Digsig4

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1 Problem 1

1.1 a)

the $a = 0,9$ variant is a lowpass filter. the $a = -0.9$ variant is a highpass filter. this is can be seen because the low frequencies start at $\omega = 0$ and the high frequencies are at $\omega = \pi$. We only need to consider the upper half of the unit circle. it is then easy to see that the amplitude is highest near the poles and the frequency response becomes appearent.

1.2 b)

2 problem 2

2.1 a

The insverse filter$G(z)$ is thefilter such that $H(z) \cdot G(z) = 1$

$$G(z) = 1/H(z) = (1 - 1/2z^{-1})(1 + 1/2z^{-1}) \tag{1}$$

2.2 b

The filter contains no poles and as such is stable.

2.3 c

Yes, the definition of a minimum phase filter is that both the original and its inverse are causal and stable. Since both filters are stable. and the system is causal it must be minimum phase.

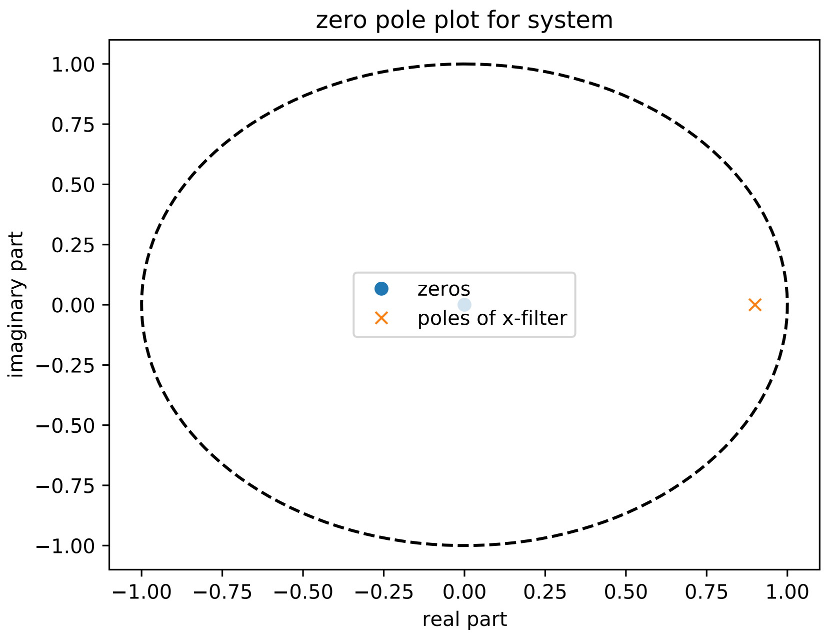

Figure 1: Zero pole plot for system in task $1a$ with $a = 0.9$

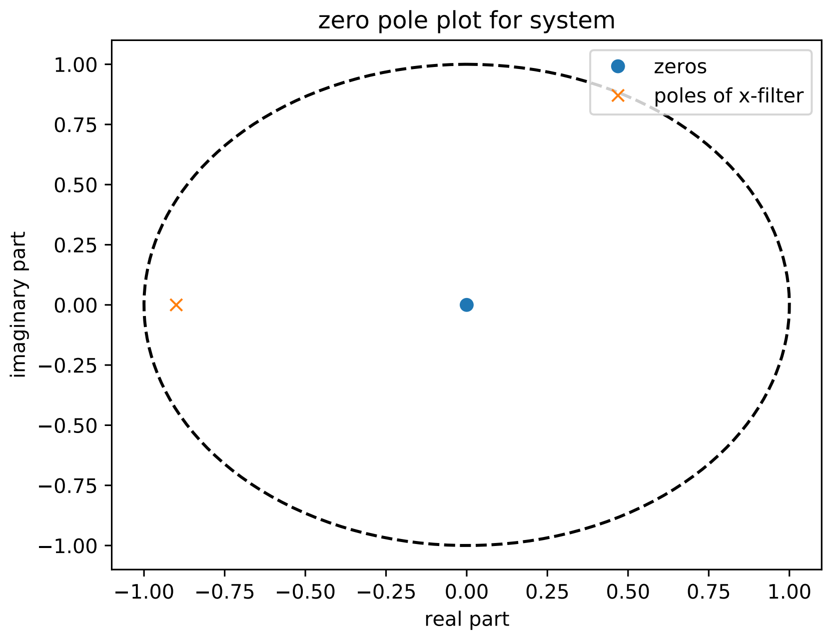

