Hybrid Methods for Simulation of Muon Ionization Cooling Channels

Josiah D. Kunz

Thesis Defense

Monday, August 22nd, 2016

Muon Accelerator Program

ILLINOIS INSTITUTE OF TECHNOLOGY

Outline

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- Motivation
  - Fundamental particle physics
  - The Higgs Boson
  - Neutrinos
  - Muon facilities

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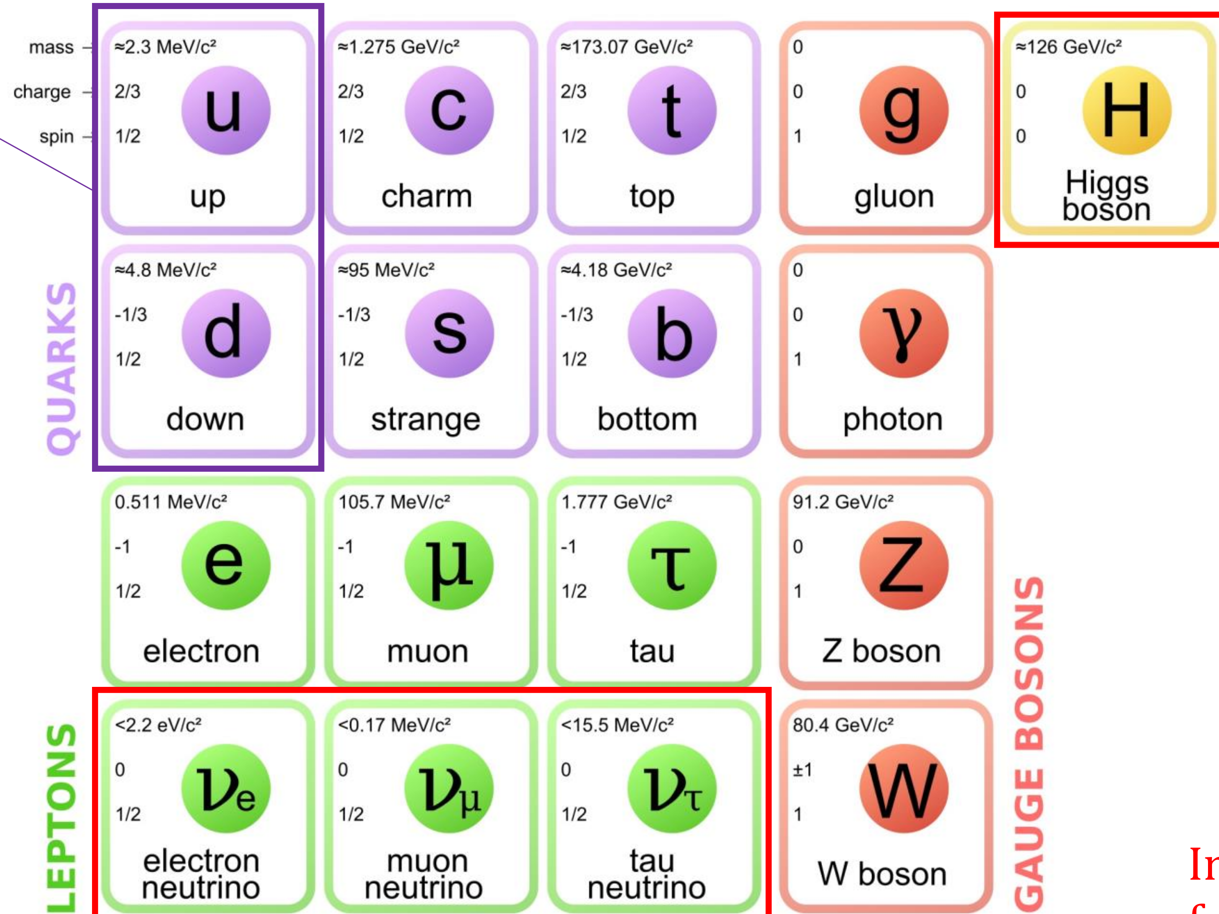
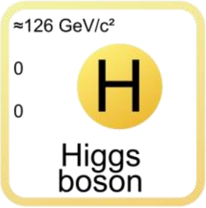
Motivation: Fundamental Particle Physics

Protons, neutrons

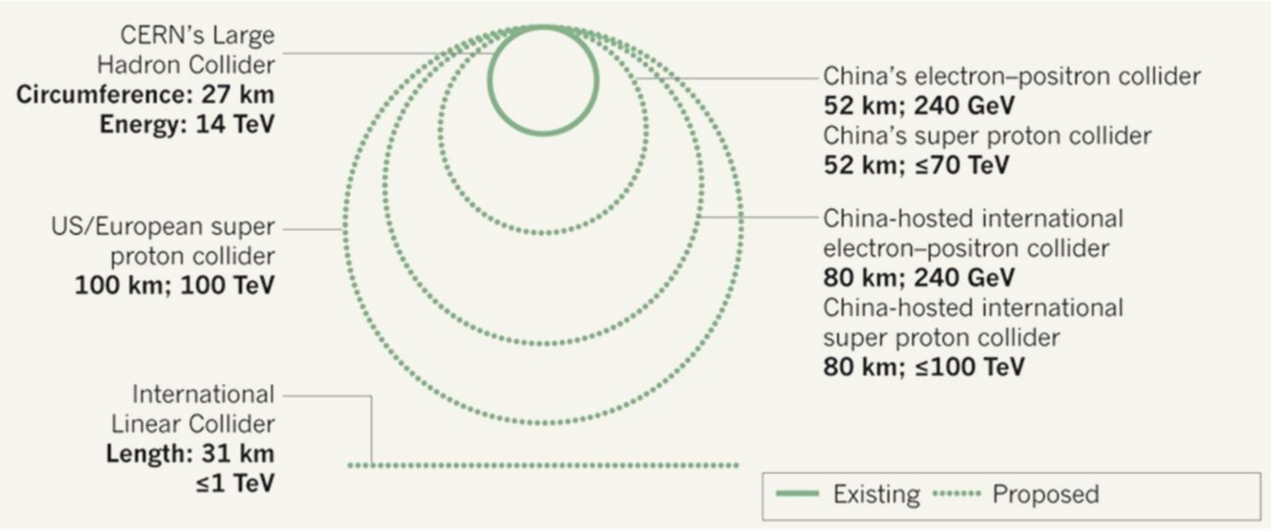


Interesting fundamental particles!

Motivation: Higgs Challenges

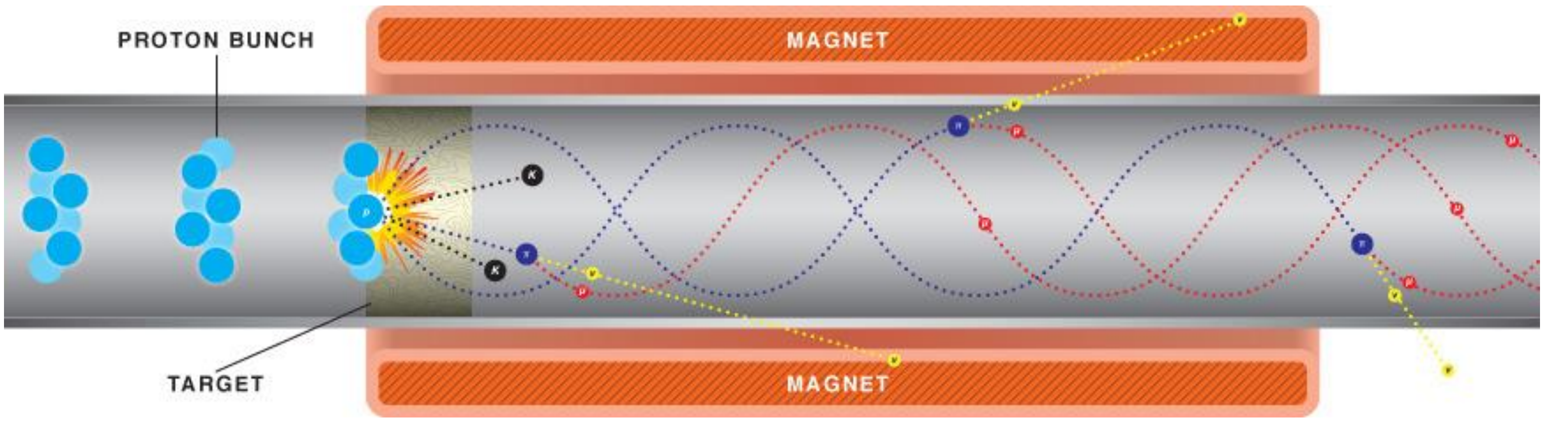


- Want to study the Higgs boson
- Discovered via protons
- Requires large facility
  - Large Hadron Collider (LHC) is 8.4 km in diameter
  - Proposed accelerators only get bigger

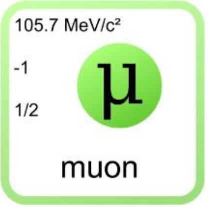


Motivation: Neutrino Challenges

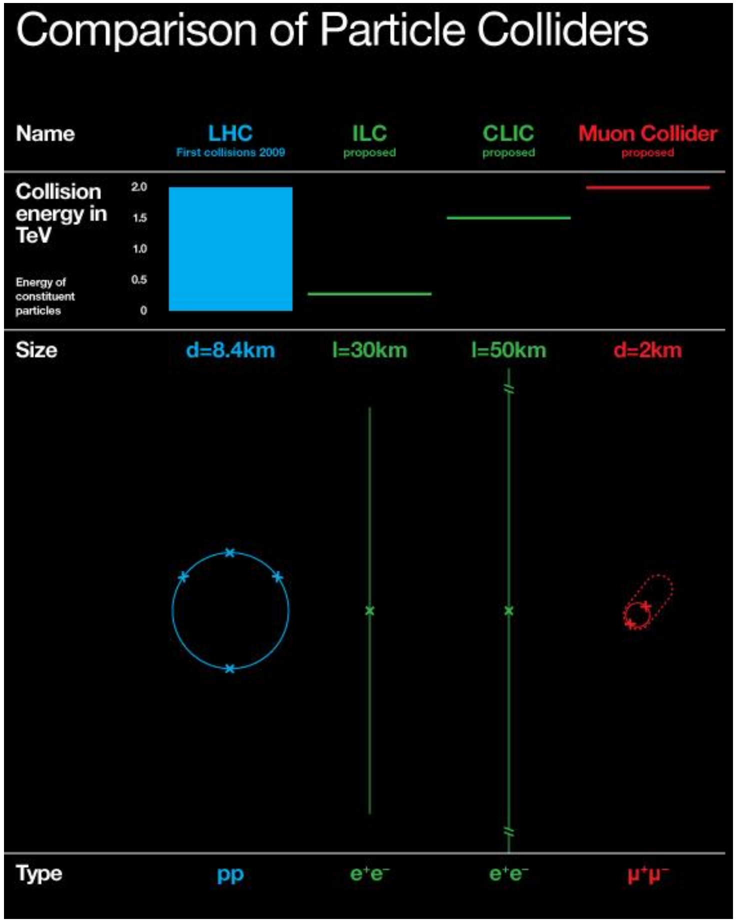
- Neutrinos can mix flavors
  - Need long experiment to allow flavor mixing
- Neutrinos are neutral
  - Cannot focus
- Neutrinos produced via decay



Motivation: Muon Facilities



- Recap:
  - Higgs facilities are too large and costly
  - Neutrino facilities have potential for improvement
- Muon facilities solve both problems
  - Muons are fundamental (unlike protons), so small facility is possible
  - Muons decay to produce neutrinos, but can be focused first



