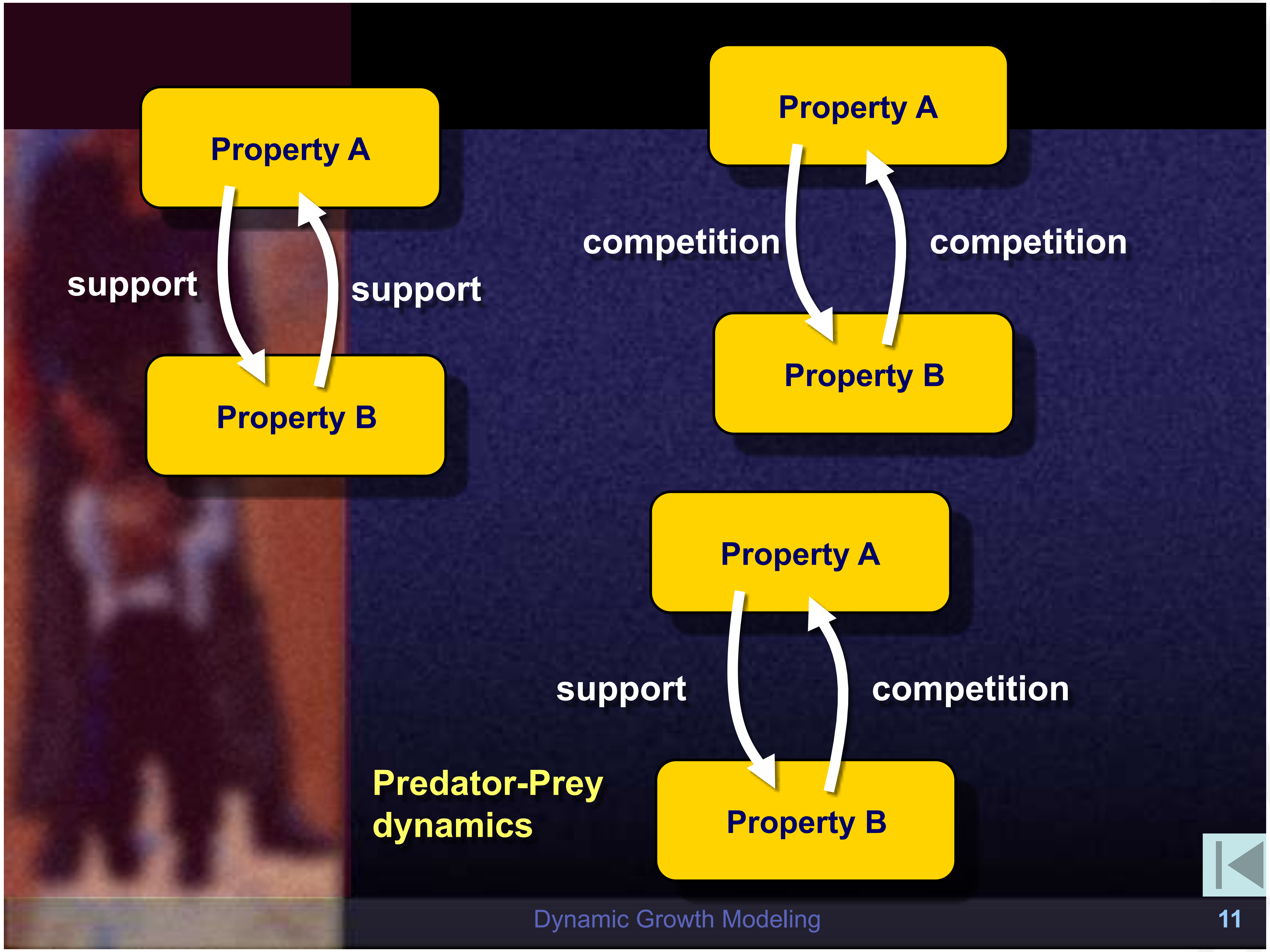
Coupled dynamics

Simple Interaction dynamics



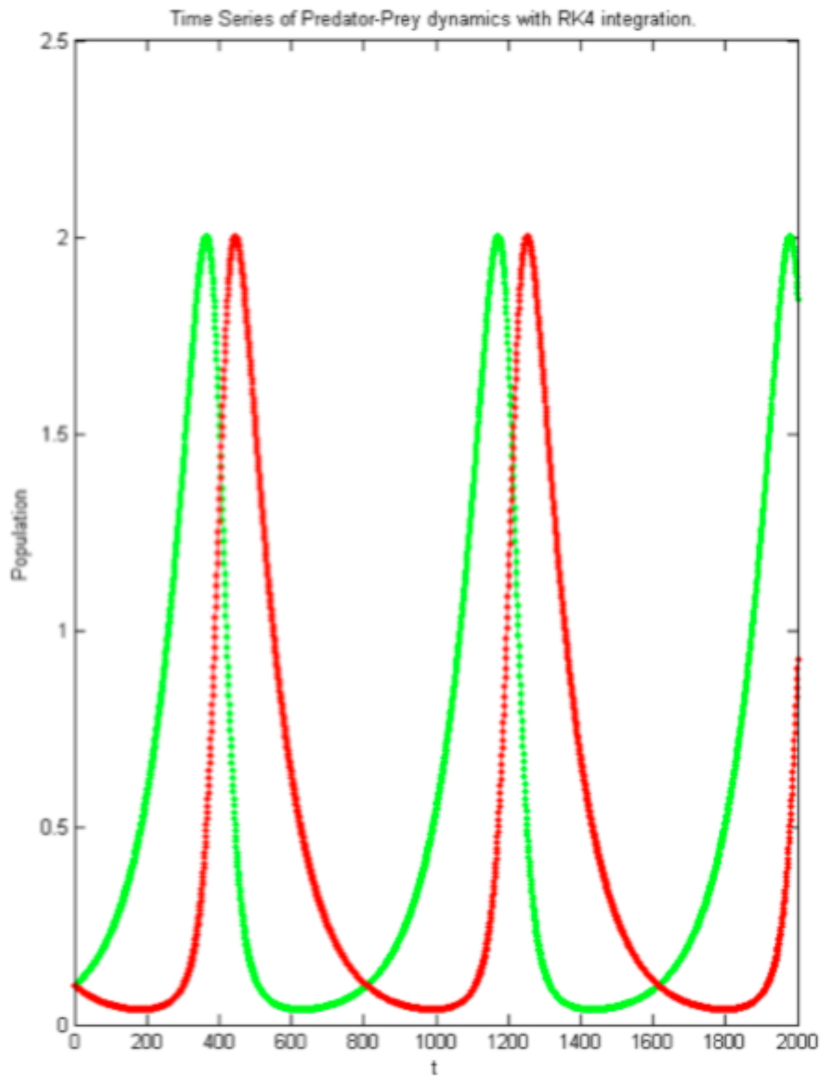
Predator-Prey model (Lotka-Volterra)

$$\frac{dR}{dt} = (a - b \times F) \times R,$$
$$\frac{dF}{dt} = (c \times R - d) \times F.$$

