

$$X_{n+1} = 4X_n - X_n^2 \quad X_0 = 4\sin^2 \theta$$

a)

$$X_{n+1} = 4\sin^2(2^{n+1}\theta) \quad \theta \in (0, \pi/2)$$

$$4X_n - X_n^2 = 4\sin^2(2^{n+1}\theta)$$

$$X_{K+1} = 4X_K - X_K^2 \quad X_{K+1} = 4\sin^2(2^{K+1}\theta)$$

$$X_{K+2} = 4X_{K+1} - X_{K+1}^2$$

$$X_{K+2} = 4\sin^2(2^{K+2}\theta)$$

$$X_{K+2} = 16\sin^2(2^{K+1}\theta) - 16\sin^4(2^{K+1}\theta)$$

$$X_{K+2} = 16\sin^2(2^{K+1}\theta)(1 - \sin^2(2^{K+1}\theta))$$

$$16\sin^2(2^{K+2}\theta) = 16\sin^2(2^{K+1}\theta)\cos^2(2^{K+1}\theta)$$

$$\sin^2(2^{K+2}\theta) = 4\sin^2(2^{K+1}\theta)\cos^2(2^{K+1}\theta)$$

$$\sin^2(4^{K+1}\theta) = 4\sin^2(2^{K+1}\theta)\cos^2(2^{K+1}\theta)$$

$$\sin^2(4^{K+1}\theta) = (2\sin(2^{K+1}\theta)\cos(2^{K+1}\theta))^2$$

$$\sin^2(4^{K+1}\theta) = \left(\frac{2\sin(4^{K+1}\theta)}{2} \right)^2$$

$$\sin^2(4^{K+1}\theta) = \sin^2(4^{K+1}\theta) \quad \checkmark$$

b)

$$x_{n+1} = 4x_n - 4x_n^2, \quad x_0 = \sin^2 \theta,$$

$$x_{n+1} = \sin^2(2^{n+1}\theta), \quad \theta \in [0, \pi/2].$$

$$x_{0+1} = 4\sin^2\theta - 4\sin^4\theta$$

$$x_{0+1} = 4\sin^2\theta(1 - \sin^2\theta)$$

$$x_{0+1} = 4\sin^2\theta(\cos^2\theta)$$

$$x_{0+1} = 2\sin^2(2\theta) \quad \times$$

no conduce