INTERNATIONAL REGIONAL SCIENCE REVIEW 26, 2: 153–166 (April 2003)

SPATIAL EXTERNALITIES, SPATIAL MULTIPLIERS, AND SPATIAL ECONOMETRICS

LUC ANSELIN
Regional Economics Applications Laboratory (REAL) and Department of Agricultural and Consumer Economics, University of Illinois at Urbana-Champaign, Urbana, IL, anselin@uiuc.edu

This article outlines a taxonomy of spatial econometric model specifications that incorporate spatial externalities in various ways. The point of departure is a reduced form in which local or global spillovers are expressed as spatial multipliers. From this, a range of familiar and less familiar specifications is derived for the structural form of a spatial regression.

Keywords: *spatial econometrics; spatial externalities; model specification; spatial multiplier; spillovers*

1. INTRODUCTION

Spatial externalities play a central role in the recent emergence of "spatial thinking" in the mainstream social sciences (Goodchild et al. 2000). For example, in economics, greatly increased attention is being paid to models of social interaction, which introduce a dependence among actors in a system (e.g., Glaeser, Sacerdote, and Scheinkman 1996; Akerlof 1997). Similarly, in sociology, the renaissance of the "Chicago School"–type analyses of neighborhood processes has led to the introduction of formal notions of spatial spillovers and dependence (Abbot 1997; Sampson, Morenoff, and Earls 1999). Empirical verification of such spatial externalities, measurement of their strength and range, requires the specification and estimation of spatial econometric models.

This research was supported in part by NSF Grant BCS-9978058 to the Center for Spatially Integrated Social Science (CSISS) and by NSF/EPA Grant SES-0084213. Earlier versions were presented at the CSISS Specialist Meeting on Spatial Externalities, Santa Barbara, California, 11-13 January 2001; the ICPSR-CSISS Advanced Workshop on Spatial Analysis in Social Research, Ann Arbor, Michigan, 17-20 May 2001; and as the Joseph Havlicek Jr. Memorial Lecture in Applied Econometrics at The Ohio State University, Columbus, 25 April 2002. Comments from participants as well as suggestions from Raymond Florax and Jan Brueckner are greatly appreciated. Special thanks to Jeff Morenoff for providing the original stimulus to write this article.

DOI: 10.1177/0160017602250972

Conceptually, the principle underlying the resulting spatial dependence is fairly straightforward. However, the precise way in which this dependence should be included in a regression specification to mimic the salient features of the process under consideration is complex. This has implications for the identifiability of parameters of interest and the types of estimation methods that need to be applied (Manski 1993). The standard taxonomy of *spatial autoregressive lag* and *error* models commonly applied in spatial econometrics (Anselin 1988) is perhaps too simplistic and leaves out other interesting possibilities for mechanisms through which phenomena or actions at a given location affect actors and properties at other locations.

In this article, I outline an alternative approach to develop a simple taxonomy of formal models of spatial externalities in cross-sectional data. This is in part a review as well as an extension of some ideas presented in Anselin and Bera (1998) and Anselin (2001). The primary emphasis is on distinguishing between a *global* and a *local* range of dependence and the way in which this translates into the incorporation in a regression specification of spatially lagged dependent variables (Wy), spatially lagged explanatory variables (WX), and spatially lagged error terms (Wu). Due to space constraints, I will limit the scope of the discussion to the standard linear regression model with continuous dependent variables.

In the remainder of the article, I first provide the general background for the formal framework that lies at the basis for the taxonomy and consider global and local forms of spatial autocorrelation explicitly. This is followed by an outline of a taxonomy of reduced forms and structural forms that reflect different types of spatial externalities. I close with a summary and some extensions.

2. Global and Local Spatial Correlation

Consider the familiar linear regression model

$$y = X\beta + \varepsilon, \tag{1}$$

where y is an n by 1 vector of observations on a dependent variable, X is an n by k matrix of observations on exogenous (explanatory) variables, with an associated k by 1 vector of regression coefficients, and ε is an n by 1 vector of random disturbance terms. To focus the discussion and review the main concepts, I first consider the error terms in isolation from the other elements in the model specification.

