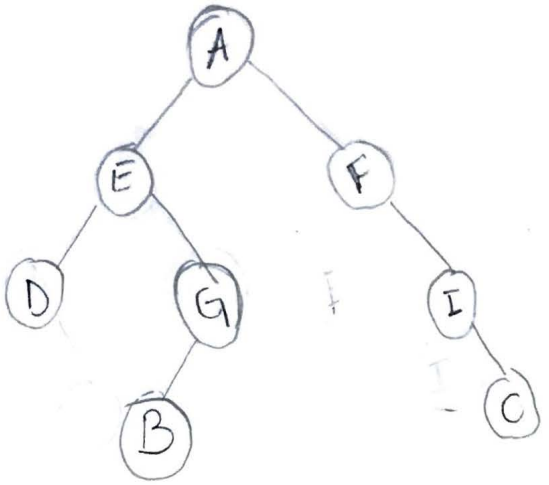
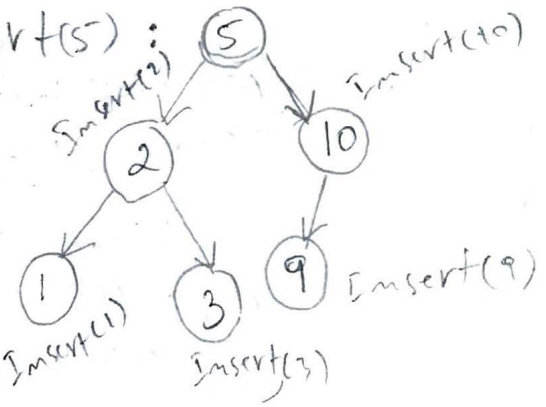
(1)

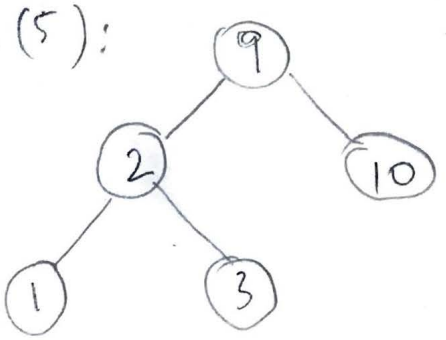



(2) BST is Empty

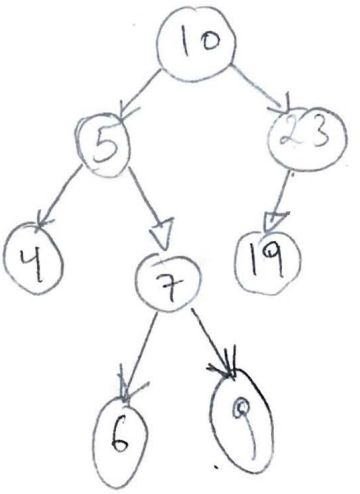
Insert(5):



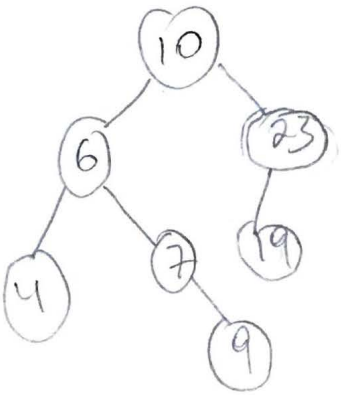

Remove (5):




(3)




Remove (5)



(4) A) height = 4

B) Depth of node 90 = 3

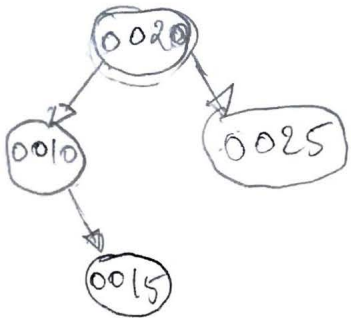
C) height of node 90 = 1

D) Pre-order: 0100, 0050, 0003, 0001, 0020, 0080, 0052, 0090, 0083, 0099, 0150, 0125, 0152

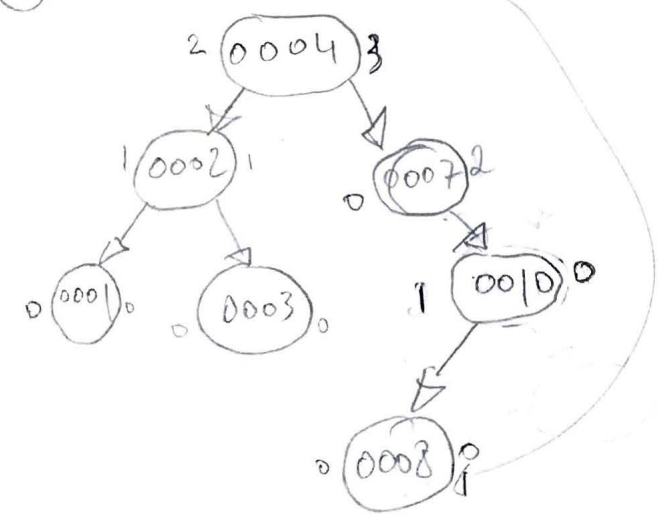
In-order: 0001, 0003, 0020, 0050, 0052, 0080, 0083, 0090, 0099, 0100, 0125, 0150, 0152

