HYBRID METHODS FOR SIMULATION OF MUON IONIZATION COOLING CHANNELS*

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Abstract

COSY Infinity is an arbitrary-order beam dynamics simulation and analysis code. It uses high-order transfer maps of combinations of particle optical elements of arbitrary field configurations. New features have been developed and implemented in COSY to follow charged particles through matter. To study in detail the properties of muons passing through a material, the transfer map approach alone is not sufficient. The interplay of beam optics and atomic processes must be studied by a hybrid transfer map–Monte Carlo approach in which transfer map methods describe the average behavior of the particles including energy loss, and Monte Carlo methods are used to provide small corrections to the predictions of the transfer map, accounting for the stochastic nature of scattering and straggling of particles. This way the vast majority of the dynamics is represented by fast application of the high-order transfer map of an entire element and accumulated stochastic effects. The gains in speed simplify the optimization of muon cooling channels which are usually very computationally demanding. Progress on the development of the required algorithms is reported.

INTRODUCTION

A prime example of why matter-dominated lattices are relevant comes from the prospect of a neutrino factory or a muon collider [1]. As muon branching fractions are 100% $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$ and $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$, there are obvious advantages of a muon-sourced neutrino beam. Also, because muons are roughly 200 times heavier than electrons, synchrotron radiation is not an issue. As a result, a high-energy muon collider ($\sqrt{s} \approx 6$ TeV) could be built on a compact site. Such energy levels are experimentally unprecedented in the leptonic sector, since a circular electron accelerator is restricted by synchrotron radiation. At lower energy, a muon collider could serve as a Higgs Factory ($\sqrt{s} \approx 126$ GeV), with possible new physics via the observation of Higgs to lepton coupling. This is advantageous since the Higgs coupling to leptons scales as mass squared.

However, muon-based facilities are not without their challenges. Synthetic muon creation comes from the collision of protons with a fixed target. The resultant spray of particles largely contains kaons (which decay primarily into pions and muons), pions (which decay primarily into muons), and rogue protons. High-intensity collection entails a large initial phase space volume. The resultant cloud of muons must be collected, focused, and accelerated well within the muon lifetime (2.2 μs at rest). The only technique that is fast enough to be relevant on that time scale is ionization cooling [2].

For a neutrino factory only a modest amount of cooling is required, predominantly in the transverse plane. However, neutrino factories could benefit from full six-dimensional cooling, where in addition to the transverse cooling emittance exchange is used to reduce longitudinal beam size in addition to transverse. Current muon collider designs assume a significant, $O(10^6)$, six-dimensional cooling.

Cooling channels required for a high-energy high-luminosity muon collider could be up to a thousand meters long. Designing, simulating, and optimizing performance of those channels involves using high-performance clusters and multi-objective genetic optimizers. Typically, the codes used for simulations belong to the class of particle-by-particle integrators, where each particle is guided through the length of the cooling channel independently. That takes its toll on genetic optimizers, especially with a large number of particles per run. Transfer map methods could solve this problem, since the nonlinear map of the system is calculated once, and then can be applied to any number of particles at a very low computational cost. On the other hand, the transfer map approach alone is not sufficient to study the passage of muons through a material. This study implements a hybrid transfer map–Monte Carlo approach in which transfer map methods describe the deterministic behavior of the particles, and Monte Carlo methods are used to provide corrections accounting for the stochastic nature of scattering and straggling of particles.

COSY INFINITY

COSY Infinity (COSY) [3] is a simulation tool used in the design, analysis, and optimization of particle accelerators, spectrographs, beam lines, electron microscopes, and other such devices, with its use in accelerator lattice design being of particular interest here. COSY uses the transfer map approach, in which the overall effect of the optics on a beam of particles is evaluated using differential algebra. Along with tracking of particles through a lattice, COSY includes many analysis and optimization tools.

Currently supported elements in COSY include various magnetic and electric multipoles (with fringe effects), homogeneous and inhomogeneous bending elements, Wien filters, wigglers and undulators, cavities, cylindrical electromagnetic lenses, general particle optical elements, and *deterministic* absorbers of intricate shapes described by polynomials of arbitrary order, with the last element being of particular interest for this study. This element only takes into

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account deterministic effects (producing the same final result every time for a given initial condition), not stochastic effects (intrinsically random effects such as multiple scattering and energy straggling).

To take into account stochastic effects, the transfer map paradigm needs to be augmented by implementing the corrections from stochastic effects directly into the fabric of COSY.

