ASSIGNMENT COVER SHEET

Name	Jos Craw
Student Number	35046080
Student Email	jcr124@uclive.ac.nz
Date of Submission	20/05/2020

Question	Max [%]	Mark [%]
Transmitter	/20	
a	5	
b	5	
c	5	
d	5	
Receiver	/40	
e	10	
f	10	
g	20	
Equaliser	/40	
h	20	
i	10	
j	10	
TOTAL	100	

ASIDE: Running the code

The source code for this document as well as my code can be found at:
`https://git.sys-io.net/scm/enel422/assignment-1`
This will also have all of the fully rendered README files

Contents

1 Transmitter and pulse shaping

1.1 Real World Feasibility

1.1.1 Binary Polar Signalling

Binary polar signalling is a very robust signalling method as is has a much higher noise tolerance. However, it has a much lower throughput compared to other options such as 4-PAM and 8-PAM. The noise tolerance makes binary polar signalling suitable for wireless communications as this environment can have much more noise over a much further distance.

1.1.2 4-PAM

4-PAM is also a reasonably robust signalling method as there is still some noise tolerance as the different levels are reasonably well defined. This signaling method also allows double the data rate when compared to binary polar signalling. These properties make this method well suited to wired communication as noise is less noticeable in the signal as the medium can be controlled.

1.1.3 8-PAM

8-PAM however, has comparatively poor noise performance, which could require significant error correction with large overhead which could negate the improved throughput compared to other methods.

1.1.4 Recommedation

Overall, I would recommend using 4-PAM as it has a strong balance between data through put and noise tolerance. As 8-PAM has only 50% more throughput but half the separation between signals of 4-PAM. Overall the double throughput of 4-PAM to binary signalling is worth the decreased noise resilience.

1.2 Power Spectral Density of 4-PAM (Rectangular Pulse Shaping)

With a rectangular pulse shaping the PSD of a 4-PAM signal is shown in () and the values for S_x and R_n can be found in x and x, x respectively.

$$R_n = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k-n} \tag{1}$$

Assuming that the data is random:

$$P_{a_k} = \begin{cases} \frac{1}{4} & a_k = 3 \\ \frac{1}{4} & a_k = 1 \\ \frac{1}{4} & a_k = -1 \\ \frac{1}{4} & a_k = -3 \end{cases} \quad \text{Therefore,} \quad R_n = 0 \tag{2}$$

As in the previous equation the probabilities are equal for all a_k,

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k {a_k}^2 \quad \text{With,} \quad P_{{a_k}^2} = \begin{cases} \frac{1}{2} & a_k^2 = 9 \\ \frac{1}{2} & a_k^2 = 1 \end{cases} \tag{3}$$

Therefore,

$$R_0 = 5 \tag{4}$$

$$S_x(f) = \frac{1}{T_s}[R_0 + 2\sum_{n=1}^{\infty} R_n \cos 2\pi n f T_s] \tag{5}$$

As,

$$T_b = \frac{1}{R_b} \quad \text{Therefore with a bitrate of 1Mbps,} \quad T_b = \frac{1}{1 \times 10^6} \tag{6}$$

As 4-PAM has two bits per pulse $T_s = 2T_b$ so $T_s = 2\mu s$ Therefore,

$$S_x(f) = \frac{1}{T_s}[5 + 0] \quad \text{For } T_s = 2\mu s \quad S_x = 2500000 \tag{7}$$

The equation for PSD is,

$$S_y(f) = \left|\frac{T_s}{2}\text{sinc}(\frac{f\pi T_s}{2})\right|^2 S_x(f) \tag{8}$$

So,

$$S_y(f) = \left|\frac{T_s}{2}\text{sinc}(\frac{f\pi T_s}{2})\right|^2 (2500000) \tag{9}$$

