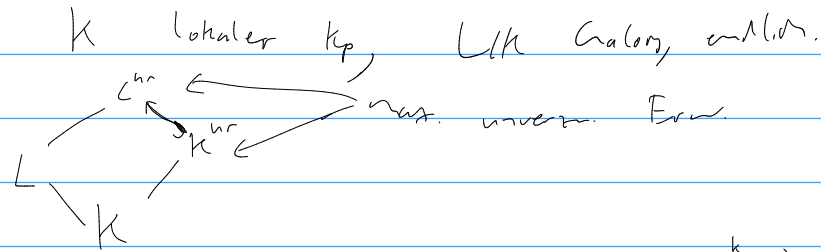


7)



$$\text{Frob}(L/K) = \{ \sigma \in \text{Gal}(L^{nr}/K) \mid \sigma|_{K^{nr}} = \text{id}_{K^{nr}} \}$$

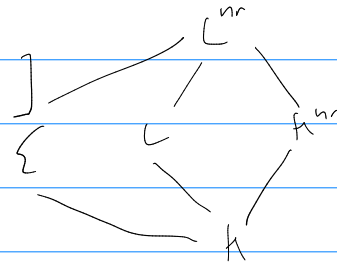
$$\text{Gal}(K^{nr}/K) \cong \text{Gal}(\overline{\mathbb{F}_q}/\mathbb{F}_q) \cong \langle \text{id}_K \rangle$$

for $\sigma \in \text{Frob}(L/K)$ determine $\Sigma = (L^{nr})^{\langle \sigma \rangle}$

(a) z.z. $[\Sigma:K] < \infty$

$$[\Sigma:K] = [L^{nr \langle \sigma \rangle} : K]$$

$$= [L^{nr \langle \sigma \rangle} : (K^{nr})^{\langle \sigma \rangle}] \cdot [(K^{nr})^{\langle \sigma \rangle} : K] \leq [L:K] \cdot [K^{nr}:K] = [L:K] \cdot \infty < \infty$$



$$(*) : \text{Gal}((L^{nr})^{\langle \sigma \rangle} / K) = \frac{\text{Gal}(L^{nr}/K)}{\text{Gal}(K^{nr}/(K^{nr})^{\langle \sigma \rangle})} \cong \frac{\hat{\mathbb{Z}}}{\langle \sigma \rangle} \cong \hat{\mathbb{Z}} / \langle \text{id}_K \rangle$$

$$\cong \hat{\mathbb{Z}} / \langle \sigma \rangle \cong \frac{\varprojlim \mathbb{Z}/n\mathbb{Z}}{\varprojlim n\mathbb{Z}} \cong \varprojlim \frac{\mathbb{Z}/n\mathbb{Z}}{n\mathbb{Z}} \cong \varprojlim \mathbb{Z}/(nK)\mathbb{Z} \cong \mathbb{Z}/K\mathbb{Z}$$

(b) z.z. $\Sigma^{nr} = L^{nr} \Leftrightarrow L^{nr}/\Sigma$ unversagt $\Leftrightarrow \text{Gal}(L^{nr}/\Sigma) \cong \text{Gal}(\overline{\mathbb{F}_q}/\mathbb{F}_q)$

Bew:

oder stattd.

$$\hat{\mathbb{Z}} \cong \langle \sigma \rangle = \text{Gal}(L^{nr}/\Sigma)$$

$$\hat{\mathbb{Z}} \cong \langle \text{id}_K \rangle = \text{Gal}(\overline{\mathbb{F}_q}/\mathbb{F}_q)$$

$$\text{Gal}(L^{nr}/\Sigma) = \varprojlim_{\Sigma' \in \mathcal{C}} \text{Gal}(\Sigma'/\Sigma)$$

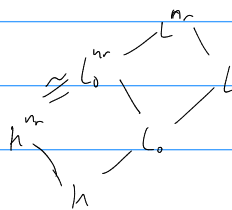
$$\text{Gal}(\Sigma'/\Sigma) \subseteq \text{Gal}(L^{nr}/\Sigma) \cong \mathbb{Z}/n\mathbb{Z}$$

c) Folgt aus dem Diagramm in b)

d) $\text{Frob}(L/K) \rightarrow \text{Gal}(L/K)$ ist surj.

