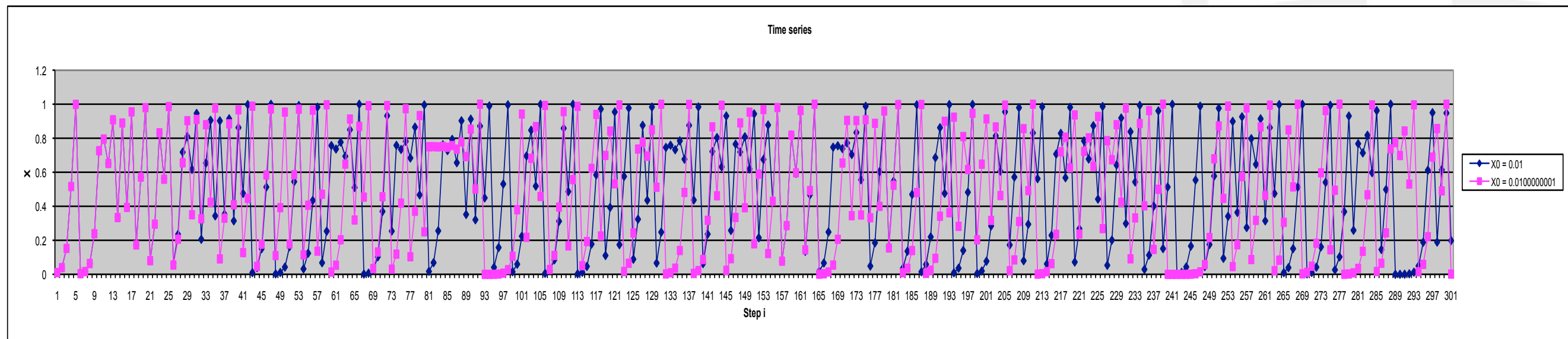


Sensitive Dependence on Initial Conditions

What *can* we say about deterministic chaos and complexity?

$$X_0 = 0.01$$

$$X_0 = 0.01000000001$$

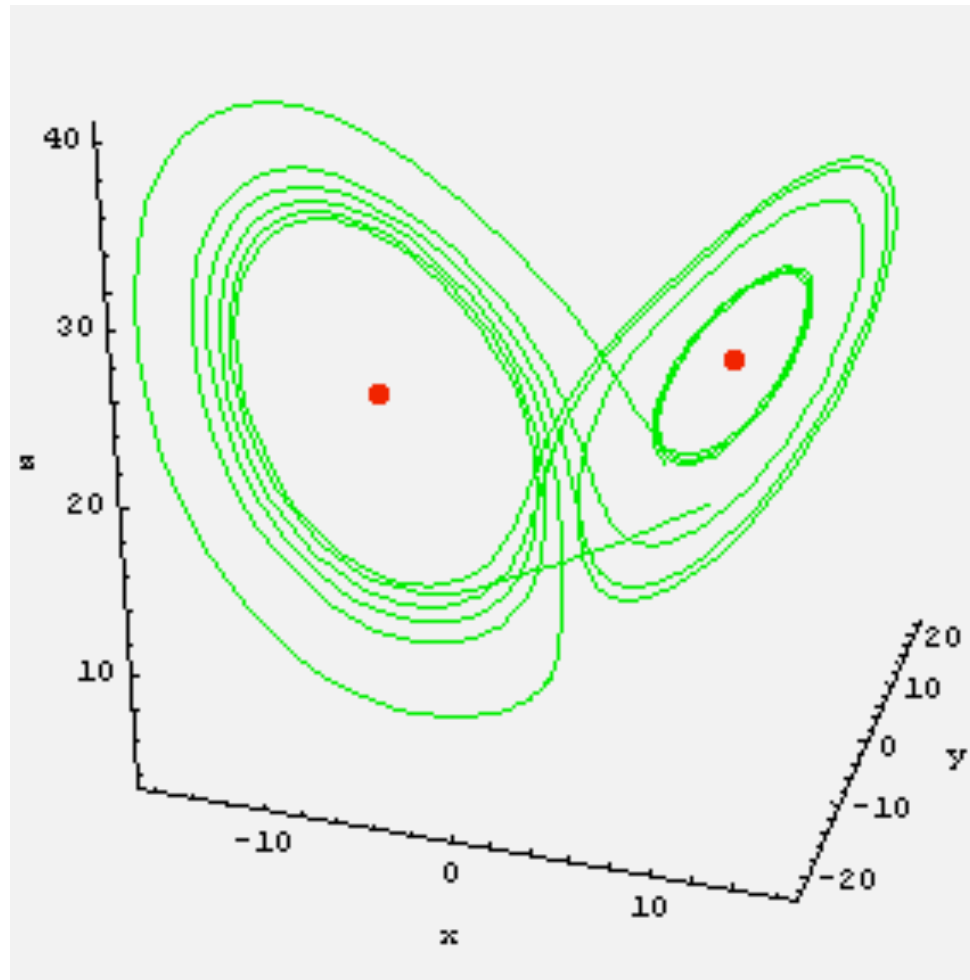


Tiny differences in initial conditions can yield diverging time-evolutions of system states

