Homework 3 (Due: October 9th, 2021)

Math-M-Addicts Group A

2021-2022

Problem 1. (Answer only) Let $x = \dfrac{\sum\limits_{n=1}^{44} \cos n^\circ}{\sum\limits_{n=1}^{44} \sin n^\circ}$. What is the greatest integer that does not exceed $100x$?

Problem 2. (Answer only) Five distinct integers are chosen from set $\{1, 2, ..., 90\}$ are chosen at random and ordered $V < W < X < Y < Z$. Find $\lfloor 10\mathbb{E}[Y] \rfloor$.

Problem 3. Let $P(x)$ be a polynomial with positive coefficients. Prove that if $P(\frac{1}{x}) \geq \frac{1}{P(x)}$ holds for $x = 1$ then it holds for all $x > 0$.

Problem 4. Some parliament set up several committees, so that any committee has at least two members, and every two committees have at least one person in common. Prove that you can give all member of the parliament a red, white or blue scarf, so each committee has at least two colors of scarves among their members.

Problem 5. Prove that any polynomial $P(x)$ with real coefficients can be represented as a difference of polynomials $P(x) = Q(x) - R(x)$, where both Q and R are increasing functions of x.

Problem 6. Let $x, y, z \geq 0$. Prove that $8(x^3 + y^3 + z^3)^2 \geq 9(x^2 + yz)(y^2 + zx)(z^2 + xy)$

Problem 7. Show that for $a, b, c > 0$ we have $\dfrac{1}{(a+b)^2} + \dfrac{1}{(b+c)^2} + \dfrac{1}{(c+a)^2} \geq \dfrac{9}{4(ab+bc+ca)}$.