Hybrid Methods for Simulation of Muon Ionization Cooling Channels

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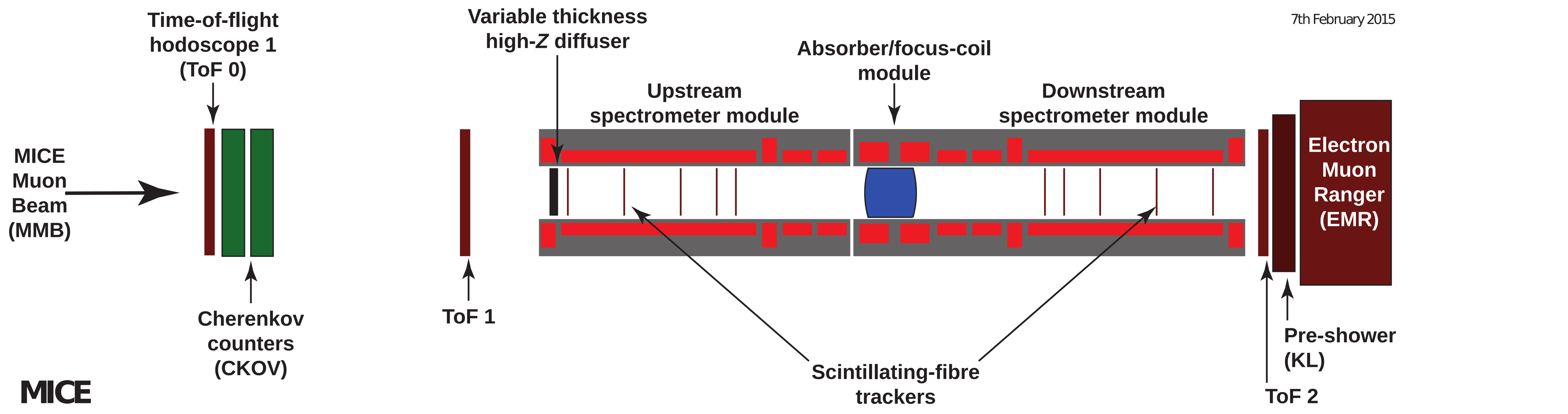
Muon Accelerator Program

Muon Ionization Cooling Experiment (MICE) Layout



Stochastic Processes

The stochastic processes of interest are straggling (fluctuation about a mean energy loss) and multiple angular scattering [?]. Straggling follows Landau theory and has the form [?]

$$f(\lambda) = \frac{1}{\xi} \cdot \frac{1}{2\pi i} \int_{c+i\infty}^{c-i\infty} \exp(x \ln x + \lambda x) dx, \quad (1)$$

where $\xi \propto Z\rho L/\beta^2 A$, and $\lambda \propto dE/\xi - \beta^2 - \ln \xi$. Here Z, A, and ρ are the atomic charge, atomic mass, and density of the material; L is the amount of material that the particle traverses; $\beta = v/c$; and dE is the fluctuation about the mean energy. The algorithm based on Eq. (1) has been implemented in COSY.

The derivation of the scattering function $g(u)$ (where $u = \cos\theta$) is done separately for small angles and large angles. For small angles, the shape is very nearly Gaussian in θ [5]. For large angles, the distribution follows the Mott scattering cross section and is Rutherford-like [?]. The resulting peak and tail are continuous and smooth at some critical u_0, which yields the final form of $g(u)$:

$$g(u) = \begin{cases} \exp\left(-\frac{1}{2}\frac{1-u}{1-u_\sigma}\right) & | \, u_0 < u \\ \zeta \cdot \frac{1+\frac{1}{2}(\beta\gamma)^2(1+u-b)}{(1-u+b)^2} & | \, u \leq u_0 \end{cases} \quad (2)$$

