Logical Clocks

dDistSys

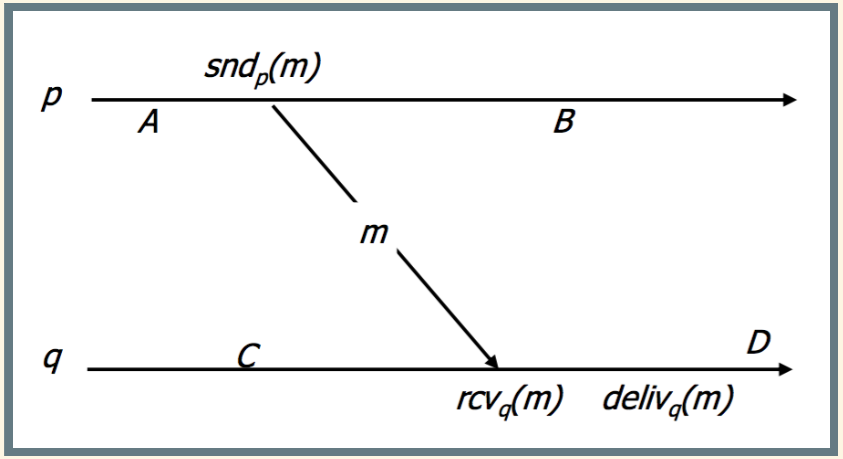
11/05/2015

Johan K. Jensen

Happened-before relationship

Happened-before relationship

- If a and b are two events in the same process, and a comes before b, then $a \rightarrow b$.
- If a is the sending of a message, and b is the receipt of that message, then $a \rightarrow b$.
- If $a \rightarrow b$ and $b \rightarrow c$, then $a \rightarrow c$.



$$A \to^{p} B$$
$$C \to^{q} D$$
$$\mathrm{snd}_p(m) \to^{M} \mathrm{rcv}_q(m)$$
$$A \to^{p} \mathrm{snd}_p(m) \to^{M} \mathrm{rcv}_q(m) \to^{q} D$$
$$A \to D$$

Lamport clocks

Lamport clocks

Attach a timestamp $C(e)$ to each event e

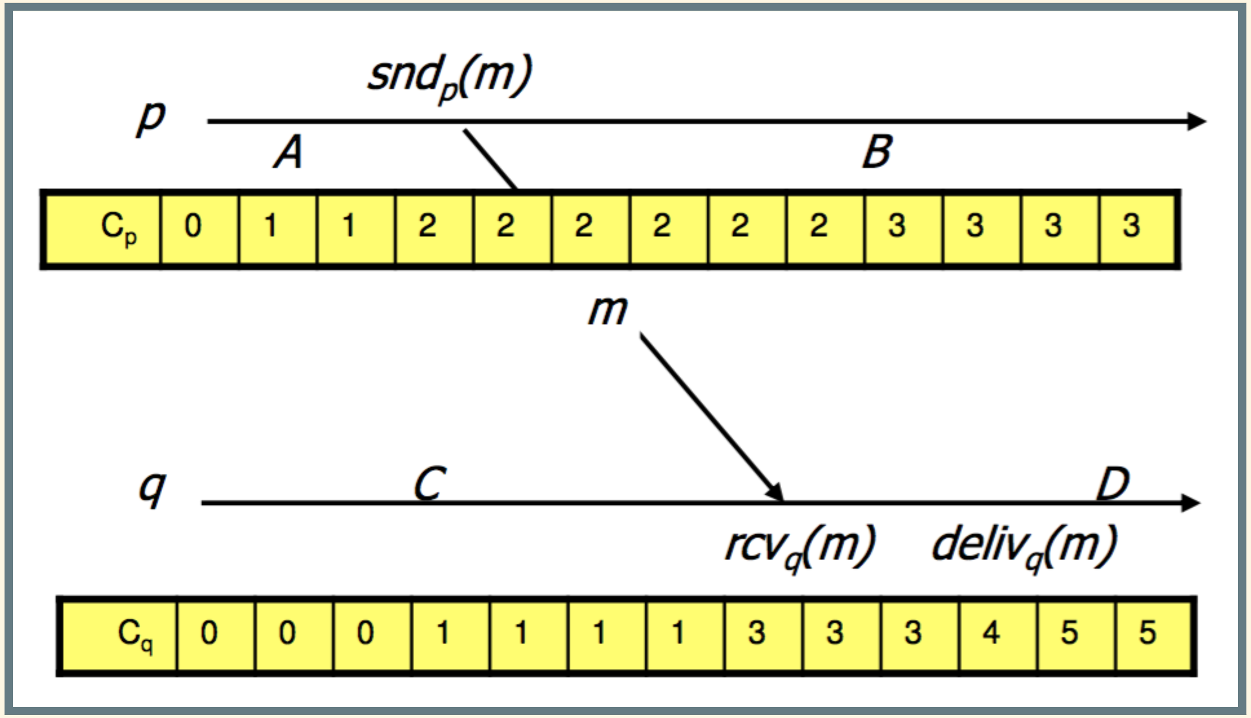
- If $A \rightarrow^p B$ then $C(A) < C(B)$
- If A corresponds to sending a message m, and B to the receipt of that message, then also $C(A) < C(B)$.

Lamport clocks

Process P_i maintains a counter C_i and adjusts this counter:

- C_i is incremented for any two successive events within P_i
- Each $snd(m)$ by process P_i, receives a timestamp $ts(m) = C_i$
- When $recv(m)$ in process P_j, then $C_j = max(C_j, ts(m))$

Lamport clocks



So if $A \to B$ then $C(A) < C(B)$
but **not**: if $C(A) < C(B)$ then $A \to B$.

