dig sig øving 2

$$x[n] = 2\delta[n] + \delta[n \pm 1]$$

$$X_{2\pi}(\omega) = \sum_{-\infty}^{\infty} x[n]\, e^{-i\omega n}$$

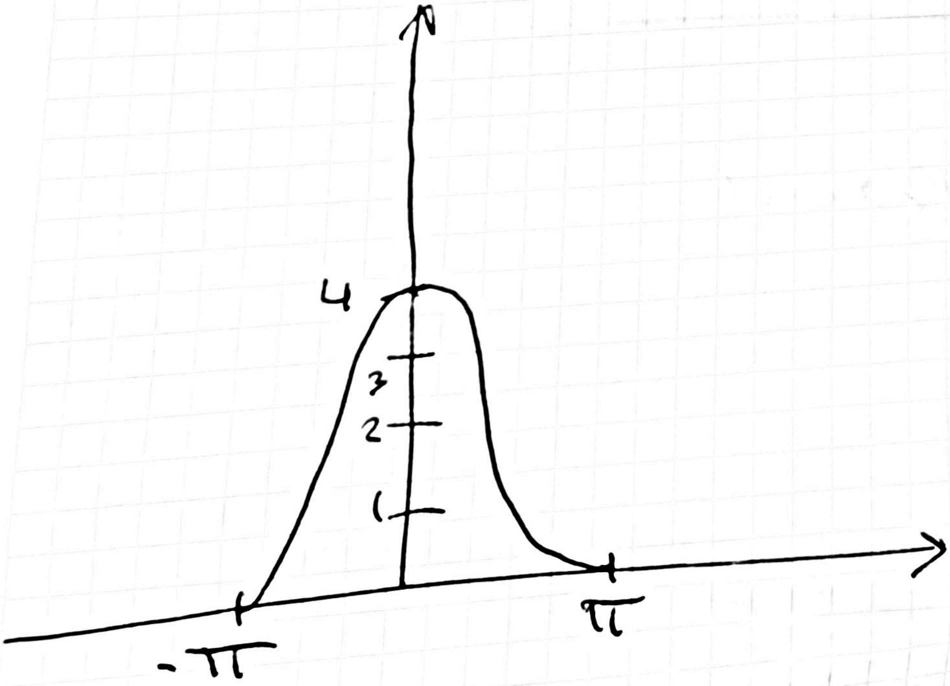
$$\Rightarrow \mathcal{F}_{2\pi}\{\delta[n-m]\} = e^{-i\omega m}$$

gives

$$\mathcal{F}_{2\pi}\{\cancel{\delta[n-m]}\, x[n]\} = e^{i\omega} + e^{-i\omega} + 2 = X(\omega)$$

$$\cos(\omega) = \frac{e^{i\omega} + e^{-i\omega}}{2}$$

$$\Rightarrow \underline{\underline{2 + 2\cos(\omega)}}$$



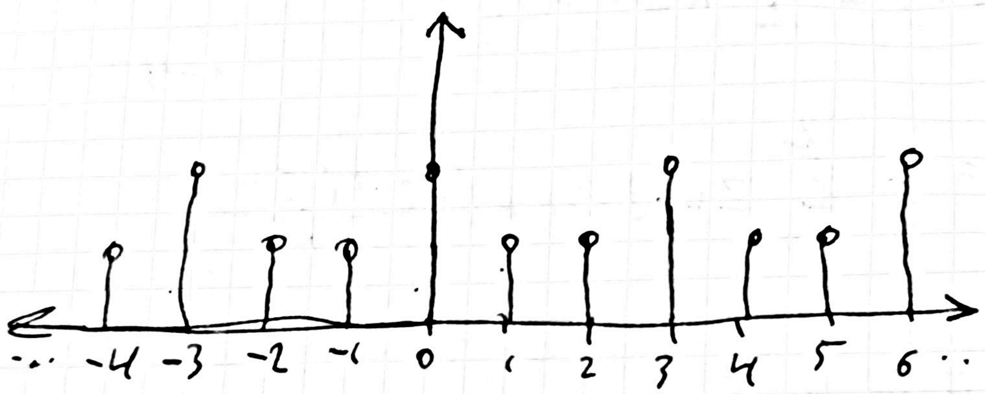
c) cause they are even.

d) $z[n] = \sum_{l=-\infty}^{\infty} x[n-lN]$

periodic extension of $x[n]$.

