ASSIGNMENT COVER SHEET

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Date of Submission	20/05/2020

Question	Max [%]	Mark [%]
Transmitter	/20	
a	5	
b	5	
c	5	
d	5	
Receiver	/40	
e	10	
f	10	
g	20	
Equaliser	/40	
h	20	
i	10	
j	10	
TOTAL	100	

ASIDE: Running the code

The source code for this document as well as my code can be found at:
`https://git.sys-io.net/scm/enel422/assignment-1`
This will also have all of the fully rendered README files

ENEL422 Communications Baseband Processing Assignment

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Contents

1 Transmitter and pulse shaping

1.1 Real World Implementation

To meet the specifications the system must have a data rate of 1Mbps and a bandwidth of no greater then 500kHz. This results in a bit period (T_s) of 1μs, As Nyquist pulse shaping is used the Nyquist bandwidth is $B_w/2 = 500$kHz. Using Nyquist pulse shaping allows data transfer over a bandwidth limited channel such as the one given in the specifications. For real world use the ideal Nyquist pulse cannot be used as it is not physically realisable due to the infinite frequencies contained within the sinc pulse produced in the frequency domain. This can be solved by not using an ideal Nyquist pulse such as a root raised cosine or a raised cosine pulse as these have the same characteristics as the Nyquist pulse where they are zeros at all time periods other than at '0', this stops a past symbol from adding to the current signal .

1.1.1 Binary Polar Signalling

Binary polar signalling is a very robust signalling method as is has a high noise tolerance. This is due to the large distance between the values for '1' and '0' this is advantageous when transmitting over a noisy medium as the signal is less likely to have bit errors when it is received. However, it has a much lower throughput compared to other options such as 4-PAM and 8-PAM. This transmission type also will introduce distortion due to the bandwidth limitations of the channel as there will be tail frequencies that are removed. The noise tolerance makes binary polar signalling suitable for wireless communications as this environment can have much more noise over a much further distance.

1.1.2 4-PAM

PAM signalling has the advantage of being able to transfer more data per signal period, in the case of 4-PAM two data bits are transferred per symbol period. These bit are encoded in the amplitude of the pulse the encoding is found in Table 1.1.2. This encoding allows 4-PAM to have double the transmission rate of binary polar signaling. This extra bandwidth allows the Nyquist pulse to use more bandwidth to reduce the intersymbol interference (ISI) of the system by using a Nyquist pulse with a higher roll off, reducing the impact of ISI for future bits if the receiver samples incorrectly. 4-PAM however, has a lower noise tolerance when compared to binary polar signalling as the distance between the symbols is less as there a more values period making the signal less noise tolerant.

Table 1: 4-PAM Encoding

'00'	-3
'01'	-1
'10'	1
'11'	3

1.1.3 8-PAM

8-PAM has the same advantages as 4-PAM in regards to throughput with double the levels of 4-PAM, 8-PAM is able to transmit 3 bits per symbol. This however, comes with worse noise tolerance than 4-PAM as the different levels are even closer together this causes the signal to become much less noise tolerant than either 4-PAM or binary polar signalling.

1.1.4 Recommedation

Overall, 4-PAM best matches the requirements as it would have low distortion due to the excess bandwidth available to use a Nyquist pulse with a high roll off factor. It also has higher noise tolerance when compared to 8-PAM and as the extra spectral efficiency gained through 8-PAM is not required by the specifications 4-PAM is the most suitable choice.

1.2 Power Spectral Density of 4-PAM (Rectangular Pulse Shaping)

The equation for the power spectral density (PSD) of a line code signal is shown in (1) where $|P(f)|^2$ is the energy spectral density of the pulse and S_Y is the power spectral density of the data.

$$S_Y(f) = |P(f)|^2 S_X(f) \tag{1}$$

S_X is defined as (2) where R_n is the average of average power of all of the signal data levels as shown in (3).

$$S_X = \frac{1}{T_s}[R_0 + 2\sum_{n=1}^{\infty} R_n \cos 2\pi n f T_s] \tag{2}$$

$$R_n = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k a_{k-n} \tag{3}$$

Assuming that the data is random the probability of each data level is equal.

$$P_{a_k} = \begin{cases} \frac{1}{4} & a_k = 3 \\ \frac{1}{4} & a_k = 1 \\ \frac{1}{4} & a_k = -1 \\ \frac{1}{4} & a_k = -3 \end{cases} \tag{4}$$

Therefore as R_n is the average power of all data and each of the symbols is equally likely the average is $R_n = 0$

As in the (4 the probabilities are equal for all a_k therefore a_k^2 results in two equally likely options as shown in (5),

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\sum_k {a_k}^2 \quad \text{With,} \quad P_{{a_k}^2} = \begin{cases} \frac{1}{2} & a_k^2 = 9 \\ \frac{1}{2} & a_k^2 = 1 \end{cases} \tag{5}$$

Therefore as $a_k^2 = 9 \quad \text{or} \quad a_k^2 = 1$ the average comes to,

$$R_0 = 5 \tag{6}$$

The required bit period can be derived from the required data rate.

$$T_b = \frac{1}{R_b} \quad \text{Therefore with a bitrate of 1Mbps,} \quad T_b = \frac{1}{1\times 10^6} \tag{7}$$

