Hybrid Methods for Simulation of Muon Ionization Cooling Channels

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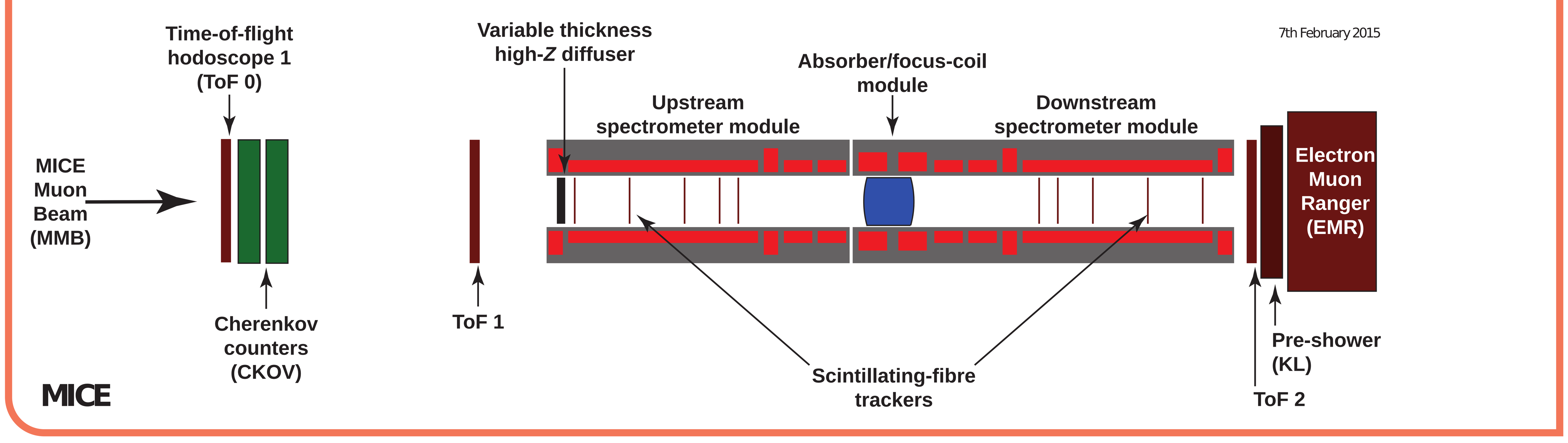
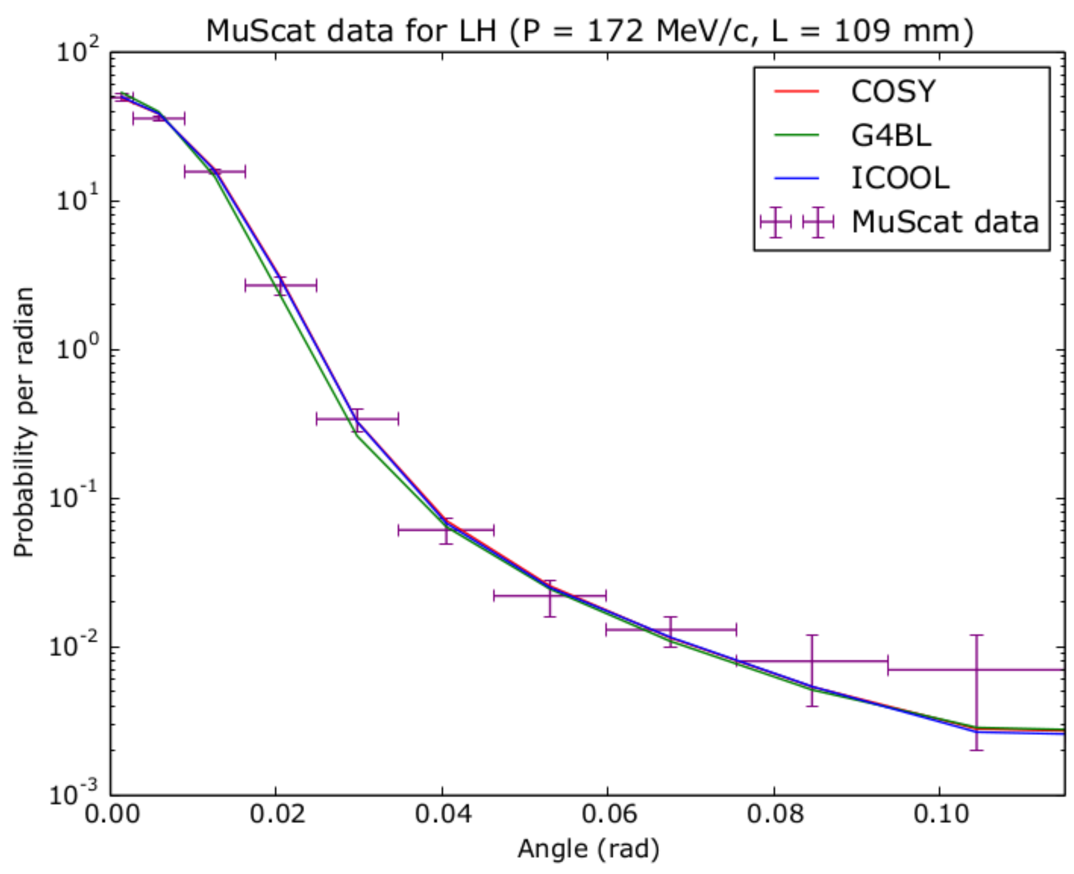
Muon Ionization Cooling Experiment (MICE) Layout