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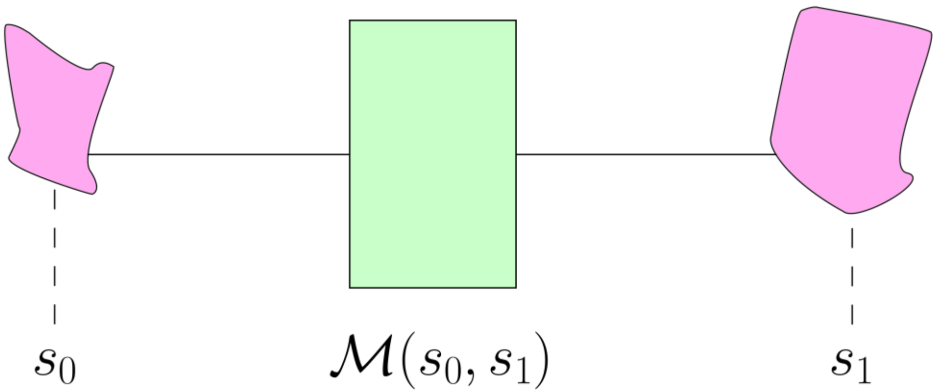
Introduction: Transfer Map Methods

- Muon facility design uses simulations first
- COSY Infinity is a simulation tool
- Excels in the design and optimization of accelerator channels
- COSY Infinity uses the so-called "transfer map methods"
- A transfer map is a relation that allows propagation of particles
- A transfer map $\mathcal{M}$ relates initial coordinates to final coordinates
- Possible to construct $\mathcal{M}$ due to <u>uniqueness</u> of trajectories



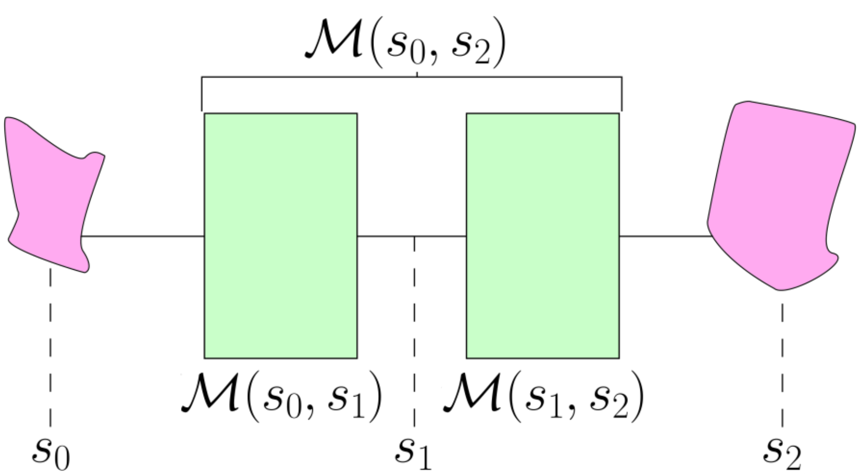
Introduction: Transfer Map Methods

- Transfer map $\mathcal{M}$ based on a beamline element
- Given the phase space vector $\vec{Z}$, map $\mathcal{M}$ relates initial coordinates to final coordinates
- Moreover, the composition of two maps yields another map

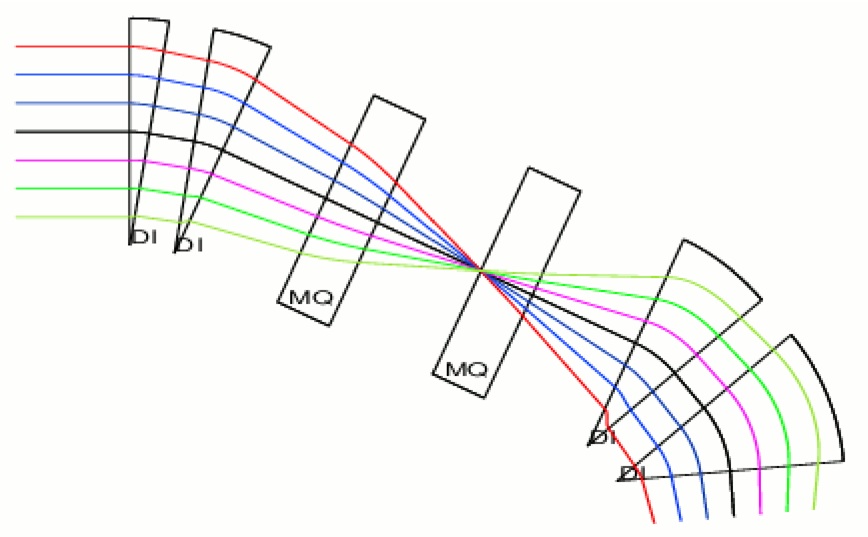
$$\vec{Z} = \begin{pmatrix} x \\ y \\ l = k\left(t - t_0\right) \\ a = p_x / p_0 \\ b = p_y / p_0 \\ \delta = \left(E - E_0\right) / E_0 \end{pmatrix}$$

$$\vec{Z}\left(s\right) = \mathcal{M}\left(s_0, s\right)\left(\vec{Z}\left(s_0\right)\right)$$

$$\mathcal{M}\left(s_1, s_2\right) \circ \mathcal{M}\left(s_0, s_1\right) = \mathcal{M}\left(s_0, s_2\right)$$



Introduction: Transfer Map Example



Introduction: Transfer Map Example

x_i a_i y_i b_i

x_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4999999999930050       1    1 0  0 0
2   3.068671838461174       1    0 1  0 0
```

a_f

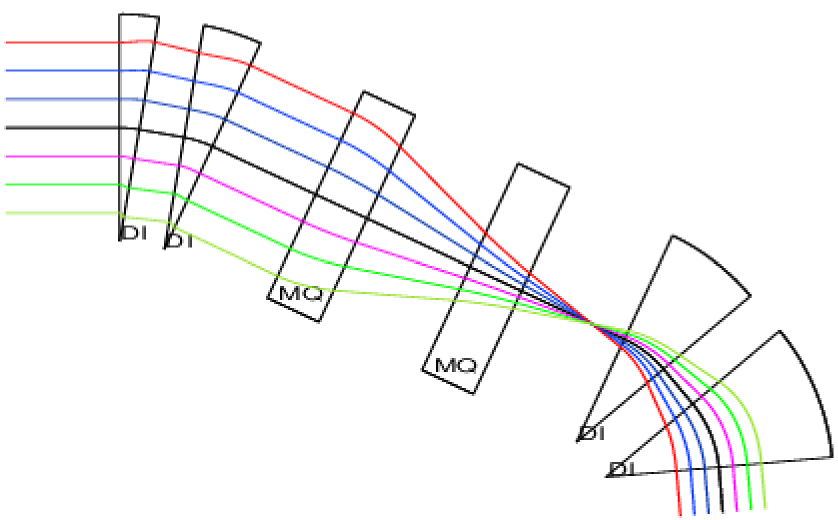
```
I  COEFFICIENT            ORDER EXPONENTS
1  -2.000000000564872       1    0 1  0 0
```

y_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4100788419716958       1    0 0  1 0
2  -.6763170711579435       1    0 0  0 1
```

b_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  0.3826251428718294E-01   1    0 0  1 0
2  -2.375451519806499       1    0 0  0 1
```



Introduction: Transfer Map Example

x_i a_i y_i b_i

x_f, a_f, y_f, b_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4999999999930050       1    1 0  0 0
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1  0.3826251428718294E-01   1    0 0  1 0
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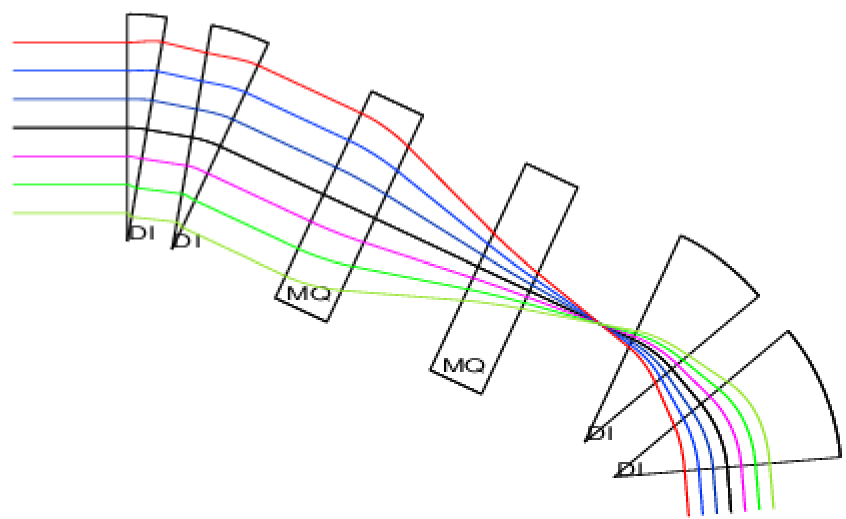
------------------------------------------
```

First box: x_f =



Introduction: Transfer Map Example

$x_i \; a_i \; y_i \; b_i$

x_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4999999999930050       1    1 0  0 0
2   3.068671838461174       1    0 1  0 0
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```

a_f

```
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1  -2.000000000564872       1    0 1  0 0
-----------------------------------------
```

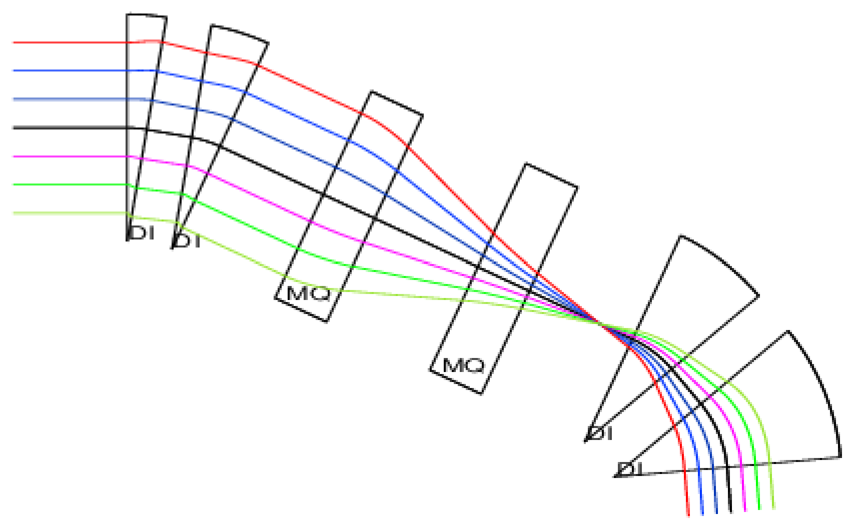
y_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4100788419716958       1    0 0  1 0
2  -.6763170711579435       1    0 0  0 1
-----------------------------------------
```

b_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  0.3826251428718294E-01   1    0 0  1 0
2  -2.375451519806499       1    0 0  0 1
-----------------------------------------
```

First box: $x_f = -0.5x_i$



Introduction: Transfer Map Example

$x_i \; a_i \; y_i \; b_i$

x_f

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2   3.068671838461174        1    0 1  0 0
------------------------------------------
```

a_f

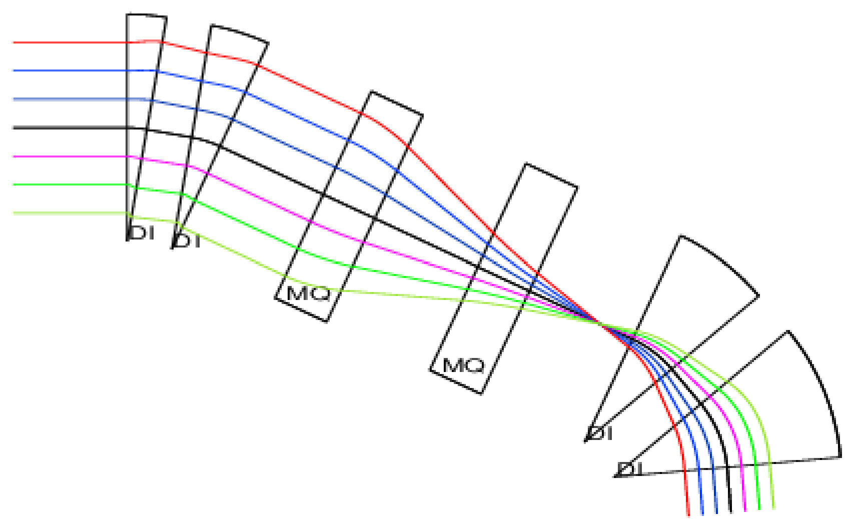
```
I  COEFFICIENT             ORDER EXPONENTS
1  -2.000000000564872        1    0 1  0 0
------------------------------------------
```

y_f