As 4-PAM has two bits per pulse $T_s = 2T_b$ so $T_s = 2\mu$s Therefore,

$$S_X(f) = \frac{1}{T_s}[5+0] \quad \text{For } T_s = 2\mu\text{s} \quad S_x = 2500000 \tag{8}$$

The rectangular pulse is defined as,

$$p(t) = \text{rect}(\frac{t}{T_s}) \tag{9}$$

$$P(f) = T_s \text{sinc}(\pi f T_s) \tag{10}$$

The equation for PSD is,

$$S_Y(f) = \left| T_s \text{sinc}(f\pi T_s) \right|^2 S_x(f) \tag{11}$$

So,

$$S_Y(f) = \left| T_s \text{sinc}(f\pi T_s) \right|^2 (2500000) \tag{12}$$

As shown in the PSD in Figure 1.2 the rectangular pulse signal has frequencies present outside the bandlimits, the removal of these would result in a distorted signal. This shows that this signal does not meet bandwidth specification.

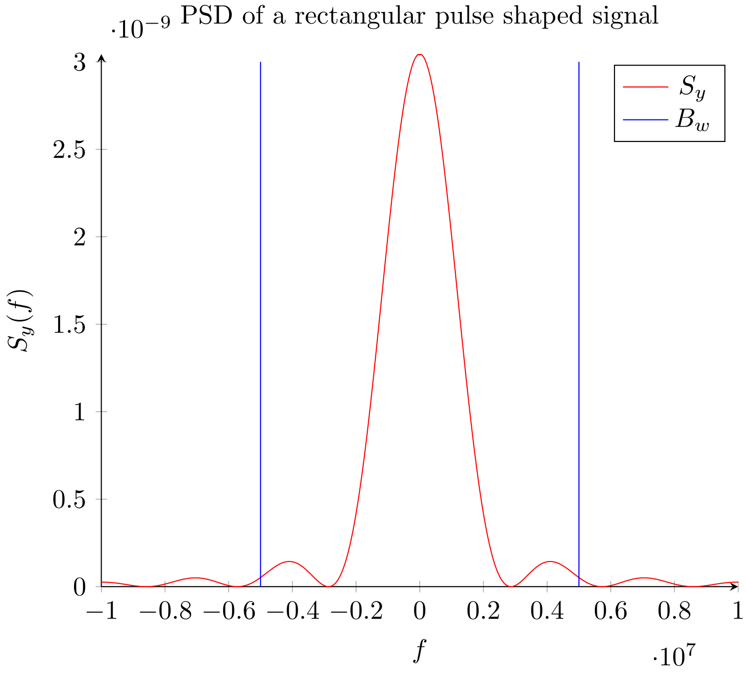

Figure 1: PSD of a 4-PAM Signal with rectangular pulse shaping

The simulated PSD of a RCOS pulse is shown in Figure 1.2.