Figure 2: Zero pole plot for system in task 1a with $a = -0.9$

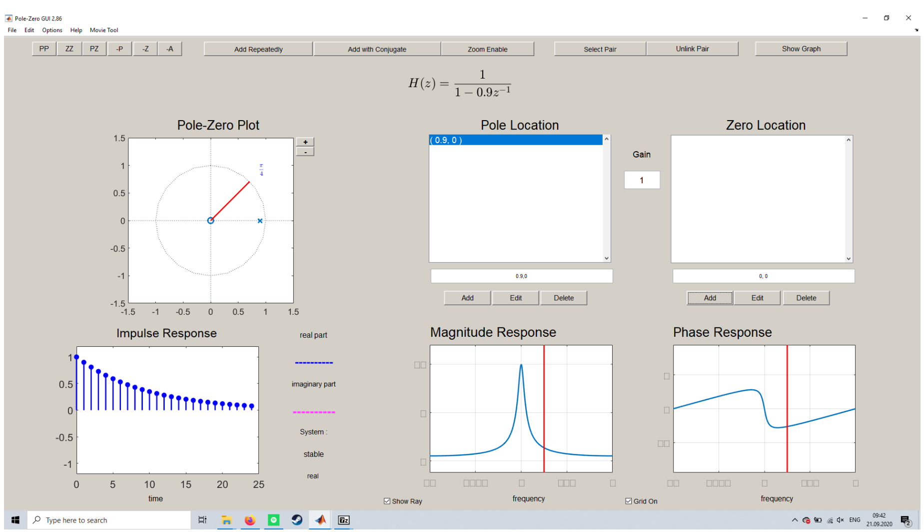

Figure 3: Pezdemo demonstration of system with $a = 0.9$

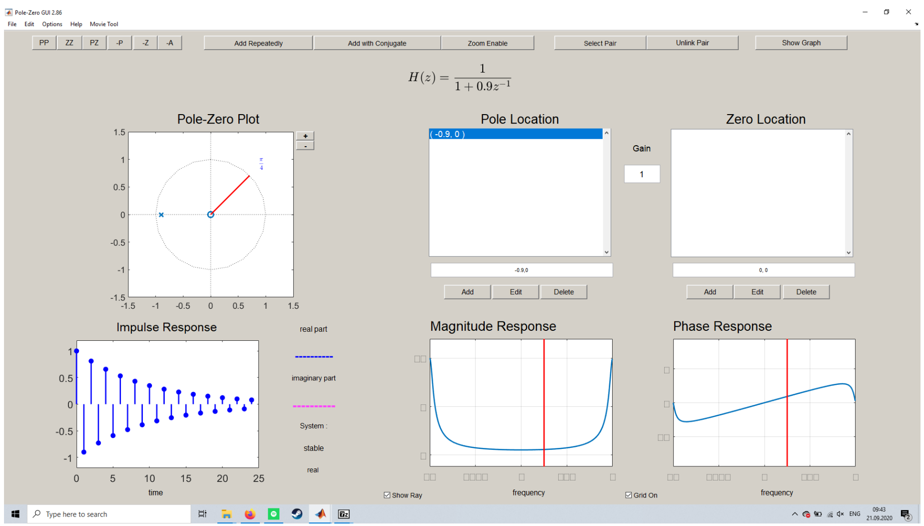

Figure 4: pezdemo demonstration of system with $a = -0.9$

2.4 d

phase response of $G(z)$:

$$|G(e^{-j\omega})| = |(1 - 1/2e^{-j\omega})||(1 + 1/2e^{-j\omega}| \tag{2}$$

$$1 - e^{-2j\omega}/4 \tag{3}$$

$$\text{impossible to write as} A(\omega)e^{i}\phi \text{with realvalued} A(\omega) \tag{4}$$

3 problem 3

3.1 a

It is a lowpass filter for $a < 1$, allpass for $a = 1$ and highpass for $a > 1$.

3.2 b

3.3 c

a controls the attenuation of the system. K seems to control the system mode. leting its highpass lowpass or allpass characteristics dominate.

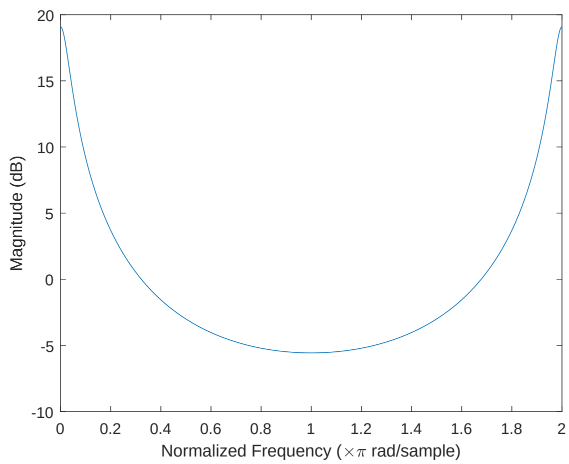

Figure 5: bottom side system frequency response

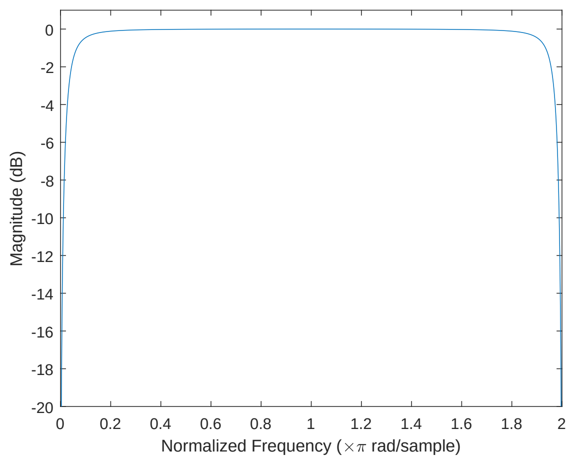

Figure 6: top side system frequency response

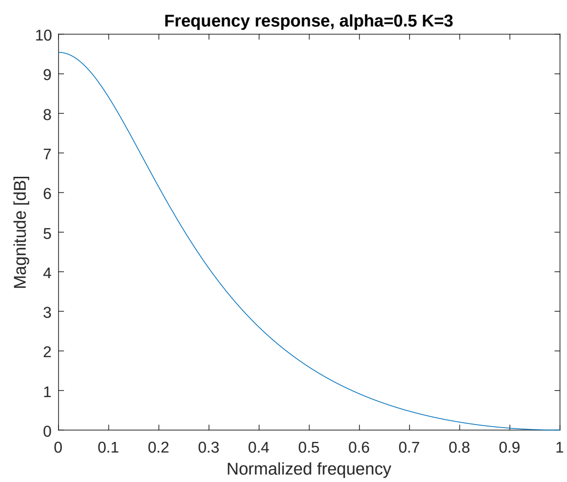

Figure 7: Pluto signal with $K = 3$ and $a = 0.5$. We see the system is a lowpass system

