

Homework 1

Jose Luiz Magallanes

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1 Prove or give a counter-example to the following hypothesis $L^2 = L; \epsilon \in L$

If $L^2 = L$ then $\epsilon \in L$.

Proof:

base case (show $L^i = L$ for $i \geq 1$): $i = 1, L = L$.

assume: $L^i = L$ and consider L^{i+1} ,

$$L^{i+1} = L^i L = L^2,$$

since $L^2 = L$, $L^{i+1} = L$.

or

base case (show that $L^i = \{\epsilon\}$ for $i \geq 0$): $i = 0, L = \{\epsilon\}$

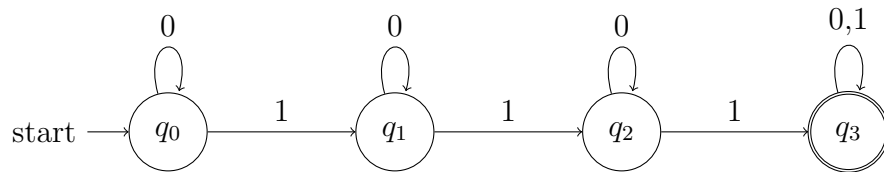
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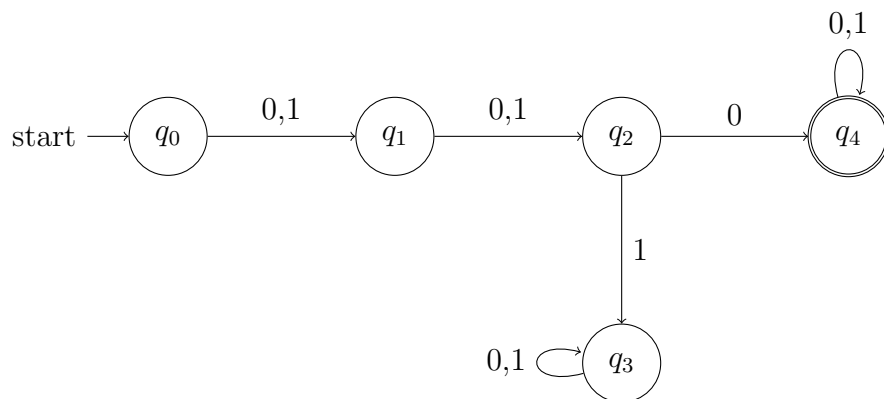
since $L^2 = L$, $L^{i+1} = L$.

2 Give state diagrams of DFAs recognizing L.

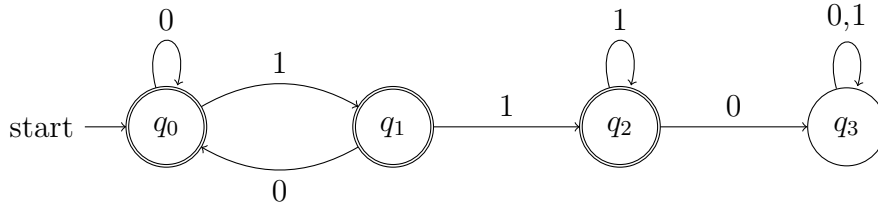
2.1 b) $L = \{w \mid w \text{ contains at least three 1s}\}$



2.2 d) $L = \{w \mid w \text{ has length at least 3 and its 3rd symbol is a 0}\}$



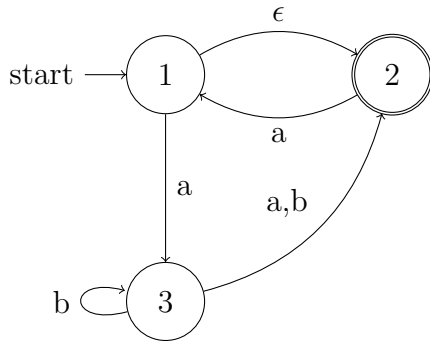
2.3 f) $L = \{w | w \text{ does not contain the substring } 110\}$



Place garbage state beyond accepting states

3 Use the construction given in Theorem 1.39 to convert the following nondeterministic finite automata to equivalent deterministic automata

3.1 b) $N_{nfa} = \{\{1,2,3\}, \{\epsilon, a, b\}, \delta, 1, \{2\}\}$



We know that there will be 2^k states total when NFA is converted to DFA. Since $k = 3$, there are 2^3 states or 8 states (k represents number of nodes in NFA).

Let's start converting the NFA:

$$Q' = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Since we have empty string as input on one or more edges:

$$E(\phi) = \phi$$

$$E(\{1\}) = \{1,2\}$$

$$E(\{2\}) = \{2\}$$

$$E(\{3\}) = \{3\}$$

$$E(\{1,2\}) = \{1,2\}$$

$$E(\{1,3\}) = \{1,2,3\}$$

$$E(\{2,3\}) = \{2,3\}$$

$$E(\{1,2,3\}) = \{1,2,3\}$$

Now let's work on the transition functions:

