ES6 Advanced macroeconomics 2, part 2[1]

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[1] Based on Fabrizio Perri's lecture notes, University of Minnesota

The problem of the individual in recursive form

$$v(a,\varepsilon;\lambda) = \max_{c,a'} \left\{ u(c) + \beta \sum_{\varepsilon' \in \mathcal{E}} v(a',\varepsilon';\lambda')\pi(\varepsilon',\varepsilon) \right\} \tag{1}$$

$$s.t.$$

$$c + a' = (1+r)a + \varepsilon \tag{2}$$

$$a' \geq -\bar{a} \tag{3}$$

Aggregate saving supply function

$$A(r) = \int_{A\times E} a'(a,\varepsilon;r)\, d\lambda^*(a,\varepsilon;r) \tag{4}$$

Firm's problem

$$\max_{K,L} \; K^{\alpha} L^{1-\alpha} + (1-\delta)K - wL - (1+r)K \tag{5}$$

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$$K: \quad K(r) = \left(\frac{\alpha L^{1-\alpha}}{\delta + r} \right)^{\frac{1}{1-\alpha}} \tag{6}$$

Asset market clearing condition

$$K(r) = A(r) \tag{7}$$

An Algorithm for the Computation of the Equilibrium

1. Fix an initial guess for the interest rate $r^0 \in (-\delta, \frac{1}{\beta} - 1)$. The interest rate r^0 is our first candidate for the equilibrium (the superscript denotes the iteration number).
2. Given the interest rate r^0, obtain the wage rate $w(r^0)$ (recall that L is given exogenously with inelastic labor supply).
3. Given prices $(r^0, w(r^0))$, you can now solve the dynamic programming problem of the agent to obtain $a'(a, \varepsilon; r^0)$ and $c(a, \varepsilon; r^0)$. Notice that these functions will be, in general, nonlinear functions of the states, hence you will have to find suitable approximations for those (either polynomial or piece-wise linear).
4. Given the policy function $a'(a, \varepsilon; r^0)$ and the Markov transition over productivity shocks $\pi(\varepsilon', \varepsilon)$, we can construct the transition function $Q(r^0)$ and obtain the fixed point distribution λ_{r^0}, conditional on the candidate interest rate r^0.

Recall the general definitions

$$Q\left((a,\varepsilon),\mathcal{A}\times\mathcal{E}\right)=\sum_{\varepsilon'\in\mathcal{E}}\mathcal{I}\left\{a'(a,\varepsilon)\in\mathcal{A}\right\}\pi(\varepsilon',\varepsilon) \tag{8}$$

$$\lambda_{n+1}(\mathcal{A}\times\mathcal{E})=T^*(\lambda_n)=\int_{A\times E}Q\left((a,\varepsilon),\mathcal{A}\times\mathcal{E}\right)\,d\lambda_n(a,\varepsilon) \tag{9}$$

$$\lambda^*(\mathcal{A}\times\mathcal{E})=\int_{A\times E}Q\left((a,\varepsilon),\mathcal{A}\times\mathcal{E}\right)\,d\lambda^*(a,\varepsilon) \tag{10}$$

An Algorithm for the Computation of the Equilibrium

5. Compute the aggregate demand of capital $K(r^0)$ from the optimal choice of the firm who takes as given r^0, i.e.
$$K(r^0) = F_k^{-1}(r^0 + \delta)$$
6. Compute the integral
$$A(r^0) = \int_{A \times E} a'(a, \varepsilon; r^0)\, d\lambda^*_{r^0}$$
which gives the aggregate supply of assets.
7. Compare $K(r^0)$ with $A(r^0)$ to verify the asset market clearing condition. If $A(r^0) > (<)\, K(r^0)$, then the next guess of the interest rate should be lower (higher), i.e. $r^1 < (>)\, r^0$.

An Algorithm for the Computation of the Equilibrium

8. Update your guess to r^1 and go back to step 1). Keep iterating until one reaches convergence of the interest rate, i.e. until

$$\left|r^{n+1} - r^n\right| < \varepsilon,$$

   for ε small.
9. All the equilibrium statistics of interest, like aggregate savings, inequality measures, etc. can be then easily computed using the stationary distribution.