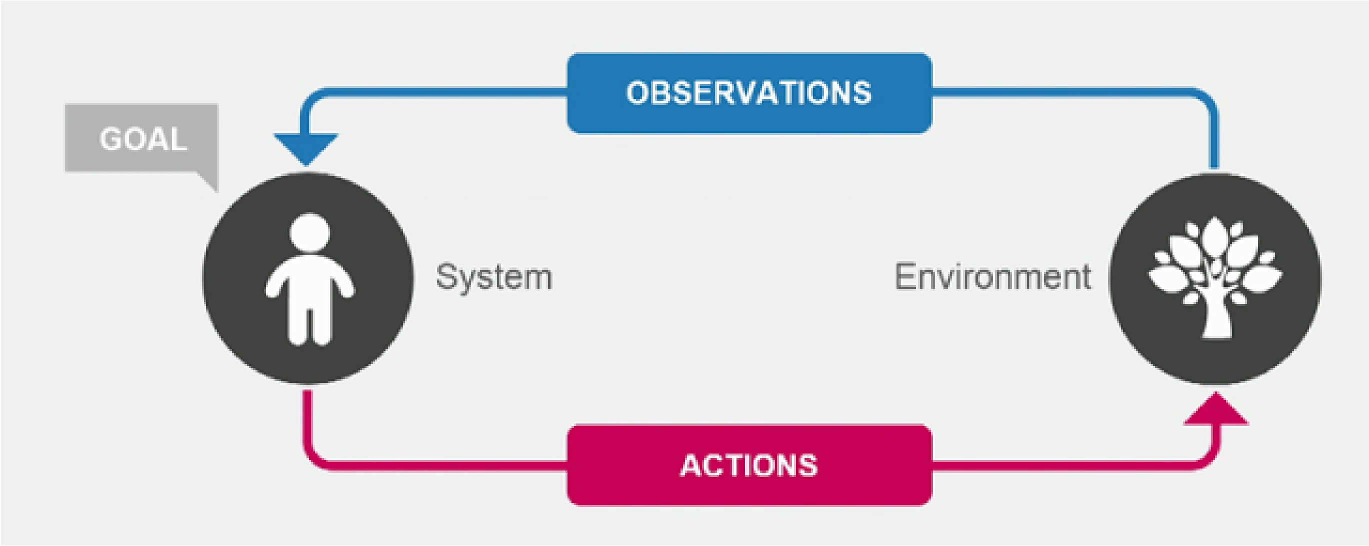


Cracking computational modelling with Stan: Using Rescorla-Wagner model as an example

Lei Zhang
Social, Cognitive and Affective Neuroscience Unit (SCAN-Unit)
Department of Cognition, Emotion, and Methods in Psychology
Faculty of Psychology, University of Vienna

April 10, 2020

https://github.com/lei-zhang/RL_tutorial_webinar

lei.zhang@univie.ac.at
lei-zhang.net
@lei_zhang_lz

universität wien
Fakultät für Psychologie

Outline

- About me
- What is computational modeling?
- The idea of the simple Rescorla-Wagner (RW) model
- Implementing RW model for one subject in Stan
- Fitting multiple subjects with the hBayesDM package
- Summary

About me

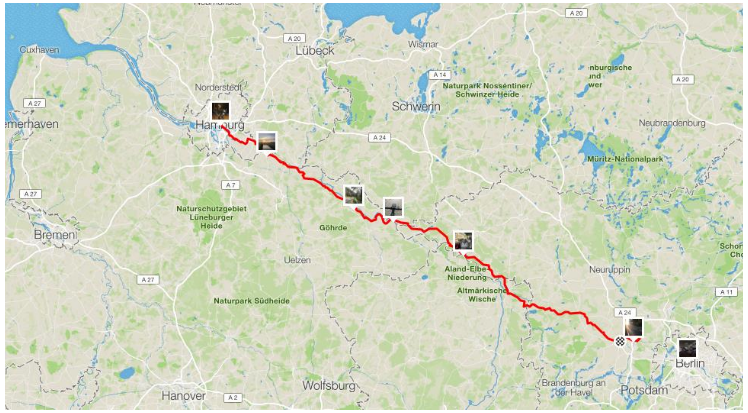
Postdoc @univie

PhD + Postdoc @UKE

RiSE intern@Roche

MSc @BCBL

BSc @BNU

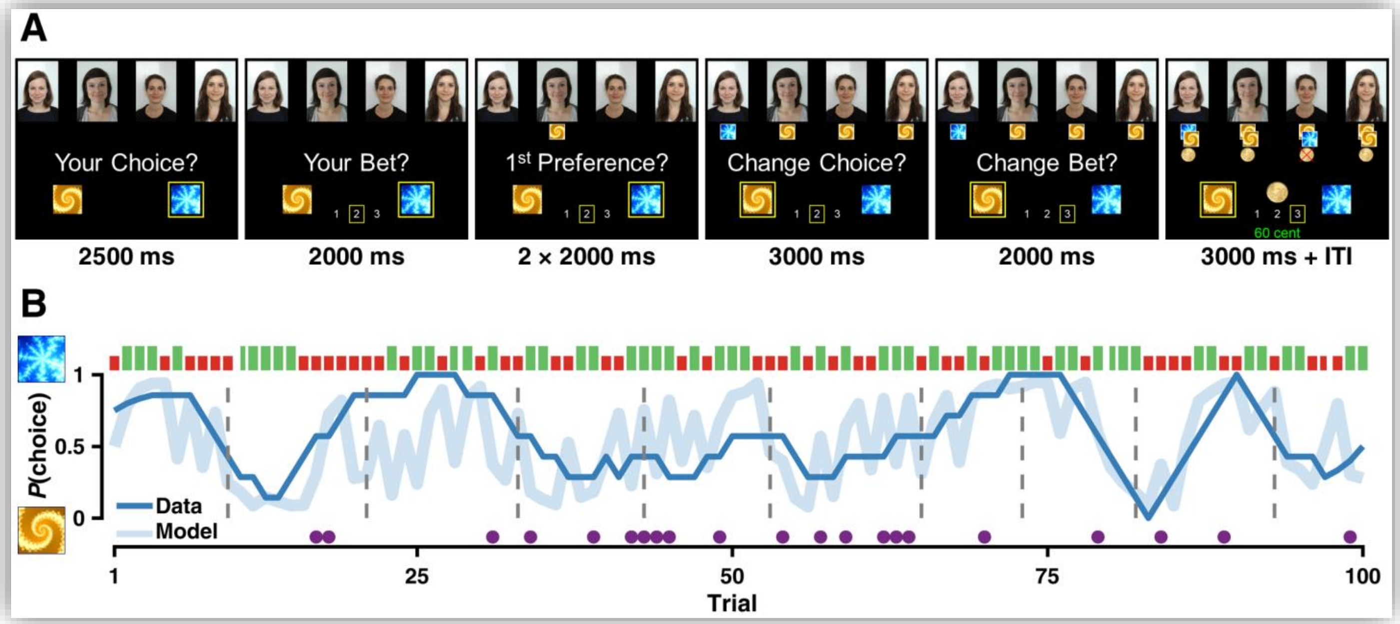


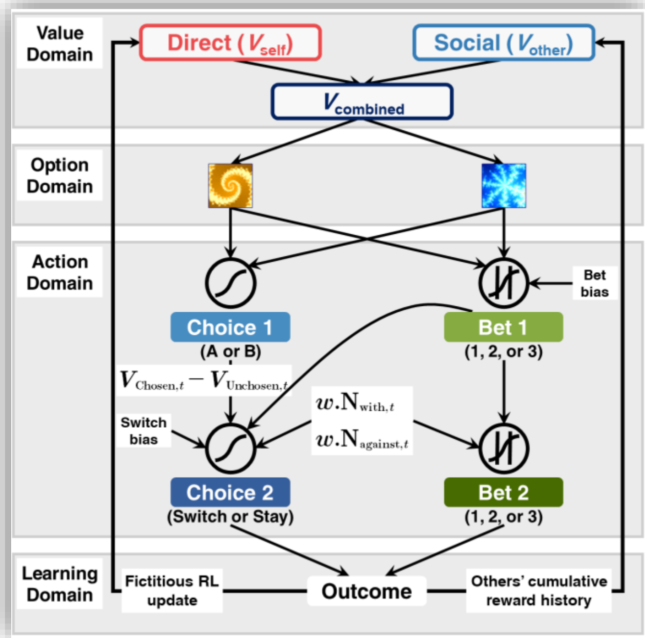
310km in one day!

STRAVA https://www.strava.com/athletes/leizhang

My research

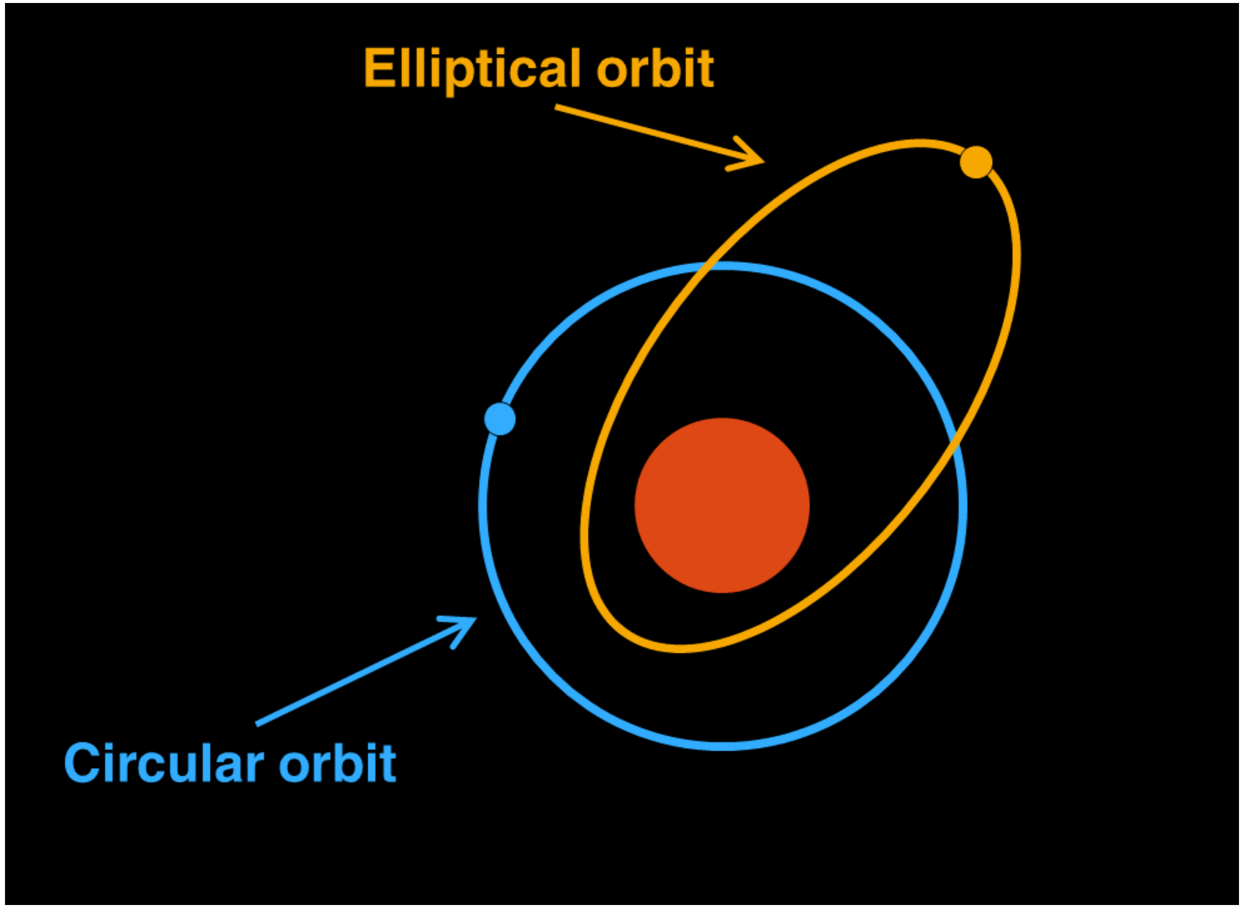
- Overarching goal: uncover the neuro-computational mechanisms underlying social decision-making
- Methods: behavioral/physiological measurement, cognitive modeling, fMRI
- Example work: A brain network supporting social influences in human decision-making (Zhang & Gläscher, bioRxiv)



