COMP90054 — AI Planning for Autonomy

4. Generating Heuristic Functions

How to Relax: Formally, and Informally, and During Search

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Semester 2

Agenda

1 Motivation

2 How to Relax Informally

3 How to Relax Formally

4 How to Relax During Search

5 Conclusion

Motivation

→ "Relax"ing is a methodology to construct heuristic functions.

- You can use it when programming a solution to some problem you want/need to solve.
- Planning systems can use it to derive a heuristic function **automatically** from the planning task description (the PDDL input).
  - **Note 1:** If the user had to supply the heuristic function by hand, then we would lose our two main selling points (generality & autonomy & flexibility & rapid prototyping, cf. → **Lecture 1-2**).
  - **Note 2:** It can of course be of advantage to give the user the *possibility* to (conveniently) supply additional heuristics. Not covered in this course.

How to Relax Informally

How To Relax:

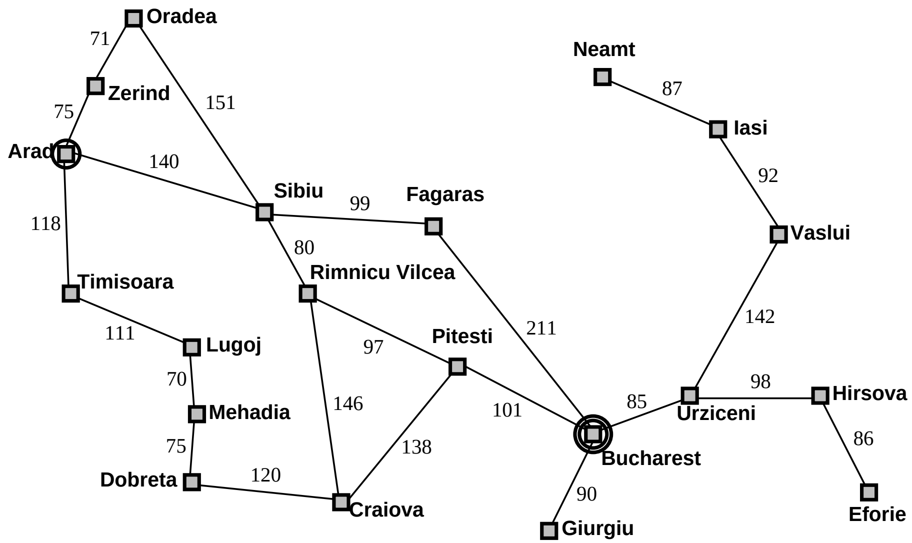
family

- You have a problem, $\mathcal{P}$, whose perfect heuristic h^* you wish to estimate.
- You define a simpler problem, $\mathcal{P}'$, whose perfect heuristic h'^* can be used to estimate h^*.
- You define a transformation, r, that simplifies instances from $\mathcal{P}$ into instances $\mathcal{P}'$.

  ↳ relaxation
- Given $\Pi \in \mathcal{P}$, you estimate $h^*(\Pi)$ by $h'^*(r(\Pi))$.

$\rightarrow$ Relaxation means to simplify the problem, and take the solution to the simpler problem as the heuristic estimate for the solution to the actual problem.

Relaxation in Route-Finding



Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Dobreta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

How to derive straight-line distance by relaxation?

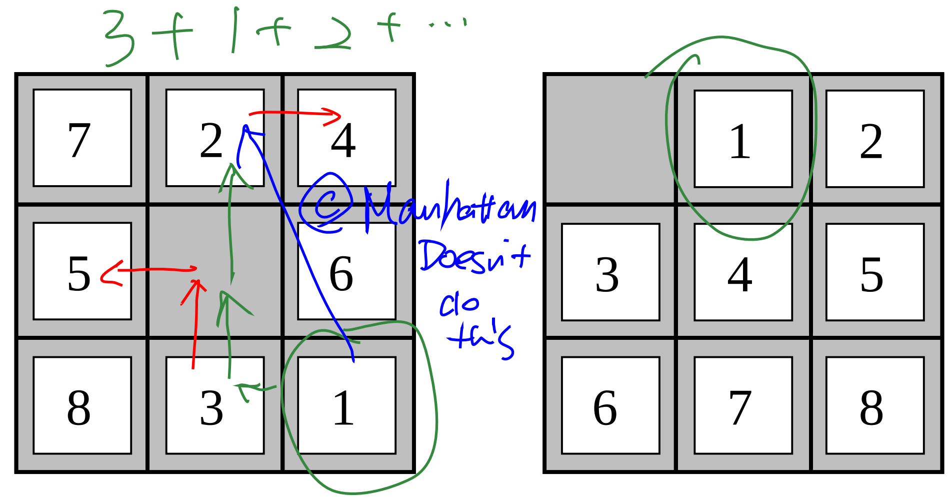
- Problem $\mathcal{P}$: Route finding.
- Simpler problem $\mathcal{P}'$: Route finding for birds.
- Perfect heuristic h'^* for $\mathcal{P}'$: Straight-line distance.
- Transformation r: Pretend you're a bird.

Relaxation in the 8-Puzzle

A move 1

B move 2 ✓

C move 1 & 2



Start State Goal State

Perfect heuristic h^* for $\mathcal{P}$: Actions = "A tile can move from square A to square B if A is adjacent (constraints 1) to B and B is blank." (2)

how to move the constraints (simplify) so that $h'^*(r(\pi))$ = Manhattan Dist

- How to derive the Manhattan distance heuristic? $\mathcal{P}'$: Actions = "A tile can move from square A to square B if A is adjacent to B."
- How to derive the misplaced tiles heuristic? $\mathcal{P}'$: Actions = "A tile can move from square A to square B."
  Is it in destination? No → Count 1. (For each cell)
- h'^* (resp. r) in both: optimal cost in $\mathcal{P}'$ (resp. use different actions).
- Here: Manhattan distance = 18, misplaced tiles = 8.

"Goal-Counting" Relaxation in Australia

$P' = D(x,y)$: $Pre = del = \phi$, $add = \{v(x), at(x)\}$

$S(State) = \{at(Sy), v(Sy), at(Pe), v(Pe), at(Br), v(Br), at(Ad), v(Ad), at(Da), v(Da)\}$ $\Rightarrow$ $G = \forall_x\, v(x)\ at(Sy)$ (All)

$\pi = \{D(Ad, Pe), D(Sy, Br), \cancel{D(Ad, Sy)}, D(Ad, Da)\}$; $D(Sy, Ad)$

Goal = $\underbrace{v(Da).\ v(Pe).\ v(Ad).\ v(Br)}_{4}.\ v(Sy).\ at(Sy)$

- Propositions P: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Da\}$; $v(x)$ (visit) for $x \in \{Sy, Ad, Br, Pe, Da\}$.
- Actions $a \in A$: $drive(x, y)$ where x, y have a road; $pre_a = \{at(x)\}$, $add_a = \{at(y), v(y)\}$, $del_a = \{\cancel{at(x)}\}$.
- Initial state I: $at(Sy), v(Sy)$. → we are at Sy and we've visited Sy
- Goal G: $at(Sy), v(x)$ for all x.

Let's "act as if we could achieve each goal directly":
- Problem $\mathcal{P}$: All STRIPS planning tasks.
- Simpler problem $\mathcal{P}'$: All STRIPS planning tasks with empty preconditions and deletes.
- Perfect heuristic h'^* for $\mathcal{P}'$: Optimal plan cost ($= h^*$).
- Transformation r: Drop the preconditions and deletes.
- Heuristic value here? 4. $h'^*(r(\pi)) = 4$ (Drive) **How many goals we have achieved now**

→ Optimal STRIPS planning with empty preconditions and deletes is still **NP**-hard! (Reduction from MINIMUM COVER, of goal set by add lists.)

→ Need to approximate the perfect heuristic h'^* for $\mathcal{P}'$. Hence goal counting: just approximate h'^* by number-of-false-goals.

How to Relax Formally: Before We Begin

- The definition on the next slide is not to be found in any textbook, and not even in any paper.
- Methods generating heuristic functions differ widely, and it is quite difficult (impossible?) to make *one* definition capturing them *all* in a natural way.
- Nevertheless, a formal definition is useful to state precisely what are the relevant distinction lines in practice.
- The present definition does, I think, do a rather good job of this.

  $\rightarrow$ It nicely fits what is currently used in planning.

  $\rightarrow$ It is flexible in the distinction lines, and it captures the basic construction, as well as the essence of all relaxation ideas.

