digsig7

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1 Problem 1

1.1 a

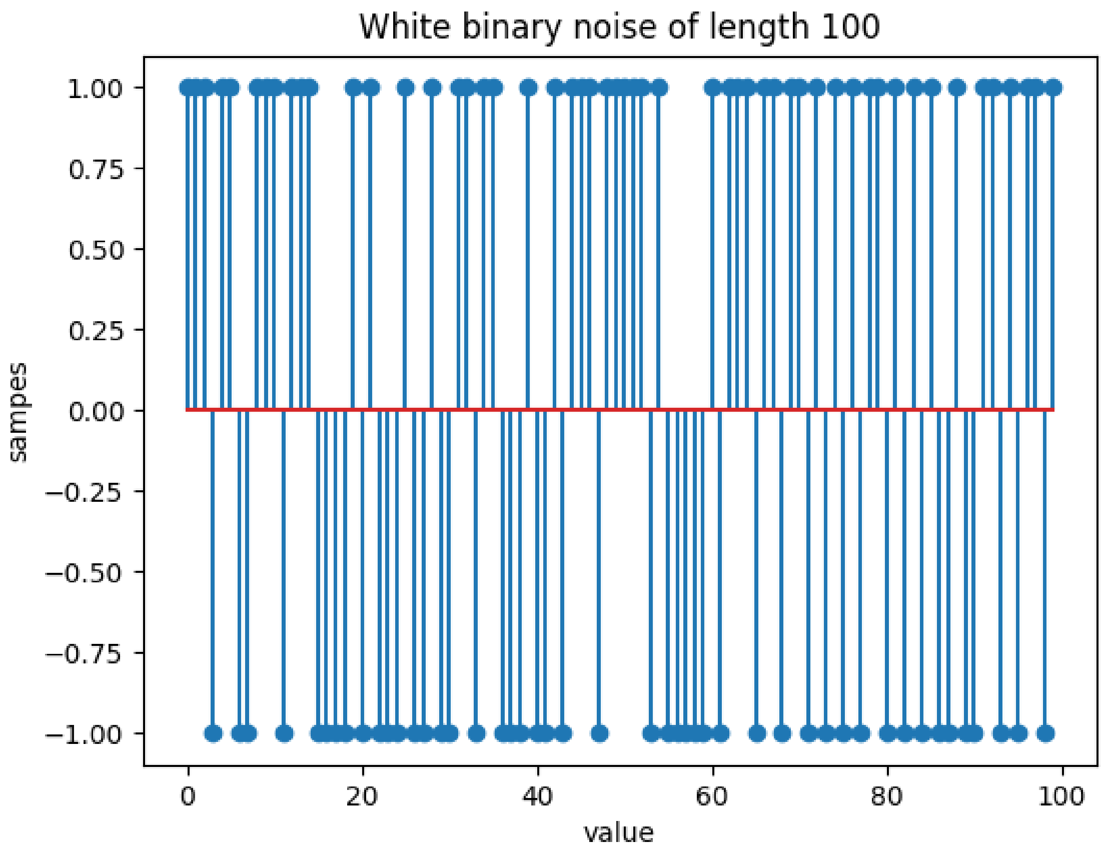

Figure 1: White binary noise

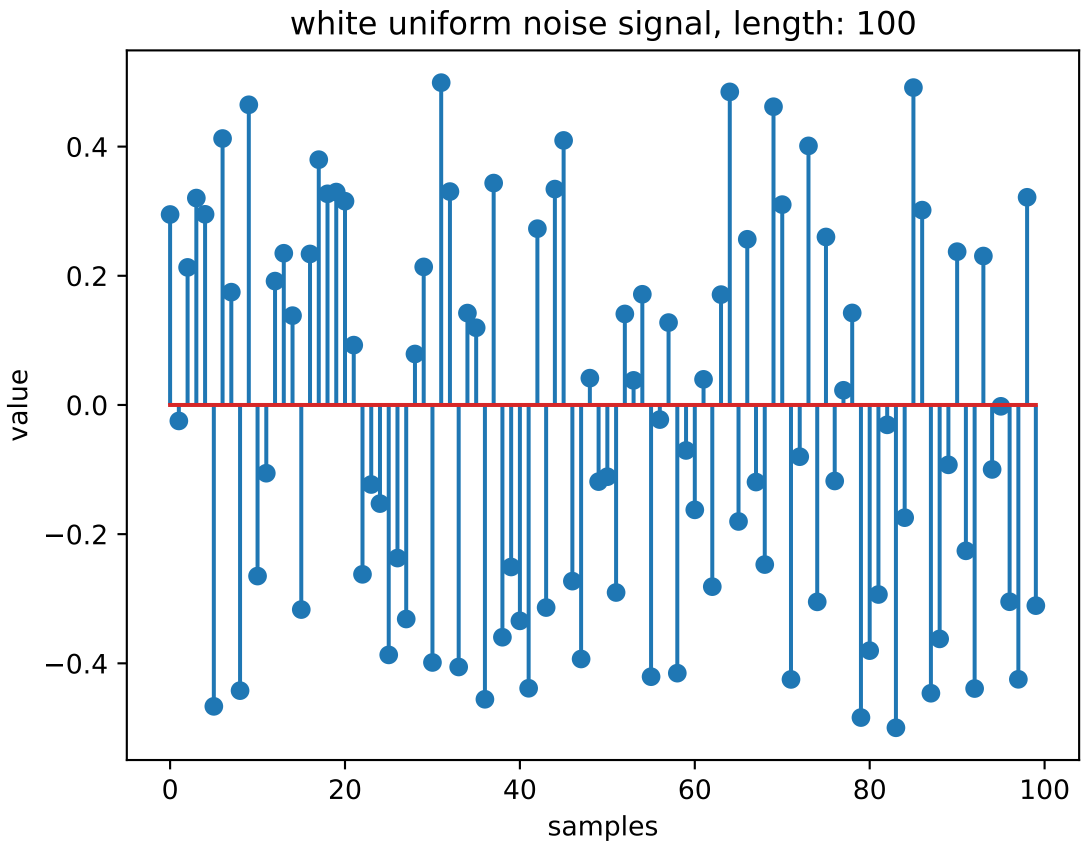

Figure 2: White uniform noise

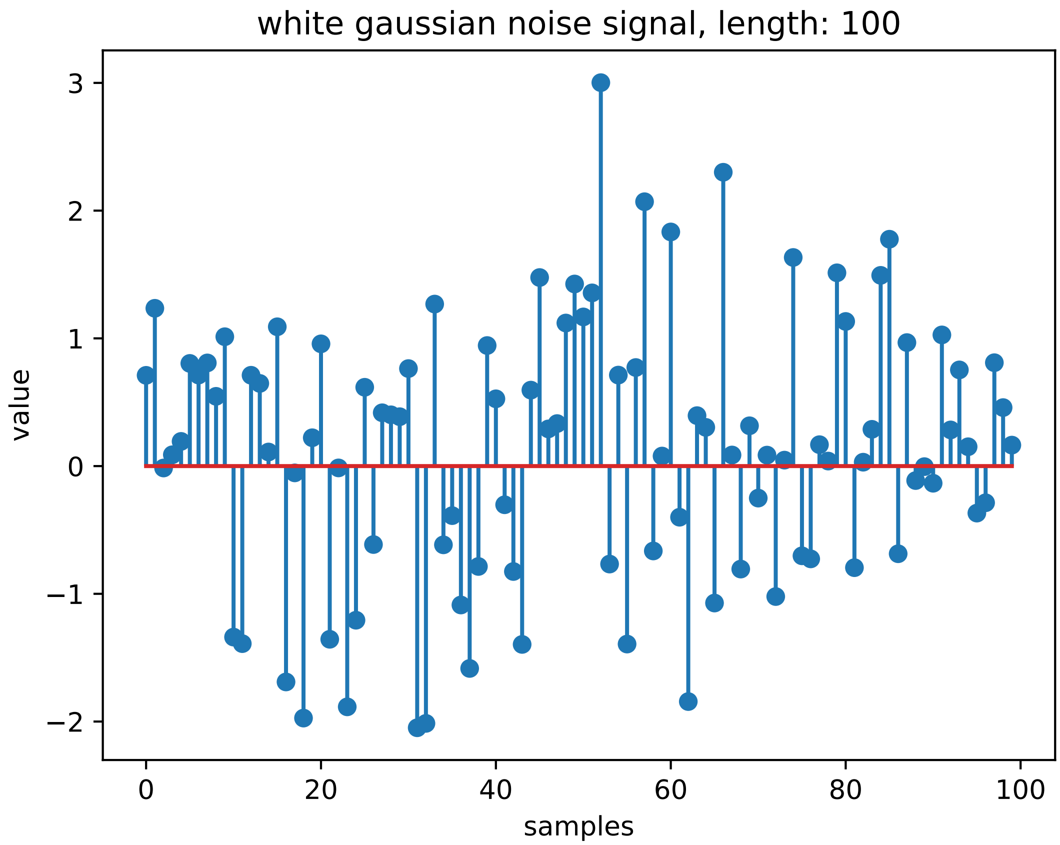

Figure 3: White gaussian noise

The noise signals are similar in that they are all white, ie have no freqency dependencies and are stationary.

They differ in that the gaussian signal preffers smaller values and ist technically unbounded, while the uniform one is bound to the interval $[-\sqrt{3}, \sqrt{3}]$. The binary nosise is the most restriced and can only take values of -1 or 1.

1.2 b

$$p_{WGN}(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \tag{1}$$

$$p_{WUN}(x) = \frac{rect(x/2)}{2} \tag{2}$$

$$p_{WBN}(x) = \frac{\delta(x-1}{2} + \frac{\delta(x+1}{2} \tag{3}$$

equations (1) gives the probability density functions for each distribution respectively.

Symmetry properties give that they all have mean 0. They are all white so their ACF must be given by $ACF(X_t, X_{t+\tau} = \sigma\delta(l) = \delta(l)$. this in turn gives PDS given by $\{\{\delta(l) =\} = 1$.