This results in the following PSD,

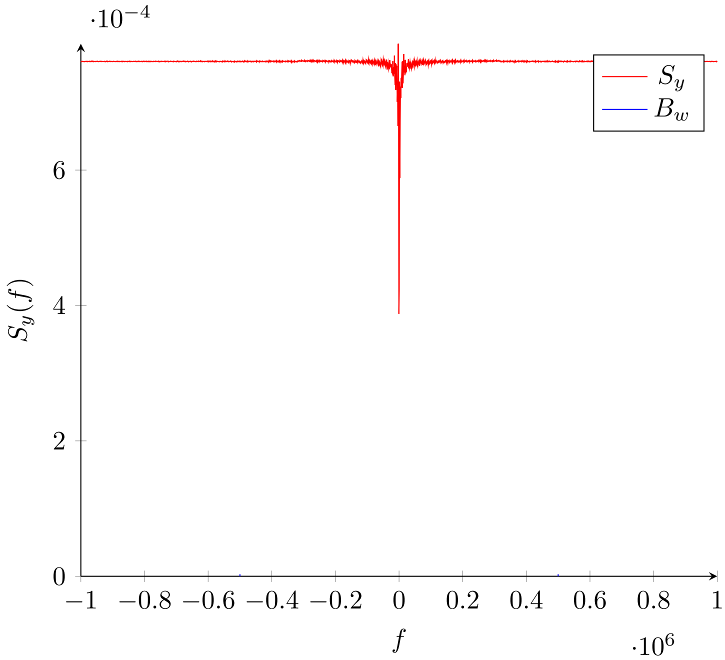


Figure 1: PSD of a 4-PAM Signal with rectangular pulse shaping

1.3 Power Spectral Density of 4-PAM (Nyquist Pulse Shaping)

The for the design of a Niquist pulse a roll off factor of $\alpha = 1$ was used as there is adequite bandwidth for the signal and the high roll off causes the tail oscillations of the pulse to fade more quickly, reducing the chance of intersymbol interference (ISI)

The S_x value is the same as above, using a root raised cosine pulse shaping the pulse is:

$$P(f) = \begin{cases} \frac{1}{4W}[1+\cos(\frac{\pi f}{2W})] \quad \text{when} \quad 0 < |f| < 2W \\ 0 \qquad \text{when} |f| \geq 2W \end{cases} \tag{10}$$

Therefore $|P(f)|^2$ is,

$$|P(f)|^2 = \begin{cases} \frac{1}{16W^2}[1+\cos(\frac{\pi f}{2W})]^2 \quad \text{when} \quad 0 < |f| < 2W \\ 0 \quad \text{when} \quad |f| \geq 2W \end{cases} \tag{11}$$

As $2W = \frac{1}{T_s}$,

$$|P(f)|^2 = \begin{cases} \frac{T_s^2}{4}[1+\cos(\pi f T_s)]^2 \quad \text{when} \quad 0 < |f| < \frac{1}{T_s} \\ 0 \quad \text{when} \quad |f| \geq \frac{1}{T_s} \end{cases} \tag{12}$$

