1. Overview

M^n = smooth n-manifold

Line field: vector field

$X: M \longrightarrow TM$

$p \longmapsto X(p) \in T_pM$

$X \neq 0$

$$L := \bigcup_{p \in M} \langle X(p) \rangle$$

2-plane fields:

Two lin. ind. vector fields

$X_1, X_2 : M \longrightarrow TM$

$$\xi := \bigcup_{p\in M} \langle X_1(p), X_2(p) \rangle$$

Ex: $M = \mathbb{R}^3 \ (x,y,z)$

(1) $X_1 = \partial_x \qquad X_2 = \partial_y$

Foliation: surfaces Σ_t s.t. $T\Sigma_t = \xi_p$

(2) $X_1 = \partial_x \qquad X_2 = \partial_y - x\partial_z$

see handout for picture

Contact structure = no surface tangent to ξ

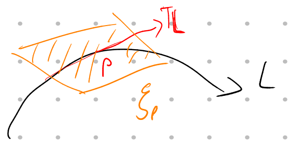
No local invariants: $\forall p \in (M^3, \xi)$ contact $\exists$ NBHD U of p s.t. $(U, \xi) \cong (\mathbb{R}^3, \xi_{\text{standard}})$

"bridge between topology and geometry"

Legendrian knots

$$L : S^1 \hookrightarrow (M^3, \xi)$$

is called Legendrian $:\Longleftrightarrow$ $TL \subset \xi$



* Can approximate any smooth knot by a Legendrian knot

Thurston–Bennequin–invariant

$tb(L) \approx$ # Turns of ξ_p along L

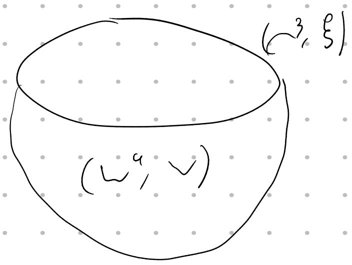
Mathematical Physics

Phase space $\longleftrightarrow$ symplectic mfd

$\cup \qquad\qquad \cup$

Energy hypersurface $\longleftrightarrow$ contact structure

Symplectic filling



Does there exist a symplectic filling for a 3-manifold?

Not it is ordinary!

For all tight ones??

Rigid (Geometry)	Flexible (Topology)
Ex: Riemannian manifolds; TIGHT (not overtwisted)	Overtwisted; Ex: Smooth manifolds
Not detected by Alg. Top. $\rightarrow$ more interesting	$\exists$ a Leg. unknot s.t. $tb(L) = 0$
	Ts (Eliashberg):
	{OT contact str.}/Isotopy $\updownarrow$ 1:1 {2-plane fields}/Homotopy
	ALG TOP

3- & 4-MFD TOP.

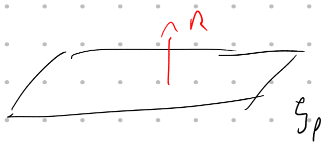
$n \neq 4$: $\mathbb{R}^n$ has a unique smooth structure

$n = 4$: $\exists$ uncountably many Top. manifolds R s.t. $R \overset{C^0}{\cong} \mathbb{R}^4$ but $R \overset{C^\infty}{\ncong} \mathbb{R}^4$

* Invariants of 3- & 4-manifolds Ex:: Heegaard-Floer homo

* $\forall (M^3, \xi)$ contact can be obtained from (S^3, ξ_{st}) by surgery on Leg. Link

Dynamical systems



REEB Vector Field

Weinstein conjecture: M closed $\overset{?}{\Rightarrow}$ R admits a periodic orbit

Nearby Lagrangian Conjecture

unit cotangent bundle

$$M^n \overset{C^\infty}{\cong} N^n \Rightarrow (S^*M, \xi_{can}) \overset{cont}{\cong} (S^*N, \xi_{can})$$

$\Leftarrow$ Conjecture

Remark: If $M^n \overset{C^0}{\cong} N^n \Rightarrow S^*M \overset{C^\infty}{\cong} S^*N$ [Milnor]