Homework 1

Jose Luiz Magallanes

September 29, 2020

1 Prove or give a counter-example to the following hypothesis $\mathbf{L}^2 = L$; $\epsilon \in \mathbf{L}$

If $L^2 =$ L then $\epsilon \in$ L.

Proof:

base case (show $L^i = L$ for $i >= 1$): $i = 1, L = L$.

assume: $L^i = L$ and consider L^{i+1},

$L^{i+1} = \mathrm{L}^i L = L^2$,

since $L^2 = L$, $L^{i+1} =$ L.

or

base case (show that $L^i = \{\epsilon\}$ for $i >= 0$): $i = 0, L = \{\epsilon\}$

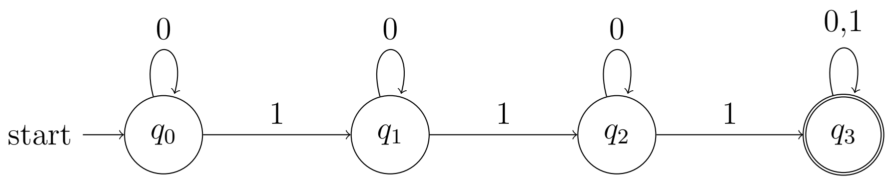
assume: $L^i = L$ and consider L^{i+1},

$L^{i+1} = \mathrm{L}^i L = L^2$,

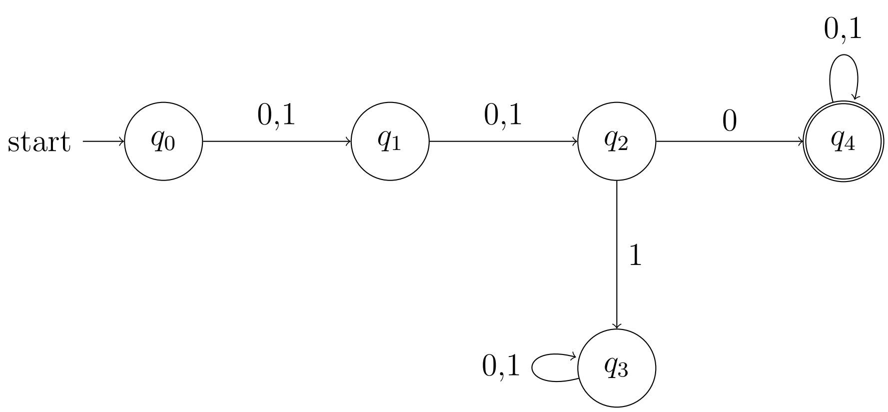
since $L^2 = L, L^{i+1} =$ L.

2 Give state diagrams of DFAs recognizing L.

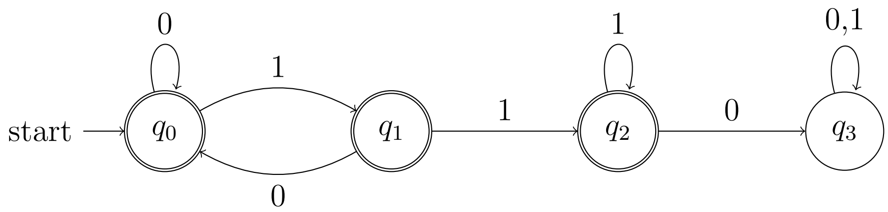
2.1 b) L = {w|w contains at least three 1s}



2.2 d) L = {w|w has length at least 3 and its 3rd symbol is a 0}



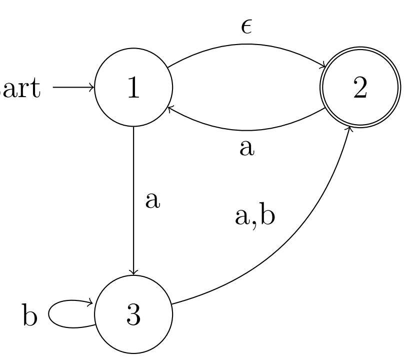
2.3 f) L = {w|w does not contain the substring 110}



Place garbage state beyond accepting states

3 Use the construction given in Theorem 1.39 to convert the following nondeterministic finite automata to equivalent deterministic automata

