

$$a) L_2(x) = \frac{1}{2}(x^2 - 4x + 2)$$

$$\Rightarrow L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x}) \quad \text{Formule de Rodrigues}$$

$n=2$

$$L_2(x) = \frac{e^x}{2} \frac{d^2}{dx^2} (x^2 e^{-x})$$

$$= \frac{e^x}{2} \frac{d}{dx} (2x e^{-x} - x^2 e^{-x})$$

$$= \frac{e^x}{2} (2e^{-x} - 4x e^{-x} + x^2 e^{-x})$$

$$= \frac{2e^{-x+x^2} - 4xe^{-x+x^2} + x^2 e^{-x+x^2}}{2}$$

$$= \frac{2 - 4x + x^2}{2}$$

$$\Rightarrow L_2(x) = \frac{1}{2}(x^2 - 4x + 2)$$

$$b) \frac{1}{2}(x^2 - 4x + 2) = 0$$

$$x^2 - 4x + 2 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 1}$$

$$x = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$x = \frac{4 \pm \sqrt{8}}{2}$$

$$x = \frac{4 \pm 2\sqrt{2}}{2}$$

$$x_1 = \frac{4 + 2\sqrt{2}}{2}$$

$$x_2 = \frac{4 - 2\sqrt{2}}{2}$$

$$= 3.1142$$

$$= 0.5857$$

$$c) w_1 = \int_0^\infty \phi(x) \left(\frac{x - x_2}{x_1 - x_2} \right) dx$$

$$\phi(x) = e^{-x}$$

$$w_1 = \int_0^\infty e^{-x} \frac{(x - x_2)}{(x_1 - x_2)} dx$$

$$= \int_0^\infty e^{-x} \left(\frac{x - 0.6}{3.4 - 0.6} \right) dx$$

$$= \int_0^\infty e^{-x} \left(\frac{x - 0.6}{2.8} \right) dx$$

$$= \frac{1}{2.8} \int_0^\infty e^{-x} (x - 0.6) dx$$

$$= \frac{1}{2.8} \left[\lim_{t \rightarrow +\infty} \int_0^t e^{-x} (x - 0.6) dx \right]$$

$$= \frac{1}{2.8} \left[\lim_{t \rightarrow \infty} -\frac{5t+2}{5e^t} + \frac{2}{5} \right]$$

$$= \frac{1}{2.8} \left[\frac{2}{5} \right] = 0.1428$$

$$w_2 = \int_0^{\infty} \theta(x) \left(\frac{x - x_1}{x_2 - x_1} \right) dx \quad x_1 = 3.4$$

$$x_2 = 0.6$$

$$\theta(x) = e^{-x}$$

$$w_2 = \int_0^{\infty} e^{-x} \left(\frac{x - 3.4}{0.6 - 3.4} \right) dx$$

$$= \frac{1}{-2.8} \int_0^{\infty} e^{-x} (x - 3.4) dx$$

$$= \frac{1}{-2.8} \left[\lim_{t \rightarrow \infty} \int_0^t e^{-x} (x - 3.4) dx \right]$$

$$= \frac{1}{-2.8} \left[\lim_{t \rightarrow \infty} \frac{-5t + 12}{5e^t} - \frac{12}{5} \right]$$

$$= \frac{1}{-2.8} \left[-\frac{12}{5} \right] = \underline{0.85714}$$

d) Usando el código tenemos

$$\sum w_i f(x_i) = 6$$

$$\Rightarrow w_1 (x_1)^3 + w_2 (x_2)^3 + w_3 (x_3)^3$$

\Rightarrow Demostrado en código