Therefore the power spectral density is,

$$S_y(f) = \frac{5T_s}{4}[1+\cos(\pi f T_s)]^2 \tag{13}$$

This results in the following plot,

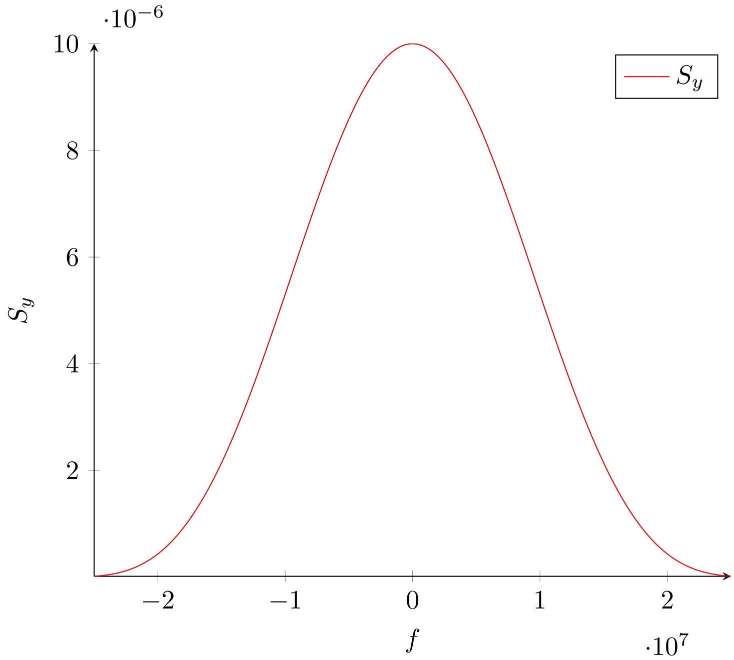


Figure 2: The PSD of a RRCOS pulse shaped 4-PAM Signal

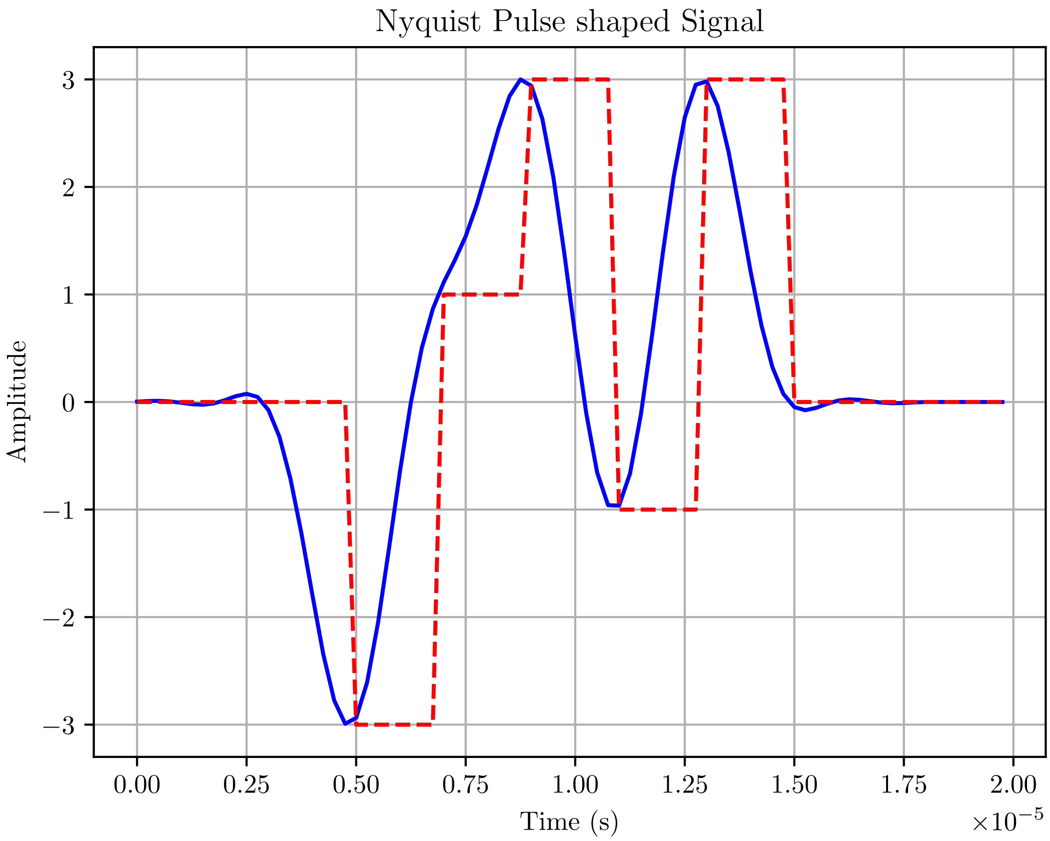


Figure 3: A Nyquist Pulse Shaped Signal carrying the data '0010110111'

1.4 Pulse shaping a signal with a Nyquist pulse

2 Receiver

2.1 Matched Reciever Eye Diagram

The eye diagram of a signal comprised of 10,000 random bits is shown in Figure, this example has a SNR= ∞.