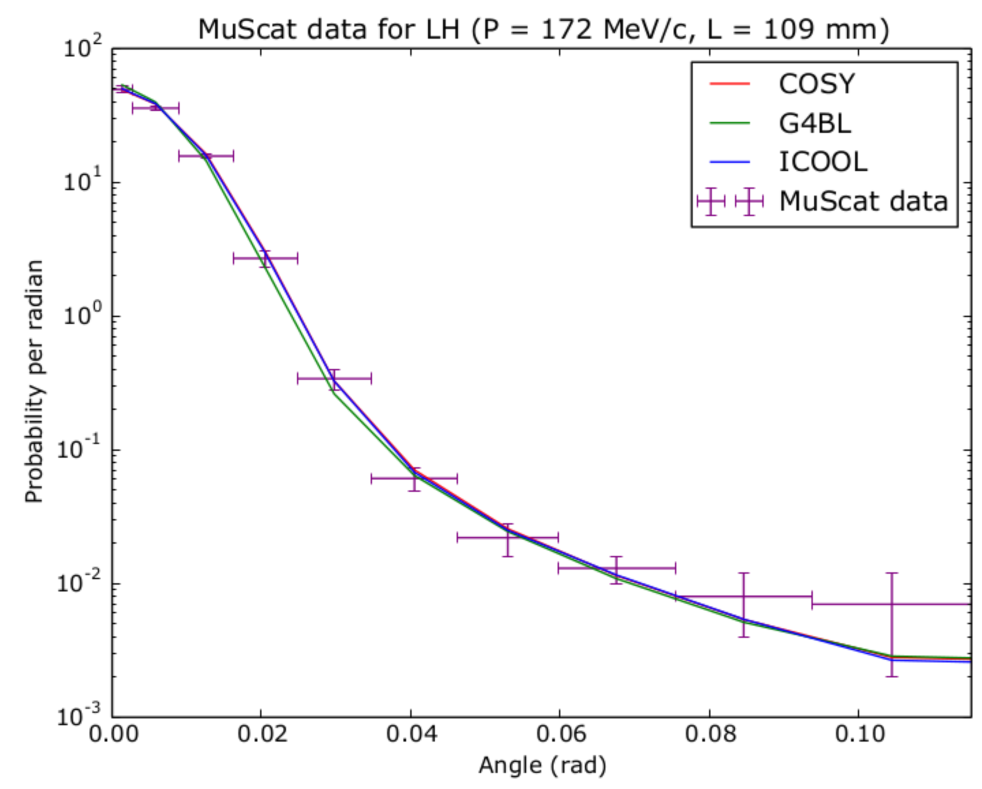
Here the parameters ζ and b are chosen to ensure continuity and smoothness, $\gamma = 1/\sqrt{1-\beta^2}$, u_0 is a fitted parameter, and u_σ is the σ-like term for a Gaussian in θ. It is another fitted parameter based on [6] and taking the form

$$u_\sigma = \cos\left(\frac{13.6\text{ MeV}}{\beta pc}\left(\frac{L}{L_0}\left(1+0.103\ln\frac{L}{L_0}\right) + 0.0038\left(\ln\frac{L}{L_0}\right)^2\right)^{\frac{1}{2}}\right).$$

Absorbers

Recently, COSY Infinity [1] has been outfitted with new simulations tools for matter-dominated lattices [3], with the application of cooling absorbers as the motivation. Some of these results are reproduced here.

Excellent agreement has been achieved between COSY, G4Beamline [7], and ICOOL [4] for pencil beams of $p = (100, 200, 300, 400)$ MeV/c through liquid hydrogen absorbers of lengths $L = (1, 10, 100)$ mm. Also, agreement has been shown with the MuScat results [2], with one example shown below.



