DMK comments

Mostly quite good
clearly written
Some more justification
of the math would help,
and some math details
could be omitted
Some stylistic issues.

HYBRID METHODS FOR SIMULATION OF MUON IONIZATION COOLING CHANNELS

BY

JOSIAH D. KUNZ

Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy in Physics
in the Graduate College of the
Illinois Institute of Technology

Approved ______________________
Advisor

Chicago, Illinois
August 2016

ACKNOWLEDGMENT

Not if you say it

It goes without saying that I am grateful to God, who has guided me in my academic path and has allowed me to fulfil my dream of getting my Ph.D. in Physics. I would also like to thank my wife, Meredith Kunz, who has been by my side even before graduate school. I am very fortunate to have an advisor like Pavel Snopok, who puts in so much time and thought on my behalf. Pavel has worked with me on every turn. I could be no luckier than to have an advisor like him.

I would like to acknowledge the committee for their valuable time and selflessness: Dan Kaplan, Linda Spentzouris, Yagmur Torun, and Xiaofan Li. The Department of Energy is also acknowledged for funding me through Pavel.

On a less serious note, I would like to thank my late cat Scout, who slept on my lap through many of my COSY days. Although he did not make it to see the end result, he will not be forgotten. I would also like to thank my puppy Oliver for his patience with me while writing my thesis, and for keeping my mind fresh with breaks for playtime.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

Symbol	Definition
$\mathcal{A}, \mathcal{B}$	Matrix, general
a, b_r, d_r, p_g, q_g	GEANT4 angular parameters
b	Impact parameter
b_c	COSY offset of scattering tail
C, C_i	Normalization constant; shell correction parameter)
C_{Euler}	Euler's constant (≈ 0.577)
c	Speed of light in a vaccuum
$\langle dE/dx \rangle$	Energy loss per unit length, mean (Bethe Bloch)
E	Energy (general; context dependent)
e	Fundamental charge (such that $z_{ch} = eQ$)
e^-, e^+	Electron or antielectron
F	Distribution function antiderivative (general; context dependent)
f	Distribution function (general; context dependent)
f_i	Oscillator strengths
$g(E)$	Energy loss distribution function
$g(u)$	Angular distribution function
H	Distribution function antiderivative (general; context dependent)
h	Distribution function (general; context dependent); Planck's constant
h_i	Highland corrections ($i = 1, 2$)

$\hbar$	Planck's constant divided by 2π
I	Ionization energy (mean)
I_j	Unit matrix of rank j
K, K_i	Constant (context dependent)
L	Length (of absorber or step size)
ℓ	Time-of-flight in units of length (COSY coordinate)
M	Moments of a function
$\mathcal{M}$	Scattering amplitude; transfer map
m	Mass (general; context dependent)
m_e	Electron mass
m_μ	Muon mass
N	Atomic density
N_A	Avagadro's Number
N_{el}	Electron density
P	Four-momentum
P_k	Legendre polynomials
p	Momentum (total)
(p_x, p_y, p_z)	Beamline momentum coordinate system
Q_i	Charge number of particle i
r_e	Electron radius
s	Arc length coordinate
T	Transverse coordinate

T_{max}	Maximum transferrable kinetic energy
t	Time (variable)
t_z	True path length
U	Spinor
u	Angular cosine variable ($u = \cos\theta$); dummy variable
u_0	Characteristic cosine variable
$\vec{v}$	Unit vector
v	Velocity
w	Weight for a distribution function (context dependent)
X_0	Radiation length
(x, y, z)	Beamline position coordinate system
Z	Atomic charge
z_{ch}	Electric charge
z_g	Geometric path length
α	Dummy variable
β	Relativistic velocity ($\beta = v/c$)
γ	Loretnz factor
γ^α	Dirac gamma matrix
δ	Density correction parameter; Dirac function
ϵ	Energy loss fluctuation ($\epsilon = \Delta E$); emittance
ζ	COSY amplitude of scattering tail