Post-order: 0001, 0020, 0003, 0052, 0083, 0099, 0090, 0080, 0050, 0125, 0152, 0150, 0100

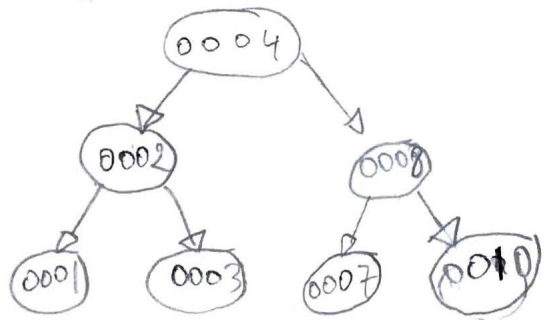
(5)



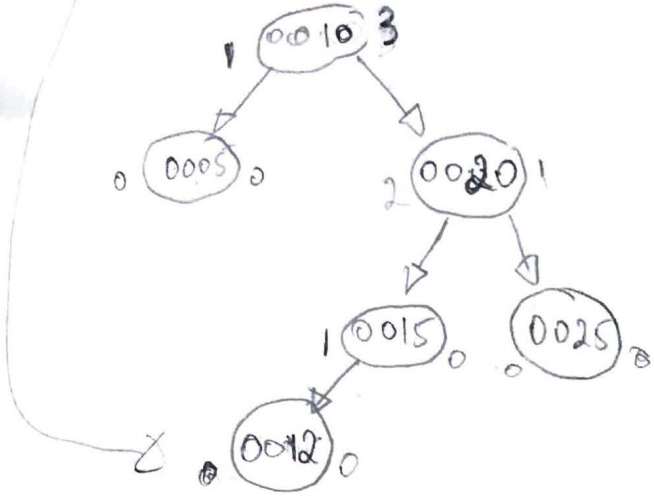

(6) Insert (8)



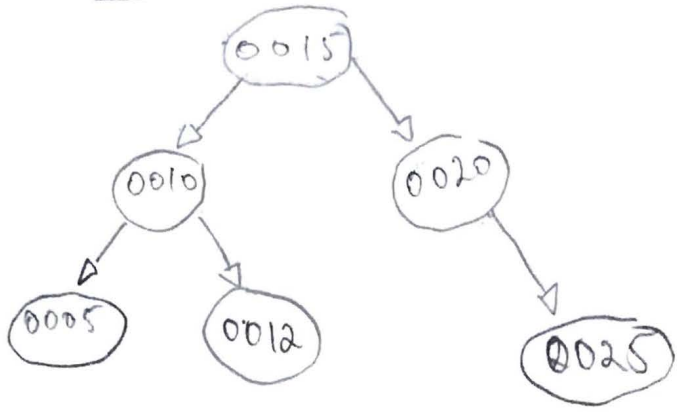

After Rotate (Right-Left)



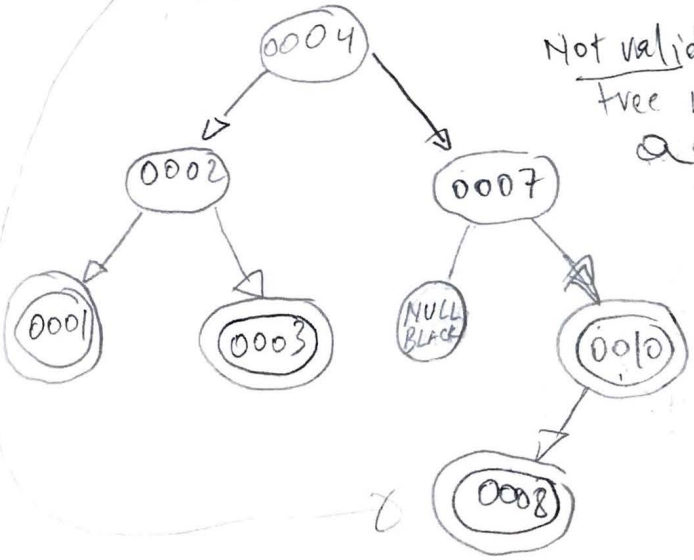
(7) Insert (12)




After Rotate (Right-Left)

