Sustitución hacia adelante:

$$\underbrace{\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{b} \quad 2\times 2$$

$$\underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{b} \quad 3\times 3$$

Donde:

$x_1 = b_1 / l_{11}$

$x_2 = (b_2 - l_{21} x_1) / l_{22}$

Entonces:

$$x_i = \frac{b_i - \sum_{j=1}^{i-1} l_{ij} x_j}{l_{ii}}$$

con $i = 2, 3, \ldots, n$

donde

$l_{ii} = A_{ii}$
$l_{ij} = A_{ij}$

$$\Rightarrow x_i = \frac{b_i - \sum_{j=1}^{i-1} A_{ij} x_j}{A_{ii}}$$

Donde:

$x_1 = b_1 / l_{11}$

$x_2 = (b_2 - l_{21} x_1) / l_{22}$

$x_3 = (b_3 - l_{31} x_1 - l_{32} x_2) / l_{33}$

$\vdots$

$$\Rightarrow [\, l_{i1} \ l_{i2} \ l_{i3} \ \ldots \ l_{ii} \ldots 0 \ldots 0 \,] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ x_{i+1} \\ \vdots \\ x_n \end{bmatrix}$$

$$\Rightarrow \sum_{j=1}^{i} l_{ij} x_j = b_i$$

$$l_{ii} x_i + \sum_{j=1}^{i-1} l_{ij} x_j = b_i$$

Sustición hacia Atrás

$$\begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \\ 0 & U_{22} & \cdots & U_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$X_n = b_n / U_{nn}$$

$$\Rightarrow X_i = \frac{b_i - \sum_{j=i+1}^{n} U_{ij} X_j}{U_{ii}}$$

donde : $A_{ij} = U_{ij}$

$$\Rightarrow x_i = \frac{b_i - \sum_{j=i+1}^{n} A_{ij} x_j}{A_{ii}}$$