```
I  COEFFICIENT             ORDER EXPONENTS
1  -.4100788419716958        1    0 0  1 0
2  -.6763170711579435        1    0 0  0 1
------------------------------------------
```

b_f

```
I  COEFFICIENT             ORDER EXPONENTS
1  0.3826251428718294E-01    1    0 0  1 0
2  -2.375451519806499        1    0 0  0 1
------------------------------------------
```



First box: $x_f = -0.5x_i + 3.07a_i$

Introduction: Transfer Map Example

$x_i\ a_i\ y_i\ b_i$

x_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4999999999930050       1    1 0  0 0
2   3.068671838461174       1    0 1  0 0
-----------------------------------------
```

a_f

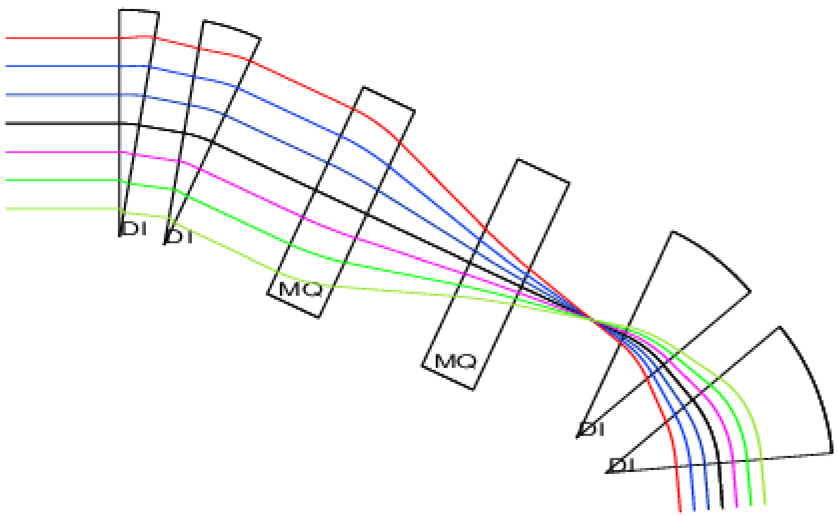
```
I  COEFFICIENT            ORDER EXPONENTS
1  -2.000000000564872       1    0 1  0 0
-----------------------------------------
```

y_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  -.4100788419716958       1    0 0  1 0
2  -.6763170711579435       1    0 0  0 1
-----------------------------------------
```

b_f