Ionization cooling

- Muons are tertiary production particles ($p \to \pi \to \mu$). As a result, the muon beam phase space volume is very large.
- Muons must be focused and accelerated within the muon lifetime (2.2 μs in the rest frame). The only cooling technique that is fast enough is ionization cooling.
- Muons traverse an absorber, momentum is lost both in the longitudinal and transverse directions due to ionization. The energy is then restored in the longitudinal direction only, leading to an overall reduction in the transverse beam size.
- To achieve cooling in the longitudinal direction, emittance exchange is used, typically involving wedge-shaped absorbers.
- For some applications such as a high-energy high-luminosity muon collider, cooling has to be very aggressive: 6D emittance reduction over six orders of magnitude to reach design goals.
- Recently, an arbitrary-order transfer map simulation code COSY Infinity [1] has been outfitted with new hybrid transfer map–Monter Carlo tools for matter-dominated muon ionization cooling lattices.
- Excellent agreement has been achieved between COSY, G4Beamline [3], and ICOOL [2] for pencil beams of $p = (100, 200, 300, 400)$ MeV/c through liquid hydrogen absorbers of lengths $L = (1, 10, 100)$ mm.
- Agreement has been shown with the MuScat results [4]. An example of a collimated beam of 172 MeV/c muons passing through 109 mm of liquid hydrogen is shown below.



Stochastic Processes

- The stochastic processes of interest are energy straggling and multiple scattering.
- Straggling follows Landau theory.
- The derivation of the scattering function is done separately for small and large angles: Gaussian for small angles and Rutherford-like for large angles.
- Both algorithms with time-of-flight and multiple scattering offset corrections were implemented in COSY and applied to MICE [5].