Relaxations

Definition (Relaxation). *Let* $h^* : \mathcal{P} \mapsto \mathbb{R}^+_0 \cup \{\infty\}$ *be a function. A relaxation of* h^* *is a triple* $\mathcal{R} = (\mathcal{P}', r, h'^*)$ *where* $\mathcal{P}'$ *is an arbitrary set, and* $r : \mathcal{P} \mapsto \mathcal{P}'$ *and* $h'^* : P' \mapsto \mathbb{R}^+_0 \cup \{\infty\}$ *are functions so that, for all* $\Pi \in \mathcal{P}$*, the relaxation heuristic* $h^{\mathcal{R}}(\Pi) := h'^*(r(\Pi))$ *satisfies* **heuristic value** $h^{\mathcal{R}}(\Pi) \leq h^*(\Pi)$*. The relaxation is:*

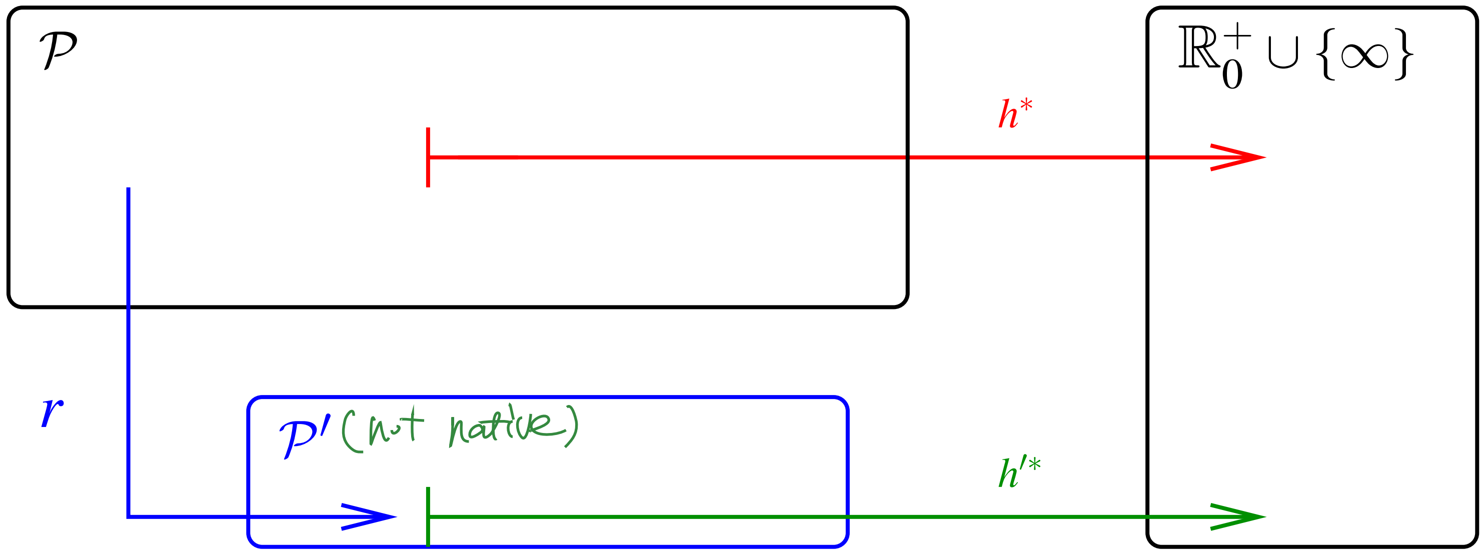
- *native if* $\mathcal{P}' \subseteq P$ *and* $h'^* = h^*$*;* $r =$ polynomial
- *efficiently constructible if there exists a polynomial-time algorithm that, given* $\Pi \in \mathcal{P}$*, computes* $r(\Pi)$*;*
- *efficiently computable if there exists a polynomial-time algorithm that, given* $\Pi' \in \mathcal{P}'$*, computes* $h'^*(\Pi')$*.* $h'^* =$ polynomial

Reminder:

- You have a problem, $\mathcal{P}$, whose perfect heuristic h^* you wish to estimate.
- You define a simpler problem, $\mathcal{P}'$, whose perfect heuristic h'^* can be used to (admissibly!) estimate h^*
- You define a transformation, r, from $\mathcal{P}$ into $\mathcal{P}'$.
- Given $\Pi \in \mathcal{P}$, you estimate $h^*(\Pi)$ by $h'^*(r(\Pi))$.

Relaxations: Illustration

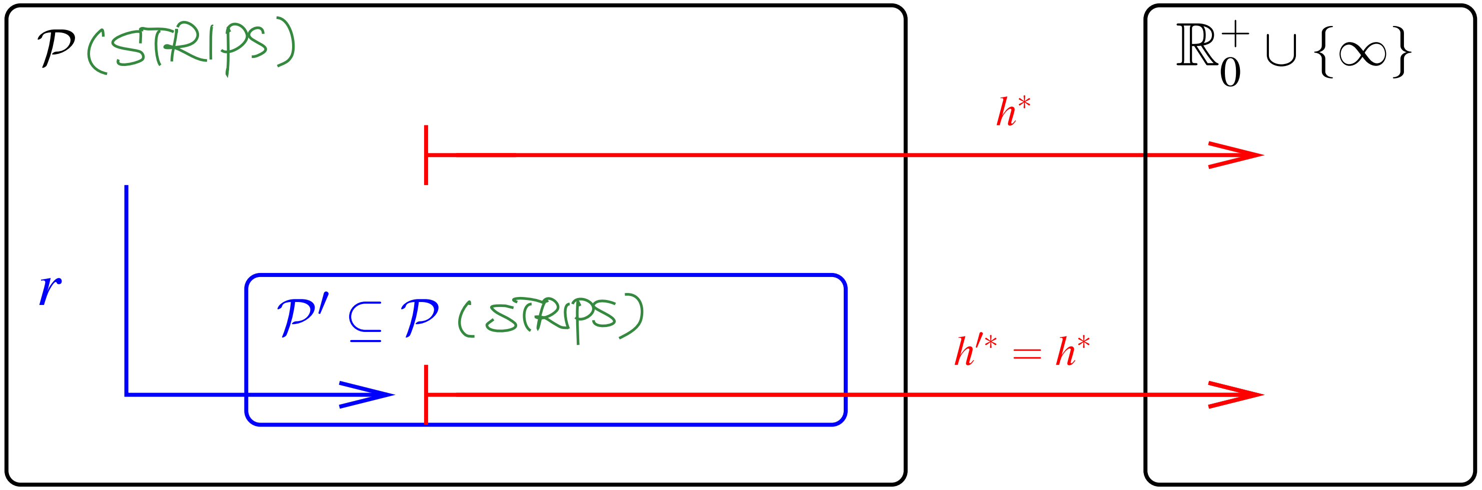
P & P' are diff



Example route-finding:

- Problem $\mathcal{P}$: Route finding.
- Simpler problem $\mathcal{P}'$: Route finding for birds.
- Perfect heuristic h'^* for $\mathcal{P}'$: Straight-line distance.
- Transformation r: Pretend you're a bird.

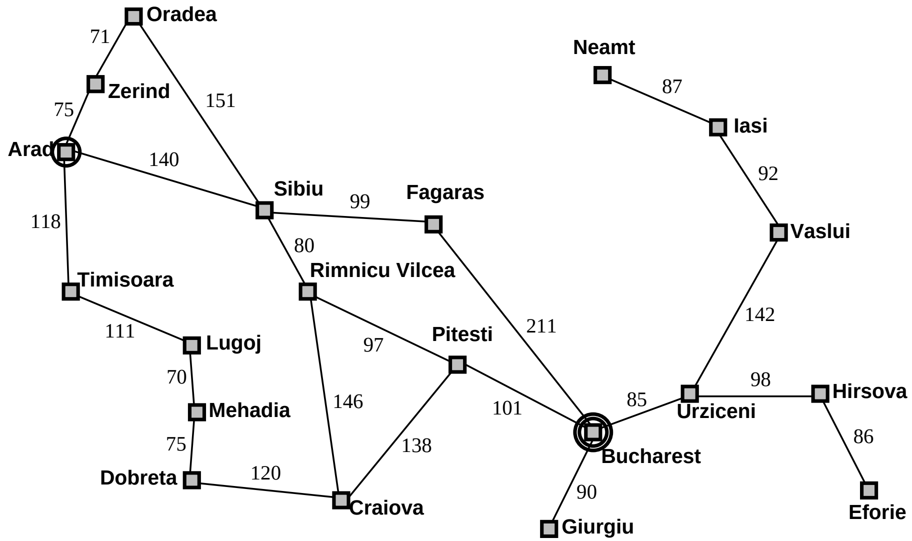
Native Relaxations: Illustration



Example "goal-counting":

- Problem $\mathcal{P}$: All STRIPS planning tasks.
- Simpler problem $\mathcal{P}'$: All STRIPS planning tasks with empty preconditions and deletes.
- Perfect heuristic h'^* for $\mathcal{P}'$: Optimal plan cost $= h^*$.
- Transformation r: Drop the preconditions and deletes.

Relaxation in Route-Finding: Properties



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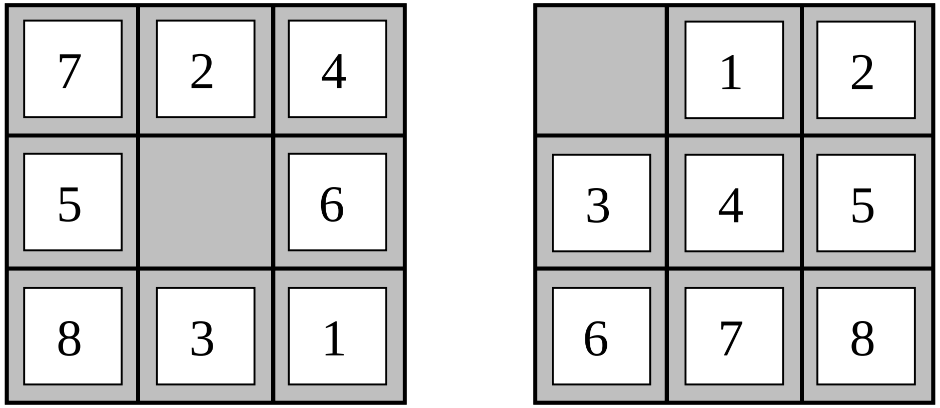
Relaxation $\mathcal{R} = (\mathcal{P}', r, h'^*)$**:** Pretend you're a bird.