1.3 c

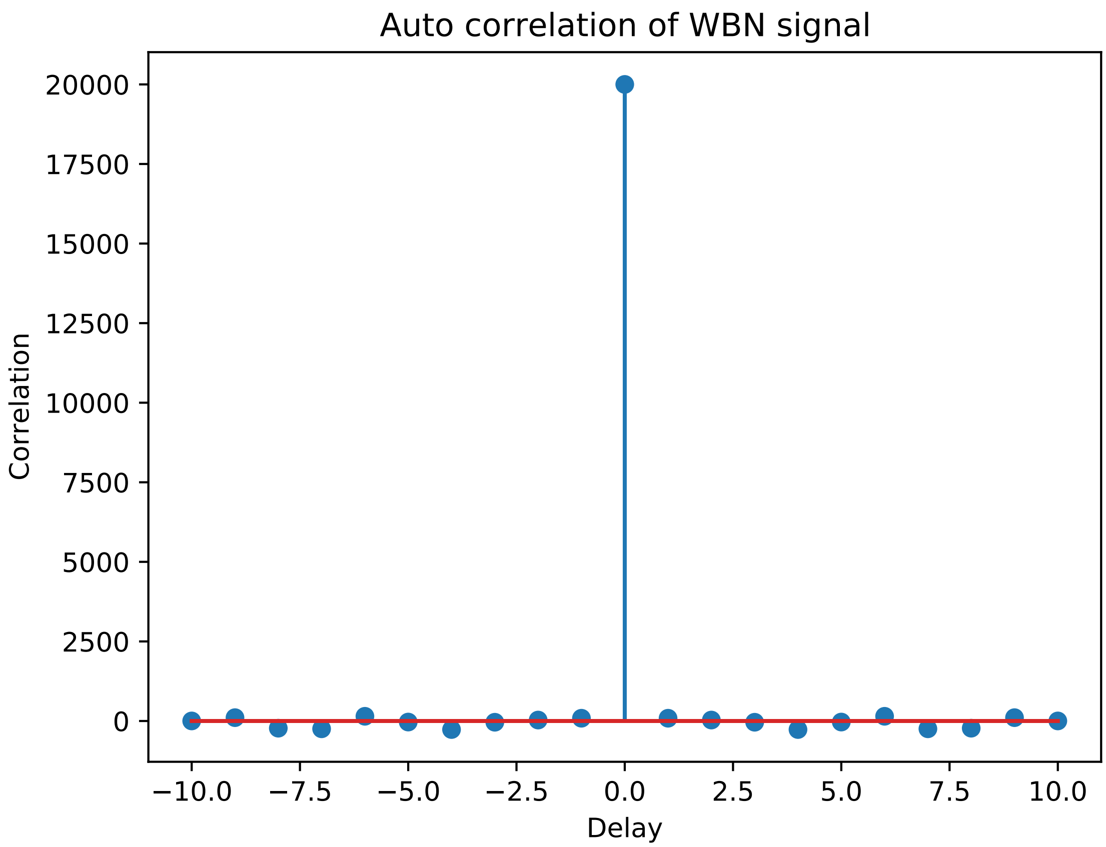

Figure 4: Autocorrelation estimate of white binary noise

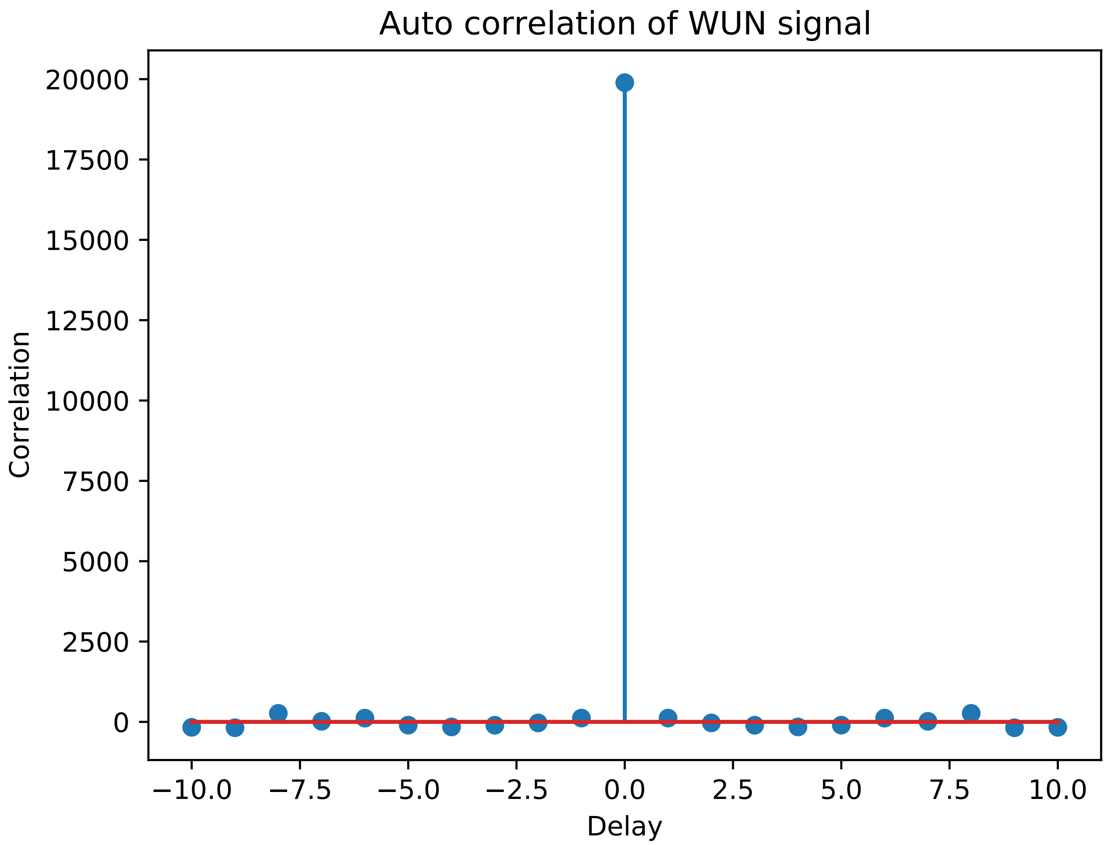

Figure 5: Autocorrelation estimate of white uniform noise

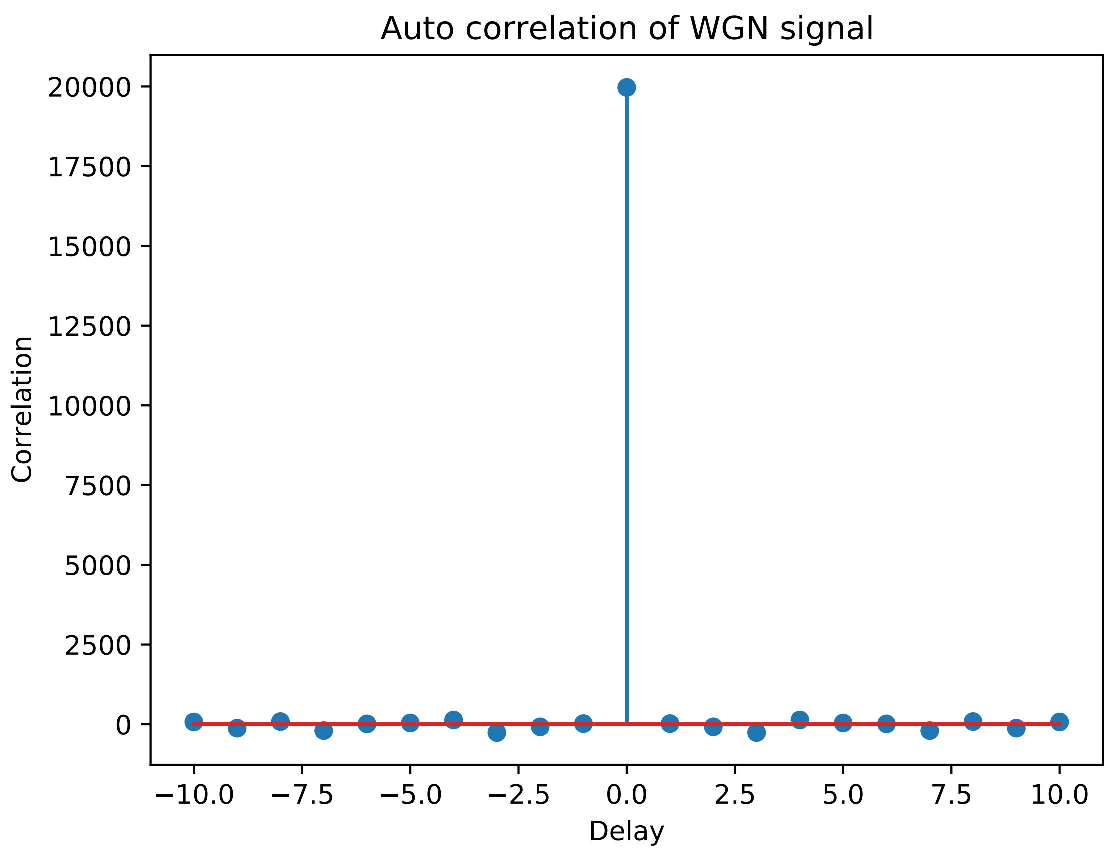

Figure 6: Autocorrelation estimate of white gaussian noise

2 Problem 2

2.1 a

Symmetry properties of normal distribution gives

$$mean_{W(x)} = 0 \tag{4}$$

White Gaussian noise is white, this implies autocorrelation,

$$\gamma_{ww} = \sigma^2 \delta(l) = \frac{3}{4}\delta(l). \tag{5}$$

the filter given by

$$H(z) = \frac{1}{1 + \frac{z^{-1}}{2}} \tag{6}$$

this gives a time-domain impulse response

$$h(t) = \left(-\frac{1}{2}\right)^n u(n) \tag{7}$$

We can then calculate the autocorrelation of the filter for $l > 0$.

$$\begin{aligned}
\gamma_{hh}(l) &= \sum_{n=-\infty}^{\infty} \left(-\frac{1}{2}\right)^n u(n) \left(-\frac{1}{2}\right)^{n+l} u(n+l) && (8)\\
&= \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^{2n+l} && (9)\\
&= \left(-\frac{1}{2}\right)^l \sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n && (10)\\
&= \left(-\frac{1}{2}\right)^l \frac{4}{3} \text{ for } l > 0 && (11)
\end{aligned}$$

$$\text{Autocorrelation is even} \implies \gamma_{hh} \left(-\frac{1}{2}\right)^{|l|} \frac{4}{3} \tag{12}$$

These values give

$$E(y) = E(W(x)) * h(t) = 0 * h(t) = 0 \tag{13}$$

$$\gamma_{yy} = \gamma_{hh} * \gamma_{ww} = \sigma^2 \delta(l) * \frac{4}{3} \left(-\frac{1}{2}\right)^{|l|} = \left(-\frac{1}{2}\right)^{|l|} \tag{14}$$

We can then calculate the Power density spectrum. Given by

$$\begin{aligned}