$x[n] = \delta[n] + 2\,\delta[n\pm1]$

$\cancel{z[n]=2}$



$z[n] = 1 + \delta[n-3k]$, $k \in \mathbb{Z}$

$$\mathcal{F}\{z[n]\} = \sum_{n=-\infty}^{\infty} z[n]\, e^{-iwn}$$

$2 + 2\cos(w) + 2\cos(2w) + 4\cos(3w) + \dots$

difference is the c_k spectrum is an infinite series.

c)

$$z[n] = \sum_{l=-\infty}^{\infty} x[n-lN]$$

$$N = 3$$

$$z[n] = \sum_{l=-\infty}^{\infty} x[n-l3]$$

$$\cancel{z[0] = \sum_{-\infty}^{\infty} x[0-3l] = x[-3l] = x[-1] + x[+1] + x}$$

$$\cancel{= 2 + 2 = 4}$$

$$\cancel{z[1] = \sum x[1-3l] = 4}$$

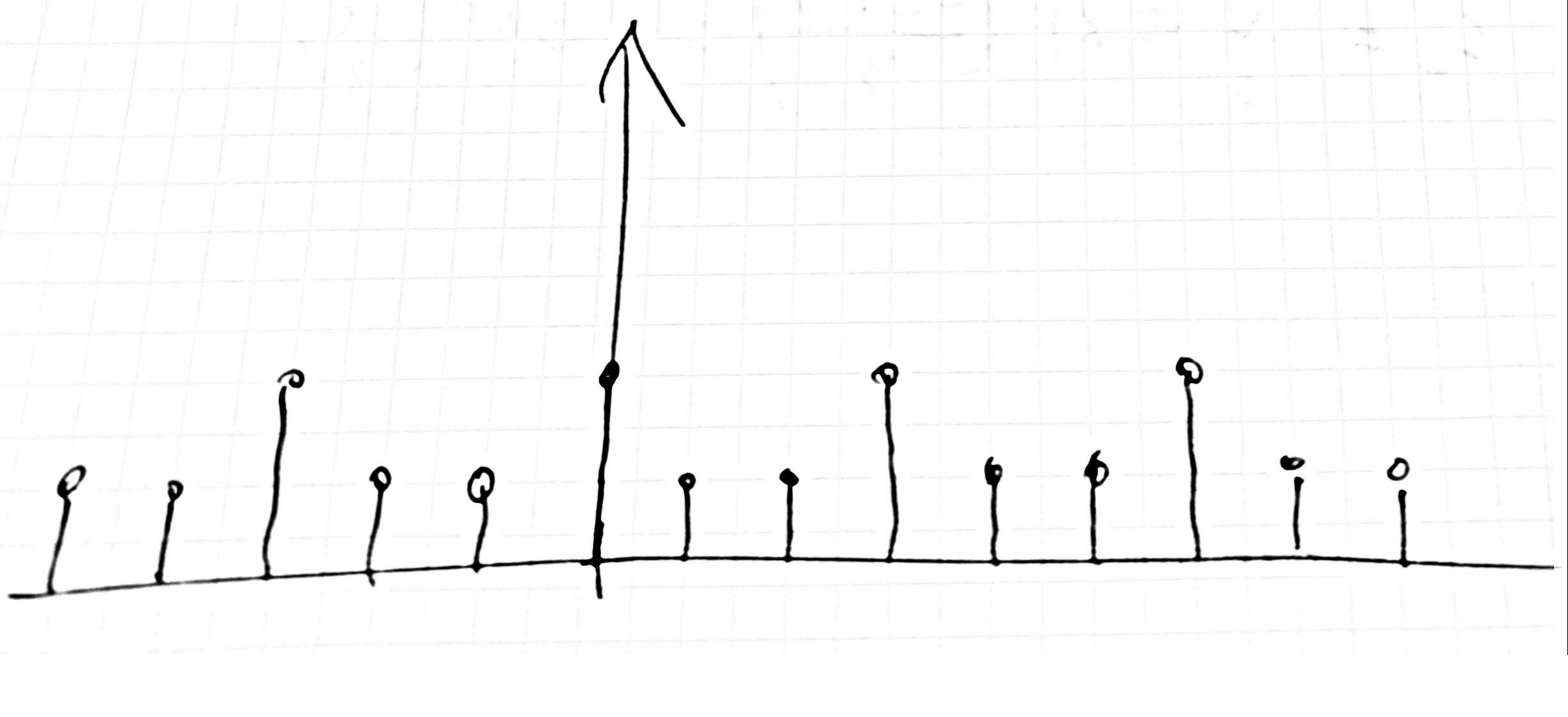
$$z[0] = x[-3l] = x[0] = 2$$

$$z[1] = x[1-3l] = 1$$

$$z[-1] = x[-1-3l] = x[-1] = 1$$

$$z[2] = x[2-3l] = x[-1] = 1$$

$$z[3] = x[3-3l] = x[0] = 2$$



$N = 3$

$$c_k = \frac{1}{N} \sum_{k=0}^{N-1} x[n] \exp(-iwkn/N)$$

$$c_{-1} = \frac{1}{3} \exp\left(\frac{-iw(-1)}{3}\right) = \frac{\exp\left(\frac{wk}{3}\right)}{3}$$

$$c_0 = \frac{2}{3} \exp(0)$$

$$c_1 = \frac{1}{3} \exp\left(\frac{-iw}{3}\right)$$