A 2-D state space
2 coupled flows ~

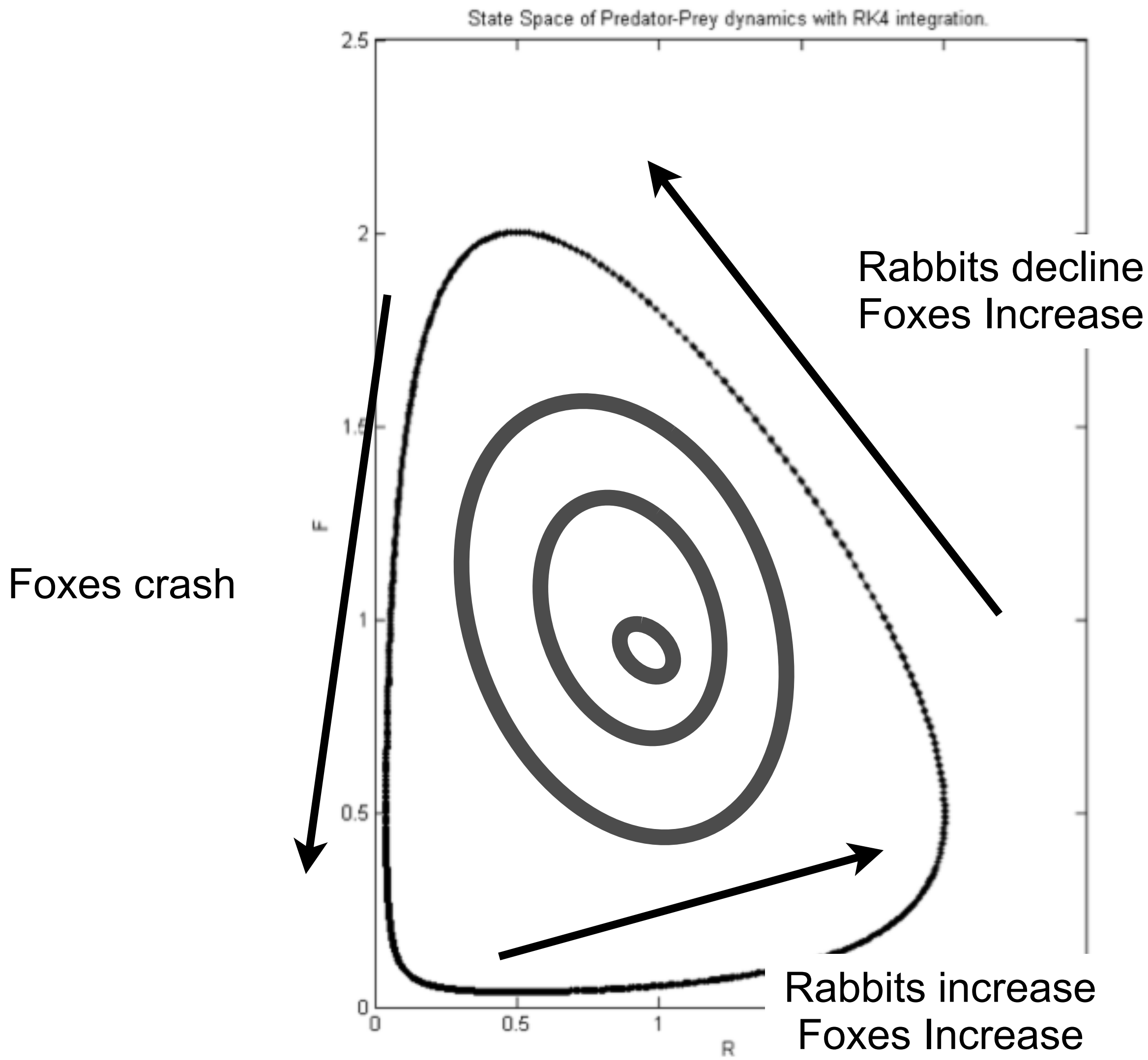
- *R* is the ***number of rabbits*** in a year
- *F* is the ***number of foxes*** in a year

