# Probabilistic Context-Free Grammar

COMP90042

Natural Language Processing

Lecture 15

Semester 1 2022 Week 8
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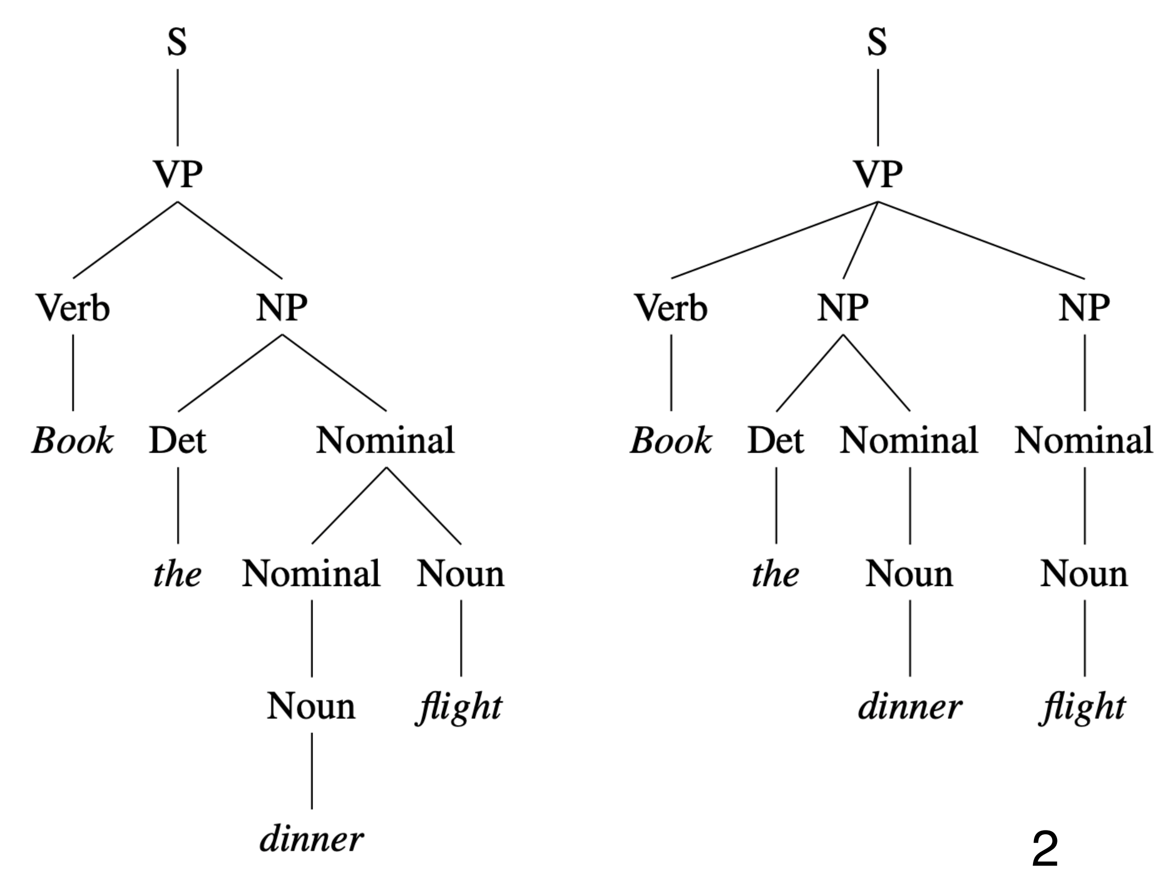
# Ambiguity In Parsing

- Context-free grammars assign hierarchical structure to language
  - Formulated as generating all strings in the language
  - Predicting the structure(s) for a given string
- Raises problem of ambiguity — which is better?
  - Probabilistic CFG!



# Outline

- Basics of Probabilistic CFGs (PCFGs)
- PCFG parsing
- Limitations of CFG

# Basics of PCFGs

# Basics of PCFGs

- Same symbol set:
  - Terminals: words such as *book*
  - Non-terminal: syntactic labels such as NP or NN
- Same productions (rules)
  - LHS non-terminal → ordered list of RHS symbols
- In addition, store a **probability** with each production
  - NP → DT NN [p = 0.45]
  - NN → cat [p = 0.02]
  - NN → leprechaun [p = 0.00001]
  - …

# Basics of PCFGs

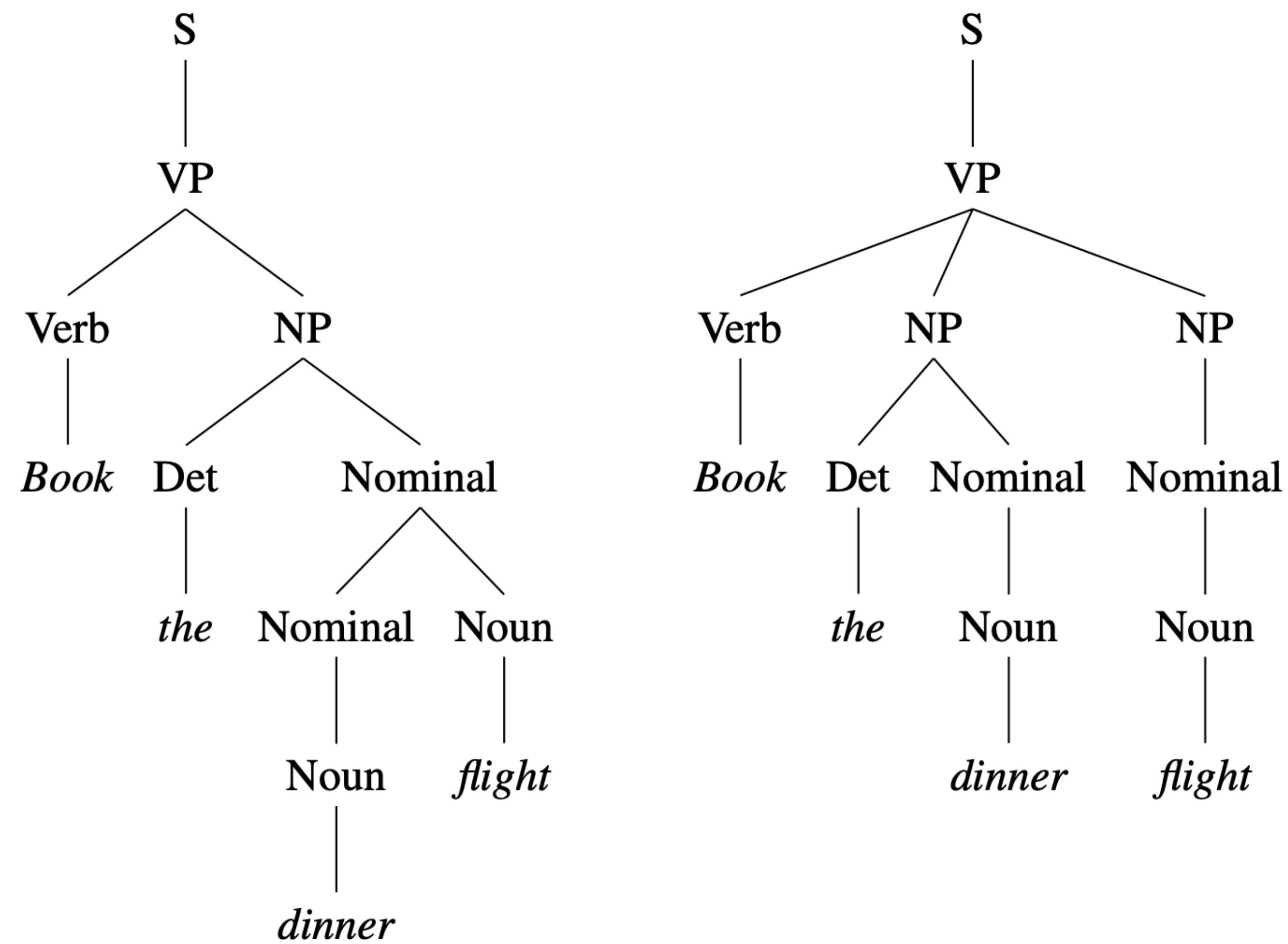
- Probability values denote **conditional**
  - P(LHS → RHS)
  - P(RHS | LHS)
- Consequently they:
  - must be positive values, between 0 and 1
  - must sum to one for given LHS
- E.g.,
  - NN → aadvark [p = 0.0003]
  - NN → cat [p = 0.02]
  - NN → leprechaun [p = 0.0001]
  - $\sum_{x} P(NN \rightarrow x) = 1$