- Native? No: Birds don't do route-finding. (Well, it's equivalent to trivial maps with direct routes between everywhere.)
- Efficiently constructible? Yes (pretend you're a bird).
- Efficiently computable? Yes (measure straight-line distance).

Relaxation in the 8-Puzzle: Properties



Start State Goal State

Relaxation $\mathcal{R} = (\mathcal{P}', r, h'^*)$**:** Use more generous actions rule to obtain Manhattan distance.

- Native? No: With the modified rules, it's not the "same puzzle" anymore. (Well, one could be generous in defining what the "same puzzle" is.)
- Efficiently constructible? Yes (exchange action set).
- Efficiently computable? Yes (count misplaced tiles/sum up Manhattan distances).

What shall we do with the relaxation?

What if $\mathcal{R}$ is not efficiently constructible? (rare, coz if so nobody would use it)

- Either (a) approximate r, or (b) design r in a way so that it will typically be feasible, or (c) just live with it and hope for the best.
- Vast majority of known relaxations (in planning) are efficiently constructible.

hard to compute (normal)

What if $\mathcal{R}$ is not efficiently computable?

- Either (a) approximate h'^*, or (b) design h'^* in a way so that it will typically be feasible, or (c) just live with it and hope for the best.
- Many known relaxations (in planning) are efficiently computable, some aren't. The latter use (a); (b) and (c) are not used anywhere right now.

"Goal-Counting" Relaxation in Australia: Properties

- Propositions P: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Da\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Da\}$.
- Actions $a \in A$: $drive(x, y)$ where x, y have a road; $pre_a = \{at(x)\}$, $add_a = \{at(y), v(y)\}$, $del_a = \{at(x)\}$.
- Initial state I: $at(Sy), v(Sy)$.
- Goal G: $at(Sy), v(x)$ for all x.

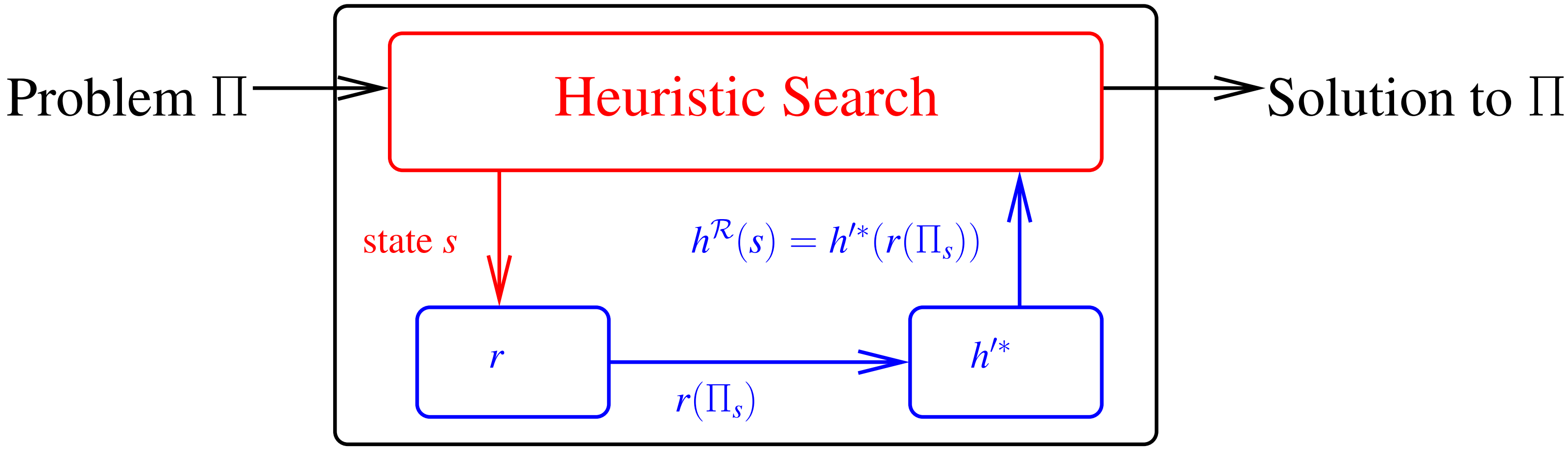
Relaxation $\mathcal{R} = (\mathcal{P}', r, h'^*)$: Remove preconditions and deletes, then use h^*.

- Native? Yes: Planning with empty preconditions and deletes is a special case of planning (i.e., a sub-class of $\mathcal{P}$).
- Efficiently constructible? Yes (drop preconditions and deletes).
- Efficiently computable? **No!** Optimal planning is still **NP**-hard in this case (MINIMUM COVER of goal set by add lists).

What shall we do with the relaxation? $\rightarrow$ Use method (a): Approximate h^* in $\mathcal{P}'$ by counting the number of goals not currently true.

How to Relax During Search: Diagram

Using a relaxation $\mathcal{R} = (\mathcal{P}', r, h'^*)$ during search:

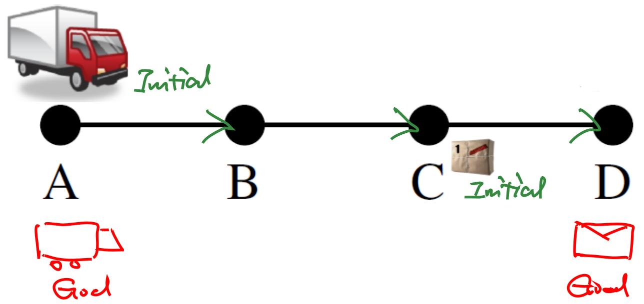


→ Π_s: Π with initial state replaced by s, i.e., $\Pi = (F, A, c, I, G)$ changed to (F, A, c, s, G).

→ The task of finding a plan for search state s.

→ We will be using this notation in the course!

How to Relax During Search: Goal-Counting



Greedy best-first search:
(tie-breaking: alphabetic)

We are here

AC

$h = ?$ "1"

3 ⇒ D(A,D). L(C). U(D)
2 ⇒ L(C). U(D)
1 ⇒ U(D)

We don't have Pre & Del

(T) Truck Package(P) Truck at A

Real problem:

- Initial state I: AC; goal G: AD. → Package at D
- Actions A: pre, add, del.
- $drXY$, loX, ulX.

drive from X to Y   load P into a T   unload P into a certain location

negation

$Drive(X,Y)$: $\cancel{T(X)} \longrightarrow T(Y),\ \cancel{\neg T(X)}$

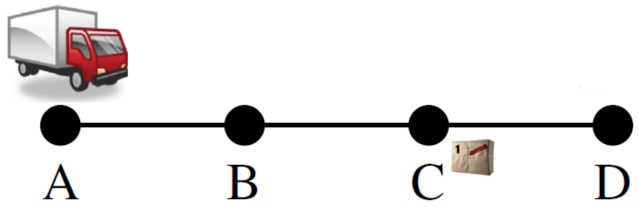
$L(X)$: $T(X)\ \cancel{P(X)} \longrightarrow P(T),\ \cancel{\neg P(X)}$

$U(X)$: $\cancel{P(T)\ T(X)} \longrightarrow P(X),\ \cancel{\neg P(T)}$

Pre (constraints)
(X can happen any time where
Just do/go)

del
(X in both places)

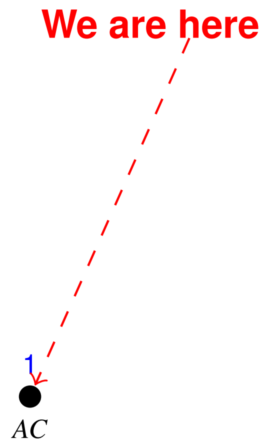
How to Relax During Search: Goal-Counting



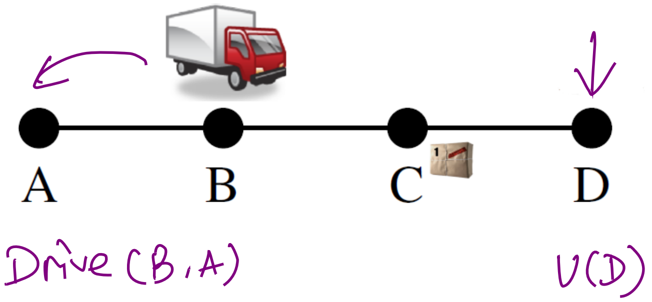
Relaxed problem:

- State s: AC; goal G: AD.
- Actions A: *add*.
- $h^{\mathcal{R}}(s) = 1$.

Greedy best-first search:
(tie-breaking: alphabetic)



How to Relax During Search: Goal-Counting



Real problem:

- State s: BC; goal G: AD.
- Actions A: pre, add, $\cancel{del}$.
- $AC \xrightarrow{drAB} BC$.