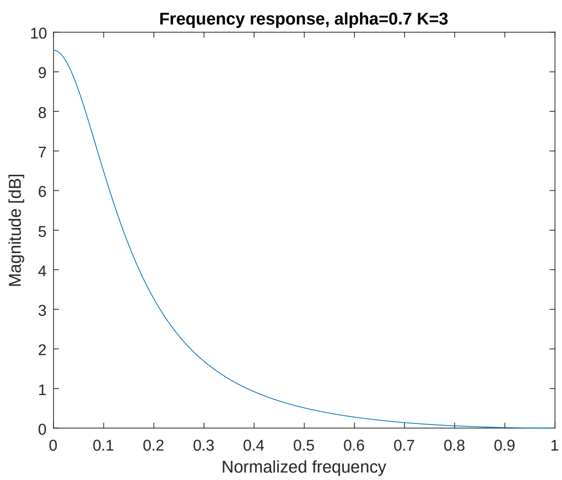

Figure 8: Pluto signal with $K = 3$ and $a = 0.7$. We see the system is still a lowpass system, but with stronger attenuation of higher frequencies

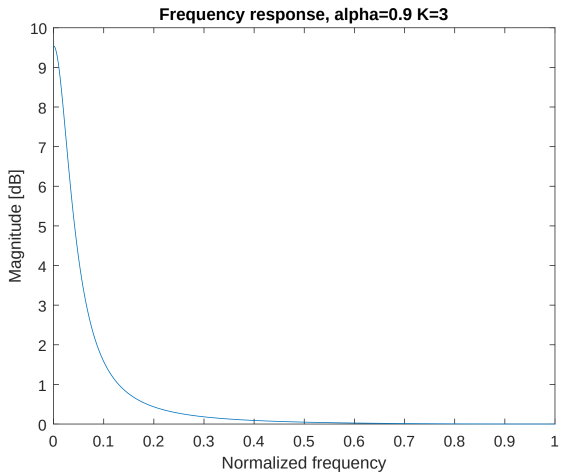

Figure 9: Pluto signal with $K = 3$ and $a = 0.9$. We see the system is still a lowpass system, but with the strongest attenuation of higher frequencies

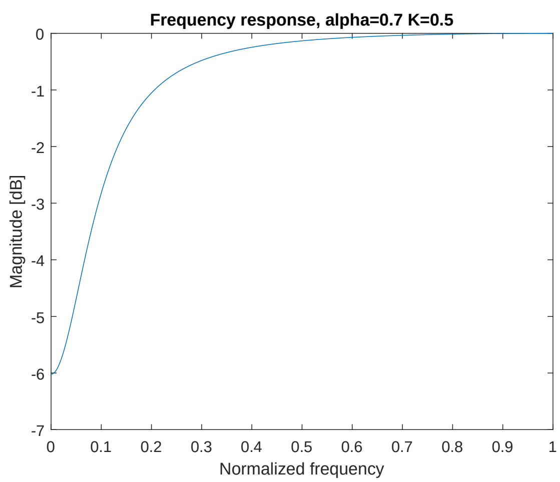

Figure 10: Pluto signal with $K = 0.5$ and $a = 0.7$. The system is now a highpass filter.

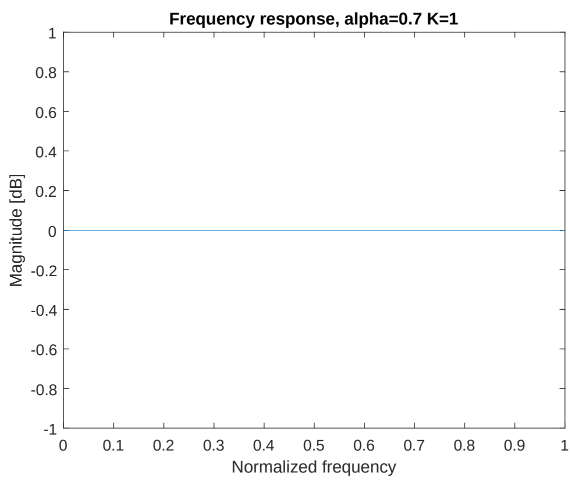

Figure 11: Pluto signal with $K = 1$ and $a = 0.7$. The system is now an allpass filter. with a flat unity system response

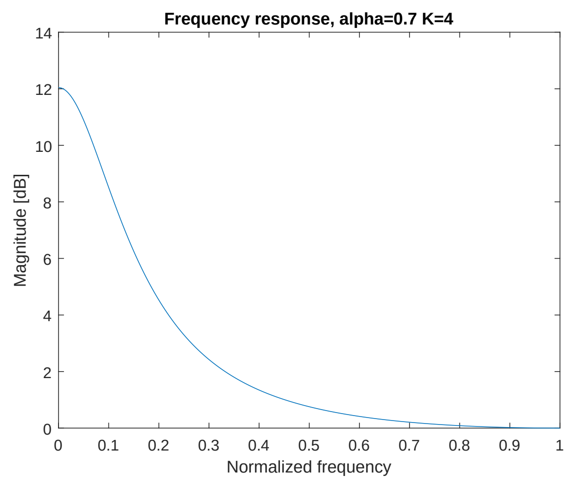

Figure 12: Pluto signal with $K = 4$ and $a = 0.7$. The system is now an lowpass filter, with a very strong filter response.