# Stochastic Generation with PCFGs

Almost the same as for CFG, with one twist:

1. Start with S, the sentence symbol
2. Choose a rule with S as the LHS
   - **Randomly select a RHS** according to P(RHS | LHS) e.g., S → VP
   - Apply this rule, e.g., substitute VP for S
3. Repeat step 2 for each non-terminal in the string (here, VP)
4. Stop when no non-terminals remain

Gives us a tree, as before, with a sentence as the yield

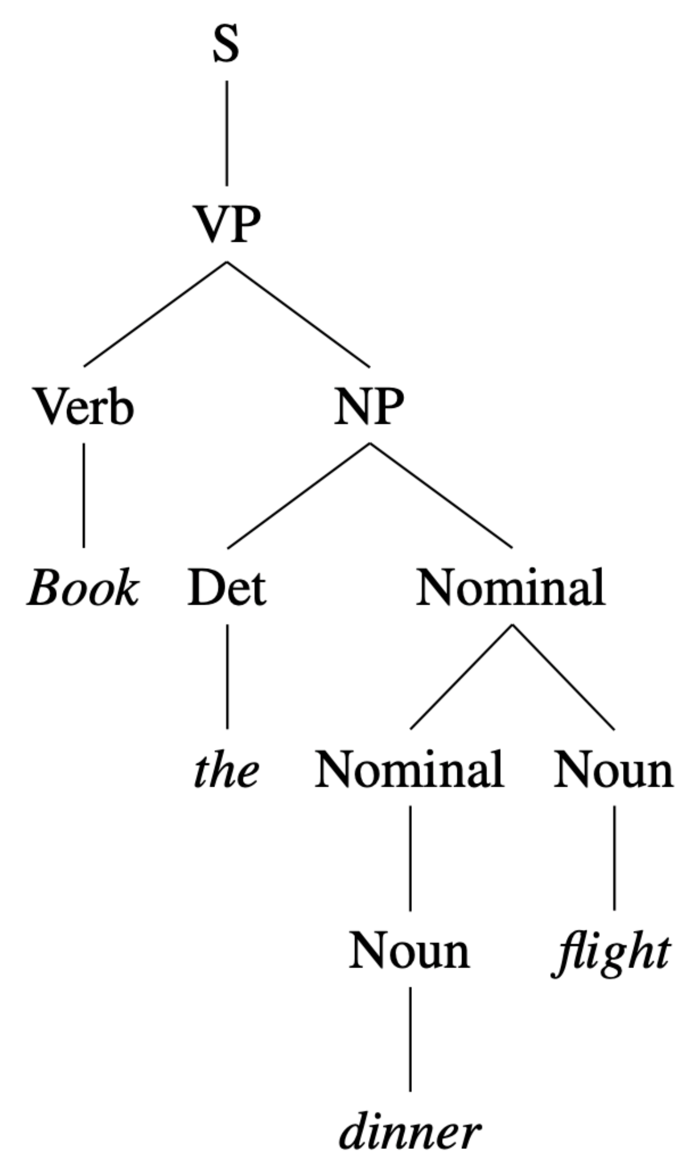



| Rules | | | P | Rules | | | P |
|---|---|---|---|---|---|---|---|
| S | → | VP | .05 | S | → | VP | .05 |
| VP | → | Verb NP | .20 | VP | → | Verb NP NP | .10 |
| NP | → | Det Nominal | .20 | NP | → | Det Nominal | .20 |
| Nominal | → | Nominal Noun | .20 | NP | → | Nominal | .15 |
| Nominal | → | Noun | .75 | Nominal | → | Noun | .75 |
| | | | | Nominal | → | Noun | .75 |
| Verb | → | book | .30 | Verb | → | book | .30 |
| Det | → | the | .60 | Det | → | the | .60 |
| Noun | → | dinner | .10 | Noun | → | dinner | .10 |
| Noun | → | flight | .40 | Noun | → | flight | .40 |

# How Likely Is a Tree?

- Given a tree, we can compute its probability
  - Decomposes into probability of each production
- P(tree) =

  P(S → VP) ×
  P(VP → Verb NP) ×
  P(Verb → *Book*) ×
  P(NP → Det Nominal) ×
  P(Det → *the*) ×
  P(Nominal → Nominal Noun) ×
  P(Nominal → Noun) ×
  P(Noun → *dinner*) ×
  P(Noun → *flight*)



# How Likely Is a Tree?

P(tree)

= P(S → VP) × P(VP → Verb NP) × P(Verb → Book) ×
P(NP → Det Nominal) × P(Det → the) × P(Nominal → Nominal Noun) ×
P(Nominal → Noun) × P(Noun → dinner) × P(Noun → flight)

