

Module 9: Recommended Exercises

TMA4268 Statistical Learning V2020

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Problem 1

Work through the lab in Section 9.6.1 of the course book.

Problem 2 (Book Ex.2)

We have seen that in $p = 2$ dimensions, a linear decision boundary takes the form $\beta_0 + \beta_1 X_1 + \beta_2 X_2 = 0$. We now investigate a non-linear decision boundary.

a) Sketch the curve

$$(1 + X_1)^2 + (2 - X_2)^2 = 4.$$

b) On your sketch, indicate the set of points for which

$$(1 + X_1)^2 + (2 - X_2)^2 > 4,$$

as well as the set of points for which

$$(1 + X_1)^2 + (2 - X_2)^2 \leq 4.$$

c) Suppose that a classifier assigns an observation to the blue class if

$$(1 + X_1)^2 + (2 - X_2)^2 > 4,$$

and to the red class otherwise. To what class is the observation $(0, 0)$ classified? $(-1, 1)$? $(2, 2)$? $(3, 8)$?

d) Argue that while the decision boundary in (c) is not linear in terms of X_1 and X_2 , it is linear in terms of X_1 , X_1^2 , X_2 , and X_2^2 .

Problem 3

The SVM is an extension of the support vector classifier, by enlarging the feature space using kernels. The polynomial kernel is a popular choice and has the following form

$$K(\mathbf{x}_i, \mathbf{x}'_i) = \left(1 + \sum_{j=1}^p x_{ij} x'_{ij}\right)^d$$

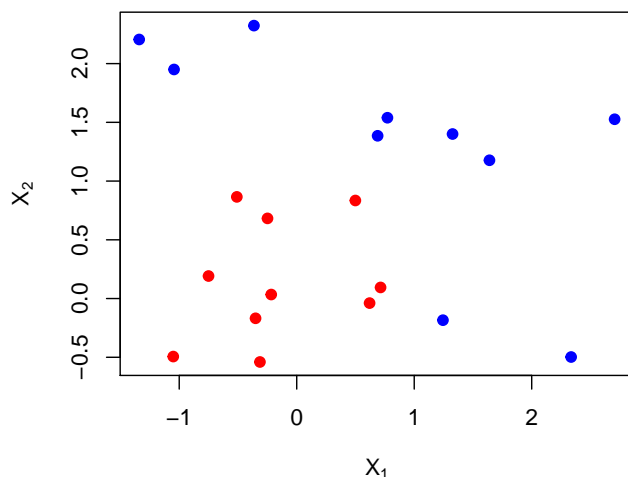
Show that for a feature space with inputs X_1 and X_2 and for degree $d = 2$, the above kernel can be represented as the inner product

$$K(\mathbf{x}_i, \mathbf{x}'_i) = \langle h(\mathbf{x}), h(\mathbf{x}') \rangle ,$$

where $h(\mathbf{x})$ is a 6-dimensional transformation function in an enlarged space.

Problem 4

This problem involves plotting of decision boundaries for different kernels and it's taken from [Lab video](#).



- (a) Plot the decision boundary of the `svmfit` model by using the function `make.grid`. Hint: Use the `predict` function for the grid points and then plot the predicted values $\{-1,1\}$ with different colors.

```
library(e1071)
svmfit = svm(y ~ ., data = dat, kernel = "linear", cost = 10, scale = FALSE)

# grid for plotting
make.grid = function(x, n = 75) {
  # takes as input the data matrix x and number of grid points n in
  # each direction the default value will generate a 75x75 grid

  grange = apply(x, 2, range) # range for x1 and x2
  x1 = seq(from = grange[1, 1], to = grange[2, 1], length.out = n) # sequence from the lowest to the
  x2 = seq(from = grange[1, 2], to = grange[2, 2], length.out = n) # sequence from the lowest to the
  expand.grid(X1 = x1, X2 = x2) #create a uniform grid according to x1 and x2 values
}
```

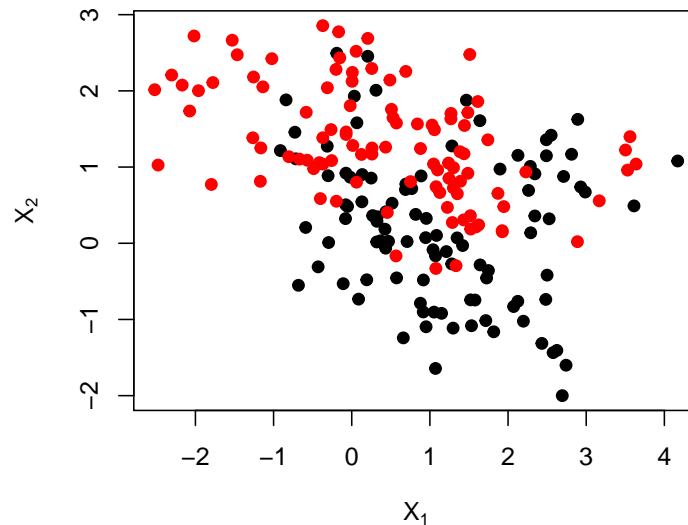
- (b) On the same plot add the training points and indicate the support vectors.
(c) The solutions to the SVM optimization problem is given by

$$\hat{\beta} = \sum_{i \in S} \hat{\alpha}_i y_i x_i ,$$

where S is the set of the support vectors. From the `svm()` function we cannot extract $\hat{\beta}$, but instead we have access to $\text{coef}_i = \hat{\alpha}_i y_i$, and $\hat{\beta}_0$ is given as `rho`. For more details see [here](#).

Calculate the coefficients $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$. Then add the decision boundary and the margins using the function `abline()` on the plot from (b).

- (d) Now we fit an svm model with radial kernel to the following data taken from Hastie, Tibshirani, and Friedman (2009):



```
fit = svm(factor(y) ~ ., data = dat, scale = FALSE, kernel = "radial",  
          cost = 5)
```

Using the same idea as in (a) plot the non-linear decision boundary, and add the training points. Furthermore if you want to create the decision boundary curve you can use the argument `decision.values=TRUE` in the function `predict`, and then you can plot it by using the `contour()` function.

Problem 5 - optional (Book Ex. 7)

This problem involves the OJ data set which is part of the ISLR package. (a) Create a training set containing a random sample of 800 observations, and a test set containing the remaining observations.

- (b) Fit a support vector classifier to the training data using `cost=0.01`, with `Purchase` as the response and the other variables as predictors. Use the `summary()` function to produce summary statistics, and describe the results obtained.
- (c) What are the training and test error rates?
- (d) Use the `tune()` function to select an optimal cost. Consider values in the range 0.01 to 10.
- (e) Compute the training and test error rates using this new value for `cost`.
- (f) Repeat parts (b) through (e) using a support vector machine with a radial kernel. Use the default value for `gamma`.
- (g) Repeat parts (b) through (e) using a support vector machine with a polynomial kernel. Set `degree=2`.
- (h) Overall, which approach seems to give the best results on this data?

Hastie, Trevor, Robert Tibshirani, and Jerome Friedman. 2009. *The Elements of Statistical Learning*. 2nd ed. Vol. 1. Springer series in statistics New York.