Advanced Macroeconomics 2/Part 2, CERGE-EI, Spring 2025
Problem set 5

Instructor: Ctirad Slavik
TA: Samvel Margaryan

Hugget type of model in Matlab

Consider the following model:

The representative agent maximizes the following utility function over streams of consumption c_t:

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \tag{1}$$

The budget constraint is:

$$c_t + a_{t+1} = a_t(1+r) + y_t$$
$$a_0 \quad given$$

Assume an ad-hoc borrowing constraint: $a_{t+1} \geq \underline{a} = -1$

Furthermore, y_t is stochastic and it is given by a process:

$$ln(y_t) = \theta_t + \epsilon_t$$

where

$$\theta_t = \rho\theta_{t-1} + \kappa_t$$

ϵ_t and κ_t and independent and identically distributed shocks with mean 0 and standard deviations: $\sigma_\epsilon = 0.067$, $\sigma_\kappa = 0.016$.

Other parameter values are: $\sigma = 2$, $\beta = 0.96$, and $\rho = 0.985$.

1. Discretise and approximate the processes by Markov chain with 3 states for θ and 3 states for ϵ. *Suggestion: You can use the Rouwenhorst method.*

2. Assume that there is a continuum of agents with mass 1. Furthermore, assume that $r = 0.015$. Define wealth (cash in hand): $w_t = a_t(1+r) + y_t$. Define $\underline{w} = \underline{a}(1+r) + \underline{y}$, where $\underline{y}$ is the lowest possible realization of y. Let ψ be the probability measure on (S, β_s), where $S = [\underline{w}, \bar{w}] \times \Theta$, and β_s is the Borel σ-algebra. Therefore, for $B \in \beta_s$, $\psi(B)$ indicates the mass of agents whose individual states fall in B.
   Compute the stationary distribution of agents over wealth w and permanent productivity θ (A, Θ).

To do so, you will need to first compute the transition function which will tell you the distribution of agents over (W, Θ) in period $t+1$, given the distributions of agents over (W, Θ) in period t. Θ is the set of all possible θs. Transition function P ($P : S \times \beta_s \rightarrow [0,1]$) provides us with $P(x, B)$, which is the the probability that an agent with state x will have the individual state lying in B the next period. The probability measure ψ is stationary if:

$$\psi(B) = \int_S P(x, B) d\psi \quad \forall \quad B \in \beta_s$$

3. Now, endogenise r. We will assume that the bonds (a) are in zero net supply. That means that r has to be such that $\int_S a_{t+1}(x) d\psi = 0$. (For example, to find the endogenous r, you can use bisection algorithm.) When you find the endogenous r, define the equilibrium. Compute the long run Welfare, and the earnings (y_t) GINI coefficient. Compute the wealth (w) GINI coefficient.

4. Consider the same model as in c) , but now set $\sigma_\epsilon = 0.005$. Compute long-run Welfare and GINI coefficients for earnings and wealth. Comment the differences between c) and this setup.

5. Consider the same model as in c) , but now set $\underline{a} = -0.3$. Compute long-run Welfare, and GINI coefficients for earnings and wealth. Comment the differences between c) and this setup.