Multivariate Models... Multivariate State Space

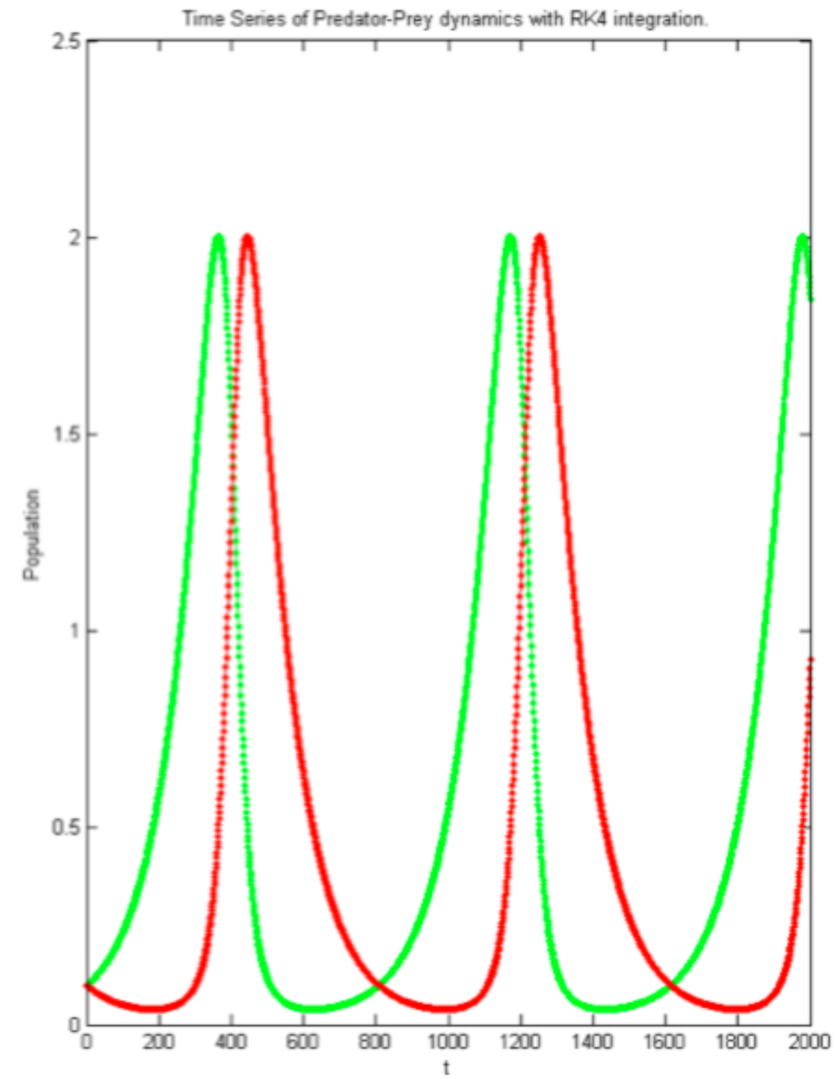


Time Series

Multivariate Models... Multivariate State Space



State Space



Time Series

Lorenz System

$$\frac{dx}{dt} = a(y - x),$$