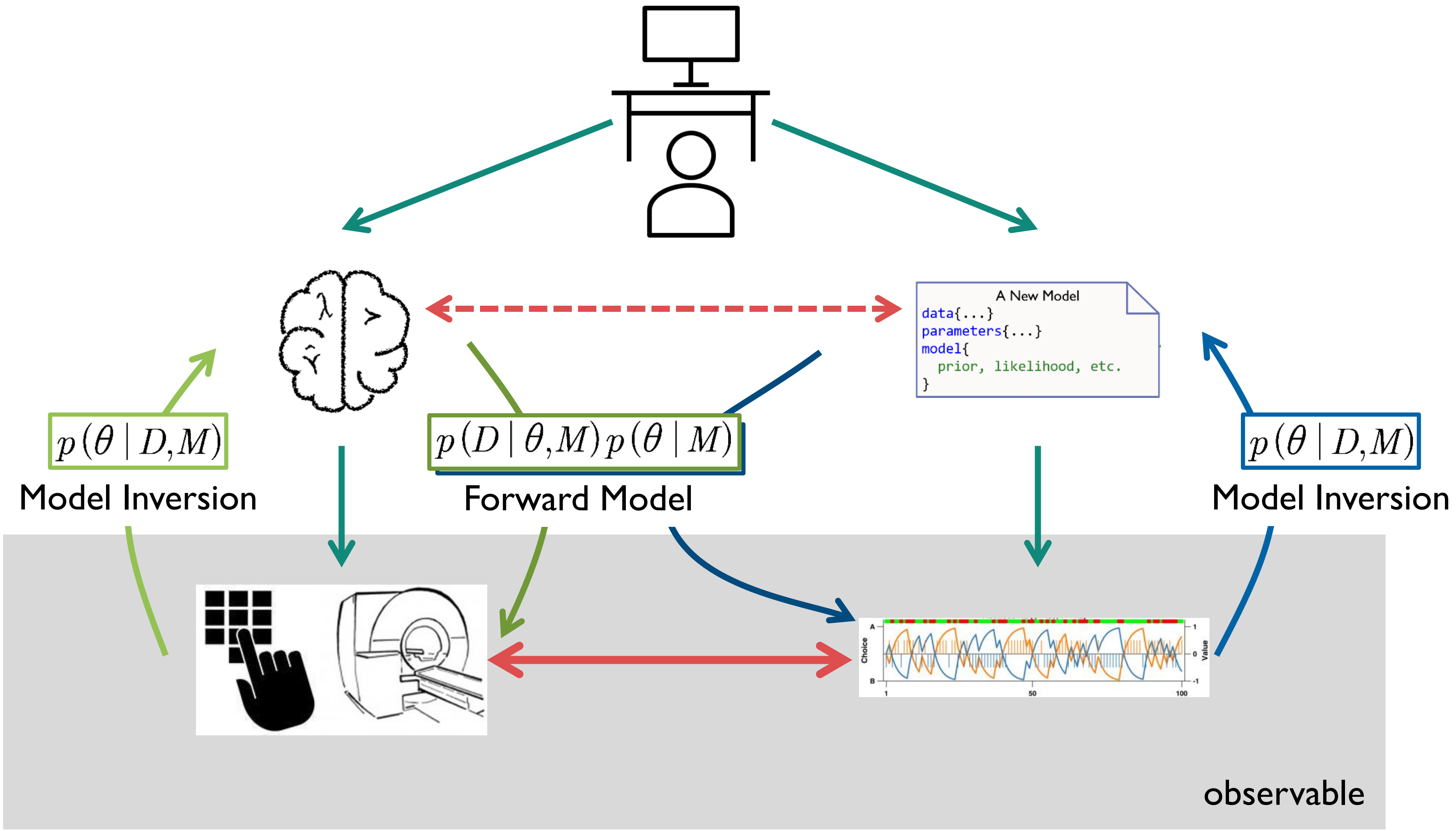


The idea of computational modeling is never new

Scientists use mathematical models to approximate certain processes (physical or mental), in order to explain and to predict.

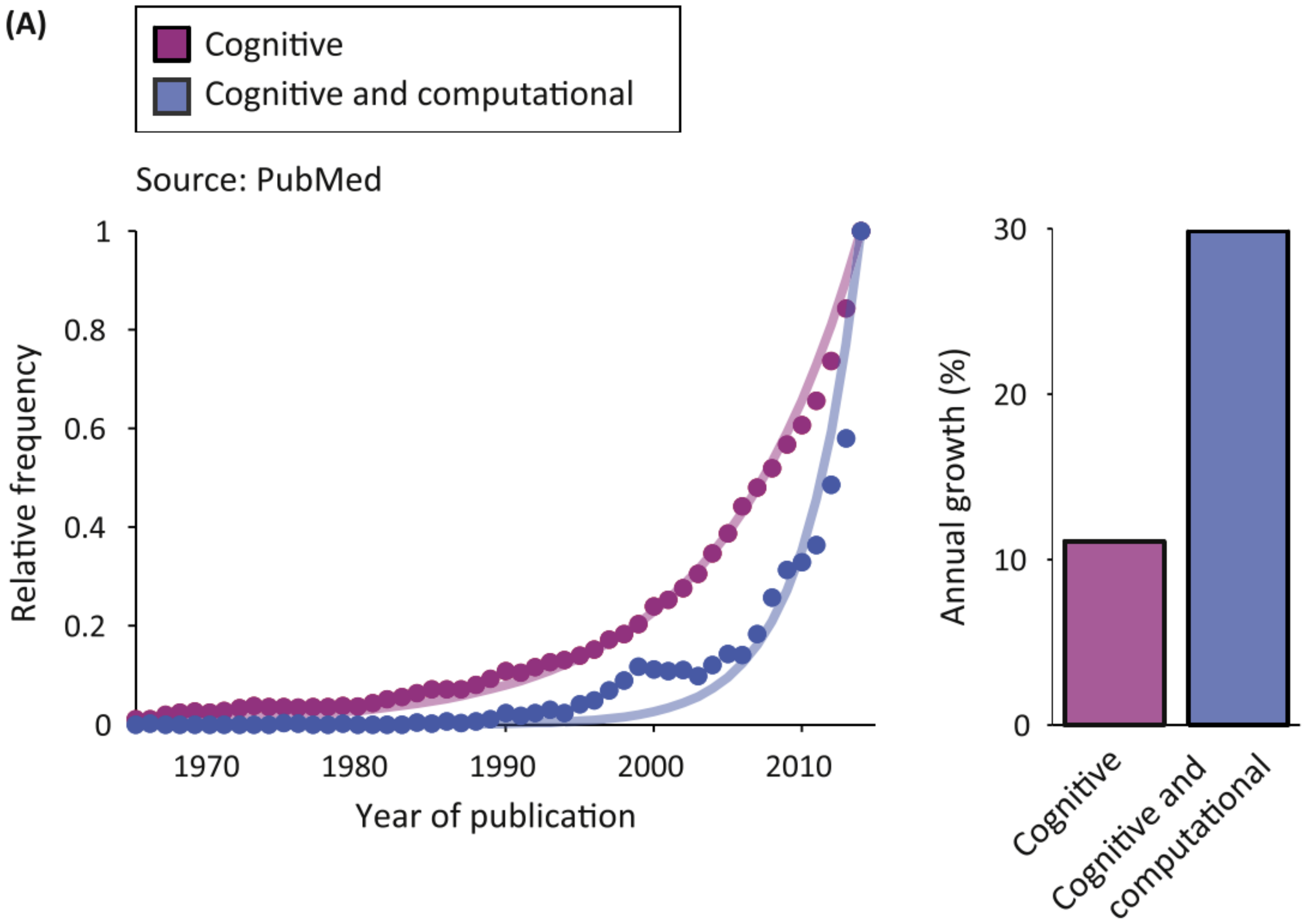


Computational modeling of Cognition



Lewandowsky & Farrell (2011); Stephan et al. (2016); Ahn et al., (2017)

Boom in Computational Modeling



Palminteri et al. (2017)

Very recent examples

REPORT

Dopamine promotes cognitive effort by biasing the benefits versus costs of cognitive work

A. Westbrook[1,2,3,*], R. van den Bosch[2,3], J. I. Määttä[2,3], L. Hofmans[2,3], D. Papadopetraki[2,3], R. Cools[2,3,†], M. J. Frank[1,4,†]

+ See all authors and affiliations

Science 20 Mar 2020:
Vol. 367, Issue 6484, pp. 1362-1366
DOI: 10.1126/science.aaz5891

Neuron

Available online 17 March 2020

In Press, Corrected Proof

CellPress

Article

A Neuro-computational Account of Arbitration between Choice Imitation and Goal Emulation during Human Observational Learning

Caroline J. Charpentier[1,2], Kiyohito Iigaya[1], John P. O'Doherty[1]