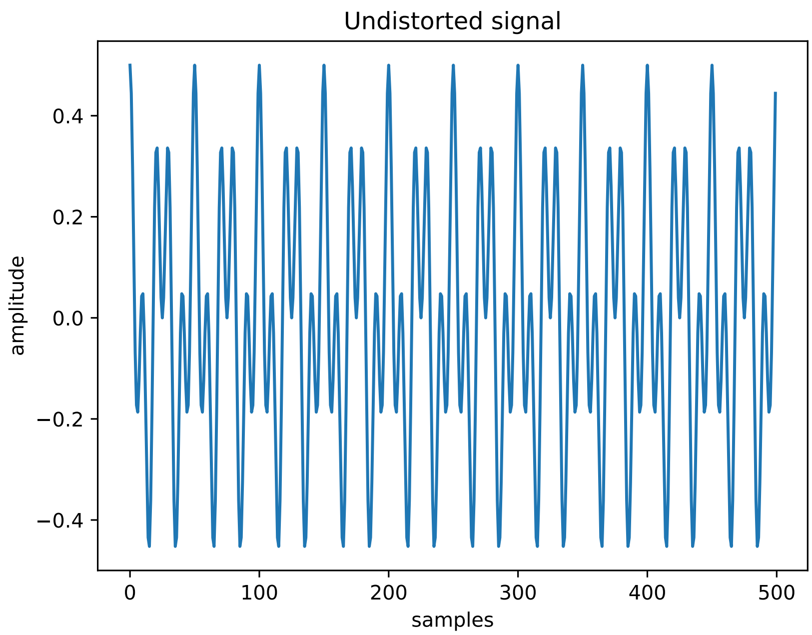

Figure 13: undistorted signal $d[n]$

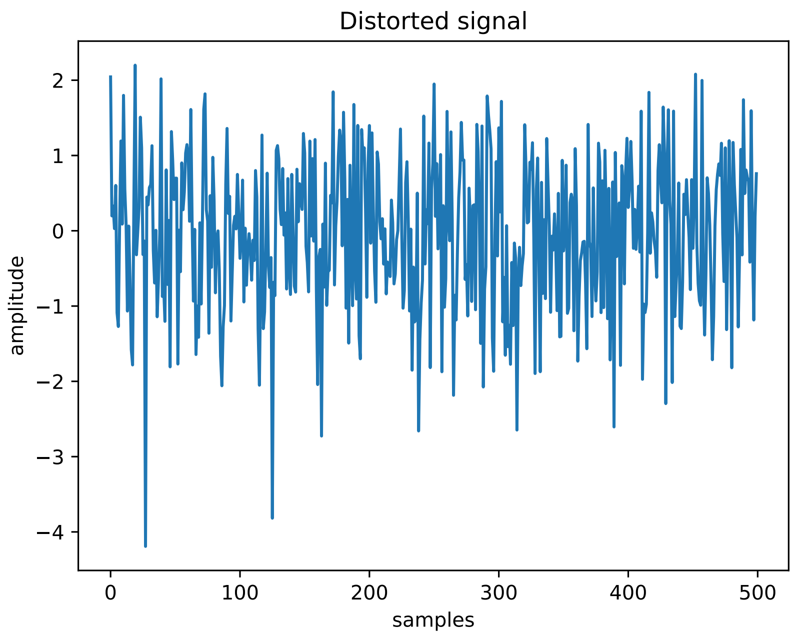

Figure 14: distorted signal $g[n]$

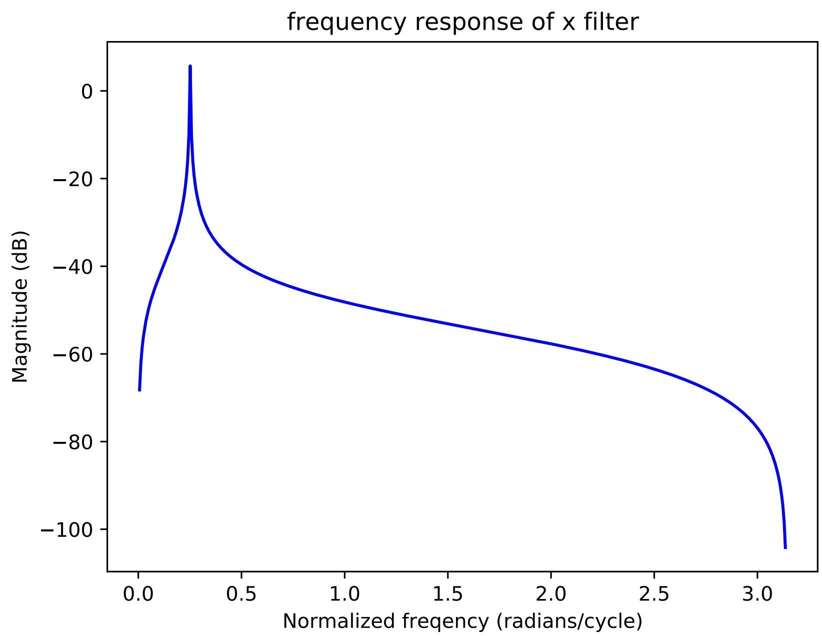

Figure 15: freqeuency response of x-filter

4 problem 4

4.1 a)

4.2 b)

The expression for $H_x(z)$ and $H_y(z)$ can be found in equations 5 and **??** with $r = 0.999$, $f_x = 0.04$, $f_y = 0.10$

$$H_x(z) = \left(\frac{(1 + r^2 - 2r\cos(2\pi f_x)}{2(1 - cos(2\pi f_x)}\right)^{1/2} \cdot \frac{(z-1)(z+1)}{(z - re^{j2\pi f_x})(z - re^{-j2\pi f_x})} \tag{5}$$

$$H_x(z) = \left(\frac{(1 + r^2 - 2r\cos(2\pi f_y)}{2(1 - cos(2\pi f_y)}\right)^{1/2} \cdot \frac{(z-1)(z+1)}{(z - re^{j2\pi f_y})(z - re^{-j2\pi f_y})} \tag{6}$$