$$\frac{dy}{dt} = x(b - z) - y,$$

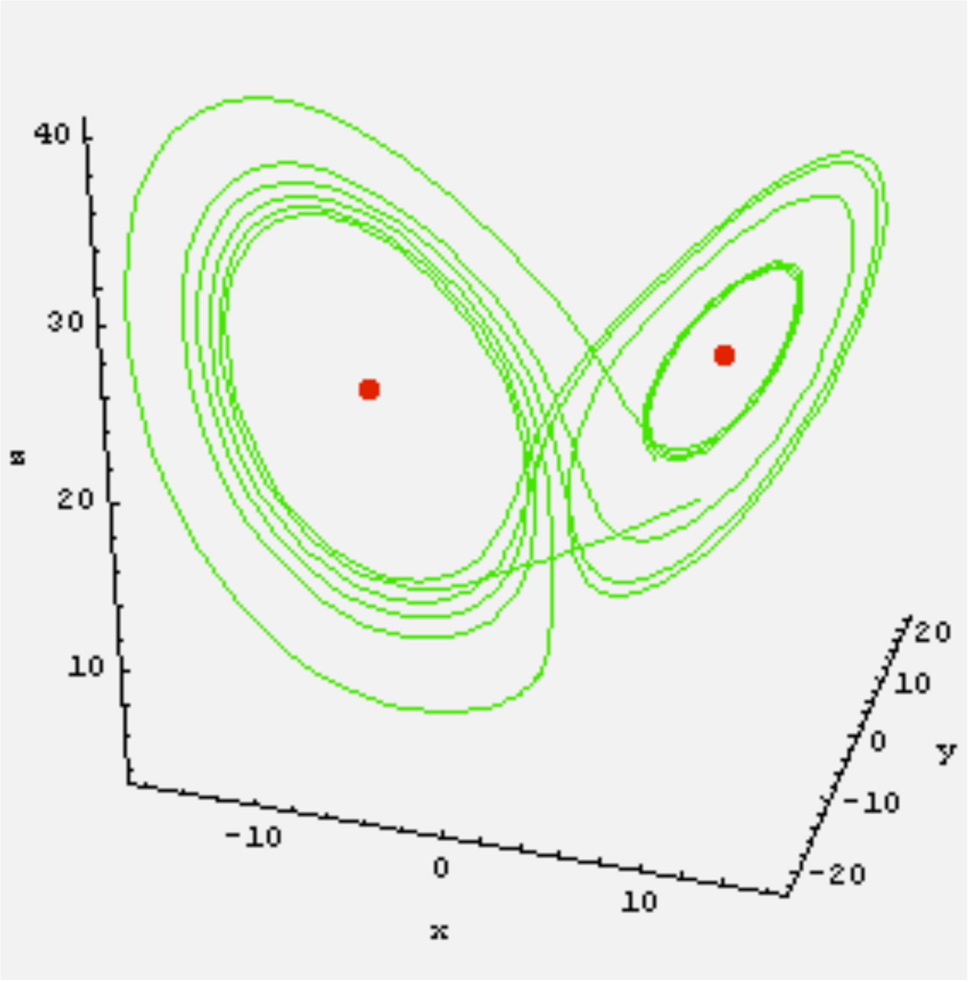
$$\frac{dz}{dt} = xy - cz.$$

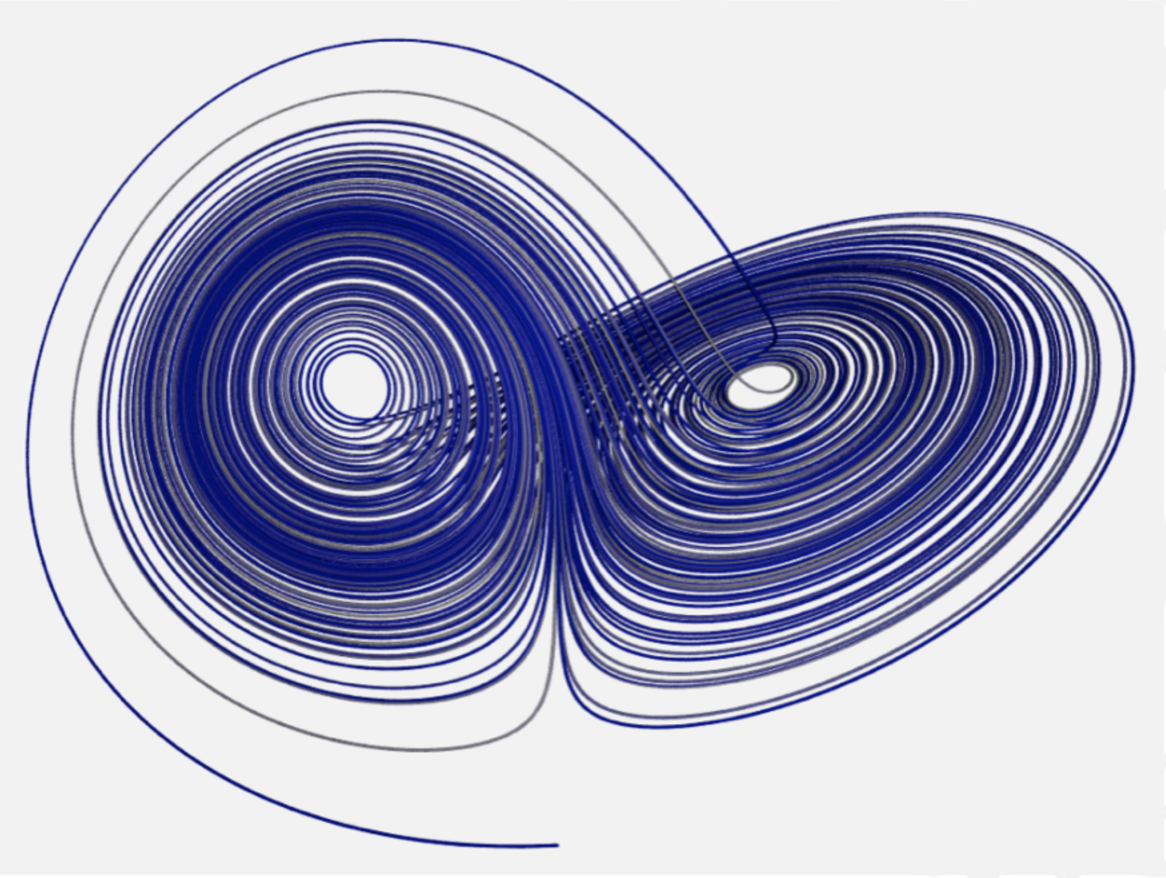
Interaction dominant dynamics
Multiple processes (3)
Multiple Scales (time)