3 out of 4 focused on Reinforcement Learning models!

nature reviews neuroscience

Review Article | Published: 12 March 2020

The neural and computational systems of social learning

Andreas Olsson, Ewelina Knapska & Björn Lindström

Nature Reviews Neuroscience **21**, 197–212(2020) | Cite this article

Translational Psychiatry

Article | Open Access | Published: 17 March 2020

Neurocomputational mechanisms underpinning aberrant social learning in young adults with low self-esteem

Geert-Jan Will, Michael Moutoussis, Palee M. Womack, Edward T. Bullmore, Ian M. Goodyer, Peter Fonagy, Peter B. Jones, NSPN Consortium, Robb B. Rutledge & Raymond J. Dolan

Simple reinforcement learning: 2-armed bandit task

a simple task often used in the laboratory:

- repeated choice between N options (N-armed bandit)
- …whose properties (reward amounts, probabilities) are learned through trial-and-error
- …with a goal in mind: maximize the overall reward

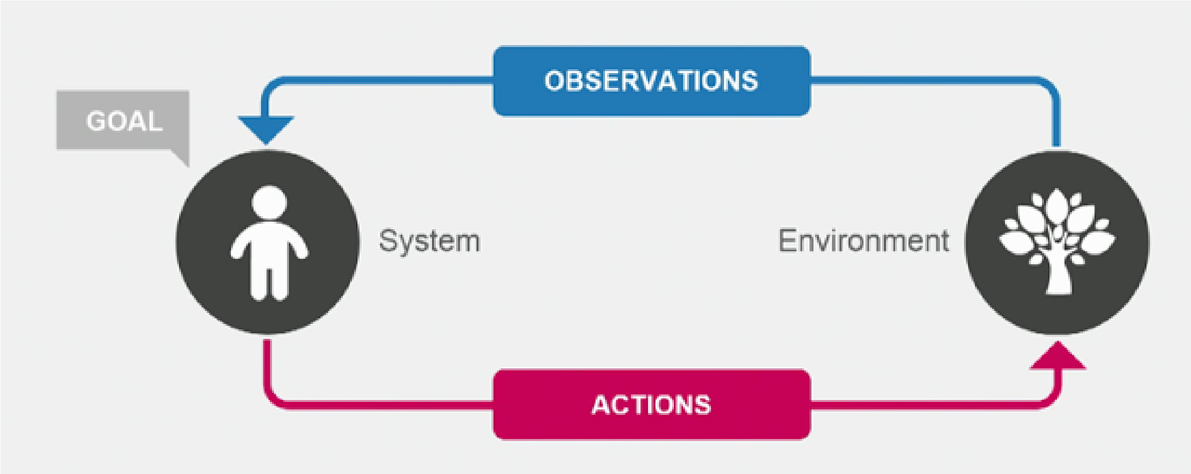
2-armed bandit task

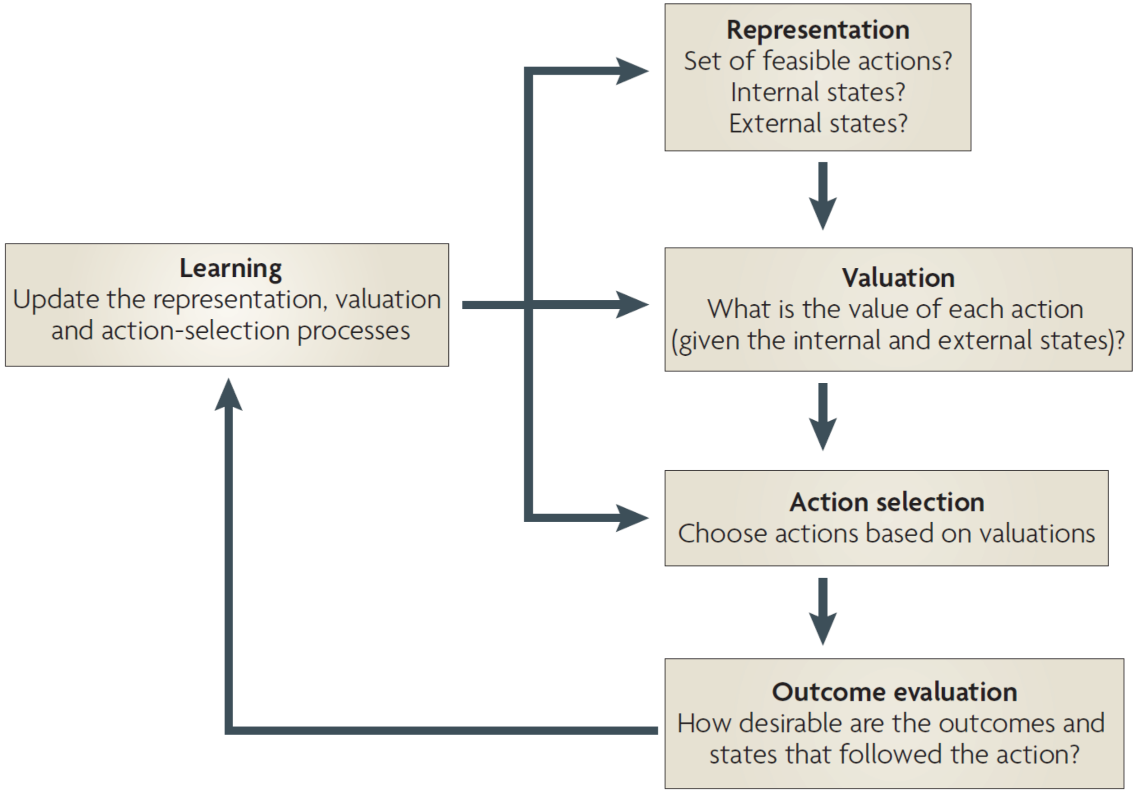
What can be your strategies:

1. predict the value of each deck
2. choose the best
3. learn from outcome to update predictions

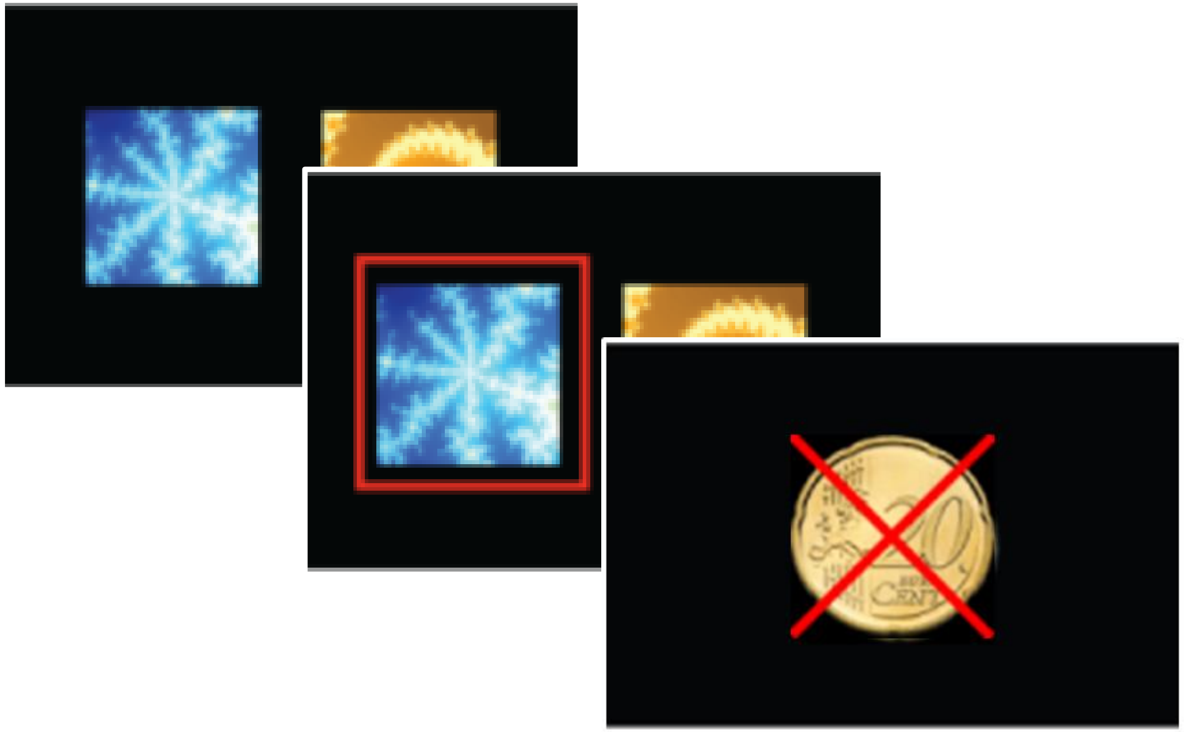
(repeat)

How prediction is shaped by learning?

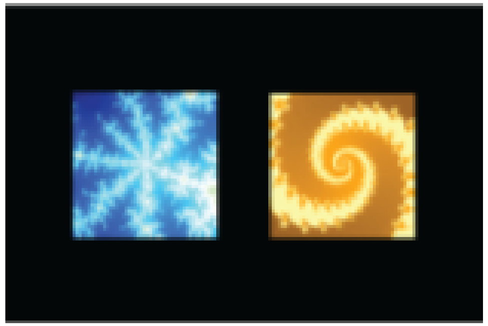




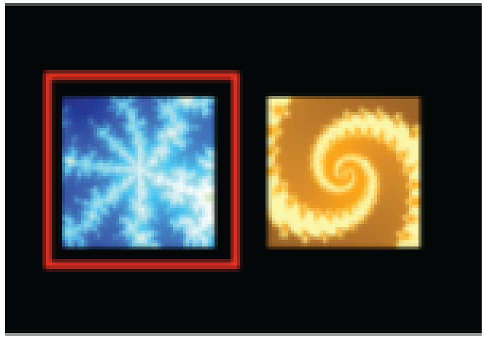
Sutton & Barto (1998); Rangel et al (2008); Daw & Niv workshop 2015 reward contingency 80:20



