# EEE933 - Design and Analysis of Experiments

## Case Study 02

Danilo de Mattos Batista
Undergraduate Program in
Control and Automation Engineering
Universidade Federal de Minas Gerais
Email: dmb.mattos@gmail.com

Sadraque Silva Viana
Undergraduate Program in
Control and Automation Engineering
Universidade Federal de Minas Gerais
Email: viana.sadraque@gmail.com

## I. Problem Description

A student is interested in comparing 8 different materials accordingly to their mean thermal emissivity. The number of layers of material used is another factor that influences the thermal emissivity, so the student decided to block it.

To perform the test, different aluminum ingots are used, which are divided in 8 different pieces, randomly assigned to each material. On each piece there will be painted 4 stripes of one material, and every stripe will have a number of layers varying from $1...4$.

Each piece is put in an oven at $300^o$C until stabilization. After that, the thermal emissivity is measured.

The student is interested in detecting a minimum standardized effect size of:

$$d = \delta^\star/\sigma = 0.5 \tag{1}$$

The defined characteristics of the test are:

- Significance level: 0.05
- Power level: 0.9

As the student only has access to the equipment during one day, he decided to obtain enough observations to give his pairwise comparisons the required power [1].

## II. Experimental Design

The parameter of interest is the difference of thermal emissivity means of 8 different materials. Thus, we can obtain the following hypothesis:

- $H_0 : \tau_i = 0 \ \forall i \in \{1, 2, ...., a\}$
- $H_1 : \exists \tau_i \neq 0$

In the hypotheses defined above, $\tau_i$ is the effect of each group $i$.

The defined hypothesis will be tested using an ANOVA. If the ANOVA indicates the rejection of the null hypothesis, we are going to perform sequential comparisons between the materials. Hence, we are going to perform multiple paired t-tests, trying to detect differences in both sizes.

Because the student wants to calculate the required sample size to perform comparisons, we need to adjust $\alpha$ accordingly. Using Bonferroni's method:

$$\alpha_{adj} = \alpha/K = \alpha/(a-1) = 0.05/7 = 0.007 \tag{2}$$

Using the `power.t.test` [4] routine, we calculate the required sample size as $n = 68$. This is the total number of blocks required to perform the analysis. However, each ingot possesses 4 blocks, so the student will need a number of 17 ingots.

The statistical analysis of the data can be found in the following section.

## III. Statistical Analysis

### A. Investigation of Effect Differences

To gain familiarity with the database, we drew the boxplot of emissivity modeled by ingot, layers and materials, showed respectively in figures 1, 2 and 3.

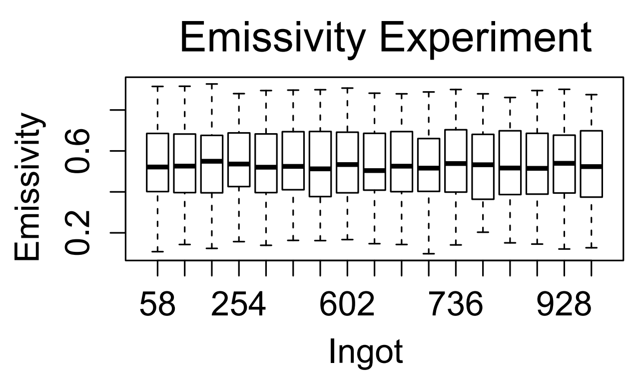


Fig. 1. Boxplot emissivity modeled by ingot.

It seems to indicate that there are no differences of emissivity caused by ingots. However, material and layers greatly affect emissivity, which can be better understood by looking at figure 4. As the student is interested only in the difference between materials, blocking the effect of layers appears to have been a good idea.

To confirm our initial thoughts, we performed an ANOVA on the data. In relation to material, we obtaind a F statistic of $F = 4913.526$ and a p-value of $p < 2e^{-16}$; in relation to layers, we obtained $F = 5815.364$ and $p < 2e^{-16}$. Thus, we can strongly reject the null hypothesis for both material and layers.

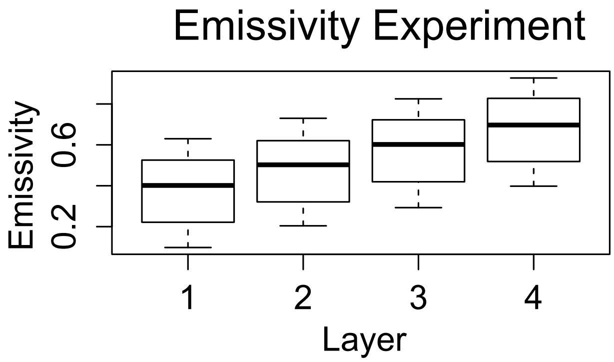


Fig. 2. Boxplot emissivity modeled by layer.