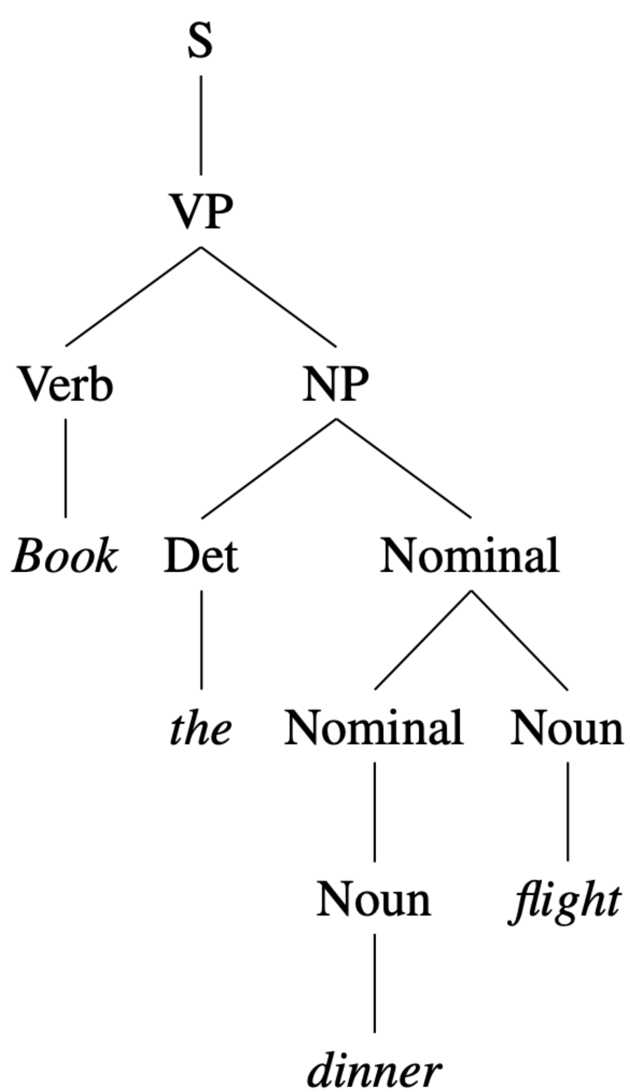
= 0.05 × 0.20 × 0.30 ×
0.20 × 0.60 × 0.20 ×
0.75 × 0.10 × 0.40

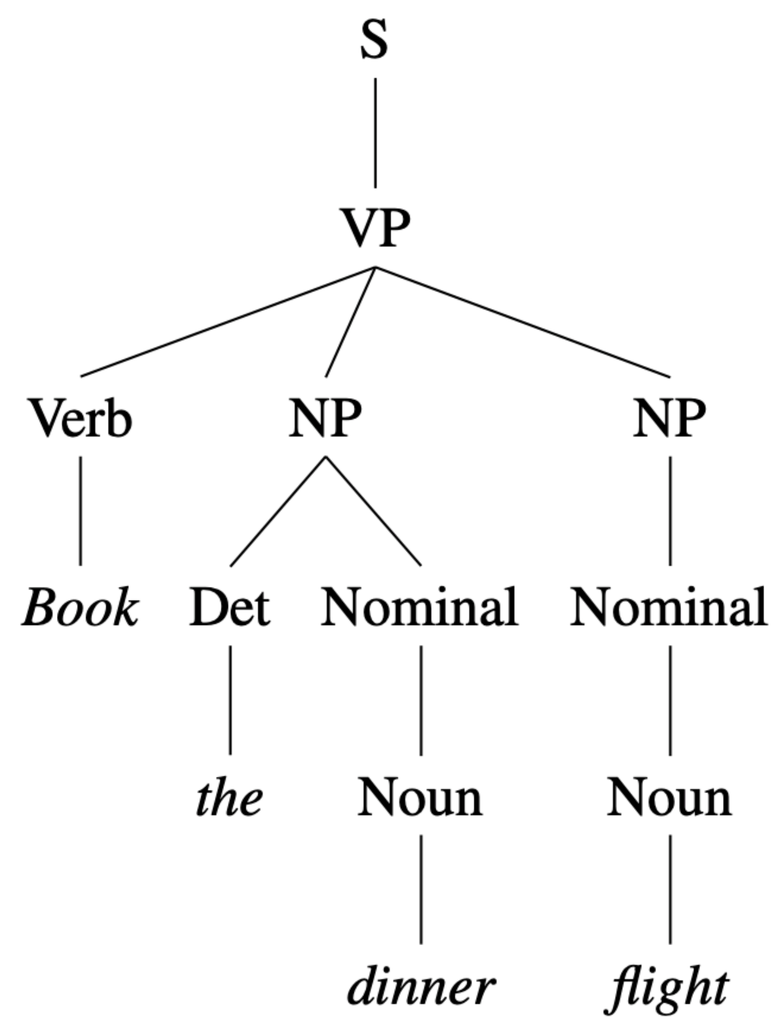
= 2.2 × 10-6

| Rules | | | P |
|---|---|---|---|
| S | → | VP | .05 |
| VP | → | Verb NP | .20 |
| NP | → | Det Nominal | .20 |
| Nominal | → | Nominal Noun | .20 |
| Nominal | → | Noun | .75 |
| | | | |
| Verb | → | book | .30 |
| Det | → | the | .60 |
| Noun | → | dinner | .10 |
| Noun | → | flight | .40 |

# Resolving Parse Ambiguity

- Can select between different trees based on P(T)
- $P(T_{left}) = 2.2 \times 10^{-6}$ $P(T_{right}) = 6.1 \times 10^{-7}$



# PCFG Parsing

# Parsing PCFGs

- Before we looked at
  - CYK
  - for unweighted grammars (CFGs)
  - finds **all possible trees**
- But there are often 1000s, many completely nonsensical
- Can we solve for the **most probable tree**?

# CYK for PCFGs

- CYK finds **all trees** for a sentence; we want **best** tree
- Prob. CYK follows similar process to standard CYK
- Convert grammar to Chomsky Normal Form (CNF)
  - VP → Verb NP NP [0.10]
  - VP → Verb NP+NP [0.10]
    NP+NP → NP NP [1.0]
  - where NP+NP is a new symbol.