The error variance-covariance matrix, $\mathrm{Cov}[\varepsilon\varepsilon']$, or equivalently, $E[\varepsilon\varepsilon']$, expresses *spatial* covariance when the off-diagonal elements are nonzero in accordance with a given spatial structure or "spatial ordering" (Kelejian and Robinson 1992). The spatial ordering specifies those pairs of locations $i - j$ (with $i \neq j$) for which the covariance will be nonzero, or $E[\varepsilon_i\varepsilon_j] \neq 0$. One way to obtain this structure is directly (referred to in the literature as *direct representation*), by specifying the covariance as a function of the distance d_{ij} that separates any two location pairs.

This is typically done in a geostatistical approach to the problem. It requires the specification of a smooth distance decay function and a parameter space that ensures a positive definite variance-covariance matrix.[1] Alternatively, the precise nature of the distance decay function can be left unspecified and approximated by a step function, in a nonparametric fashion. For this to work, the nature of the spatial covariance should be smooth and relatively constant in a small number of distance bands from which it can be estimated, as in Conley (1999). The direct representation approach only applies to error covariance structures. It is not further pursued here, because the objective is to develop a general taxonomy that applies to spillovers for all the variables in the model.

A second fundamental strategy to obtain the spatial structure of the nonzero elements in the variance-covariance matrix is *indirect*. This follows when a spatial stochastic process is specified that relates the value of a random variable at a location (such as ε_i) to the values of random variable at *neighboring* locations. Instead of linking all pairs of observations through a distance decay function, the neighbors for each individual location are specified by means of a so-called spatial weights matrix, W. The weights matrix consists of positive elements, with $w_{ij} \neq 0$ for "neighbors" and $w_{ij} = 0$ for others (by convention, the diagonal elements w_{ii} are set to zero).[2]

2.1. Global Autocorrelation

The most commonly used spatial process specification is the autoregressive model (SAR). Formally, for a vector of error terms,

$$\varepsilon = \lambda W \varepsilon + u, \tag{2}$$

where λ is the spatial autoregressive parameter, W is the weights matrix, and u is a vector of i.i.d. errors with variance σ^2.[3] As is well known in the spatial econometrics literature, after solving equation (2) for ε as

$$\varepsilon = [I - \lambda W]^{-1} u, \tag{3}$$

the variance-covariance matrix for the random vector ε follows as

$$E[\varepsilon\varepsilon'] = \sigma^2[(I - \lambda W)^{-1}(I - \lambda W)^{-1\prime}]. \tag{4}$$

The structure of this variance-covariance matrix is such that every location is correlated with every other location in the system, but closer locations more so, in effect following Tobler's (1979) first law. This can been seen by considering the expanded form of equation (3). Since (in most cases) $|\lambda| < 1$ and the elements of W are less than 1 as well (for row-standardized spatial weights), a "Leontief expansion" of the matrix inverse in equation (3) follows as

$$[I - \lambda W]^{-1} = I + \lambda W + \lambda^2 W^2 + \ldots \tag{5}$$

and its transpose is obtained by applying the transpose operation to every matrix in equation (5). The complete structure of the variance-covariance matrix then follows as the product of equation (5) with its transpose, yielding a sum of terms containing matrix powers and products of W, scaled by powers of λ. Specifically, the lowest-order term is I, followed by λW and $\lambda W'$, $\lambda^2(W^2 + WW' + W^{2\prime})$, and so on. For a spatial weights matrix corresponding to first-order contiguity, each of the powers involves a higher order of contiguity, in effect creating bands of ever larger reach around each location, relating every location to every other one. Moreover, the powers of the autoregressive parameter (with $|\lambda| < 1$) ensure that the covariance decreases with higher orders of contiguity, hence satisfying the second condition of Tobler's law.[4]

In addition to the structure for the covariance terms, it is also interesting to note that the diagonal elements in equation (4), or the variance of the process at each location, depend on the diagonal elements in W^2, WW', and so on. These terms are directly related to the number of neighbors for each location. If the neighborhood structure is not constant across the landscape (as is the case in most irregular lattice structures), heteroskedasticity results, even though the initial process (2) is *not* heteroskedastic.

