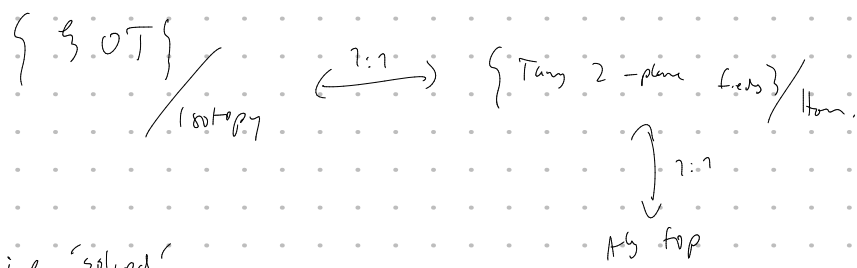


5. Existence & classification of contact 3-manifolds & leg knots

5.1 by picture

OT = flexible:



i.e. 'solved'

Tight = rigid

\exists knots with a unique tight c.s.

\exists knots with n c.s.

\exists knots with ∞ c.s.

5.2 classification of tight contact structures

Thm 1: $(\mathbb{R}^3, \xi_{\text{st}}), (S^2, \xi_{\text{st}}), (S^2 \times \mathbb{R}, \xi_{\text{st}}), (S^2 \times S^2, \xi_{\text{st}})$ are tight

proof: in sect 6

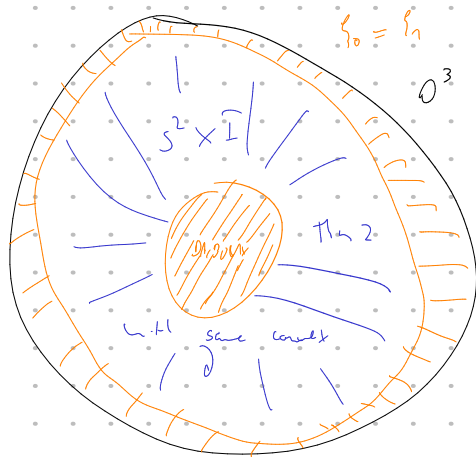
Thm 2: A tight contact structure ξ on $S^2 \times E(2)$ with convex boundary is determined (up to isotopy fixing ∂) by the dividing set of ∂

Thm 3 [Bourbaki]

Let ξ_0, ξ_1 be tight c.s. on D^3 s.t. ∂D^3 is convex and $\Gamma_{\partial D^3}^0 = \Gamma_{\partial D^3}^1$

$\Rightarrow \xi_0$ is isotopic to ξ_1 rel ∂D^3

proof:



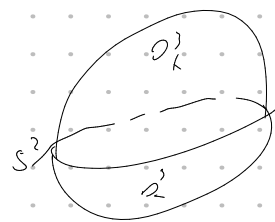
Thm 4 [Eckberg]

$S^3, \mathbb{R}^3, S^1 \times S^2$ admit unique total C.S.

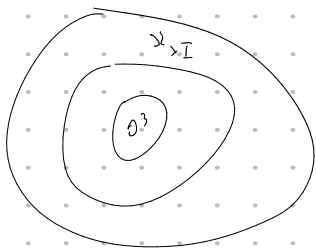
Proof: (1) $S^3 = D^3_+ \cup_{S^2} D^3_-$ why: S^2 convex

$\xrightarrow{\text{q.t.}} \Gamma_{S^2}$ is connected

Thm 3
 $\xrightarrow{\text{q.t.}} q_0$ is isotopic to q_1



(2) $\mathbb{R}^3 = D^3(0) \cup S^2 \times [0,1] \cup S^2 \times [1,2] \cup \dots$



why $S^2 \times n$ convex $\forall n \in \mathbb{Z}$
 $\xrightarrow{\text{q.t.}} \Gamma_{S^2 \times n}$ is connected
 $\xrightarrow{\text{Thm 2.8.2}} q_0$ is isotopic to q_1

(3) $S^1 \times S^2 = [-1,1] \times S^2$
 $-1 \times S^2 \sim 1 \times S^2$

\Rightarrow why $\pm 1 \times S^2$ is convex with connected Γ

Let V be a solid torus with tight C.S. st. ∂V is convex

Fix: meridian μ : S.C.C. on ∂V non-trivial in ∂V but trivial in V .