x depends on y and x
y depends on x, y and z
z depends on x, y and z

A 3-D state space
3 coupled flows ~

Lorenz Attractor





A note on simulating differential equations: ~flows ~

Differential equations are **continuous**…

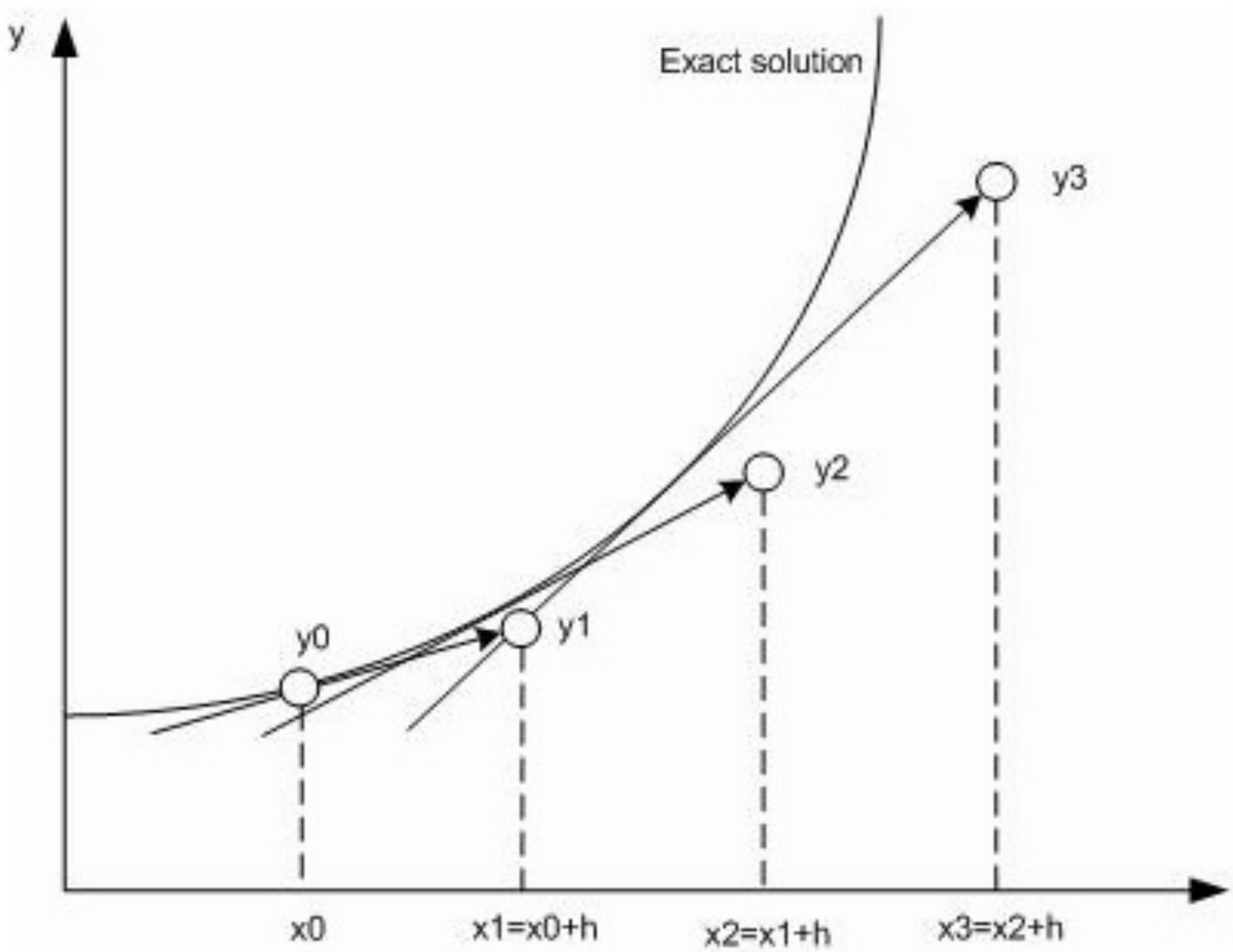
To find out how they behave when there is no solution we need to 'discretize' them and approximate the solution with a difference equation: Numerical integration

The easiest (but most error prone) method is Euler's method (18th century):