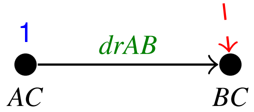
Greedy best-first search:
(tie-breaking: alphabetic)

D(B,A)   U(D)

BC → B A C → B A C D

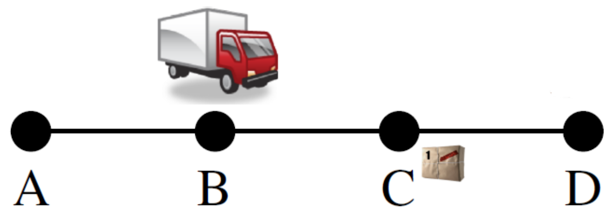
T   P

We are here



h= ? "2"

How to Relax During Search: Goal-Counting

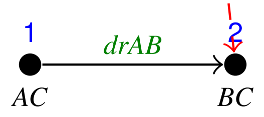


Relaxed problem:

- State s: BC; goal G: AD.
- Actions A: add.
- $h^{\mathcal{R}}(s) = 2$.

Greedy best-first search:
(tie-breaking: alphabetic)

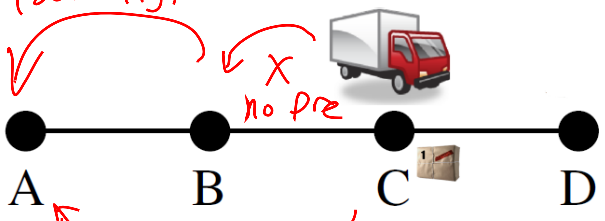
We are here



How to Relax During Search: Goal-Counting

(directly)

X no pre



X must move pt to pt

Real problem:

- State s: CC; goal G: AD.
- Actions A: pre, add, del.
- $BC \xrightarrow{drBC} CC$.

Greedy best-first search:
(tie-breaking: alphabetic)

We are here

$$\underset{AC}{\overset{1}{\bullet}} \xrightarrow{drAB} \underset{BC}{\overset{2}{\bullet}} \xrightarrow{drBC} \underset{CC}{\bullet}$$

h = ? "2"

D(B,A) U(P)

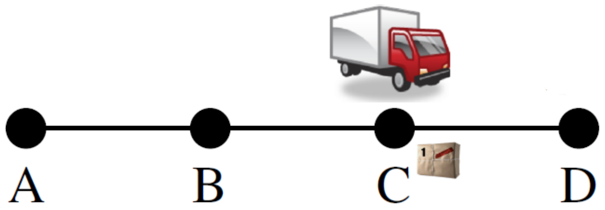
↓ ↓

Same ⟷ I can do U(D) w/o P in D

↓

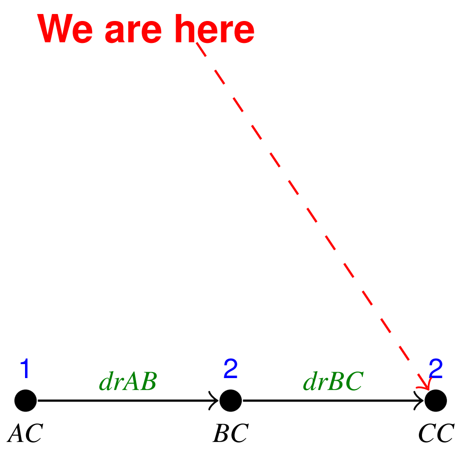
can drive from B to A w/o T in B

How to Relax During Search: Goal-Counting

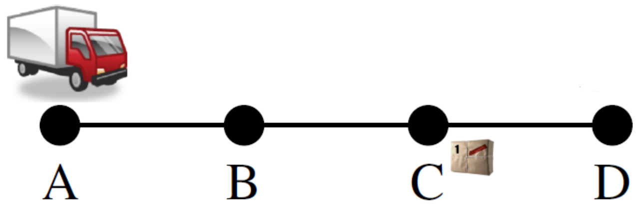


Relaxed problem:
- State s: CC; goal G: AD.
- Actions A: *add*.
- $h^{\mathcal{R}}(s)$ =2.

Greedy best-first search:
(tie-breaking: alphabetic)



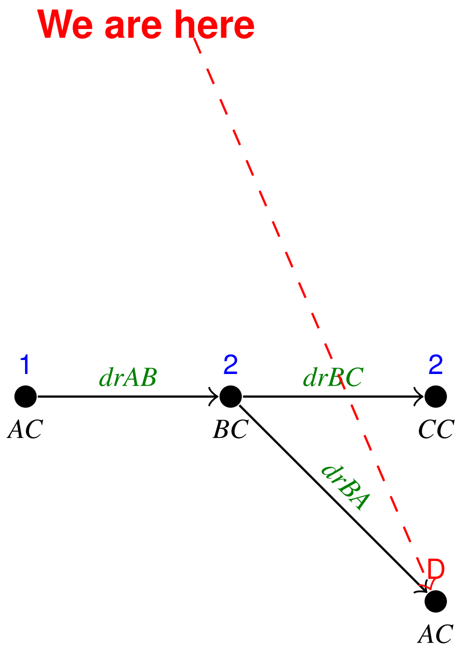
How to Relax During Search: Goal-Counting



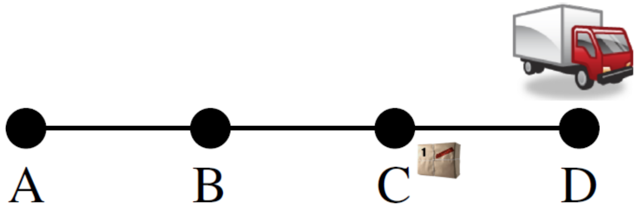
Real problem:

- State s: AC; goal G: AD.
- Actions A: pre, add, del.
- Duplicate state, prune.

Greedy best-first search:
(tie-breaking: alphabetic)



How to Relax During Search: Goal-Counting

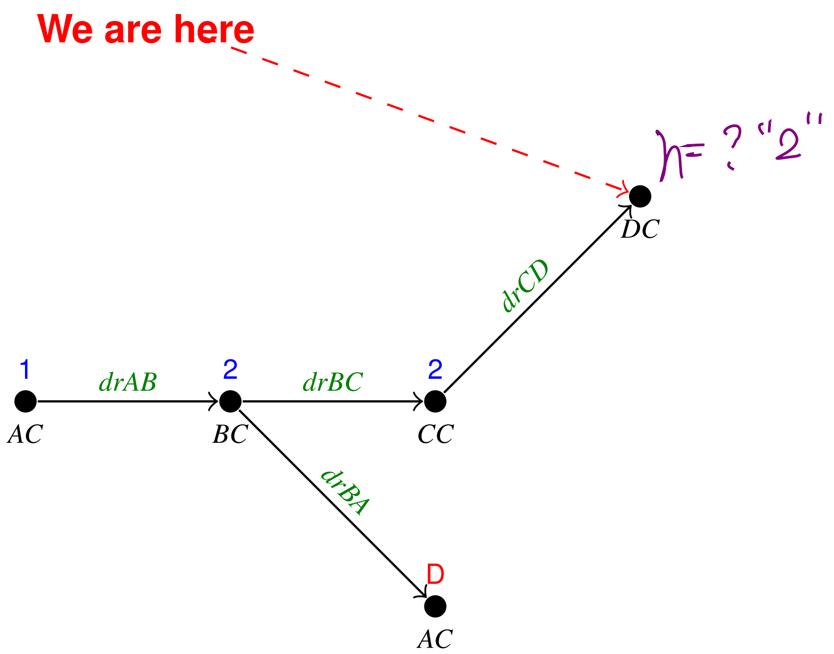


Real problem:

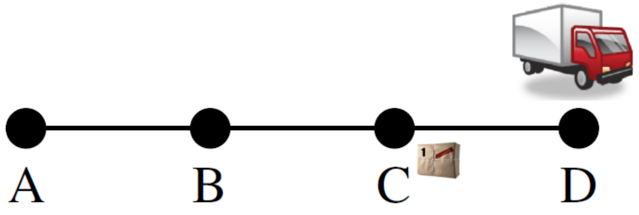
- State s: DC; goal G: AD.
- Actions A: pre, add, del.
- $CC \xrightarrow{drCD} DC$.

Greedy best-first search:
(tie-breaking: alphabetic)

$D(B,A).\quad U(D)$



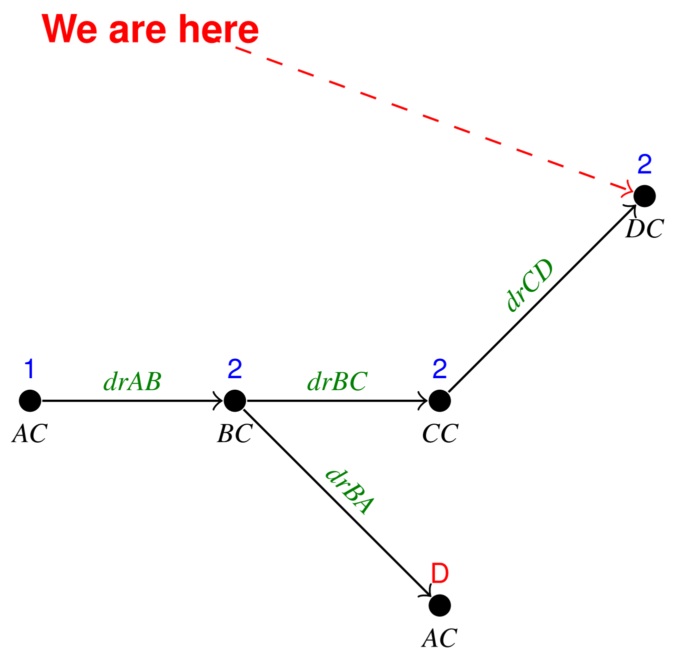
How to Relax During Search: Goal-Counting



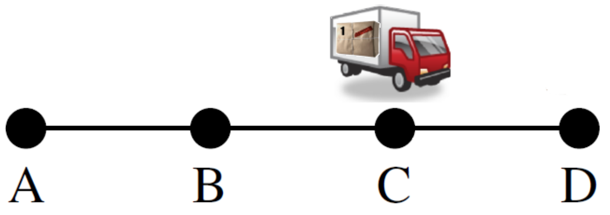
Relaxed problem:

- State s: DC; goal G: AD.
- Actions A: add.
- $h^{\mathcal{R}}(s)$ $=2$.

