

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{3}{5-4\cos(w)} dw = 3 \int_{-\pi/2}^{\pi/2} \frac{1}{5-4\cos(w)} dw$$

$$u = 5-4\cos(w) = 4\sin(w)$$

$$u = \frac{1}{5-4\cos(w)} = \frac{-4\sin(w)}{(5-4\cos(w))^2}$$

$$\frac{3}{2\pi} \int_{-\pi}^{\pi} \frac{1}{5-4\cos(w)} dw =$$

$$u = \tan\left(\frac{w}{2}\right)$$

$$\sin(w) = \frac{2u}{u^2+1}$$

$$du = \frac{dw}{2\cos^2(w/2)}$$

$$\cos(w) = \frac{1-u^2}{u^2+1}$$

$$\Rightarrow dw = \frac{2du}{u^2+1}$$

$$\frac{3}{2\pi} \int_{u(-\pi)}^{u(\pi)} \frac{2du}{(u^2+1)(5-4(\frac{1-u^2}{1+u^2}))} = \frac{3}{\pi} \int_{u(-\pi)}^{u(\pi)} \frac{du}{9u^2-1}$$

$$\frac{ds}{du} = 3$$

$$s = 3u$$

$$= \frac{3}{\pi} \int_{-\infty}^{\infty} \frac{du}{9u^2-1} = \frac{2 \cdot 3}{\pi} \int_0^{\infty} \frac{du}{9u^2-1} = \frac{2 \cdot 3}{\pi} \int_0^{\infty} \frac{1}{s^2-1} ds$$

$$\frac{2}{\pi} \int_0^{\infty} \frac{1}{s^2+1} = [\tan^{-1}(s)]_0^{\infty} = \frac{2}{\pi} \tan^{-1}(\infty) - \frac{2}{\pi} \tan^{-1}(0)$$

$$= \underline{\underline{1}}$$