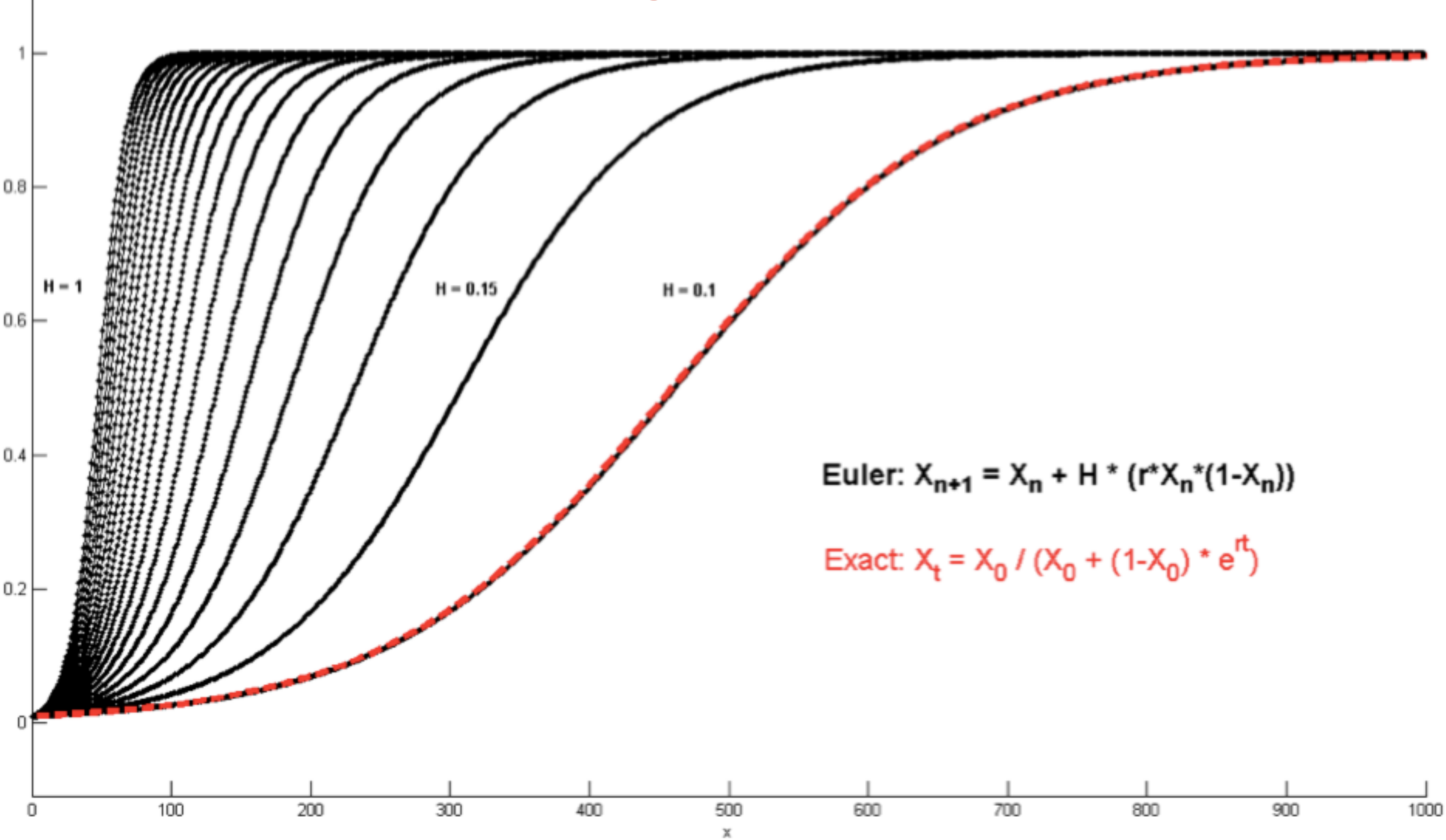
$\mathbf{X_{n+1} = X_{n+H} * f(X_n)}$ where **H** = step length

Checking how well the approximation is, can easily be done
if we know an analytic algebraic solution

A note on simulating differential equations: ~flows ~
Euler's method



Euler’s method for the logistic flow



Runge-Kutta 4th Order Method

$$\mathbf{k}_1 = h \cdot f(\mathbf{y}_n)$$

$$\mathbf{k}_2 = h \cdot f\left(\mathbf{y}_n + \frac{\mathbf{k}_1}{2}\right)$$

$$\mathbf{k}_3 = h \cdot f\left(\mathbf{y}_n + \frac{\mathbf{k}_2}{2}\right)$$

$$\mathbf{k}_4 = h \cdot f(\mathbf{y}_n + \mathbf{k}_3)$$

$$\Rightarrow \mathbf{y}_{n+1} = \mathbf{y}_n + \frac{\mathbf{k}_1}{6} + \frac{\mathbf{k}_2}{3} + \frac{\mathbf{k}_3}{3} + \frac{\mathbf{k}_4}{6}$$

Comparison of methods