| we | eat | sushi | with | chopsticks |
|---|---|---|---|---|
| [0,1] | [0,2] | [0,3] | [0,4] | [0,5] |
| | [1,2] | [1,3] | [1,4] | [1,5] |
| | | [2,3] | [2,4] | [2,5] |
| | | | [3,4] | [3,5] |
| | | | | [4,5] |

| | | | |
|---|---|---|---|
| S | → | NP VP | 1 |
| NP | → | NP PP | ⅛ |
| | → | we | ¼ |
| | → | sushi | ⅛ |
| | → | chopsticks | ½ |
| PP | → | IN NP | 1 |
| IN | → | with | 1 |
| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

| we | eat | sushi | with | chopsticks |
|---|---|---|---|---|
| **NP** 1/4 [0,1] | [0,2] | [0,3] | [0,4] | [0,5] |
| | **V** 1 [1,2] | [1,3] | [1,4] | [1,5] |
| | | **NP** 1/8 [2,3] | [2,4] | [2,5] |
| | | | **IN** 1 [3,4] | [3,5] |
| | | | | **NP** 1/2 [4,5] |

| | | | |
|---|---|---|---|
| S | → | NP VP | 1 |
| NP | → | NP PP | ⅛ |
| | → | we | ¼ |
| | → | sushi | ⅛ |
| | → | chopsticks | ½ |
| PP | → | IN NP | 1 |
| IN | → | with | 1 |
| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

| we | eat | sushi | with | chopsticks |
|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | [0,3] | [0,4] | [0,5] |
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| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

| we | eat | sushi | with | chopsticks |
|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | [0,3] | [0,4] | [0,5] |
| | **V** 1 [1,2] | **VP** 1/16 (1/2 * 1 * 1/8) [1,3] | [1,4] | [1,5] |
| | | **NP** 1/8 [2,3] | [2,4] | [2,5] |
| | | | **IN** 1 [3,4] | [3,5] |
| | | | | **NP** 1/2 [4,5] |

| | | | |
|---|---|---|---|
| S | → | NP VP | 1 |
| NP | → | NP PP | ⅛ |
| | → | we | ¼ |
| | → | sushi | ⅛ |
| | → | chopsticks | ½ |
| PP | → | IN NP | 1 |
| IN | → | with | 1 |
| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

| we | eat | sushi | with | chopsticks |
|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | **S** 1/64 (1 * 1/4 * 1/16) [0,3] | [0,4] | [0,5] |
| | **V** 1 [1,2] | **VP** 1/16 [1,3] | [1,4] | [1,5] |
| | | **NP** 1/8 [2,3] | [2,4] | [2,5] |
| | | | **IN** 1 [3,4] | [3,5] |
| | | | | **NP** 1/2 [4,5] |

| | | | |
|---|---|---|---|
| S | → | NP VP | 1 |
| NP | → | NP PP | ⅛ |
| | → | we | ¼ |
| | → | sushi | ⅛ |
| | → | chopsticks | ½ |
| PP | → | IN NP | 1 |
| IN | → | with | 1 |
| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

| we | eat | sushi | with | chopsticks |
|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | **S** 1/64 [0,3] | Ø [0,4] | [0,5] |
| | **V** 1 [1,2] | **VP** 1/16 [1,3] | Ø [1,4] | [1,5] |
| | | **NP** 1/8 [2,3] | Ø [2,4] | [2,5] |
| | | | **IN** 1 [3,4] | [3,5] |
| | | | | **NP** 1/2 [4,5] |

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|---|---|---|---|
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| IN | → | with | 1 |
| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

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|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | **S** 1/64 [0,3] | Ø [0,4] | [0,5] |
| | **V** 1 [1,2] | **VP** 1/16 [1,3] | Ø [1,4] | [1,5] |
| | | **NP** 1/8 [2,3] | Ø [2,4] | [2,5] |
| | | | **IN** 1 [3,4] | **PP** 1/2 (1 * 1 * 1/2) [3,5] |
| | | | | **NP** 1/2 [4,5] |

| | | | |
|---|---|---|---|
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| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

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|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | **S** 1/64 [0,3] | Ø [0,4] | [0,5] |
| | **V** 1 [1,2] | **VP** 1/16 [1,3] | Ø [1,4] | [1,5] |
| | | **NP** 1/8 [2,3] | Ø [2,4] | **NP** 1/128 (1/8 * 1/8 * 1/2) [2,5] |
| | | | **IN** 1 [3,4] | **PP** 1/2 [3,5] |
| | | | | **NP** 1/2 [4,5] |

| | | | |
|---|---|---|---|
| S | → | NP VP | 1 |
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| IN | → | with | 1 |
| VP | → | V NP | ½ |
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| | → | MD V | ¼ |
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|---|---|---|---|---|
| **NP** 1/4 [0,1] | Ø [0,2] | **S** 1/64 [0,3] | Ø [0,4] | [0,5] |
| | **V** 1 [1,2] | **VP** 1/16 [1,3] | Ø [1,4] | $1 \times 1/128 \times 1/2 = 1/256$ VP **?** $1/16 \times 1/2 \times 1/4 = 1/128$ [1,5] |
| | | **NP** 1/8 [2,3] | Ø [2,4] | **NP** 1/128 [2,5] |
| | | | **IN** 1 [3,4] | **PP** 1/2 [3,5] |
| | | | | **NP** 1/2 [4,5] |

| | | | |
|---|---|---|---|
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| IN | → | with | 1 |
| VP | → | V NP | ½ |
| | → | VP PP | ¼ |
| | → | MD V | ¼ |
| V | → | eat | 1 |

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# Prob CYK: Retrieving the Parses

- S in the top-right corner of parse table indicates success
- Retain back-pointer to best analysis
- To get parse(s), follow pointers back for each match
- Convert back from CNF by removing new non-terminals

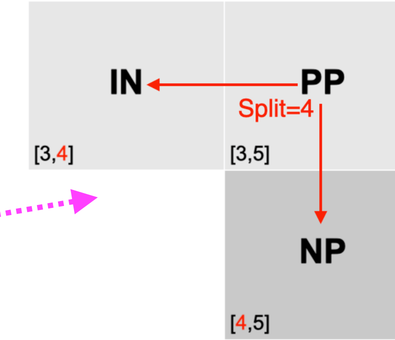
# Prob. CYK

```
function PROBABILISTIC-CKY(words,grammar) returns most probable parse
                                                  and its probability
  for j←from 1 to LENGTH(words) do
     for all { A | A → words[j] ∈ grammar}
        table[j−1, j,A]←P(A → words[j])
     for i←from j−2 downto 0 do
        for k←i+1 to j−1 do
              for all { A | A → BC ∈ grammar,
                               and table[i,k,B] > 0 and table[k, j,C] > 0 }
                    if (table[i,j,A] < P(A → BC) × table[i,k,B] × table[k,j,C]) then
                       table[i,j,A]←P(A → BC) × table[i,k,B] × table[k,j,C]
                       back[i,j,A]←{k,B,C}
     return BUILD_TREE(back[1, LENGTH(words), S]), table[1, LENGTH(words), S]
```

Source: JM3 Ch 14

```
function CKY-PARSE(words, grammar) returns table

  for j ← from 1 to LENGTH(words) do
      for all {A | A → words[j] ∈ grammar}
          table[j−1, j] ← table[j−1, j] ∪ A
      for i ← from j−2 downto 0 do
          for k ← i+1 to j−1 do
              for all {A | A → BC ∈ grammar and B ∈ table[i,k] and C ∈ table[k, j]}
                  table[i,j] ← table[i,j] ∪ A
```