Greedy best-first search:
(tie-breaking: alphabetic)



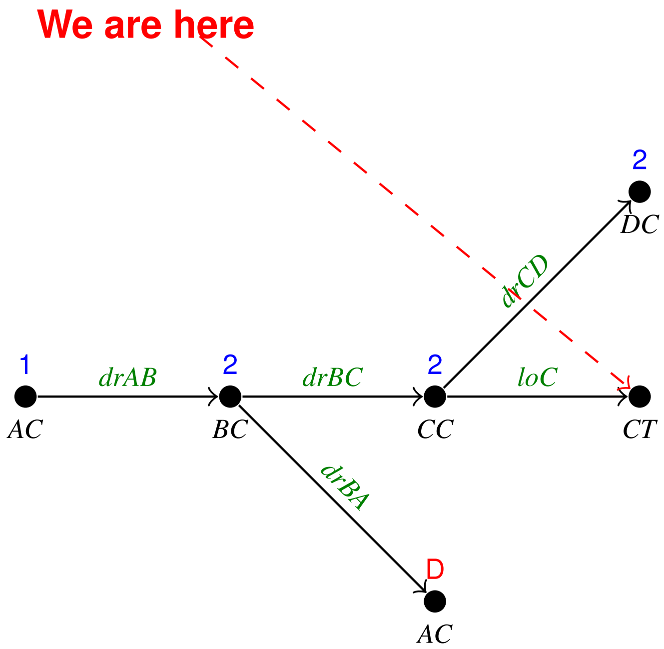
How to Relax During Search: Goal-Counting



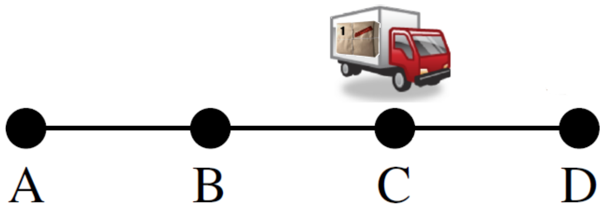
Real problem:

- State s: CT; goal G: AD.
- Actions A: pre, add, del.
- $CC \xrightarrow{loC} CT$.

Greedy best-first search:
(tie-breaking: alphabetic)

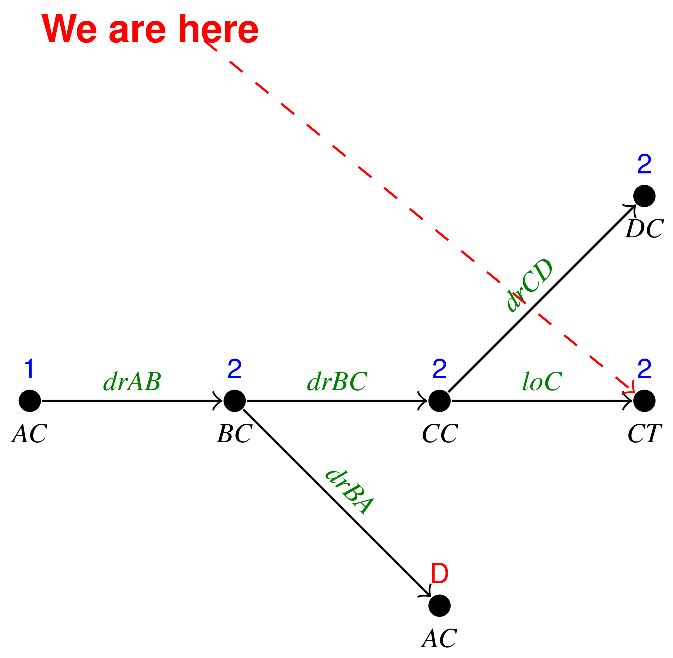


How to Relax During Search: Goal-Counting

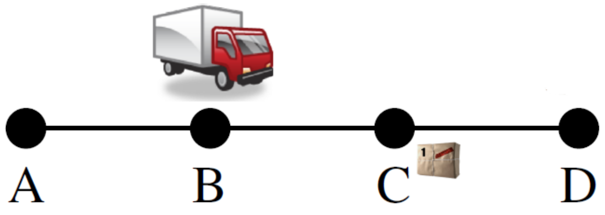


Relaxed problem:
- State s: CT; goal G: AD.
- Actions A: add.
- $h^{\mathcal{R}}(s) = 2$.

Greedy best-first search:
(tie-breaking: alphabetic)



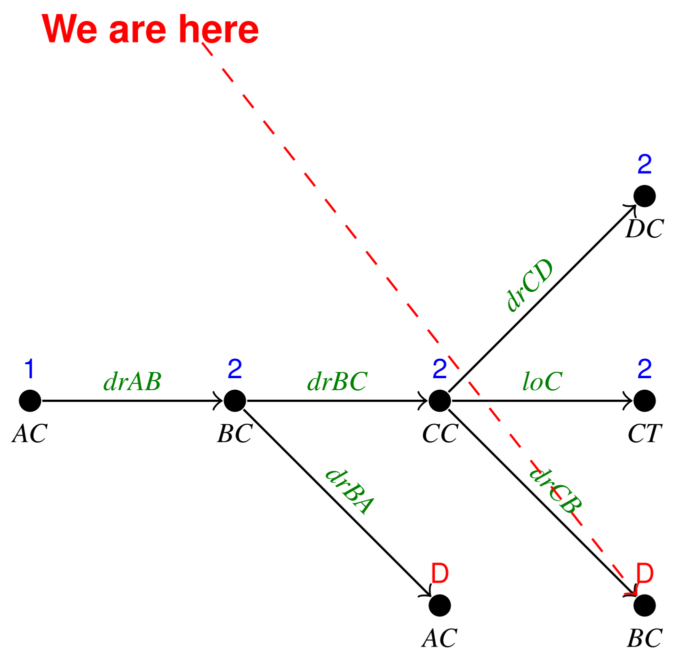
How to Relax During Search: Goal-Counting



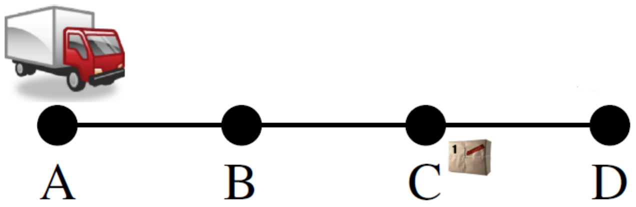
Real problem:

- State s: BC; goal G: AD.
- Actions A: pre, add, del.
- Duplicate state, prune.

Greedy best-first search:
(tie-breaking: alphabetic)



