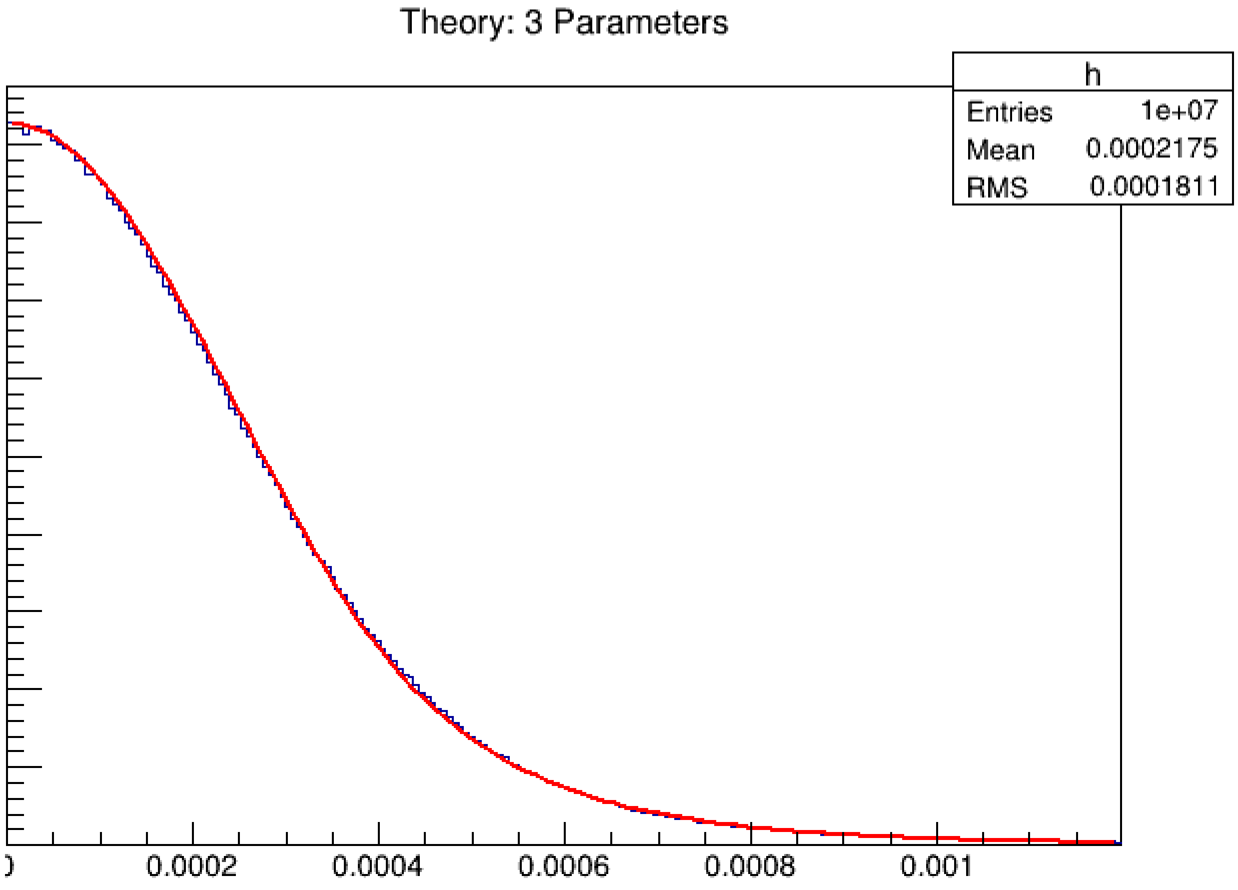
- Gauss+Enge (3 parameters): $\chi^2 = 779^*$
- Gaussian+Enge (5 parameters):

$$GE(x) = \begin{cases} A_0 e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} & ,x < \sigma \\ A_1/(1 + e^{a_0 + a_1 x + a_2 x^2}) & ,x \geq \sigma \end{cases}$$

*Simulations done using 1E7 muons @ Pz = 200 MeV/c through 1 cm liquid hydrogen

Transverse Momentum: Which Model Is Best?