**CYK**

Both CYK and prob. CYK store all possible NTs

validity test now looks to see that the child chart cells have non-zero probability

```
function PROBABILISTIC-CKY(words,grammar) returns most probable parse
                                                and its probability
  for j ← from 1 to LENGTH(words) do
      for all { A | A → words[j] ∈ grammar}
          table[j−1, j,A] ← P(A → words[j])
      for i ← from j−2 downto 0 do
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              for all { A | A → BC ∈ grammar,
                          and table[i,k,B] > 0 and table[k, j,C] > 0 }
                  if (table[i,j,A] < P(A → BC) × table[i,k,B] × table[k,j,C]) then
                      table[i,j,A] ← P(A → BC) × table[i,k,B] × table[k,j,C]
                      back[i,j,A] ← {k,B,C}
      return BUILD_TREE(back[1, LENGTH(words), S]), table[1, LENGTH(words), S]
```

Instead of storing set of symbols, store the probability of best scoring tree fragment covering span [i,j] with root symbol A

Overwrite lower scoring analysis if this one is better, and record the best production

**Probabilistic CYK**

Time complexity in terms of sentence length n?

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# Limitations of CFG

上下文无关语法的独立假设

# CFG Problem 1: Poor Independence Assumptions

有问题

- Rewrite decisions made independently, whereas inter-dependence is often needed to capture global structure.
  - NP → DT NN [0.28]
  - NP → PRP [0.25]
  - Probability of a rule independent of rest of tree
  - No way to represent this contextual differences in PCFG probabilities

只考虑左边的当前状态就能预测出左边 没有考虑global

这是根据上下文统计出来的.

| | Pronoun | Non-Pronoun |
|---|---|---|
| 主语 Subject | 91% | 9% |
| 宾语 Object | 34% | 66% |

即不是0.25

NP statistics in the Switchboard corpus

# Poor Independence Assumptions

| | Pronoun | Non-Pronoun |
|---|---|---|
| Subject | 91% | 9% |
| Object | 34% | 66% |

NP statistics in the Switchboard corpus

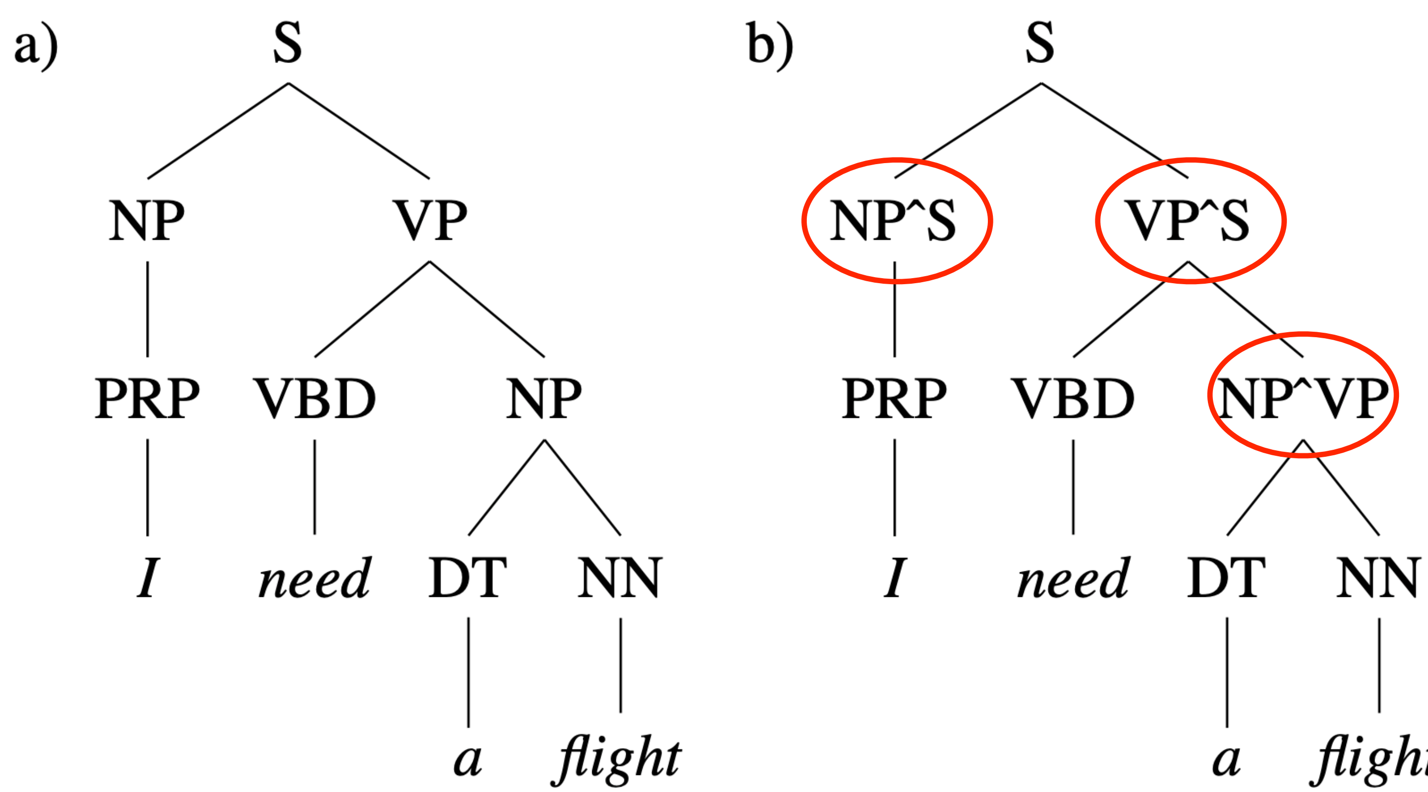
$$NP \rightarrow DT\ NN \quad .28$$
$$NP \rightarrow PRP \quad .25$$

PCFG probabilities based on Switchboard corpus

- NP → PRP should go up to 0.91 as a subject
- NP → DT NN should be 0.66 as an object
- Solution: add a condition to denote whether NP is a subject or object

# Solution: Parent Conditioning

- Make non-terminals more explicit by incorporating parent symbol into each symbol