η	Minkowski metric
θ	Scattered angle
θ_0	Angular distribution Gaussian width
κ	Vavilov limit parameter
λ	Landau parameter
λ_k	Transport free mean paths, k^{th} value
λ_v	Vavilov parameter
μ	Mean; muon
$\nu_e, \bar{\nu}_e$	Electron neutrino or electron antineutrino
ν_μ	Muon neutrino
π	Circle constant
ρ	Density
Σ	Cross section
σ	Standard deviation
σ^j	Pauli matrices
ϕ	Laplace transformed function
ψ	Wavefunction, time-independent component
Ω	Solid angle
*	Complex conjugate
†	Transpose conjugate
^{T}	Transpose of a matrix

ABSTRACT

COSY Infinity is an arbitrary-order beam dynamics simulation and analysis code. It can determine high-order transfer maps of combinations of particle optical elements of arbitrary field configurations. For precision modeling, design, and optimization of next-generation muon beam facilities, its features make it a very attractive code. New features are being developed for inclusion in COSY to follow the distribution of charged particles through matter. To study in detail some of the properties of muons passing through material, the transfer map approach alone is not sufficient. The interplay of beam optics and atomic processes must be studied by a hybrid transfer map–Monte Carlo approach in which transfer map methods describe the average behavior of the particles in the accelerator channel including energy loss, and Monte Carlo methods are used to provide small corrections to the predictions of the transfer map accounting for the stochastic nature of scattering and straggling of particles. The advantage of the new approach is that it is very efficient in that the vast majority of the dynamics is represented by fast application of the high-order transfer map of an entire element and accumulated stochastic effects as well as possible particle decay. The gains in speed shown in this work are expected to simplify the optimization of muon cooling channels which are usually very computationally demanding due to the need to repeatedly run large numbers of particles through large numbers of configurations. This work describes the development of the required algorithms and their application to the simulation of muon ionization cooling channels. The code is benchmarked against other codes, validated with experimental results, and predicts results for current muon ionization cooling efforts.

CHAPTER 1

INTRODUCTION

Better to cite original source

1.1 Muon-based Accelerators

Muons (μ) were first discovered experimentally in 1947 by Powell *et al.* [1] who were looking for the Yukawa meson. It is now known that muons fit into a particle group called leptons, and fit into the standard model (along with other fundamental particles) as shown in Figure 1.

-antimuon?

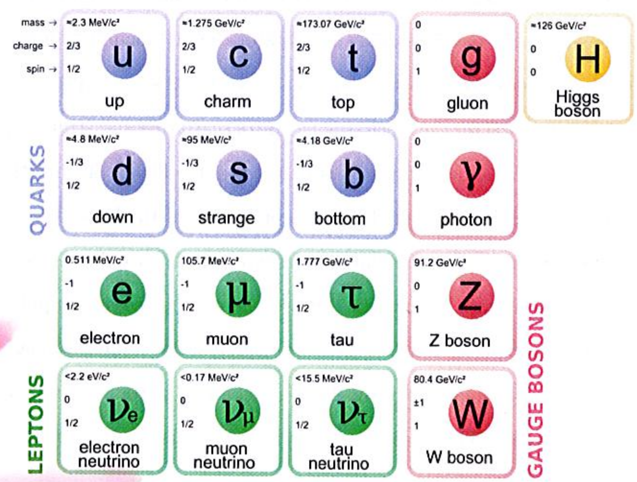

Figure 1.1: The current model of particle physics. Image courtesy of [2].

Silly American conventions period always inside quotes (some ignore)

- positron?

Like

~~Similar to~~ the electron (e), the muon carries a fundamental charge of ± 1, a total spin of $1/2$, and observes the electromagnetic and weak forces. Moreover, the muon also has a corresponding neutrino: the muon neutrino (ν_μ). However, the muon (mass $= 105.7$ MeV$/c^2$) is about 200 times heavier than the electron (mass $= 0.511$ MeV$/c^2$). Indeed, sometimes it is useful to think of a muon simply as a heavy electron, but the mass implies several unique characteristics. One of these is the instability of muons, and results in muons decaying into an electron, an electron antineutrino, and a muon neutrino:

$$\mu \rightarrow e + \bar{\nu}_e + \nu_\mu.$$

This is quite interesting, as it means muons are a double-edged sword. On the one hand, their point-like nature means that ~~the~~ muon collisions are clean. That is to say, muon colliders have a great advantage over, e.g., proton colliders since protons are composed of three quarks. Each quark may have a different flavor or energy level and hence adds more variables to the analysis. Furthermore, each quark is bound and so gluon interactions must also be considered. These quarks and gluons may undergo hadronization when interacting with one another, creating a plethora of possible hadrons. Hadronization is not fully understood, and so these sprays of hadrons are typically lumped together as a single "jet". While there are many working models for jet analysis, none are exact. Conversely, any data from muon interactions will have relatively little noise and will not produce jets.

Clean collisions can also be achieved with linear electron colliders. Yet unlike the electron, the muon does not emit a large amount of synchrotron radiation as it is accelerated. This is because the power irradiated ~~off~~ by a particle due to synchrotron radiation is inversely proportional to the mass of the particle to the fourth power [3]:

$$P \propto 1/m^4.$$

Therefore, the relative power loss due to synchrotron radiation for electrons and muons is $P_e/P_\mu \propto m_\mu^4/m_e^4 \approx 1.8$E9. Since muons lose power via synchrotron radiation at roughly one-billionth the rate of electrons, it is possible to have a circular muon accelerator. Furthermore, due to the small mass of a muon compared to a proton

$\times 10^9$

should use human rather than computer notation

also possible for electrons $\Rightarrow$ need more quantitative discussion

(~105 MeV/c^2 vs. ~938 MeV/c^2), muons are easier to accelerate. This means that a muon facility can be much smaller than its proton counterpart.

wrong reason

However, there is one problem with a muon accelerator. A rest frame lifetime of 2 .2 μs requires the muons to be accelerated quickly before they decay. This is a challenge for circular colliders ~~which~~ that require a high-intensity beam.

Overall, it appears that there are several advantages and one key disadvantage: the 2 μs mean rest frame lifetime of the muon. However, this is only a disadvantage for muon colliders. Another application, which turns the moderately short lifetime into an advantage, is a neutrino factory. This is a facility that is dedicated to the output of a neutrino beam. Muons have two primary advantages over fission reactor neutrino sources. The first is that muons decay into exactly two flavors of neutrino: electron and muon. Therefore, the initial composition of the neutrino beam would be well-defined. This is important since neutrinos can change their flavors over time. Secondly, since neutrinos have no electric charge, they cannot be manipulated via electromagnetic focusing methods. However, the beam of muons can be focused into a high-intensity beam, and the intensity of the neutrino beam will reflect this.

wrong comparison
produce only $\bar{\nu}_e$
(assumed small)

Also true of $\pi^\pm$ ⟹ better to be a bit more quantitative

1.2 COSY Infinity

COSY Infinity is a beamline simulation tool used in the design, analysis, and optimization of particle accelerators [4]. COSY uses the transfer map approach, which evaluates the overall effect of a system on a beam of particles using differential algebra. This involves expanding an ordinary differential equation into multivariate Taylor polynomials up to arbitrary order [5]. Each particle in this work is represented by its coordinates as a phase space vector. The form of phase space vectors used in this work is

of what?

is what?

$$\mathbf{Z} = \begin{pmatrix} x \\ y \\ l = k(t - t_0) \\ a = p_x/p_0 \\ b = p_y/p_0 \\ \delta = (E - E_0)/E_0 \end{pmatrix}, \tag{1.1}$$

where the coordinates are transverse positions (x, y), time-of-flight in units of length (l), transverse angles w.r.t. the reference particle (a, b), and kinetic energy deviations w.r.t. the reference particle (δ). The 0 subscript in the definitions denotes the reference particle properties.

In beam physics, phase space vectors $\mathbf{Z}$ are subject to physics processes. This subjugation can usually be represented by a differential equation. For example, a particle in an electric field is subject to the force law [3]

?

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

or in terms of the momentum,

$$\frac{d}{dt}\mathbf{p} = Q(\mathbf{E} + \frac{1}{m}\mathbf{p} \times \mathbf{B}).$$

non-relativistic?