STOCHASTIC PROCESSES

The stochastic processes of interest are straggling (fluctuation about a mean energy loss) and multiple angular scattering. The general outline to simulate these two beam properties is discussed more thoroughly in [4]. Straggling follows Landau theory and has the form [5]

$$f(\lambda) = \frac{1}{\xi} \cdot \frac{1}{2\pi i} \int_{c+i\infty}^{c-i\infty} \exp(x \ln x + \lambda x) dx, \quad (1)$$

where $\xi \propto Z\rho L/\beta^2 A$, and $\lambda \propto dE/\xi - \beta^2 - \ln \xi$. Here Z, A, and ρ are the atomic charge, atomic mass, and density of the material; L is the amount of material that the particle traverses; $\beta = v/c$; and dE is the fluctuation about the mean energy. The algorithm based on Eq. (1) has been implemented in COSY.

The derivation of the scattering function $g(u)$ (where $u = \cos\theta$) is done separately for small angles and large angles. For small angles, the shape is very nearly Gaussian in θ [6]. For large angles, the distribution follows the Mott scattering cross section and is Rutherford-like [7]. The resulting peak and tail are continuous and smooth at some critical u_0, which yields the final form of $g(u)$:

$$g(u) = \begin{cases} \exp\left(-\frac{1}{2}\frac{1-u}{1-u_\sigma}\right) & \mid u_0 < u \\ \zeta \cdot \dfrac{1+\frac{1}{2}(\beta\gamma)^2(1+u-b)}{(1-u+b)^2} & \mid u \le u_0 \end{cases}. \quad (2)$$

Here the parameters ζ and b are chosen to ensure continuity and smoothness, $\gamma = 1/\sqrt{1-\beta^2}$, u_0 is a fitted parameter chosen as $u_0 = 9u_\sigma - 8$, u_σ is the σ-like term for a Gaussian in θ. It is another fitted parameter based off [11] and taking the form

$$u_\sigma = \cos\left(\frac{13.6\text{ MeV}}{\beta pc}\left(\frac{L}{L_0}\left(1+0.103\ln\frac{L}{L_0}\right)+0.0038\left(\ln\frac{L}{L_0}\right)^2\right)^{\frac{1}{2}}\right).$$

In [4], the hybrid method presented here was benchmarked against two other beamline simulation codes, ICOOL [8] and G4Beamline [9], and (in the case of angular scattering) against experimental data obtained by MuScat [10].

THE MUON IONIZATION COOLING EXPERIMENT

The Muon Ionization Cooling Experiment (MICE [14]) is an experiment currently being developed at the Rutherford Appleton Laboratory in Oxfordshire, U.K. Its goal is to show a proof-of-principle demonstration of muon ionization cooling. MICE Step IV configuration is explored in this work. The Step IV cell includes 12 magnetic coils positioned symmetrically around a flat absorber. Figure 1 shows a schematic of this lattice with 350 mm of liquid hydrogen as the absorber.

MICE Step IV Coil Parameters

Name	z position mm	Length mm	Inner radius mm	Outer radius mm	Current density A/mm^2
End2	∓3200	111	258	326	±126
Center	∓2450	1314	258	280	±148
End1	∓1700	111	258	319	±133
Match2	∓1300	199	258	289	±132
Match1	∓861	201	258	304	±133
Focus	∓202	213	268	362	±104

Table 1: MICE Step IV coil parameters.

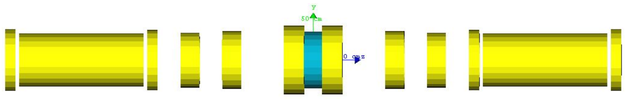

Figure 1: MICE Step IV cell. Magnetic coils are shown in yellow and the absorber is shown in blue. The green and blue axes are the y and z axes, here drawn to scale as 500 mm each. The aperture (invisible for display purposes) is 300 mm. Image rendered via G4Beamline [9].

10^6 muons were simulated through the cell shown in Figure 1. The coil parameters may be found in Table 1. The absorber was a 350 mm cylindrical block of liquid hydrogen centered at $z = 0$. The aperture was set to 300 mm. Note that other materials such as safety windows were not accounted for in this simulation. The decay process was disabled in all simulation codes. The beam started at −2.45 m and ended at 2.45 m. The initial distribution was Gaussian with the following parameters: $\sigma_x = \sigma_y = 32$ mm, $\sigma_{p_x} = \sigma_{p_y} = 20$ MeV/c, $\sigma_{p_z} = 30$ MeV/c.

In COSY, it was found that a 5th order simulation was sufficient. Through the coil-only portion of the simulation, 50 steps were taken on each side of the absorber (or roughly a step size of 46 mm both upstream and downstream). The particles were tracked through the momentary transfer map after each step and then the transfer map was set to unity. It was noted that for the coil-only section, a single transfer map was not sufficient even at the 9th order. This is due to the relatively large phase space volume of the beam and the complexity of the magnetic field. Through the absorber-coil region (−350/2 mm to 350/2 mm), it was found that a 1st order map with 5 steps was sufficient. This is due to the transverse phase space of the beam reaching a minimum and the magnetic field passing through the point of symmetry.