上位词

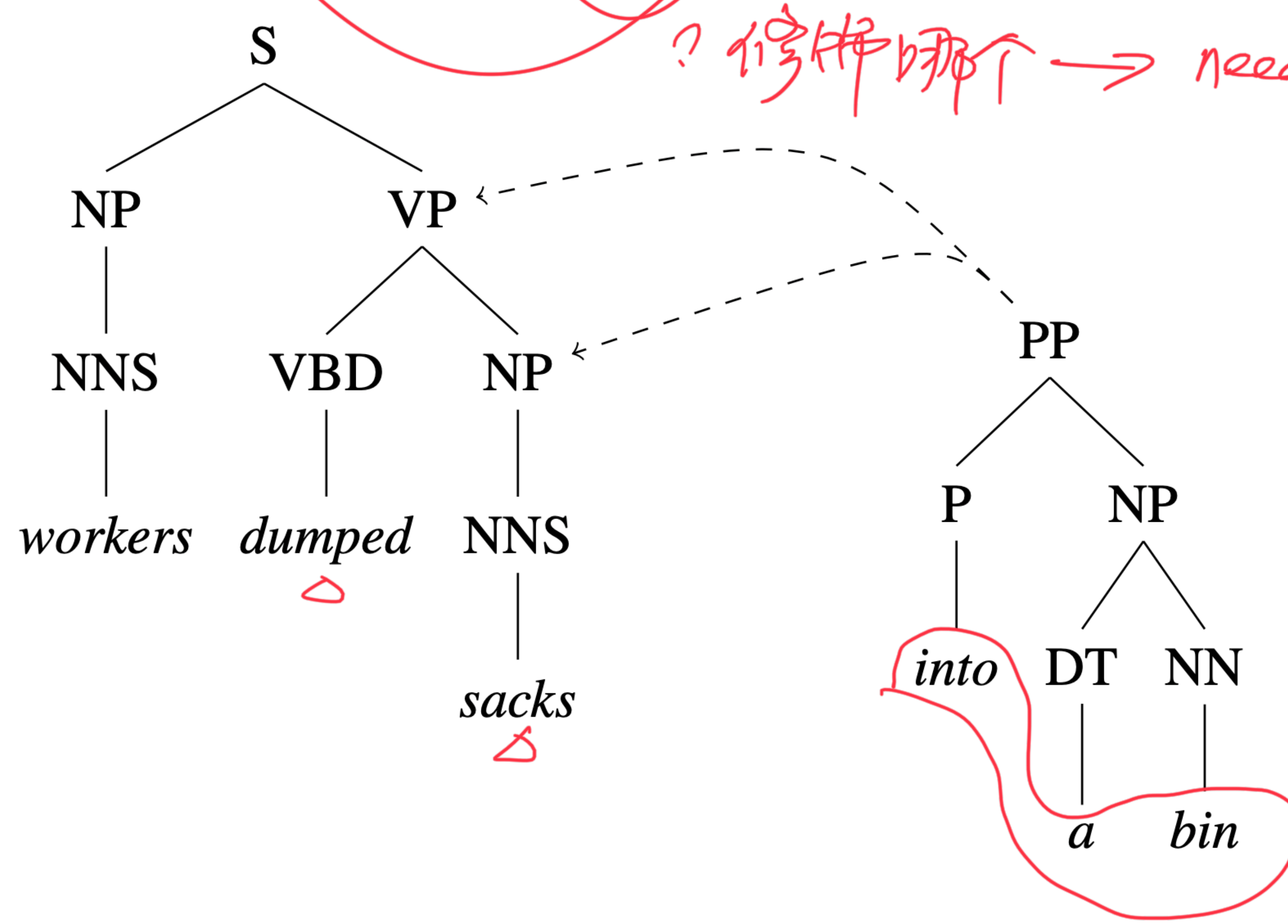
- NP^S represents subject position (left)
- NP^VP denotes object position (right)

# CFG Problem 2:
# Lack of Lexical Conditioning

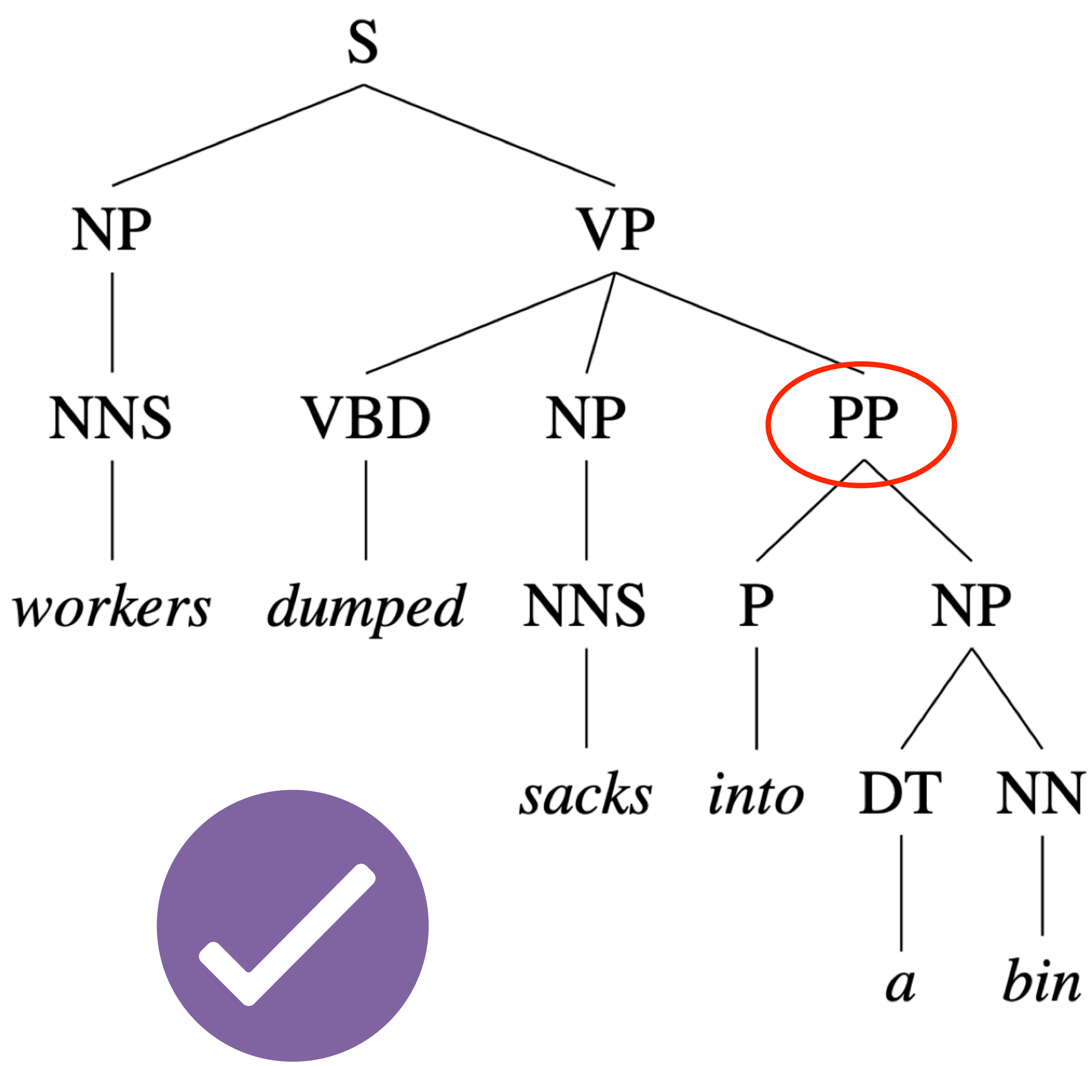
缺乏词汇依赖

- Lack of sensitivity to words in tree
- Prepositional phrase (PP) attachment ambiguity
  - *Worker dumped sacks into a bin*