Lorenz observed this in his models of the upper atmosphere:

The divergence was so extreme it resembled a butterfly flapping its wings -or not- could be the difference between weather developing as a hurricane or a summer breeze

Lorenz Attractor

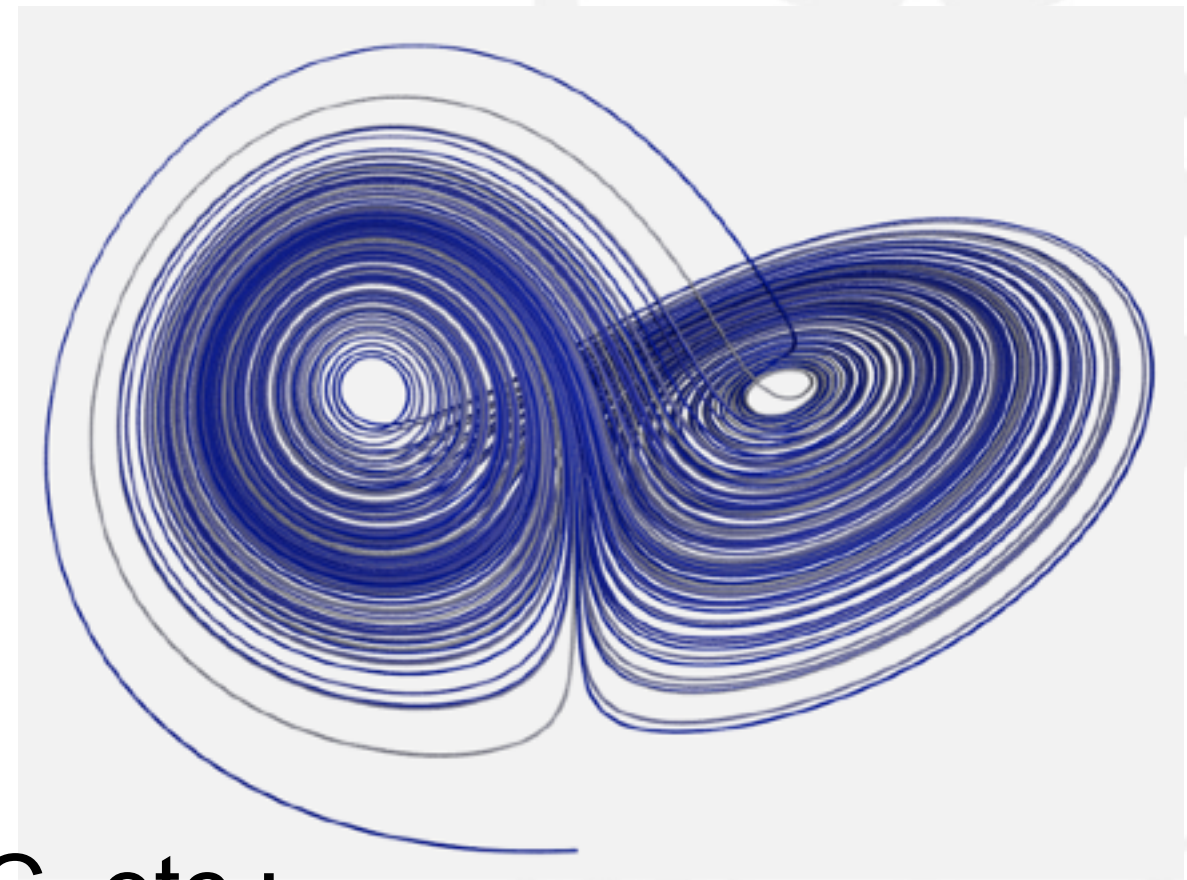


$$\begin{aligned}\frac{dx}{dt} &= a(y - x), \\ \frac{dy}{dt} &= x(b - z) - y, \\ \frac{dz}{dt} &= xy - cz.\end{aligned}$$

Deterministic Chaos

Maps: linear map, 1D state space

Flows: Need 3 coupled ODEs (ordinary differential equations)
Minimum is 3D state space



Lorenz about chaos, fractals, SOC, etc.:
“Study of things that look random -but are not”