I will refer to the type of spatial covariance structure induced by the SAR model as *global*, since it relates all the locations in the system to each other. In practice, for small values of λ, the covariance may approach zero after a relatively small number of powers, but "in principle," the covariance matrix is a dense and full matrix.[5]

The global nature of the covariance also implies the existence of a *spatial multiplier* in SAR processes. For an error process, a shock in the error u at any location will be transmitted to all other locations following the multiplier expressed in equation (5). The same principle applies to spatial autoregressive processes in y and X.

2.2. Local Autocorrelation

The second most commonly used spatial process implements a moving average specification (SMA). Formally,

$$\varepsilon = \gamma W u + u, \tag{6}$$

where γ is the SMA parameter, and the other notation is as in equation (2). In contrast to the SAR model, there is no inverse involved in the reduced form, since equation (6) is in fact the reduced form. The lack of this inverse results in a *local* range for the induced spatial covariance,

$$\begin{aligned} E[\varepsilon\varepsilon'] &= \sigma^2[(I + \gamma W)(I + \gamma W)'], \\ &= \sigma^2[I + \gamma(W + W') + \gamma^2 WW']. \end{aligned} \tag{7}$$

As can be seen from the structure of equation (7), the only off-diagonal nonzero elements in the variance-covariance matrix are those corresponding to nonzero elements in W (or, equivalently, W') and WW'. For W defined as first-order contiguity, such elements consist of location pairs that are first- and second-order neighbors, but no higher orders of contiguity. In other words, beyond two "bands" of neighbors, the spatial covariance is zero.[6] Consequently, the range of the effect of the spatial multiplier is much smaller than for a corresponding SAR model. Similar to the SAR case, an unequal number of neighbors induces heteroskedasticity, since the diagonal terms in WW' will not be constant.

2.3. Conditional and Simultaneous Autocorrelation

The importance of the distinction between *global* and *local* forms of spatial correlation becomes obvious when one tries to relate model specifications and associated spatial multipliers to particular types of spatial spillovers. I use this as the basis for constructing a taxonomy of models for spatial externalities in the next section.

Before proceeding to this, it may be useful to draw the distinction between the SAR and SMA models, on one hand, which are "simultaneous," and a "conditional" specification on the other. The latter is more common in spatial statistics, and referred to as CAR (Cressie 1993).

In a simultaneous model, the focus is on the explanation of the *complete* spatial pattern, that is, the interaction between *all* observations or locations, observed simultaneously. Typically, this is conceptualized in the form of a joint multivariate distribution that possesses a covariance structure compatible with a spatial ordering. The simultaneity follows from the nature of dependence in space, which, in contrast to the time series context, is two-directional. As a consequence, each location is in turn a "neighbor" for its neighbors, so that the effect of the neighbors has to be treated as *endogenous*.

In contrast, in a conditional model, a random variable at a given location is conditioned on the observations on that random variable at neighboring locations. Since the latter are treated as *exogenous*, they are not explained by the model. Instead, they can be exploited to construct optimal predictions for the random variable at unobserved locations. The set of conditional distributions for each location is only compatible with a spatially correlated joint density under restrictive conditions. These are considered in the so-called Hammersley-Clifford theorem (Cressie 1993). From a practical point of view, it is important to note that it is only possible in special circumstances (multivariate normal distribution, symmetric weights matrix) to move between a conditional specification and a proper joint (simultaneous) specification.

One can also distinguish between the two models by considering that they are based on different information sets. In the simultaneous model, only exogenous variables (X) are used in the explanation of the complete spatial pattern of the dependent variable y. In the conditional model, both exogenous X as well as (assumed) exogenous Wy are used in the model for the spatial pattern of the y.