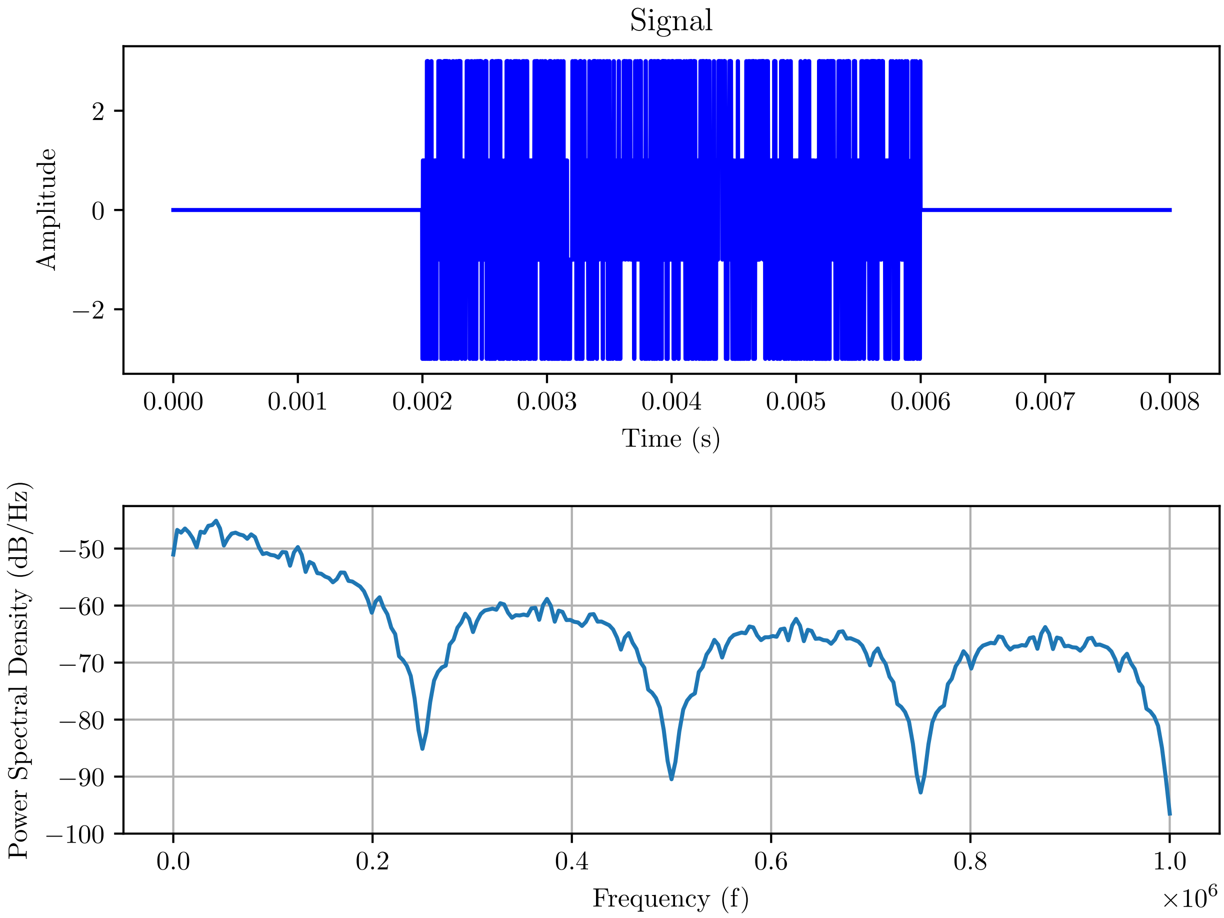

Figure 2: The simulated PSD of a 1000 bit long message

1.3 Power Spectral Density of 4-PAM (Nyquist Pulse Shaping)

The for the design of a Nyquist pulse a roll off factor of $\alpha = 1$ was used as there is adequate bandwidth for the signal and the high roll off causes the tail oscillations of the pulse to fade more quickly, reducing the chance of intersymbol interference (ISI)

The S_x value is the same as above, using a root raised cosine pulse shaping the pulse is:

$$P(f) = \begin{cases} \frac{1}{4W}[1 + \cos(\frac{\pi f}{2W})] \quad \text{when} \quad 0 < |f| < 2W \\ 0 \qquad \text{when} |f| \geq 2W \end{cases} \tag{13}$$

Therefore $|P(f)|^2$ is,

$$|P(f)|^2 = \begin{cases} \frac{1}{16W^2}[1 + \cos(\frac{\pi f}{2W})]^2 \quad \text{when} \quad 0 < |f| < 2W \\ 0 \quad \text{when} \quad |f| \geq 2W \end{cases} \tag{14}$$

As $2W = \frac{1}{T_s}$,

$$|P(f)|^2 = \begin{cases} \frac{T_s^2}{4}[1+\cos(\pi f T_s)]^2 & \text{when} \quad 0 < |f| < \frac{1}{T_s} \\ 0 \quad \text{when} \quad |f| \geq \frac{1}{T_s} \end{cases} \tag{15}$$

Therefore the power spectral density is,

$$S_y(f) = \frac{5T_s}{4}[1+\cos(\pi f T_s)]^2 \tag{16}$$

As shown is the PSD in Figure **??** there is no signal energy outside of the band limits given in the specifications, this means that the signal will not be distorted when bandlimited. This shows that using a root raised cosine pulse shape for a bandlimited channel will have better performance than the rectangular pulse shaping in Figure 1.2

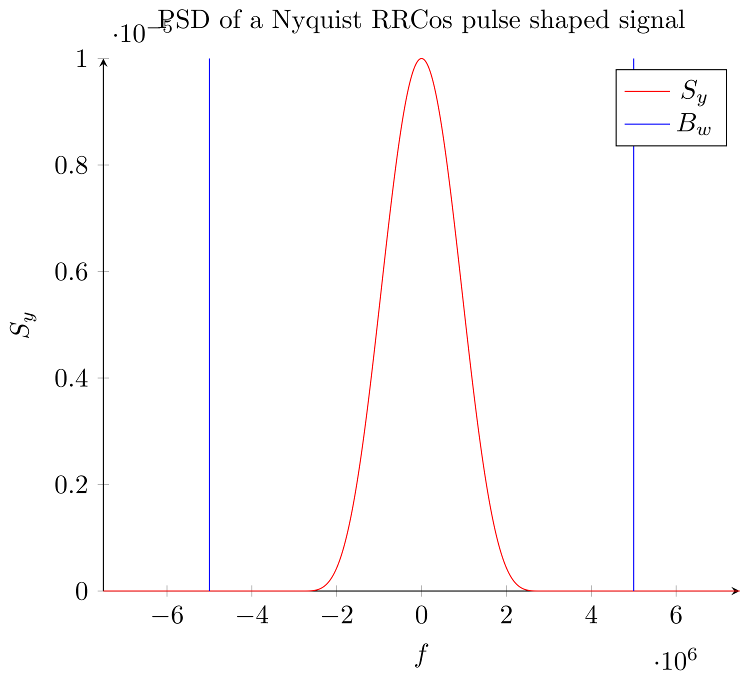

Figure 3: The PSD of a RRCOS pulse shaped 4-PAM Signal

1.4 Pulse shaping a signal with a Nyquist pulse

Figure 1.4 shows a raised cosine pulse shaped signal carrying the data '0010110111', this corresponds with the 4-PAM levels: [-3, 1, 3, -1, 3].