One simple experiment: two choice task



choice presentation



action selection

outcome

what do we know? — choice & outcome

what can we measure? — choice accuracy

what do we not know? — RL update

	subjID	trialID	choice	outcome
1	1	1	1	1
2	1	2	1	1
3	1	3	1	1

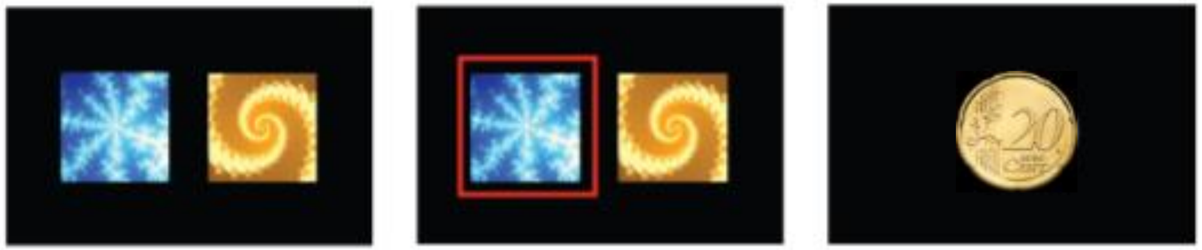
$p(\text{choosing the better option})$

Rescorla-Wagner (1972)

- The idea: error-driven learning
- Change in value is proportional to the difference between actual and predicted outcome

Robert A. Rescorla

Allan R. Wagner



Value update: $V_t = V_{t-1} + \alpha * \mathrm{PE}_{t-1}$

Prediction error: $\mathrm{PE}_{t-1} = R_{t-1} - V_{t-1}$

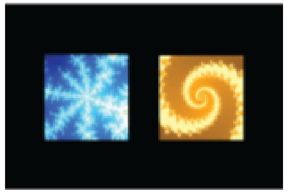
*Expectations on the next trial = the expectation on the current trial + learning rate * prediction error (reward – current expectation)*

Rescorla & Wagner (1972)

Understand the learning rate

Value update: $V_t = V_{t-1} + \alpha * \mathrm{PE}_{t-1}$

Prediction error: $\mathrm{PE}_{t-1} = R_{t-1} - V_{t-1}$



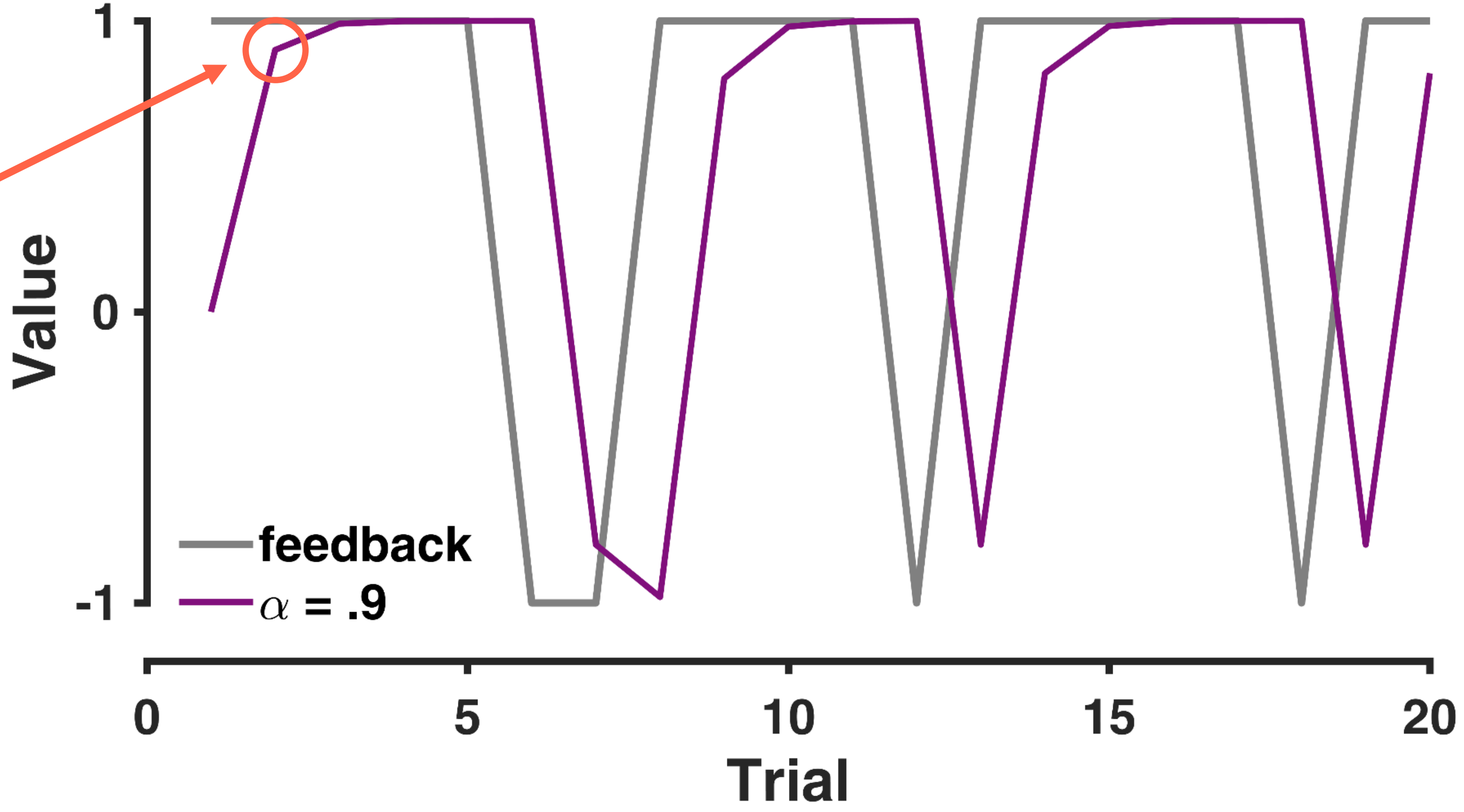
if $\alpha = 0.9$

$V_1 = 0$

$V_2 = V_1 + 0.9 * (1 - V_1)$

$= 0 + 0.9 * (1 - 0)$

$= 0.9$

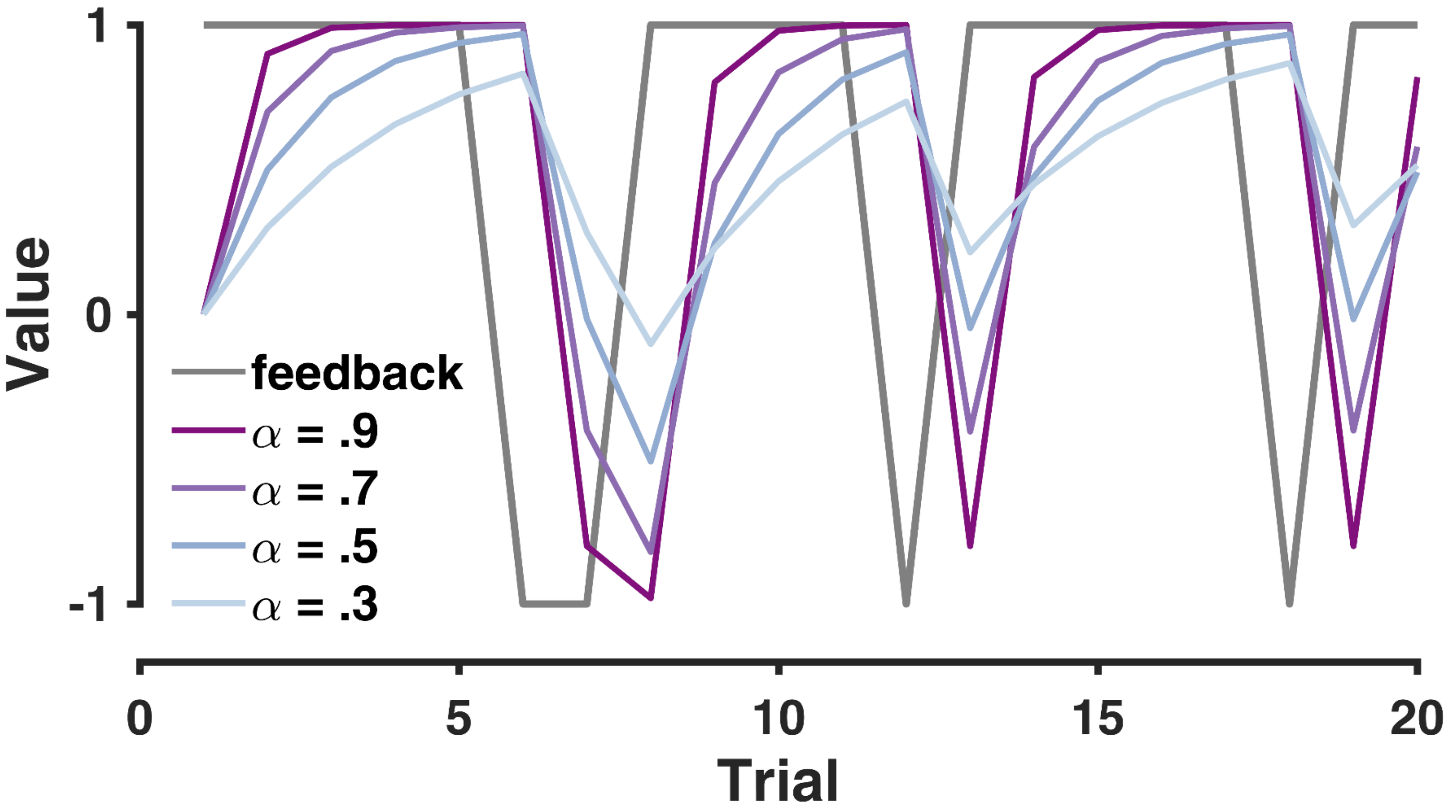


reward contingency – 80:20

Understand the learning rate

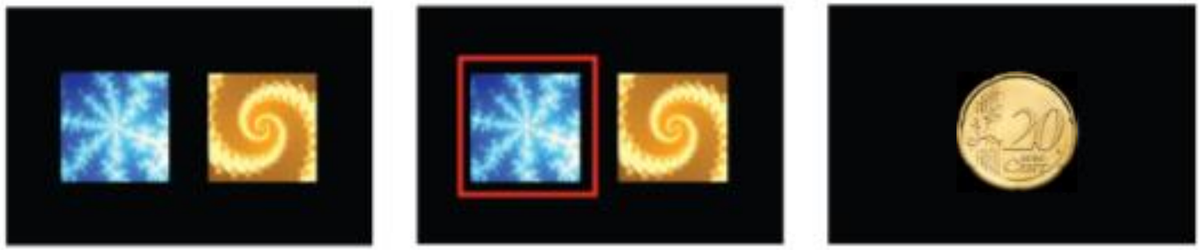
Value update: $V_t = V_{t-1} + \alpha * \mathrm{PE}_{t-1}$