A conditional perspective is common in much of the recent literature in spatial statistics, especially pertaining to hierarchical models.[7] In contrast, in spatial econometric applications, especially in applied economics, the focus is typically on modeling the complete simultaneity of the spatial interaction. Thus, inference on the parameters allows one to explain the pattern for all locations as a function of exogenous variables. This is also the perspective taken in the remainder of this article.

3. Reduced Forms and Structural Forms

A classification of spatial externalities in regression models can be approached by considering the *reduced* form, in which the right hand side (RHS) contains only expressions in the exogenous variables (X) and error terms (u) and spatial transformations of them. However, the spatial econometric specification used for estimation and inference is typically not the reduced form. Instead, a *structural* form is employed, in which various expressions in spatially lagged variables (Wy, WX, or Wu) may appear on the RHS.

A possibly confusing issue in this regard is the interpretation of the role of the spatially lagged dependent variable Wy on the RHS of the equation. While it may be intuitive to interpret such a variable as relating values for y at i to its neighbors, this is only partially the case, since the neighboring values in turn depend on y_i. More precisely, the particular spatial pattern between locations and their neighbors can be considered to be an equilibrium outcome of a process that follows from global spatial correlation in the X and error terms. Hence, any economic interpretation of y_i depending on y_j actually works through the spatial patterns in the X and u. Other interpretations sometimes unintentionally imply a conditional perspective that is incompatible with the simultaneous framework considered here.

The primary dimension in the taxonomy then becomes whether the spatial correlation *in the reduced form* pertains only to the unmodeled effects (error terms), to the modeled effects (included explanatory variables), or to both. Clearly, discriminating between these effects can be approached from a purely empirical perspective, by letting specification tests dictate which is the proper alternative.[8] However, equally valid and more accepted in the mainstream social science literature is the view that a substantive theoretical argument should suggest the nature of the externalities. For example, in real estate economics, "neighborhood effects" are typically relegated to the error term on a priori grounds, inducing spatial error autocorrelation when such effects show a spatial structure.[9] Alternatively, in neighborhood analysis in sociological studies, any externalities could be constrained to pertain to the neighborhood characteristics themselves, such as crime in one area being a function of poverty in adjoining areas.[10]

The second dimension in the taxonomy is the distinction between global and local spillovers, outlined in the previous section. In the reduced form specification, this boils down to the inclusion of a spatial multiplier effect of the form $(I - \lambda W)^{-1}$ versus a simple spatial lag term using spatial weights W.

3.1. SPATIAL EXTERNALITIES IN UNMODELED EFFECTS

First, consider the introduction of externalities in the error term of our standard regression model (1). This is the case when the unmodeled variables that are subsumed in the error term jointly follow a spatial random process. An example would be the effect of air quality or neighborhood quality in a hedonic model for house prices when no precise measures are available for these variables. Any spatial patterns in them would be reflected in the error terms.

Global effects are specified as a SAR model, as in equation (2). The combination of equations (1) and (2) is the familiar *structural* form of the model. The corresponding reduced form is also straightforward, with equation (3) to substitute for ε in the regression specification,

$$y = X\beta + (I - \lambda W)^{-1}u. \tag{8}$$

Further manipulation of equation (8) yields an alternative structural form as the so-called spatial Durbin or common factor model. This specification includes both a spatially lagged dependent variable as well as spatially lagged explanatory variables:[11]

$$y = \lambda Wy + X\beta - \lambda WX\beta + u. \tag{9}$$

In addition, model (9) contains k nonlinear constraints on the parameters, the so-called common factor constraints:

$$\lambda.\beta = -\lambda B. \tag{10}$$

When these constraints are not satisfied, equation (9) is no longer a common factor model but instead implies a more complex structure of spatial externalities. In general, we can write equation (9) in unconstrained form as

$$y = \lambda Wy + X\beta + WX\gamma + u, \tag{11}$$

where some of the elements of γ could be zero such that not all the lagged X variables need to be included in the model. Instead of equation (8), this formulation yields a very different reduced form:

$$y = (I - \lambda W)^{-1}X\beta + (I - \lambda W)^{-1}WX\gamma + (I - \lambda W)^{-1}u. \tag{12}$$

In this specification, the effects of bands of neighbors is no longer λWX, λ^2W^2X, and so on. Instead, the effect of WX becomes $(\gamma + \lambda\beta)$, that of W^2X becomes $(\lambda\gamma + \lambda^2\beta)$, and similarly for the higher-order lags. Whether this is a viable *substantive* model would depend on the theoretical and empirical context, but it clearly implies a very different concept of spatial externalities than the familiar SAR.