3.1 b) N_{nfa}= $\{$**{1,2,3},{**$\epsilon, a, b\}, \delta, 1, \{2\}$**}**



We know that there will be 2^k states total when NFA is converted to DFA. Since $k = 3$, there are 2^3 states or 8 states (k represents number of nodes in NFA).
Let's start converting the NFA:

$Q' = \{\phi$, {1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}

Since we have empty string as input on one or more edges:

E(ϕ) = ϕ

E({1}) = {1,2}

E({2}) = {2}

E({3}) = {3}

E({1,2}) = {1,2}

E({1,3}) = {1,2,3}

E({2,3}) = {2,3}

E({1,2,3}) = {1,2,3}

Now let's work on the transition functions:

$\delta'(\phi, a) = E(\phi) = \phi$

$\delta'(\phi, b) = E(\phi) = \phi$

$\delta'(\{1\}, a) = E(\{3\}) = \{3\}$

$\delta'(\{1\}, b) = E(\phi) = \phi$

$\delta'(\{2\}, a) = E(\{1\}) = \{1, 2\}$

$\delta'(\{2\}, b) = E(\phi) = \phi$

$\delta'(\{3\}, a) = E(\{2\}) = \{2\}$

$\delta'(\{3\}, b) = E(\{2, 3\}) = \{2, 3\}$

$\delta'(\{1, 2\}, a) = E(\{3\}) \cup$ E({1}) = {1,2,3}

$\delta'(\{1, 2\}, b) = E(\phi) \cup$ E(ϕ) = ϕ

$\delta'(\{1, 3\}, a) = E(\{3\}) \cup$ E({2}) = {2,3}

$\delta'(\{1, 3\}, b) = E(\phi) \cup$ E({2,3}) = {2,3}

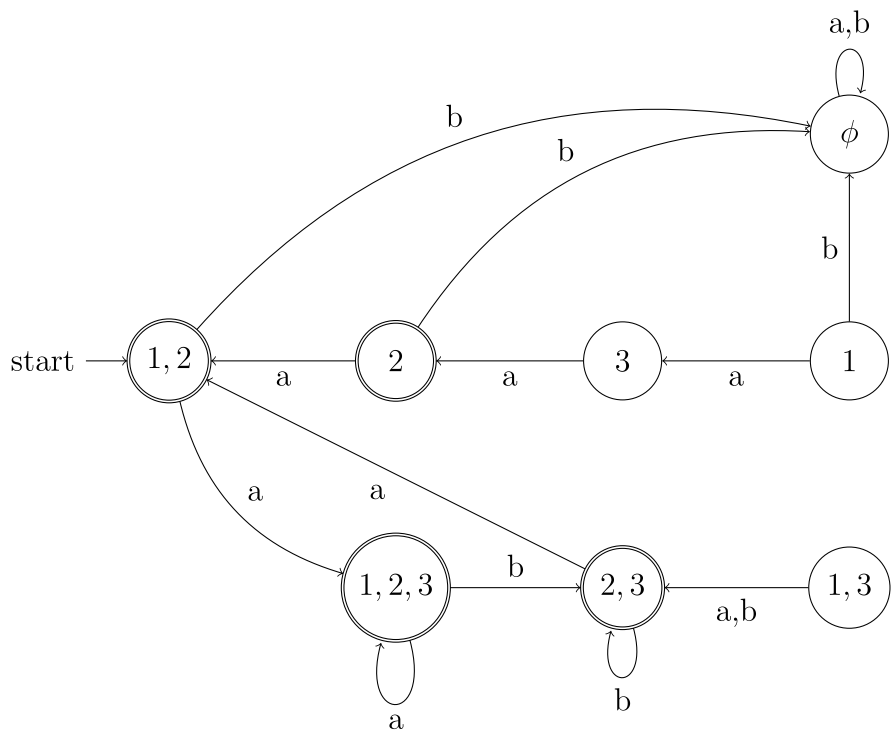
$\delta'(\{2, 3\}, a) = E(\{1\}) \cup$ E({2}) = {1,2}