Gauss+Enge: Determination of Model Parameters

- According to Lewis theory*,

$$\cos^{-1}\left(1-\left(\frac{\sigma}{2P_z}\right)^2\right) \sim t^{0.555} \text{ (true pathlength).}$$

- We may approximate the true pathlength by assuming the distance between the true pathlength t and stepsize L goes like:

$$t^{0.555} = A * L^{0.555} + B$$

- Then:

$$\cos^{-1}\left(1-\left(\frac{\sigma}{2P_z}\right)^2\right) \sim L^{0.555}$$

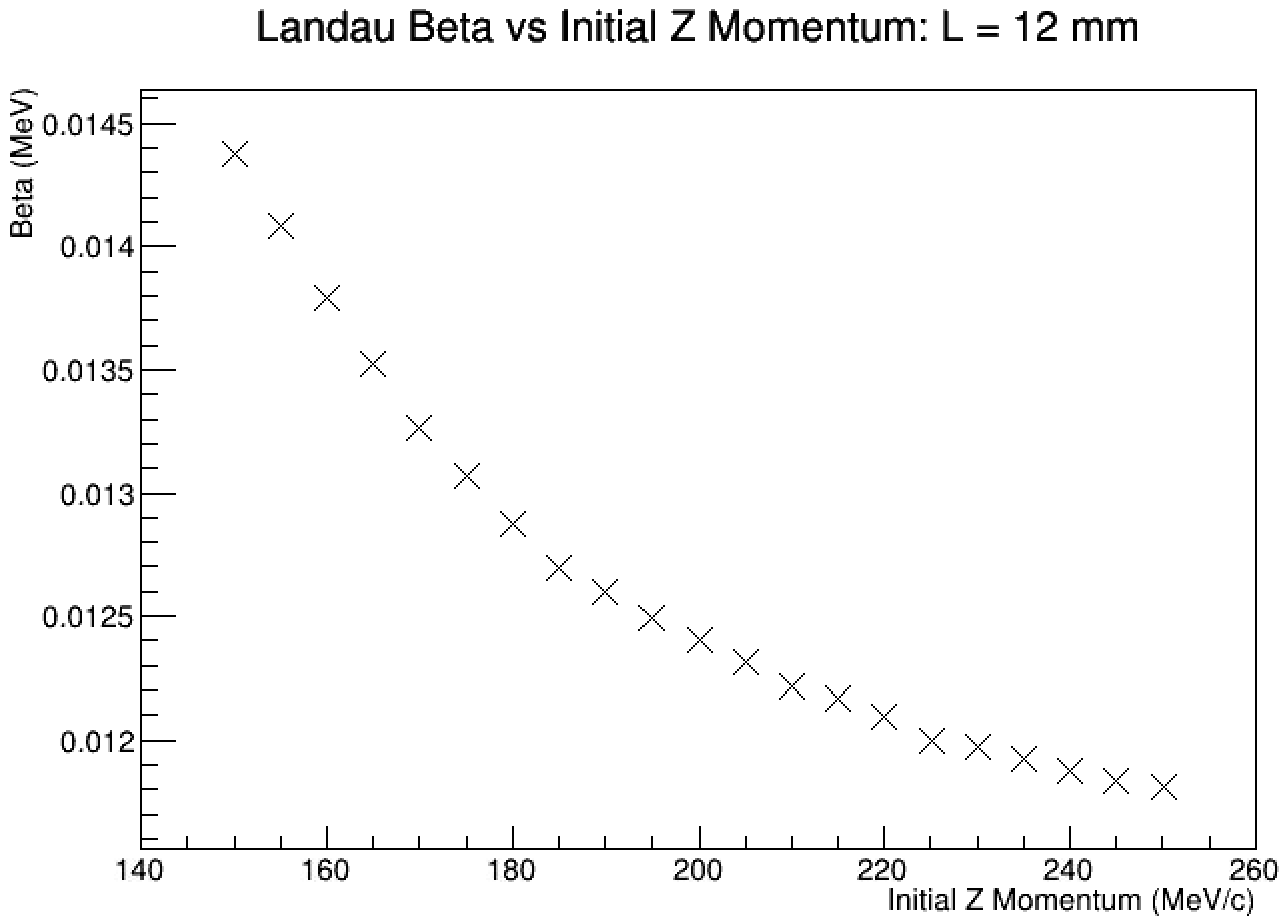
*Implied (not explicit) relations were used.

Landau: Determination of Model Parameters

- In a similar fashion:

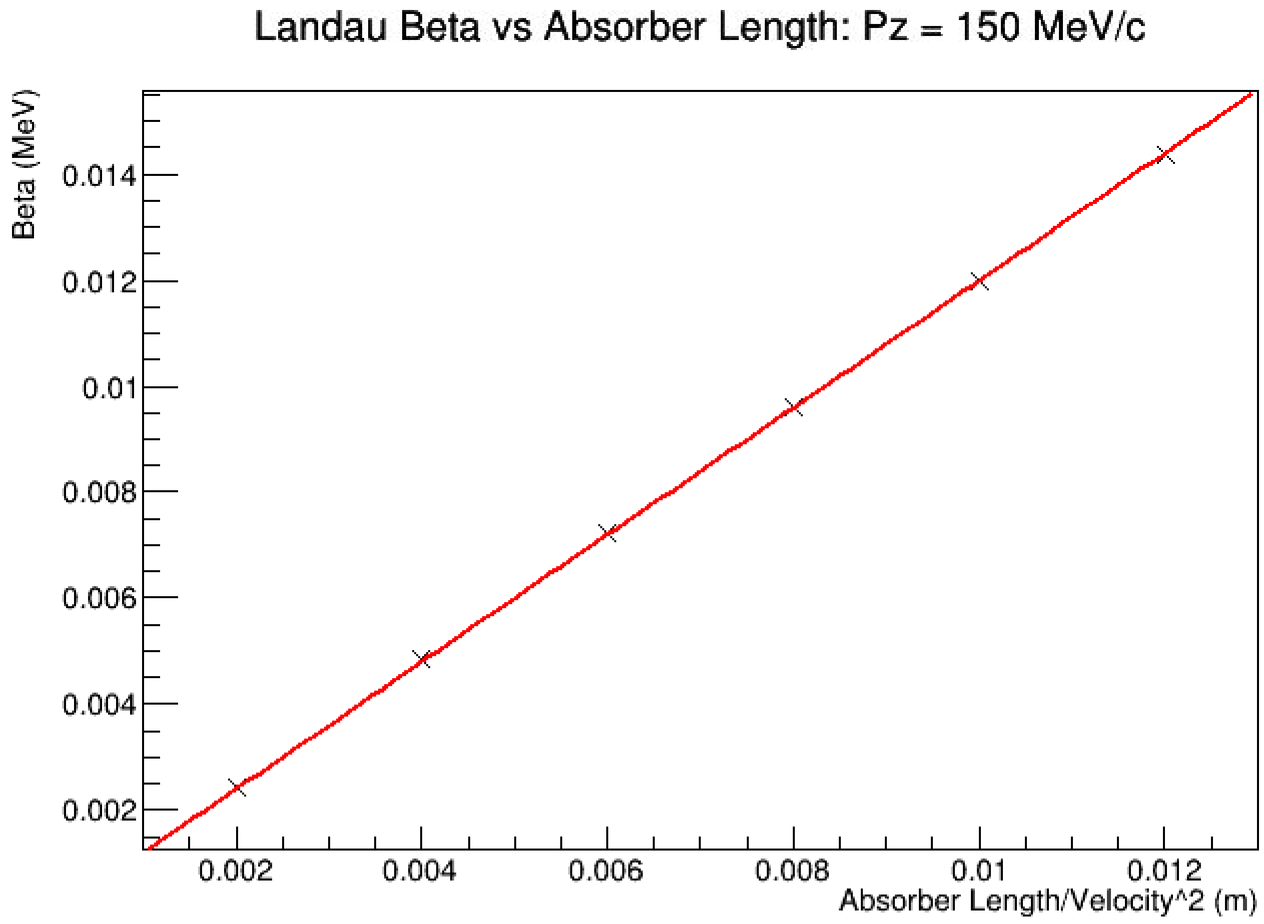
$$\alpha = \bar{\epsilon} + \beta(L, E_i) * \left[2 - \gamma_{euler} - \left(\frac{m}{E_i}\right)^2 + \ln\left(\frac{\beta(L, E_i)}{E_{max}}\right)\right]$$

$$\beta = \xi = coeff * \frac{L}{1 - \left(\frac{m}{E_i}\right)^2},$$



Landau: Determination of Model Parameters

- $\beta = \xi = coeff * \frac{L}{1-\left(\frac{m}{E_i}\right)^2}$
- $\chi^2 = 5.17E-9$ for $P_{z_i} = 150\ MeV/c$; similar for all others
- $coeff$ changes depending on initial momentum (disagreement with theory)



Parameter Coupling: X, PX

- Treatment of X is different than the treatment of PX. From the GEANT4 manual:

In its present form the model samples the path length correction and angular distribution from model functions, while for the lateral displacement only the mean value is used and the correlations are neglected. However, the model is general enough to incorporate other random quantities and correlations in the future.

- The square of the mean value is given by:

$$< x^2 + y^2 > = \frac{4\lambda_1^2}{3}\left[\tau - \frac{\kappa+1}{\kappa} + \frac{\kappa}{\kappa-1}e^{-\tau} - \frac{1}{\kappa(\kappa-1)}e^{-\kappa\tau}\right],$$

where λ is tabulated, κ is a model parameter (2.5), and τ is sampled.