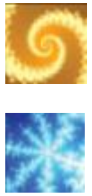
Prediction error: $\mathrm{PE}_{t-1} = R_{t-1} - V_{t-1}$



reward contingency – 80:20

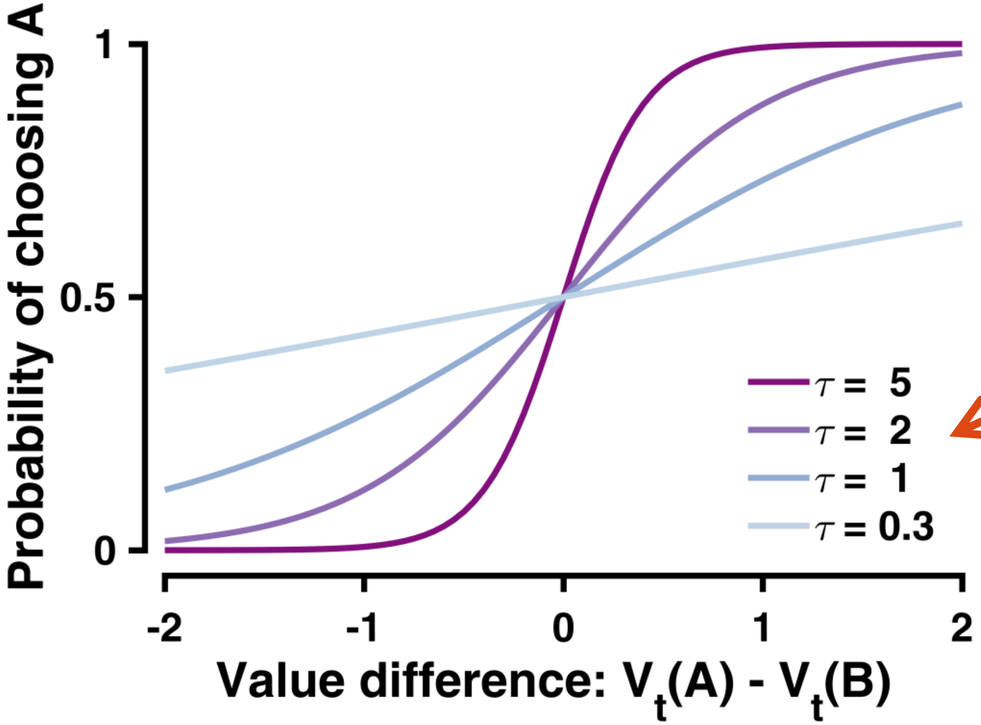
Choice rule: softmax



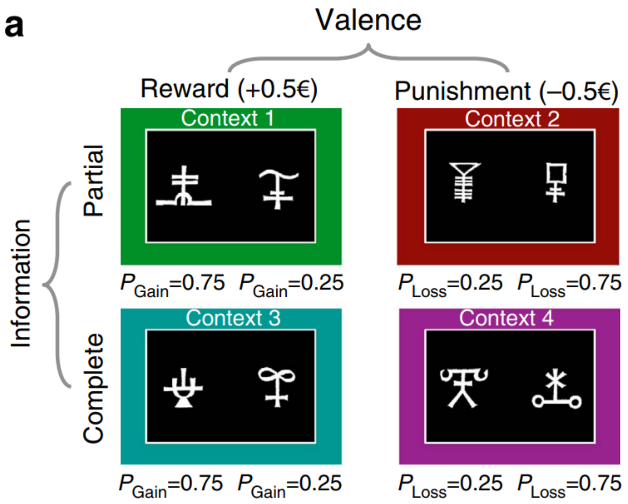
V()
V( )

$$\boxed{p_t(A) = \frac{e^{\tau * V_t(A)}}{e^{\tau * V_t(A)} + e^{\tau * V_t(B)}}}$$

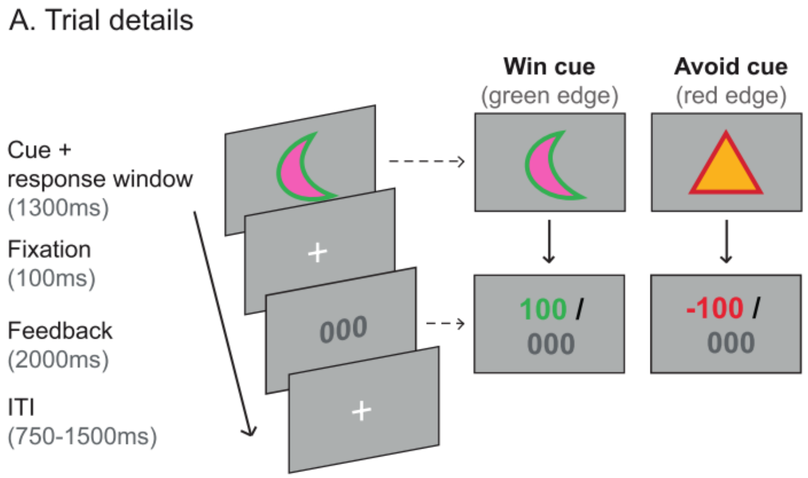
$$= \frac{1}{1 + e^{-\tau * (V_t(A) - V_t(B))}}$$



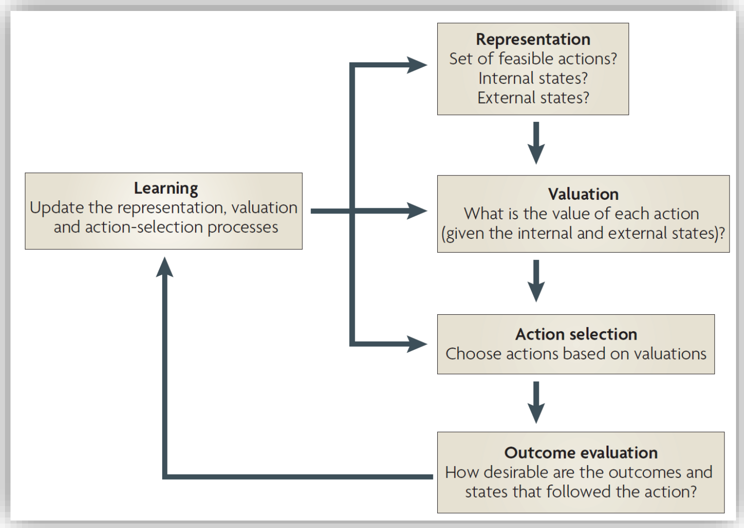
Generalizing RL framework

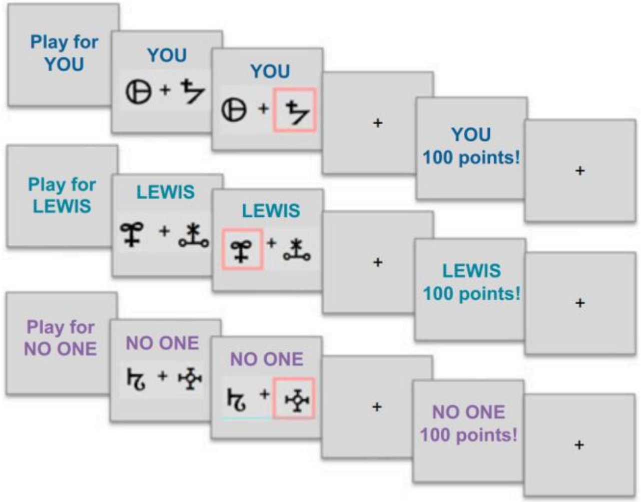


Palminteri et al. (2015)

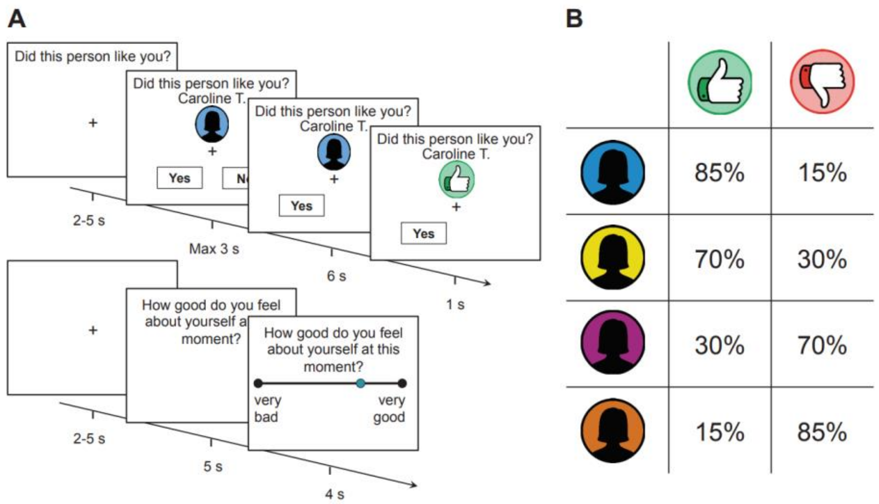


Swart et al. (2017)





Lockwood et al. (2016)



Will et al. (2017)

Bayesian analysis in Stan

Likelihood
How plausible is the data given our parameter is true?

Prior
How plausible is our parameter before observing the data?

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)}$$

Posterior
How plausible is our parameter given the observed data?

Evidence
How plausible is the data under all possible parameters?

Getting rid of the denominator

$$p(\theta \mid D) = \frac{p(D \mid \theta)\, p(\theta)}{\boxed{\int p(D \mid \theta^*)\, p(\theta^*)\, d\theta^*}}$$

$$p(data) = \int_{\text{All}\theta_1} \int_{\text{All}\theta_2} p(data, \theta_1, \theta_2) \mathrm{d}\theta_1 \mathrm{d}\theta_2$$

$$p(data) = \int_{\mu_1} \int_{\sigma_1} \cdots \int_{\mu_{100}} \int_{\sigma_{100}} \underbrace{p(data \mid \mu_1, \sigma_1, \ldots, \mu_{100}, \sigma_{100})}_{\text{likelihood}} \times \underbrace{p(\mu_1, \sigma_1, \ldots, \mu_{100}, \sigma_{100})}_{\text{prior}}$$

$$\mathrm{d}\mu_1 \mathrm{d}\sigma_1 \ldots \mathrm{d}\mu_{100} \mathrm{d}\sigma_{100},$$

$$\boxed{p(\theta \mid D) \propto p(D \mid \theta)\, p(\theta)}$$

important property that can be made use of by algorithms, e.g., Markov Chain Monte Carlo (MCMC)