Longitude λ : S.C.C. on ∂V non-trivial in ∂V st. $\mu \neq \lambda = p\mu$

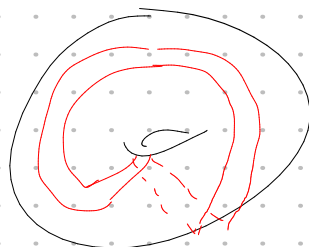
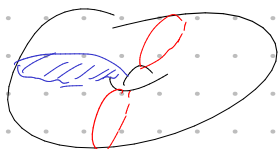
$$\lambda' = \lambda + n\mu$$

$\xrightarrow{\text{q.t.}} \Gamma_{\partial V}$ consists of (non) curves st. parallel copies of $p\mu + q\lambda$

$$\text{slope}(\partial V) = \text{slope}(\Gamma_{\partial V})$$

$$\text{Ex: } \text{slope}(S^1 \times D^2, q_1) = -\frac{1}{n}$$

no slope 0:

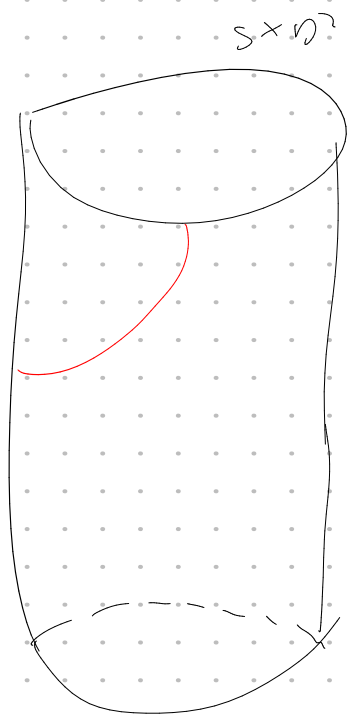


Thm 5: [convex hulls]

(1) $\forall n \in \mathbb{Z} \exists!$ Tight C.S. on $S^n \times D^2$ w/ (convex) hull with 2 boundary curves of slope $= \frac{2}{n}$

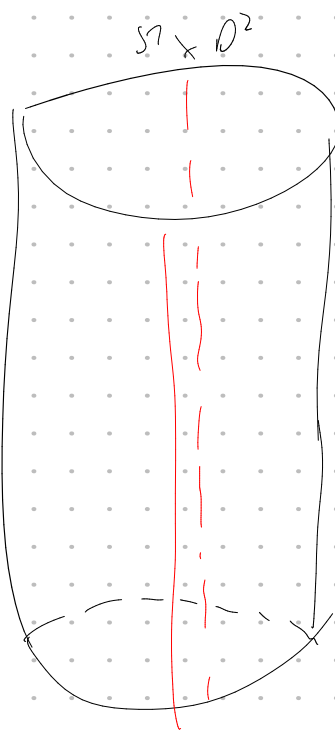
(2) $\forall r \in \mathbb{R} \setminus \{0\} \exists$ further many tight C.S. w/ of slope $= r$

proof sketch:



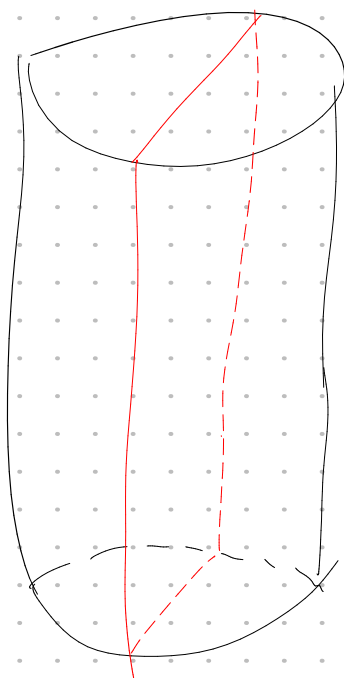
$2/n$

(cut along D^2 /
hust
&
reglue



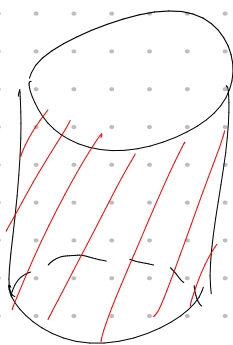
$2/n-1$

(cut
along
diag

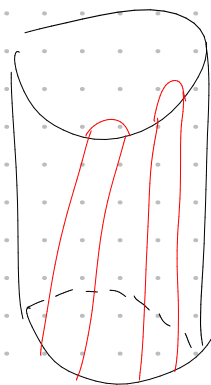


∂D^2 is convex
 $\overline{Th_3}$ is unique

(2)



D^3



or