In contrast to the global model, local effects do not require the solution of a reduced form and are specified as an SMA model for the errors. Such a specification would be appropriate when the unmodeled effects are constrained to the immediate neighboring observations. Continuing with the real estate example, this would be appropriate if (unmodeled) noise and other nuisances at a given location do not affect the price for houses more than two units away (as opposed to affecting the price for all the houses in the system).

The corresponding structural form is the familiar SMA error model,

$$y = X\beta + u + \gamma W u. \tag{13}$$

3.2. *Spatial Externalities in Modeled Effects*

When modeling spatial externalities in the explanatory variables, two specifications result that are less common in the literature. The first follows when the X terms affect the left hand side (LHS) through a global spatial multiplier effect, that is, both x_i as well as a set of x_j throughout the spatial system affect y_i.[12] An example would be where there is copycatting in a system such that tax rates are set in function not only of the local income, but of the income of the neighbors, and their neighbors' neighbors, and so on. In other words, this copycatting travels throughout the whole system rather than being limited to the immediate neighbors (as in equation [16] below). Importantly, the externalities are restricted to modeled effects only (the X), and any error spillover is specifically excluded.

Formally, such global spillover is obtained by replacing X in equation (1) by $(I - \rho W)^{-1}X$. This yields a *reduced* form as

$$y = (I - \rho W)^{-1} X\beta + u. \tag{14}$$

More interestingly, after multiplying both sides by $(I - \rho W)$, the *structural* form follows as

$$(I - \rho W)y = X\beta + (I - \rho W)u,$$

or

$$y = \rho W y + X\beta + u - \rho W u. \tag{15}$$

This model contains both a spatially lagged dependent variable as well as an SMA error, that is, it is a special case of the SARMA model (Anselin and Bera 1998). There is also a single parameter constraint to ensure that the coefficient of the spatial lag equals minus the parameter in the spatial moving average. Note that in a general SARMA model, there is no reason for such a parameter constraint to be imposed.[13]

Local spillovers in the explanatory variables are expressed in a mixed regressive, spatial cross-regressive model (Florax and Folmer 1992). For example, this would be appropriate when the proper spatial range of the explanatory variables is the location and its immediate neighbors (but not beyond). In the copycatting example above, this would imply a constraint on the range of neighbors considered in the reference space, for example, only the direct neighbors, but not the neighbors' neighbors.

This model adds a set of spatially lagged explanatory variables WX on the RHS of equation (1), as

$$y = X\beta + WX\rho + u, \tag{16}$$

where, in contrast to equation (15), ρ is now not a scalar but a $k - 1$ by 1 vector (matching the column dimension of WX, which excludes a constant term). If necessary, additional constraints may be introduced by forcing a scalar relationship between the coefficients of X and those of its spatial lag, as

$$y = X\beta + \rho WX\beta + u, \tag{17}$$

where ρ is now again the familiar scalar.

3.3. Spatial Externalities in Both Modeled and Unmodeled Effects

When there are no strong a priori theoretical reasons to limit global externalities to either the errors or the explanatory variables, the standard approach is to include both. This is the usual interpretation given to the reduced form of the *spatial lag* model. However, one interesting aspect of this specification that is typically ignored is that this involves additional constraints. Specifically, to obtain the spatial lag model as the structural form, it is necessary to constrain the spatial autoregressive parameter ρ as well as the spatial weights to be the same for the spatial multiplier for both X and u in the reduced form. The *constrained* reduced form then follows as