The MICE cell

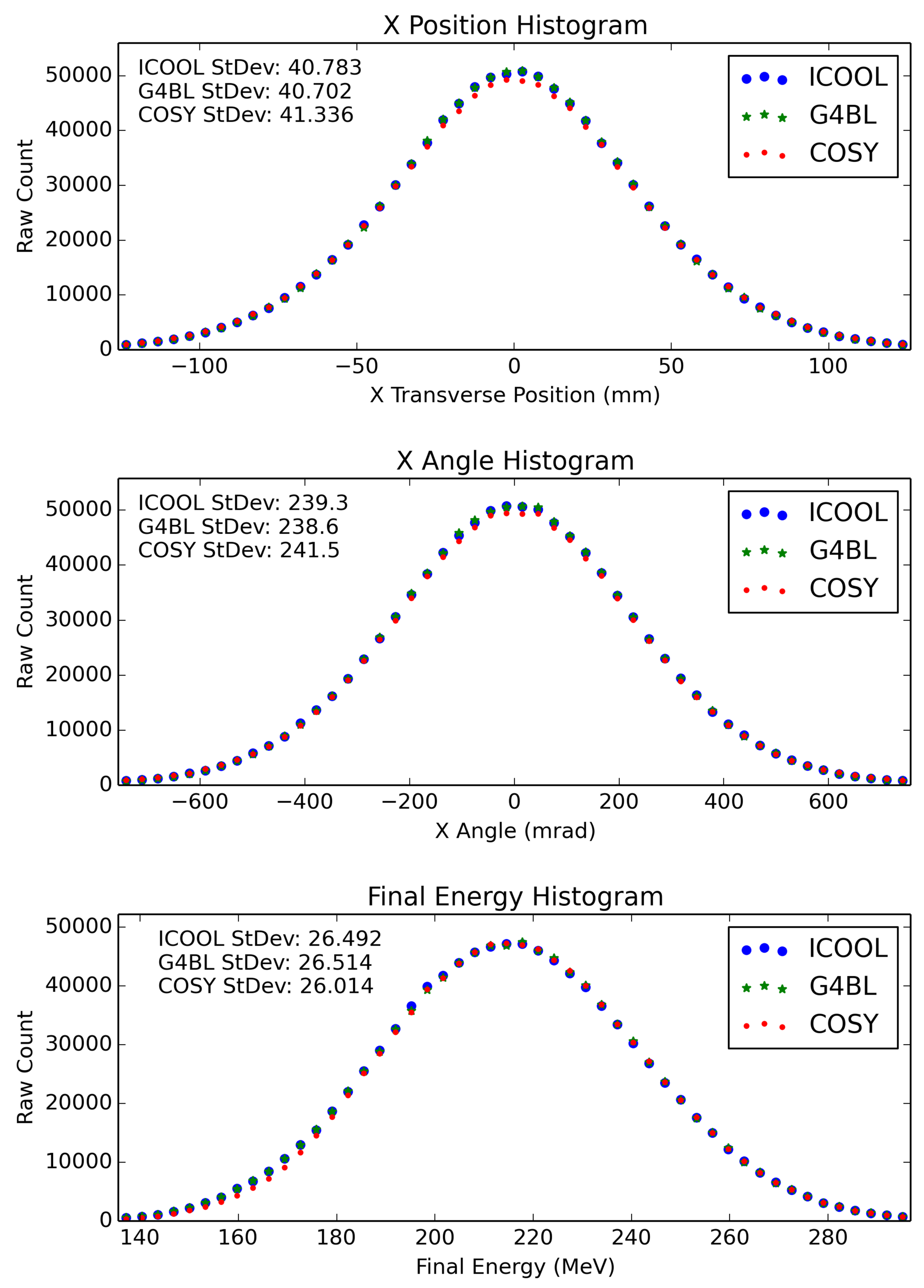
MICE was simulated in-parts by COSY and G4Beamline. The initial distribution can be seen in the table below, along with the results of these separate simualtions. The absorber was a cylindrical lithium hydride block of 65 mm.

The coils (as seen in the MICE layout figure) were simulated to and from the scintillating-fibre trackers.