4.3 c)

It is somewhat as expected, i do not understand why the response seems to increase with time, if i have understood everything correctly then then the filter should be highly frequency sensitive, which it is, but it shouldnt attenuate

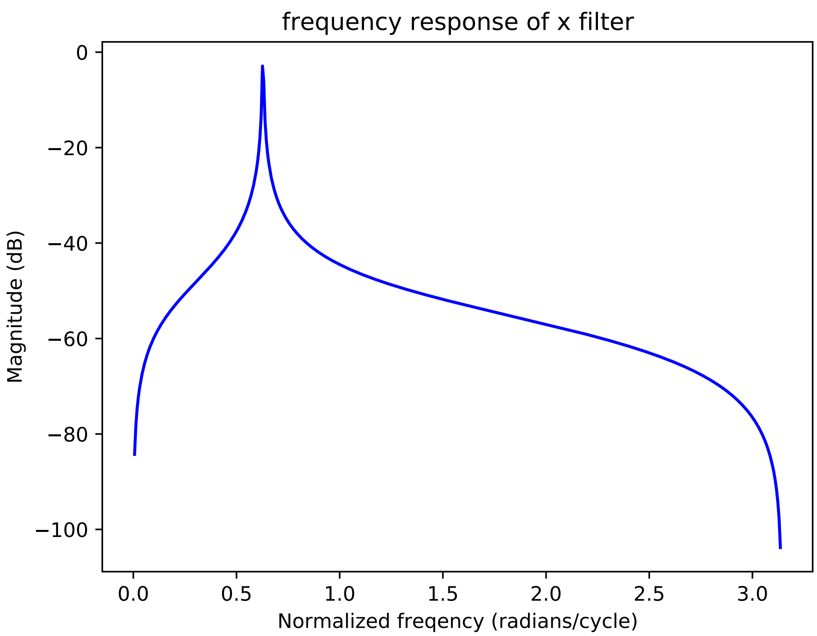

Figure 16: freqeuency response of y-filter

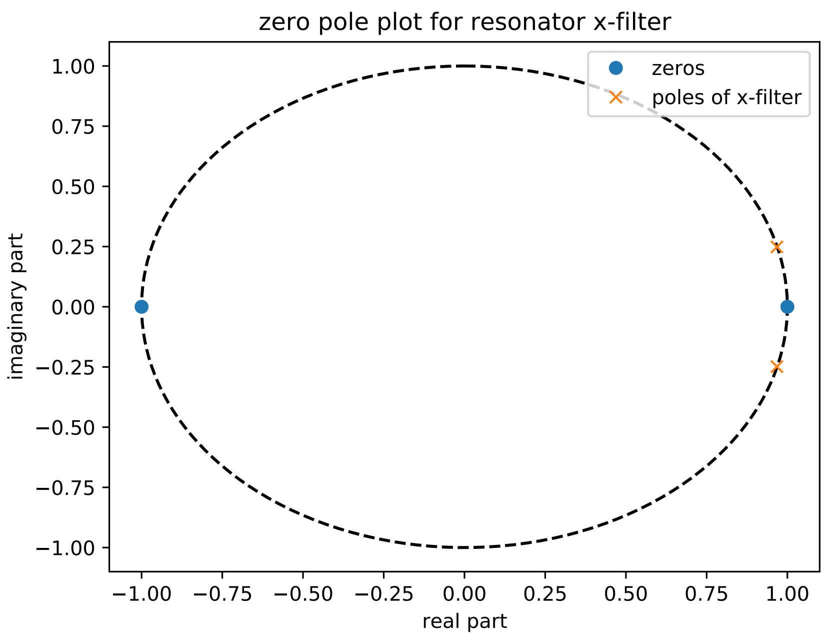

Figure 17: Zero-pole plot of x-filter

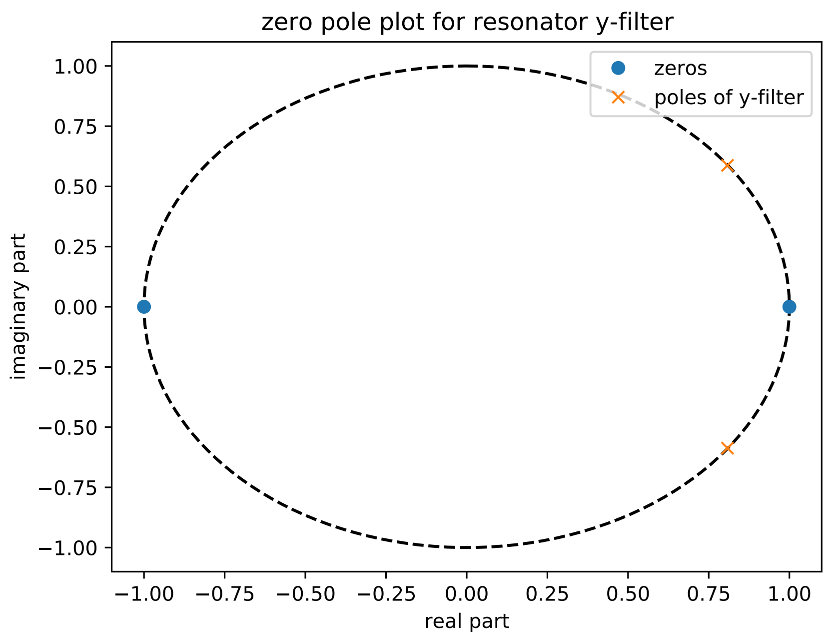