2.2 Eye Diagram with a SNR=10dB

As noise is added to the received signal the 'eye' in the diagram will appear to close. This is because the opening of the eye shows the separations between the signal levels at the ideal sampling instances but as noise is introduced the different levels become harder to distinguish from each other, this is shown on the eye diagram as the 'eye' starting to close. An eye diagram from a signal of 10,000 random bits and a SNR=10dB can be found in Figure

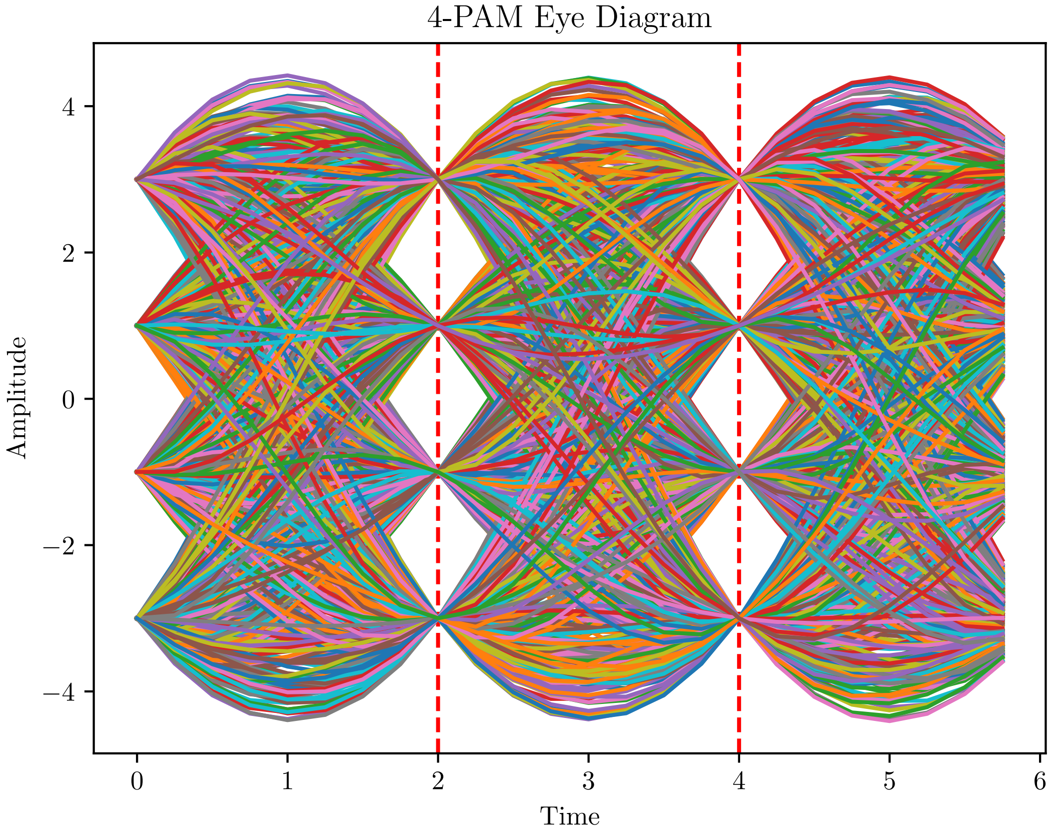


Figure 4: Eye Diagram of a signal passed through a matched filter with a SNR=∞

2.3 Bit error rate perfomance

2.4 Bit and symbol errors

I am unlikely to be able to accurately predict the bit error rate with E_b/N_0 of greater than 10dB as the probability of the noise being large enough to cause a bit error is very low, this means that a huge number of message bits would have to be simulated before any bit errors could occur. This would cause the simulation to take far to long to complete the simulation. To reliably predict the bit error rate the efficiency of the simulation would have to be increased or use greater computing power to simulate more data quickly.

3 Equaliser

3.1 Intersymbol Interference

$$R_y[m] = E[(\sum_k a_k h[i+m-k] + w[i+m])(\sum_j a_j^* h_j^*[i-j] + w[i])] \tag{14}$$

$$E[a_k a_j^*] = \begin{cases} E_a & j = k \\ 0 & j \neq k \end{cases} \tag{15}$$

