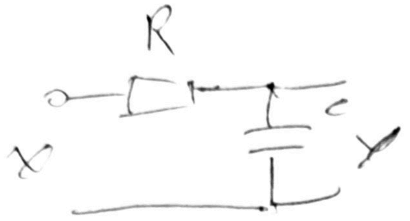
P1 ~~dig sig~~ 3

a) 

$$V = R\,I(t) + \frac{\int_{t_0}^{t} I(\tau)\,d\tau}{C} \Rightarrow R\,\dot{I}(t) + \frac{I(t)}{C} - I_0 = 0$$

$$V = RI$$

$$\dot{V} + \frac{V - V_0}{RC} = 0$$

$$H(s) = \frac{Z_2}{Z_1 + Z_2}$$

$$Z_2 = \frac{1}{sC} \qquad Z_1 = R$$

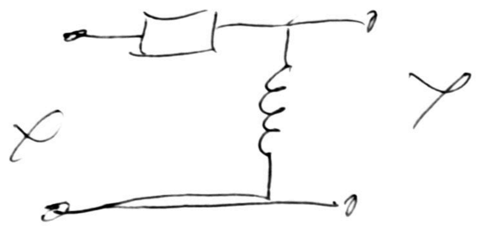
$$\frac{\frac{1}{sC} \cdot sC}{R + \frac{1}{sC} \cdot sC} = \underline{\frac{1}{1 + RsC}}$$

$$V = V_0 - \dot{V} RC$$

$$\underline{y = x(t) - \frac{\partial y}{\partial t} RC}$$

$\cancel{V = RI + L\frac{dI}{dt}}$

$\cancel{V_I}$



$$H(s) = \frac{Z_2}{Z_1 + Z_2}$$

$$X = RI + L\,dI$$

$$Z_1 = R$$

$$Z_2 = sL$$

$$V = RI + L\dot{I}$$

$$H(s) = \frac{sL}{R + sL} \frac{\frac{1}{L}}{\frac{1}{L}}$$

$$\cancel{V_s = V_e + \frac{L}{R}\frac{dV_e}{dt}}$$

$$\frac{s}{\frac{R}{L} + s}$$

b) capacitor circuit is lowpass

inductor ~~c~~ circuit is high pass.

c)

$$h(t)_1 = \mathcal{L}^{-1}\left\{\frac{1}{1 + RCs}\right\} = e^{-\frac{t}{RC}}$$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{s}{\frac{R}{L} + s}\right\} = \frac{-R}{L} e^{-\frac{Rt}{L}}$$

P2) a) $H(z) = \dfrac{1}{1-\frac{2}{3}z^{-1}}$ causal

$h[n] = \left(\frac{2}{3}\right)^n u[n]$ $\quad a = \frac{2}{3} \Rightarrow ROC: |z| > |a|$

$= |z| > \left|\frac{2}{3}\right|$

b)

$H(z) = \dfrac{1}{\ldots} = \dfrac{A}{\ldots} + \dfrac{B}{\ldots}$

b) find ROC of

$$H(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-z^{-1})} = 1 + \frac{(z+1)}{2z^2 - z - 1}$$

$$\cancel{H = 1 + \frac{A}{2z^2 \; 2z^2 - 1} + \frac{B}{2z^2}}$$

$$2z^2 - z - 1 = \Rightarrow z_0 = \frac{-(-1) \pm \sqrt{1 - 4 \cdot 2 \cdot (-1)}}{2} = \frac{1 \pm 3}{2} = 1,\ -2$$

$$H(z) = 1 + \frac{\cancel{A}(z+1)\frac{1}{2}}{(z-1)(z+\frac{1}{2})} = 1 + \frac{z+1}{(z-1)(2z+1)}$$

Partial fractions.

$$\frac{A}{z-1} + \frac{B}{2z+1} = \frac{z+1}{(z-1)(2z+1)}$$

$$A(2z+1) + B(z-1) = z+1$$

$z = \frac{-1}{2}$

$z = 1 \Rightarrow 3A = 2$

$A = \frac{2}{3}$

$$\underbrace{A\left(\frac{2 \cdot 1}{-2} + 1\right)}_{0} + B\left(-\frac{1}{2} - 1\right) = \frac{1}{2}$$

$$-\frac{3B}{2} = \frac{1}{2} \qquad B = \frac{-1}{3}$$

$$H(z) = 1 + \frac{z}{3(z-1)} - \frac{1}{3\frac{(2z+1)}{2} \cdot 2}$$

$$Z^{-1}\{H(z)\} = \delta[n] + \frac{2}{3} Z^{-1}\left\{\frac{1}{z-1}\right\} - \frac{1}{3 \cdot 2} Z^{-1}\left\{\frac{1}{z+\frac{1}{2}}\right\}$$

$$\boxed{\begin{array}{l} Z^{-1}\left\{\frac{z}{z \pm a}\right\} = a^n u[n] \\ Z^{-1}\{x[n-k]\} = X(z) z^{-k} \end{array}}$$

$$Z^{-1}\{H(z)\} = \delta[n] + \frac{2}{3} \cancel{a^{n-1}} u[n-1] - \frac{1}{6}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$u[n-1]$ ROC: $|z| > 1$

