1 General Equations

Kinematic Equations:

$$X(t) = X_i + V_{xi} \times t + \frac{a_x \times t^2}{2}$$
$$V(t) = V_{xi} + a_x \times t$$
$$V_x^2 = V_{xi}^2 + 2a_x \times (\Delta X)$$

1.1 Springs (X = displacement from rest position)

$$F_x = -kx$$

1.2 Friction

$$f_{smax} = \mu_s \cdot F_N$$
$$f_k = \mu_k \cdot F_N$$
$$F_N = mg$$
$$f_{smax} \geq f_k$$

1.3 Dot Products

$$\vec{A} \cdot \vec{B} = AB \times \cos\Theta$$

$$\vec{A} \cdot \vec{B} = \begin{cases} 0 & \text{When } \vec{A} \text{ and } \vec{B} \text{ are perpendicular} \\ AB & \text{When } \vec{A} \text{ and } \vec{B} \text{ are parrallel} \end{cases}$$

$$\vec{A} \cdot \vec{A} = A^2$$
$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$
$$(\vec{A} + \vec{B}) \cdot \vec{C} = (\vec{A} \cdot \vec{C}) + (\vec{B} \cdot \vec{C})$$

1.4 Conservative and Nonconservative forces

Conservative Forces - Path taken is irrelevant
Nonconservative Forces - Path matters

$$\oint_c (\vec{F}) \cdot d\vec{l} = W_{total}$$

C is the closed path length. This problem can be broken into pieces and solved.

1.5 Linear Motion

Displacement:

$$\Delta X$$

Velocity:

$$V_x = \frac{dx}{dt}$$

Acceleration:

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

Constant-acceleration equations:

$$V_x = V_{xi} + a_x t$$
$$\Delta X = V_{xavg} \Delta t$$
$$V_{xavg} = \frac{1}{2}(V_{xi} + V_x$$
$$X = X_i + V_{xi}t + \frac{1}{2}(a_x t^2)$$
$$V_x^2 = V_{xi}^2 + 2a_x \Delta X$$

Force: F_x
Mass: m
Work: $dW = F_x dx$
Kinetic Energy: $K = \frac{1}{2}mv^2$
Power: $P = F_x V_x$
Momentum: $p_x = mV_x$
Newton's Second Law:

$$F_{xnet} = ma_x = \frac{dp_x}{dt}$$

1.6 Rotational Motion

Angular Displacement:

$$\Delta\Theta$$

Angular Velocity:

$$\omega = \frac{d\Theta}{dt}$$

Angular Acceleration:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\Theta}{dt^2}$$

Constant Angular Acceleration equations:

$$\omega = \omega_i + \alpha t$$
$$\Delta\Theta = \omega_{avg} \Delta t$$
$$\omega_{avg} = \frac{1}{2}(\omega_i + \omega)$$
$$\Theta = \Theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_i^2 + 2\alpha\Delta\Theta$$

Torque: τ
Moment of inertia: I
Work: $dW = \tau d\Theta$
Kinetic Energy: $K = \frac{1}{2}I\omega^2$
Power: $P = \tau\omega$
Angular Momentum: $L = I\omega$
Newton's Second Law:

$$\tau_{net} = I\alpha = \frac{dL}{dt}$$

2 General Maths

Quadratic Equation:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vector Composition:

$$\vec{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

2D Vector Decomposition:

$$V_y = \bar{V} \times \sin\Theta$$
$$V_x = \bar{V} \times \cos\Theta$$
$$\bar{V} = \sqrt{V_x^2 + V_y^2}$$

Standard Units:

- Time $\Rightarrow$ Seconds (s)
- Mass $\Rightarrow$ Kilograms (kg)
- Length $\Rightarrow$ Meter (m)
- Force $\Rightarrow$ Newton (N) $= \frac{1kg/1m}{1s^2}$
- $g = 9.81m/s^2$

Geometry:

$$\Theta = \tan^{-1}\frac{opp}{hyp}$$

3 Calculus

General Rules:

(a) As long as $n \neq -1$

$$\int t^n dt = \frac{t^{n+1}}{n+1} + C$$

(b) For when $n = 1$

$$\int t^{-1} dt = \ln t + C$$

4 Work-Kinetic Energy Theorum

W-KE Equations:

$$W_{total} = \Delta K$$
$$K = \frac{1}{2}mv^2$$
$$F_{netx} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$W_{total} = \Delta K$$

Kinematics:

$$V_f^2 = V_i^2 + 2a_x\Delta X$$
$$a_x = (V_f^2 - V_i^2) \times \frac{1}{2\Delta X}$$

Solving Problems:

(a) Draw everything at initial and final positions

(b) Add axis (x,y,z)