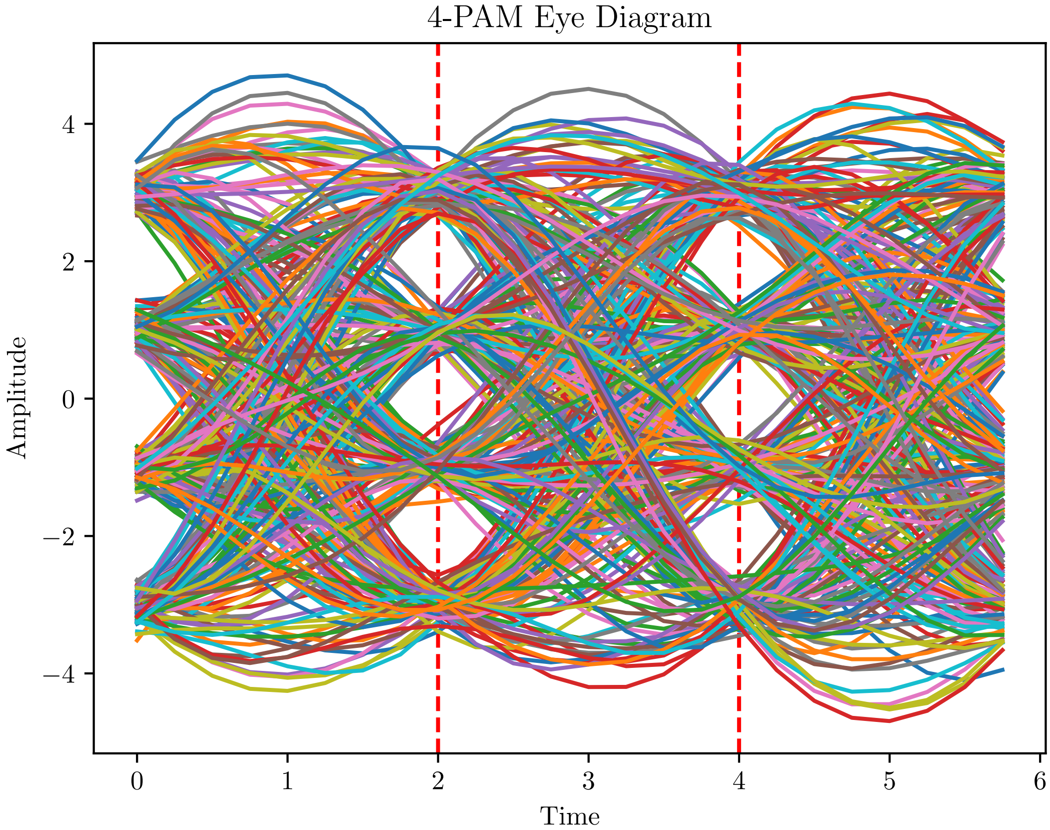


Figure 5: Eye Diagram of a signal passed through a matched filter with a SNR=10dB

$$E[a_k w^*[j]] = 0 \tag{16}$$

$$E[w[j]w^*[k]] = \begin{cases} \sigma_w^2 = \frac{N_0}{2} & j = k \\ 0 \quad j \neq k \end{cases} \tag{17}$$

$$\therefore R_y[m] = \sum_k E_a h[i+m-k]h^*[i-k] + \frac{N_0}{2}\delta[m] \tag{18}$$