$$y = (I - \rho W)^{-1}X\beta + (I - \rho W)^{-1}u, \tag{18}$$

which, after the usual manipulations, yields a matching structural form of the familiar spatial lag model (or, mixed regressive, spatial autoregressive model),

$$y = \rho Wy + X\beta + u. \tag{19}$$

Instead of these constraints, a more general approach would allow different parameters and spatial weights for the spillover in the X and u, as

$$y = (I - \rho W_1)^{-1}X\beta + (I - \lambda W_2)^{-1}u. \tag{20}$$

The corresponding structural form follows from premultiplying both sides of equation (20) by $(I - \rho W_1)(I - \lambda W_2)$, which yields

$$(I - \rho W_1)(I - \lambda W_2)y = (I - \lambda W_2)X\beta + (I - \rho W_1)u, \tag{21}$$

or, after some manipulation,

$$y = \rho W_1 y + \lambda W_2 y - \rho\lambda W_1 W_2 y + X\beta - \lambda W_2 X\beta + u - \rho W_1 u. \tag{22}$$

Allowing different parameter values, but enforcing the same spatial weights, yields a slightly simpler form as[14]

$$y = (\rho + \lambda)Wy - \rho\lambda W^2 y + X\beta - \lambda WX\beta + u - \rho Wu. \tag{23}$$

Both equations (22) and (23) are higher-order SARMA models with several nonlinear constraints on the parameters.

The specification with local spillover in both X and u is much simpler. It contains a spatial cross-regressive term WX as well as an SMA for the error term. In general, both can take on different parameter values and pertain to different spatial weights, as

$$y = X\beta + W_1 X\rho + u + \gamma W_2 u, \tag{24}$$

where, as in equation (16), ρ is a vector. As before, equality constraints may be imposed, yielding (with ρ as a scalar),

$$y = (I + \rho W)(X\beta + u). \tag{25}$$

4. Summary and Extensions

The six types of structural forms for spatial econometric models are summarized in Table 1. For models with externalities in both X and u, the unconstrained as well as the constrained forms are listed (for the sake of simplicity, the spatial weights are taken to be the same for both X and u). Several unfamiliar specifications result and some interesting patterns can be discerned.

First, Wy only appears on the RHS for models that incorporate *global* externalities. Second, while the unconstrained forms for both local and global models are nested within their own category (i.e., a simple model can be obtained by imposing a zero restriction on a parameter of a more complex model), this is not the case across categories.[15] In other words, none of the local models result from imposing constraints on the parameters of the global models. Third, all models incorporating global externalities involve parameter constraints, either nonlinear (of the common factor variety) or linear (between the spatial lag in y and the error structure).

It is also striking that the often cited model with both a spatially autoregressive dependent variable and a spatial autoregressive error structure (e.g., Kelejian and Prucha 1998) does not follow from either global or local spatial externalities as

TABLE 1. Taxonomy of Structural Forms

	Local Externalities	*Global Externalities*
u	$y = X\beta + u + \gamma Wu$	$y = \lambda Wy + X\beta - \lambda WX\beta + u$
X	$y = X\beta + WX\rho + u$	$y = \rho Wy + X\beta + u - \rho Wu$
Both	$y = X\beta + WX\rho + u + \gamma Wu$	$y = (\rho + \lambda)Wy - \rho\lambda W^2 y + X\beta - \lambda WX\beta + u - \rho Wu$
	$y = X\beta + WX\rho + u + \rho Wu$	$y = \rho Wy + X\beta + u$

specified here. Instead, this model suggests a rather more complex reduced form. Consider

$$y = \rho W_1 y + X\beta + (I - \lambda W_2)^{-1} u. \tag{26}$$

After some algebraic manipulation, the reduced form follows as

$$y = (I - \rho W_1)^{-1} X\beta + (I - \rho W_1)^{-1}(I - \lambda W_2)^{-1} u. \tag{27}$$

While this model does include a familiar global spatial multiplier in X, the induced pattern of spatial dependence for the error term is much more complex and involves the interaction between the two spatial parameters as well as the two spatial weights. The latter is hard to interpret in terms of the spatial multiplier terminology used here. It remains an empirical matter whether or not it may be more appropriate in given situations, but its interpretation involves a more complex spatial pattern of externalities than for the models included in Table 1.