\Gamma_{yy} = |H(w)|^2 \gamma_{yy} &= \frac{1}{1 + \frac{e^{-iw}}{2}} \frac{1}{1 + \frac{e^{iw}}{2}} \sigma^2 && (15)\\
&= \frac{3}{4\left(\frac{5}{4} - \cos(\omega)\right)} = \frac{3}{5 - 4\cos(\omega)} && (16)\\
& && (17)
\end{aligned}$$

and finally the power given by

$$\int_{-0.5}^{0.5} \Gamma_{yy}(\omega) d\omega = \int_{-\pi}^{\pi} \frac{3}{5 - 4\cos(\omega)} d\omega ...(\text{Se eget skriv} = 1 \tag{18}$$

2.2 b

assuming a signal x[n] of length N

$$\text{mean} \tag{19}$$

$$\frac{\sum_0^{N-1} x[n]}{N} = \mu_x \tag{20}$$

$$\text{autocorrelation} \tag{21}$$

$$\sum_{n=-\infty}^{\infty} x[n] * x[n+l] = \gamma_{xx} \tag{22}$$

$$\text{power density} \tag{23}$$

$$\mathcal{F}\{\gamma_{xx}\}[k] = \sum_{n=-\infty}^{\infty} \gamma_{yy} e^{-iwkn/N} \tag{24}$$

$$\text{power density} \tag{25}$$

$$\mathcal{F}\{\gamma_{xx}\}[k] = \sum_{n=-\infty}^{\infty} \gamma_{yy} e^{-iwkn/N} \tag{26}$$

$$\text{power} \tag{27}$$

$$\gamma_{xx}[0] \tag{28}$$

2.3 c

Power estimate: 1.00249. mean estimate: 0.00322

2.4 d

The Bartlett estimate is much better. The original naive approach seems to blow up to extreme amplituedes. The 10 bin version seems to fit best, altough selecting bin count seems to be a problem for cross-validation to find an optimal count.

2.5 e

The estimates dont seem to vary very much with each realization. probably due to the signal length being quite long.

3 Problem 3

mean estimate K=20: 0.000934. variance estimate K=20: 0.0079.

mean estimate K=40: -0.003769. variance estimate K=20: 0.00333.

mean estimate K=100: 0.000346. variance estimate K=20: 0.00129. The variance decreases with larger bins so it seems to be inline with theoretical expectations.