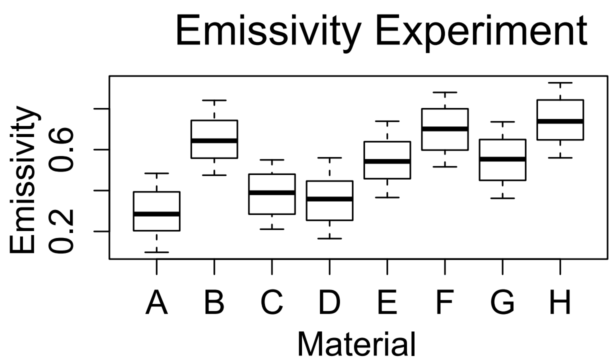


Fig. 3. Boxplot emissivity modeled by material.

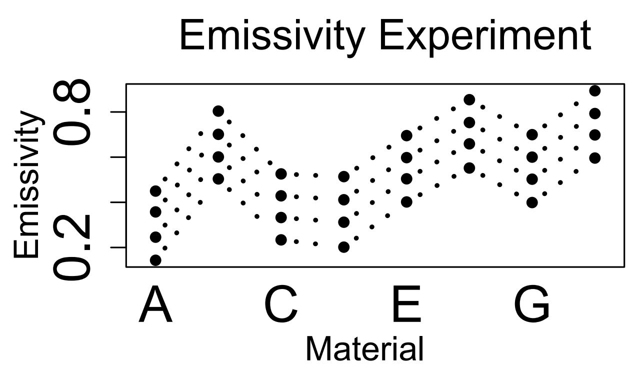


Fig. 4. Interaction plot of emissivity modeled by material and layers.

*B. Validation of Assumptions*

To check our assumptions, we used the Shapiro-Wilk test to check for normality, the Fligner-Killeen test to check for homogeneity of variances and the Durbin-Watson test to check for auto-correlation (an indicative of independence of residuals).

The Shapiro-Wilk test gives $p = 0.07215$, which confirms the normality assumption; Fligner-Killeen test gives $p = 0.7743$, confirming the homoscedascity assumption; Durbin-Watson test gives $p = 0.0.05428$, confirming the assumption of independence. Therefore, all of the assumptions we considered were correct.

*C. Comparisons of Effects*

The ANOVA indicated that there is a difference of effect between materials. To further investigate this, we performed a sequential comparison of the materials, using `glht` [4]. The results are summarized in figure 5:

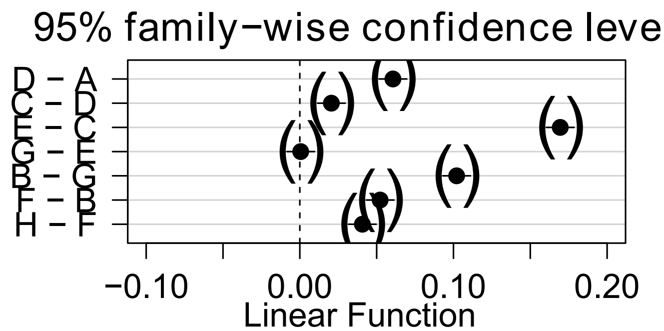


Fig. 5. Comparisons of different materials.

We found significant difference of effects in all comparisons ($p < 1e^{-7}$), except for the difference of materials $G - E$ ($p = 1$).

Hence, the student can conclude that the best materials, in terms of thermal emissivity, are: material H, followed by F, B and G, all of which presents statistically significant difference in performance among each other. Materials G and E presented no significant difference, but are better than material C, followed by D and A, which presented significant difference among them.

## IV. Discussions and Conclusions

The best material in terms of thermal emissivity is material H, which presented a statistically significant better performance than other all materials. Also, the student can conclude than thermal emissivity is directly dependent on the number of layers of material used.

## V. Specific Activities

Both authors performed the design of experiment, the sample size calculations, and the statistical analysis. Author 1 wrote the report while author 2 wrote the R [4] code.

## References

[1] Felipe Campelo, *Case Study 03 Description*. UFMG, Belo Horizonte, Brasil (2014).
[2] Felipe Campelo, *Lecture Notes on Design and Analysis of Experiments*. UFMG, Belo Horizonte, Brasil (2014).
[3] D. C. Montgomery, and G. C. Runger, *Applied Statistics and Probability for Engineers*. John Wiley and Sons, (2003).
[4] R Development Core Team, *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria (2008).