

$$1.) \int_a^b \frac{x-b}{a-b} f(a) + \frac{x-a}{(b-a)} f(b) dx = \frac{1}{a-b} \int (x-b) f(a) dx - \int (x-a) f(b) dx$$

$$b-a = -(a-b)$$

$$= \frac{1}{a-b} \left[ f(a) \left( \frac{x^2}{2} - bx \right) \Big|_a^b - f(b) \left( \frac{x^2}{2} - ax \right) \Big|_a^b \right]$$

$$= \frac{1}{a-b} \left[ f(a) \left[ \frac{b^2}{2} - b^2 - \frac{a^2}{2} + ab \right] - f(b) \left[ \frac{b^2}{2} - ab - \frac{a^2}{2} + a^2 \right] \right]$$

$$= \frac{1}{a-b} \left[ f(a) \left[ -\frac{b^2}{2} - \frac{a^2}{2} + ab \right] + f(b) \left[ -\frac{b^2}{2} + ab - \frac{1}{2}a^2 \right] \right]$$

$$= \frac{1}{a-b} \left[ [f(a) + f(b)] \left[ -\frac{b^2}{2} - \frac{a^2}{2} + ab \right] \right]$$

$$= \frac{-1}{2(a-b)} \left[ (f(a) + f(b)) [b^2 + a^2 - 2ab] \right]$$

$$= \frac{-1}{2(a-b)} \left[ \text{" " } [(a-b)^2] \right]$$

$$= - \frac{[f(a) + f(b)](a-b)^2}{2(a-b)}$$

$$= \frac{-(a-b)(f(a) + f(b))}{2}$$

$$= \frac{(b-a)(f(a) + f(b))}{2}$$