```
I  COEFFICIENT            ORDER EXPONENTS
1  0.3826251428718294E-01   1    0 0  1 0
2  -2.375451519806499       1    0 0  0 1
-----------------------------------------
```



First box: $x_f = -0.5x_i + \cancelto{0}{3.07}a_i$

Introduction

- Want to use COSY transfer map approach to study muon facilities
  - Fast!
  - Particles can be simulated through many meters via single map
  - Matrix multiplication instead of particle-by-particle propagation

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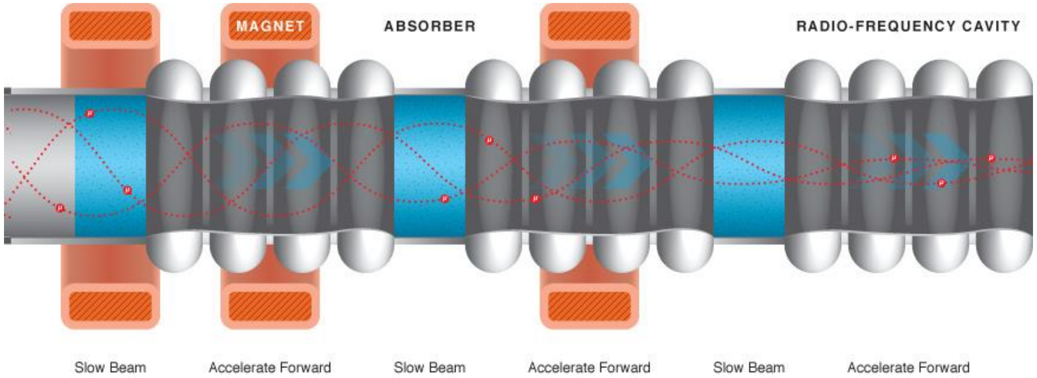
Stochastic Processes: Ionization Cooling

- Recall: muons decay
- Best method of collecting, focusing, and accelerating muons is "ionization cooling"
- "Absorber" – low-*Z* material (e.g. liquid hydrogen)
- Beam ionizes the material; beam loses momentum
- Radio frequency cavity restores longitudinal momentum
- Result: reduced transverse momentum

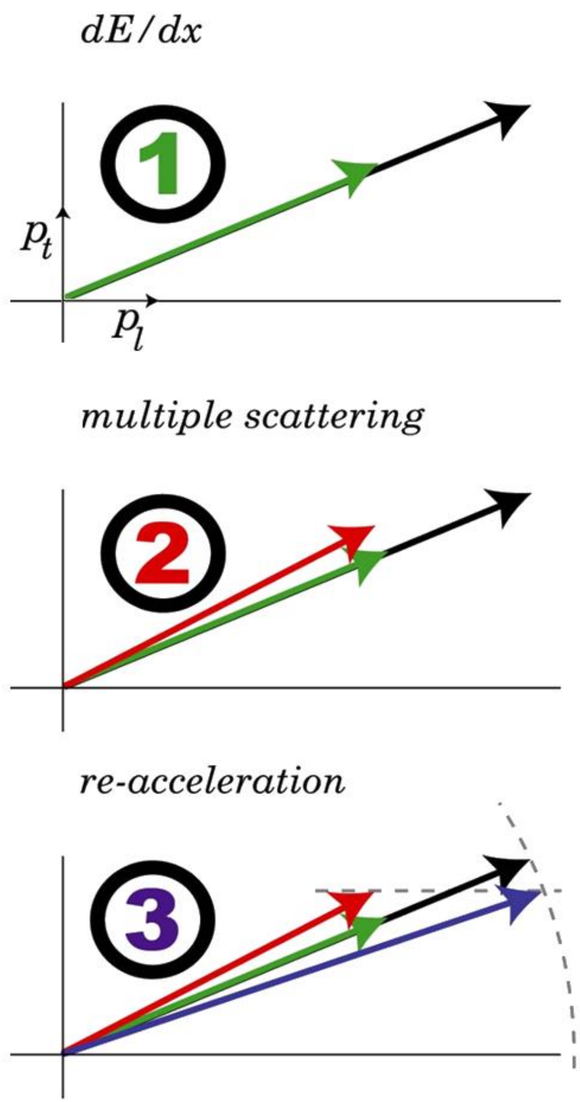


Stochastic Processes: Ionization Cooling

1. **dE/dx**
   Beam loses momentum, both transverse and longitudinal.

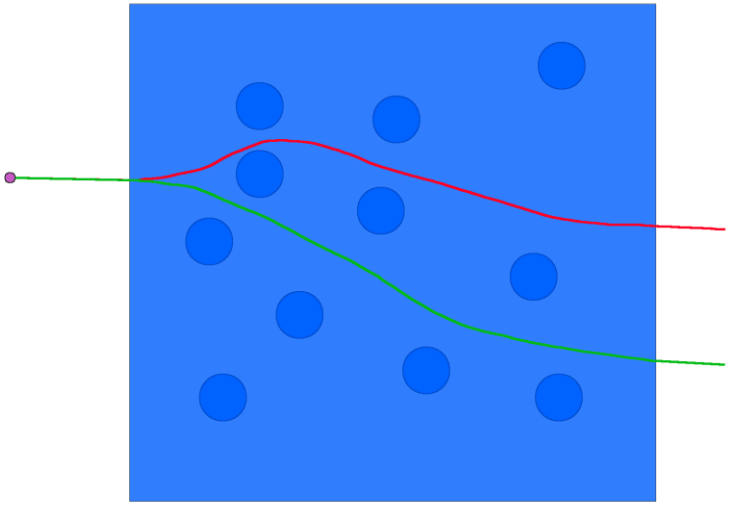
2. **Multiple scattering**
   Transverse momentum grows due to scattering effects.

3. **Re-acceleration**
   Longitudinal momentum increased via RF cavity. Total momentum remains the same. Transverse momentum reduced.



Stochastic Processes: Why not use maps?

- It is only possible to construct a transfer map if the particle has a unique evolution
- This is not true due to energy straggling, multiple scattering
  - Quantum effects → intrinsically random
- Stochastic processes do not fit into the transfer map paradigm

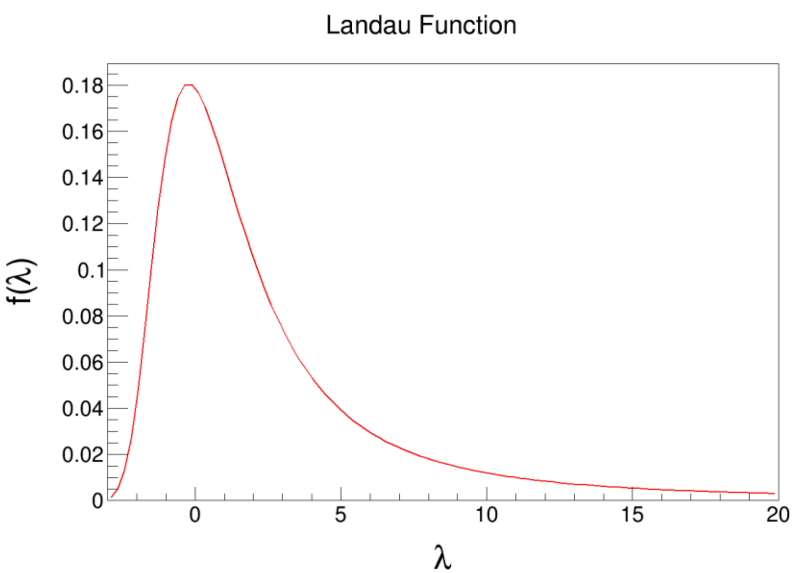


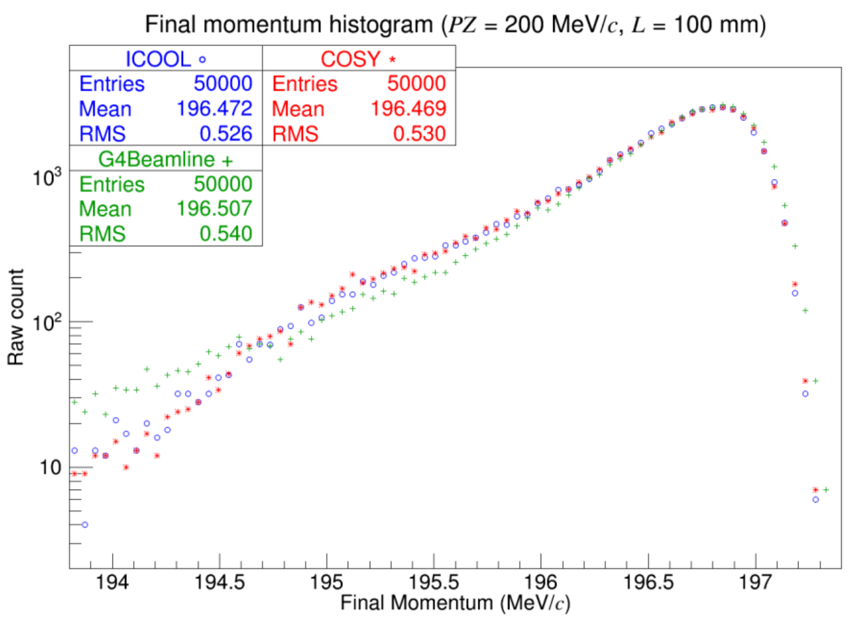
Goal

- Outfit COSY Infinity with tools to handle stochastic processes
  - Energy straggling
  - Multiple scattering
- Should be particle-by-particle propagation, not transfer maps
  - Still use transfer maps where possible
