

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$f''(x_j) = \frac{f(x_{j+1}) - 2f(x_j) + f(x_{j-1}))}{h^2}$$

$$f^{(4)}(x_j) = \left[\left(\frac{f(x_{j+1}+h) - 2f(x_j+h) + f(x_{j-1}+h))}{h^2} \right) 2 \left(\frac{-f(x_{j+1}) - 2f(x_j) + f(x_{j-1}))}{h^2} \right) \dots \right. \\ \left. \dots + \left(\frac{f(x_{j+1}-h) - 2f(x_j-h) + f(x_{j-1}-h))}{h^2} \right) \right] / h^2$$

Order $\mathcal{O}(h^5)$:

$$= \frac{\left(\frac{f(x_{j+2}) - 2f(x_{j+1}) + f(x_j) - 2f(x_{j+1}) + 4f(x_j) - 2f(x_{j-1}) + f(x_j) - 2f(x_{j-1}) + f(x_{j-2}))}{h^2} \right)}{h^2}$$

$$\Rightarrow \frac{f(x_{j+2}) + 6f(x_j) - 8f(x_{j-1}) + f(x_{j-2}))}{h^4}$$

$$= \frac{f(x_{j+2}) + 6f(x_j) - 8f(x_{j-1}) + f(x_{j-2}))}{h^4}$$