Figure 18: Zero-pole plot of y-filter

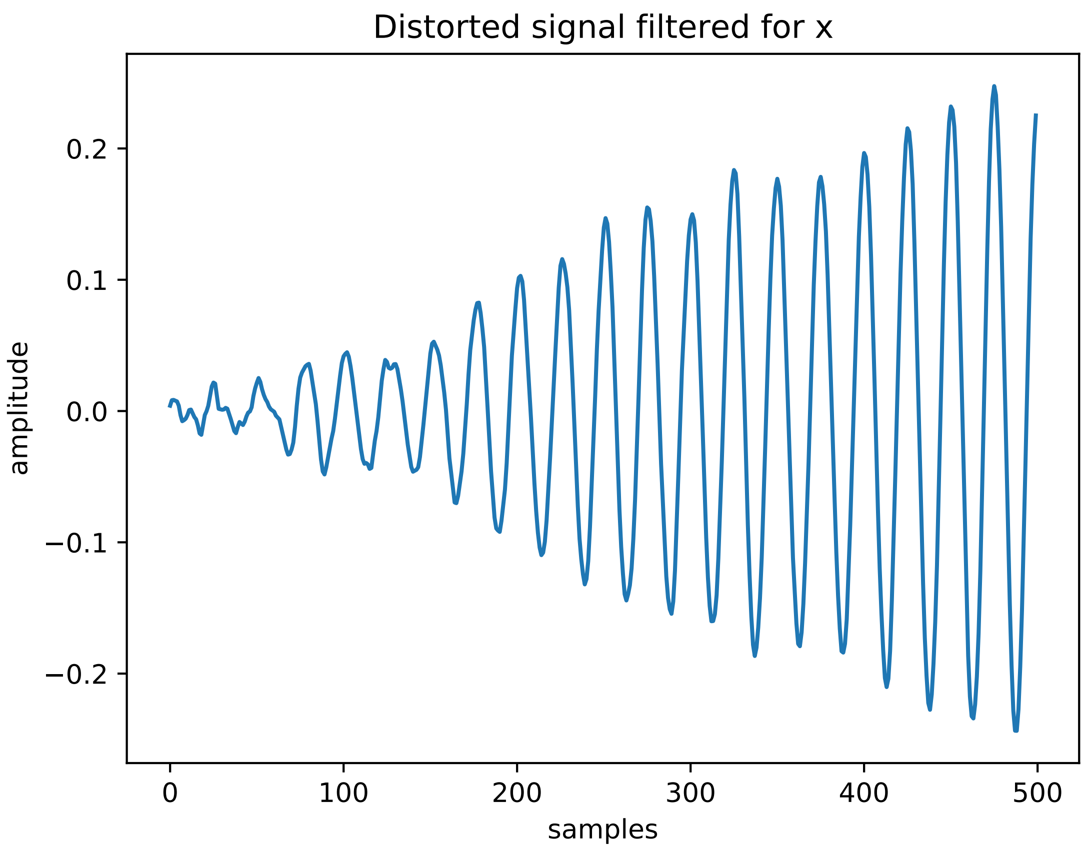

Figure 19: System filtered with the x-sensistive filter

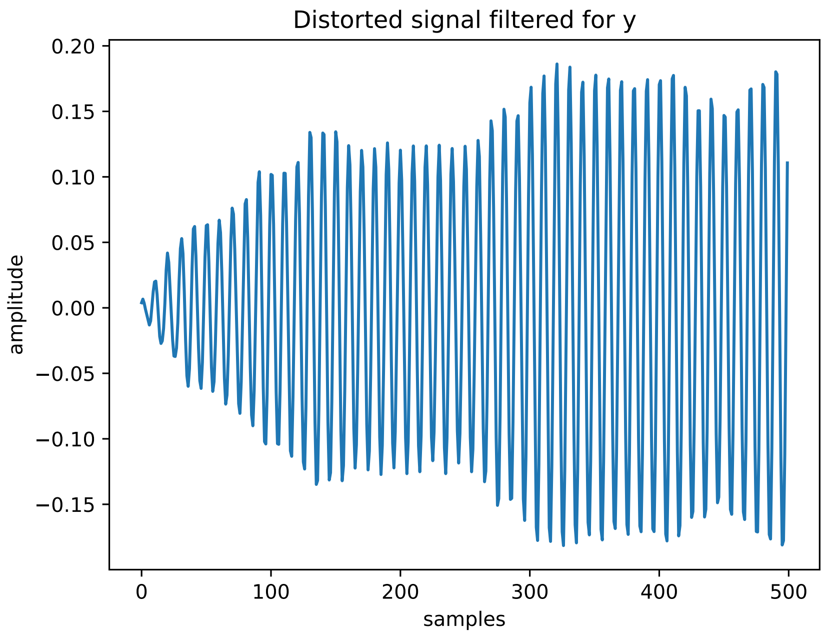

Figure 20: System filtered with the y-sensistive filter

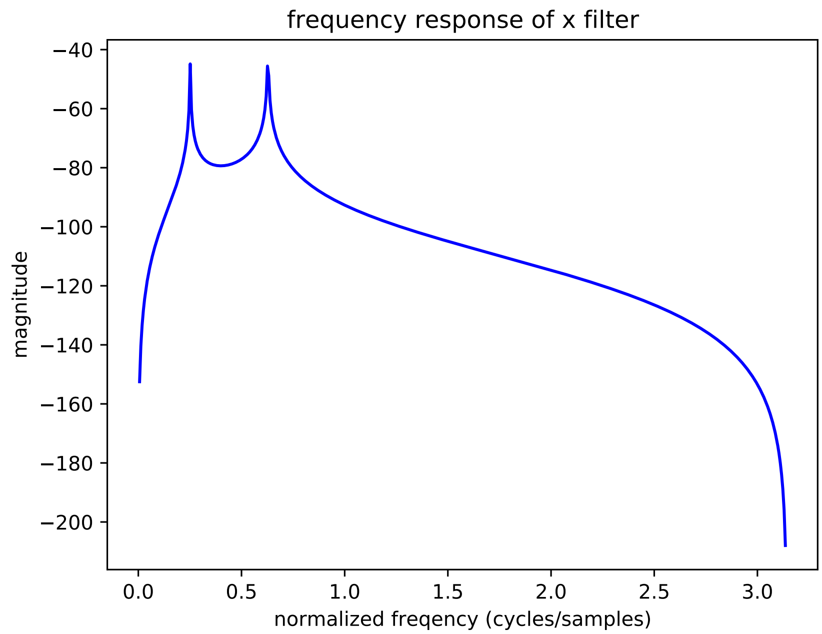

Figure 21: magnitude response of double filter

signals at their start. I will admit i dont understand what mechanism is at play here, when we have a LTI filter.