$$c_0 = \frac{2}{3} \exp(0)$$

$$c_1 = \frac{1}{3} \exp\left(\frac{-iw}{3}\right)$$

$$c_2 = \frac{1}{3} \exp\left(\frac{-2iw}{3}\right) = \frac{1}{3} \exp\left(\frac{iw}{3}\right)$$

P2)

a $x_1[n] = x[n+3]$

use time shift

$X_1(w) = X_{2\pi}(w) \, e^{3w}$

b) $x_2[n] = x[-n]$

$X_2(w) = X(-w)$

c) $x_3[n] = x[3-n]$

$X_3 = X(-w)e^{3w}$

d) $x_4[n] = x[n] * x[n]$

$X_4 = (X(w))^2$

P3/

$y_1[n] = x[n] + 2x[n-1] + x[n-2]$

$y_2[n] = -0{,}9\,y[n-1] + x[n]$

$Y_1(w) = X(w) + 2X(w)e^{-iw} + X(w)e^{-2wi}$

$H(w) = 1 + 2e^{-iw} + e^{-2iw}$

$= (1 + 2\cos(w))\,e^{-iw}$

$|H(w)| = |1 + 2\cos(w)| = \underline{\underline{1 + |2\cos(w)|}}$ $\underline{\text{even}}$

$\angle(H(w)) = \underline{\underline{-w}}$ $\underline{\text{odd}}$

$Y_2(w) = -0{,}9\,Y(w)e^{-iw} + X$

$(1 + 0{,}9e^{-iw})\,Y_2(w) = X(w)$

$H(w) = \dfrac{1}{1 + 0{,}9\,\cancel{e}\exp(-iw)}$

$H(w) = \dfrac{\exp(iw)}{\exp(iw) + 0{,}9}$

$$|H(w)| = \left| \frac{\exp(iw)}{\exp(iw)+0{,}9} \right| = \frac{1}{|\exp(iw)+0{,}9|}$$

