Homework 2 (Due: October 2nd, 2021)

Math-M-Addicts Group A

2021-2022

Problem 1. (Answer only) Let $ABCD$ be a square with $|AB| = 5$. Let r_1 and r_2 be two rays starting at point A such that the angle between them is 45°. Let S be the area of the piece of $ABCD$ contained between r_1 and r_2. Find the maximal possible value of S.

Problem 2. (Answer only) An ordered pair of positive integers (m,n) is called neat if there exists a partition of the set of positive integers into sets A and B, such that no difference between two elements of A or two elements of B is either m or n. Find the number of neat pairs (m,n) with $1 \le m,n \le 32$.

Problem 3. Let A be a 101-element subset of $\{0,1,2,....,1000\}$. Let B be the set of pairwise differences between elements of A. Prove that B contains at least 10 positive numbers no greater than 100.

Problem 4. Let $ABCD$ be a convex quadrilateral inscribed in a semicircle with diameter AB. The lines AC and BD intersect at E and the lines AD and BC meet at F. The line EF meets the semicircle at G and AB at H. Prove that E is the midpoint of GH if and only if G is the midpoint of the line segment FH.

Problem 5. Find all integers of the form 2^n (with positive integer $n > 3$) which would remain a power of two after you delete their leftmost digit.

Problem 6. Let $ABCD$ be a quadrilateral such that all sides have equal length and $\angle ABC = 60^\circ$. Let k be a line through D and not intersecting the quadrilateral (except at D). Let E and F be the intersection of k with lines AB and BC respectively. Let M be the point of intersection of CE and AF. Prove that $CA^2 = CM \cdot CE$.

Problem 7. ♦ Let ABC be a fixed acute triangle inscribed in a circle ω with center O. A variable point X is chosen on minor arc AB of ω, and segments CX and AB meet at D. Denote by O_1 and O_2 the circumcenters of triangles ADX and BDX, respectively. Determine all points X for which the area of triangle OO_1O_2 is minimized.