Fortunately, any phase space vector $\mathbf{Z}$ in an arbitrary order ordinary differential equation (ODE) can be rewritten as a first-order ODE [5]. For an order n ODE, a first-order ODE is constructed by introducing $n - 1$ new variables. This is to say that

$$\frac{d^n}{dt^n}\mathbf{Z} = \mathbf{f}(\frac{d^0}{dt^0}\mathbf{Z}, ..., \frac{d^{n-1}}{dt^{n-1}}\mathbf{Z})$$

No, it's not the vector that's rewritten

use \left, \right

can be rewritten as

$$\frac{d}{dt}\begin{pmatrix} \mathbf{Z} \\ \mathbf{Z}_1 \\ \vdots \\ \mathbf{Z}_{n-1} \end{pmatrix} = \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \vdots \\ \mathbf{f}(\mathbf{Z}, ..., \mathbf{Z}_{n-1}) \end{pmatrix}.$$

Here, $\mathbf{f}$ represents the physics processes. For the Maxwell's equations example, the equation is already first order in a and b, with the momentum component of $\mathbf{f}$ as

$$\mathbf{f}(\mathbf{p}) = Q(\mathbf{E} + \frac{1}{m}\mathbf{p} \times \mathbf{B}) = Q(\mathbf{E} + \frac{1}{m}\left[ap_0\hat{x} + bp_0\hat{y} + p_z\hat{z}\right] \times \mathbf{B}).$$

Furthermore, COSY does not use time as the independent variable, but rather arc length s (see Figure 1.2).

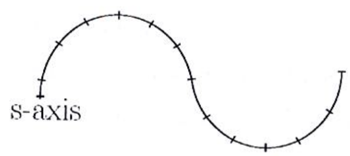

Figure 1.2: The reference orbit. Figure courtesy of [6].

If there exists a unique evolution of $\mathbf{Z}$ then it is possible to construct the so-called transfer map $\mathcal{M}$. Mathematically, this relationship is $\mathbf{Z}(s) = \mathcal{M}(s_0, s) * \mathbf{Z}(s_0)$, with $*$ representing the application of the transfer map to the phase space vector $\mathbf{Z}$ at s_0.

It is possible to construct a transfer map for most cases in beamline physics. This is because most beamline elements follow differential equations which yield unique solutions dependent on initial conditions (such as Maxwell's equations). If there does not exist a unique evolution of $\mathbf{Z}$ then it is not possible to construct the transfer map. Systems which produce a unique evolution of $\mathbf{Z}$ are called "deterministic".

An example of the relationship between the initial phase space vector, the transfer map, and the final phase space vector can be seen in Figure 1.3. The initial phase space occupied by the beam of particles is at the coordinate s_0. Physically, there exists some deterministic beamline element between s_0 and s_1. This element can be represented by the map $\mathcal{M}$, which creates a bijection for the phase space vectors $\mathbf{Z}(s_0)$ and $\mathbf{Z}(s_1)$ between the initial coordinate s_0 and the final coordinate s_1.

Entire lattices may also be represented by a single transfer map. This is done by dividing the lattice into its base components or elements. A transfer map for each corresponding element may then be produced. For two example elements between coordinates s_0 and s_2 (see Figure 1.4), the composition of two maps yields another map: $\mathcal{M}(s_1, s_2) \times \mathcal{M}(s_0, s_1) = \mathcal{M}(s_0, s_2)$. Therefore, it is possible to simplify the middle part s_1. In this way, the transfer maps from small components build up into a single transfer map for the whole system. Computationally this is advantageous because once calculated, it is much faster to apply a single transfer map to a distribution of particles than to simulate that same distribution through many meters of individual lattice elements.

Along with the tracking of particles through a lattice, COSY also has a plethora of analysis and optimization tools, including (but not limited to) lattice aberration and correction tools, support for Twiss parameters, support for tunes and nonlinear tune shifts, built-in optimizers for lattice design, and spin tracking.

Valid elements are any beamline elements that are deterministic. Elements used in this study are magnetic multipoles (dipoles, quadrupoles, etc.), solenoidal coils, radiofrequency (RF) cavities, and drifts. Currently supported elements in COSY