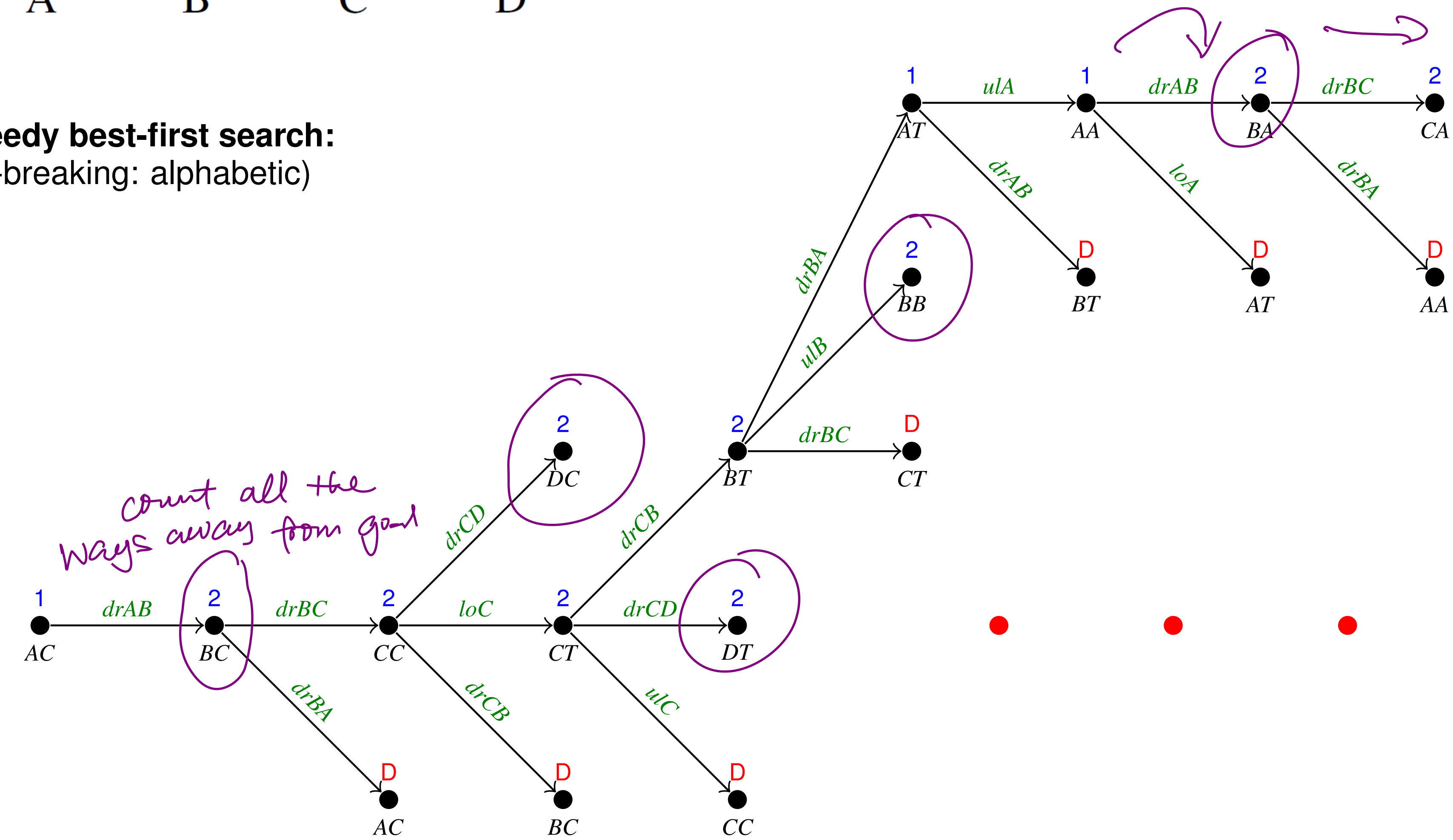
How to Relax During Search: Goal-Counting



Real problem:

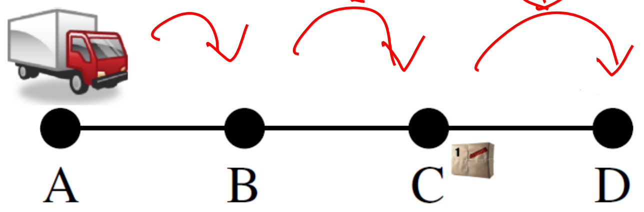
- Initial state I: AC; goal G: AD.
- Actions A: pre, add, del.
- $drXY$, loX, ulX.

Greedy best-first search:
(tie-breaking: alphabetic)



How to Relax During Search: Ignoring Deletes

Keep the pre, forget about del (No)



① ② ④ U(D) ⑤

X X L(C) ③ X

Real problem:

- Initial state I: AC; goal G: AD.
- Actions A: $pre, add, \color{red}{del}$.
- $drXY, loX, ulX$.

Object can be in both places (no negation in this world)

Greedy best-first search:
(tie-breaking: alphabetic)

Why we're not driving back?

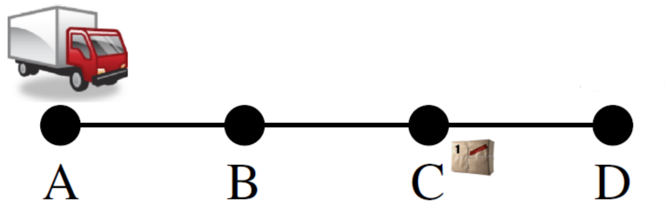
⇒ no del here

⇒ the footprint was kept.

We are here

AC h= ? "5"

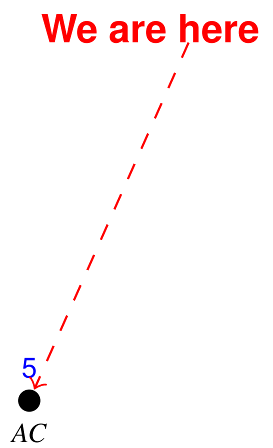
How to Relax During Search: Ignoring Deletes



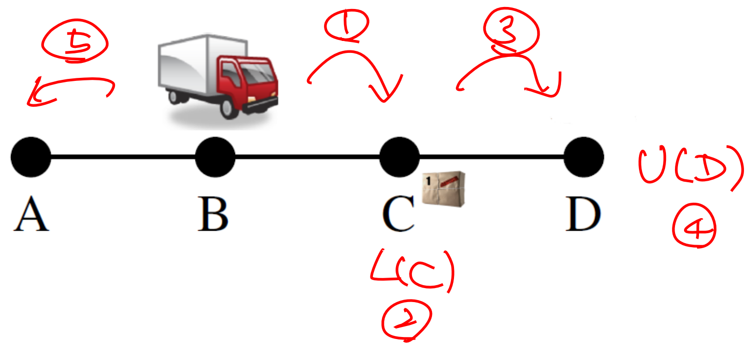
Relaxed problem:

- State s: AC; goal G: AD.
- Actions A: pre, add.
- $h^{\mathcal{R}}(s) = h^{+}(s) = 5$.

Greedy best-first search:
(tie-breaking: alphabetic)



How to Relax During Search: Ignoring Deletes

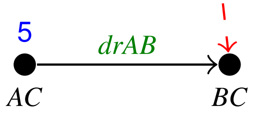


Real problem:

- State s: BC; goal G: AD.
- Actions A: pre, add, del.
- $AC \xrightarrow{drAB} BC$.

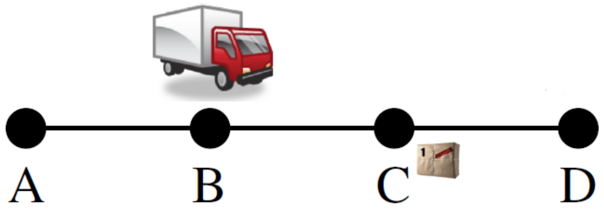
Greedy best-first search:
(tie-breaking: alphabetic)

We are here



h="5"

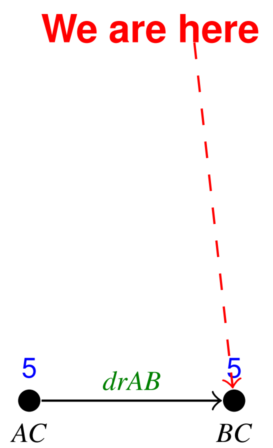
How to Relax During Search: Ignoring Deletes



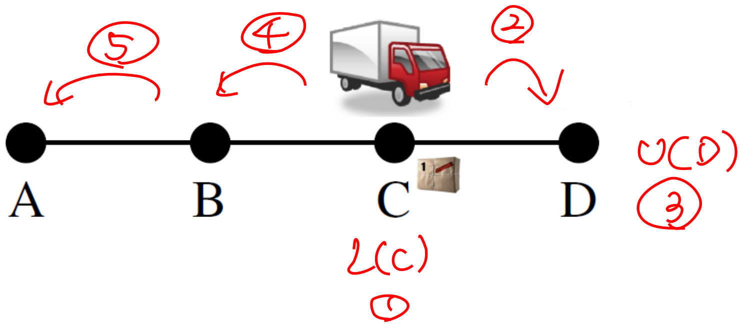
Relaxed problem:

- State s: BC; goal G: AD.
- Actions A: pre, add.
- $h^{\mathcal{R}}(s) = h^{+}(s) =5$.

Greedy best-first search:
(tie-breaking: alphabetic)



How to Relax During Search: Ignoring Deletes

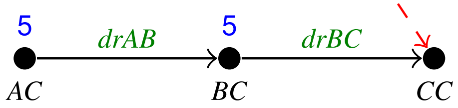


Real problem:

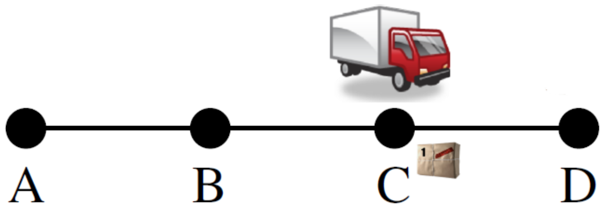
- State s: CC; goal G: AD.
- Actions A: pre, add, del.
- $BC \xrightarrow{drBC} CC$.

Greedy best-first search:
(tie-breaking: alphabetic)

We are here



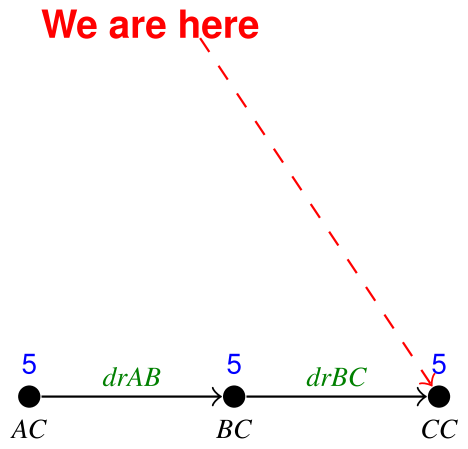
How to Relax During Search: Ignoring Deletes



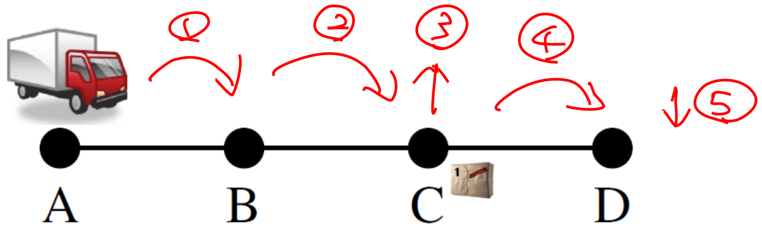
Relaxed problem:

- State s: CC; goal G: AD.
- Actions A: pre, add.
- $h^{\mathcal{R}}(s) = h^{+}(s) =5$.