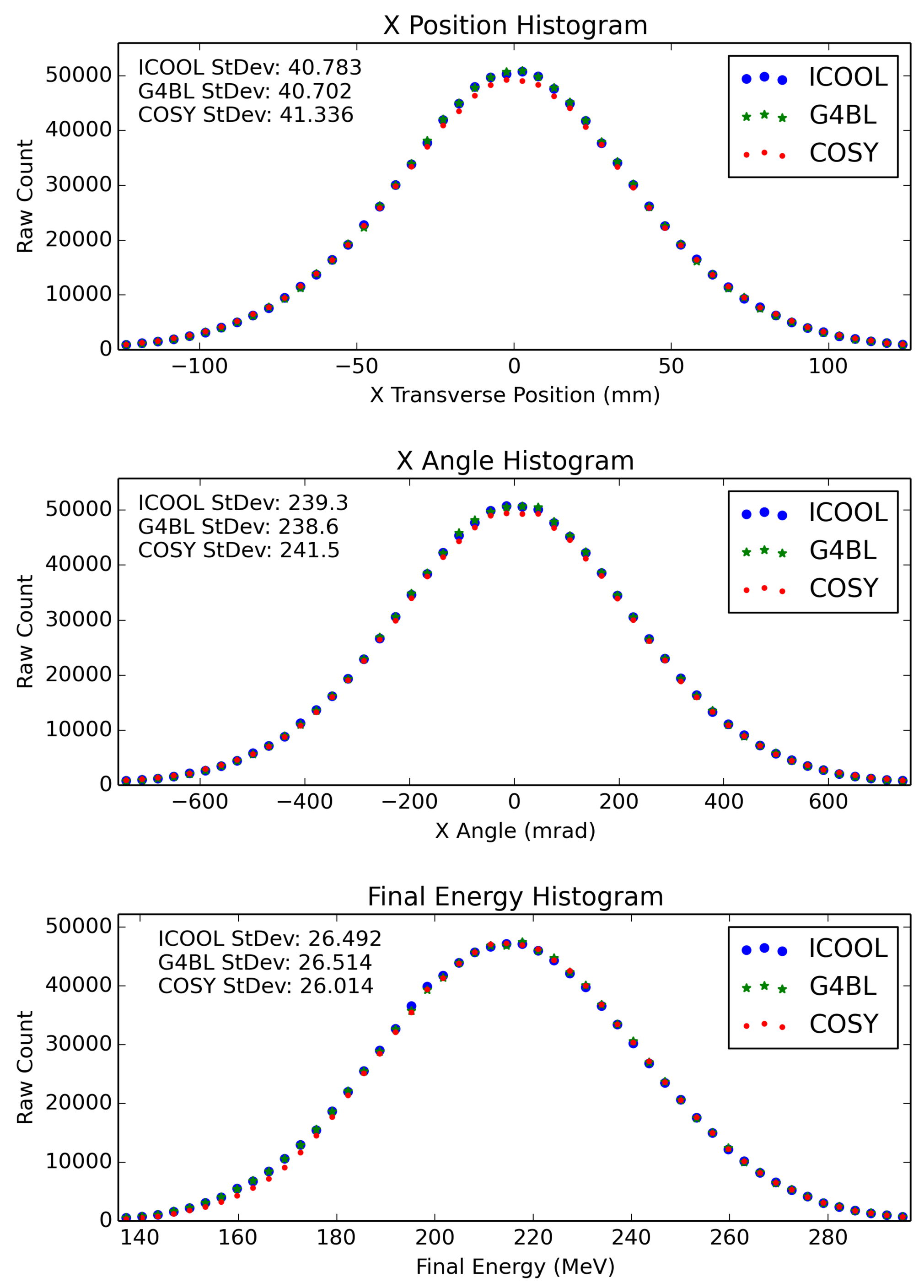
MICE cell

- MICE simulations use the Step IV geometry shown in the figure above.
- Piecewise transfer maps of order 5 were used to represent the complex magnetic field accurately.
- The initial distribution was Gaussian with the following parameters: $\sigma_{x,y} = 32$ mm, $\sigma_{p_x,p_y} = 20$ MeV$/c$, $\sigma_{p_z} = 30$ MeV$/c$.
- The absorber was a cylindrical lithium hydride block of 65 mm or a 350 mm volume of liquid hydrogen.
- The beam was propagated from the center of the upstream spectrometer module to the center of the downstream spectrometer module (see figure above).
- Coil parameters are summarized in the table below. Inner radius is 258 mm for all coils.