- Validate against experimental data
- Benchmark against other codes
- Apply to simulations of cooling channels

Stochastic Processes: Energy Straggling

- Energy straggling: random fluctuations about a mean energy loss
- Landau theory describes this well

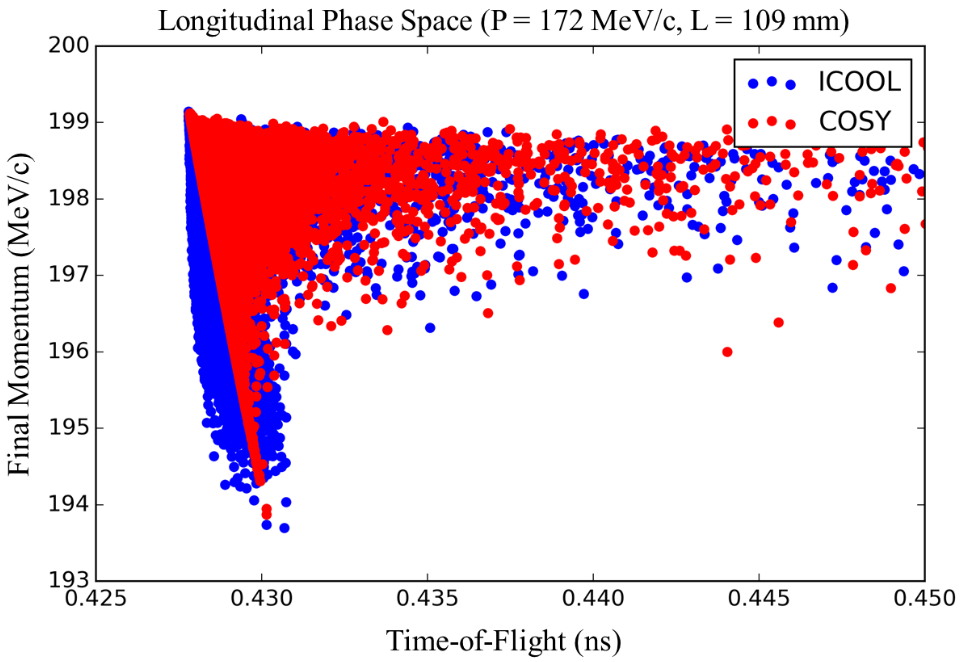




Stochastic Processes: Energy Straggling

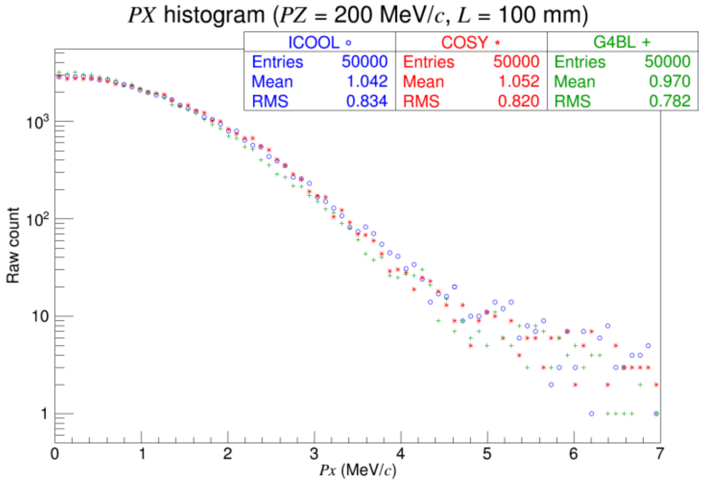
- Time-of-Flight depends on energy loss through the absorber
- Assuming a constant deceleration,

- $\Delta t = \frac{v_f - v_o}{v_f^2 - v_o^2} \cdot 2L$
    - $v = \frac{pc^2}{E}$



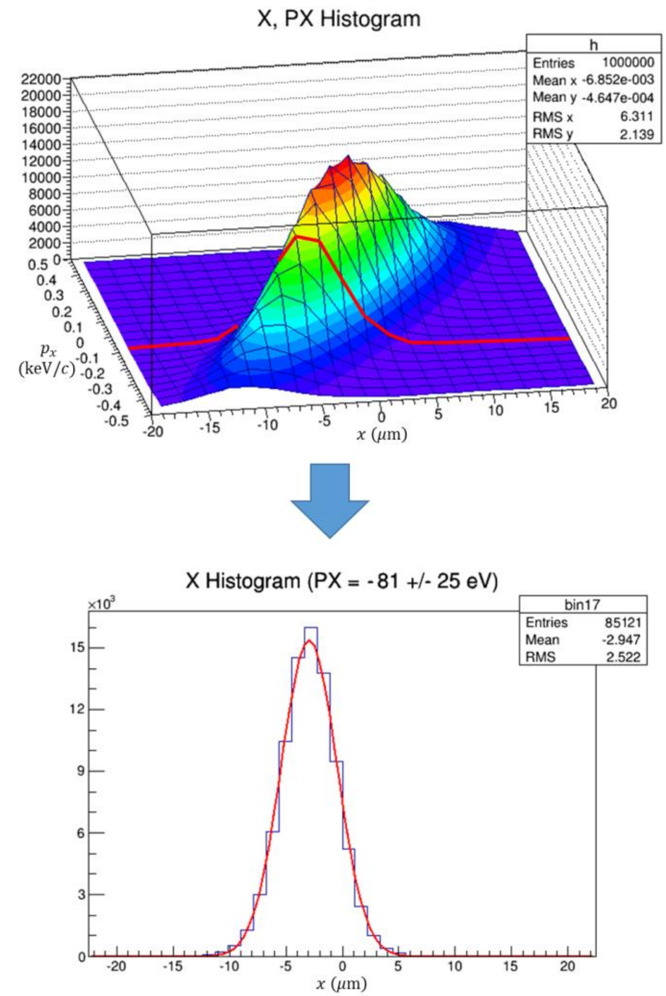
Stochastic Processes: Multiple Scattering

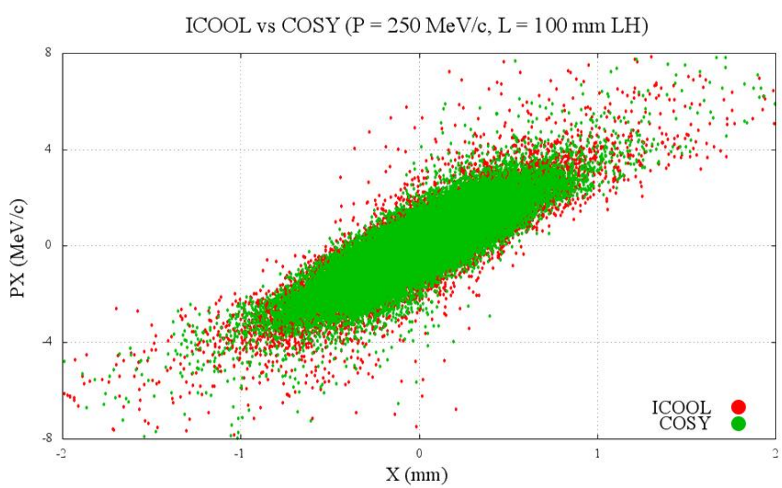
- Piecewise function
  - Gaussian-like peak
  - Mott tail
  - $u = \cos\theta$
- $g(u) = \begin{cases} e^{-a(1-u)} & \theta \text{ close to 0 (peak)} \\ \dfrac{1+\frac{(\beta\gamma)^2}{2}(1+u)}{(1-u)^2} & \text{otherwise (tail)} \end{cases}$
- For $\theta \to 0$,
  - $1 - u = 1 - \cos\theta \approx 1 - \left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}$
  - $e^{-a(1-u)} \to e^{-\theta^2/2a}$
- For $\beta \to 0$, the tail tends to a Rutherford distribution:
  - $\dfrac{1+\frac{(\beta\gamma)^2}{2}(1+\cos\theta)}{(1-\cos\theta)^2} \to \dfrac{1}{(1-\cos\theta)^2}$



Stochastic Processes: Multiple Scattering

- Stochastic transverse displacement Δx_{sto} is related to multiple scattering:
  - $\Delta x_{sto} = \text{Gaus}\left(\frac{(\theta_{final} - \theta_o)L}{2}, \frac{\theta_c}{2\sqrt{3}}\right)$
- Determined by comparison with other codes





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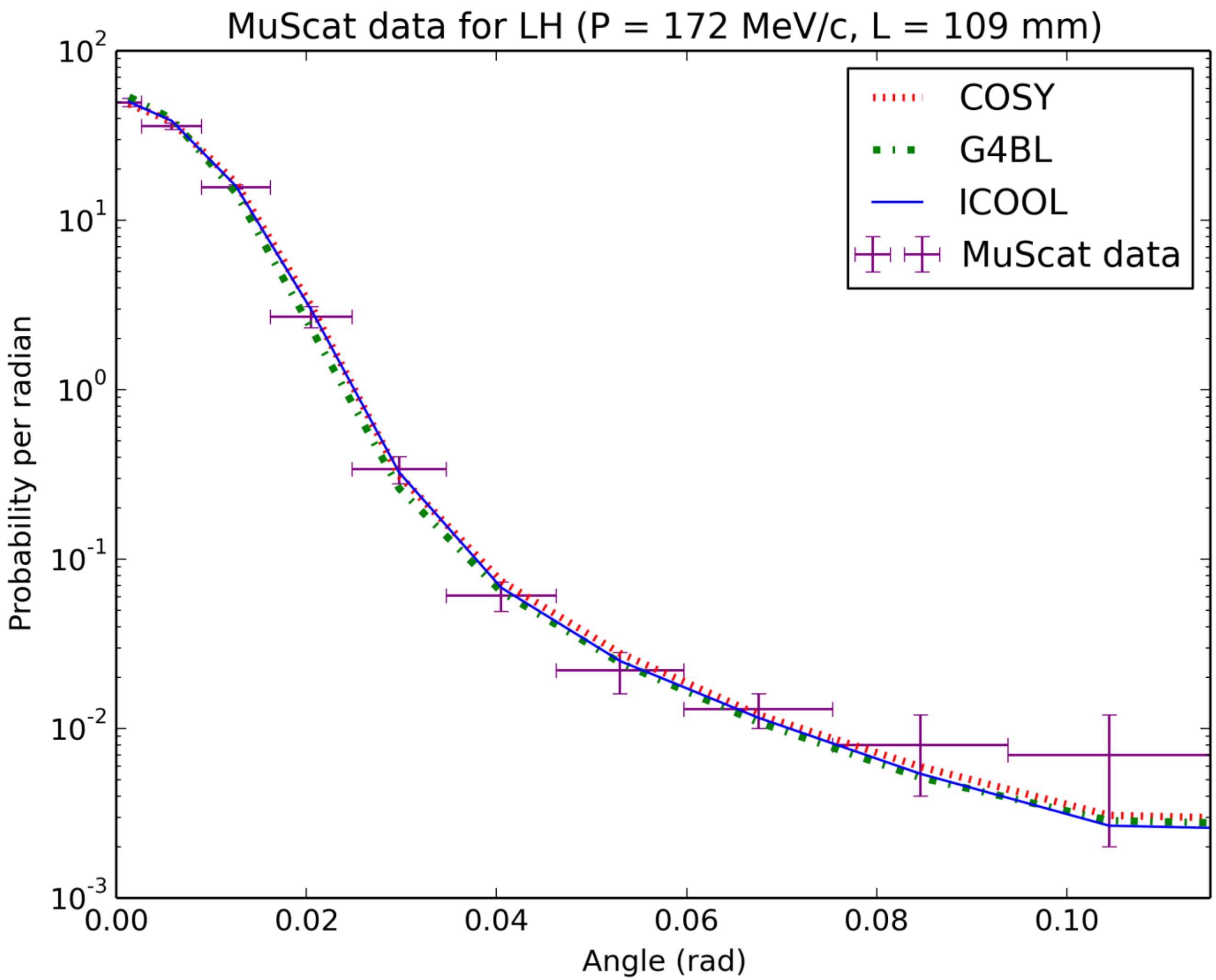
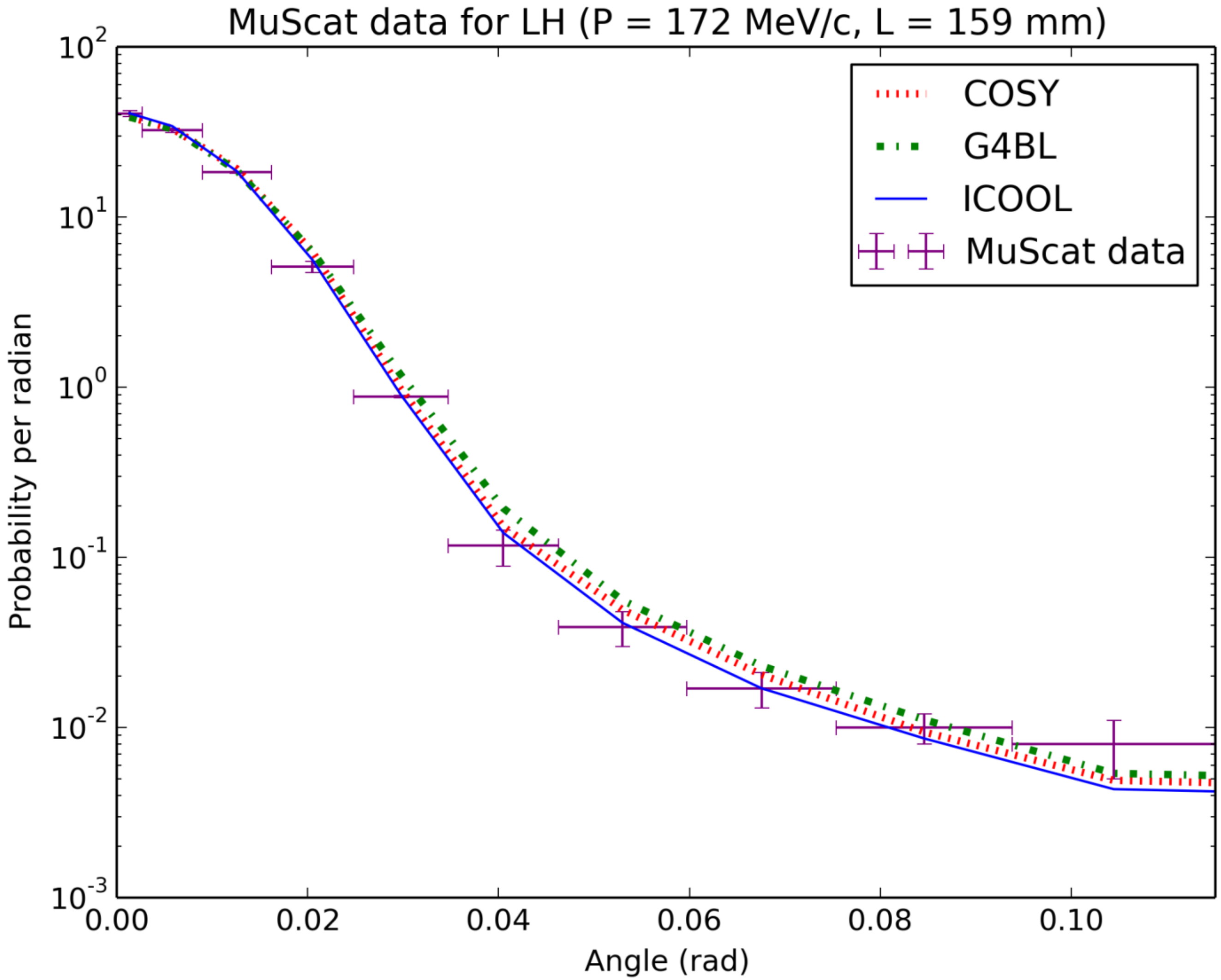
Results: Validation

- **Validation:** testing new software against experimental results
- Stochastic processes were further altered to give agreement with experimental results from MuScat

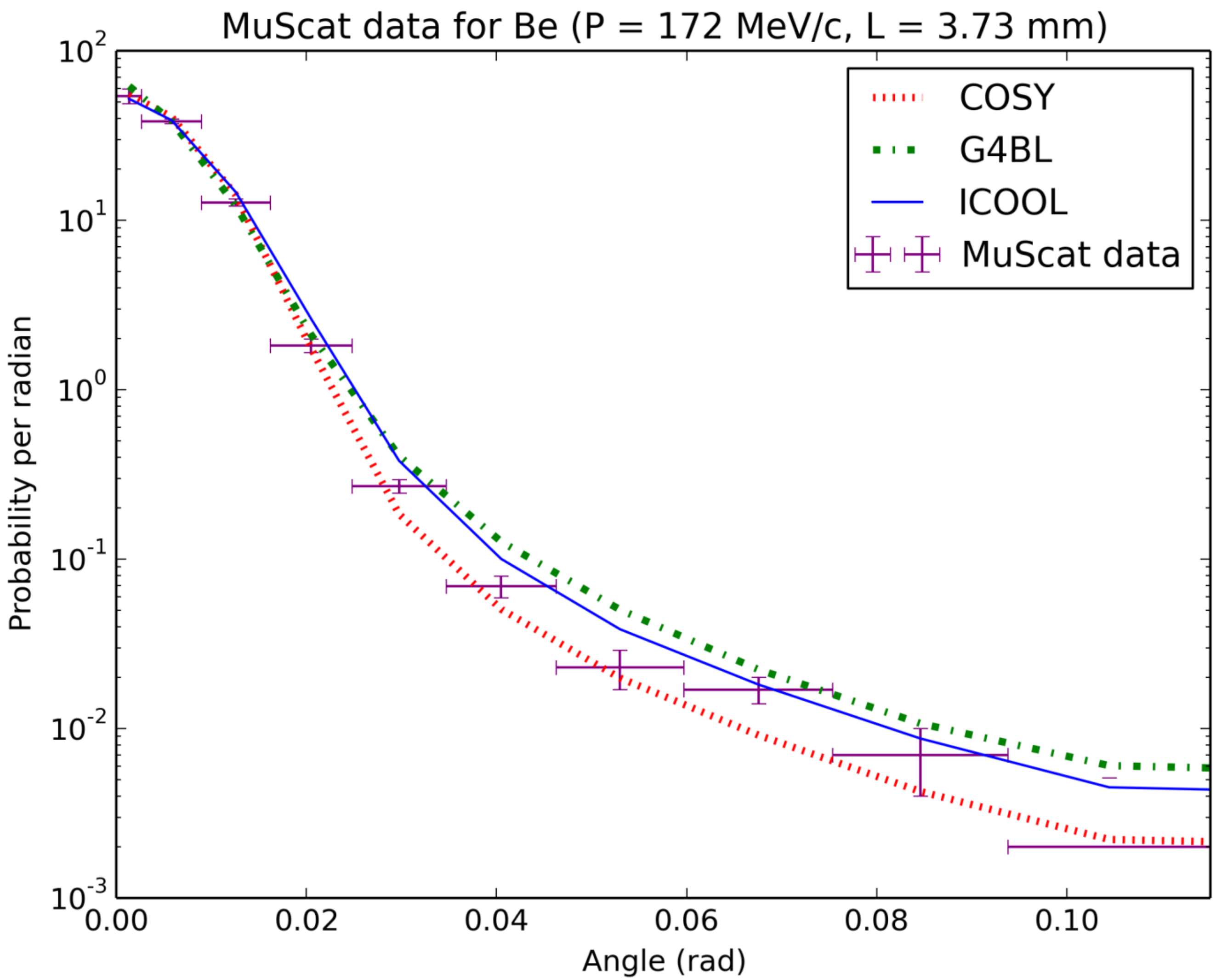
Results: Validation



Results: Validation



