ENEL420 Formula Sheet

The bitchiest of all the exams

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1 Interpolation

Interpolation, convolution with a kernel function:

$$f(k_0) = \sum_k f(k) \underbrace{K(k_0 - k)}_{\text{convolution kernel}} \tag{1}$$

Where $f(k_0)$ is the interpolated signal and $f(k)$ is the data

1.1 Kernels

Nearest Neighbour:

$$K(x) = \mathrm{rect}(x) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Linear:

$$K(x) = \mathrm{tri}(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Sinc:

$$K(x) = \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \tag{4}$$

2 Aliasing

General Case to avoid aliasing:

$$\frac{2f_u}{n} < f_s < \frac{2f_l}{n-1} \tag{5}$$

Where,

$$1 \leq n \leq I_{MLT}\left(\frac{f_u}{B}\right) \tag{6}$$

I_{MLT} is Maximum integer less than

3 Z-Transform

$$F(z) = \sum_{n=0}^{\infty} f(n) z^{-n} \tag{7}$$

Or,

$$F(s) = \sum_{n=0}^{\infty} f(nT) e^{-nTs} \tag{8}$$

4 Quantisation Noise

RMS quantisation noise:

$$\sigma = \frac{\Delta}{\sqrt{12}} = \frac{\Delta}{2\sqrt{3}} \tag{9}$$

Noise Power:

$$\text{NP} \propto \Delta^2 \tag{10}$$

Therefore is noise power is decreased by $\frac{1}{2}$,

$$10\text{log}_{10} \left(\frac{1}{2}^2 \right) = 6\text{dB} \tag{11}$$

5 Filters

Direct realisation:

$$H(z) = \frac{\sum_{k=0}^{K} b_k z^{-k}}{1 + \sum_{m=1}^{M} a_m z^{-m}} \tag{12}$$

Linear Phase, if a filter has linear phase it can be expressed in the form:

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)} \tag{13}$$

Where,

$$\theta(\omega) = -(\alpha\omega + \beta) \tag{14}$$

Windowing the impulse response is defined by the inverse Fourier Transform,

$$h_D(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_D(\omega) e^{j\omega n} \quad d\omega \tag{15}$$

non-recursive FIR Filters,

$$y(n) = b_0 x(n) + b_1 x(n-1) \tag{16}$$

Recursive,

$$y(n) = ay(n-1) + bx(n) \tag{17}$$

So,

$$H(z) = \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{\frac{j2\pi kn}{N}} z^{-n} \tag{18}$$

6 Detection Theory

Likelihood Ratio,

$$\frac{p(\mathbf{x}|H_0)}{p(\mathbf{x}|H_1)} \tag{19}$$

6.1 Likelihood ratio test

Choose null hypothesis if,

$$\frac{p(\mathbf{x}|H_0)}{p(\mathbf{x}|H_1)} > \gamma \tag{20}$$

Choose alternitive hypothesis if,

$$\frac{p(\mathbf{x}|H_0)}{p(\mathbf{x}|H_1)} \leq \gamma \tag{21}$$

6.2 Log Likelihood

Choose null hypothesis if,

$$-x[0] + \frac{1}{2} > 0 \leftrightarrow x[0] < \frac{1}{2} \tag{22}$$

Choose alternitive hypothesis if,

$$-x[0] + \frac{1}{2} \leq 0 \leftrightarrow x[0] \geq \frac{1}{2} \tag{23}$$

Solve for γ using,

$$P_{FA} = \int_{\mathbf{x}:L(\mathbf{x})<\gamma} p(\mathbf{x}|H_0) d\mathbf{x} \tag{24}$$

6.3 Missed Detection

The probability of missed detection is,

$$P_M = p(x[0] \leq \frac{1}{2}|H_1) = p(x[0] \leq \frac{1}{2}|u = 1) \tag{25}$$

6.4 Detection Error Rate

$$P_{FA}P(H_0) + P_M P(H_1) \tag{26}$$

Where P_{FA} is the probability of falsely selecting the alternative hypothesis and P_M is the probability of falsely selecting the null hypothesis.

6.5 Detection Threshold