Bew Sei $\sigma \in \text{Gal}(L/k)$. Definiere $\tilde{\sigma} \in \text{Frob}(L/k)$ s.t. $\tilde{\sigma}|_{k^n} = \sigma|_{k^n} \in \text{Gal}(k^n/k) \in \text{Gal}(L^n/k)$

$$L_0 = L \cap k^n.$$



$$\text{Gal}(L_0/k) \cong \text{Gal}(k^n/k) \Rightarrow \sigma|_{L_0} = \sigma^n|_{L_0} \text{ for } n \in \mathbb{N}$$

Definiere $\tilde{\tau} \in \text{Gal}(L^n/k)$ durch $\tilde{\tau} = \sigma \tilde{\sigma}^n$

$$\tilde{\tau} = \sigma \tilde{\sigma}^n|_{L_0} \text{ ist die Identität auf } L_0 \Rightarrow \tilde{\tau} \in \text{Gal}(L/L_0)$$

Es gilt $\text{Gal}(L^n/L_0^n) \twoheadrightarrow \text{Gal}(L/L_0)$ ist surjektiv.

$$\text{Wähle } \tilde{\tau} \text{ s.t. } \tilde{\tau}|_{L_0} = \tau.$$

$$\text{Definiere } \tilde{\sigma} = \tilde{\tau} \tilde{\sigma}^n \quad \tilde{\sigma}|_{k^n} = \tilde{\tau}|_{k^n} \tilde{\sigma}^n|_{k^n} = \sigma^n|_{k^n} = \sigma^n|_{k^n} \stackrel{\text{id}}{=} \sigma^n|_{k^n}$$

$$\tau|_{L_0} = \sigma|_{L_0} \quad \tilde{\sigma}^{-n}|_{L_0} = \sigma^{-n}|_{L_0} \quad \tilde{\sigma}^n|_{L_0} = \text{id}$$

$$\tilde{\sigma}|_L = \tilde{\tau}|_L \tilde{\sigma}^n|_L = \sigma \tilde{\sigma}^{-n}|_L \tilde{\sigma}^n|_L = \sigma$$

42) Betrachte $\tilde{\sigma} \in \text{Frob}(L/K)$, $\Sigma = (L^{\text{un}})^{\langle \tilde{\sigma} \rangle}$

$\Pi_{\Sigma_1}, \Pi_{\Sigma_2}$ Uniformisierende von Σ . Es gilt $\Pi_{\Sigma_1} = \Pi_{\Sigma_2} \epsilon \epsilon \in U_{\Sigma}$

z.z. $N_{\Sigma/K} \Pi_{\Sigma_1} = N_{\Sigma/K} \Pi_{\Sigma_2}$ mod $N_{L/K} L^{\times}$

zu

Sei $E = \Sigma \cdot L$.

$E \in U_{\Sigma} \subseteq U_E \subseteq N_{E/K} E^{\times}$ ($N_{E/\Sigma} E^{\times} = U_E \times \langle \Pi_E \rangle$),
d.h. $E = N_{E/\Sigma} \sigma, \sigma \in E^{\times}$

$$N_{E/K}^{\times} = N_{E/K} N_{E/L}^{\times}$$

$$\begin{aligned} N_{\Sigma/K} \Pi_{\Sigma_2} &= N_{\Sigma/K} \Pi_{\Sigma_1} \underbrace{N_{\Sigma/K} \Sigma}_{N_{\Sigma/K} N_{E/K} \sigma} \\ &\equiv N_{E/K} \sigma = N_{L/K} (N_{E/L} \sigma) \end{aligned}$$

b) Ist $\tilde{\sigma}|_L = \text{id}_L$, so gilt $L \subseteq \Sigma$, d.h. $N_{\Sigma/K} \Pi_{\Sigma} = N_{L/K} (N_{\Sigma/L} (\Pi_{\Sigma})) \in N_{L/K} L^{\times}$.

c) $\text{Frob}(L/K) \xrightarrow{\tilde{\gamma}_{L/K}} K^{\times} / N_{L/K} L^{\times}$ Gruppenhom. (siehe Hinweis)

\downarrow
 $\text{Aut}(L/K) \dashrightarrow \exists \gamma_{L/K}$

$$\begin{aligned} &\ker(\text{Frob}(L/K) \rightarrow \text{Aut}(L/K)) \\ &= \{\tilde{\sigma} \in \text{Frob}(L/K) \mid \tilde{\sigma}|_L = \text{id}_L\} \subseteq \ker(\gamma_{L/K}) \end{aligned}$$