Small Open Economy Model with Endogenous Discount Factor and Debt-Elastic Interest Rate

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Based on the content provided from the paper *"Closing small open economy models"* by Schmitt-Grohe and Uribe (2003), we can construct a model that combines features of **Model 1 (endogenous discount factor)**, **Model 1a (simplified endogenous discount factor without internalization)**, and **Model 2 (debt-elastic interest rate)**. The goal is to build a **small open economy model** where:

- Agents have an **endogenous discount factor** that depends on both consumption and working hours.
- The domestic interest rate is **endogenously determined** and increases with the level of net foreign debt.

This setup ensures stationarity in the equilibrium dynamics while maintaining realism in capturing agents' behavior and market imperfections.

1. Preferences

The household's preferences are given by the recursive utility function:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t)$$

with:

$$\theta_0 = 1, \quad \theta_{t+1} = \beta(c_t, h_t)\theta_t$$

where $\beta(c_t, h_t)$ is the endogenous discount factor depending on aggregate consumption and labor supply. Assume the functional forms:

$$U(c_t, h_t) = \frac{[c_t - \frac{h_t^\omega}{\omega} - 1]^{1-\gamma}}{1-\gamma}$$

$$\beta(c_t, h_t) = \left[1 + c_t - \frac{h_t^\omega}{\omega}\right]^{-\psi_1}$$

$$F(k_t, h_t) = k_t^\alpha h_t^{1-\alpha}$$

$$\Phi(x_t) = \frac{\phi}{2} x_t^2, \quad \phi > 0$$

Where:
- $\psi_1 > 0$: elasticity of the discount factor
- $\omega > 0$: elasticity parameter for labor.
- $\gamma > 0$: elasticity parameter for utility.
- $\alpha \in (0, 1)$: capital share.

2. Technology and Production

Output is produced using capital k_t and labor h_t via Cobb–Douglas production:

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

Capital evolves as:

$$k_{t+1} = i_t + (1-\delta)k_t$$

Investment subject to adjustment costs:

$$F(k_{t+1} - k_t) = \frac{\phi}{2}\left(\frac{k_{t+1} - k_t}{k_t}\right)^2$$

3. Budget Constraint and Foreign Debt

The budget constraint:

$$d_t = (1 + r_t)d_{t-1} + y_t - c_t - i_t - F(k_{t+1} - k_t)$$

Domestic interest rate:

$$r_t = r^* + p(d_t), \quad p(d_t) = \psi_2(e^{d_t - \bar{d}} - 1)$$

4. First-Order Conditions (Household Optimization)

Let λ_t be the Lagrange multiplier on the budget constraint.

Euler Equation:

$$\lambda_t = \beta(c_t, h_t)(1 + r_t)\mathbb{E}_t[\lambda_{t+1}]$$

Labor Supply Condition:

$$-U_h(c_t, h_t) = \lambda_t A_t (1-\alpha) k_t^{\alpha} h_t^{-\alpha}$$

Investment Condition:

$$\lambda_t[1 + F'(k_{t+1} - k_t)] = \beta(c_t, h_t)\mathbb{E}_t\left[\lambda_{t+1}\left(A_{t+1}(1-\delta) + \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha}\right)\right]$$

5. Steady state

We need to impose the following conditions for the steady state:

Euler equation steady state:

$$\beta(c, h)(1 + r(d)) = 1$$

where

$$r(d) = r^* + \psi_2(d - \bar{d})$$

Resource constraint steady state:

$$y = c + i + \text{adjustment costs}$$

(no adjustment costs at SS if $k = k^{-1}$).

Production function steady state:

$$y = Ak^{\alpha} h^{1-\alpha} \quad \text{with} \quad A = 1$$

Labor FOC steady state:

$$\left(c - \frac{\omega}{h^{\omega}}\right) - \gamma \frac{h^{\omega} - 1}{h^{\omega}} = (1 - \alpha)\frac{y}{h}$$

Investment steady state:

$$i = \delta k$$

5. Equilibrium Definition

A competitive equilibrium satisfies:
- Household optimization conditions,
- Resource constraints,
- Productivity shock process: $\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}$,
- Interest rate determination: $r_t = r^* + p(d_t)$,
- No-Ponzi condition: $\lim_{j\to\infty} \mathbb{E}_t \left[\frac{d_{t+j}}{(1+r)^j}\right] \leq 0$

6. Calibration Strategy

To ensure comparability with original models:
- Use parameter values from Tables 1 and 2 in the original paper. - Set χ, v, γ so that steady-state trade balance-to-GDP ratio matches empirical data. - Choose ξ and η so that current account volatility matches observed Canadian data.

7. Key Features of the Combined Model

- Stationarity via endogenous discount factor and debt-elastic interest rate.
- Enriched behavioral dynamics: agents respond to both consumption and labor supply.
- Financial frictions: interest rate premium affects borrowing behavior.

8. Computational Implementation

To solve the model numerically:

1. Log-linearize equilibrium conditions around deterministic steady state.
2. Use perturbation methods or value function iteration.
3. Simulate to compute second moments and impulse responses.

Use `Dynare`, and `MATLAB`.

Conclusion

This combined model integrates two key mechanisms—endogenous time preferences and debt-dependent interest rates—to induce stationarity in a small open economy framework. It preserves the core structure of real business cycle models while incorporating richer behavioral and financial dynamics. As shown in the original paper, these types of modifications yield similar business-cycle implications, suggesting that this hybrid model will perform similarly in terms of macroeconomic volatility and comovement, while offering a more nuanced description of agent behavior and financial frictions.