$$\delta'(\phi, a) = E(\phi) = \phi$$

$$\delta'(\phi, b) = E(\phi) = \phi$$

$$\delta'(\{1\}, a) = E(\{3\}) = \{3\}$$

$$\delta'(\{1\}, b) = E(\phi) = \phi$$

$$\delta'(\{2\}, a) = E(\{1\}) = \{1, 2\}$$

$$\delta'(\{2\}, b) = E(\phi) = \phi$$

$$\delta'(\{3\}, a) = E(\{2\}) = \{2\}$$

$$\delta'(\{3\}, b) = E(\{2, 3\}) = \{2, 3\}$$

$$\delta'(\{1, 2\}, a) = E(\{3\}) \cup E(\{1\}) = \{1, 2, 3\}$$

$$\delta'(\{1, 2\}, b) = E(\phi) \cup E(\phi) = \phi$$

$$\delta'(\{1, 3\}, a) = E(\{3\}) \cup E(\{2\}) = \{2, 3\}$$

$$\delta'(\{1, 3\}, b) = E(\phi) \cup E(\{2, 3\}) = \{2, 3\}$$

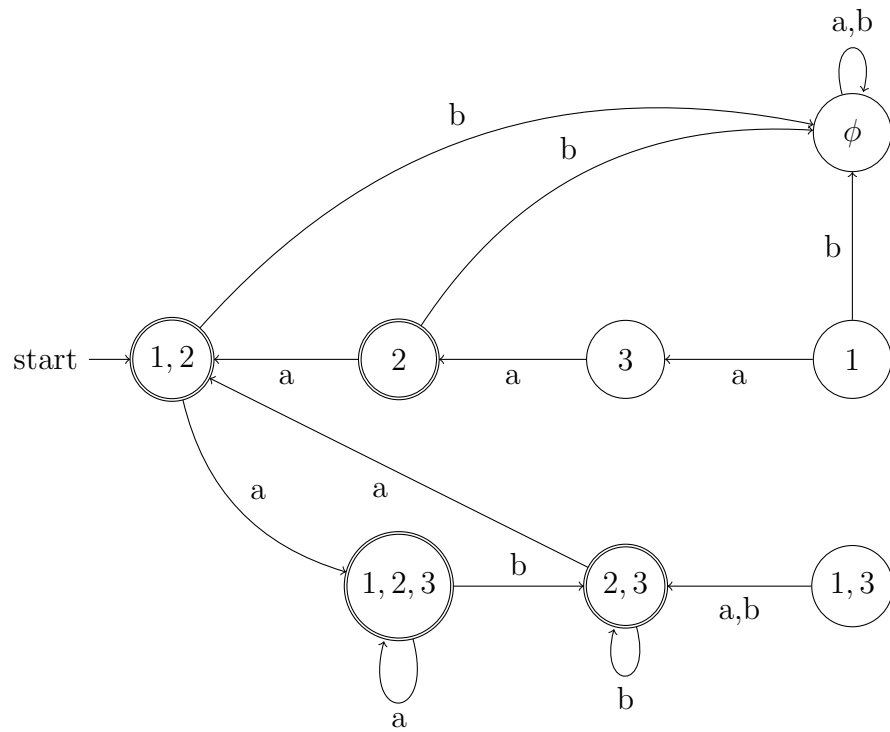
$$\delta'(\{2, 3\}, a) = E(\{1\}) \cup E(\{2\}) = \{1, 2\}$$

$$\delta'(\{2, 3\}, b) = E(\phi) \cup E(\{2, 3\}) = \{2, 3\}$$

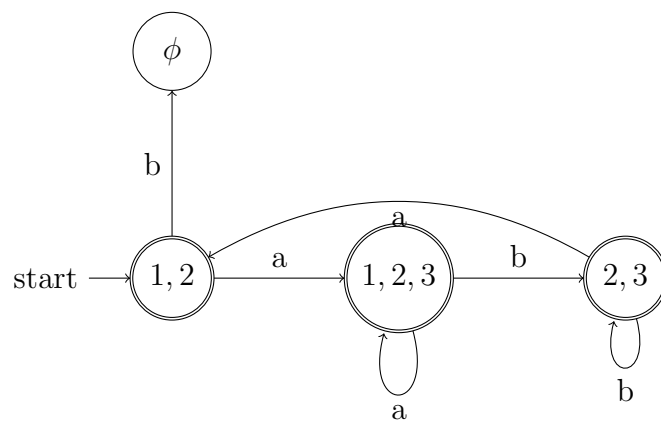
$$\delta'(\{1, 2, 3\}, a) = E(\{3\}) \cup E(\{1\}) \cup E(\{2\}) = \{1, 2, 3\}$$

$$\delta'(\{1, 2, 3\}, b) = E(\phi) \cup E(\phi) \cup E(\{2, 3\}) = \{2, 3\}$$

The following is the temporary state diagram:



If we remove the states without an incoming arrow, we get a simplified DFA state diagram:



4 Consider the state diagrams of the two Automata in Figure 1.6 on page 36 and in Figure 1.10 on page 38. If M_1 is the machine in Figure 1.6 and M_2 is the machine in Figure 1.10, then give the automata to accept:

4.1 $L(M_1) \cup L(M_2): \{q_1r_1, q_2r_1, q_2r_2, q_3r_1\}$

4.2 $L(M_1) \cap L(M_2): \{q_2r_1\}$

4.3 $L(M_1) - L(M_2): \{q_2r_2\}$