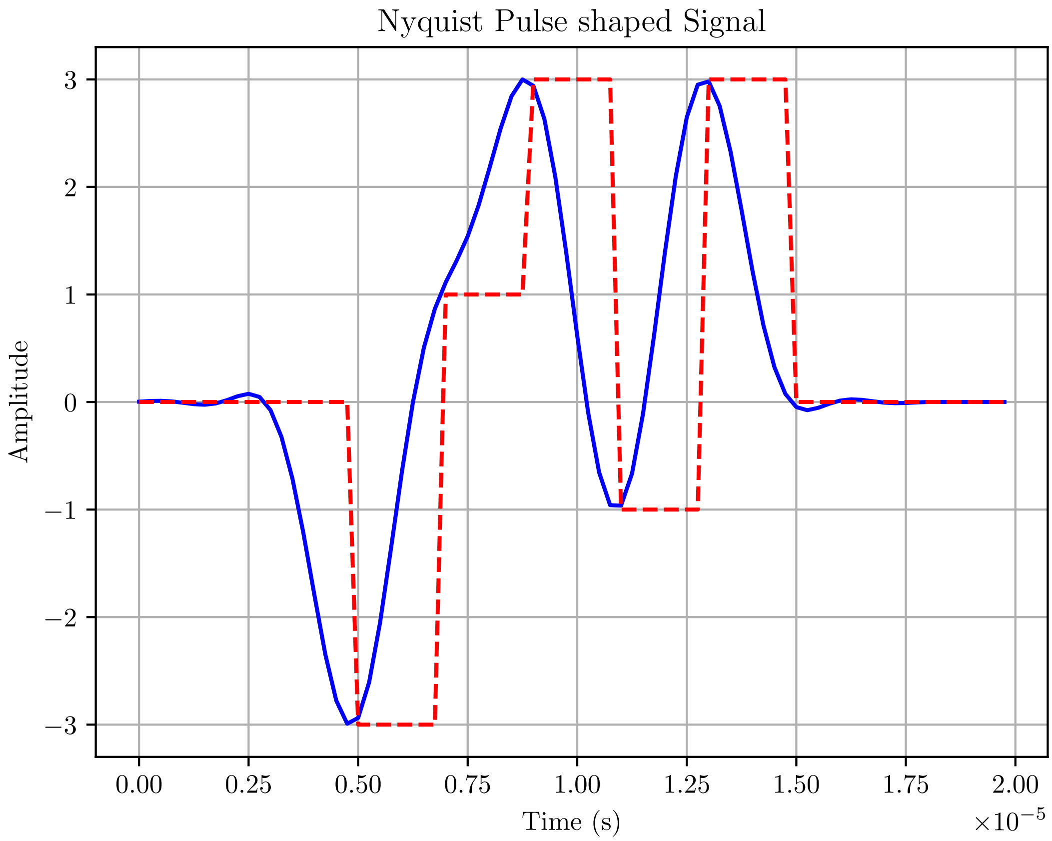

Figure 4: A Nyquist Pulse Shaped Signal carrying the data '0010110111'

2 Receiver

2.1 Matched Reciever Eye Diagram

The eye diagram of a signal comprised of 10,000 random bits is shown in Figure 5, this example has a SNR$=\infty$. The ideal sampling instances are indicated by the red dashed lines, these are the points that the eyes are the most open. This means that these points are sampling instances that have the most difference between the different levels and therefore that points that have the highest noise tolerance.