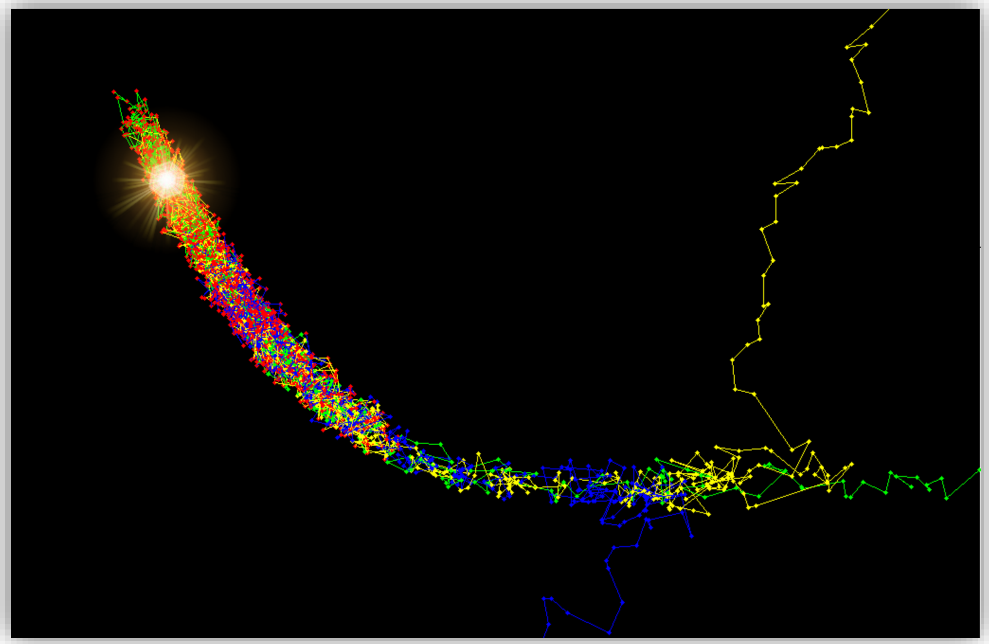
MCMC Sampling Algorithms

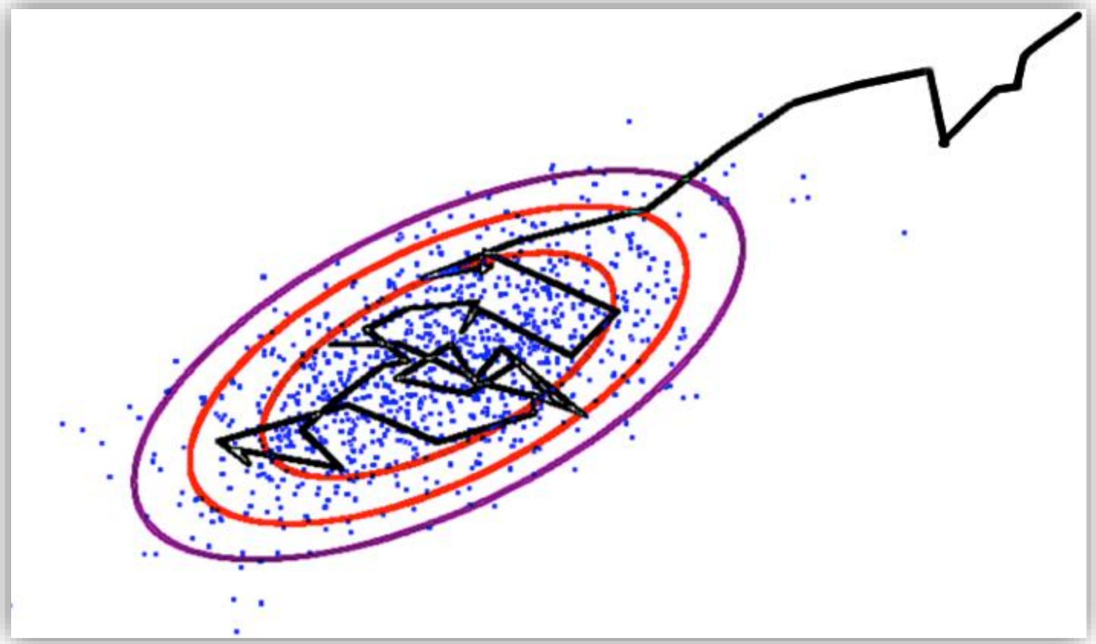
- Rejection sampling
- Importance sampling
- Metropolis algorithm
- Gibbs sampling → JAGS
- HMC sampling*

Stan!