Hence,

$$R_y[m] = E_a \sum_j h[m+j]h^*[j] + \frac{N_0}{2}\delta[m] \tag{19}$$

3.2 MMSE Equaliser

3.3 Displaying a eye diagram with lab equipment

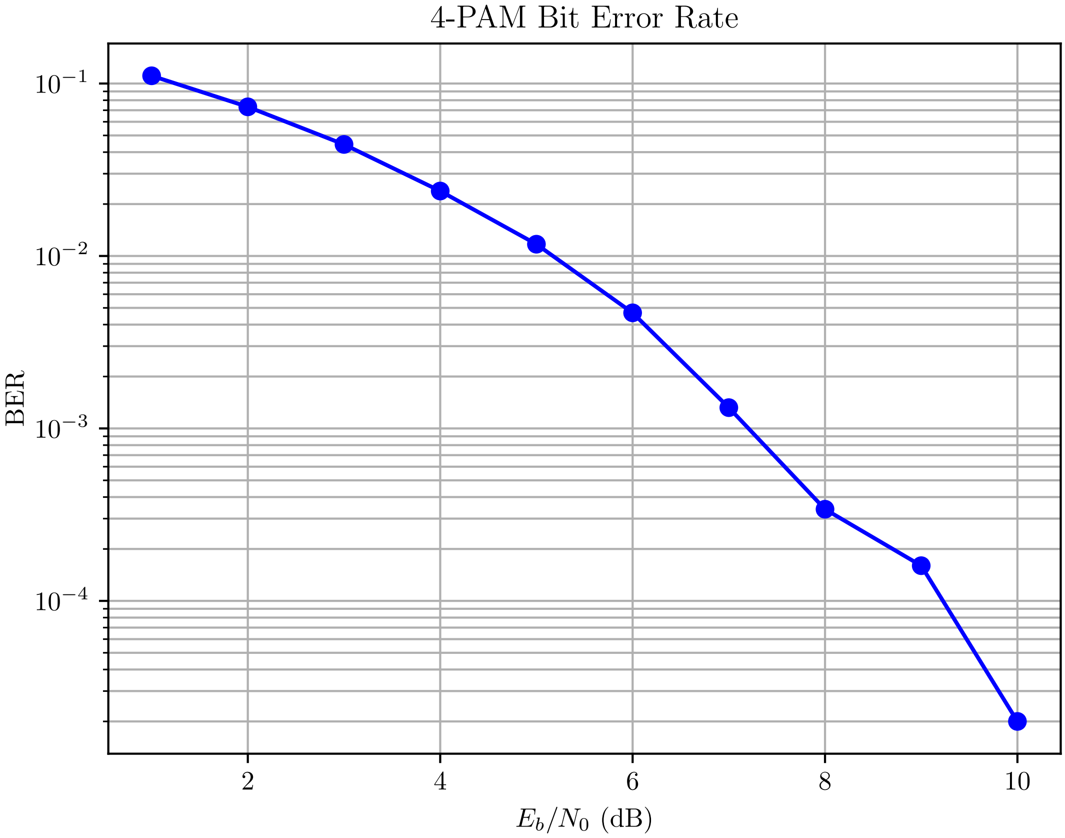


Figure 6: BER perfomance of 100,000 bits with an SNR from 1 to 10dB with a step of 1dB

