

Table A1 (continued)

Measure	Binary and undirected definitions	Weighted and directed definitions
Modularity	<p>Modularity of the network (Newman, 2004b),</p> $Q = \sum_{u \in M} \left[ e_{uu} - \left( \sum_{v \in M} e_{uv} \right)^2 \right],$ <p>where the network is fully subdivided into a set of nonoverlapping modules <math>M</math>, and <math>e_{uv}</math> is the proportion of all links that connect nodes in module <math>u</math> with nodes in module <math>v</math>. An equivalent alternative formulation of the modularity (Newman, 2006) is given by <math>Q = \frac{1}{l} \sum_{i,j \in N} \left( a_{ij} - \frac{k_i k_j}{l} \right) \delta_{m_i, m_j}</math>, where <math>m_i</math> is the module containing node <math>i</math>, and <math>\delta_{m_i, m_j} = 1</math> if <math>m_i = m_j</math>, and 0 otherwise.</p>	<p>Weighted modularity (Newman, 2004),</p> $Q^w = \frac{1}{l^w} \sum_{i,j \in N} \left[ w_{ij} - \frac{k_i^w k_j^w}{l^w} \right] \delta_{m_i, m_j}.$ <p>Directed modularity (Leicht and Newman, 2008),</p> $Q^{\rightarrow} = \frac{1}{l} \sum_{i,j \in N} \left[ a_{ij} - \frac{k_i^{\text{out}} k_j^{\text{in}}}{l} \right] \delta_{m_i, m_j}.$
Measures of centrality		
Closeness centrality	<p>Closeness centrality of node <math>i</math> (e.g. Freeman, 1978),</p> $L_i^{-1} = \frac{n-1}{\sum_{j \in N, j \neq i} d_{ij}}.$	<p>Weighted closeness centrality, <math>(L_i^w)^{-1} = \frac{n-1}{\sum_{j \in N, j \neq i} d_{ij}^w}.</math> Directed closeness centrality, <math>(L_i^{\rightarrow})^{-1} = \frac{n-1}{\sum_{j \in N, j \neq i} d_{ij}^{\rightarrow}}.</math></p>
Betweenness centrality	<p>Betweenness centrality of node <math>i</math> (e.g., Freeman, 1978),</p> $b_i = \frac{1}{(n-1)(n-2)} \sum_{\substack{h,j \in N \\ h \neq j, h \neq i, j \neq i}} \frac{\rho_{hj}(i)}{\rho_{hj}},$ <p>where <math>\rho_{hj}</math> is the number of shortest paths between <math>h</math> and <math>j</math>, and <math>\rho_{hj}(i)</math> is the number of shortest paths between <math>h</math> and <math>j</math> that pass through <math>i</math>.</p>	<p>Betweenness centrality is computed equivalently on weighted and directed networks, provided that path lengths are computed on respective weighted or directed paths.</p>
Within-module degree z-score	<p>Within-module degree z-score of node <math>i</math> (Guimera and Amaral, 2005),</p> $z_i = \frac{k_i(m_i) - \bar{k}(m_i)}{\sigma^{k(m_i)}},$ <p>where <math>m_i</math> is the module containing node <math>i</math>, <math>k_i(m_i)</math> is the within-module degree of <math>i</math> (the number of links between <math>i</math> and all other nodes in <math>m_i</math>), and <math>\bar{k}(m_i)</math> and <math>\sigma^{k(m_i)}</math> are the respective mean and standard deviation of the within-module <math>m_i</math> degree distribution.</p>	<p>Weighted within-module degree z-score, <math>z_i^w = \frac{k_i^w(m_i) - \bar{k}^w(m_i)}{\sigma^{k^w(m_i)}}</math> Within-module out-degree z-score, <math>z_i^{\text{out}} = \frac{k_i^{\text{out}}(m_i) - \bar{k}^{\text{out}}(m_i)}{\sigma^{k^{\text{out}}(m_i)}}</math> Within-module in-degree z-score, <math>z_i^{\text{in}} = \frac{k_i^{\text{in}}(m_i) - \bar{k}^{\text{in}}(m_i)}{\sigma^{k^{\text{in}}(m_i)}}</math></p>



# Network / Graph topology: It's a Small World After All



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Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature*, 393(6684), 440-442.

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