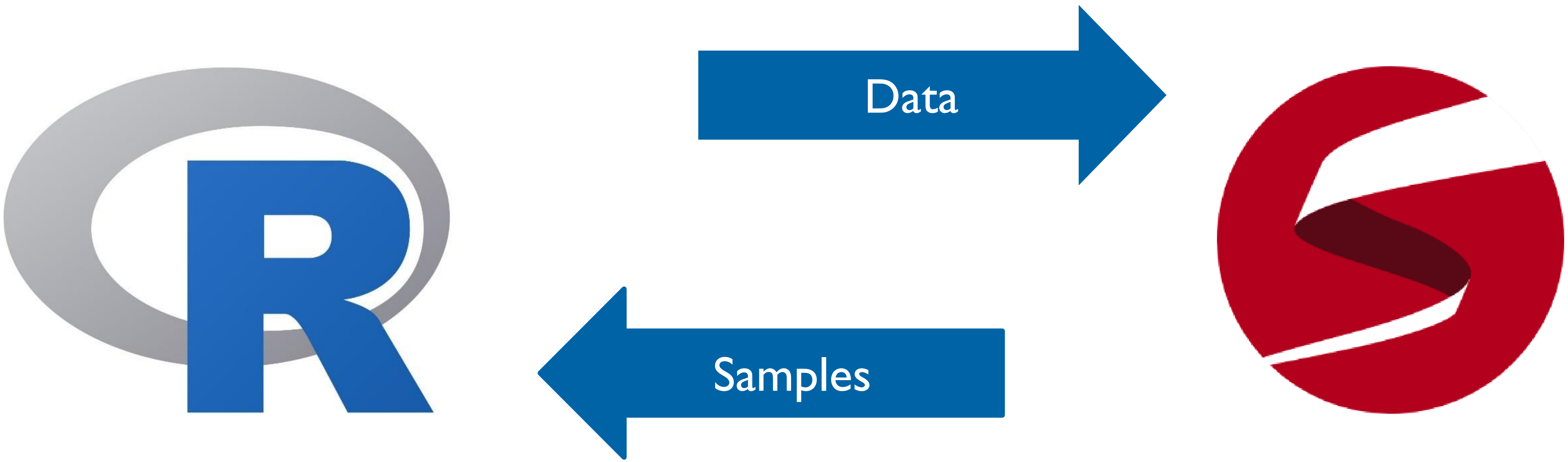
*: also PyMC3

Visual Example





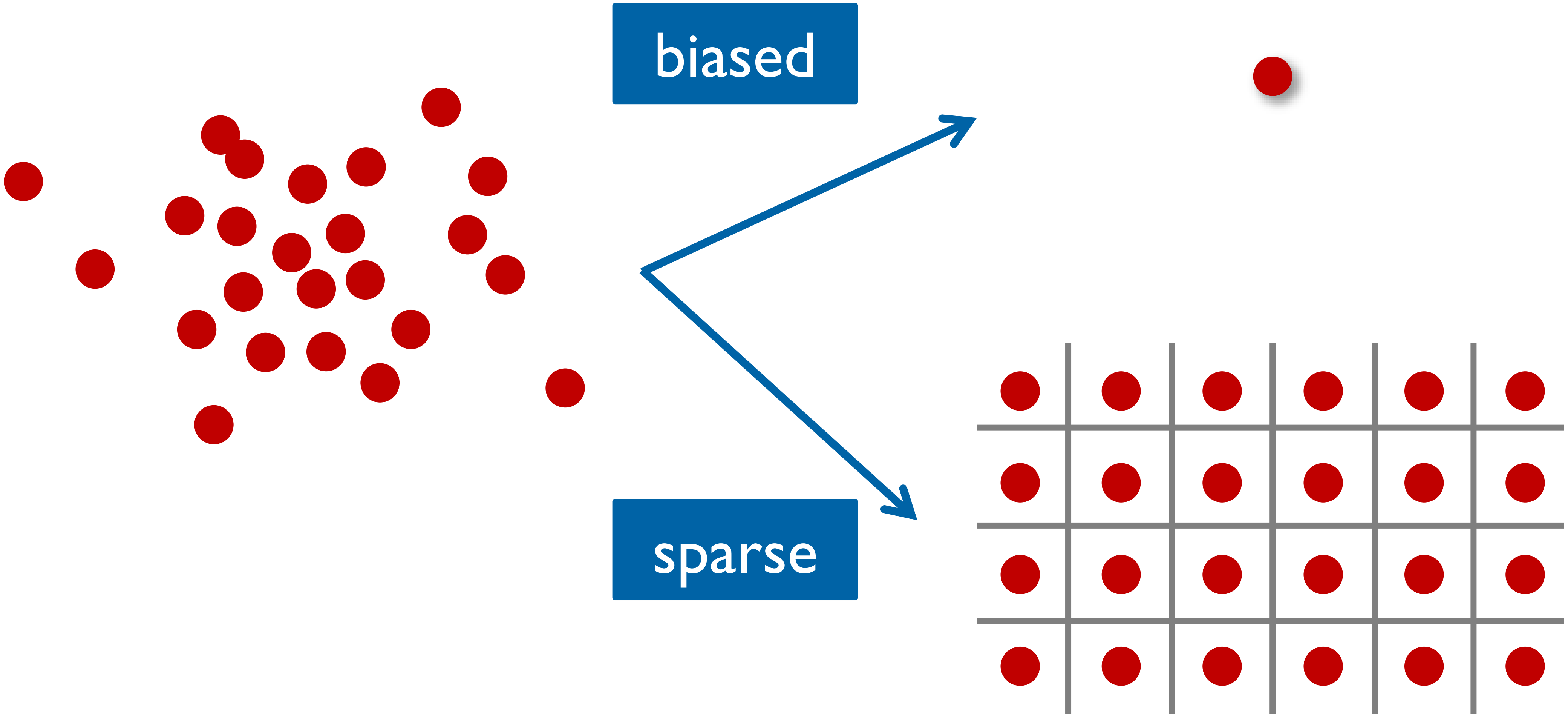
Stan and RStan



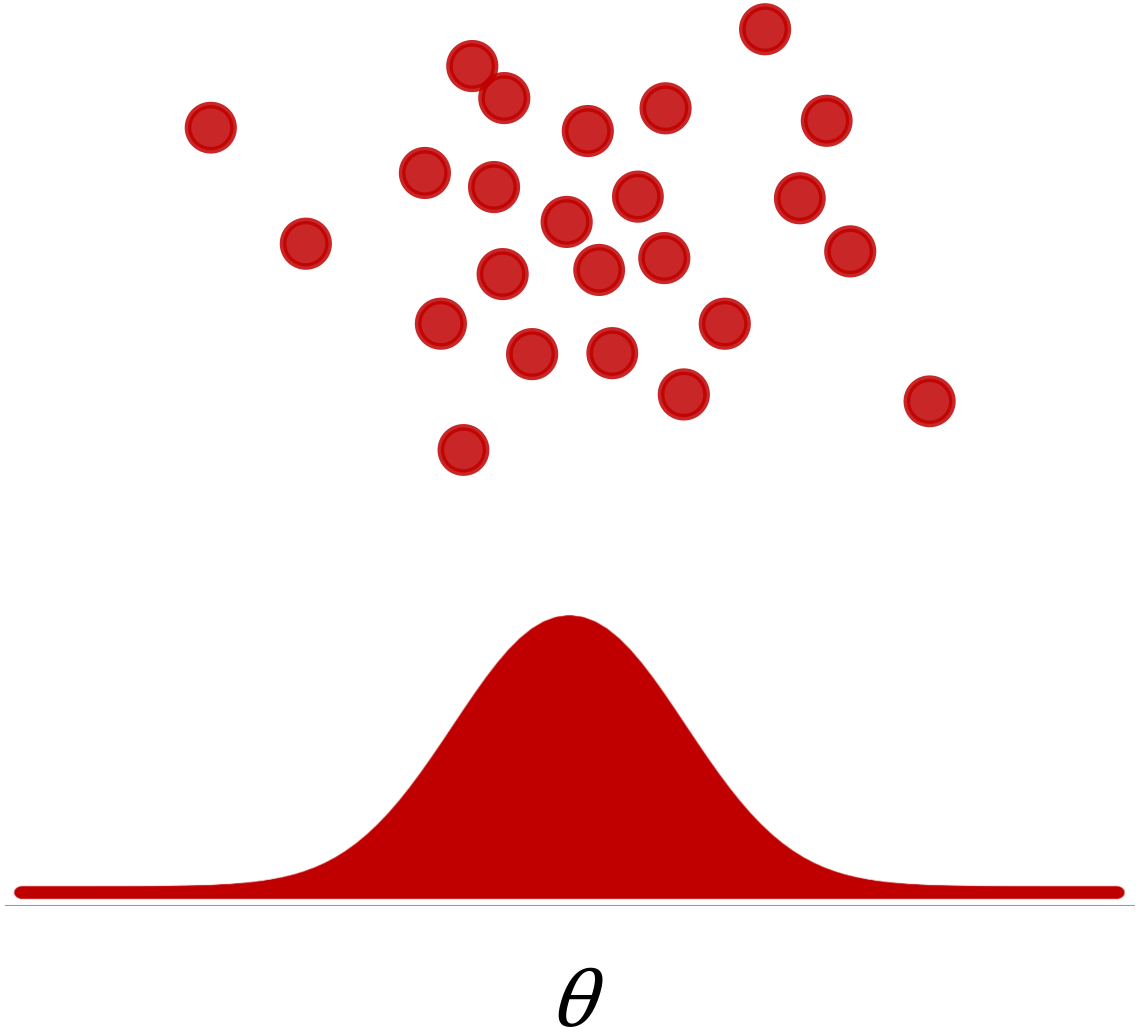
Stan Language

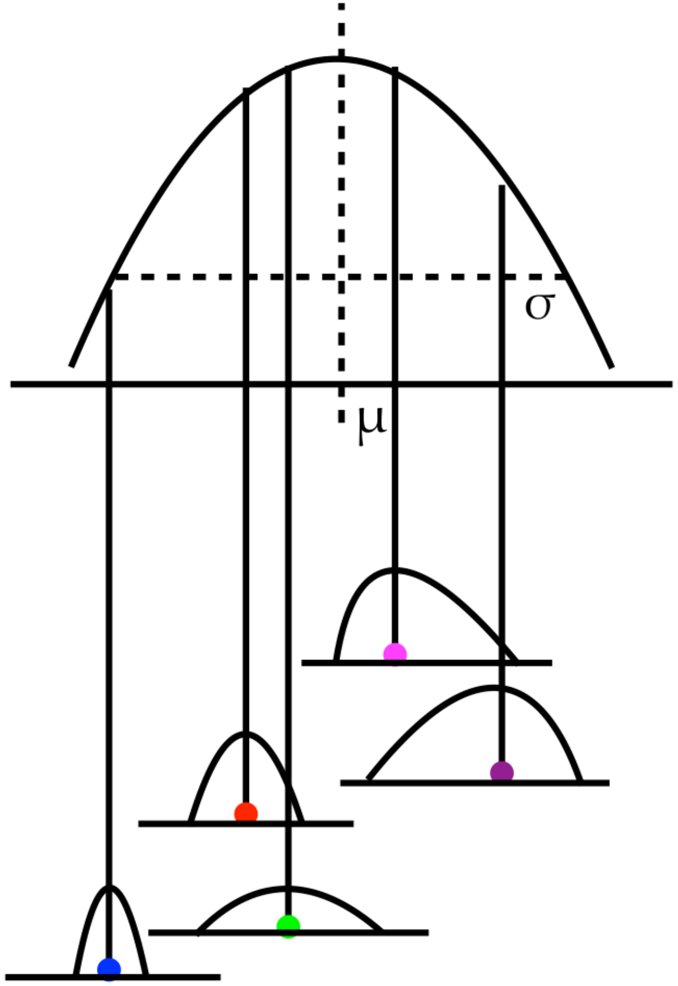
```
data {
 //... read in external data...
}

transformed data {
 //... pre-processing of data ...
}

parameters {
 //... parameters to be sampled by HMC ...
}

transformed parameters {
 //... pre-processing of parameters ...
}

model {
 //... statistical/cognitive model ...
}

generated quantities {
 //... post-processing of the model ...
}
```

Fitting Multiple Participants

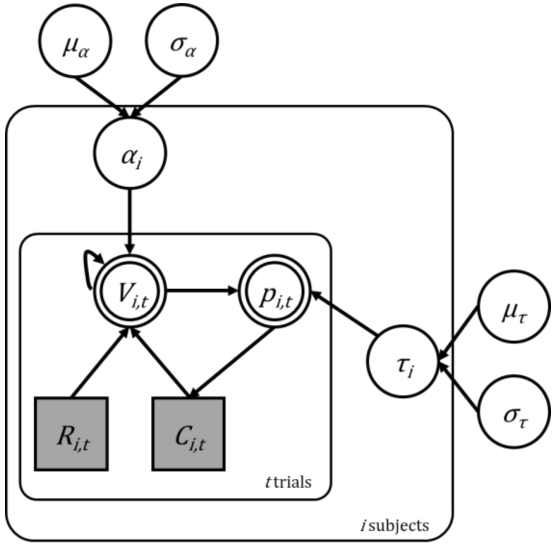


Fitting Multiple Participants with hierarchical Bayesian analysis (HBA)





HBA sounds good, but...



$$\mu_\alpha \sim Uniform(0,1)$$

$$\sigma_\alpha \sim halfCauchy(0,1)$$

$$\mu_\tau \sim Uniform(0,3)$$

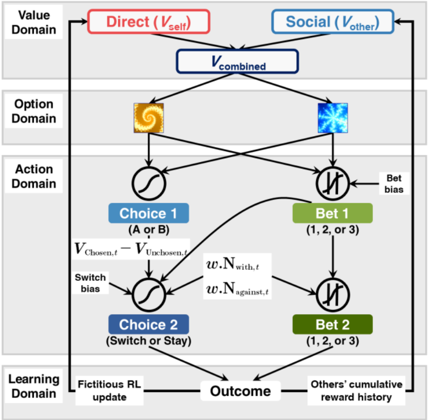
$$\sigma_\tau \sim halfCauchy(0,3)$$

$$\alpha_i \sim Normal(\mu_\alpha, \sigma_\alpha)_{\mathcal{T}(0,1)}$$

$$\tau_i \sim Normal(\mu_\tau, \sigma_\tau)_{\mathcal{T}(0,3)}$$

$$p_{i,t}(C=A) = \frac{1}{1+e^{\tau_i(V_{i,t}(B)-V_{i,t}(A))}}$$

$$V^c_{i,t+1} = V^C_{i,t} + \alpha_i(R_{i,t} - V^C_{i,t})$$



$$
\begin{aligned}
\mathbb{V}_{self,t} &= [V_{self,t}(\mathrm{A}), V_{self,t}(\mathrm{B})] \\
\mathbb{V}_{other,t} &= [V_{other,t}(\mathrm{A}), V_{other,t}(\mathrm{B})] \\
\mathbb{V}_t &= \beta_{vself}\,\mathbb{V}_{self,t} + \beta_{vother}\,\mathbb{V}_{other,t} \\
C1_t &\sim Categorical(Softmax(\mathbb{V}_t)) \\
U_{bet1,t} &= \beta_{bias_{b1}} + \beta_{vdiff_{b1}}(V_{chosen,C1,t} - V_{unchosen,C1,t}) \\
B1_t &\sim OrderedLogistic(U_{bet1,t} \mid \theta) \\
w.\mathrm{N}_{against,t} &= \frac{\sum_{s=1}^{K} w_{s,t}}{\sum_{s=1}^{4} w_{s,t}}, K = 0,1,\ldots,4 \\
w.\mathrm{N}_{with,t} &= \frac{\sum_{s=1}^{4-K} w_{s,t}}{\sum_{s=1}^{4} w_{s,t}} \\
V_t(\mathrm{switch}) &= \beta_{bias_{c2}} + \beta_{vdiff_{c2}}(V_{chosen,C1,t} - V_{unchosen,C1,t}) + \beta_{against}\,w.\mathrm{N}_{against,t} \\
C2 &\sim Bernoulli(V_t(\mathrm{switch})) \\
U_{bet2,t} &= \begin{cases} U_{bet1,t} + \beta_{with_{agr}}\,w.\mathrm{N}_{with,t} + \beta_{against_{agr}}\,w.\mathrm{N}_{against,t} & \text{, if } C1 = C2 \\ U_{bet1,t} + \beta_{with_{disagr}}\,w.\mathrm{N}_{with,t} + \beta_{against_{disagr}}\,w.\mathrm{N}_{against,t}, & \text{if } C1 \neq C2 \end{cases} \\
B2_t &\sim OrderedLogistic(U_{bet2,t} \mid \theta) \\
\Phi(x) &= \frac{1}{1+e^{-x}} \\
\delta_{self,chosen,C2,t} &= R_{self,t} - V_{self,chosen,C2,t} \\
\delta_{self,unchosen,C2,t} &= -R_{self,t} - V_{self,unchosen,C2,t} \\
V_{self,chosen,C2,t+1} &= V_{self,chosen,C2,t} + \alpha\delta_{self,chosen,C2,t} \\
V_{self,unchosen,C2,t+1} &= V_{self,unchosen,C2,t} + \alpha\delta_{self,unchosen,C2,t}
\end{aligned}
$$

cpsy

an open access journal

RESEARCH

Revealing Neurocomputational Mechanisms of Reinforcement Learning and Decision-Making With the hBayesDM Package

Woo-Young Ahn[1], Nathaniel Haines[1], and Lei Zhang[2]

[1]Department of Psychology, The Ohio State University, Columbus, OH

[2]Institute for Systems Neuroscience, University Medical Center Hamburg-Eppendorf, Hamburg, Germany

Keywords: reinforcement learning, decision-making, hierarchical Bayesian modeling, model-based fMRI

https://ccs-lab.github.io/hBayesDM/articles/getting_started.html

hBayesDM 1.0.2 | Reference | Articles | Changelog

Getting Started

Source: `vignettes/getting_started.Rmd`

hBayesDM (***h***ierarchical ***Bayes***ian modeling of ***D***ecision-***M***aking tasks) is a user-friendly R package that offers hierarchical Bayesian analysis of various computational models on an array of decision-making tasks. Click here to download its help file (reference manual). Click here to read our paper published in Computational Psychiatry. Click here to download a poster we presented at several conferences/meetings. You can find hBayesDM on CRAN and GitHub.