A second extension is to consider hybrid models that combine global externalities for X with the local type for u and vice versa. Both such models yield a structural form with a spatially lagged dependent variable Wy as well as higher-order spatial lags in either X or u, with nonlinear parameter constraints. Specifically, a model with global externalities in X and local externalities in u has a reduced form (for simplicity, assuming the same spatial weights)

$$y = (I - \rho W)^{-1} X\beta + (I + \lambda W)u, \tag{28}$$

and matching structural form

$$y = \rho Wy + X\beta + u + (\lambda - \rho)Wu - \lambda\rho W^2 u. \tag{29}$$

And the reverse has a reduced form of

$$y = X\beta + WX\rho + (I - \lambda W)^{-1} u, \tag{30}$$

yielding as structural form

$$y = \lambda Wy + X\beta - \lambda WX\beta + WX\rho - \lambda W^2 X\rho + u. \tag{31}$$

Neither of these models has so far received attention in the literature.

5. Conclusion

The main objective of this article was to demonstrate how various perspectives on spatial externalities translated into specific spatial econometric models. Several unfamiliar specifications resulted, many characterized by a spatial moving average structure for the error terms, in combination with spatial lags for y and/or X. Some limitations of more familiar models in terms of their interpretation as models for spatial externalities became apparent as well. The ultimate test of the relevance of the suggested taxonomy will have to be based on empirical studies. Ongoing work focuses on developing the appropriate tests and estimation methods to guide in a specification search. A related agenda is to implement these techniques in user-friendly software.

Notes

1. See Dubin (1988) for an early application of this approach.

2. For an extensive discussion of spatial weights, see Cliff and Ord (1981) and Anselin (1988).

3. Different forms of heteroskedasticity can be introduced as well, in the usual manner.

4. For other types of spatial weights, the specific interpretation in terms of higher-order neighbors does not hold in a strict sense, although the general principle of increasing denseness of the weights and smaller importance for higher orders applies.

5. See Smirnov and Anselin (2001) for a technical discussion of the use of the power expansion to approximate the matrix inverse.

6. For more general types of weights, there is a similar cutoff, although it can no longer be related to orders of contiguity. Also note that this cutoff is similar to the notion of a *range* in the variogram used in geostatistics.

7. For example, see the reviews in Wikle, Berliner, and Cressie (1998) and Royle and Berliner (1999).

8. For example, the forward specification search strategy outlined in Anselin et al. (1996) or the backward model selection (Hendry approach) reviewed in Florax, Folmer, and Rey (forthcoming) can be used to effectively distinguish between alternatives of the spatial lag and spatial error variety.

9. See, for example, the review of real estate applications in Dubin, Pace, and Thibodeau (1999), which are limited to specifications for spatial error correlation.

10. See Morenoff, Sampson, and Raudenbush (2000).

11. In practice, the spatially lagged constant is not included in WX, since there is an identification problem for row-standardized W (the spatial lag of a constant is the same as the original variable). The coefficient of the constant term in X is therefore not β_0 but the combined $\beta_0 - \lambda$.

12. This is a strict interpretation of the "proximity hypothesis" in the model of neighborhood crime of Morenoff, Sampson, and Raudenbush (2000).

13. The reduced form associated with a general SARMA specification is

$$y = (I - \rho W)^{-1} X\beta + (I - \rho W)^{-1}(I + \lambda W)u,$$

implying a more complex spatial structure for the error externalities.

14. Even further simplification follows for orthogonal weights, such that $W_1 W_2 = 0$. Then the structural form is the standard SARMA with a spatial cross-regressive component, although with additional constraints on the parameters.

15. Note that the standard spatial lag model and the spatial error model, though both related to global externalities, are nonnested. This is due to the parameter constraints needed to obtain the specific lag form.

