



THE UNIVERSITY OF
MELBOURNE

Comp90042

Workshop

Week 8

9 May





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1. Language theory

2. Parsing



Regular language

What are regular grammar and regular language? How are they different?



Regular language

What are regular grammar and regular language? How are they different?

- Language: set of acceptable strings (e.g., sentences)
- Grammar: a generative description of a language
- Regular language: a formal language accepted by a regular expression
- Regular grammar: a formal grammar defined by a set of productions rules
 - $A \rightarrow xB$,
 - $A \rightarrow x$
 - $A \rightarrow \varepsilon$

where A and B are non-terminals, x is a terminal and ε is the empty string.



Regular grammar example

- Rules:
 - $S \rightarrow A$
 - $A \rightarrow a A$
 - $A \rightarrow \epsilon$

Where S is the start symbol, A is non-terminal, a is terminal and ϵ is empty

- Example strings?
 - $a, aa, aaa, aaaa.$
- Regular expression?
 - $(a)^*$



Properties of regular languages

- L_1 : regular language
- L_2 : regular language
- $L_1 \cup L_2$:?
- $L_1 \cap L_2$:?
- Concatenation of L_1 and L_2 :?



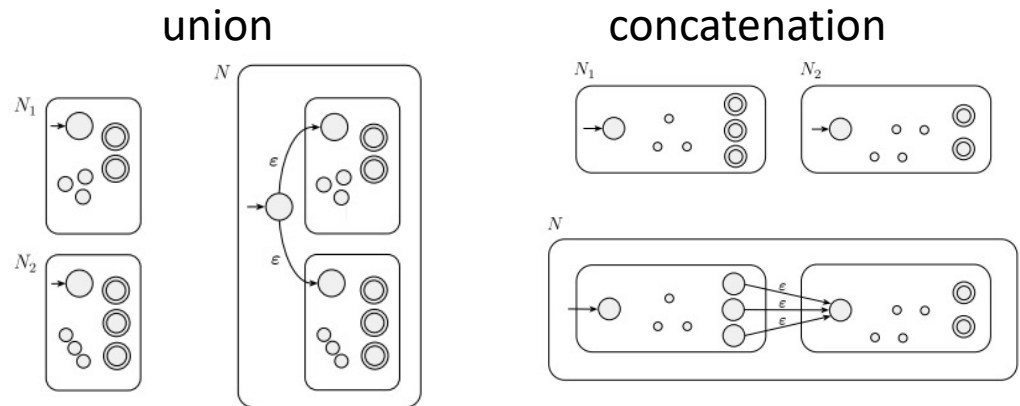
Properties of regular languages

- L_1 : regular language
- L_2 : regular language
- $L_1 \cup L_2$: regular language
- $L_1 \cap L_2$: regular language
- Concatenation of L_1 and L_2 : regular language

Properties of regular languages

Regular languages are closed under union, intersection and concatenation. What does it mean? Why is it important?

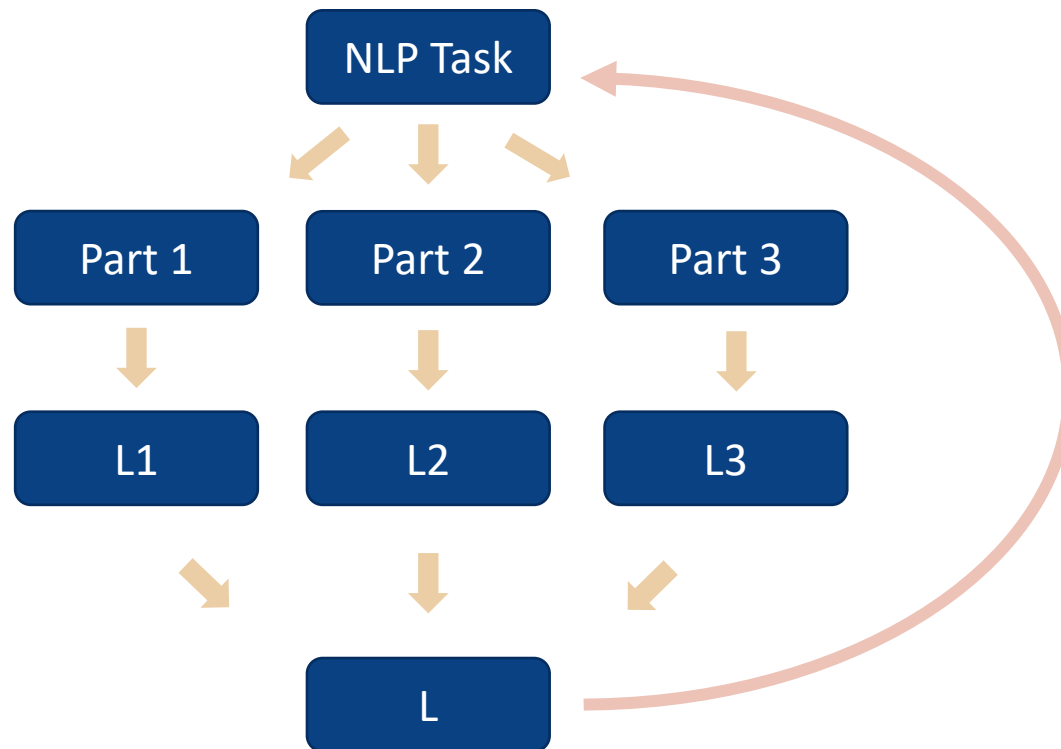
If L_1 and L_2 are two regular languages, then the union, intersection and concatenation of L_1 and L_2 are also regular languages



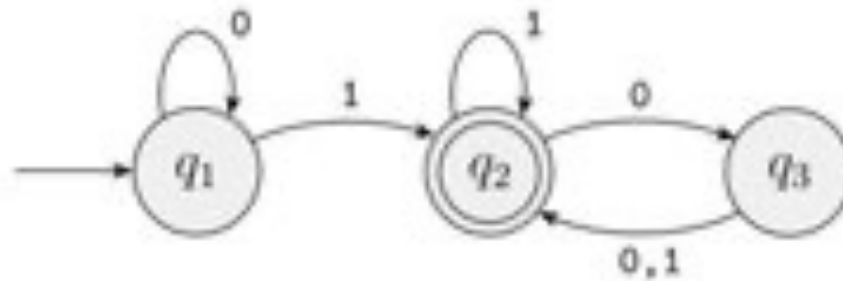
- NLP problems can be factored into small simple parts,
- we can develop regular languages for each part,
- and combine them into a complex system to handle the NLP problems.

Properties of regular languages

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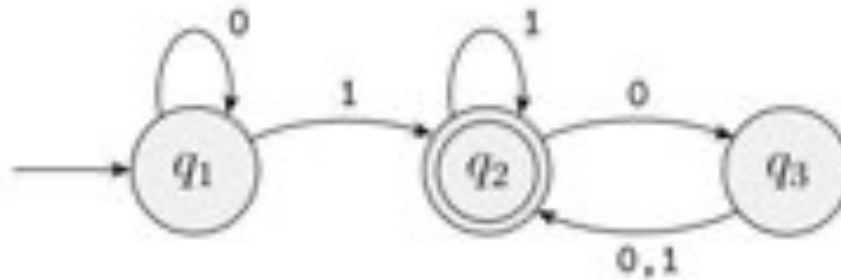
Finite State Acceptor



An FSA is formally represented as a 5-tuple $M1 = (Q, \Sigma, \delta, q_1, F)$

1. States $Q =$
2. Input alphabet $\Sigma =$
3. Transition function δ
4. q_1 is the start state, and
5. Final state $F =$

Finite State Acceptor



An FSA is formally represented as a 5-tuple $M1 = (Q, \Sigma, \delta, q_1, F)$

1. States $Q = \{q_1, q_2, q_3\}$,
2. Input alphabet $\Sigma = \{0,1\}$,
3. Transition function δ
4. q_1 is the start state, and
5. Final state $F = \{q_2\}$

Transition function

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

What language does $M1$ accept?



Example: Word Morphology

Draw a Finite State Acceptor (FSA) for word morphology to show the possible derivations from root forms using the words:

play, played, playing;

walk, walked, walking;

sit, sat, sitting.

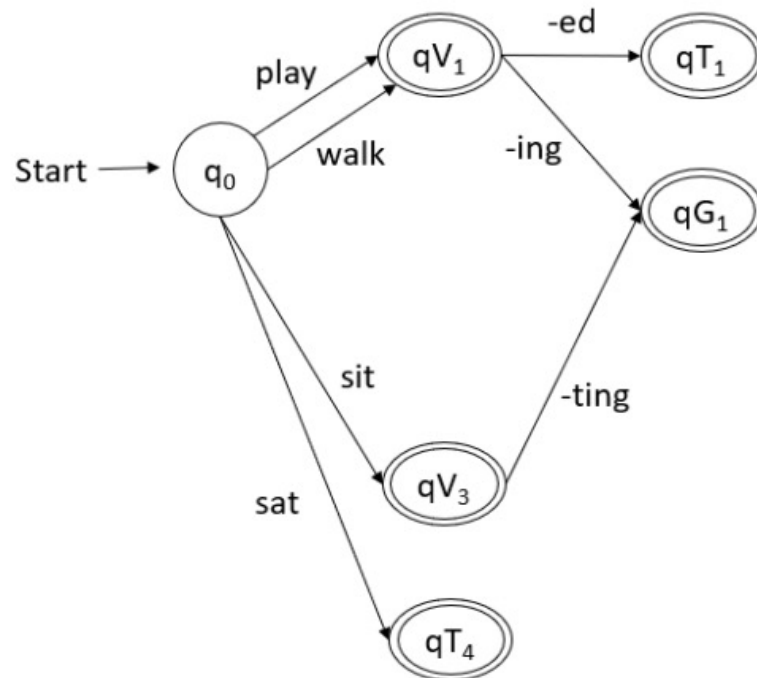
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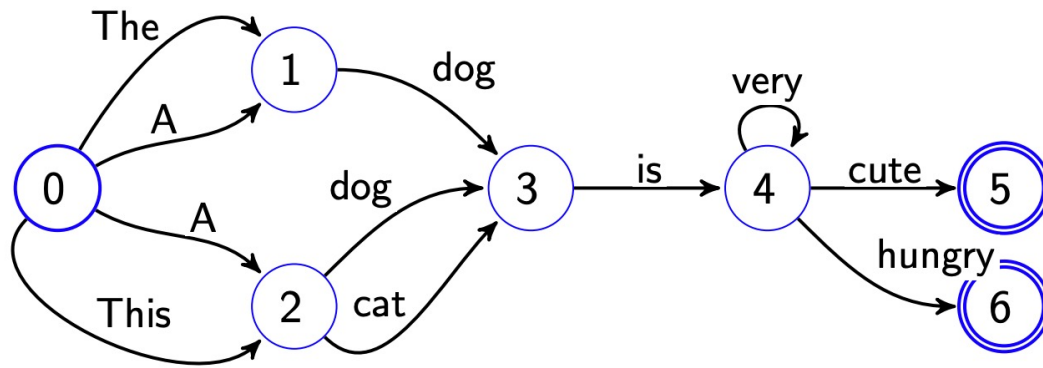
FSAs vs Weighted Finite State Acceptors (WFSAs)

What are Weighted Finite State Acceptors (WFSAs)? When and why are they useful?

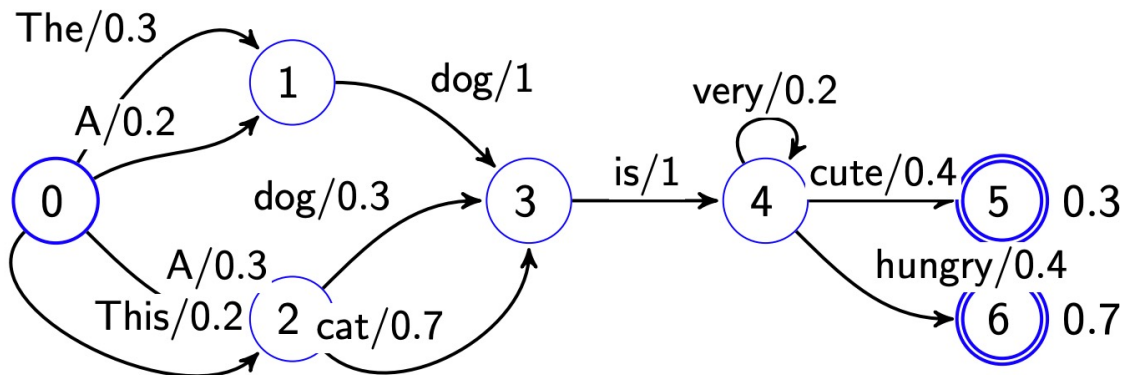
Weighted FSA (WFSA)

A **WFSA** is an FSA with **weights** on the edges.

FSA



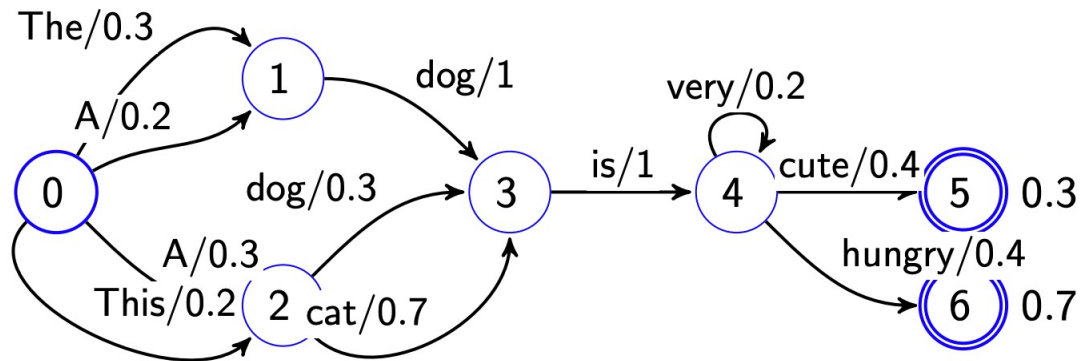
WFSA



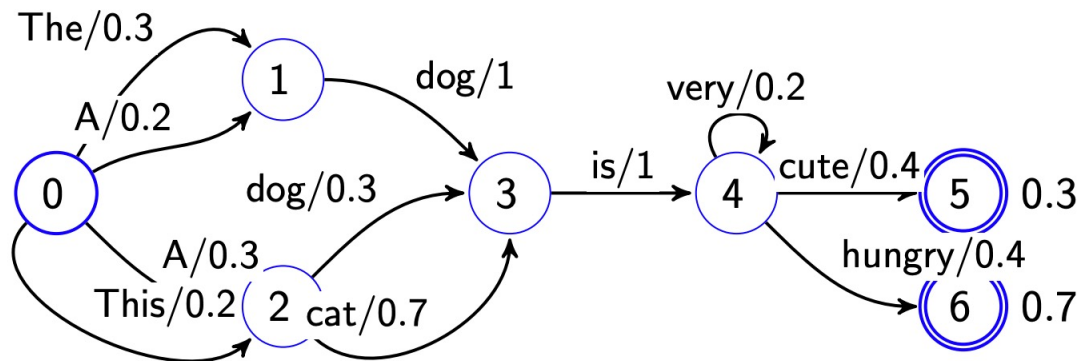
How does a WFSA work?

Add weights to initial/final states of FSA

- λ : assign weights to initial state
- ρ : assign weight to final state
- δ : assign weight to transitions



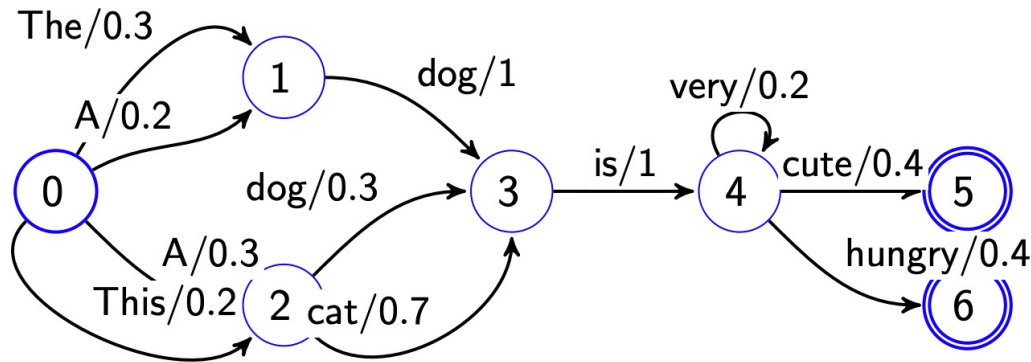
How does a WFSA work?



A WFSA assign a score for any path $\pi = t_1, t_2, \dots, t_N$

- $S(\pi) = \lambda(t_0) + N \times \delta(t_n) + \rho(t_N)$.
- Dijkstra algorithm for shortest path

How to accumulate the path score?



An example: there are two matched paths for “A dog is hungry”.

The scores are calculated by multiplying the edge scores:

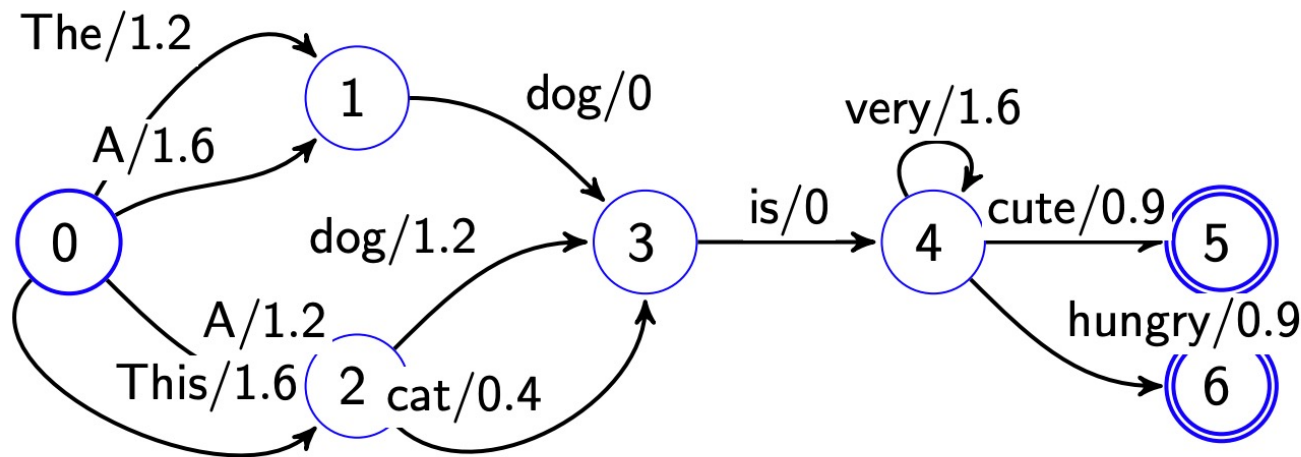
$$p(\text{Path through state 1}) = (0.2)(1)(1)(0.4) = 0.08$$

$$p(\text{Path through state 2}) = (0.3)(0.3)(1)(0.4) = 0.036$$

How to accumulate the path score?

The Issue: WFSA's have floating point underflow problems

The solution: negative log probabilities



“A dog is hungry”.

The negative log probs for two paths:

- $\ln p(\text{Path through state 1}) = 1.6 + 0 + 0 + 0.9 = 2.5$
- $\ln p(\text{Path through state 2}) = 1.2 + 1.2 + 0 + 0.9 = 3.3$



FSA vs WFSA

FSA:

- can only accept/reject input strings: is it grammatical/valid? (binary)
- returns **VALID** if a string corresponds to a valid path from start to end, and **NOT-VALID** otherwise

WFSA:

- decomposes a sequence (a word or a sentence) into sub-sequences
- gives a score: how grammatical/likely a sequence is (eg. N-gram language model;)



When and why WFSAs are useful?

Words: assign scores to all strings of characters forming words to handle:

- Spelling mistakes
- New words
- Strange but acceptable words

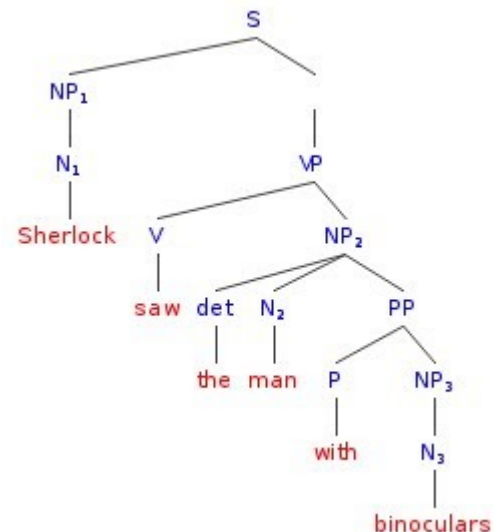
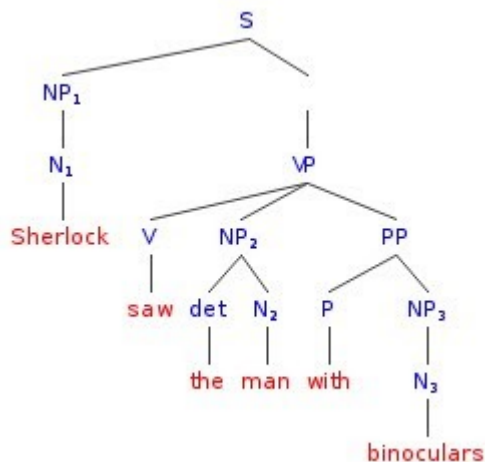
Sentences: assign scores to all strings of words forming sentences to handle:

- Sentence generation, speech recognition, OCR, translation

Parsing

Parsing in general is the process of identifying the structure(s) of a sentence, according to a grammar of the language.

Example parse trees





Context-free grammar

This is the dog that worried the cat that killed the rat
that ate the malt that lay in the house that Jack built

VS

A man that a woman that a child that a bird that I
heard saw knows loves

Context-free grammar

A context-free grammar is a 4-tuple $G = (V, \Sigma, R, S)$

1. V , a set of non-terminal symbols (or variables)
2. Σ , a set of terminal symbols (disjoint from V)
3. R a set of rules in the form $A \rightarrow \beta$, where A is a non-terminal, β is a string of variables or terminals
4. $S \in V$, the start variable



Parsing

(a) What changes need to be made to this CFG to make it suitable for CYK parsing?

- $S \rightarrow NP VP$
- $VP \rightarrow V NP \mid V NP PP$
- $PP \rightarrow P NP$
- $V \rightarrow \text{"saw"} \mid \text{"walked"}$
- $NP \rightarrow \text{"John"} \mid \text{"Bob"} \mid \text{Det } N \mid \text{Det } N PP$
- $\text{Det} \rightarrow \text{"a"} \mid \text{"an"} \mid \text{"the"} \mid \text{"my"}$
- $N \rightarrow \text{"man"} \mid \text{"cat"} \mid \text{"telescope"} \mid \text{"park"}$
- $P \rightarrow \text{"on"} \mid \text{"by"} \mid \text{"with"}$

Step 1: Chomsky Normal Form: Each rule consists of either:

- $A \rightarrow BC$ a (single) non-terminal which re-writes as exactly two non-terminals
- $A \rightarrow a$ a (single) non-terminal which re-writes as a single terminal



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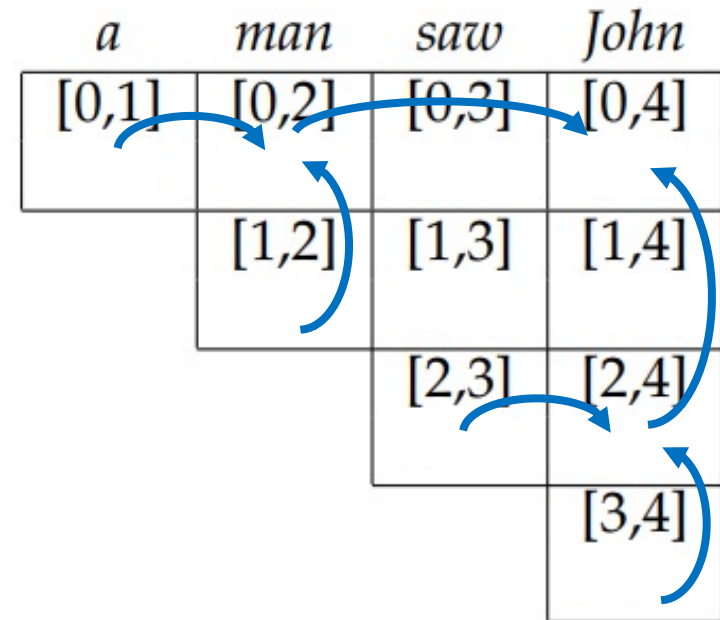
• $VP \rightarrow V X$
• $X \rightarrow NP PP$

• $NP \rightarrow \text{Det } Y$
• $Y \rightarrow N PP$

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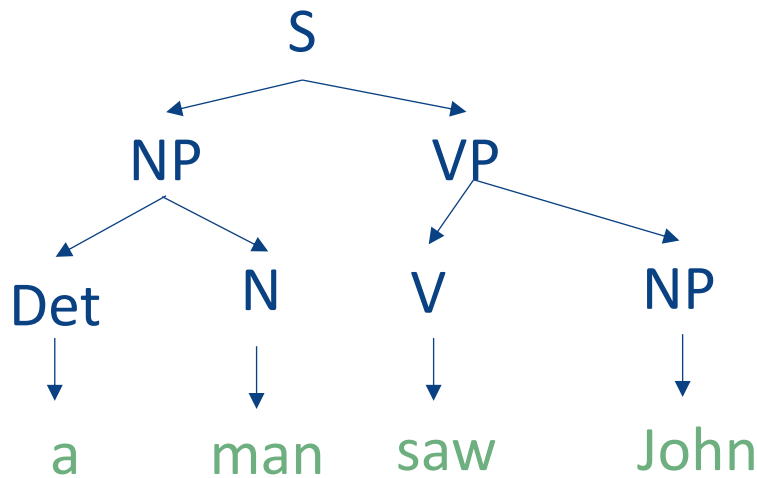
- $A \rightarrow B C$ a (single) non-terminal which re-writes as exactly two non-terminals
- $A \rightarrow a$ a (single) non-terminal which re-writes as a single terminal

CYK Example 1



1. $S \rightarrow NP VP$
2. $VP \rightarrow V NP \mid V X$
3. $PP \rightarrow P NP$
4. $X \rightarrow NP PP$
5. $Y \rightarrow N PP$
6. $NP \rightarrow \text{"John"} \mid \text{"Bob"} \mid \text{Det } N \mid \text{Det } Y$
7. $V \rightarrow \text{"saw"} \mid \text{"walked"}$
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CYK Example 1



<i>a</i>	<i>man</i>	<i>saw</i>	<i>John</i>
[0,1] Det	[0,2] NP	[0,3] -	[0,4] S
	[1,2] N	[1,3] -	[1,4] -
		[2,3] V	[2,4] VP
			[3,4] NP

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CYK example 2

<i>an</i>	<i>park</i>	<i>by</i>	<i>Bob</i>	<i>walked</i>	<i>an</i>	<i>park</i>	<i>with</i>	<i>Bob</i>
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	[0,6]	[0,7]	[0,8]	[0,9]
	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]	[1,7]	[1,8]	[1,9]
		[2,3]	[2,4]	[2,5]	[2,6]	[2,7]	[2,8]	[2,9]
			[3,4]	[3,5]	[3,6]	[3,7]	[3,8]	[3,9]
				[4,5]	[4,6]	[4,7]	[4,8]	[4,9]
					[5,6]	[5,7]	[5,8]	[5,9]
						[6,7]	[6,8]	[6,9]
							[7,8]	[7,9]
								[8,9]

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CYK example 3

<i>park</i>	<i>by</i>	<i>the</i>	<i>cat</i>	<i>with</i>	<i>my</i>	<i>telescope</i>
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]	[0,6]	[0,7]
	[1,2]	[1,3]	[1,4]	[1,5]	[1,6]	[1,7]
		[2,3]	[2,4]	[2,5]	[2,6]	[2,7]
			[3,4]	[3,5]	[3,6]	[3,7]
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Takeaways

1. Regular languages:
 1. Regular Grammar
 2. Finite state acceptors (FSA)
 3. Weighted finite state acceptors (WFSA)
2. Context Free Languages
 1. Context Free Grammar
 2. CYK algorithm for parsing