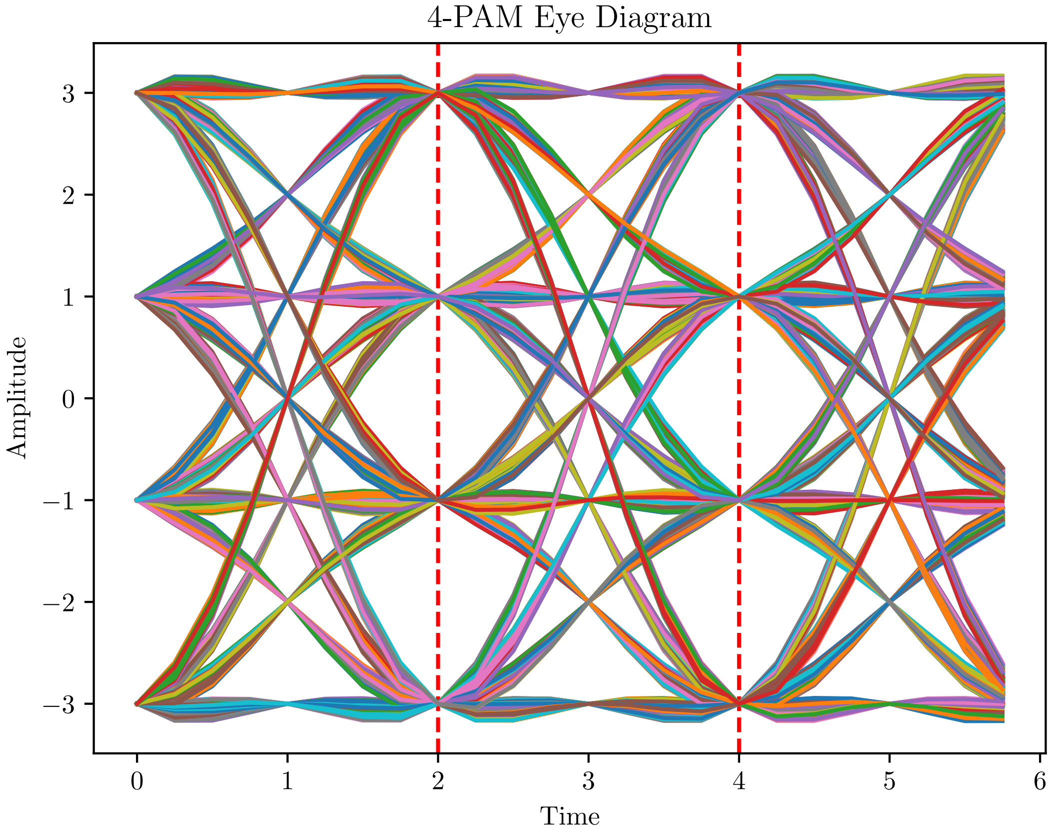

Figure 5: Eye Diagram of a signal passed through a matched filter with a SNR=∞

2.2 Eye Diagram with a SNR=10dB

As noise is added to the received signal the ‘eye’ in the diagram will appear to close. This is because the opening of the eye shows the separations between the signal levels at the ideal sampling instances but as noise is introduced the different levels become harder to distinguish from each other, this is shown on the eye diagram as the ‘eye’ starting to close. An eye diagram from a signal of 10,000 random bits and a SNR=10dB can be found in Figure 6

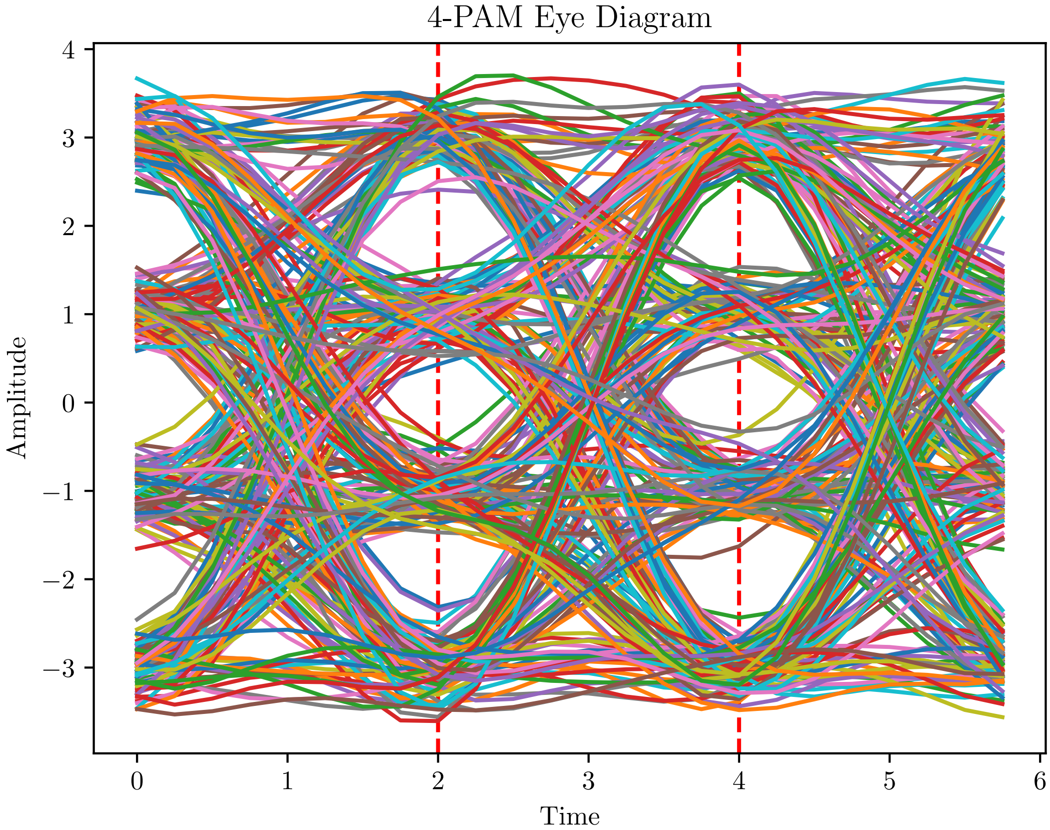

Figure 6: Eye Diagram of a signal passed through a matched filter with a SNR=10dB

2.3 Bit error rate performance

Figure 7 shows a simulation of 100,000 bits (50,000 symbols) that have been pulse shaped, had noise applied then passed through a match filter with the bit errors detected and the bit error rate plotted against the introduced noise. Figure 8 shows the same data, this time with only 10,000 bits (5000 symbols) simulated. These plots show that with only 10,000 bits the simulation is unable to detect any bit errors at 9 or 10 dB. However in the 100,000 bit simulation is able to dectect some bit errors at these levels. This shows the scale of bits required to show bit errors at just 10dB.