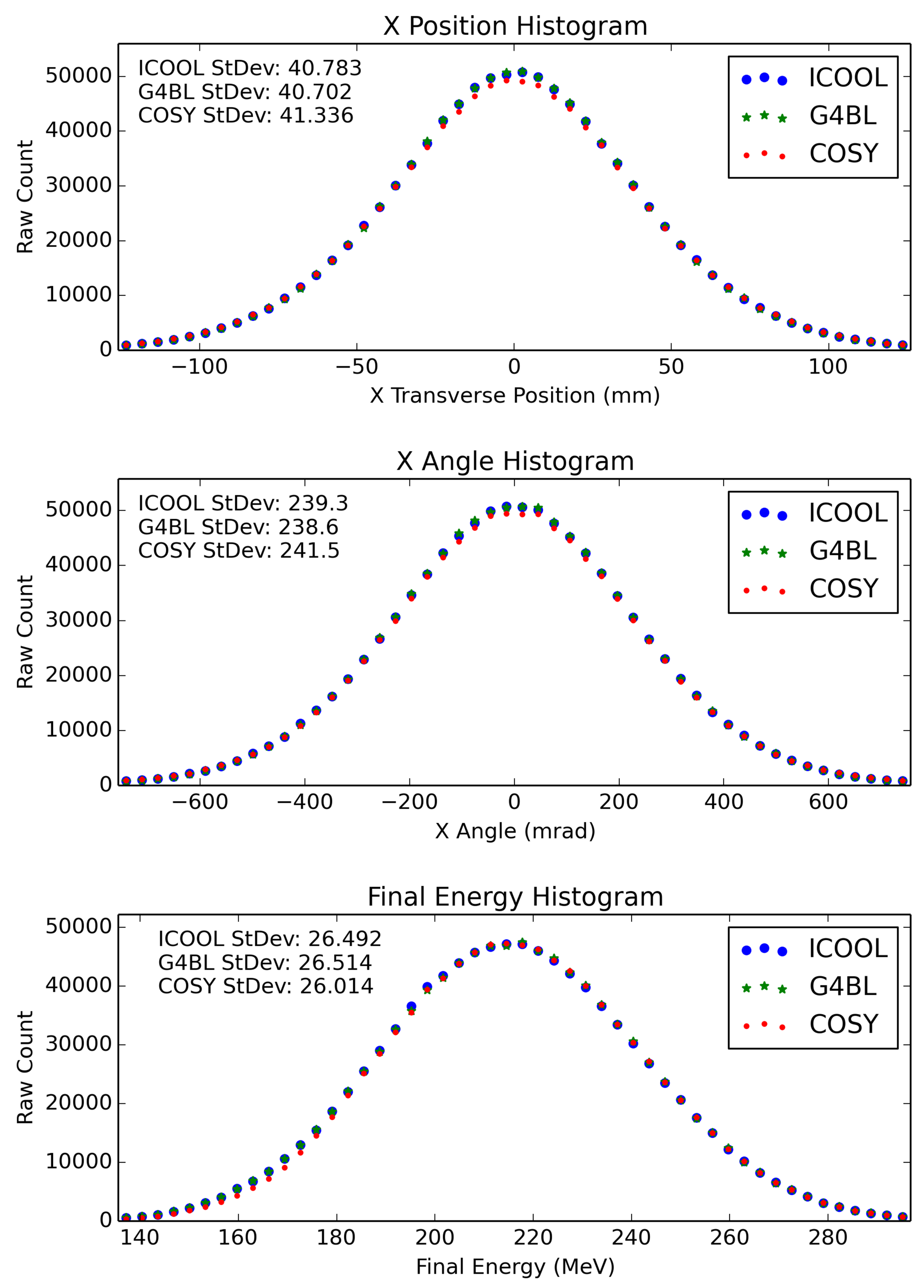
Name	z position mm	Length mm	Outer radius mm	Current density A/mm^2
End2	$\mp$3200	111	326	$\pm$126
Center	$\mp$2450	1314	280	$\pm$148
End1	$\mp$1700	111	319	$\pm$133
Match2	$\mp$1300	199	289	$\pm$132
Match1	$\mp$861	201	304	$\pm$133
Focus	$\mp$202	213	362	$\pm$104

MICE Results

The results of the MICE simulation with 350 mm of liquid hydrogen:



Simulation with 65 mm of lithium hydride:



Computational times for liquid hydrogen are summarized below. The new hybrid approach in COSY is about twice as fast as G4Beamline and about three times as fast as ICOOL.

Number of particles:	10^6	10^5	10^4	10^3
COSY:	367	31	6	4
G4BL (coils):	3973	392	40	6
G4BL (field map):	662	75	15	9
ICOOL (field map):	1091	117	19	9

References

[1] K. Makino, M. Berz, COSY INFINITY Version 9, *Nuclear Instruments and Methods* **A558** (2005) 346-350, see also `http://www.cosyinfinity.org`

[2] R. C. Fernow *et al.*, ICOOL Simulation Code, `http://www.cap.bnl.gov/ICOOL/`

[3] T. Roberts, G4Beamline, `http://www.muonsinternal.com/muons3/G4Beamline`

[4] D. Attwood *et al.* (2006), The scattering of muons in low-Z materials, NIM B251, p. 41

[5] A. Blondel *et al.*, (2003), Tech. rep., MICE-NOTE-21, `http://mice.iit.edu/mnp/MICE0021.pdf`