(c) Draw arrows, initial and final velocities, ΔK, etc

(d) Add force arrows to inital position drawing

(e) Calculate $W_{total} = \Delta K$, $F_{net}d = etc$

5 Examples

5.1 Kinematics and Integration

The a_x of a rocket is given by $a_x = bt$ where b is a positive constant. Find the position function.
Givens:

$$a_x = bt$$

$$X = X_0 + V_x t = V_{xi} \text{ at } t = 0$$

Find the solution:

$$V_x(t) - V_x(0) = \int_0^t (bt)\mathrm{d}t = (\frac{1}{2}bt^2) \mid_0^t$$

$$V_x(t) = V_{xi} + \frac{1}{2}bt^2$$

$$X(t) - X(0) = \int_0^t (V_{xi} + \frac{1}{2}bt^2)\mathrm{d}t$$

$$X(t) = X_i + V_{xi}t + (\frac{1}{2}b + \frac{1}{3}t^3)\min_0^t$$

Solution:

$$X(t) = X_i + V_{xi}t + \frac{1}{6}bt^3$$

5.2 Projectile Motion

Governing Equations:

$$V_{xi} = V_i\cos\Theta$$

$$V_{yi} = V_i\sin\Theta$$

$$V_x = V_{xi} \text{ (Assuming no drag)}$$

$$V_y = V_{yi} - gt$$

Derived Equations:

$$X(t) = X_i + V_{xi}t$$

$$Y(t) = Y_i + V_{yi}t + \frac{1}{2}(-g)t^2$$

$$Y(X) = \frac{V_{yi}}{V_{xi}}X - \frac{g}{2 \times V_{xi}^2}X^2$$

5.3 Circular Motion

$a_c = \frac{v^2}{r}$ Inward a needed for const. circ. motion.

$$v = \sqrt{a_c r}$$

$$v = \frac{2\pi r}{t} \text{ t = time for 1 rev.}$$

$$F = ma = m\frac{v^2}{r} \text{ if } a = a_c$$

5.4 Summing Vectors

Givens:

$$\vec{F_a} = 40.0\text{N at } 45\deg$$

$$\vec{F_b} = 30.0\text{N at } 37\deg$$

Find sum of vectors:

$$\vec{F_t} = \sum(F_x)\hat{\mathbf{i}} + \sum(F_y)\hat{\mathbf{j}}$$

$$\sum F_{tx} = 40\cos(45\deg) + 30\cos(37\deg) = 52.3\text{N}$$

$$\sum F_{tx} = 40\sin(45\deg) + (-30\sin(37\deg)) = 10.2\text{N}$$

$$\vec{F_t} = \sqrt{10.2^2 + 52.3^2} \text{ in } \Theta$$
$$= \tan^{-1}\frac{10.2}{52.3} = 53.3\text{N at } 11\deg \text{ towards A}$$

5.5 Angle when g overcomes friction

A coin is resting on an incline of increasing angle. At what angle does it begin to slide?
Begins to slide when $F_x = F_{fric}$

$$\sum F_y = ma_y = 0 = F_n - mg\cos\Theta$$
$$\sum F_x = mg\sin\Theta - f_s = 0$$

$$mg\sin\Theta - \mu_s F_n = 0$$
$$mg\sin\Theta - \mu_s(mg\cos\Theta) = 0$$
$$mg\sin\Theta = \mu_s mg\cos\Theta$$

Coin starts to slide:

$$\frac{\sin\Theta}{\cos\Theta} = \tan(\Theta_m ax) = \mu_s$$

5.6 Speed at bottom of incline

A block is sliding down an incline. It slides 1.5m, what is it's final speed?
Givens:

$$\Theta = 60\deg$$

$$V_i = 0\text{m/s}$$

$$\mu_k = \mu_s = 0$$

Set-up Equations:

$$W = \Delta K$$
$$K = \frac{1}{2}mv^2$$
$$76\text{Joules} = \frac{1}{2}mv_f^2 - 0$$

Work Done:

$$W = F_x \Delta X$$
$$F_x = \sum F_x = F_g = mg\sin(60\deg)$$
$$W = (1.5\text{m})(6.0\text{kg})(9.81m/s^2)\sin(60\deg) = 76\text{Joules}$$
$$V_{final} = \sqrt{\frac{2\times 76}{m}} = 5.0m/s$$

5.7 Potential Energy Example

ΔU = Work done by a conservative force.

$$\Delta U = -\int(\vec{F})\cdot d\vec{l} = U_2 - U_1$$
$$U_0 = 0 \text{ when } y = 0$$
$$dU = -\vec{F}\cdot d\vec{l}$$
$$\Delta U = mgh$$
$$Wg = \int(\vec{F})d\vec{l} = -\Delta U$$