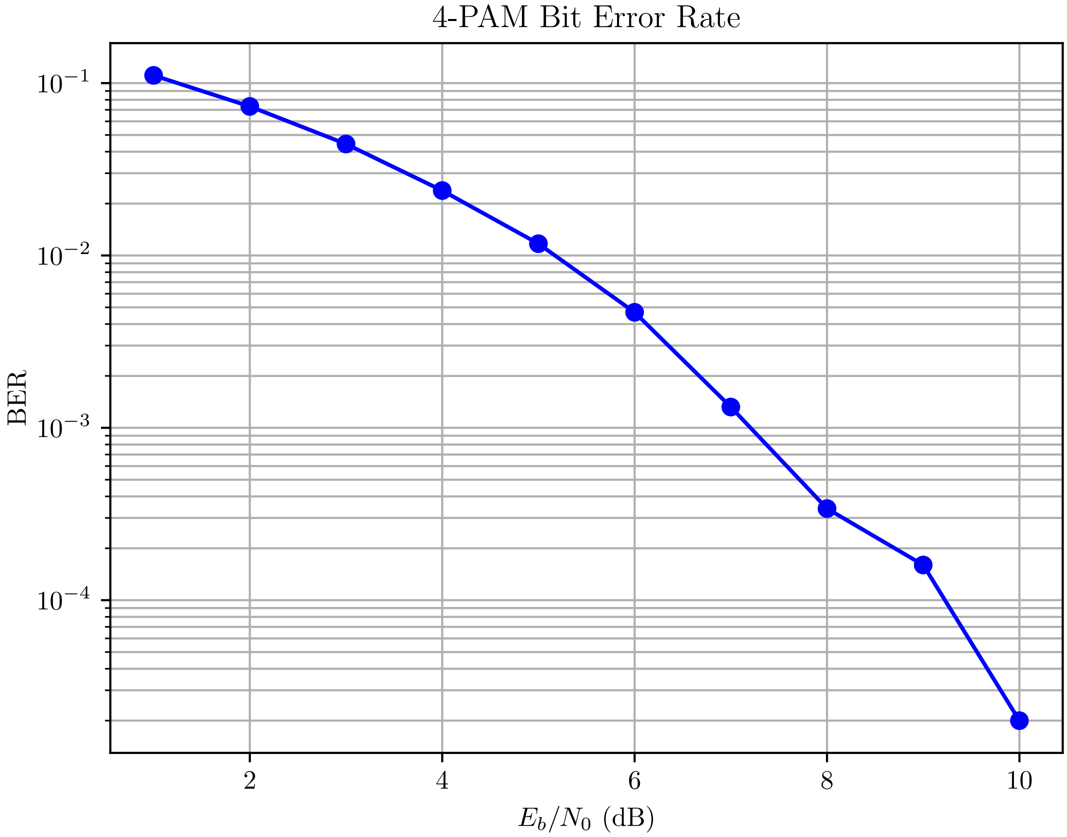

Figure 7: BER perfomance of 100,000 bits with an SNR from 1 to 10dB with a step of 1dB

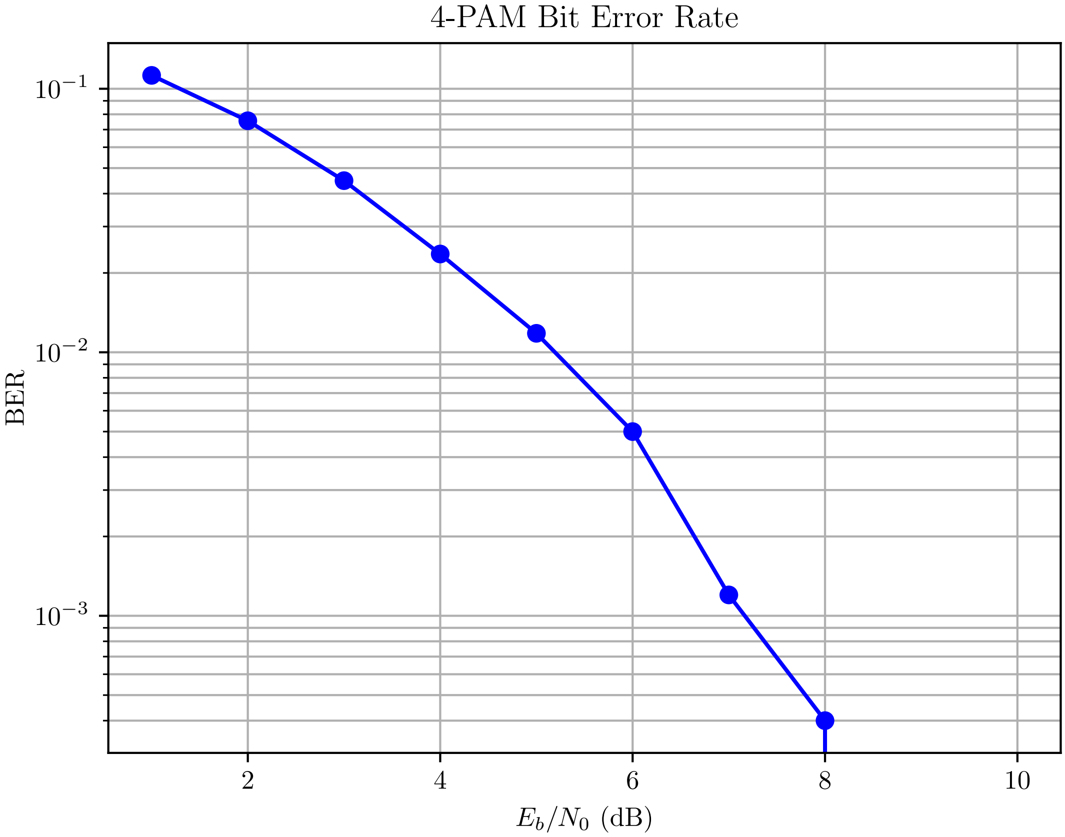

Figure 8: BER perfomance of 10,000 bits with an SNR from 1 to 10dB with a step of 1dB

2.4 Bit and symbol errors

A simulation unlikely to be able to accurately predict the bit error rate with E_b/N_0 of greater than 10dB as the probability of the noise being large enough to cause a bit error is very low, this means that a huge number of message bits would have to be simulated before any bit errors could occur. This is shown above where 100,000 bits had to be simulated to detect any bit errors for $E_b/N_0 =$ 10dB. This would cause the simulation to take far to long to complete the simulation. To reliably predict the bit error rate the efficiency of the simulation would have to be increased or use greater computing power to simulate more data quickly.