= even function

$$\angle H(w) = \arg(\exp(iw)/(\exp(iw)+0{,}9))$$

$$= \arg(\exp(iw)) - \arg(\exp(iw+0{,}9))$$

$$\arg(H(w)) = w - \arg(\exp(iw+0{,}9))$$

odd

sys 1 low pass

sys 2 high pass

e/

~~dig sig el~~

$$x[n] = \frac{1}{2}\sin\left(\frac{\pi}{2}n + \frac{\pi}{2}\right)$$

$$H_1(w) = e^{2iw} + 2e^{iw} + 1$$

$$X(w) = \int \sum x[n]\, e^{-iwn}$$

$$x[n] = \frac{1}{2}\left(\frac{i}{2}\left(e^{-\left(i\frac{\pi}{2}n + \frac{\pi}{4}\right)} - e^{\left(\frac{\pi}{2}in + \frac{\pi}{4}\right)}\right)\right)$$

$$\cancel{X(w) = \sum x[n]\, e^{iwn}}$$

$$\cancel{= \frac{i}{4}\left(\exp\left(\frac{-i\pi}{2}n + \frac{\pi}{4}\right) - \exp\left(\frac{\pi i n}{2} + \frac{\pi}{4}\right)\cdot\exp(iwn)\right)}$$

$$\cancel{= \frac{i}{4}\left(\exp\left(-\frac{\pi}{4}\right) - \exp\left(\frac{\pi}{4}\right)\right)}$$

$$\cancel{X(\omega) = \Sigma \frac{i}{4} \exp\left(-\left(\frac{i\pi n}{2} + \frac{\pi}{4}\right)\right) + \Sigma \frac{i}{4} \exp}$$

$$X(\omega) = \mathcal{F}\left\{\frac{i}{4}\left(\exp\left(-\left(\frac{i\pi}{2} n + \frac{\pi}{4}\right)\right) - \exp\left(\frac{\pi i n}{2} + \frac{\pi}{4}\right)\right.\right.$$

$$= \mathcal{F}\left\{\frac{i}{4} \exp\left(-\frac{\pi}{4} i\right) \exp\left(-\frac{i\pi n}{2}\right)\right\} - \mathcal{F}\left\{\frac{i}{4} \exp\left(\frac{\pi}{4} i\right) \exp\left(\frac{\pi i n}{4}\right)\right.$$

$$\cancel{= \frac{i}{4} \exp\left(-\frac{\pi}{4}\right) \mathcal{F}\left\{\exp\left(\frac{-i\pi n}{2}\right)\right\} - \frac{i}{4} \exp\left(\frac{\pi i}{4}\right) \mathcal{F}\left\{\exp\left(\frac{\pi i n}{4}\right.\right.}$$

$$\cancel{= \frac{i}{4} \exp\left(-\frac{\pi i}{4}\right) \delta\left(\omega + \frac{\pi}{2}\right) - \frac{i}{4} \exp\left(\frac{\pi i}{4}\right) \delta\left(\omega - \frac{\pi}{2}\right)}$$

$$\cancel{= \mathcal{F}\left\{\frac{1}{4} \exp\left(\frac{\pi}{2} i\right) \exp\left(\frac{\pi}{4} i\right) \exp\left(\frac{-i\pi n}{2}\right)\right\}}$$

$$\cancel{- \mathcal{F} \frac{1}{4} \exp\left(\frac{\pi}{2} i\right) \exp}$$

$$= \mathcal{F}\left\{\frac{i \exp\left(\frac{\pi}{4} i\right)}{4} \exp\left(\frac{-i\pi}{2} n\right)\right\} - \mathcal{F}\left\{\frac{i \exp\left(\frac{\pi}{4} i\right)}{4} \exp\left(i\right.\right.$$

$$\underline{\underline{\frac{i \exp \frac{\pi i}{4}}{4} \delta\left(\omega + \frac{\pi}{2}\right) - \frac{i}{4} \exp \frac{\pi}{4} i \, \delta\left(\omega - \frac{\pi}{2}\right)}}$$