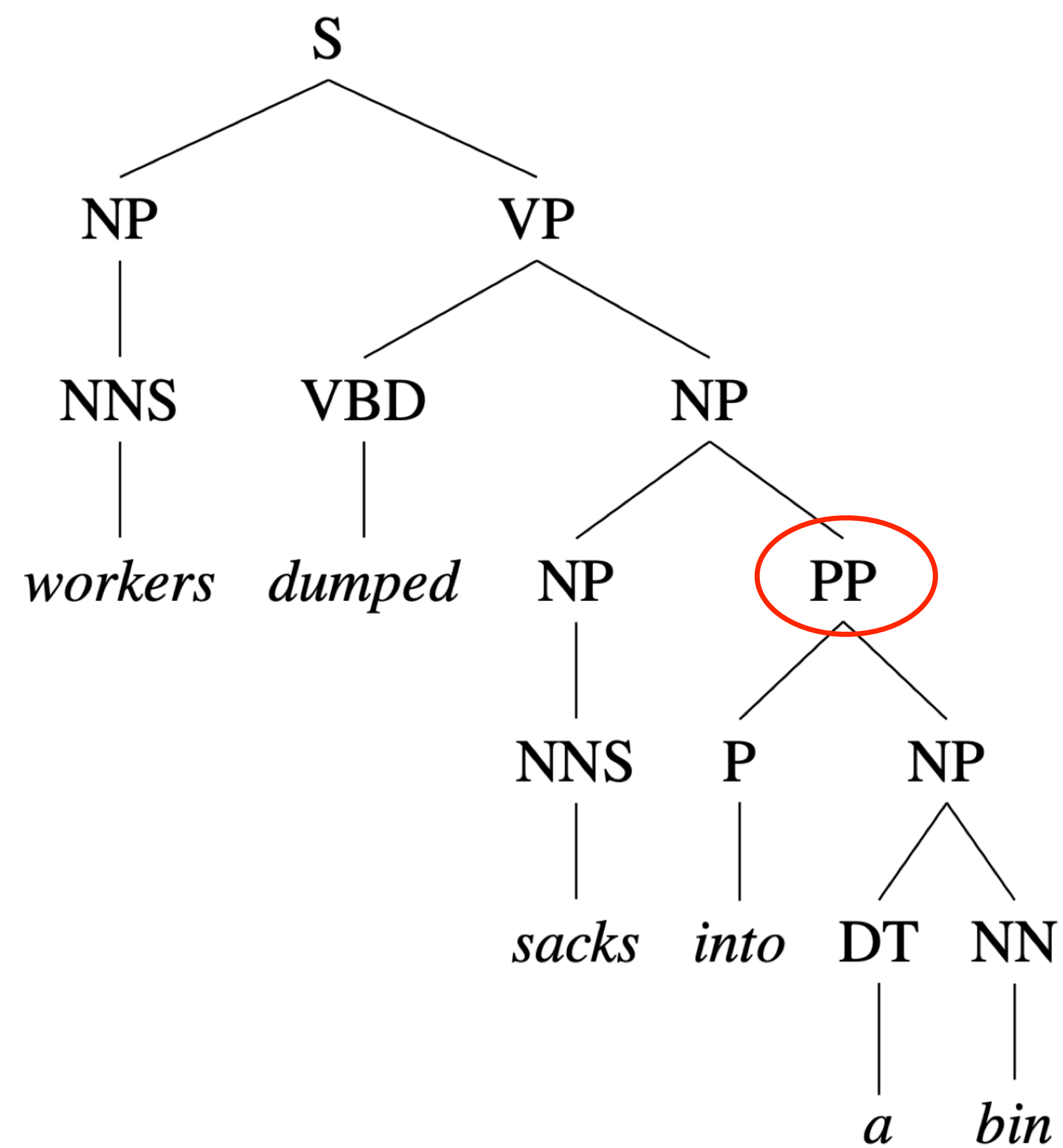
? 修饰哪个 → need Lexical info.