Precalculated field maps were used by both G4Beamline (which used the `fieldmap` command) and ICOOL (which used the `GRID` command operating in `G43D` mode).

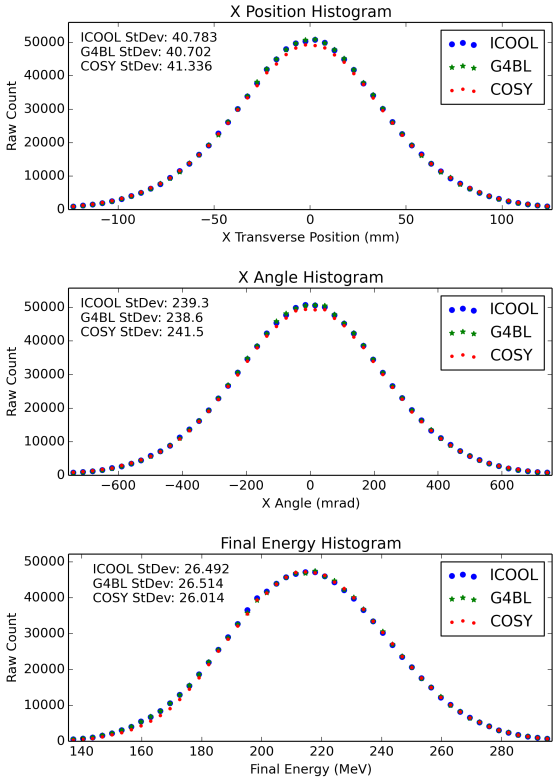

Figure 2: MICE Step IV x position, x angle, and final total energy results for 350 mm of liquid hydrogen.

Run Times (in seconds) for MICE Step IV Simulation

Number of particles:	10^6	10^5	10^4	10^3
COSY:	367	31	6	4
G4BL (coils):	3973	392	40	6
G4BL (field map):	662	75	15	9
ICOOL (field map):	1091	117	19	9

Table 2: Run times for MICE Step IV simulation for liquid hydrogen. Note that the G4Beamline initialization time was not added to the run time values. G4BL (coils) represents the simulation in G4Beamline when the `coil` parameter was used. G4BL (field map) represents the simulation when G4Beamline (like ICOOL) read the field map from a file.

The runtimes of ICOOL, G4Beamline, and COSY are listed in Table 2. Note that the initialization time for G4Beamline to create the field maps was 33 seconds. The time it took to create a text file for ICOOL input was 11 seconds. Since G4Beamline only has to create the field map once, the initialization time is added to neither ICOOL nor G4Beamline the run times in Table 2. COSY did not have any initialization time.

As a second test, MICE configuration in Figure 1 was simulated using 65 mm of lithium hydride. Lithium hydride is an attractive material because, unlike liquid hydrogen, it does not require cryogenic conditions, but still maintains a low Z value. It can be seen from Figure 3 that 65 mm of lithium hydride has a similar effect on the beam as 350 mm of liquid hydrogen.

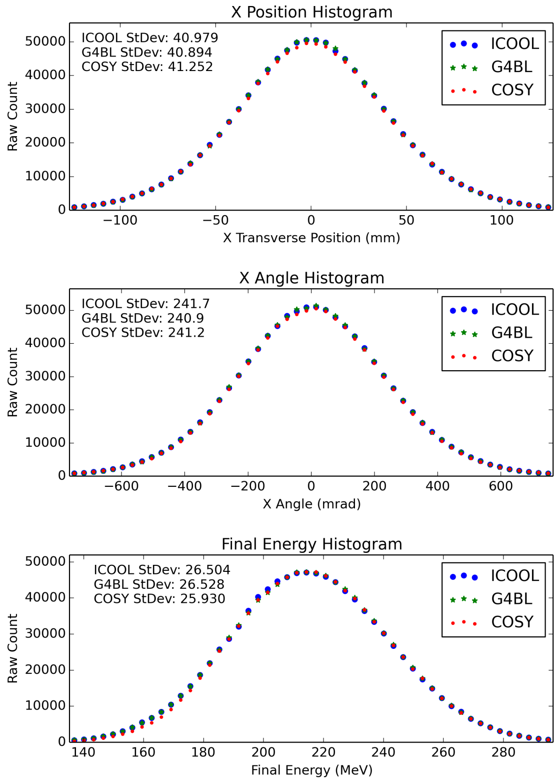

Figure 3: MICE Step IV x position, x angle, and final total energy results for 65 mm of lithium hydride.

SUMMARY

The addition of stochastic processes in COSY Infinity for the use of muon ionization cooling has been successful. Using MICE as a case study, Figures 2 and 3 show agreement within 1%. The hybrid method implemented into COSY works well for both liquid hydrogen and lithium hydride. Furthermore, Table 2 shows that COSY uses roughly half the computational time as G4Beamline and a third of the computational time of ICOOL.

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