The double filtered system seems to work well enuogh though it still has some wierd amplituede responses that are present in both the single filteres aswell.

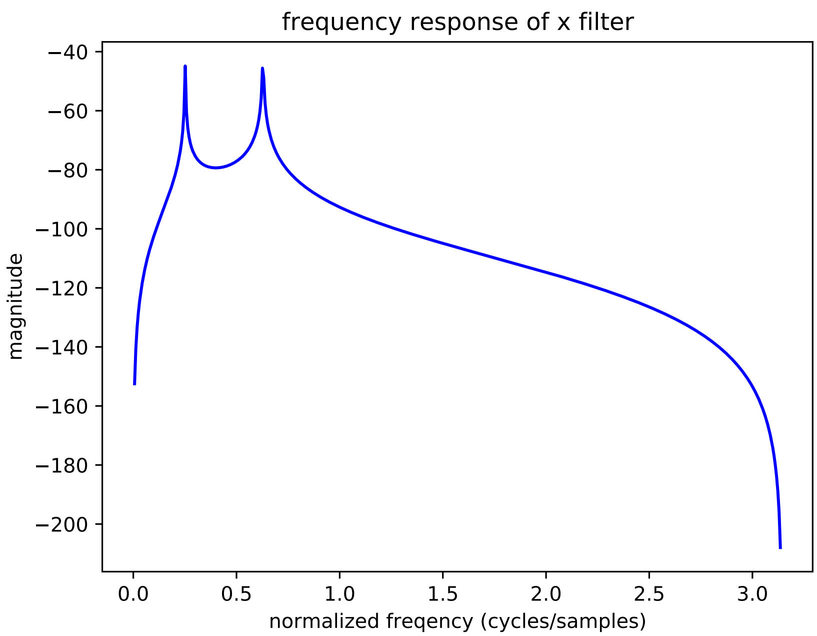

Figure 22: magnitude response of double filter

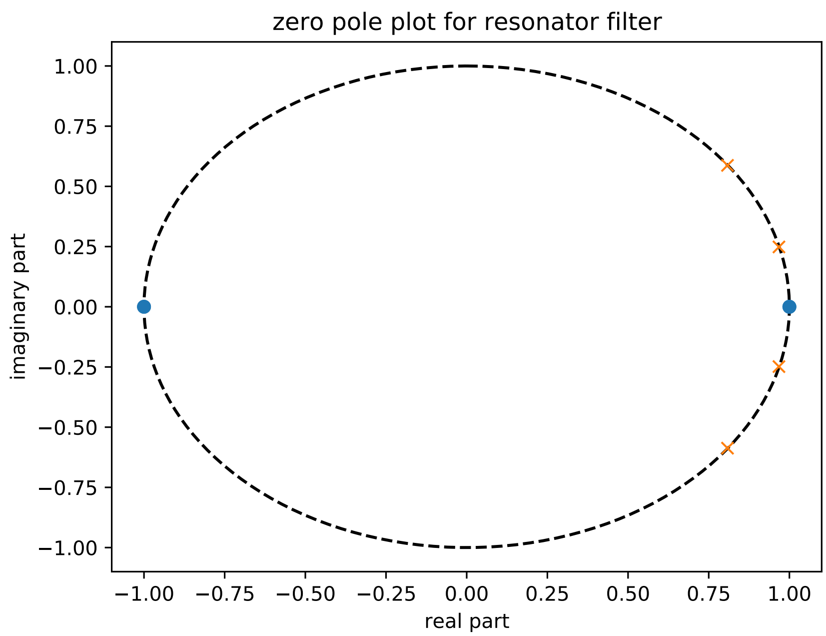

Figure 23: Zero-Pole plot of full filter.