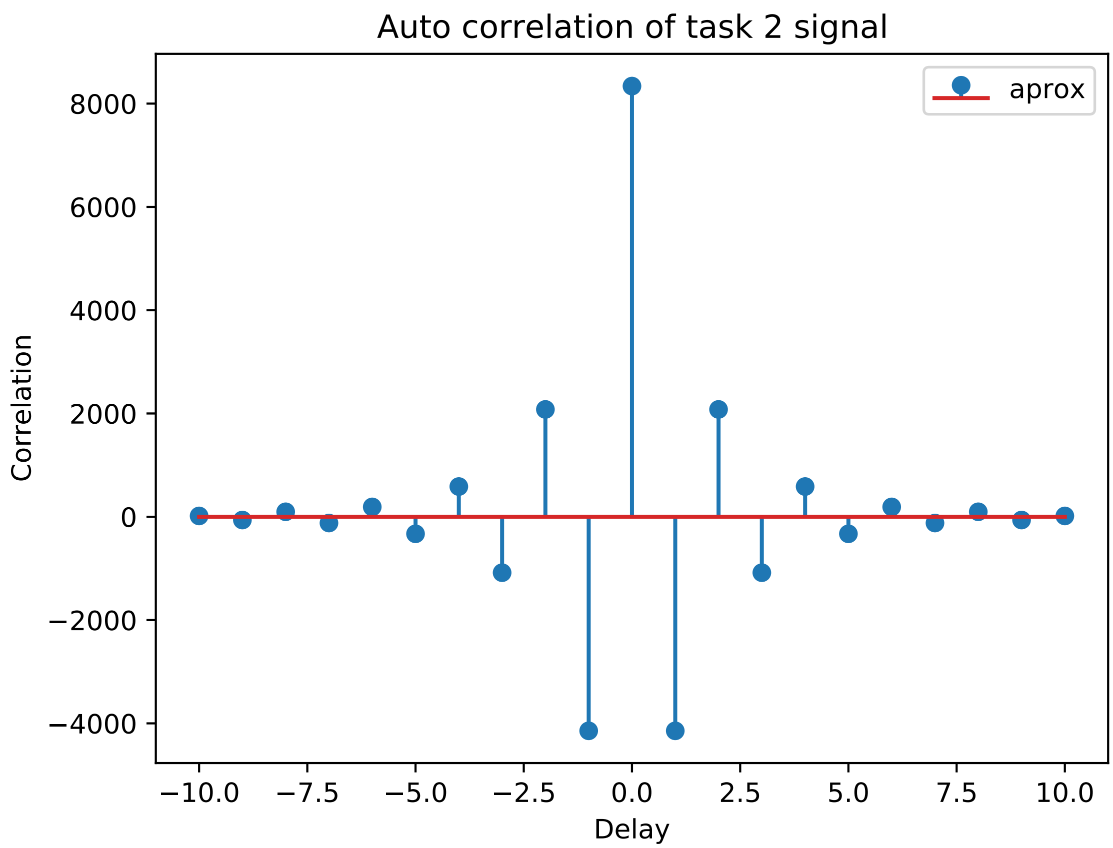

Figure 7: Estimated ACF

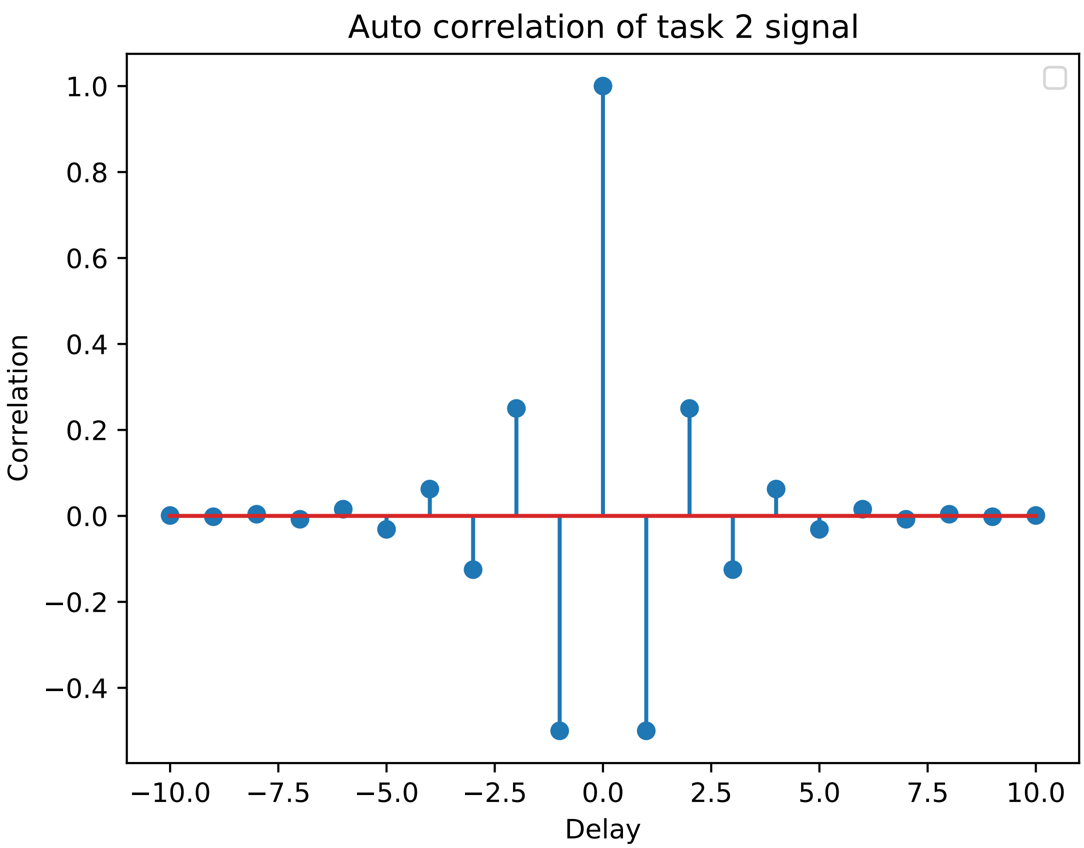

Figure 8: exact ACF

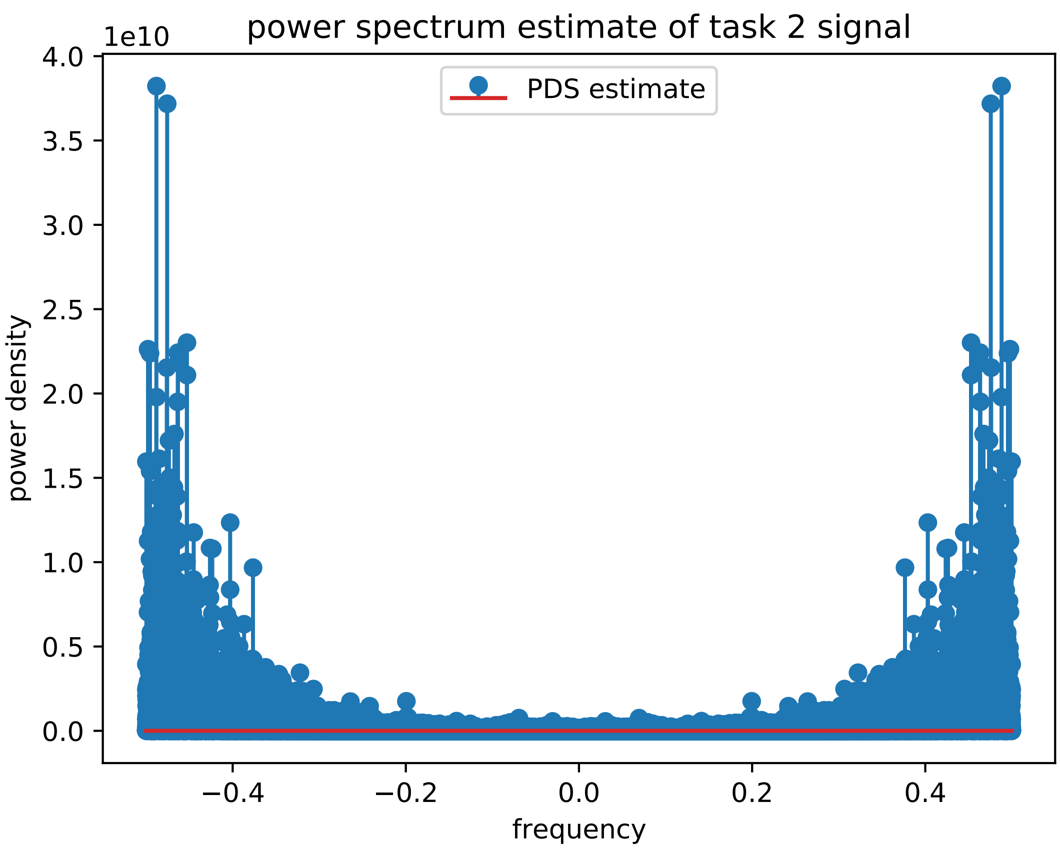