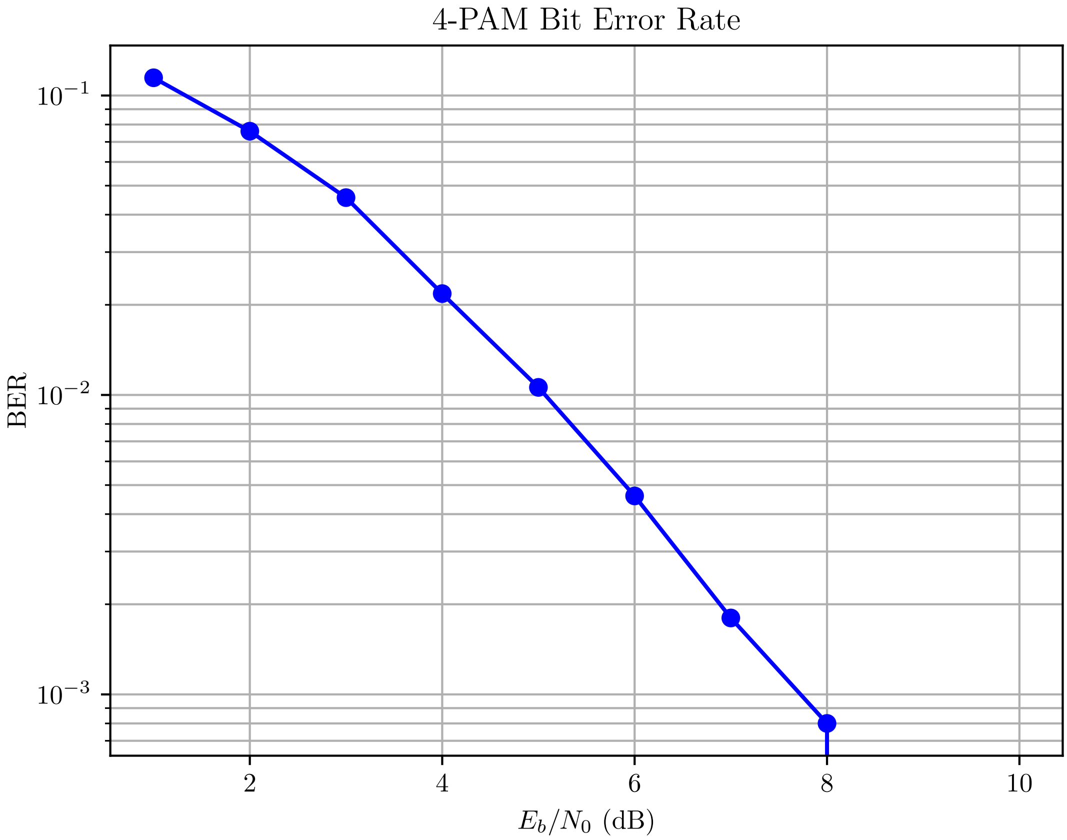
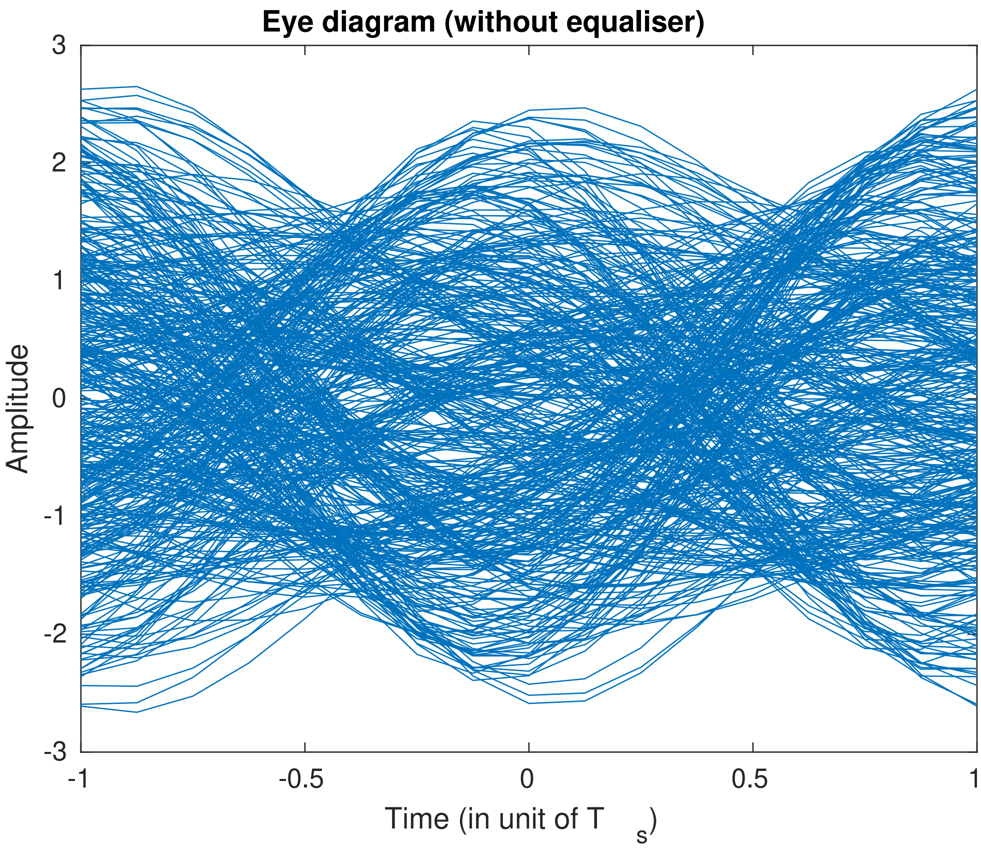
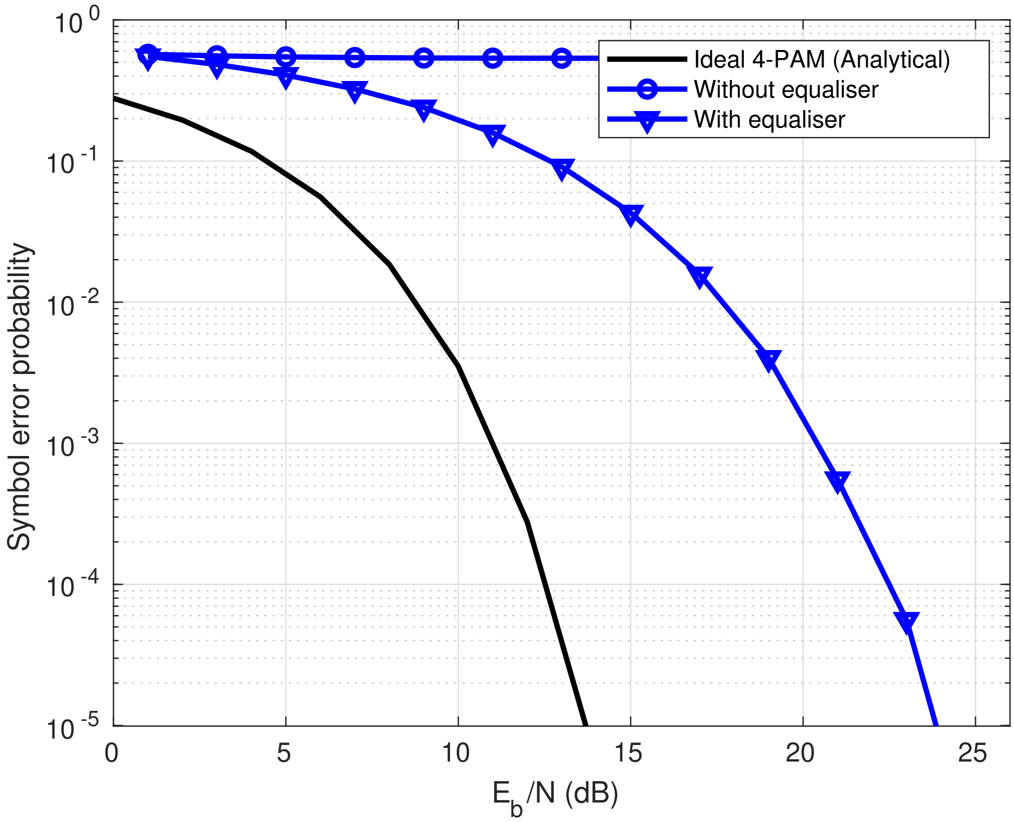


