Tight and non-Fillable Contact Structures on the Sphere

Contact topology is the study of contact manifolds up to isotopy. Contact structures are divided into two classes: Overtwisted (flexible) and tight (rigid) contact structures. While the former are fully classified by their topological properties [Eli89, BEM15], the latter turn out to be very interesting. A first step in understanding tight contact structures was to show that fillable contact structures are tight [Gro85, Eli91]. However, there are several known contact manifolds where the contrary is wrong [EH02, MNW13]. In the talk, I explain recent results by Bowden–Gironella–Moreno–Zhou [BGMZ22] showing that any contact manifold that admits a tight contact structure also admits a non-strongly-fillable tight contact structure in the same almost contact class.

References

[BEM15] Matthew Strom Borman, Yakov Eliashberg, and Emmy Murphy. Existence and classification of overtwisted contact structures in all dimensions. *Acta Mathematica*, 215(2):281–361, January 2015.

[BGMZ22] Jonathan Bowden, Fabio Gironella, Agustin Moreno, and Zhengyi Zhou. Tight contact structures without symplectic fillings are everywhere. 2022.

[EH02] John B. Etnyre and Ko Honda. Tight contact structures with no symplectic fillings. *Inventiones mathematicae*, 148(3):609–626, June 2002.

[Eli89] Y. Eliashberg. Classification of overtwisted contact structures on 3-manifolds. *Inventiones Mathematicae*, 98(3):623–637, October 1989.

[Eli91] Yakov Eliashberg. Filling by holomorphic discs and its applications. volume 2 of *London Mathematical Society Lecture Note Series*, pages 45–68. Cambridge University Press, Cambridge, 1991.

[Gro85] M. Gromov. Pseudo holomorphic curves in symplectic manifolds. *Inventiones mathematicae*, 82(2):307–347, June 1985.

[MNW13] Patrick Massot, Klaus Niederkrüger, and Chris Wendl. Weak and strong fillability of higher dimensional contact manifolds. *Inventiones mathematicae*, 192(2):287–373, May 2013.