$A(x_1, x_2, \ldots, x_n)$, donde $A \sim N(\mu, \sigma^2)$

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad , \quad \sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

La función de verosimilitud esta dada por:

$$① \; l((\mu, \sigma^2); \underline{x}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2}$$

y la de Log-Verosimilitud:

$$② \; L((\mu, \sigma^2); \underline{x}) = -\frac{n}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2$$

Igualamos las expresiones:

$$\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \cdot e^{-1/2\sigma^2 \sum (x_i - \mu)^2}$$

$$= (2\pi\sigma^2)^{-n/2} e^{-1/2\sigma^2 \sum (x_i - \mu)^2}$$

$$\Rightarrow l(u) = \ln L(u) = \ln(2\pi\sigma^2)^{-n/2} + \ln\left(e^{-1/2\sigma^2 \sum \ldots}\right)$$

$$= -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2 \cdot \frac{\partial}{\partial \mu}$$

Derivamos e igualamos a 0

$$\Rightarrow \frac{\partial l(\mu)}{\partial \mu} = 0 - \frac{1}{2\sigma^2} \cdot 2 \sum_{i=1}^{n} (x_i - \mu)(-1))$$

$$0 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (x_i - \mu)$$

$$0 = \sum_{i=1}^{n} (x_i - \mu)$$

$$\Rightarrow \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \mu = \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\sum_{i=1}^{n} x_i = n\mu$$

$$\Rightarrow \boxed{\mu = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

$$\circ \frac{\partial L}{\partial \sigma} = \frac{-n}{2(2\pi\sigma^2)} 2\pi\sigma^2 + \frac{1}{2\sigma^2} \Sigma$$

$$0 = \frac{-n}{2} + \frac{1}{2\sigma^2} \Sigma$$

$$\frac{n}{2\Sigma} = \frac{1}{2\sigma^2}$$

$$\frac{\cancel{2}n}{\cancel{2}\Sigma} = \frac{1}{\sigma^2} \Rightarrow \boxed{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n} = \sigma^2}$$