Greedy best-first search:
(tie-breaking: alphabetic)



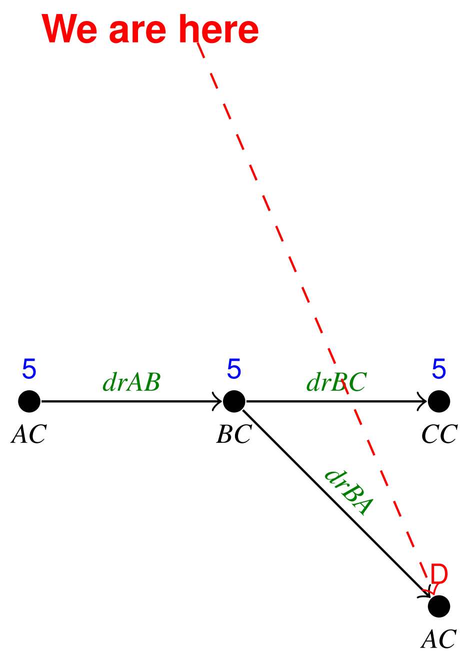
How to Relax During Search: Ignoring Deletes



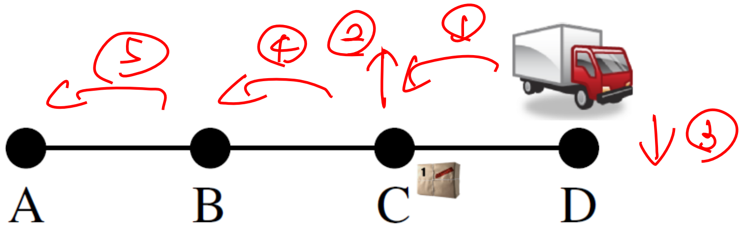
Real problem:

- State s: AC; goal G: AD.
- Actions A: pre, add, del.
- Duplicate state, prune.

Greedy best-first search:
(tie-breaking: alphabetic)

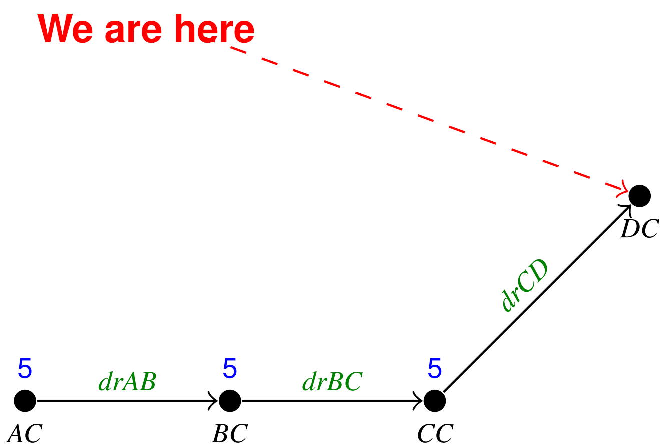


How to Relax During Search: Ignoring Deletes

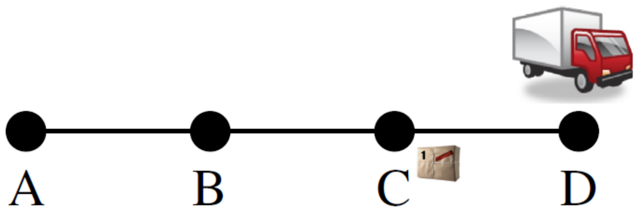


Real problem:
- State s: DC; goal G: AD.
- Actions A: pre, add, del.
- $CC \xrightarrow{drCD} DC$.

Greedy best-first search:
(tie-breaking: alphabetic)



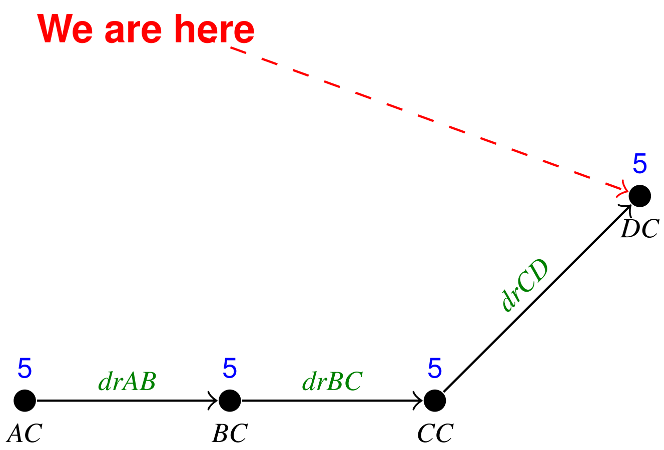
How to Relax During Search: Ignoring Deletes



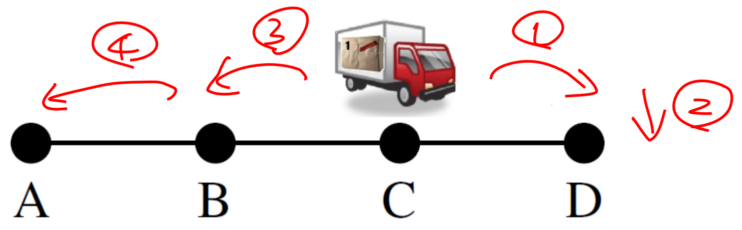
Relaxed problem:

- State s: DC; goal G: AD.
- Actions A: pre, add.
- $h^{\mathcal{R}}(s) = h^{+}(s) =$5.

Greedy best-first search:
(tie-breaking: alphabetic)



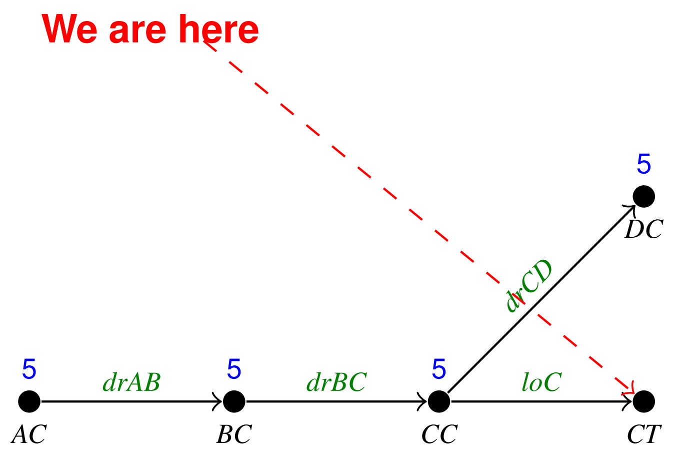
How to Relax During Search: Ignoring Deletes



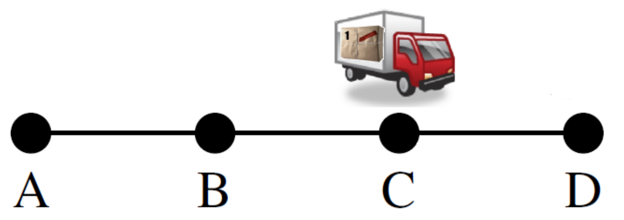
Real problem:

- State s: CT; goal G: AD.
- Actions A: pre, add, del.
- $CC \xrightarrow{loC} CT$.

Greedy best-first search:
(tie-breaking: alphabetic)



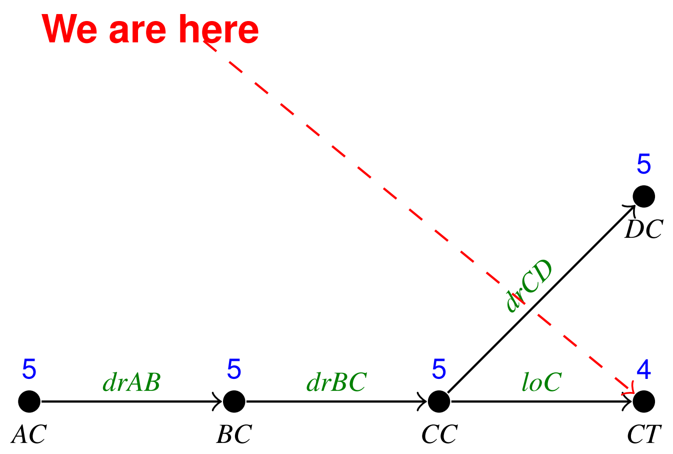
How to Relax During Search: Ignoring Deletes



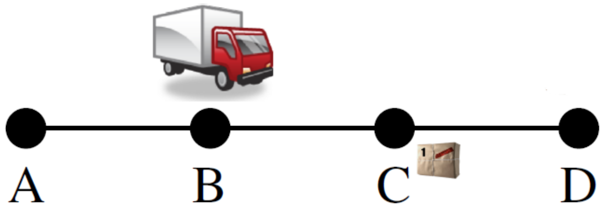
Relaxed problem:

- State s: CT; goal G: AD.
- Actions A: pre, add.
- $h^{\mathcal{R}}(s) = h^{+}(s)$ =4.