REFERENCES

Abbot, A. 1997. Of time and space: The contemporary relevance of the Chicago school. *Social Forces* 75: 1149-82.

Akerlof, G. A. 1997. Social distance and social decisions. *Econometrica* 65: 1005-27.

Anselin, L. 1988. *Spatial econometrics: Methods and models*. Dordrecht, the Netherlands: Kluwer Academic.

———. 2001. Spatial econometrics. In *A companion to theoretical econometrics*, edited by B. Baltagi, 310-30. Oxford, England: Blackwell.

Anselin, L., and A. Bera. 1998. Spatial dependence in linear regression models with an introduction to spatial econometrics. In *Handbook of applied economic statistics*, edited by A. Ullah and D. E. Giles, 237-89. New York: Marcel Dekker.

Anselin, L., A. Bera, R. J. Florax, and M. Yoon. 1996. Simple diagnostic tests for spatial dependence. *Regional Science and Urban Economics* 26: 77-104.

Cliff, A., and J. K. Ord. 1981. *Spatial processes: Models and applications*. London: Pion.

Conley, T. G. 1999. GMM estimation with cross-sectional dependence. *Journal of Econometrics* 92: 1-45.

Cressie, N. 1993. *Statistics for spatial data*. New York: John Wiley.

Dubin, R. 1988. Estimation of regression coefficients in the presence of spatially autocorrelated errors. *Review of Economics and Statistics* 70: 466-74.

Dubin, R., R. K. Pace, and T. G. Thibodeau. 1999. Spatial autoregression techniques for real estate data. *Journal of Real Estate Literature* 7: 79-95.

Florax, R., and H. Folmer. 1992. Specification and estimation of spatial linear regression models: Monte Carlo evaluation of pre-test estimators. *Regional Science and Urban Economics* 22: 405-32.

Florax, R. J., H. Folmer, and S. J. Rey. Forthcoming. Specification searches in spatial econometrics: The relevance of Hendry's methodology. *Regional Science and Urban Economics*.

Glaeser, E. L., B. Sacerdote, and J. Scheinkman. 1996. Crime and social interactions. *Quarterly Journal of Economics* 111: 507-48.

Goodchild, M., L. Anselin, R. Appelbaum, and B. Harthorn. 2000. Toward spatially integrated social science. *International Regional Science Review* 23: 139-59.

Kelejian, H. H., and I. Prucha. 1998. A generalized spatial two stage least squares procedures for estimating a spatial autoregressive model with autoregressive disturbances. *Journal of Real Estate Finance and Economics* 17: 99-121.

Kelejian, H. H., and D. P. Robinson. 1992. Spatial autocorrelation: A new computationally simple test with an application to per capita country police expenditures. *Regional Science and Urban Economics* 22: 317-33.

Manski, C. F. 1993. Identification of endogenous social effects: The reflexion problem. *Review of Economic Studies* 60: 531-42.

Morenoff, J. D., R. J. Sampson, and S. W. Raudenbush. 2000. Neighborhood structure, social processes, and the spatial dynamics of urban violence. Working paper, Population Studies Center, Institute for Social Research, University of Michigan, Ann Arbor.

Royle, J. A., and L. M. Berliner. 1999. A hierarchical approach to multivariate spatial modeling and prediction. *Journal of Agricultural, Biological and Environmental Statistics* 4: 29-56.

Sampson, R. J., J. Morenoff, and F. Earls. 1999. Beyond social capital: Spatial dynamics of collective efficacy for children. *American Sociological Review* 64: 633-60.

Smirnov, O., and L. Anselin. 2001. Fast maximum likelihood estimation of very large spatial autoregressive models: A characteristic polynomial approach. *Computational Statistics and Data Analysis* 35: 301-19.

Tobler, W. 1979. Cellular geography. In *Philosophy and geography*, edited by S. Gale and G. Olsson, 379-86. Dordrecht, the Netherlands: Reidel.

Wikle, C. K., L. M. Berliner, and N. Cressie. 1998. Hierarchical Bayesian space-time models. *Environmental and Ecological Statistics* 5: 117-54.