Figure 7: BER perfomance of 10,000 bits with an SNR from 1 to 10dB with a step of 1dB



Eye diagram (without equaliser)
Amplitude
Time (in unit of T s)
3
2
1
0
-1
-2
-3
-1
-0.5
0
0.5
1

Figure 8:



Ideal 4-PAM (Analytical)
Without equaliser
With equaliser
Symbol error probability
E_b/N (dB)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
5
10
15
20
25

Figure 9:

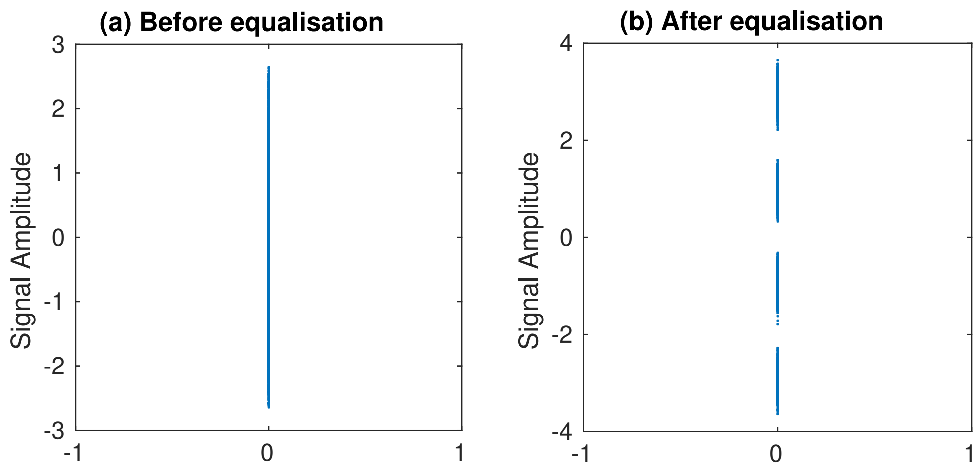

(a) Before equalisation
Signal Amplitude
3
2
1
0
-1
-2
-3
-1
0
1
(b) After equalisation
Signal Amplitude
4
2
0
-2
-4
-1
0
1

Figure 10: