Abstract

Contact geometry is the study of odd-dimensional smooth manifolds with so called contact structures, specific hyperplane distributions $\xi = \ker \alpha$ that have to satisfy a certain relation, the contact condition

$$\alpha \wedge (\mathrm{d}\alpha)^n \neq 0.$$

A manifold can have multiple different contact structures, which can be either rigid (in which case one speaks of a "tight" manifold) or flexible (in the sense that they satisfy an h-principle). Such contact manifolds are then called overtwisted. To illustrate this dichotomy, consider the sphere S^3. It has precisely one tight contact structure, but infinitely many overtwisted contact structures, corresponding to the infinitely many homotopy classes of 2-plane fields on the 3-sphere. There are other examples where there are infinitely many or no tight contact structures on a contact manifold. A further interesting property of contact manifolds comes from the fact that contact geometry is the odd-dimensional counterpart to symplectic geometry: Often, it is possible to view a contact manifold as the boundary of a symplectic manifold. Manifolds that are in this sense "fillable" are always tight. The contrary, however, doesn't need to hold and one can ask the question under which conditions such tight, but non-fillable manifolds exist. Bowden, Gironella, Moreno and Zhou have shown in a recent paper that there exist homotopically standard, non-fillable but tight contact structures on all spheres S^{2n+1} with $n >= 2$.

Starting with a specific open book decomposition of S^{2n-1}, one can construct a contact form on this manifold using a well-known construction by Thurston-Winkelnkemper. Then, according to Bourgeois, this contact structure can be extended to a tight contact structure on $S^{2n-1} \times T^2$. Applying subcritical surgery (preserving the tightness), one can kill the topology of the T^2-factor and obtain a tight contact structure on S^{2n+1}. Because of the special way of constructing it, one can show that it is non-fillable, but still homotopically standard.