$\delta'(\{2, 3\}, b) = E(\phi) \cup$ E({2,3}) = {2,3}

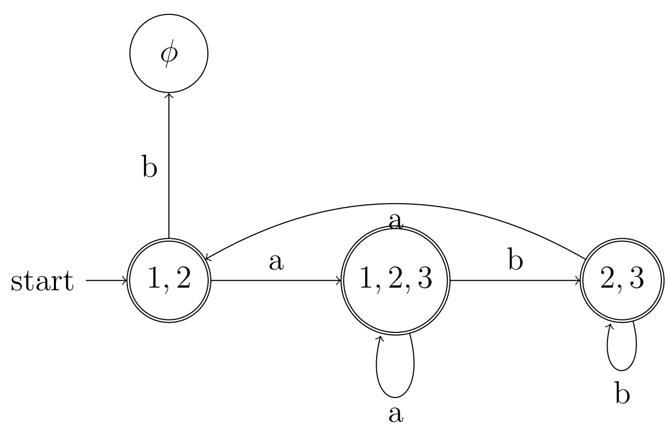
$\delta'(\{1, 2, 3\}, a) = E(\{3\}) \cup$ E({1}) $\cup$ E({2}) = {1,2,3}

$\delta'(\{1, 2, 3\}, b) = E(\phi) \cup$ E(ϕ) $\cup$ E({2,3}) = {2,3}

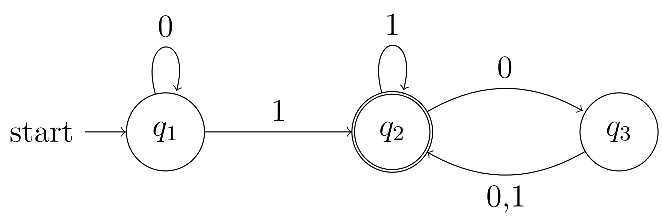
The following is the temporary state diagram:



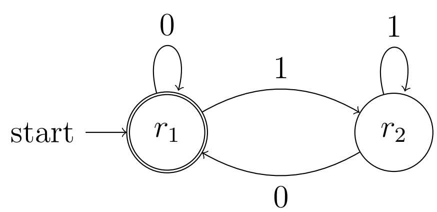
If we remove the states without an incoming arrow, we get a simplified DFA state diagram:



4 Consider the state diagrams of the two Automata in Figure 1.6 on page 36 and in Figure 1.10 on page 38. If M_1 is the machine in Figure 1.6 and M_2 is the machine in Figure 1.10, then give the automata to accept:



M1



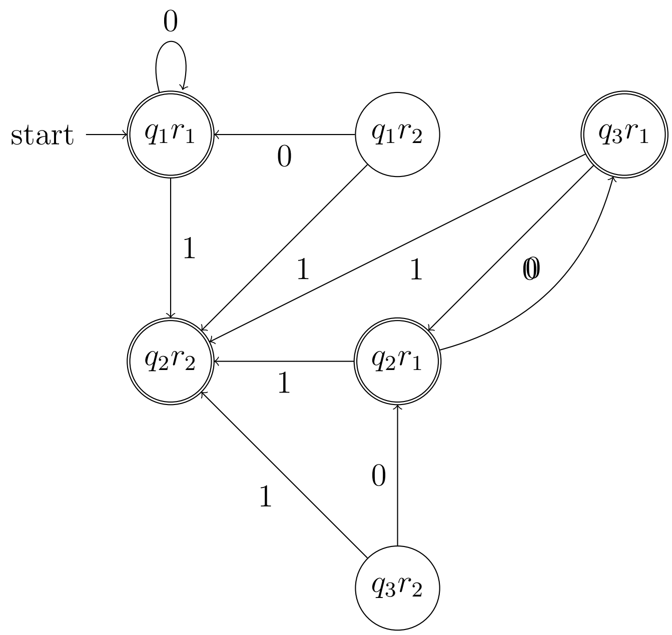
M2

4.1 L(M_1) $\cup$ L (M_2): $\{q_1r_1, q_2r_1, q_2r_2, q_3r_1\}$

4.2 L(M_1) $\cap$ L (M_2): $\{q_2r_1\}$

4.3 L(M_1) $-$ L (M_2): $\{q_2r_2\}$

Without applying triple jump method, I was able to yield this diagram:


0
start
q_1r_1
q_1r_2
q_3r_1
0
1
1
1
0
q_2r_2
q_2r_1
1
0
1
q_3r_2