$$\frac{1}{4}\exp\left(\frac{3\pi}{4}i\right)\delta\left(\omega-\frac{\pi}{2}\right)-\frac{1}{4}\exp\left(\frac{i\pi}{4}\right)$$

$$X(\omega) = \frac{1}{4}e^{\frac{3\pi i}{4}}\delta\left(\omega-\frac{\pi}{2}\right)-\frac{1}{4}e^{\frac{i\pi}{4}}\delta\left(\omega+\frac{\pi}{2}\right)$$

$$H(\omega)X(\omega) = \frac{1}{4}\exp\left(\frac{3}{4}\pi i\right)\delta\left(\omega-\frac{\pi}{2}\right)H(\omega) - \frac{\exp\left(\frac{i\pi}{4}\right)}{4}\delta\left(\omega+\frac{\pi}{2}\right)H(\omega)$$

$$= \frac{\exp\left(\frac{3}{4}\pi i\right)}{4}\cancel{\delta\left(\omega-\frac{\pi}{2}\right)}H\left(\frac{\pi}{2}\right) - \frac{\exp\left(\frac{i\pi}{4}\right)}{4}H\left(-\frac{\pi}{2}\right)$$

$$H_1 = e^{2i\omega} + 2e^{i\omega}+1$$

$$H_2 = \frac{\exp(i\omega)}{\exp(i\omega)+0{,}9}$$

$$H_1\left(\frac{\pi}{2}\right) = e^{i\pi}+2e^{i\frac{\pi}{2}}+1 = -1+2i+1 = 2i$$

~~H_2~~

$$H_1\left(-\frac{\pi}{2}\right) = e^{-2i\frac{\pi}{2}} + 2e^{-i\frac{\pi}{2}}+1 = -1-2i+1 = -2i$$

$$H_2\left(\frac{\pi}{2}\right) = \frac{i}{i+0{,}9}$$

$$H_2\left(-\frac{\pi}{2}\right) = \frac{-i}{-i+0{,}9}$$

$$H_1(\omega)X(\omega) = \frac{\exp(\frac{3}{4}\pi i)}{4} 2i - \frac{\exp(\frac{i\pi}{4})}{4} - 2i$$

$$= \frac{i}{2}\left(\exp(\tfrac{3}{4}\pi i) + \exp(\tfrac{i\pi}{4})\right)$$

$\cancel{H_2(\omega)X = \exp(3}$

$$= \frac{1}{2}\left(\exp(\tfrac{5}{4}\pi i) + \exp(\tfrac{3}{4}\pi i)\right)$$

$$H_2(\omega)X(\omega) = \frac{\exp(\frac{3}{4}\pi i)}{4}\left(\frac{i}{i+0{,}9}\right) - \frac{(-i)}{(-i+0{,}9)}\frac{\exp(\frac{\pi i}{4})}{4}$$

$$\frac{1}{4}\left(\frac{\exp(\frac{3}{4}\pi i)\, i\,(0{,}9-i)}{0{,}9^2+1^2} + \frac{\exp(\frac{\pi i}{4})\, i\,(0{,}9+i)}{0{,}9^2+1}\right)$$

$$\frac{1}{4(0{,}81+1)}\exp(\tfrac{3}{4}\pi i)(0{,}9i-i^2) + \frac{\exp(\frac{\pi}{4}i)(0{,}9i+i^2)}{4(0{,}9^2+1)}$$

