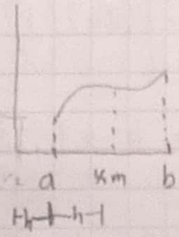
3.) $\int_a^b f(x)dx \simeq \int_a^b P_2(x)dx = \frac{h}{3}\left[f(a) + 4f(x_m) + f(b)\right]$ $\qquad x_m = \frac{a+b}{2}$

$\Rightarrow P_2(x) = f(a)\frac{(x-x_m)(x-b)}{(a-x_m)(a-b)} + f(x_m)\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} + f(b)\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}$

• Al tener $h = \frac{b-a}{2} = x_m - a = b - x_m$



$\Rightarrow P_2(x) = \frac{(x-x_m)(x-b)f(a)}{(-h)(-2h)} + f(x_m)\frac{(x-a)(x-b)}{(h)(-h)} + f(b)\frac{(x-a)(x-x_m)}{(2h)(h)}$

$= \frac{f(a)}{2h^2}(x-x_m)(x-b) - \frac{f(x_m)}{h^2}(x-a)(x-b) + \frac{f(b)}{2h^2}(x-a)(x-x_m)$

$a = h \qquad b = 3h$
$x_m = 2h$

• Integramos

$\Rightarrow \frac{y_a}{2h^2}\int_a^b (x-x_m)(x-b) - \frac{y_m}{h^2}\int_a^b (x-a)(x-b) + \frac{y_b}{2h^2}\int_a^b (x-a)(x-x_m)\,dx$

$u = x-\alpha \qquad dv = (x-\beta)dx$
$du = dx \qquad v = \frac{(x-\beta)^2}{2}$

$\int (x-\alpha)(x-\beta)dx$
$= (x-\alpha)\frac{(x-\beta)^2}{2} - \int \frac{(x-\beta)^2}{2}dx$
$= (x-\alpha)\frac{(x-\beta)^2}{2} - \frac{(x-\beta)^3}{6} + C$

$I_1 = \int_a^b (x-x_m)(x-b) = (x-x_m)\frac{(x-b)^2}{2} - \frac{(x-b)^3}{6}\Big|_a^b$

$= \left[(b-x_m)\cancelto{0}{\frac{(b-b)^2}{2}} - \cancelto{0}{\frac{(b-b)^3}{6}}\right] - \left[(a-x_m)\frac{(a-b)^2}{2} - \frac{(a-b)^3}{6}\right]$

$= -\left[(-h)\frac{(-2h)^2}{2} - \frac{(-2h)^3}{6}\right] = \frac{4h^3}{2} - \frac{8h^3}{6} = 2h^3 - \frac{4h^3}{3}$

$= \frac{2}{3}h^3$

$I_2 = \int_a^b (x-a)(x-b)dx = (x-a)\frac{(x-b)^2}{2} - \frac{(x-b)^3}{6}\Big|_a^b$

$= \left[(b-a)\cancelto{0}{\frac{(b-b)^2}{2}} - \cancelto{0}{\frac{(b-b)^3}{6}}\right] - \left[\cancelto{0}{(a-a)\frac{(a-b)^2}{2}} - \frac{(a-b)^3}{6}\right]$

$= -\left[-\frac{(-2h)^3}{6}\right] = \frac{-8h^3}{6} = \frac{-4h^3}{3}$

$$I_3 = \int_a^b (x-a)(x-x_m)\,dx = (x-a)\frac{(x-x_m)^2}{2} - \frac{(x-x_m)^3}{6}\Bigg|_a^b$$

$a = h$

$x_m = 2h$

$b = 3h$

$$= \left[(b-a)\frac{(b-x_m)^2}{2} - \frac{(b-x_m)^3}{6}\right] - \left[\overset{0}{\cancel{(a-a)}}\frac{(a-x_m)^2}{2} - \frac{(a-x_m)^3}{6}\right]$$

$$= \left[\frac{(2h)(h)^2}{2} - \frac{h^3}{6}\right] - \left[-\frac{(-h)^3}{6}\right] = h^3 - \frac{h^3}{6} - \frac{h^3}{6} = \frac{h^3 - h^3 - h^3}{6}$$

$$= h^3 + \left(\frac{-2h^3}{6}\right) - h^3 + \left(\frac{-1h^3}{3}\right) = \frac{3h^3 - h^3}{3} = \frac{2h^3}{3}$$

- Reemplazando en la formula tendriamos:

$$P_2(x) = \frac{f(a)}{2h^2} I_1 - \frac{f(x_m)}{h^2} I_2 + \frac{f(b)}{2h^2} I_3$$

$$\Rightarrow \frac{f_a}{2h^2}\left(\frac{2h^3}{3}\right) - \frac{f_{x_m}}{h^2}\left(-\frac{4}{3}h^3\right) + \frac{f(b)}{2h^2}\left(\frac{2h^3}{3}\right)$$

$$\Rightarrow \frac{f(a)h}{3} + \frac{f(x_m)4h}{3} + \frac{f(b)h}{3} \quad (\text{Factorizando})$$

$$\int_a^b P_2(x) = \frac{h}{3}\left[f(a) + f(x_m)4 + f(b)\right]$$