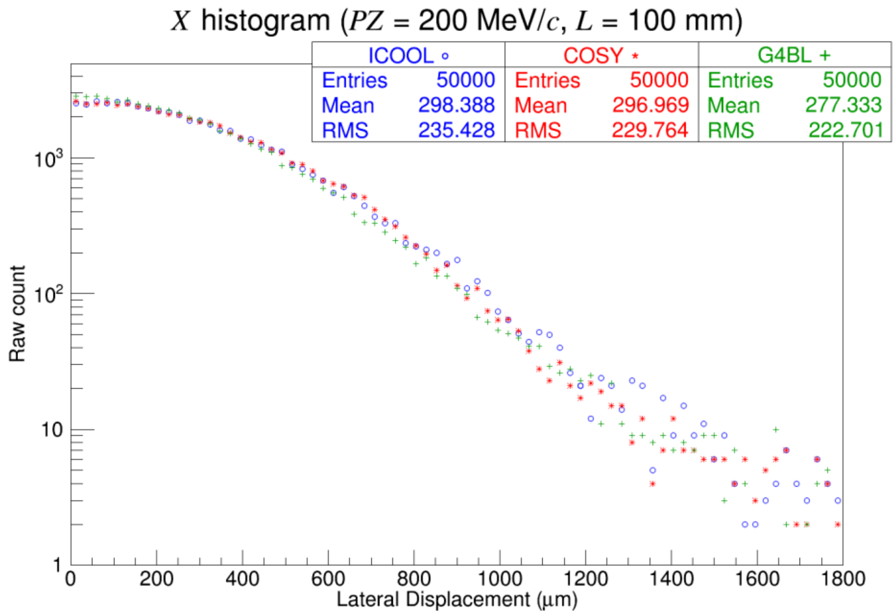
Results: Validation

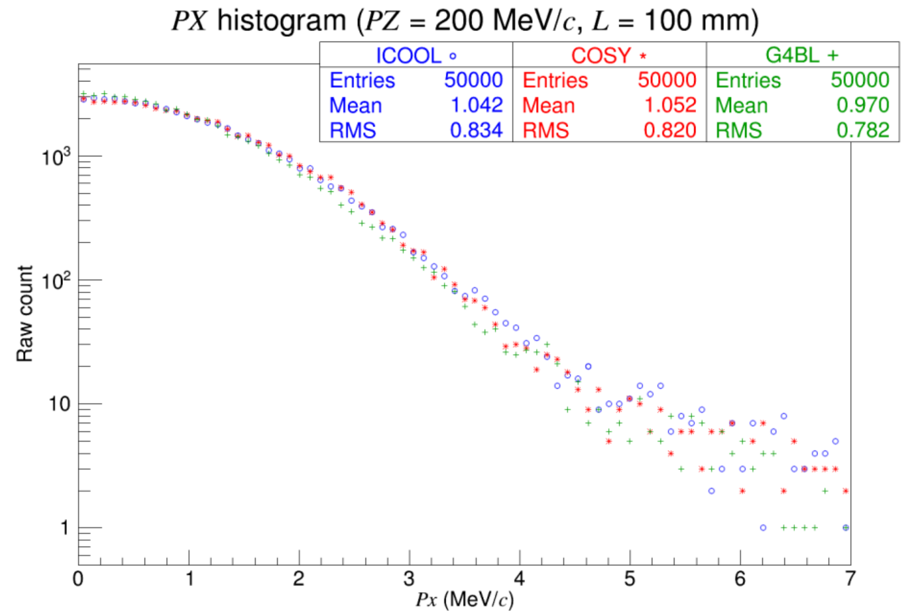


Results: Benchmarking

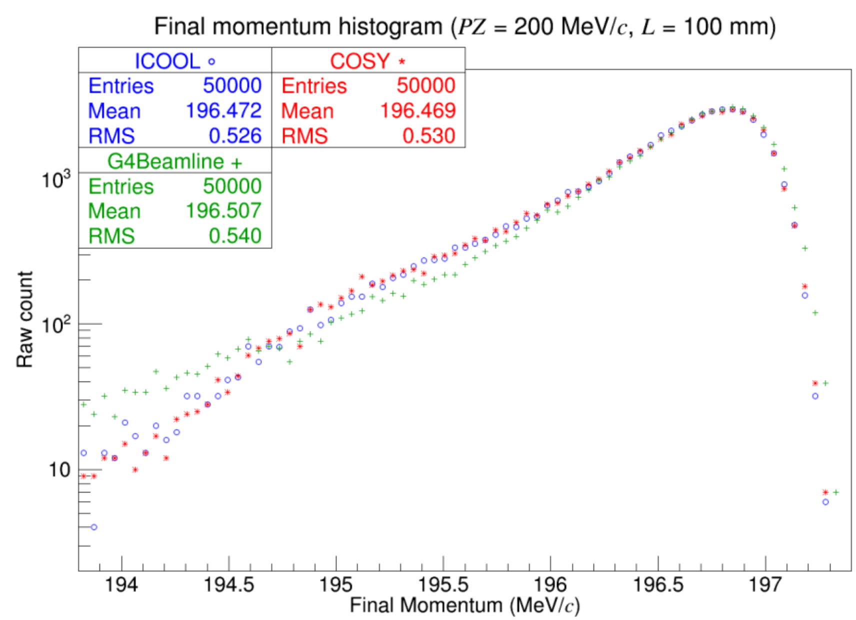
- **Benchmarking:** testing new software against known codes
- Needed because little experimental data exists

- All stochastic processes have been empirically altered to give reasonable agreement with two other codes, ICOOL and G4Beamline
- 36 benchmarking figures seen in thesis (12 figures with 3 sub-figures each)



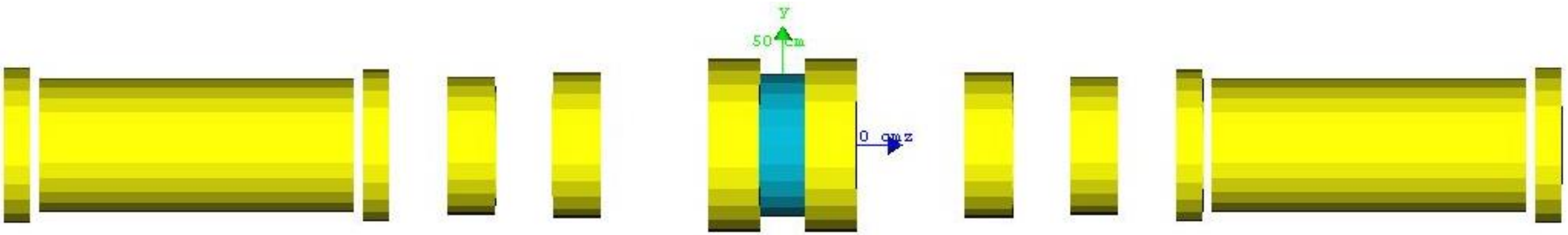


Pencil beam of 50k muons at 200 MeV/c through 100 mm of liquid hydrogen



Results: MICE

- The Muon Ionization Cooling Experiment (MICE) is an experiment being developed at Rutherford Appleton Laboratory in Oxfordshire, U.K.
- Goal: a proof-of-principle demonstration of muon ionization cooling
- MICE Step IV: 12 coils symmetric about an absorber

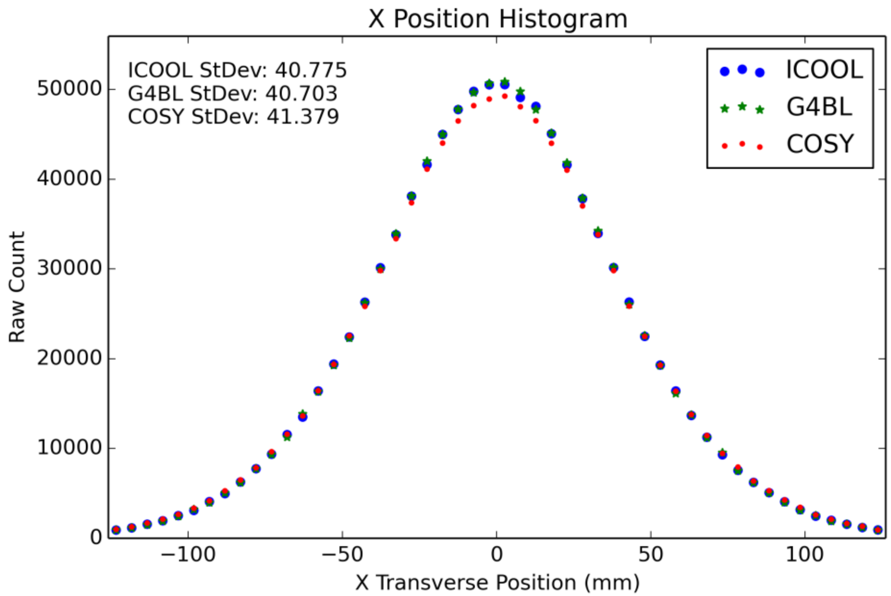


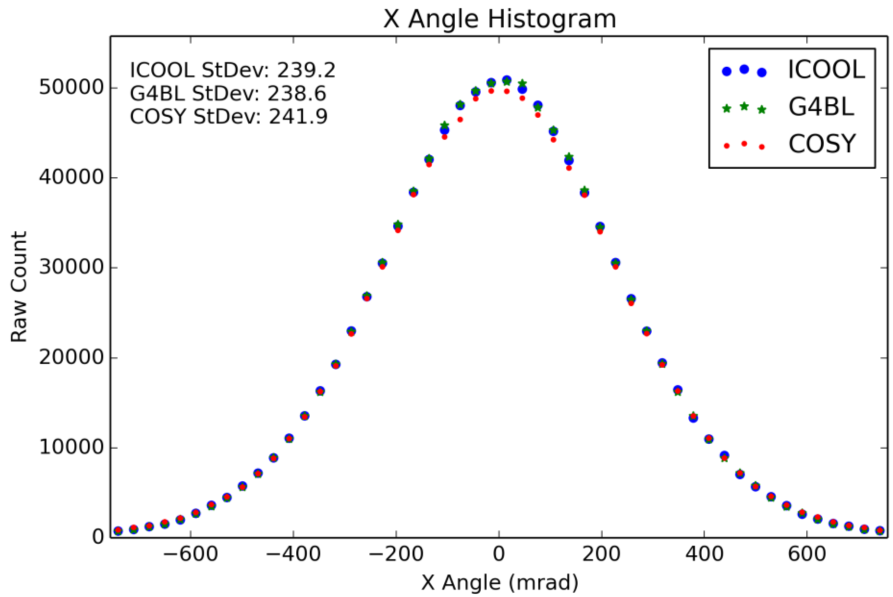
Results: MICE

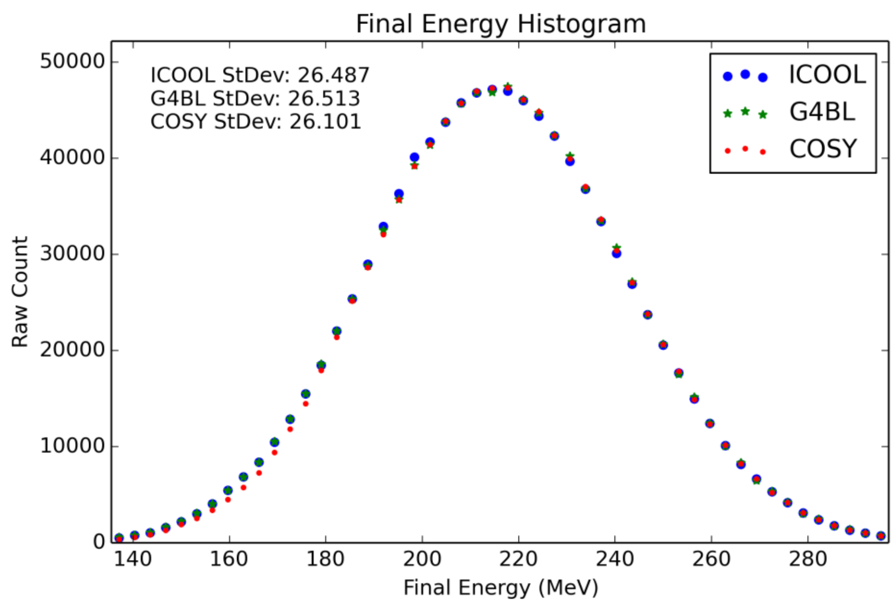
- 10^6 muons simulated through 350 mm liquid hydrogen
- Gaussian beam
- Agreement to within 1%

MICE Step IV Initial Distribution Parameters

Parameter	Mean	Standard deviation
x (mm)	0	32
y (mm)	0	32
z (mm)	0	0
p_x (MeV/c)	0	20
p_y (MeV/c)	0	20
p_z (MeV/c)	200	30

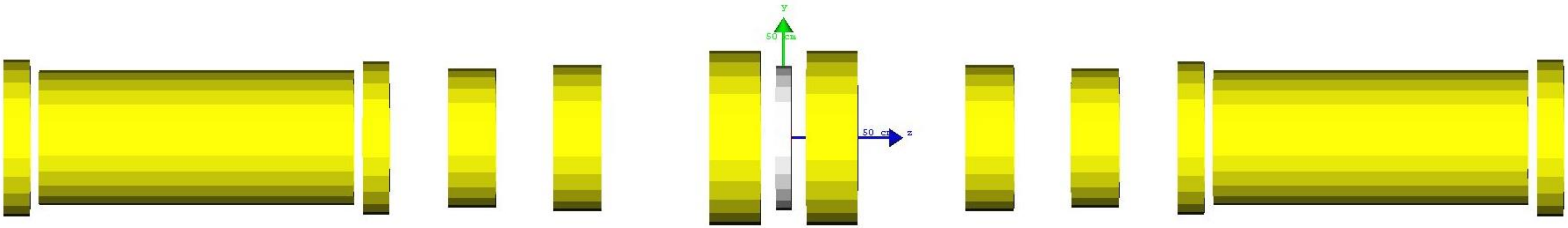

X Position Histogram
ICOOL StDev: 40.775
G4BL StDev: 40.703
COSY StDev: 41.379
ICOOL
G4BL
COSY
Raw Count
X Transverse Position (mm)
50000
40000
30000
20000
10000
0
−100
−50
0
50
100

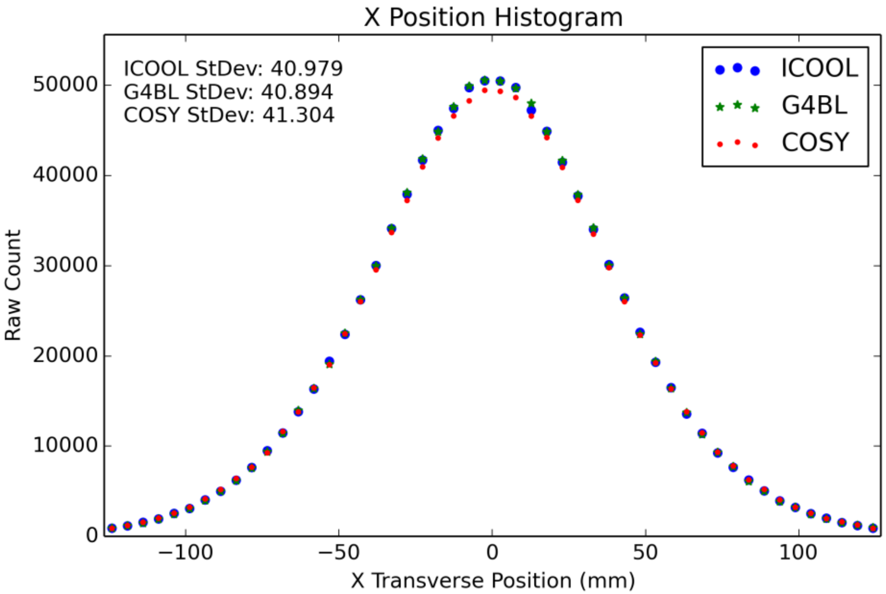

X Angle Histogram
ICOOL StDev: 239.2
G4BL StDev: 238.6
COSY StDev: 241.9
ICOOL
G4BL
COSY
Raw Count
X Angle (mrad)
50000
40000
30000
20000
10000
0
−600
−400
−200
0
200
400
600

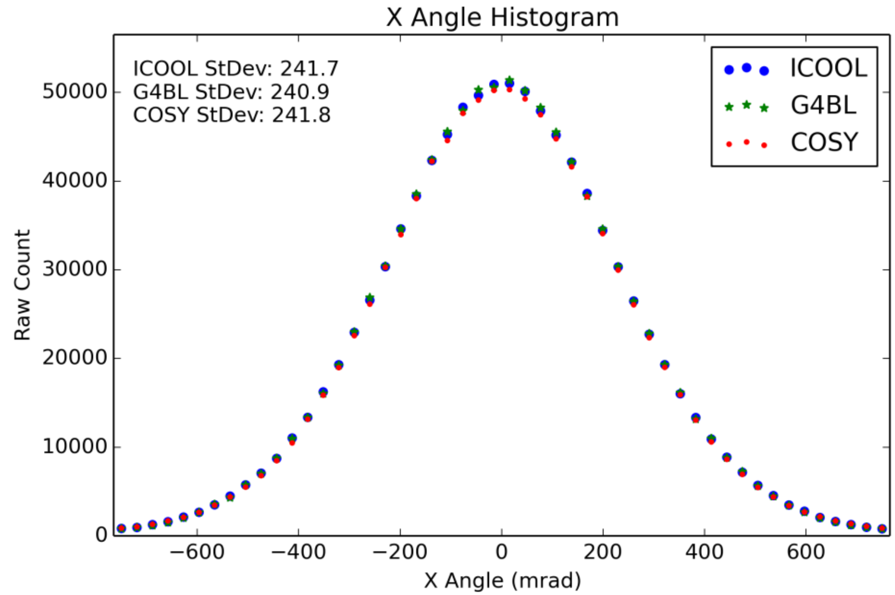

Final Energy Histogram
ICOOL StDev: 26.487
G4BL StDev: 26.513
COSY StDev: 26.101
ICOOL
G4BL
COSY
Raw Count
Final Energy (MeV)
50000
40000
30000
20000
10000
0
140
160