$\cancel{4 \cdot 1{,}81}$

$$\frac{\exp(\frac{3}{4}\pi i)(1+0{,}9i)}{4 \cdot 1{,}81} + \frac{\exp(\frac{\pi i}{4})(0{,}9i-1)}{4 \cdot 1{,}81}$$

$$\approx -0{,}37\ldots$$

$$|H_1 X| = \left|\tfrac{1}{2}\left(\exp\left(\tfrac{5}{4}\pi i\right) + \exp\left(\tfrac{3}{4}\pi i\right)\right)\right| = \left|\tfrac{1}{2} \cdot -\sqrt{2}\right| = \underline{\underline{\tfrac{1}{\sqrt{2}}}}$$

$$\angle(H_1 X) = \tfrac{1}{2}\left(\exp\left(\tfrac{5}{4}\pi i + \exp\left(\tfrac{7}{4}\pi i\right)\right) = \angle\left(\tfrac{-1}{\sqrt{2}}\right) = \underline{\underline{\pi}}$$

$$|H_2 X(w)| = \underline{\underline{0{,}37}}$$

$$\angle(H_2 X(w)) = \underline{\underline{\pi}}$$

P4

$$x_a(t) = \cos(2k\pi t) \quad \text{sampling} \Rightarrow t \to nT_s = \frac{n}{f_s}$$

$$x_1[n] = \cos\left(2000\pi \frac{n}{\cancel{1500}\,4000}\right) = \cos\left(\frac{\pi}{2} n\right)$$

$$x_2[n] = \cos\left(2000\pi \frac{n}{1500}\right) = \cos\left(\frac{4}{3}\pi n\right) = \cos\left(-\frac{2}{3}\pi n\right)$$

4.5

3.8

P4

$$x_a(t) = \cos(2000\pi t)$$

$$x_1[n] = \cancel{\cos}\ x(nT) \quad T = \frac{1}{4000}$$

$$\cos\left(\frac{\pi n}{2}\right)$$

$$x_2[n] = \cos\left(\frac{4\pi n}{3}\right)$$

$$X_1(w) = \pi\left(\delta\left(w - \frac{\pi}{2}\right) + \delta\left(w + \frac{\pi}{2}\right)\right)$$

$$X_2(w) = \pi\left(\delta\left(w - \frac{4\pi}{3}\right) + \delta\left(w + \frac{4}{3}\pi\right)\right)$$

$\cancel{w} = 2\pi f$

$$X_1(\omega) = \pi\left(\delta\left(\omega - \frac{\pi}{2}\right) + \delta\left(\omega + \frac{\pi}{2}\right)\right)$$

$$X_2(\omega) = \pi\left(\delta\left(\omega - \frac{4}{3}\pi\right) + \delta\left(\omega + \frac{4}{3}\pi\right)\right)$$

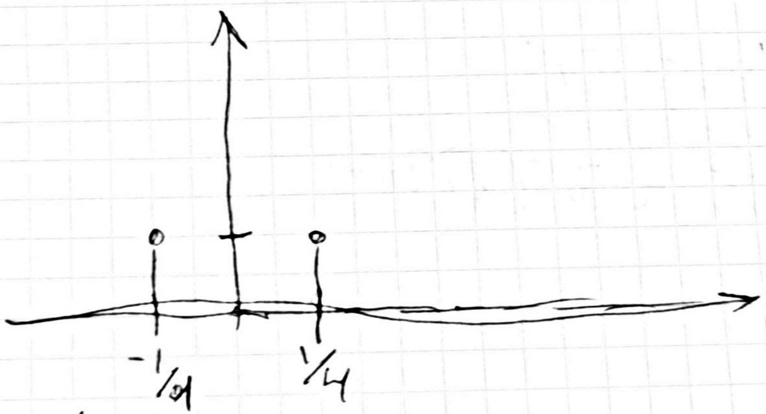
cos(x) periodic function gives $X_2 = \pi\left(\delta\left(\omega + \frac{2}{3}\pi\right) + \delta\left(\omega - \frac{2\pi}{3}\right)\right)$

