

Motivation  
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Delete Relaxation  
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Additive and Max  
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Relaxed Plans  
oooooooooooo

Conclusion  
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# COMP90054 — AI Planning for Autonomy

## 5. Delete Relaxation Heuristics

It's a Long Way to the Goal, But How Long Exactly?

Part I: *Acting As If the World Can Only Get Better*

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# Agenda

## 1 Motivation

## 2 The Delete Relaxation

## 3 The Additive and Max Heuristics

## 4 Relaxed Plans

## 5 Conclusion

# Motivation

→ Delete relaxation is a method to relax planning tasks, and thus automatically compute heuristic functions  $h$ .

→ Every  $h$  yields good performance only in some domains! (Search reduction vs. computational overhead)

→ We must come up with as many alternative methods as possible!

We cover the 4 different methods currently known:

- Critical path heuristics:
- Delete relaxation. Soon to be Done.
- Abstractions.
- Landmarks.

→ Delete relaxation is very wide-spread, and highly successful for satisficing planning!

We introduce the method in STRIPS.

Motivation  
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## Reminder: Relaxing the World by Ignoring Delete Lists

“What was once true remains true forever.”

Relaxed world: (after)



# The Delete Relaxation

## Definition (Delete Relaxation).

- (i) For a STRIPS action  $a$ , by  $a^+$  we denote the corresponding *delete relaxed action*, or short *relaxed action*, defined by  $\text{pre}_{a^+} := \text{pre}_a$ ,  $\text{add}_{a^+} := \text{add}_a$ , and  $\text{del}_{a^+} := \emptyset$ .
- (ii) For a set  $A$  of STRIPS actions, by  $A^+$  we denote the corresponding set of relaxed actions,  $A^+ := \{a^+ \mid a \in A\}$ ; similarly, for a sequence  $\vec{a} = \langle a_1, \dots, a_n \rangle$  of STRIPS actions, by  $\vec{a}^+$  we denote the corresponding sequence of relaxed actions,  $\vec{a}^+ := \langle a_1^+, \dots, a_n^+ \rangle$ .
- (iii) For a STRIPS planning task  $\Pi = (F, A, c, I, G)$ , by  $\Pi^+ := (F, A^+, c, I, G)$  we denote the corresponding *(delete) relaxed planning task*.  
→ “+” super-script = delete relaxed. We'll also use this to denote states encountered within the relaxation. (For STRIPS,  $s^+$  is a fact set just like  $s$ .)

**Definition (Relaxed Plan).** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, and let  $s$  be a state. An *(optimal) relaxed plan* for  $s$  is an *(optimal) plan* for  $\Pi_s^+$ . A relaxed plan for  $I$  is also called a relaxed plan for  $\Pi$ .

→ Anybody remember what  $\Pi_s$  is?  $\Pi_s = (F, A, c, s, G)$

# A Relaxed Plan for “TSP” in Australia

The precondition still holds.



- 1 Initial state:**  $\{at(Sy), v(Sy)\}.$
  - 2 Apply**  $drive(Sy, Br)^+ :$   $\{at(Br), v(Br), at(Sy), v(Sy)\}.$
  - 3 Apply**  $drive(Sy, Ad)^+ :$   $\{at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}.$
  - 4 Apply**  $drive(Ad, Pe)^+ :$   $\{at(Pe), v(Pe), at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}.$
  - 5 Apply**  $drive(Ad, Da)^+ :$   $\{at(Da), v(Da), at(Pe), v(Pe), at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}.$

# State Dominance

**Definition (Dominance).** Let  $\Pi^+ = (F, A^+, c, I, G)$  be a STRIPS planning task, and let  $s^+, s'^+$  be states. We say that  $s'^+$  *dominates*  $s^+$  if  $s'^+ \supset s^+$ .

→ For example, on the previous slide, who dominates who? Each state along the relaxed plan dominates the previous one, simply because the actions don't delete any facts.

**Proposition (Dominance).** Let  $\Pi^+ = (F, A^+, c, I, G)$  be a STRIPS planning task, and let  $s^+, s'^+$  be states where  $s'^+$  dominates  $s^+$ . We have:

- (i) If  $s^+$  is a goal state, then  $s'^+$  is a goal state as well.
  - (ii) If  $\vec{a}^+$  is applicable in  $s^+$ , then  $\vec{a}^+$  is applicable in  $s'^+$  as well, and  $appl(s'^+, \vec{a}^+)$  dominates  $appl(s^+, \vec{a}^+)$ . Same preconditions held in both states

**Proof.** (i) is trivial. (ii) by induction over the length  $n$  of  $\vec{a}^+$ . Base case  $n = 0$  is trivial. Inductive case  $n \rightarrow n + 1$  follows directly from induction hypothesis and the definition of  $appl(., .)$ .

→ It is always better to have more facts true.

# The Delete Relaxation and State Dominance

**Proposition.** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, let  $s$  be a state, and let  $a \in A$ . Then  $appl(s, a^+)$  dominates both (i)  $s$  and (ii)  $appl(s, a)$ .

**Proof.** Trivial from the definitions of  $appl(s, a)$  and  $a^+$ .

⇒ Optimal relaxed plans admissibly estimate the cost of optimal plans:  
 $a^+ \leq a$

**Proposition (Delete Relaxation is Admissible).** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, let  $s$  be a state, and let  $\vec{a}$  be a plan for  $\Pi_s$ . Then  $\vec{a}^+$  is a relaxed plan for  $s$ .

**Proof.** Prove by induction over the length of  $\vec{a}$  that  $appl(s, \vec{a}^+)$  dominates  $appl(s, \vec{a})$ . Base case is trivial, inductive case follows from (ii) above.

⇒ It is now clear how to find a relaxed plan:

- Applying a relaxed action can only ever make more facts true ((i) above).
  - That can only be good, i.e., cannot render the task unsolvable (dominance proposition).
- So? Keep applying relaxed actions, stop if goal is true (see next slide).

# Greedy Relaxed Planning

$$|h^{\text{greedy}} - h^*| > |h^{\text{max}} - h^*| \quad \begin{array}{l} \text{worst case: } h^{\text{greedy}} [0, 1A1] \\ \text{In practice: less useful / informed} \end{array}$$

## Greedy Relaxed Planning for $\Pi_s^+$

```

 $s^+ := s; \vec{a}^+ := \langle \rangle$ 
while  $G \not\subseteq s^+$  do: action in reverse direction is invalid (i.e sy -> br, br -> sy same)
    if  $\exists a \in A$  s.t.  $pre_a \subseteq s^+$  and  $appl(s^+, a^+) \neq s^+$  then
        randomly select one such a
         $s^+ := appl(s^+, a^+); \vec{a}^+ := \vec{a}^+ \circ \langle a^+ \rangle$  update state by next state
        else return " $\Pi_s^+$  is unsolvable" endif add action+ to the relaxed plan(action list)
    endwhile
    return  $\vec{a}^+$ 

```

**Proposition.** Greedy relaxed planning is **sound**, **complete**, and **terminates in time polynomial in the size of  $\Pi$** .

**Proof.** Soundness: If  $\vec{a}^+$  is returned then, by construction,  $G \subseteq appl(s, \vec{a}^+)$ . Completeness: If " $\Pi_s^+$  is unsolvable" is returned, then no relaxed plan exists for  $s^+$  at that point; since  $s^+$  dominates  $s$ , by the dominance proposition this implies that no relaxed plan can exist for  $s$ .

Termination: Every  $a \in A$  can be selected at most once because afterwards  $appl(s^+, a^+) = s^+$ .

⇒ It is easy to decide whether a relaxed plan exists!

# Greedy Relaxed Planning to Generate a Heuristic Function?

## Using greedy relaxed planning to generate $h$

- In search state  $s$  during forward search, run greedy relaxed planning on  $\Pi_s^+$ .
- Set  $h(s)$  to the cost of  $\vec{a}^+$ , or  $\infty$  if “ $\Pi_s^+$  is unsolvable” is returned.

→ Is this heuristic safe? Yes:  $h(s) = \infty$  only if no relaxed plan for  $s$  exists, which by admissibility of delete relaxation implies that no plan for  $s$  exists.

→ Is this heuristic goal-aware? Yes, we'll have  $G \subseteq s^+$  right at the start.

→ Is this heuristic admissible? Would be if the relaxed plans were optimal; but they clearly aren't. So  $h$  isn't consistent either.

→ To be informed (accurately estimate  $h^*$ ), a heuristic needs to approximate the *minimum effort* needed to reach the goal. Greedy relaxed planning doesn't do this because it may select arbitrary actions that aren't relevant at all.

# $h^+$ : The Optimal Delete Relaxation Heuristic

**Definition ( $h^+$ ).** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task with state space  $\Theta_\Pi = (S, A, c, T, I, G)$ . The optimal delete relaxation heuristic  $h^+$  for  $\Pi$  is the function  $h^+ : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$  where  $h^+(s)$  is defined as the cost of an optimal relaxed plan for  $s$ .

**Corollary ( $h^+$  is Admissible).** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task. Then  $h^+$  is admissible, and thus safe and goal-aware. (By admissibility of delete relaxation.)

→ To be informed (accurately estimate  $h^*$ ), a heuristic needs to approximate the *minimum effort* needed to reach the goal.  $h^+$  naturally does so by asking for the cheapest possible relaxed plans.

[→ You might rightfully ask “But won’t optimal relaxed plans usually under-estimate  $h^*$ ?” Yes, but that’s just the effect of considering a relaxed problem, and arbitrarily adding actions useless within the relaxation does not help to address it.]

# $h^+$ in “TSP” in Australia

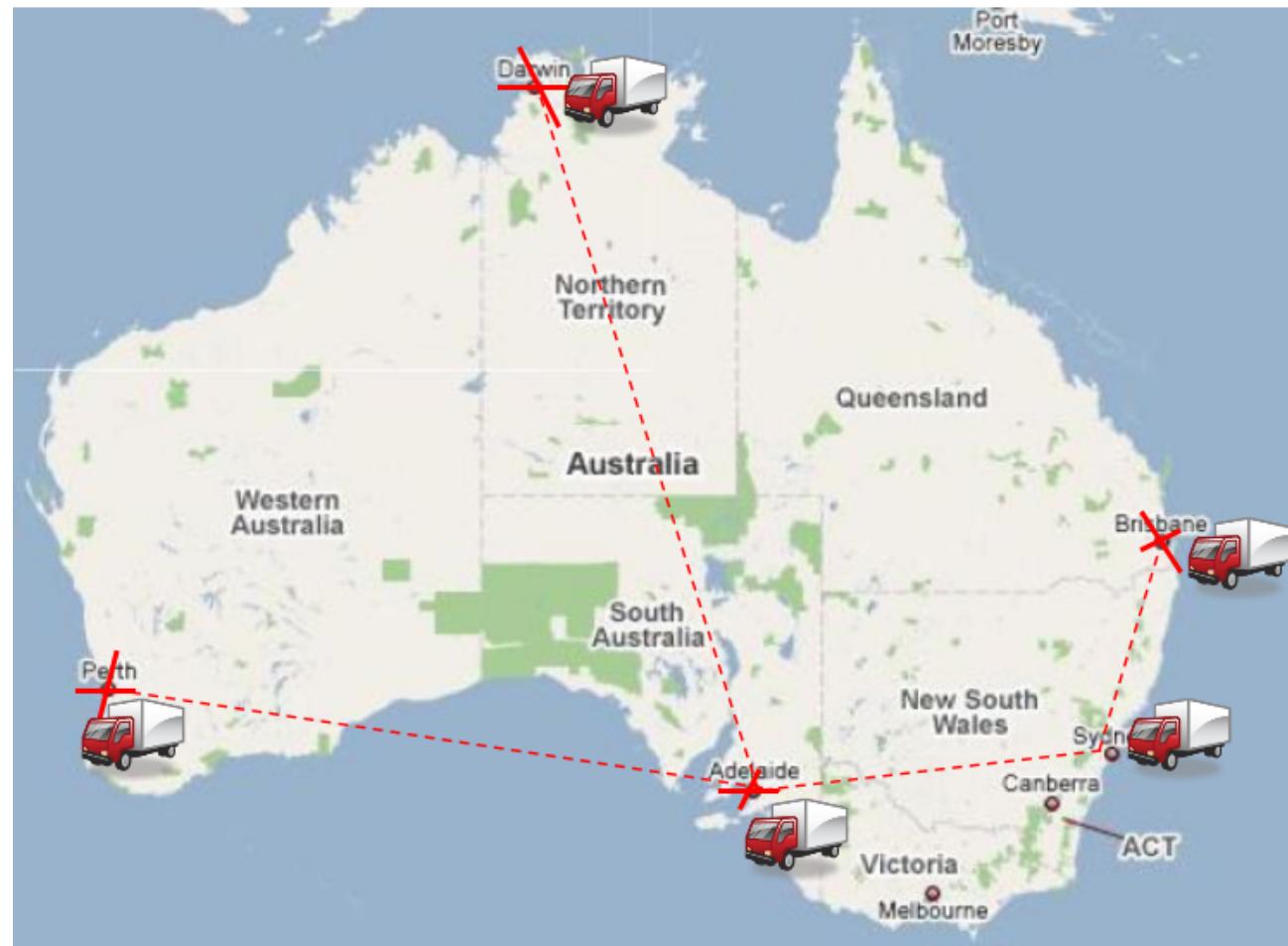


- $P$ :  $at(x)$  for  $x \in \{Sy, Ad, Br, Pe, Ad\}$ ;  $v(x)$  for  $x \in \{Sy, Ad, Br, Pe, Ad\}$ .
  - $A$ :  $drive(x, y)$  where  $x, y$  have a road.
- $$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$
- days
- $I$ :  $at(Sy), v(Sy); G$ :  $at(Sy), v(x)$  for all  $x$ .

## Planning vs. Relaxed Planning:

- **Optimal plan:**  $\langle drive(Sy, Br), drive(Br, Sy), drive(Sy, Ad), drive(Ad, Pe), drive(Pe, Ad), drive(Ad, Da), drive(Da, Ad), drive(Ad, Sy) \rangle$ .
- **Optimal relaxed plan:**  $\langle drive(Sy, Br), drive(Sy, Ad), drive(Ad, Pe), drive(Ad, Da) \rangle$ .
- $h^*(I) = 20; h^+(I) = 10$ .

## Reminder: $h^+$ in (the real) TSP



$h^+(TSP) = \text{Minimum Spanning Tree!}$

Motivation  
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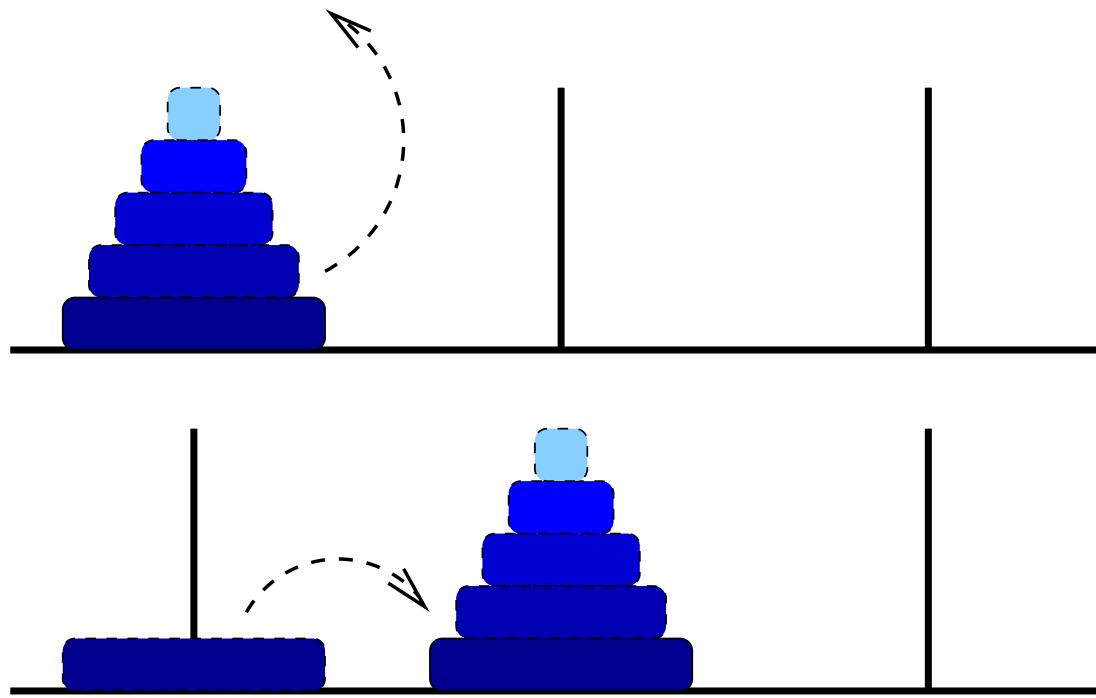
Delete Relaxation  
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Additive and Max  
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Relaxed Plans  
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Conclusion  
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## Reminder: $h^+$ in Hanoi



$$h^+(\text{Hanoi}) = n, \text{ not } 2^n$$

## But How to Compute $h^+$ ?

**Definition (Optimal Relaxed Planning).** By  $\text{PlanOpt}^+$ , we denote the problem of deciding, given a STRIPS planning task  $\Pi = (F, A, c, I, G)$  and  $B \in \mathbb{R}_0^+$ , whether there exists a relaxed plan for  $\Pi$  whose cost is at most  $B$ .

→ By computing  $h^+$ , we would solve  $\text{PlanOpt}^+$ .

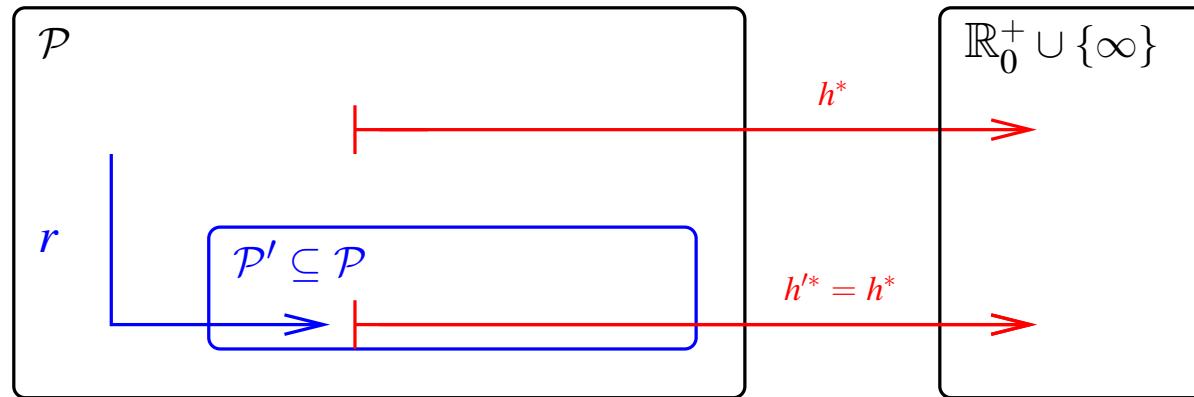
**Theorem (Optimal Relaxed Planning is Hard).**  $\text{PlanOpt}^+$  is NP-complete.

**Proof.** Membership: Guess action sequences of length  $|A|$  – in a relaxed plan, each action is applied at most once!

Hardness: By reduction from SAT.

- For each variable  $v_i \in \{v_1, \dots, v_m\}$  in the CNF, three facts  $v_i$ ,  $\text{not}v_i$ , and  $\text{set}v_i$ ; for each clause  $c_j \in \{c_1, \dots, c_n\}$  in the CNF, one fact  $\text{sat}c_j$ .
- Actions  $\text{setvtrue}_i$ :  $(\emptyset, \{v_i, \text{set}v_i\}, \emptyset)$  and  $\text{setvfalse}_i$ :  $(\emptyset, \{\text{not}v_i, \text{set}v_i\}, \emptyset)$ .
- Actions  $\text{makesat}c_j$ :  $(\{v_i\}, \{\text{sat}c_j\}, \emptyset)$  where  $v_i$  appears positively in clause  $c_j$ ;  $(\{\text{not}v_i\}, \{\text{sat}c_j\}, \emptyset)$  where  $v_i$  appears negatively in clause  $c_j$ .
- Initial state  $\emptyset$ , goal  $\{\text{set}v_1, \dots, \text{set}v_m, \text{sat}c_1, \dots, \text{sat}c_n\}$ ;  $B := m + n$ .

# $h^+$ as a Relaxation Heuristic



where, for all  $\Pi \in \mathcal{P}$ ,  $h^*(r(\Pi)) \leq h^*(\Pi)$ .

For  $h^+ = h^* \circ r$ :

- Problem  $\mathcal{P}$ : All STRIPS planning tasks.
- Simpler problem  $\mathcal{P}'$ : All STRIPS planning tasks with empty deletes.
- Perfect heuristic  $h'^*$  for  $\mathcal{P}'$ : Optimal plan cost =  $h^*$  on  $\mathcal{P}'$ .
- Transformation  $r$ : Drop the deletes.

- Is this a native relaxation? Yes.
- Is this relaxation efficiently constructible? Yes.
- Is this relaxation efficiently computable? No.

# What shall we do with this relaxation?

## Reminder:

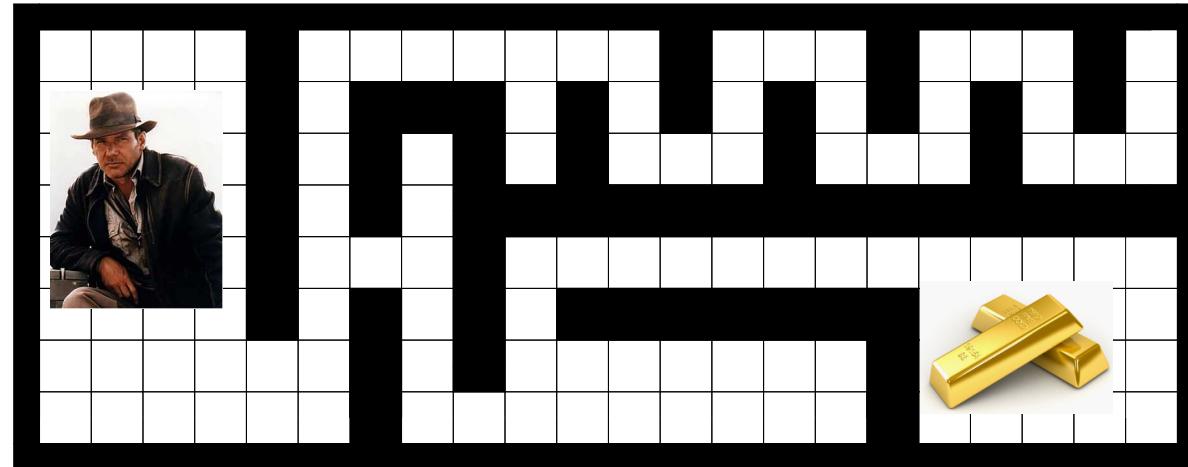
→ **Lecture 4**

### What if $\mathcal{R}$ is not efficiently computable?

- Either (a) approximate  $h'^*$ , or (b) design  $h'^*$  in a way so that it will typically be feasible, or (c) just live with it and hope for the best.
- Many known relaxations (in planning) are efficiently computable, some aren't (like  $h^+$ ). The latter use (a); (b) and (c) are not used anywhere right now.

→ The delete relaxation heuristic we want is  $h^+$ . Unfortunately, this is hard to compute so the computational overhead is very likely to be prohibitive. All implemented systems using the delete relaxation approximate  $h^+$  in one or the other way. We now look at the most wide-spread approaches to do so.

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### Question!

In this domain,  $h^+$  is equal to?

- (A): Manhattan Distance.      (B):  $h^*$ .  
(C): Horizontal distance.      (D): Vertical distance.

→ (A): No, relaxed plans can't walk through walls. (B): Yes, optimal plan = shortest path = relaxed plan (deletes do not matter because “shortest paths never walk back”). (C), (D): No, relaxed plans must move both horizontally and vertically.

# The Additive and Max Heuristics

**Definition ( $h^{\text{add}}$ )**. Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task. The additive heuristic  $h^{\text{add}}$  for  $\Pi$  is the function  $h^{\text{add}}(s) := h^{\text{add}}(s, G)$  where  $h^{\text{add}}(s, g)$  is the point-wise greatest function that satisfies  $h^{\text{add}}(s, g) =$

$G \& g \Rightarrow h^{\text{add}}$  relationship

$g \subseteq F$  ①  $g = G \subseteq F$

②  $g = \text{pre}_a \subseteq F$

$$\begin{cases} 0 & g \subseteq s \text{ (Goal)} \\ \min_{a \in A, g \in \text{add}_a} [c(a) + h^{\text{add}}(s, \text{pre}_a)] & |g| = 1 \quad g = \{\text{act}(Sg)\} \quad \text{Pre}_a = \{\text{act}(Ad), \dots, \text{act}(Br)\} \\ \sum_{g' \in g} h^{\text{add}}(s, \{g'\}) & |g| > 1 \\ & g = \{\text{act}(Sy) \cdot \text{act}(Ad)\} \\ & \quad c(a) + h(\text{act}(Ad)) \\ & h^{\text{add}} = h(\text{act}(Sy)) + h(\text{act}(Ad)) & \left. \begin{array}{l} c(a) + h(\text{act}(Br)) \\ \vdots \\ \min \end{array} \right\} \end{cases}$$

**Definition ( $h^{\text{max}}$ )**. Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task. The max heuristic  $h^{\text{max}}$  for  $\Pi$  is the function  $h^{\text{max}}(s) := h^{\text{max}}(s, G)$  where  $h^{\text{max}}(s, g)$  is the point-wise greatest function that satisfies  $h^{\text{max}}(s, g) =$

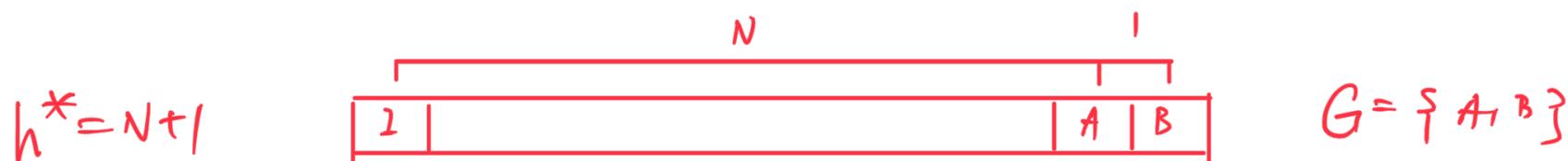
$$\begin{cases} 0 & g \subseteq s \\ \min_{a \in A, g \in \text{add}_a} [c(a) + h^{\text{max}}(s, \text{pre}_a)] & |g| = 1 \\ \max_{g' \in g} h^{\text{max}}(s, \{g'\}) & |g| > 1 \\ & h^{\text{max}} = \max [h(\text{act}(Sy)), h(\text{act}(Ad))] \end{cases}$$

# The Additive and Max Heuristics: Properties

**Proposition ( $h^{\max}$  is Optimistic).**  $h^{\max} \leq h^+$ , and thus  $h^{\max} \leq h^*$ .

**Proposition ( $h^{\text{add}}$  is Pessimistic).** For all STRIPS planning tasks  $\Pi$ ,  $h^{\text{add}} \geq h^+$ . There exist  $\Pi$  and  $s$  so that  $h^{\text{add}}(s) > h^*(s)$ .

→ Both  $h^{\max}$  and  $h^{\text{add}}$  approximate  $h^+$  by assuming that singleton sub-goal facts are achieved independently.  $h^{\max}$  estimates optimistically by the most costly singleton sub-goal,  $h^{\text{add}}$  estimates pessimistically by summing over all singleton sub-goals.



$$h^{\text{add}}(s, G) = h(s, A) + h(s, B) = N + (N + 1) = 2N + 1$$

$$h^{\max}(s, G) = \max[N, N + 1] = N + 1$$

## The Additive and Max Heuristics: Properties, ctd.

**Proposition ( $h^{\max}$  and  $h^{\text{add}}$  Agree with  $h^+$  on  $\infty$ ).** For all STRIPS planning tasks  $\Pi$  and states  $s$  in  $\Pi$ ,  $h^+(s) = \infty$  if and only if  $h^{\max}(s) = \infty$  if and only if  $h^{\text{add}}(s) = \infty$ .

→ States for which no relaxed plan exists are easy to recognize, and that is done by both  $h^{\max}$  and  $h^{\text{add}}$ . Approximation is needed only for the cost of an optimal relaxed plan, if it exists.

# Bellman-Ford for $h^{\max}$ and $h^{\text{add}}$

Bellman-Ford variant computing  $h^{\text{add}}$  for state  $s$

**new table**  $T_0^{\text{add}}(g)$ , for  $g \in F$

For all  $g \in F$ :  $T_0^{\text{add}}(g) := \begin{cases} 0 & g \in s \\ \infty & \text{otherwise} \end{cases}$

} Init table

**fn**  $c_i(g) := \begin{cases} T_i^{\text{add}}(g) & |g| = 1 \\ \sum_{g' \in g} T_i^{\text{add}}(g') & |g| > 1 \end{cases}$

} Relaxation

**fn**  $f_i(g) := \min[c_i(g), \min_{a \in A, g \in \text{add}_a} c(a) + c_i(\text{pre}_a)] \rightarrow$  Update rule

**do forever:**

**new table**  $T_{i+1}^{\text{add}}(g)$ , for  $g \in F \rightarrow$  add row

For all  $g \in F$ :  $T_{i+1}^{\text{add}}(g) := f_i(g) \rightarrow$  update row

**if**  $T_{i+1}^{\text{add}} = T_i^{\text{add}}$  **then stop endif**

$i := i + 1 \rightarrow$  check termination (if last 2 rows are the same)

**enddo**

→ Basically the same algorithm works for  $h^{\max}$ , just change  $\sum$  for  $\max$

**Proposition.** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task. Then the series  $\{T_i^{\text{add}}(g)\}_{i=0, \dots}$  converges to  $h^{\text{add}}(s, g)$ , for all  $g$ . (Proof omitted.)

# Bellman-Ford for $h^{\max}$ in “TSP” in Australia



- $F: at(x)$  for  $x \in \{Sy, Ad, Br, Pe, Ad\}$ ;  $v(x)$  for  $x \in \{Sy, Ad, Br, Pe, Ad\}$ .
- $A: drive(x, y)$  where  $x, y$  have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

- $I: at(Sy), v(Sy); G: at(Sy), v(x)$  for all  $x$ .

**Content of Tables  $T_i^1$ :**

$i$	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$	$\infty$	$\infty$	$\infty$
1	0	1.5	1	$\infty$	$\infty$	0	1.5	1	$\infty$	$\infty$
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\max}(I) = 5.5 << 20 = h^*(I).$        $\uparrow$        $\uparrow$   
 $1.5 + 3.5$        $1.5 + 4$

# Bellman-Ford for $h^{\text{add}}$ in “TSP” in Australia



- $F: at(x)$  for  $x \in \{Sy, Ad, Br, Pe, Ad\}$ ;  $v(x)$  for  $x \in \{Sy, Ad, Br, Pe, Ad\}$ .
  - $A: \text{drive}(x, y)$  where  $x, y$  have a road.
- $$c(\text{drive}(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$
- $I: at(Sy), v(Sy); G: at(Sy), v(x)$  for all  $x$ .

**Content of Tables  $T_i^{\text{add}}$ :**

$i$	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$	$\infty$	$\infty$	$\infty$
1	0	1.5	1	$\infty$	$\infty$	0	1.5	1	$\infty$	$\infty$
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

More informed  
closer to  $h^*$

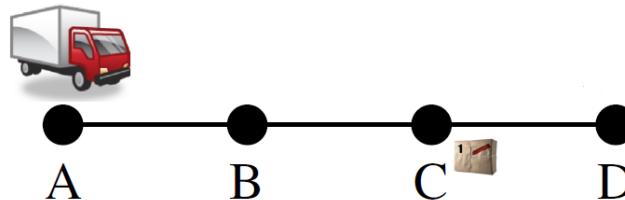
$$\rightarrow h^{\text{add}}(I) = 1.5 + 1 + 5 + 5.5 = 13 > 10 = h^+(I). \text{ But } < 20 = h^*(I).$$

not admissible

$\rightarrow h^{\text{add}}(I) > h^+(I)$  because it counts the cost of  $\text{drive}(Sy, Ad)$  3 times:

As part of  $h^{\text{add}}(I, \{v(Ad)\})$ ,  $h^{\text{add}}(I, \{v(Pe)\})$ , and  $h^{\text{add}}(I, \{v(Da)\})$ !

# Bellman-Ford for $h^{\text{add}}$ in “Logistics”

 $p(T)$ 

$$lo(c) = h(T(c)) + h(p(c))$$

$$ul(d) = h(T(d)) + h(p(T))$$

**Content of Tables**  $T_i^{\text{add}}$ : (Table content  $T_i^1$ , where different, given in red)

$i$	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$
1	0	1	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$
2	0	1	2	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$
3	0	1	2	3	3	$\infty$	$\infty$	0	$\infty$
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$ . But  $< 8 = h^*(I)$ .

$\rightarrow h^{\text{add}}(I) > h^+(I)$  because? It counts the cost of  $dr(A, B)$ ,  $dr(B, C)$  2 times, for the two preconditions  $p(T)$  and  $t(D)$  of achieving  $p(D)$ .

$\rightarrow$  So, what if  $G = \{t(D), p(D)\}$ ?  $h^{\text{add}}(I) = 10 > 5 = h^*(I) = h^+(I)$  because now  $dr(A, B)$ ,  $dr(B, C)$ ,  $dr(C, D)$  is counted also as part of the goal  $t(D)$ .

- Initial state  $I: t(A), p(C)$ . Pre:  $T(X), P(X)$
- Goal  $G: t(A), p(D)$ . Post:  $T(X), P(T)$
- Actions  $A: dr(X, Y), lo(X), ul(X)$ . cost of each action = 1

$$\begin{aligned} h^{\text{add}} \\ p(B) &= 1 + [h(T(B)) + h(p(T))] \\ &= 1 + [1 + 3] \\ &= 5 \end{aligned}$$

$$\begin{aligned} h^{\text{max}} \\ p(B) &= 1 + \max[h(T(B)), h(p(T))] \\ &= 1 + \max[1, 3] \\ &= 4 \end{aligned}$$

# The Additive and Max Heuristics: So What?

## Summary of typical issues in practice with $h^{\text{add}}$ and $h^{\text{max}}$ :

- Both  $h^{\text{add}}$  and  $h^{\text{max}}$  can be computed reasonably quickly.
- $h^{\text{max}}$  is **admissible**, but is typically **far too optimistic**.
- $h^{\text{add}}$  is **not admissible**, but is typically a lot more informed than  $h^{\text{max}}$ .
- $h^{\text{add}}$  is sometimes better informed than  $h^+$ , but for the “wrong reasons”: rather than accounting for deletes, it **overcounts** by **ignoring positive interactions**, i.e., sub-plans shared between sub-goals.
- Such overcounting can result in **dramatic over-estimates of  $h^*$ !!**

→ On slide 28 with goal  $t(D)$ , if we have 100 packages at  $C$  that need to go to  $D$ , what is  $h^{\text{add}}(I)$ ?  $703 >> 203 = h^*(I) = h^+(I)$ : For every package, a count of 7 which includes  $dr(A, B)$ ,  $dr(B, C)$  for getting the package into the truck, and  $dr(A, B)$ ,  $dr(B, C)$ ,  $dr(C, D)$  for getting the truck to  $D$ .

→ Relaxed plans (up next) are a means to reduce this kind of over-counting.

## Relaxed Plans, Basic Idea

- First compute a **best-supporter function  $bs$** , which for every fact  $p \in F$  returns an **action** that is deemed to be the **cheapest achiever of  $p$**  (within the relaxation). Then extract a **relaxed plan** from that function, by applying it to singleton sub-goals and collecting all the actions.
- The best-supporter function can be based directly on  $h^{\max}$  or  $h^{\text{add}}$ , simply selecting an action  $a$  achieving  $p$  that minimizes the sum of  $c(a)$  and the cost estimate for  $\text{pre}_a$ .

**And now for the details:**

# Popular Best-Supporter Functions

**Definition (Best-Supporters from  $h^{\max}$  and  $h^{\text{add}}$ )**. Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, and let  $s$  be a state.

The  $h^{\max}$  supporter function  $bs_s^{\max} : \{p \in F \mid 0 < h^{\max}(s, \{p\}) < \infty\} \mapsto A$  is defined by  
 $bs_s^{\max}(p) := \arg \min_{a \in A, p \in add_a} [c(a) + h^{\max}(s, pre_a)]$

The  $h^{\text{add}}$  supporter function  $bs_s^{\text{add}} : \{p \in F \mid 0 < h^{\text{add}}(s, \{p\}) < \infty\} \mapsto A$  is defined by  
 $bs_s^{\text{add}}(p) := \arg \min_{a \in A, p \in add_a} c(a) + h^{\text{add}}(s, pre_a)$ .

**Example  $h^{\text{add}}$  in "Logistics":**

Heuristic Values:

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
$h^{\text{add}}$	0	1	2	3	3	4	5	0	7

Yields best-supporter function:

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
$bs^{\text{add}}$	—	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	—	$ul(D)$

$$1 = 1 + 0 = c(a) + h(t(a))_{prea} = dr(A, B) + dr(B, C) + dr(C, D) = lo(C) = ul(A) + ul(B) = 1 + 2 = 3$$

## Relaxed Plan Extraction

↳ which action is considered to go into the relaxed plan

→ Facts need to be supported

→ Already supported in our relaxed plan

Relaxed Plan Extraction for state  $s$  and best-supporter function  $bs$

Open :=  $G \setminus s$ ; Closed :=  $\emptyset$ ; RPlan :=  $\emptyset$   
**while** Open  $\neq \emptyset$  **do:**

    select  $g \in Open$

Open := Open  $\setminus \{g\}$ ; Closed := Closed  $\cup \{g\}$ ;

RPlan := RPlan  $\cup \{bs(g)\}$ ; Open := Open  $\cup (pre_{bs(g)} \setminus (s \cup Closed))$

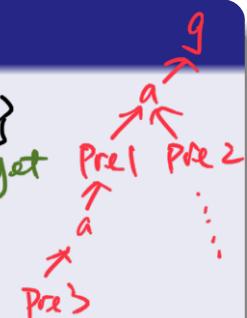
**endwhile**

**return** RPlan

$S = \{\text{initial state}\}$ .  $\text{closed} = \emptyset$ ,  $\text{RPLAN} = \emptyset$

$Open = \{\text{Goal, of which facts are in } S \text{ are EXCLUDED}\}$

not in initial state & not supported yet



↑  
Computed  
already

→ Starting with the top-level goals, iteratively close open singleton sub-goals by selecting the best supporter.

the size of the Facts (worst case)

This is fast! Number of iterations bounded by  $|P|$ , each near-constant time.

But is it correct?

→ What if  $g \notin add_{bs(g)}$ ? Doesn't make sense. → Prerequisite (A).

→ What if  $bs(g)$  is undefined? Runtime error. → Prerequisite (B).

→ What if the support for  $g$  eventually requires  $g$  itself as a precondition? Then this does not actually yield a relaxed plan. → Prerequisite (C).

# Relaxed Plan Extraction from $h^{\text{add}}$ in “Logistics”



- Initial state  $I: t(A), p(C)$ .
- Goal  $G: t(A), p(D)$ .
- Actions  $A: dr(X, Y), lo(X), ul(X)$ .

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
$bs_s^{\text{add}}$	—	$dr(A, B)$	$dr(B, C)$	$dr(C, D)$	$lo(C)$	$ul(A)$	$ul(B)$	—	$ul(D)$

Extracting a relaxed plan:

Initialization = Open =  $\{t(A), p(D)\}$ , closed =  $\emptyset$ .  $RP(I) = \emptyset$

- 1  $bs_s^{\text{add}}(p(D)) = ul(D)$ ; opens  $t(D), p(T)$ . closed × init state ✓ ↑  
Already  
True  
Initially
  - 2  $bs_s^{\text{add}}(t(D)) = dr(C, D)$ ; opens  $t(C)$ . closed ✓
  - 3  $bs_s^{\text{add}}(t(C)) = dr(B, C)$ , opens  $t(B)$ . closed ✓ init state ✓
  - 4  $bs_s^{\text{add}}(t(B)) = dr(A, B)$ ; opens nothing. closed ✓
  - 5  $bs_s^{\text{add}}(p(T)) = lo(C)$ ; opens nothing. closed ✓ init state ✓
  - 6 Anything more? No, open goals empty at this point.
- $h^{\text{FF}}(I) = 5 = h^+(I) < 7 = h^{\text{add}}(I) < 8 = h^*(I)$ . prev ✓ del ✓
- fast-forward prev del
- What if  $G = \{t(D), p(D)\}$ ?  $h^{\text{FF}}(I) = 5 = h^+(I) = h^*(I)$  because relaxed plan extraction selects the drive actions only once. By contrast,  $h^{\text{add}}(I) = 10$  overcounts these actions, cf. slide 28.

## Best-Supporter Functions

→ For relaxed plan extraction to make sense, it requires a ***closed well-founded*** best-supporter function:

**Definition (Best-Supporter Function).** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, and let  $s$  be a state. A **best-supporter function** for  $s$  is a partial function  $bs : (F \setminus s) \mapsto A$  such that  $p \in add_a$  whenever  $a = bs(p)$ .

*tuple(F, A)*

The **support graph** of  $bs$  is the directed graph with vertices  $F \cup A$  and arcs

$\{(p, a) \mid p \in pre_a\} \cup \{(a, p) \mid a = bs(p)\}$ . We say that  $bs$  is **closed** if  $bs(p)$  is defined for every  $p \in (F \setminus s)$  that has a path to a goal  $g \in G$  in the support graph. We say that  $bs$  is **well-founded** if the support graph is **acyclic**.

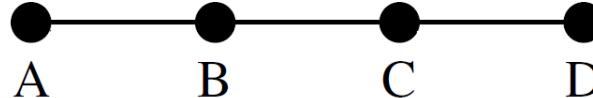
- “ $p \in add_a$  whenever  $a = bs(p)$ ”: Prerequisite (A).
- $bs$  is closed: Prerequisite (B).
- $bs$  is well-founded: Prerequisite (C).



→ Intuition for (C): Relaxed plan extraction starts at the goals, and chains backwards in the support graph. If there are cycles, then this backchaining may not reach the currently true state  $s$ , and thus not yield a relaxed plan.

Motivation  
○Delete Relaxation  
○○○○○○○○○○○○○○○○Additive and Max  
○○○○○○○○Relaxed Plans  
○○○○●○○○○○Conclusion  
○○○○○○○○

# Support Graphs and Prerequisite (C) in “Logistics”



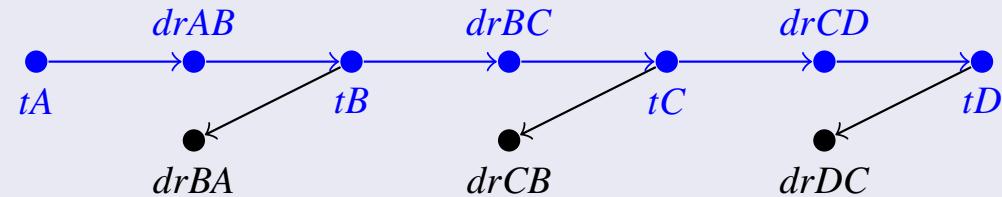
- Initial state:  $tA$ .
- Goal:  $tD$ .
- Actions:  $drXY$ .

## How to do it (well-founded)

Best-supporter function:

$p$	$bs(p)$
$t(B)$	$dr(A, B)$
$t(C)$	$dr(B, C)$
$t(D)$	$dr(C, D)$

Yields support graph backchaining:

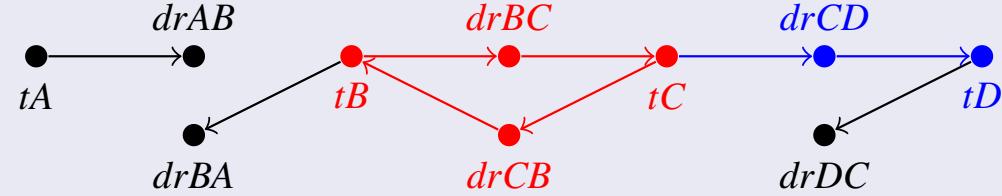


## How to NOT do it (not well-founded)

Best-supporter function:

$p$	$bs(p)$
$t(B)$	$dr(C, B)$
$t(C)$	$dr(B, C)$
$t(D)$	$dr(C, D)$

Yields support graph backchaining:



## How to obtain closed well-founded $bs$ ?

**Definition (Best-Supporters from  $h^{\max}$  and  $h^{\text{add}}$ ).** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, and let  $s$  be a state.

The  $h^{\max}$  supporter function  $bs_s^{\max} : \{p \in F \mid 0 < h^{\max}(s, \{p\}) < \infty\} \mapsto A$  is defined by  $bs_s^{\max}(p) := \arg \min_{a \in A, p \in add_a} c(a) + h^{\max}(s, pre_a)$ .

The  $h^{\text{add}}$  supporter function  $bs_s^{\text{add}} : \{p \in F \mid 0 < h^{\text{add}}(s, \{p\}) < \infty\} \mapsto A$  is defined by  $bs_s^{\text{add}}(p) := \arg \min_{a \in A, p \in add_a} c(a) + h^{\text{add}}(s, pre_a)$ .

**Proposition.** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task such that, for all  $a \in A$ ,  $c(a) > 0$ . Let  $s$  be a state where  $h^+(s) < \infty$ . Then both  $bs_s^{\max}$  and  $bs_s^{\text{add}}$  are closed well-founded supporter functions for  $s$ .

**Proof.** Since  $h^+(s) < \infty$  implies  $h^{\max}(s) < \infty$ , it is easy to see that  $bs_s^{\max}$  is closed (details omitted). If  $a = bs_s^{\max}(p)$ , then  $a$  is the action yielding  $0 < h^{\max}(s, \{p\}) < \infty$  in the  $h^{\max}$  equation. Since  $c(a) > 0$ , we have  $h^{\max}(s, pre_a) < h^{\max}(s, \{p\})$  and thus, for all  $q \in pre_a$ ,  $h^{\max}(s, \{q\}) < h^{\max}(s, \{p\})$ . Transitively, if the support graph contains a path from fact vertex  $r$  to fact vertex  $t$ , then  $h^{\max}(s, \{r\}) < h^{\max}(s, \{t\})$ . Thus there can't be cycles in the support graph and  $bs_s^{\max}$  is well-founded. Similar for  $bs_s^{\text{add}}$ .

## The Relaxed Plan Extraction: Correctness

**Proposition.** Let  $\Pi = (F, A, c, I, G)$  be a STRIPS planning task, let  $s$  be a state, and let  $bs$  be a closed well-founded best-supporter function for  $s$ . Then the action set  $RPlan$  returned by relaxed plan extraction can be sequenced into a relaxed plan  $\vec{a}^+$  for  $s$ .

**Proof.** Order  $a$  before  $a'$  whenever the support graph contains a path from  $a$  to  $a'$ . Since the support graph is acyclic, such a sequencing  $\vec{a} := \langle a_1, \dots, a_n \rangle$  exists. We have  $p \in s$  for all  $p \in pre_{a_1}$ , because otherwise  $RPlan$  would contain the action  $bs(p)$ , necessarily ordered before  $a_1$ . We have  $p \in s \cup add_{a_1}$  for all  $p \in pre_{a_2}$ , because otherwise  $RPlan$  would contain the action  $bs(p)$ , necessarily ordered before  $a_2$ . Iterating the argument shows that  $\vec{a}^+$  is a relaxed plan for  $s$ .

# The Relaxed Plan Heuristic

**Definition (Relaxed Plan Heuristic).** A heuristic function is called a relaxed plan heuristic, denoted  $h^{\text{FF}}$ , if, given a state  $s$ , it returns  $\infty$  if no relaxed plan exists, and otherwise returns  $\sum_{a \in R\text{Plan}} c(a)$  where  $R\text{Plan}$  is the action set returned by relaxed plan extraction on a closed well-founded best-supporter function for  $s$ .

→ Recall: If a relaxed plan exists, then there also exists a closed well-founded best-supporter function, see previous slide.

## The Relaxed Plan Heuristic, ctd.

Best-case (Lower Bound)

Safe

**Proposition** ( $h^{FF}$  is Pessimistic and Agrees with  $h^*$  on  $\infty$ ). For all STRIPS planning tasks  $\Pi$ ,  $h^{FF} \geq h^+$ ; for all states  $s$ ,  $h^+(s) = \infty$  if and only if  $h^{FF}(s) = \infty$ . There exist  $\Pi$  and  $s$  so that  $h^{FF}(s) > h^*(s)$ .

**Proof.**  $h^{FF} \geq h^+$  follows directly from the previous proposition. Agrees with  $h^+$  on  $\infty$ : direct from definition. Inadmissibility: Whenever  $bs$  makes sub-optimal choices. → **Exercise, perhaps**

→ Relaxed plan heuristics have the same theoretical properties as  $h^{add}$ .

### So what's the point?

- Can  $h^{FF}$  over-count, i.e., count sub-plans shared between sub-goals more than once? No, due to the set union in “ $RPlan := RPlan \cup \{bs(g)\}$ ”.
- $h^{FF}$  may be inadmissible, just like  $h^{add}$ , but for more subtle reasons.
- In practice,  $h^{FF}$  typically does not over-estimate  $h^*$  (or not by a large amount, anyway); cf. example on previous "Logistics" slide.

# Helpful Actions

**Definition (Helpful Actions)** Let  $h^{\text{FF}}$  be a relaxed plan heuristic, let  $s$  be a state, and let  $R\text{Plan}$  be the action set returned by relaxed plan extraction on the closed well-founded best-supporter function for  $s$  which underlies  $h^{\text{FF}}$ . Then an action  $a$  **applicable to  $s$**  is called **helpful** if it is **contained in  $R\text{Plan}$** .

## Remarks

- Initially introduced in **FF** [Hoffmann and Nebel (2011)], restricting **Enforced Hill-Climbing** to use **only** the helpful actions
- Expanding only helpful actions **does not guarantee completeness**.
- Other planners use helpful actions as preferred operators, expanding first nodes resulting from helpful actions.

Motivation  
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Delete Relaxation  
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Additive and Max  
oooooooooooo

Relaxed Plans  
oooooooooooo

Conclusion  
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# Questionnaire

## Question!

### How does ignoring delete lists simplify FreeCell?

- (A): You can move all cards immediately to their goal.      (B): Free cells remain free.

→ (A): No, we don't get any new moves in the relaxation. (B): Yes, when putting a card into a free cell, it's still free for another card.

## Question!

### How does ignoring delete lists simplify Sokoban?

- (A): Free positions remain free.      (B): You can walk through walls.  
(C): You can push 2 stones to same position.      (D): Nothing ever becomes blocked.

→ (A), (C), (D): Yes (similar to above). (B): No, we don't get any new moves.

## Summary

- The **delete relaxation** simplifies STRIPS by removing all delete effects of the actions.
- The cost of **optimal relaxed plans** yields the heuristic function  $h^+$ , which is admissible but hard to compute.
- We can approximate  $h^+$  optimistically by  $h^{\max}$ , and pessimistically by  $h^{\text{add}}$ .  $h^{\max}$  is admissible,  $h^{\text{add}}$  is not.  $h^{\text{add}}$  is typically much more informative, but can suffer from **over-counting**.
- Either of  $h^{\max}$  or  $h^{\text{add}}$  can be used to generate a **closed well-founded best-supporter function**, from which we can **extract a relaxed plan**. The resulting **relaxed plan heuristic**  $h^{\text{FF}}$  does not do over-counting, but otherwise has the same theoretical properties as  $h^{\text{add}}$ ; it typically does not over-estimate  $h^*$ .

# Example Systems

## HSP [*Bonet and Geffner, AI-01*]

1. Search algorithm: Greedy best-first search.
2. Search control:  $h^{\text{add}}$ .

## FF [*Hoffmann and Nebel ,JAIR-01*]

1. Search algorithm: Enforced hill-climbing.
2. Search control:  $h^{\text{FF}}$  extracted from  $h^{\max}$  supporter function; helpful actions pruning (basically expand only those actions contained in the relaxed plan).

## LAMA [*Richter and Westphal, JAIR-10*]

1. Search algorithm: Multiple-queue greedy best-first search.
2. Search control:  $h^{\text{FF}}$  + a landmarks heuristic ( $\rightarrow$  similar to goal counting); for each, one search queue all actions, one search queue only helpful actions.

## BFWS [*Lipovetzky and Geffner, AAAI-17*]

1. Search algorithm: best-first width search ( $\rightarrow$  next lecture).
2. Search control: novelty ( $\rightarrow$  next lecture) + variant of  $h^{\text{FF}}$  + goal counting.

Motivation  
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Delete Relaxation  
oooooooooooooooooooo

Additive and Max  
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Relaxed Plans  
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Conclusion  
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## Remarks

- The delete relaxation is aka [ignoring delete lists](#).
- HSP was competitive in the 1998 International Planning Competition (IPC'98); FF outclassed the competitors in IPC'00.
- The delete relaxation is still used at large, specially since the wins of LAMA in the satisficing planning tracks of IPC'08 and IPC'11.

## Remarks, ctd.

- → More generally, the relaxation principle is very generic and potentially applicable in many different contexts, as are all relaxation principles covered in this course.
- While  $h^{\max}$  is not informative in practice, other lower-bounding approximations of  $h^+$  are very important for optimal planning: **admissible landmarks heuristics** [*Karpas and Domshlak, IJCAI-09*]; **LM-cut heuristic** [*Helmert and Domshlak, ICAPS-09*].
- It has always been a challenge to take *some* delete effects into account. Recent work done to interpolate smoothly between  $h^+$  and  $h^*$ : **explicitly represented fact conjunctions** [*Keyder, Hoffmann, and Haslum ICAPS-12*].

Motivation  
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Delete Relaxation  
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Additive and Max  
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Relaxed Plans  
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Conclusion  
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## Reading

- *Planning as Heuristic Search [Bonet and Geffner, AI-01].*

Available at:

<http://www.dtic.upf.edu/~hgeffner/html/reports/hsp-aij.ps>

**Content:** This is “where it all started”: the first paper<sup>1</sup> explicitly introducing the notion of heuristic search and automatically generated heuristic functions to planning. Introduces the additive and max heuristics  $h^{\text{add}}$  and  $h^{\text{max}}$ .

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<sup>1</sup> Well, this is the first full journal paper treating the subject; the same authors published conference papers in AAAI'97 and ECP'99, which are subsumed by the present paper.

## Reading, ctd.

- *The FF Planning System: Fast Plan Generation Through Heuristic Search [Hoffmann:nebel:jair-01]. JAIR Best Paper Award 2005.*

Available at:

<http://fai.cs.uni-saarland.de/hoffmann/papers/jair01.pdf>

**Content:** The main reference for delete relaxation heuristics (cited > 1000 times). Introduces the relaxed plan heuristic, extracted from the  $h^{\max}$  supporter function.<sup>2</sup> Also introduces helpful actions pruning, and enforced hill-climbing.

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<sup>2</sup>Done in a uniform-cost setting presented in terms of relaxed planning graphs instead of  $h^{\max}$ , and not identifying the more general idea of using a well-founded best-supporter function (I used the same simpler presentation in the AI'12 core course). The notion of best-supporter functions (handling non-uniform action costs) first appears in [Keyder and Geffner, ECAI-08].

## Reading, ctd.

- *Semi-Relaxed Plan Heuristics [Keyder, Hoffmann, and Haslum ICAPS-12]*. **Best Paper Award at ICAPS'12**.

Available at: <http://fai.cs.uni-saarland.de/hoffmann/papers/icaps12a.pdf>

**Content:** Computes relaxed plan heuristics within a compiled planning task  $\Pi_{ce}^C$ , in which a subset  $C$  of all fact conjunctions in the task is represented explicitly as suggested by [Haslum, ICAPS-12].  $C$  can in principle always be chosen so that  $h_{\Pi_{ce}^C}^+$  is perfect (equals  $h^*$  in the original planning task), so the technique allows to interpolate between  $h^+$  and  $h^*$ . In practice, small sets  $C$  sometimes suffice to obtain dramatically more informed relaxed plan heuristics.