180
200
220
240
260
280

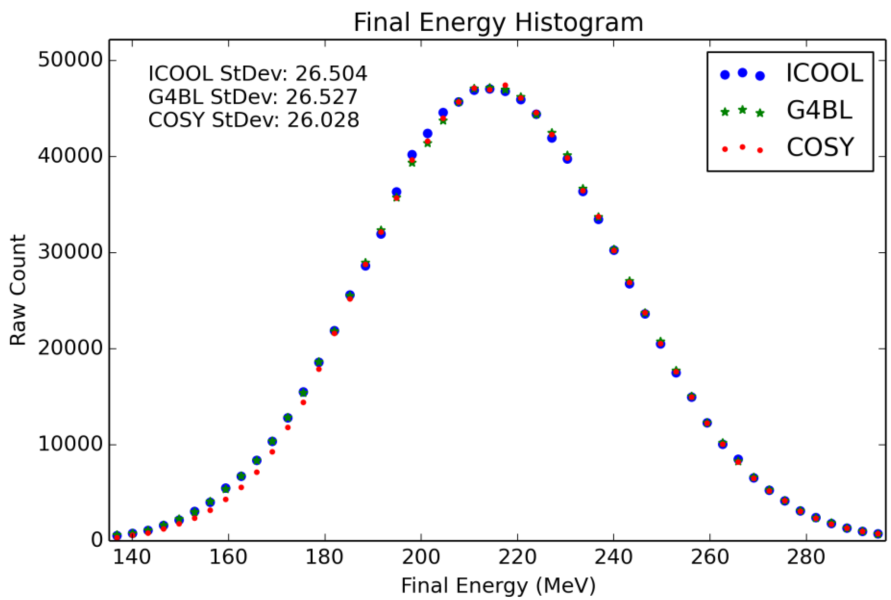
Results: MICE

- Also simulated with 65 mm of lithium hydride
  - $\rho_{lH} = 0.071\ g/cm^3$,
  - $\rho_{LiH} = 0.82\ g/cm^3$
- Agreement to within 1%




X Position Histogram
ICOOL StDev: 40.979
G4BL StDev: 40.894
COSY StDev: 41.304
ICOOL
G4BL
COSY
Raw Count
X Transverse Position (mm)


X Angle Histogram
ICOOL StDev: 241.7
G4BL StDev: 240.9
COSY StDev: 241.8
ICOOL
G4BL
COSY
Raw Count
X Angle (mrad)


Final Energy Histogram
ICOOL StDev: 26.504
G4BL StDev: 26.527
COSY StDev: 26.028
ICOOL
G4BL
COSY
Raw Count
Final Energy (MeV)

Results: MICE

- Timed simulations for COSY, G4Beamline, and ICOOL
- COSY roughly 50% the time of G4Beamline
- COSY roughly 33% the time of ICOOL

Run Times (in seconds) for MICE Step IV Simulation

Number of particles:	10^6	10^5	10^4	10^3
COSY:	367	31	6	4
G4BL :	662	75	15	9
ICOOL :	1091	117	19	9

Outline

- Motivation
  - Fundamental particle physics
  - The Higgs Boson
  - Neutrinos
  - Muon facilities
- Introduction to COSY Infinity
  - Transfer map methods
  - Transfer map example
- Stochastic Processes
  - Ionization cooling
  - Why not use transfer maps?
  - Energy straggling
  - Multiple scattering
- Results
  - Validation
  - Benchmarking
  - MICE
- Summary

Summary

- New stochastic processes implemented into COSY Infinity for particle-by-particle propagation
  - Uses hybrid transfer map–Monte Carlo method
- First principle derivations to basic theory found in thesis
- Corrections to basic theory novel in this work
- Validation shows agreement with MuScat results
- Benchmarking shows agreement with ICOOL, G4Beamline
- Prediction of MICE Step IV results within 1% of other codes
- Computation time for COSY twice as fast as other codes

Thank you!

(Questions?)

Backup Slides

Just in case!

Example: 2nd Order Map

$$x_f = -0.5x_i + 3.1a_i$$
$$+5.2x_i^2 + 21.1x_i a_i + 18.3a_i^2$$
$$-0.1y_i^2 + 0.32y_i b_i - 2.13b_i^2$$