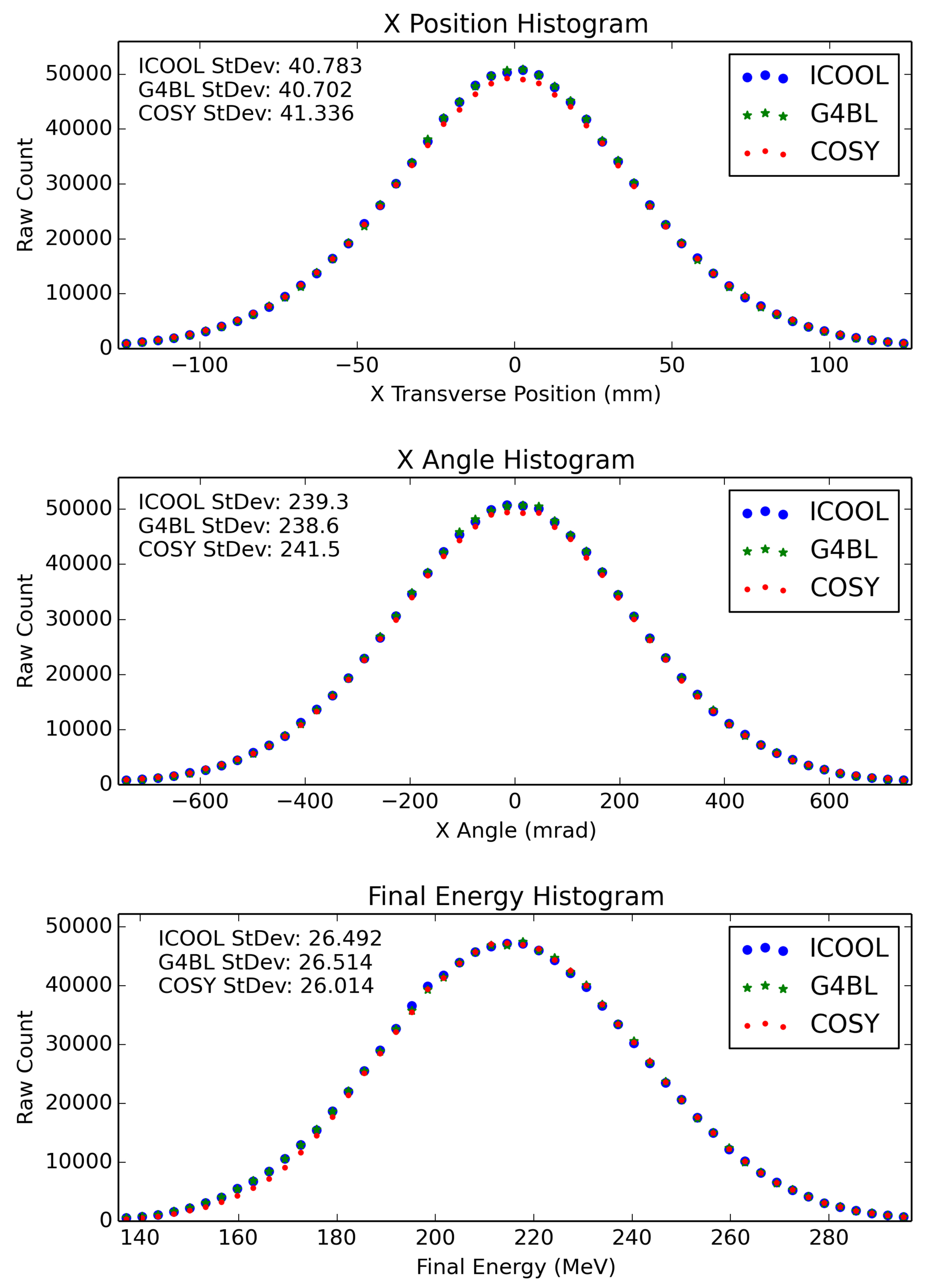
Coil parameters are summarized in the table below.

MICE Results

The results of the MICE simulation for 350 mm of liquid hydrogen are shown below.



A simulation was also ran with 65 mm of lithium hydride. These results are shown below.



Computational times for liquid hydrogen can be seen in the following table. It can be concluded that the new hybrid approach in COSY is about twice as fast as G4Beamline and about three times as fast as ICOOL.

Number of particles:	10^6	10^5	10^4	10^3
COSY:	367	31	6	4
G4BL (coils):	3973	392	40	6
G4BL (field map):	662	75	15	9
ICOOL (field map):	1091	117	19	9

References

[1] M. Berz and K. Makino. *COSY Infinity Beam Physics Manual*, 2013. Version 9.1.

[2] D. Attwood *et al.* The scattering of muons in low z materials. *NIM*, 251, 2006.

[3] J. Kunz *et al.* The advancement of cooling absorbers in cosy infinity. In *Proceedings of the 6th International Particle Accelerator Conference*, 2015.

[4] R.C. Fernow et. al. ICOOL.