Figure 9: Estimated PDS

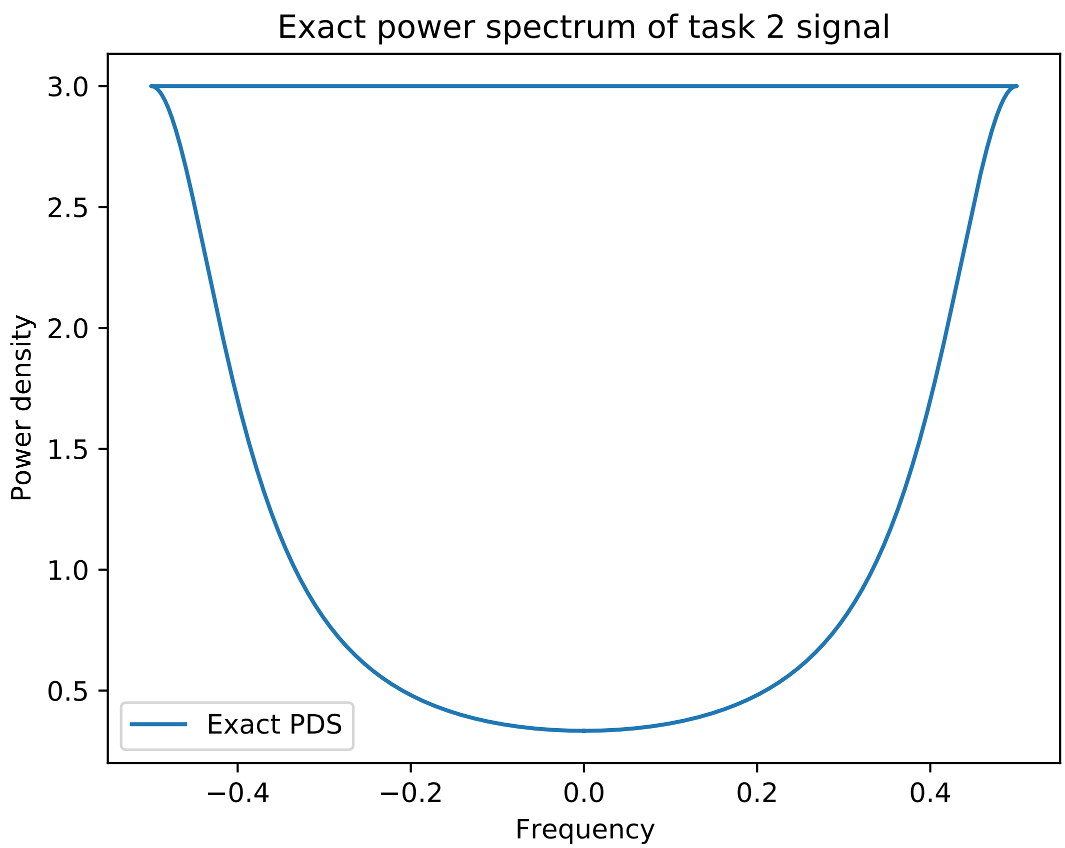

Figure 10: Exact PDS

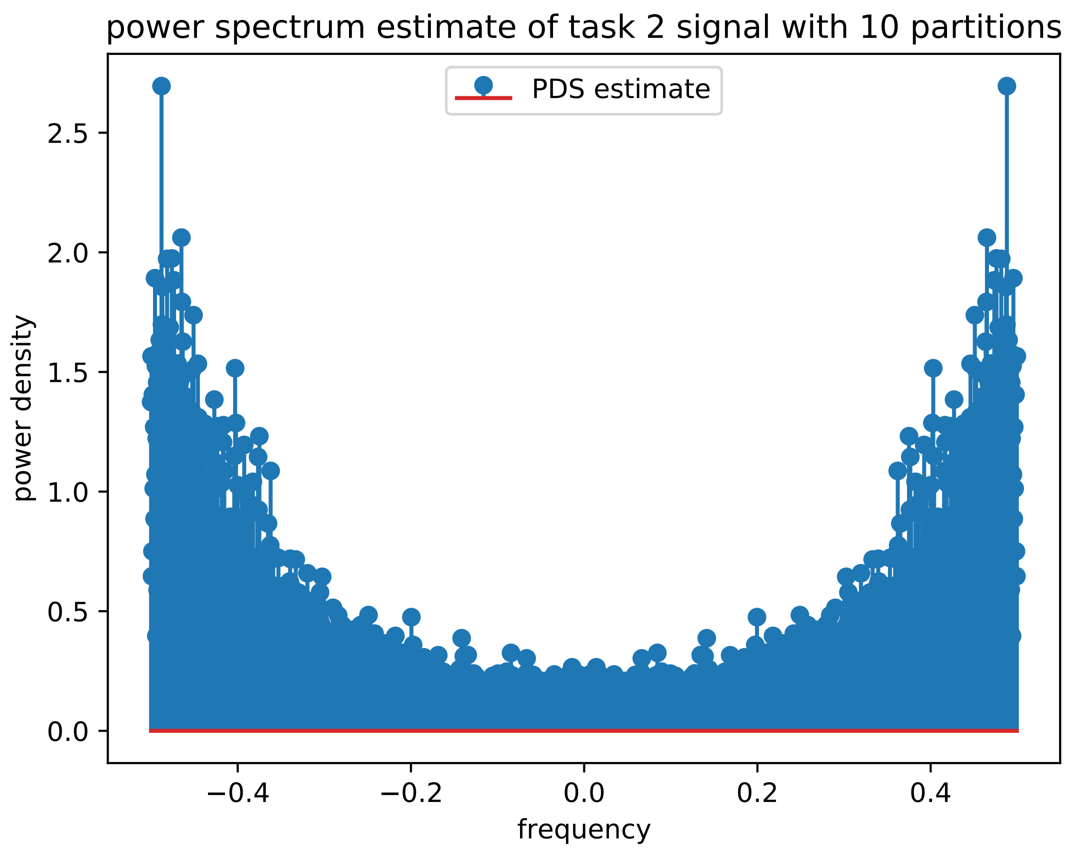

Figure 11: Bartlett PDS estimate 10 bins