```
I   COEFFICIENT               ORDER  EXPONENTS
1   -.4638328953818641E-15      0    0 0  0 0
2   -.5000000000473073          1    1 0  0 0
3    3.068671837962613          1    0 1  0 0
4    5.228585914563241          2    2 0  0 0
5    21.09726212071938          2    1 1  0 0
6    18.25557945665144          2    0 2  0 0
7   -.1414769526768841          2    0 0  2 0
8   0.3213692368536242          2    0 0  1 1
9   -2.131897663812564          2    0 0  0 2
------------------------------------------
```

Why low-Z Materials?

$$\frac{d\epsilon_x^N}{dz} = \frac{\beta_\perp}{2}\frac{E_s^2}{\beta^3 Emc^2}\frac{1}{X_0} - \frac{1}{\beta}\left|\left\langle\frac{dE}{dz}\right\rangle\right|\frac{\epsilon_x^N}{E}$$

Why low-Z Materials?

$$\frac{d\epsilon_x^N}{dz} = \frac{\beta_\perp}{2}\frac{E_s^2}{\beta^3 Emc^2}\frac{1}{X_0} - \frac{1}{\beta}\left|\left\langle\frac{dE}{dz}\right\rangle\right|\frac{\epsilon_x^N}{E}$$

heating term cooling term

Why low-*Z* Materials?

$$\frac{d\epsilon_x^N}{dz} = \frac{\beta_\perp}{2}\frac{E_s^2}{\beta^3 Emc^2}\frac{1}{X_0} - \frac{1}{\beta}\left|\left\langle\frac{dE}{dz}\right\rangle\right|\frac{\epsilon_x^N}{E}$$

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$$\frac{d\epsilon_x^N}{dz} = \frac{\beta_\perp}{2}\frac{E_s^2}{\beta^3 Emc^2}\frac{1}{X_0} - \frac{1}{\beta}\left|\left\langle\frac{dE}{dz}\right\rangle\right|\frac{\epsilon_x^N}{E}$$

$$X_0 \propto 1/Z$$

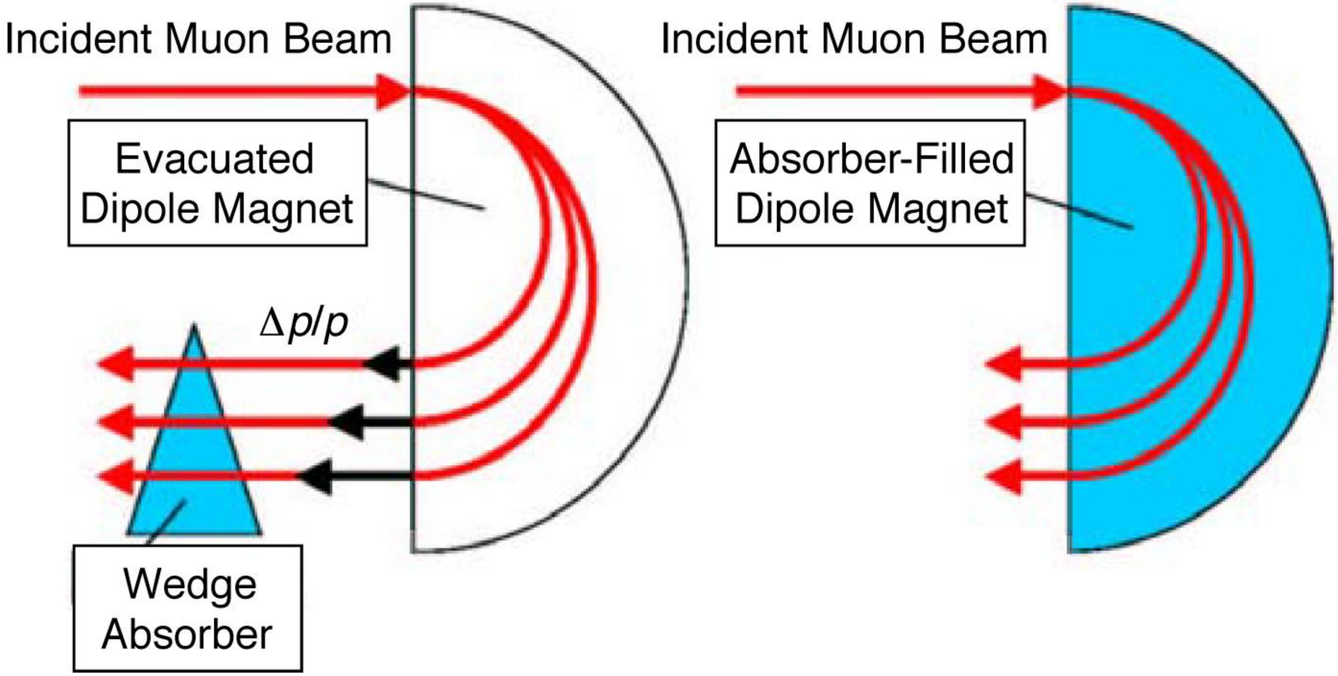
Why low-Z Materials?

$$\frac{d\epsilon_x^N}{dz} = \frac{\beta_\perp}{2}\frac{E_s^2}{\beta^3 E m c^2}\frac{1}{X_0} - \frac{1}{\beta}\left|\left\langle\frac{dE}{dz}\right\rangle\right|\frac{\epsilon_x^N}{E}$$

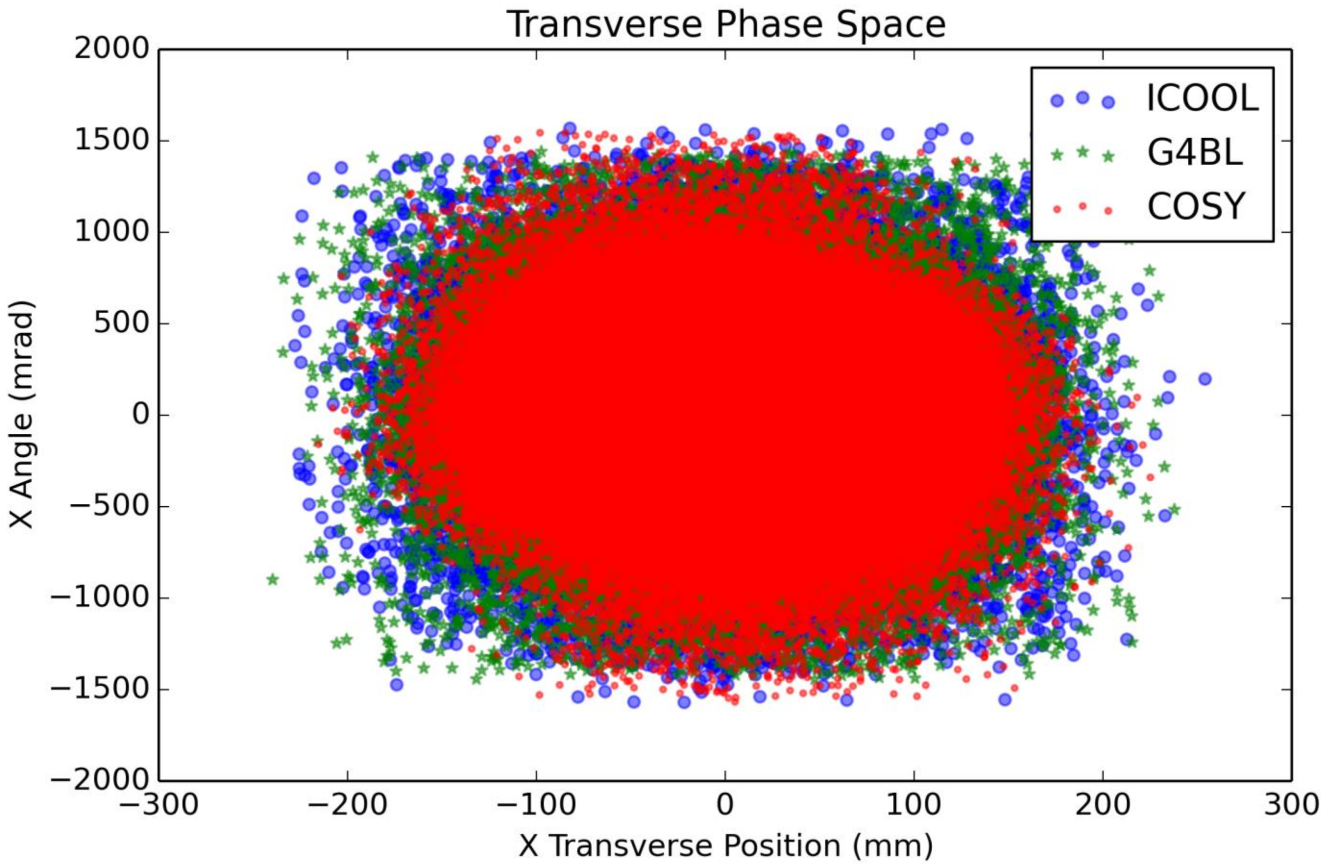
$$X_0 \propto 1/Z$$

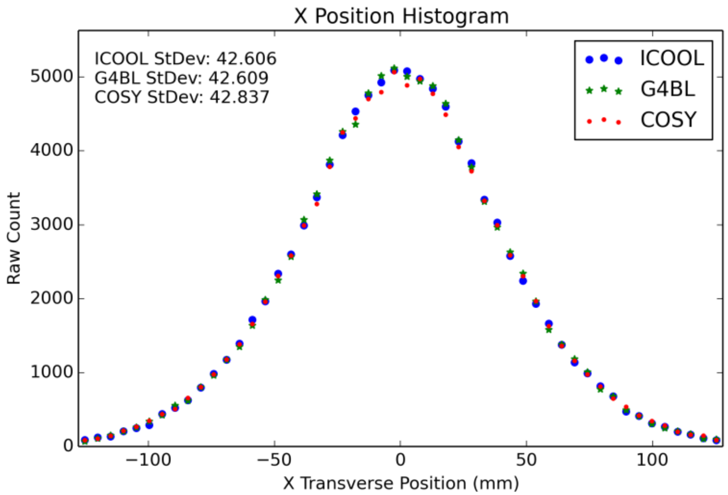
heating term $\propto Z$

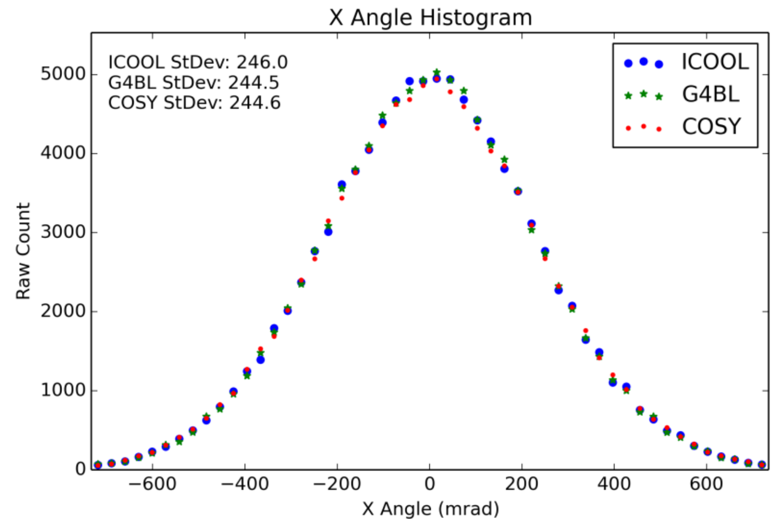
Emittance Exchange



MICE Transverse Phase Space







Order 9

Steps:

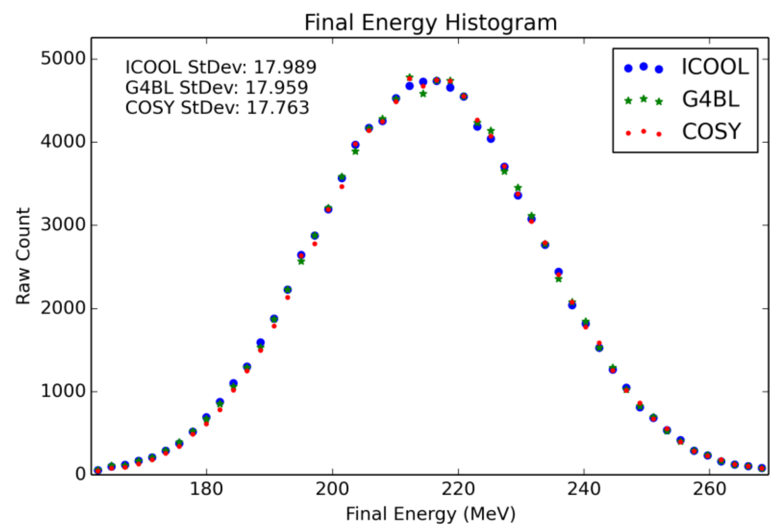
- 30 through coils
- 5 through “absorber”

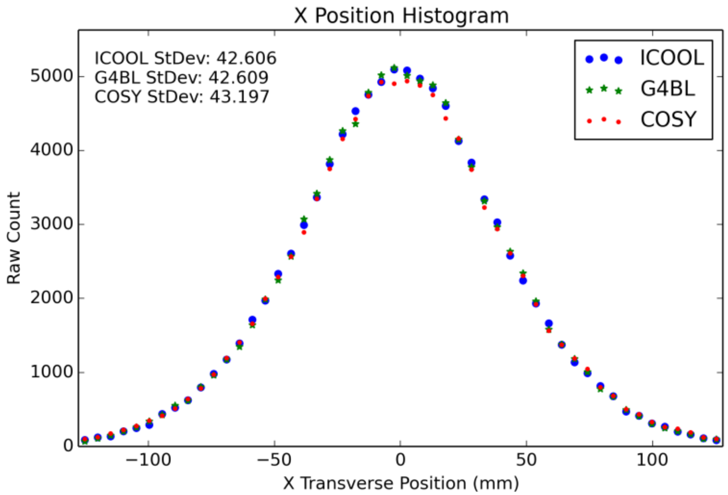
Transmission rates:

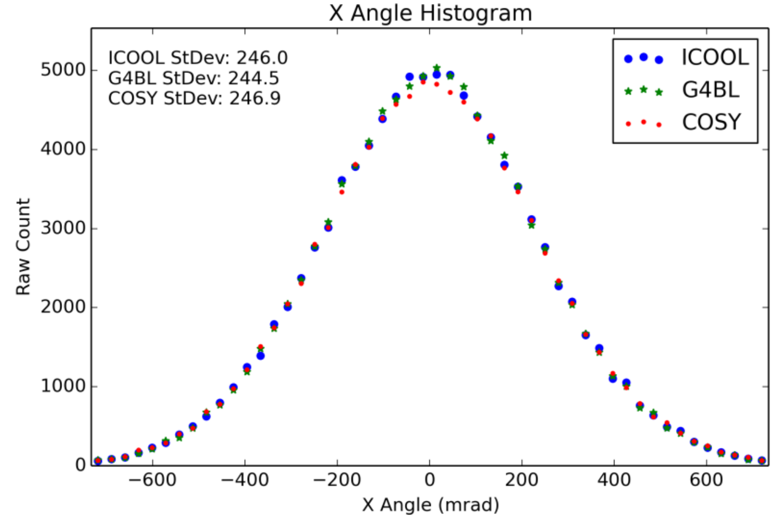
- ICOOL: 99.0%
- G4BL: 99.0%
- COSY: 98.2%

Times:

- ICOOL: 117
- G4BL: 75
- COSY: 166







Order 7

Steps:

2 upstream, 30 downstream

5 through “absorber”

Transmission rates:

ICOOL: 99.0%

G4BL: 99.0%

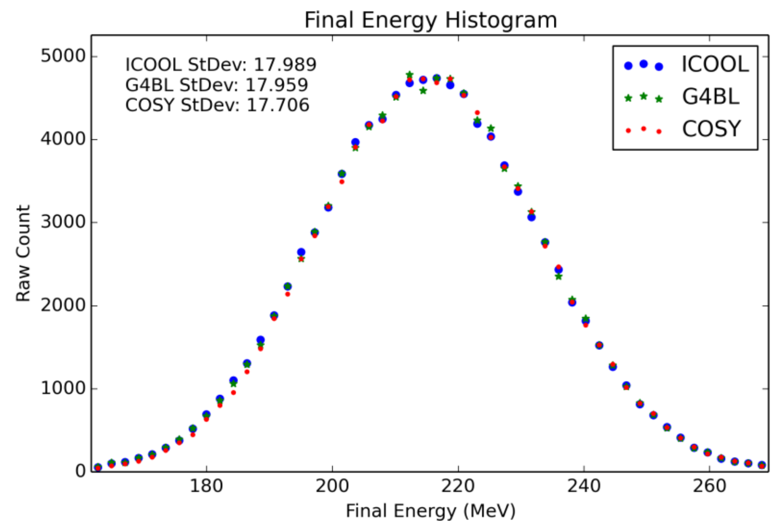
COSY: 98.0%

Times:

ICOOL: 117

G4BL: 75

COSY: 41



Publications

- J. Kunz, P. Snopok, M. Berz, K. Makino. Hybrid methods for simulation of muon ionization cooling channels. Proc. of IPAC 16 (2016).
- J. Kunz, P. Snopok, M. Berz, K. Makino. The advancement of cooling absrobers in COSY Infinity. Proc. of IPAC 15 (2015).
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- P. Snopok, J. Ellison, J. Kunz. Advanced simulation tools for muon-based accelerators. Cybernetics and Physics 3 (2014).
- J. Kunz, P. Snopok, M. Berz, K. Makino. The development of stochastic processes in COSY Infinity. Proc. of IPAC 14 (2014).
- P. Snopok, J. Kunz. Progress of the matter-dominated muon accelerator lattice simulation tools development for COSY Infinity. Proc. of NA-PAC 13 (2013).