$\left(\frac{1}{2}\right)^{n-1}$ ROC: $|z| > \frac{1}{2}$

$|z| > 1 \Rightarrow |z| > \frac{1}{2}$

ROC: $\underline{|z| > 1}$

b) anticausal

$$H(z) = \frac{z^{-1}}{\left(1+\frac{3}{2}z^{-1}\right)\left(1-3z^{-1}\right)} \frac{z^2}{z^2}$$

$$H(z) = \frac{z}{\left(z+\frac{3}{2}\right)\left(z-3\right)}$$

<u>Partial fractions:</u>

$$z = A(z-3) + B\left(z+\tfrac{3}{2}\right)$$

$$z = 3, \quad 3 = B\left(3+\tfrac{3}{2}\right) \quad 3 = \tfrac{9}{2}B, \quad \underline{B = \tfrac{2}{3}}$$

$$z = -\tfrac{3}{2} \quad -\tfrac{3}{2} = A\left(-\tfrac{3}{2} - 3\right)$$

$$-\tfrac{3}{2} = A - \tfrac{9}{2} \quad = \tfrac{1}{3}$$

$$H(z) = \frac{1}{3}\frac{1}{\left(z+\frac{3}{2}\right)} + \frac{2}{3}\frac{1}{(z-3)} \Rightarrow \cancel{z^{-1}\{H(z)\} = \frac{1}{3}\left(\frac{3}{2}\right)^{n-1}u[n] + \frac{2}{3}(-3)^{n-1}u[n]}$$

$$\cancel{\tfrac{1}{3}\left(-\tfrac{3}{2}\right)^{n-1}}$$

ROC: $|z| < \left|\frac{3}{2}\right|$

$$|z| < |3| \quad |z| < \left|\tfrac{3}{2}\right| \Rightarrow |z| < |3|$$

ROC: $\underline{\underline{|z| < \tfrac{3}{2}}}$

2a

b)

$$H(z) = \frac{1}{3}\frac{1}{z+\frac{3}{2}} + \frac{2}{3}\frac{1}{(z-3)}$$

$$Z^{-1}\{H(z)\} = \frac{-1}{3}\left(\frac{3}{2}\right)^{n-1} u[-n] + \frac{-2}{3}\, 3^{n-1} u[-n]$$

$$Z^{-1}\left\{\frac{z}{z-a}\right\} \overset{\text{anticausal}}{=} -a^n u[-n-1] = X[n]$$

$$Z\{X[n-k]\} = z^{-k} X(z)$$

c) all the causal functions are stable
the anticausal one is unstable
and will amplify the input indefinitely

P3)

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$\Updownarrow$

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$x[n] = u[n-2]$$

$$Z\{h[n]\} = \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}$$

$$Z\{x[n]\} = \frac{z^{-2}}{1-z^{-1}} = \frac{z^{-1}}{z-1}$$

b) output

$$y[n] = (h * x)[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

$$= \sum_{m=-\infty}^{\infty} \left(\frac{1}{2}\right)^m u[m]\, u[n-2-m]$$

$$\sum_{m=0}^{\infty} \left(\frac{1}{2}\right)^m u[n-2-m]$$

$y[0] = 0$ $\quad y[3] = 1 + \frac{1}{2}$

$y[1] = 0$ $\quad y[4] = 1 + \frac{1}{2} + \frac{1}{4}$

$y[2] = 1$

$$\underline{y[n] = \sum_{n=0}^{n-2} \frac{1}{2^n}}$$

c)

$\cancel{Y[z]}$

$Y(z) = H(z)\,X(z).$

$= \frac{z}{(z-1/2)}\frac{z^{-1}}{(z-1)} = \frac{1}{z^2 - z - \frac{z}{2} + \frac{1}{2}} = \frac{1}{(z-\frac{1}{2})(z-1)}$

<u>partial fraction:</u>

$\frac{A}{(z-\frac{1}{2})} + \frac{B}{(z-1)} = \frac{1}{(z-\frac{1}{2})(z-1)}$

$A(z-1) + B(z-\frac{1}{2}) = 1$

$z = 1 \quad \frac{B}{2} = 1$

$B = 2$

$\cancel{A}\ z = \frac{1}{2}$

$\cancel{B}\quad -\frac{A}{2} = 1$

$A = -2$

$$Z^{-1}\left\{\frac{-2}{z-\frac{1}{2}} + \frac{2}{z-1}\right\} = -2\left(\frac{1}{2}\right)^{n-1} u[n-1] + 2\,u[n-1]$$

$$y[n] = 2u[n-1] - \frac{1}{2}^{-1}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$$\underline{\underline{2\left(1 - \left(\frac{1}{2}\right)^{n-1}\right) u[n-1]}}$$

P4 $y[n] = x[n] - x[n-2] - \frac{1}{4} y[n-2]$

Z-transform

$$Y = X - z^{-2}X - \frac{1}{4}z^{-2}Y$$

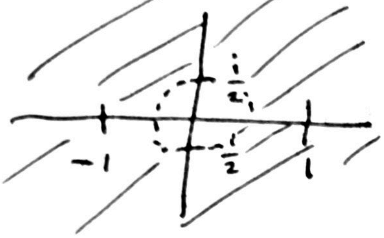
$$1 + \frac{1}{4}z^{-2}Y = 1 - z^{-2}X$$

$$\frac{Y}{X} = \frac{1 - z^{-2}}{1 + \frac{1}{4}z^{-2}} = H(z)$$

$$\frac{z^2 - 1}{z^2 + \frac{1}{4}} \Rightarrow \frac{(z-1)(z+1)}{(z - \frac{i}{2})(z + \frac{i}{2})}$$

zeros : $z = \pm 1$

Poles : $z = \pm \frac{i}{2}$



ROC

BIBO stable,

unit circle $\in$ ROC

d) ~~band pass stop~~

Band pass