3 Equaliser

3.1 Intersymbol Interference

The recieved signal can be represented as:

$$y[i] = \sum_{k=-\infty}^{\infty} a_k h[i-k] + w[i] \tag{17}$$

The definition of autocorrelation coefficients:

$$R_y[m] \triangleq E[y[i+m]y^*[i]] \tag{18}$$

(19) Is a substitution of (17) into (18) with the conjugate of y[i] and i substituted with $i+m$

$$R_y[m] = E[(\sum_k a_k h[i+m-k] + w[i+m])(\sum_j a_j^* h_j^*[i-j] + w[i])] \tag{19}$$

(20) this step shows that two dependant orthogonal vectors are zero at all time except where the vector index is equal.

$$E[a_k a_j^*] = \begin{cases} E_a & j = k \\ 0 & j \neq k \end{cases} \tag{20}$$

(21) also shows the principal of orthogonality where the expected value of the conjugate of the noise and a_k is zero a all points.

$$E[a_k w^*[j]] = 0 \tag{21}$$

(22) this step shows that two dependant orthogonal vectors are zero at all time except where the vector index is equal.

$$E[w[j]w^*[k]] = \begin{cases} \sigma_w^2 = \dfrac{N_0}{2} & j = k \\ 0 & j \neq k \end{cases} \tag{22}$$

(23) Is a simple substitution of the previous equations

$$\therefore R_y[m] = \sum_k E_a h[i+m-k]h^*[i-k] + \frac{N_0}{2}\delta[m] \tag{23}$$

Hence,
(24) simply sets $j = i - k$ to simplify the expression

$$R_y[m] = E_a \sum_j h[m+j]h^*[j] + \frac{N_0}{2}\delta[m] \tag{24}$$

3.2 MMSE Equaliser

Figure 9 shows the eye diagram of a signal after matched filtering but before equalisation. After equalisation the eye diagram would appear to open due to the signal correction in equalisation. This corresponds with the reduction in the symbol error probability as the eye diagram shows the distance between different amplitude levels. When there is not distinct levels shown on the eye diagram this shows that accuratly dectecting the symbol is no longer possible. This is also shown on the scatter plots where the values at the sampling instant are shown. The data shown in Figure 11 (b) also shows the seperation between the symbol levels, indicationg that an accurate distinction could be made, compared to Figure 11 (a) where no accurate distinction can be made between the levels.