Greedy best-first search:
(tie-breaking: alphabetic)



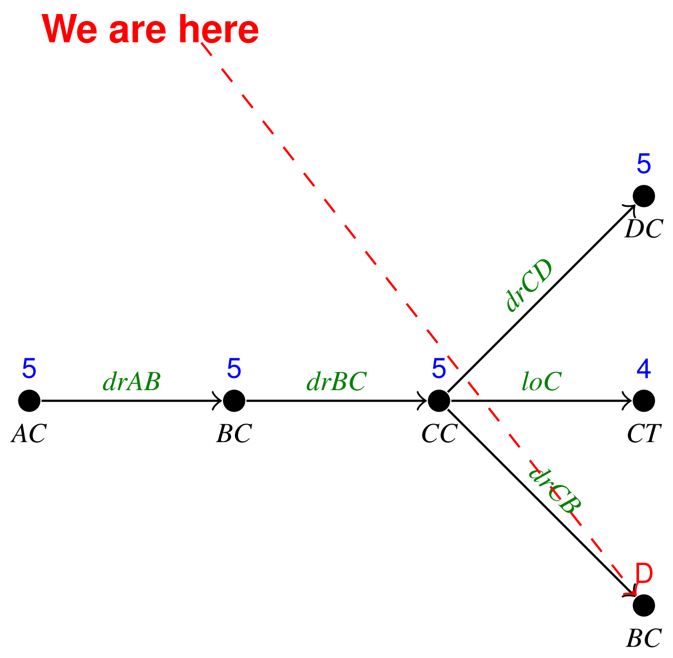
How to Relax During Search: Ignoring Deletes



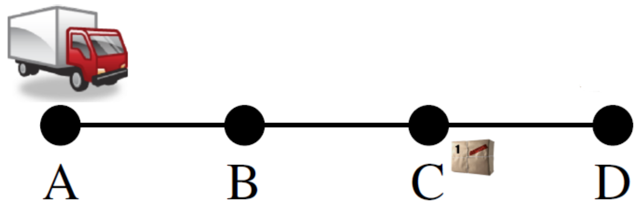
Real problem:

- State s: BC; goal G: AD.
- Actions A: pre, add, del.
- Duplicate state, prune.

Greedy best-first search:
(tie-breaking: alphabetic)



How to Relax During Search: Ignoring Deletes

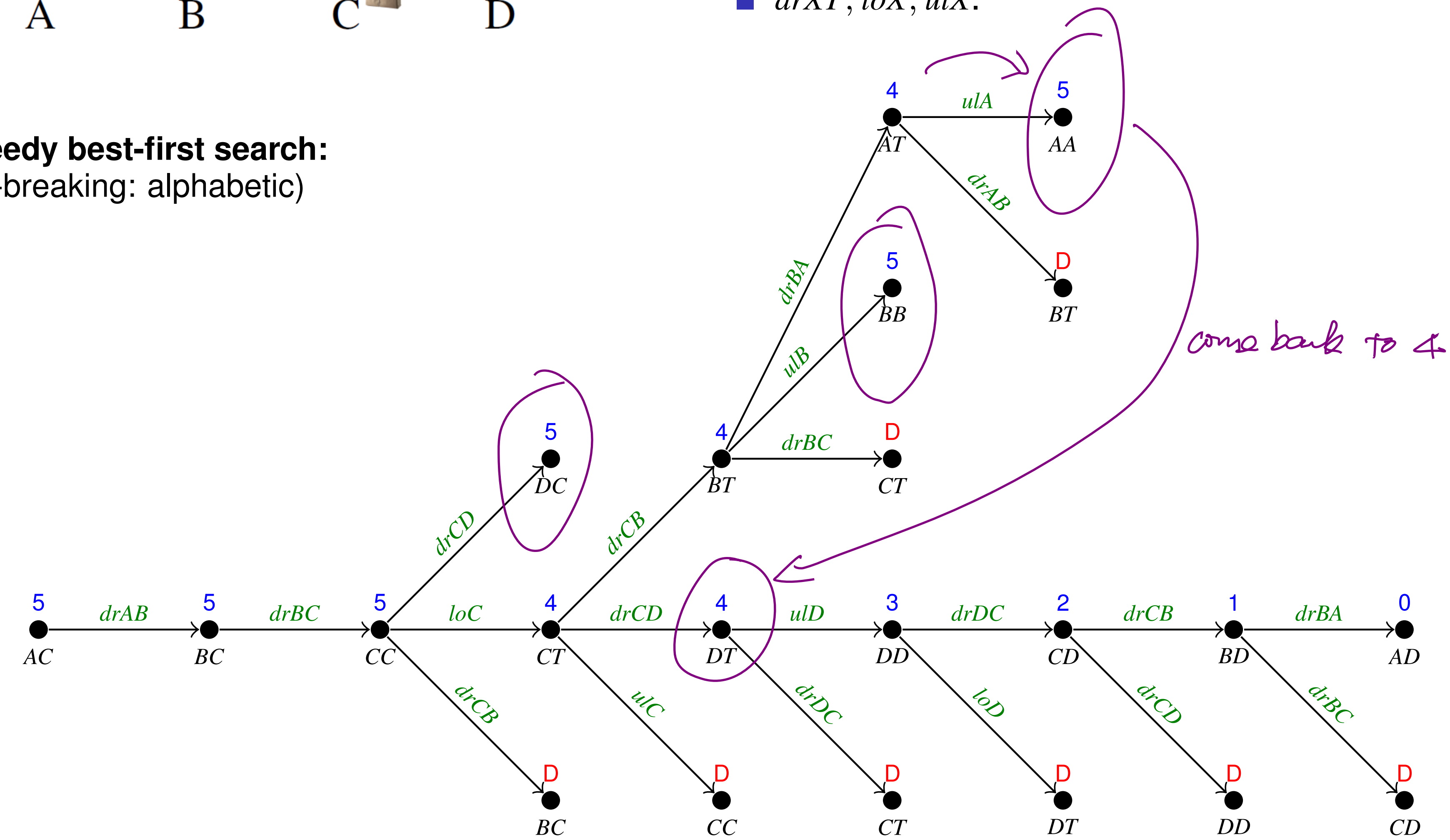


Much more powerful

Real problem:

- Initial state I: AC; goal G: AD.
- Actions A: pre, add, del.
- $drXY, loX, ulX$.

Greedy best-first search:
(tie-breaking: alphabetic)



Questionnaire

Question!

Say we have a robot with one gripper, two rooms A and B, and n balls we must transport. The actions available are $moveXY$, $pickB$ and $dropB$; say $h =$"number of balls not yet in room B". Can h be derived as $h^{\mathcal{R}}$ for a relaxation $\mathcal{R}$?

(A): No.

(B): Yes, just drop the deletes

(C): Sure, *every* admissible h can be derived via a relaxation.

(D): I'd rather relax at the beach.

$\rightarrow$ We can define $\mathcal{P}'$ as the problem of computing the cardinality of a finite set, and define r as the function that maps a state to the set of balls not yet in room B. So: (A) is incorrect, (B) is incorrect, should drop preconditions and deletes.

$\rightarrow$ (C): Yes. Admissibility of $h^{\mathcal{R}}$ is the only strict requirement made by the definition. Given admissible $h : \mathcal{P} \mapsto \mathbb{R}_0^+ \cup \{\infty\}$, we can simply define $\mathcal{P}' := \mathcal{P}$ and take r to be the identity function $id_{\mathcal{P}}$. In other words, $\mathcal{R} := (\mathcal{P}, id_{\mathcal{P}}, h)$ is a relaxation with $h^{\mathcal{R}} = h$. (And, yes, h here is admissible.)

Summary

- Relaxation is a method to compute heuristic functions.
- Given a problem $\mathcal{P}$ we want to solve, we define a relaxed problem $\mathcal{P}'$. We derive the heuristic by mapping into $\mathcal{P}'$ and taking the solution to this simpler problem as the heuristic estimate.
- Relaxations can be native, efficiently constructible, and/or efficiently computable. None of this is a strict requirement to be useful.

- During search, the relaxation is used *only inside* the computation of the heuristic function on each state; the relaxation does not affect anything else. (This can be a bit confusing especially for native relaxations like ignoring deletes.)

Remarks

The goal-counting approximation $h =$"count the number of goals currently not true" is **a very uninformative heuristic function:**

- Range of heuristic values is small ($0 \dots |G|$).
- We can transform any planning task into an equivalent one where $h(s) = 1$ for all non-goal states s. How? Replace goal by new fact g and add a new action achieving g with precondition G.
- Ignores almost all structure: Heuristic value does not depend on the actions at all!

$\rightarrow$ By the way, is h safe/goal-aware/admissible/consistent? Only safe and goal-aware.

$\rightarrow$ We will see in $\rightarrow$ **the next lecture** how to compute **much** better heuristic functions.