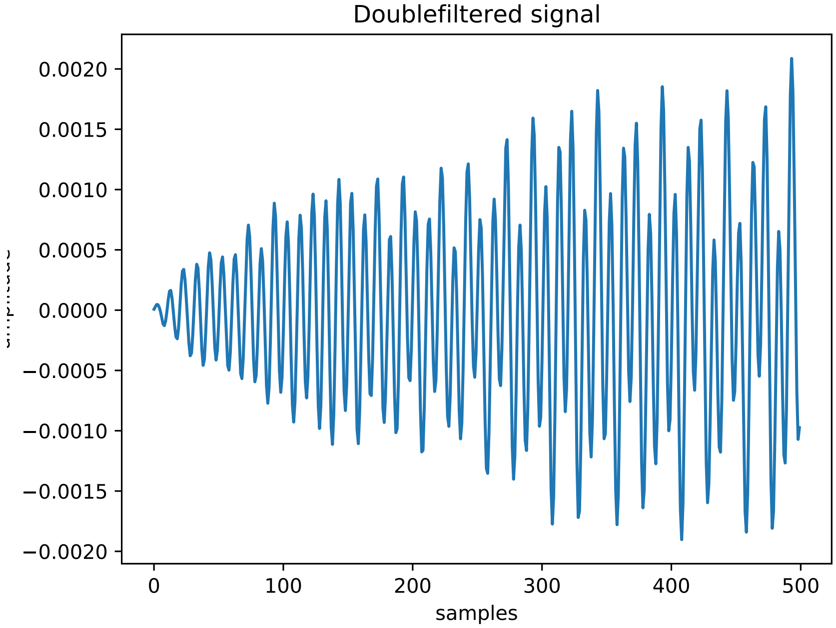

Figure 24: Double filtered signal

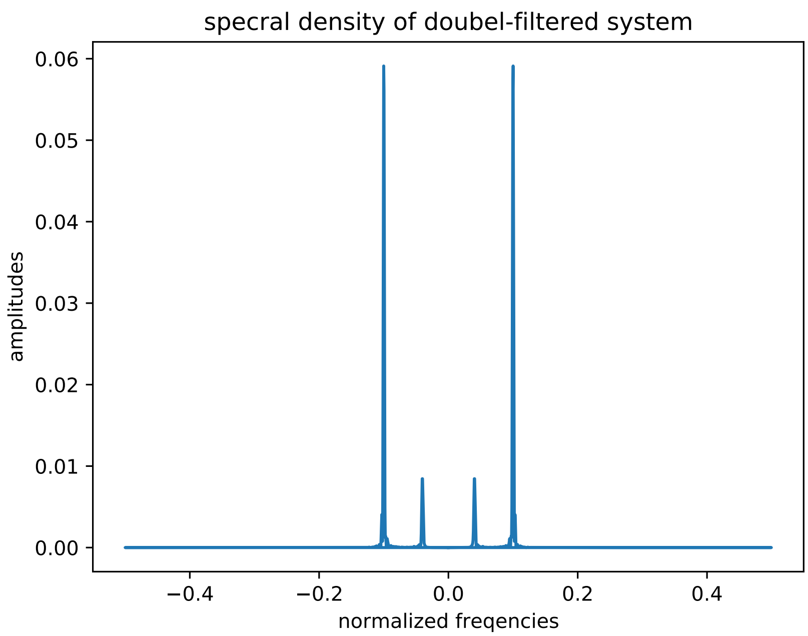

Figure 25: Double filtered spectral density

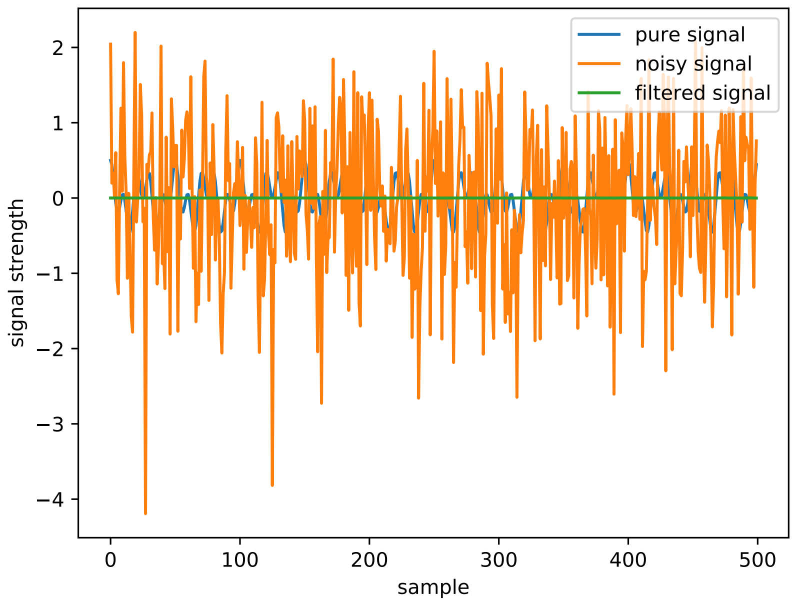

Figure 26: comparative plots of signals

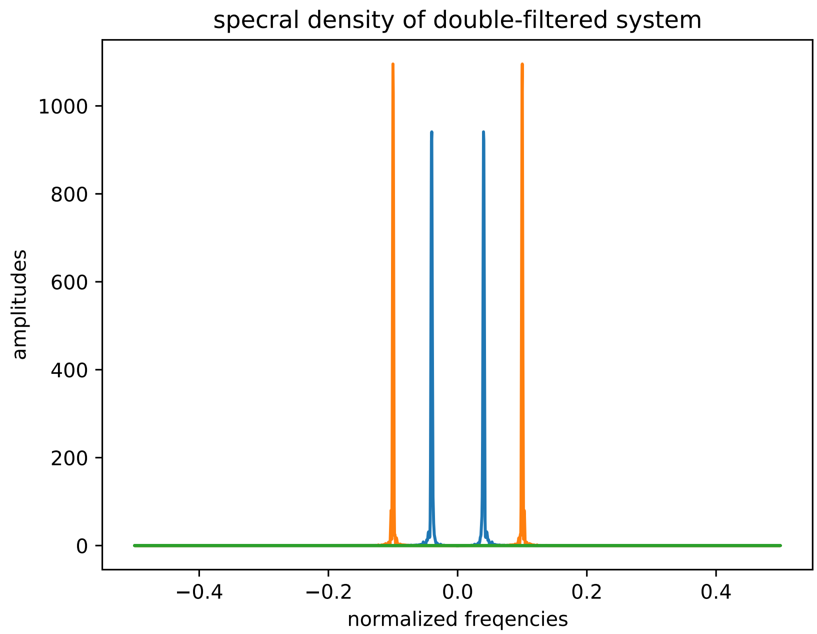

Figure 27: comparative plots of spectral densities