~~mapping t [$-\frac{1}{2}, \frac{1}{2}$]~~ from

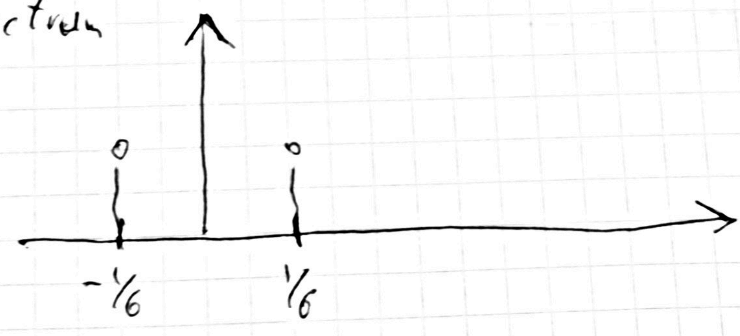
linear mapping: $[-\pi, \pi] \to [-\frac{1}{2}, \frac{1}{2}]$

$$\Rightarrow f(\omega) = \frac{\omega}{2\pi} = f$$

X_1 spectrum



X_2 spectrum



they sound different because of aliasing.
that is when we sample $x_2[n]$
the signal is too high frequent for
our sample rate, and it becomes
equivalent to a lower frequency signal.

$$Y[n] = u[n-M] - u[n+M]$$

$$\mathcal{F}\{u[n]\} = \frac{1}{1-e^{-iw}}$$

$$F\{x[n-k]\} = X_{2\pi}(w) e^{-iwk}$$

$$\frac{e^{-iwM}}{1-e^{-iw}} - \frac{e^{iwM}}{1-e^{-iw}} = \frac{-(e^{iwM}-e^{-iwM})}{(1-e^{-iw})}$$

$\leftarrow 2i \sin($

$$\frac{e^{-iwM}\, e^{-\frac{iw}{2}}}{(1-e^{-iw})e^{-\frac{iw}{2}}} - \frac{e^{iwM}\, e^{\frac{iw}{2}}}{(1-e^{-iw})e^{\frac{iw}{2}}}$$

$$\cancel{\frac{e^{-iw(M+\frac{1}{2})}}{e^{-\frac{iw}{2}}-e^{-i\frac{2w}{2}}}} - \cancel{\frac{e^{iw(M+\frac{1}{2})}}{e^{\frac{iw}{2}}-e^{-\frac{iw}{2}}}}$$

$$\frac{e^{-iwM}}{1-e^{-iw}} - \frac{e^{iwM}}{1-e^{-iw}}$$

$$\frac{e^{-iwM}\, e^{-\frac{iw}{2}}}{e^{-\frac{iw}{2}}-e^{-\frac{3iw}{2}}} - \frac{e^{iwM}\, e^{\frac{iw}{2}}}{e^{\frac{iw}{2}}-e^{-\frac{iw}{2}}}$$

$e^{-\frac{3iw}{2}}$, periodic $\Rightarrow e^{-\frac{3iw}{2}} = e^{\frac{iw}{2}}$

$$\frac{e^{-iw(M+\frac{1}{2})}}{e^{-\frac{iw}{2}}-e^{\frac{iw}{2}}} - \frac{e^{iw(M+\frac{1}{2})}}{e^{\frac{iw}{2}}-e^{-\frac{iw}{2}}}$$

$$\frac{-e^{-iw(M+\frac{1}{2})}}{e^{\frac{iw}{2}}-e^{-\frac{iw}{2}}} - \frac{e^{iw(M+\frac{1}{2})}}{e^{\frac{iw}{2}}-e^{-\frac{iw}{2}}}$$

$$\frac{e^{-iw(M+\frac{1}{2})}}{e^{-\frac{iw}{2}} - e^{\frac{iw}{2}}} - \frac{e^{iw(M+\frac{1}{2})}}{e^{\frac{iw}{2}} - e^{-\frac{iw}{2}}}$$

$$\frac{e^{-iw(M+\frac{1}{2})} - e^{iw(M+\frac{1}{2})}}{e^{-\frac{iw}{2}} - e^{\frac{iw}{2}}} = \frac{\sin(w(M+\frac{1}{2}))}{\sin(\frac{w}{2}}$$