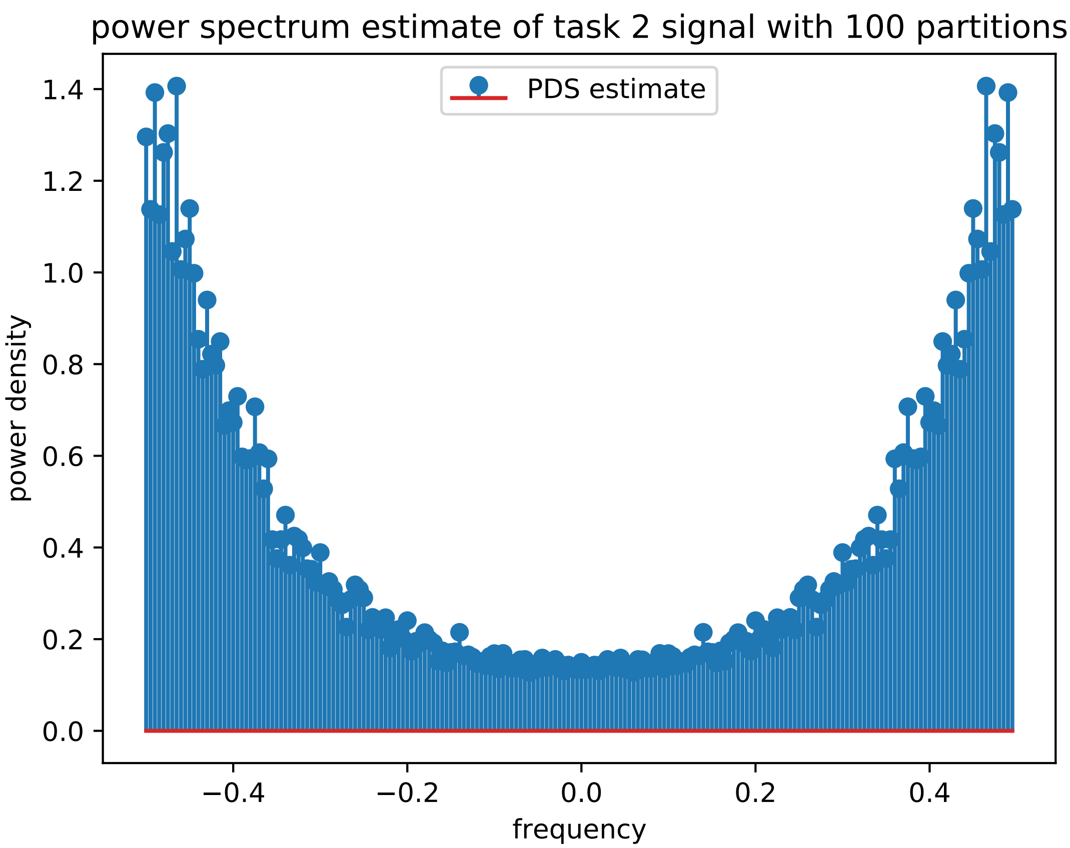

Figure 12: Bartlett PDS estimate 100 bins

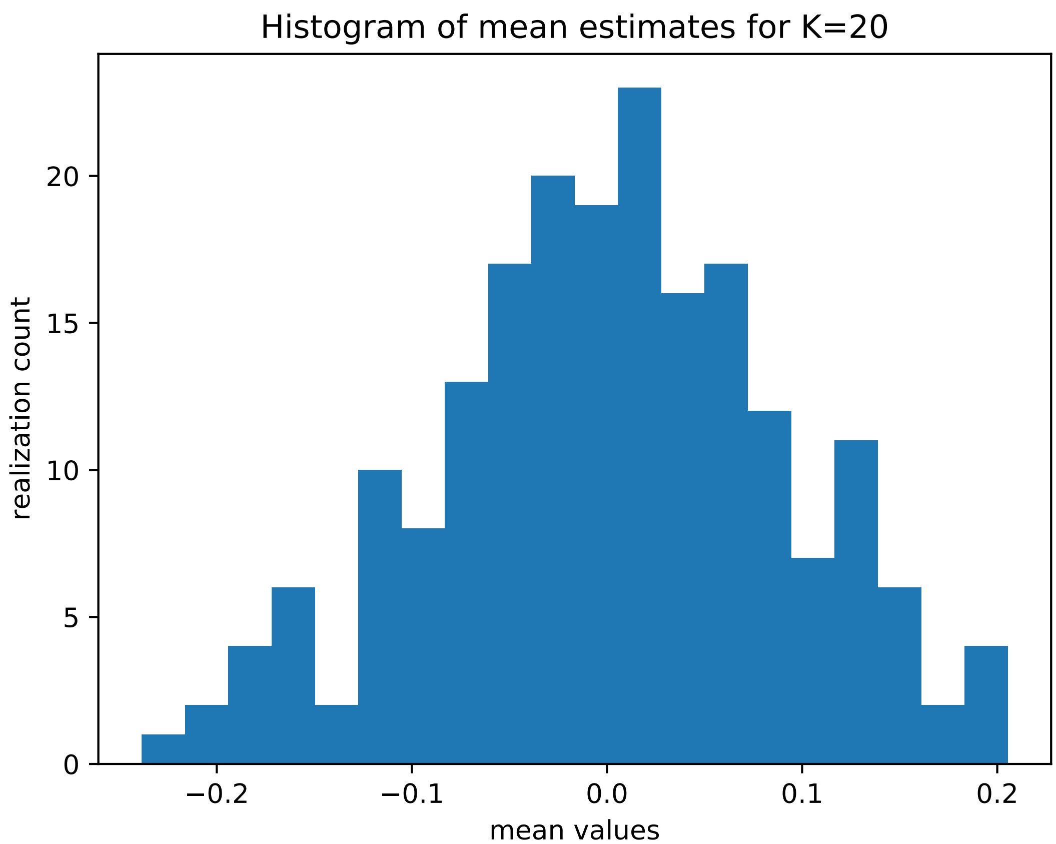

Figure 13: Mean histogram for K=20