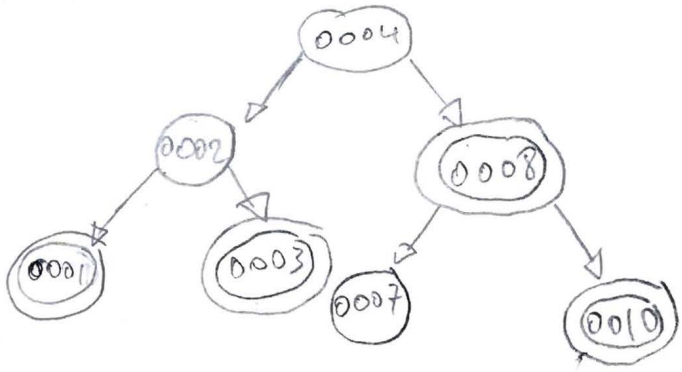



(8) Insert 8

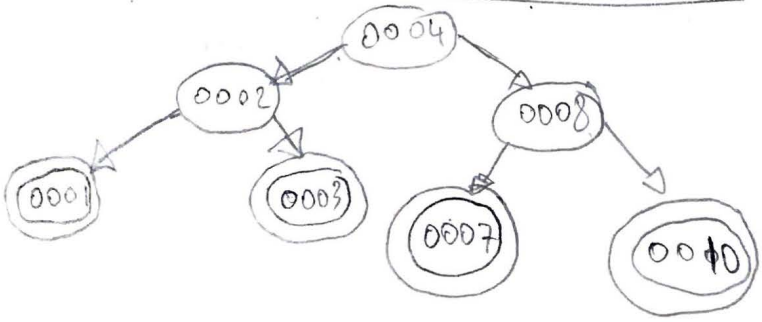



Not valid Red-black tree because we have 2 consecutive Red Node

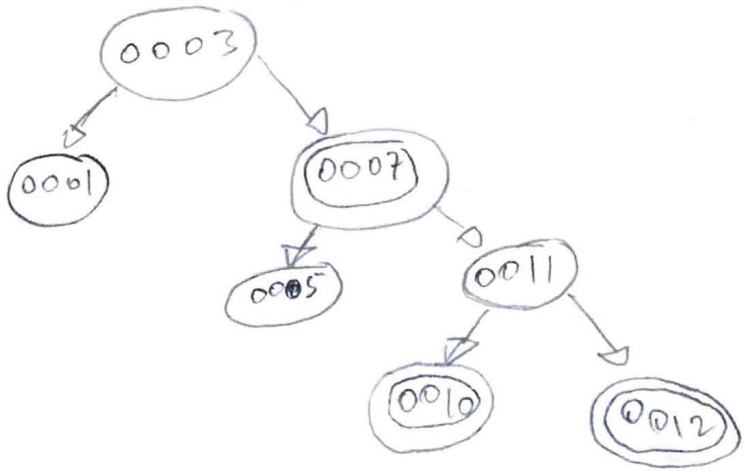
Rotation (Right-left)




After Rotate (fix color of the node)



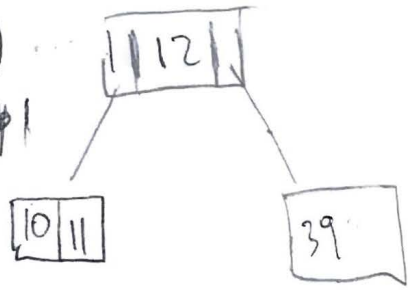
9



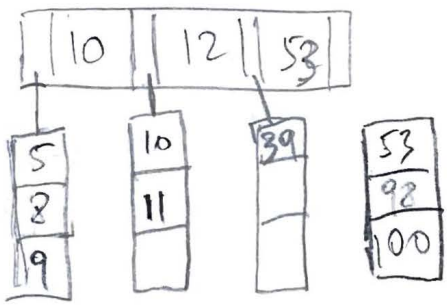

10

a)

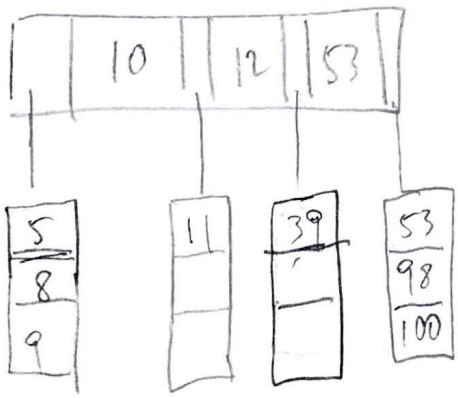
step 1




step 2




B)



(11) - size of M is 3
- the size of B-tree leaf is 6
- $5^m < n \leq 5^{m+1}$
- $5^6 < 30\,000 \leq 5^7$ $h = 6$
- $5^9 < 2\,500\,000 \leq 5^{10}$ $h = 9$