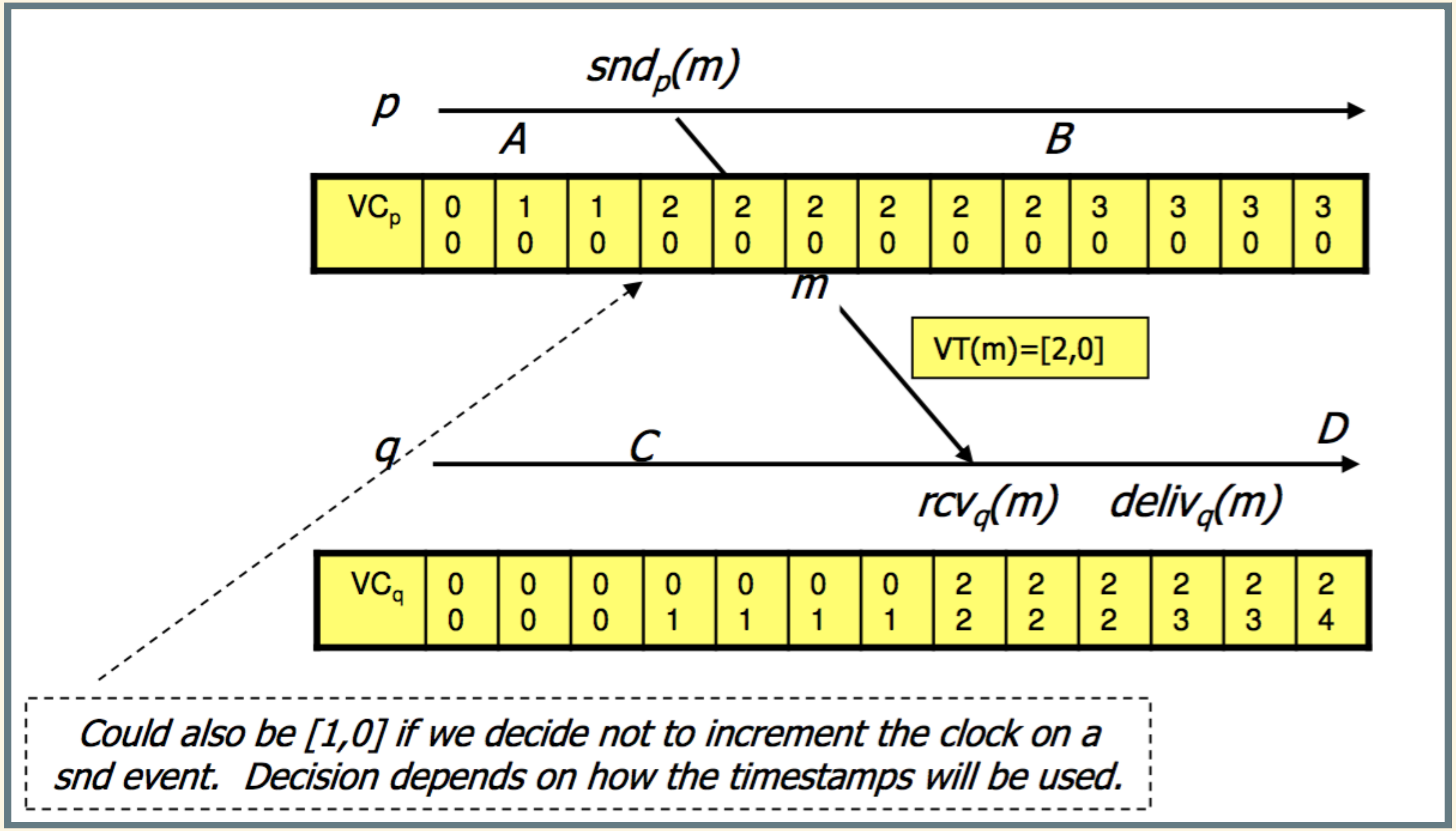
Vector clocks

Vector clocks

Extension of Logical clocks where each process has a unique vector of times for each process:
$VC(A) = [q, p]$

Sending message: $VC(m) = VC_p$
Receive message: $VC_q = max(VC_q, VC(m))$

Vector clocks



Comparing Vector clocks

- $VC(A) \leq VC(B)$ if
  - $\forall i, VC_A[i] \leq VC_B[i]$
- $VC(A) < VC(B)$ if
  - $VC(A) \leq VC(B)$ and $VC(A) \neq VC(B)$
  - $\exists i, VC_A[i] < VC_B[i]$

Properties of Vector clocks

If $A \rightarrow B$, then $VC(A) < VC(B)$ (as Lamport clocks).

But also: if $VC(A) < VC(B)$ then $A \rightarrow B$.

This is shown by:
If $\neg(A \rightarrow B)$ then $\neg(VC(A) < VC(B))$

Vector clocks

If $\neg(A \rightarrow B)$ then $\neg(VC(A) < VC(B))$

- A occurs at p and B at q
- $VC(A)[p] = k$
- There cannot be a sequence of events $A = E_0 \rightarrow E_1 \rightarrow E_2 \rightarrow \ldots \rightarrow E_n = B$ (otherwise $A \rightarrow B$)
- $VC(B)[p] < k$
- Consequently $\neg(VC(A) < VC(B))$