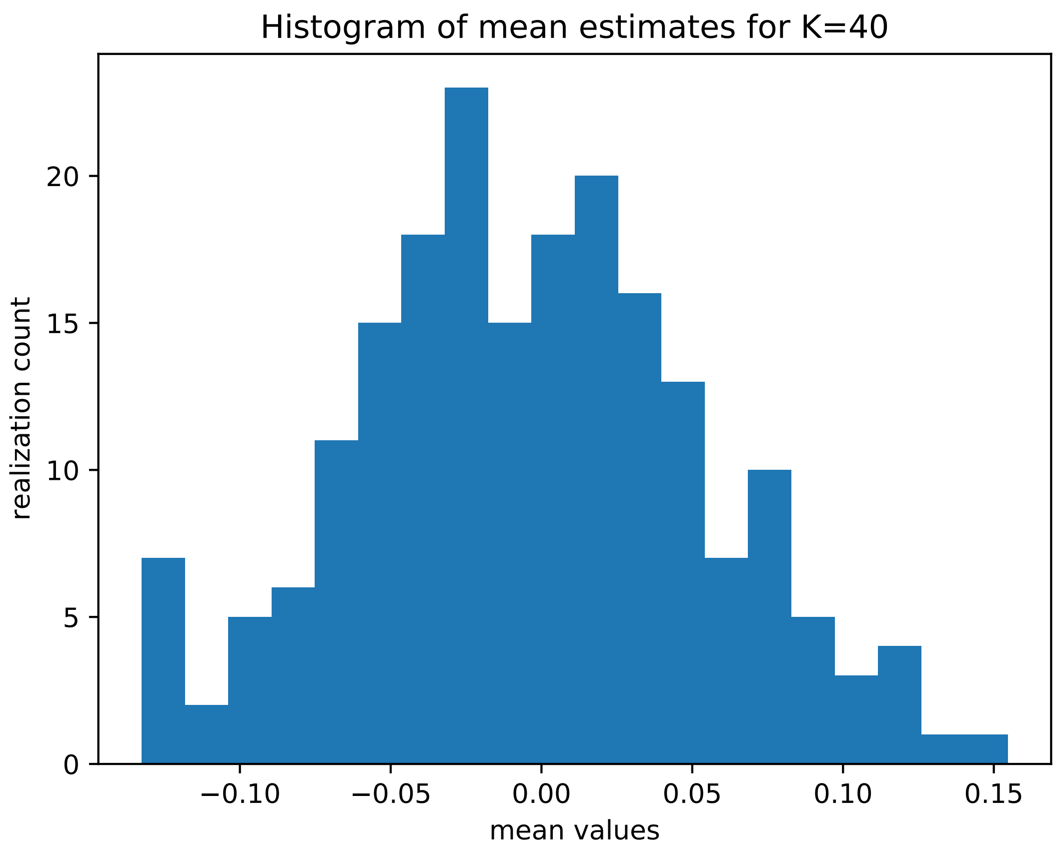

Figure 14: Mean histogram for K=40

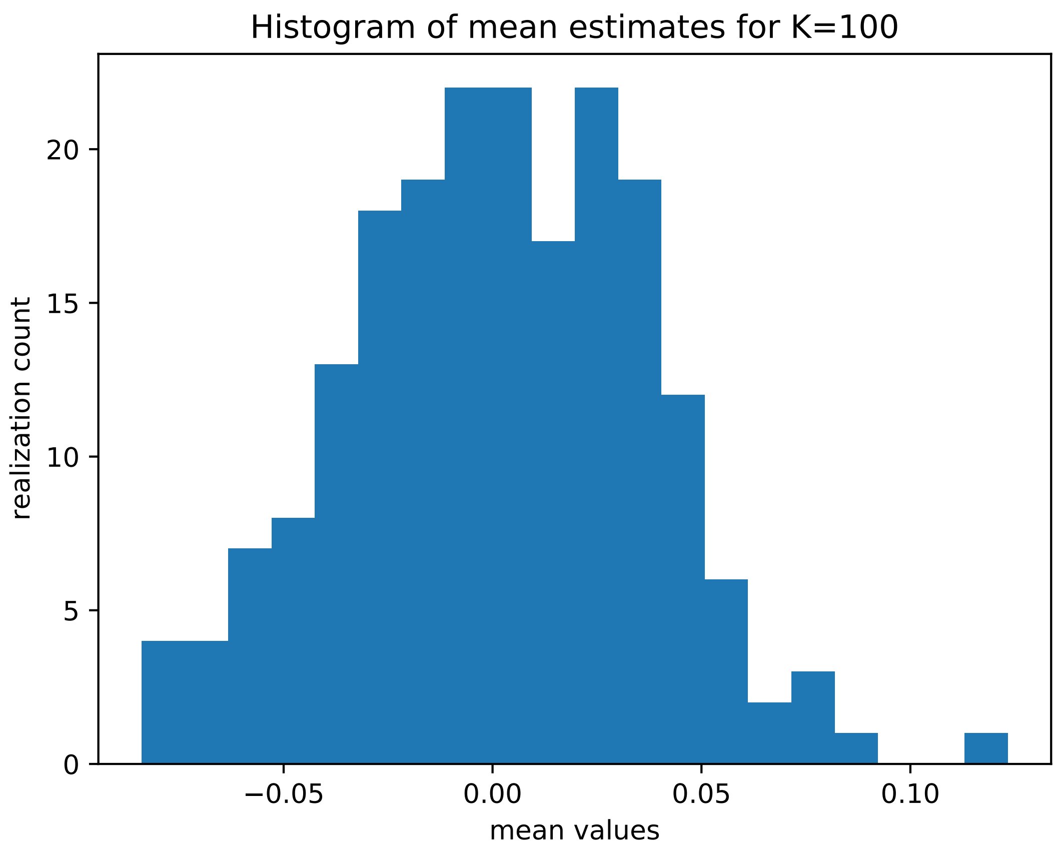

Figure 15: Mean histogram for K=100