Recommended reading: tutorial

Using reinforcement learning models in social neuroscience: frameworks, pitfalls, and suggestions of best practices

AUTHORS
Lei Zhang, Lukas Lengersdorff, Nace Mikus, Jan Gläscher, Claus Lamm

CREATED ON November 06, 2019 LAST EDITED March 19, 2020

https://psyarxiv.com/uthw2

Ten simple rules for the computational modeling of behavioral data

Robert C Wilson, Anne GE Collins
University of Arizona, United States; University of California, Berkeley, United States

https://elifesciences.org/articles/49547

ACCEPTED MANUSCRIPT
Computational modelling of social cognition and behaviour—a reinforcement learning primer

Patricia L Lockwood, Miriam Klein-Flügge

Social Cognitive and Affective Neuroscience, nsaa040, https://doi.org/10.1093/scan/nsaa040
Published: 30 March 2020 **Article history** ▾

https://academic.oup.com/scan/advance-article/doi/10.1093/scan/nsaa040/5813717

The Importance of Falsification in Computational Cognitive Modeling

Stefano Palminteri,[1,2,*,‡] Valentin Wyart,[1,2,*,‡] and Etienne Koechlin[1,2,*]

https://doi.org/10.1016/j.tics.2017.03.011

Recommended reading: empirical work

Catecholaminergic challenge uncovers distinct Pavlovian and instrumental mechanisms of motivated (in)action

Jennifer C Swart, Monja I Froböse, Jennifer L Cook, Dirk EM Geurts, Michael J Frank, Roshan Cools, Hanneke EM den Ouden
Radboud University, The Netherlands; University of Birmingham, United Kingdom; Radboud University Medical Center, The Netherlands; Linguistic and Psychological Sciences, Brown University, United States; Brown University, United States

https://elifesciences.org/articles/22169

New Results

Comment on this paper

A brain network supporting social influences in human decision-making

Lei Zhang, Jan P. Gläscher

doi: https://doi.org/10.1101/551614

https://www.biorxiv.org/content/10.1101/551614v3

Social threat learning transfers to decision making in humans

Björn Lindström[a,b,c,1], Armita Golkar[c,d], Simon Jangard[c], Philippe N. Tobler[b], and Andreas Olsson[c]

[a]Department of Social Psychology, University of Amsterdam, 1018 WT Amsterdam, The Netherlands; [b]Laboratory for Social and Neural Systems Research, Department of Economics, University of Zürich, 8001 Zürich, Switzerland; [c]Section for Psychology, Department of Clinical Neuroscience, Karolinska Institutet, 171 77 Stockholm, Sweden; and [d]Department of Clinical Psychology, University of Amsterdam, 1018 WT Amsterdam, The Netherlands

https://www.pnas.org/content/116/10/4732.abstract

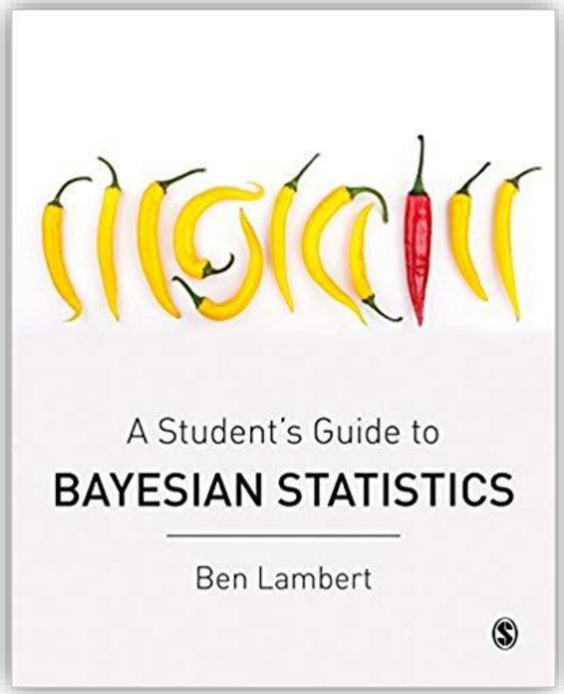
Article

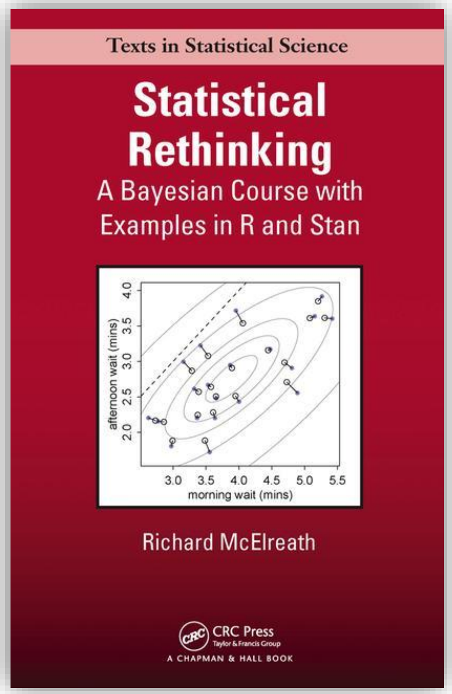
Primate Amygdala Neurons Simulate Decision Processes of Social Partners

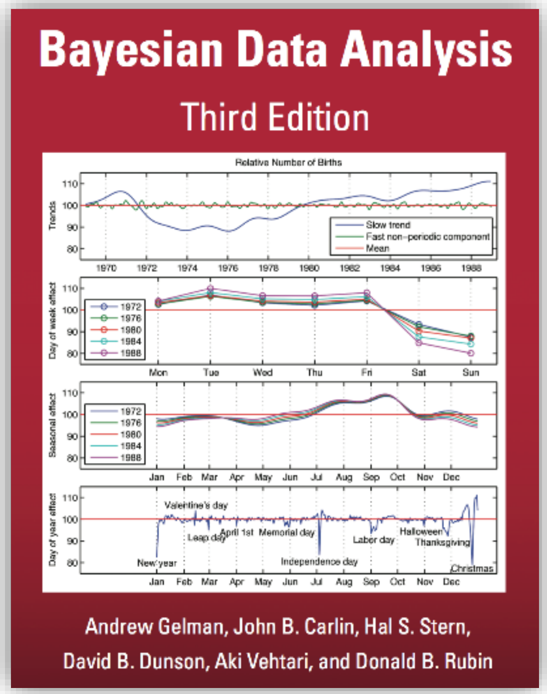
Fabian Grabenhorst[1,5], Raymundo Báez-Mendoza[1,4], Wilfried Genest[1], Gustavo Deco[2,3], Wolfram Schultz[1]

https://www.sciencedirect.com/science/article/pii/S0092867419302259

Recommended reading: book

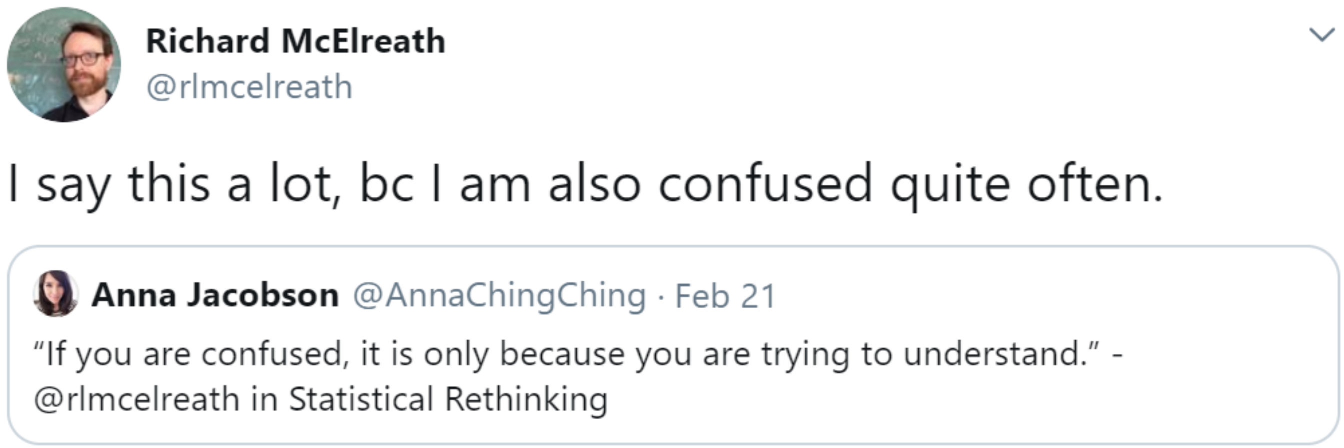






Summary

- Computational modeling is never new → don't let it fear you!
- Learn some statistics (e.g., different statistical distributions)
- Learn some math (e.g., linear algebra)
- Learn some programing (e.g., R/Python/Julia/Matlab)
- Learn to seek external help (e.g., existing packages)
- Learn in pairs; practice makes perfect!



Contact

lei.zhang@univie.ac.at

https://lei-zhang.net/

@lei_zhang_lz

@zhang-lei-44-62

@leizhang认知神经科学

@LeiZhang

@lei-zhang


ANY
QUESTIONS
?

Happy Computing!