# PP Attachment Ambiguity



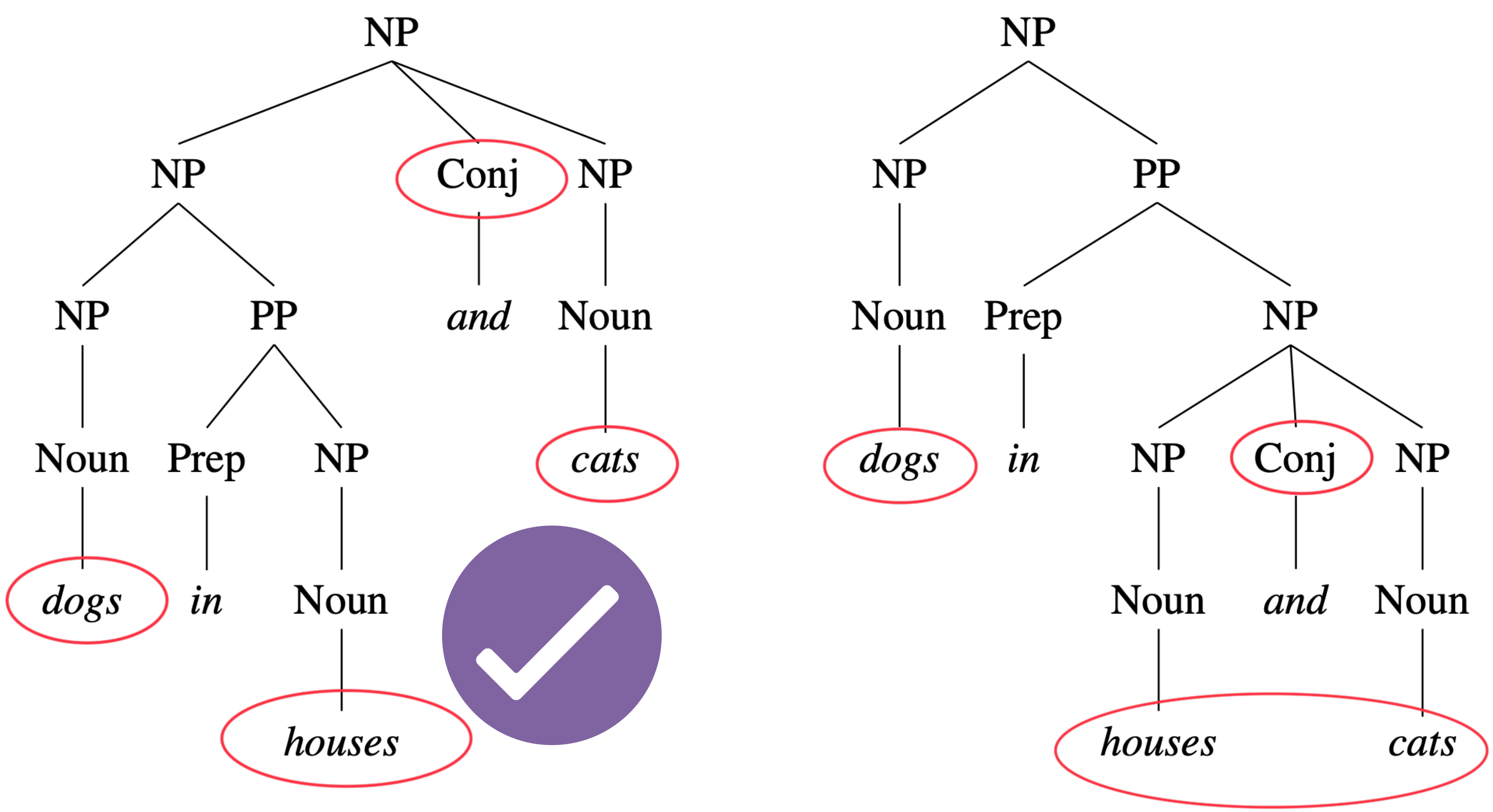

"into a bin" describes the resulting location of the sacks




sacks to be dumped are the ones which are "into a bin"

# Coordination Ambiguity

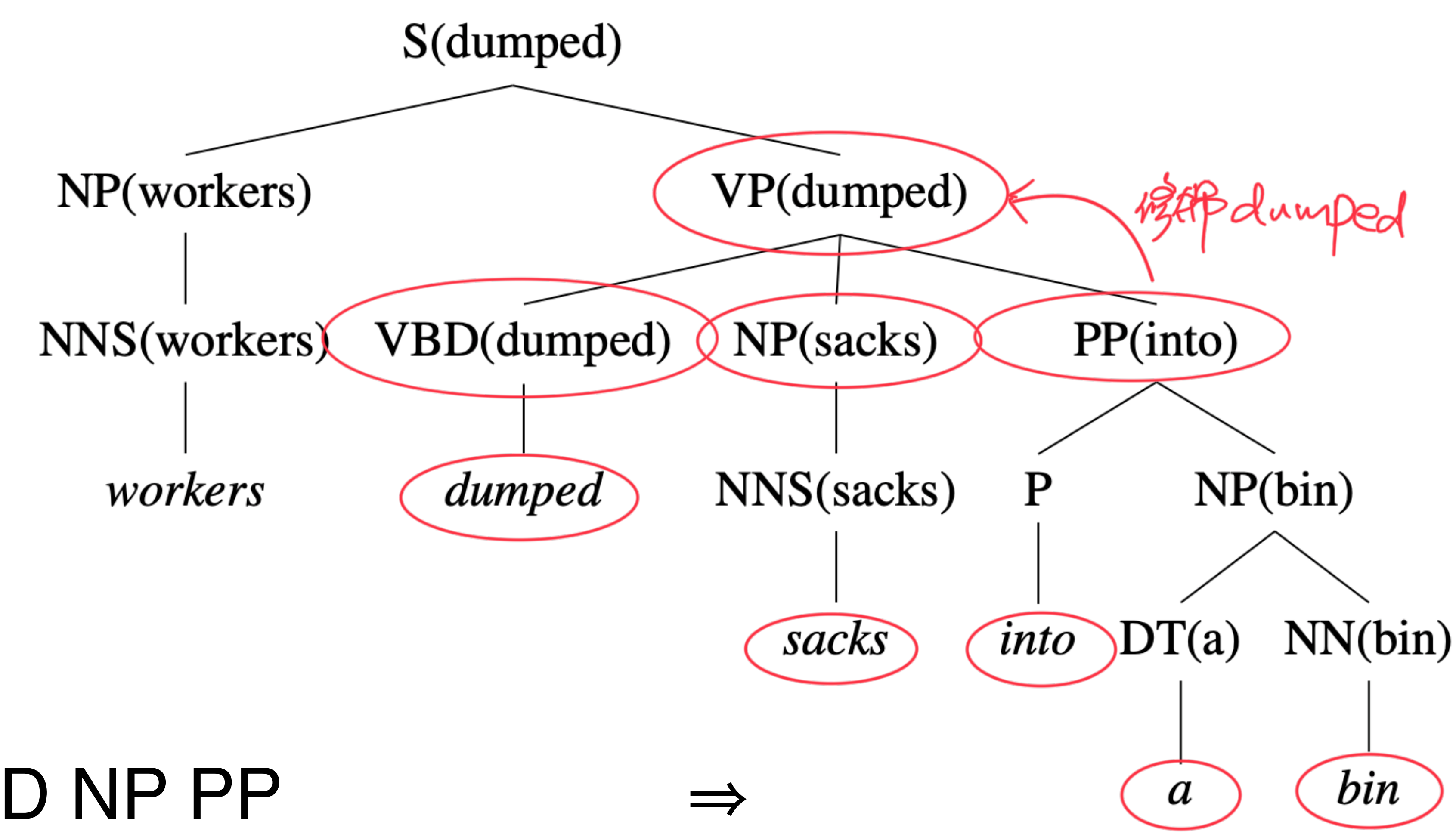
- *dogs in houses and cats*




- *dogs* is semantically a better conjunct for *cats* than *houses* (dogs can't fit into cats!)

# Solution: Head Lexicalisation

- Record head word with parent symbols
  - the most salient child of a constituent, usually the noun in a NP, verb in a VP etc




  - VP → VBD NP PP ⇒

    VP(dumped) → VBD(dumped) NP(sacks) PP(into)

加了就阻止了Context-Free. ∴只加到Head节点

# Head Lexicalisation

- Incorporate head words into productions, to <u>capture</u> the <u>most important links</u> between words
  - Captures correlations between head words of phrases
  - PP(into): VP(dumped) vs. NP(sacks)
- Grammar symbol inventory expands massively!
  - <u>Many of the productions too specific, rarely seen</u>

    规律越具体. less general. 其实规则应该更抽象更广泛. 否则就要写更多的规则

  - Learning more involved to avoid sparsity problems (e.g., zero probabilities)

# A Final Word

- PCFGs widely used, and there are efficient parsers available.
  - Collins parser, Berkeley parser, Stanford parser
  - All use some form of lexicalisation
- But there are other grammar formalisms
  - Lexical function grammar
  - Head-driven phrase structure grammar
  - Next lecture: dependency grammar!

# Required Reading

- JM3 Ch. 13-13.2