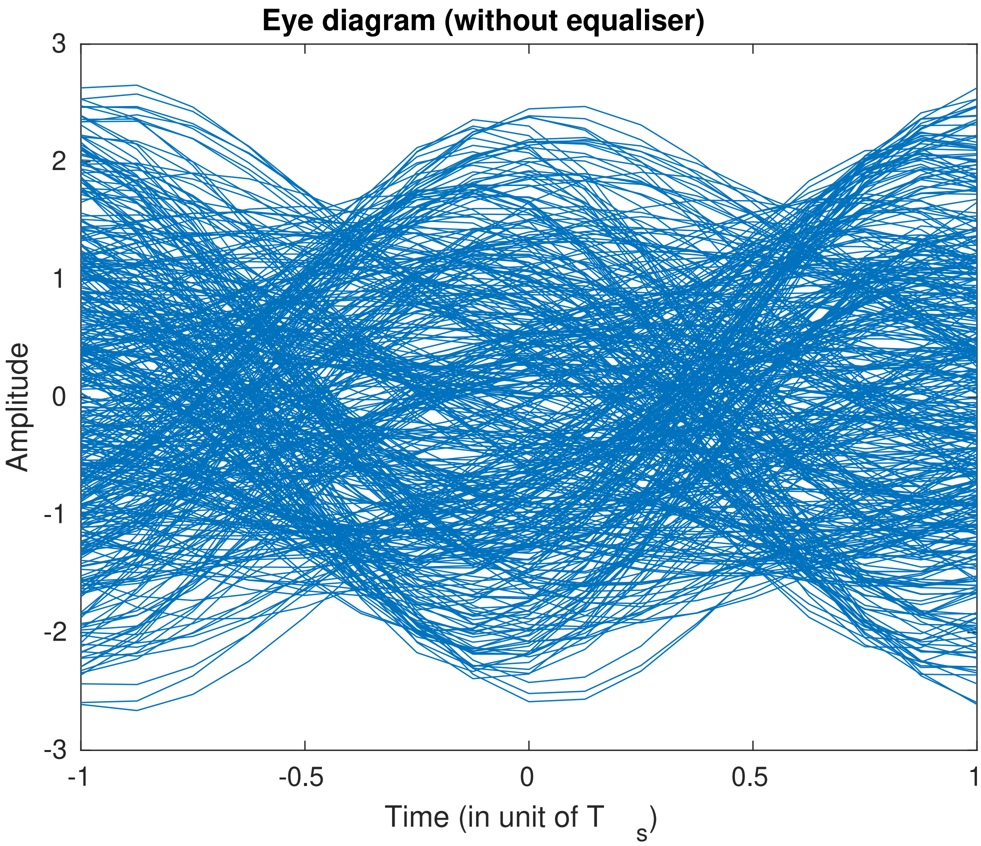

Figure 9: Eye Diagram of a 4-PAM signal before equalisation

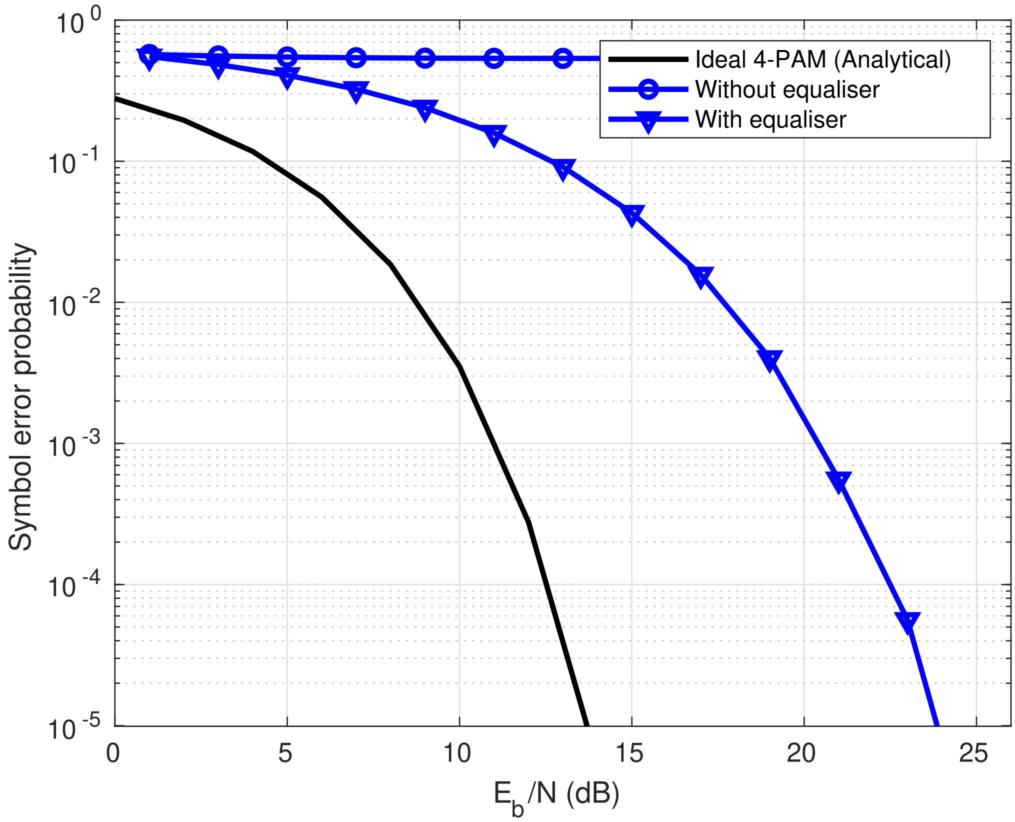

Figure 10: SER comparison of a singal with and without equalisation

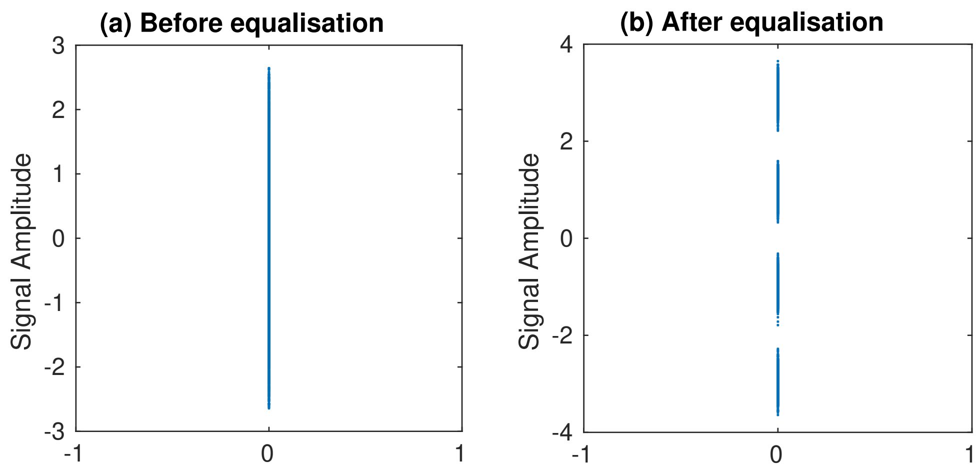

Figure 11: Before and after equalisation scatter plots showing the signal seperation after equalisation

3.3 Displaying a eye diagram with lab equipment

To display an eye diagram on the ECE lab equipment the signal generator would have the clock output connected to the channel 1 input of the oscilloscope and the data output would be connected channel 2. The data generation on the signal generator would be set to 'RANDOM' this ensures a proper eye can be generated as it is not just the same data cycling. Next set an oscilloscope trigger on the input to channel 1, this allows the oscilloscope to detect each period and know when to loop the signal on the screen. Finally, the display persistance should be set to ininity to ensure that the past pulses are not removed from the screen so a proper eye is formed.