





Radboud University Nijmegen





Behavioral Science Institute



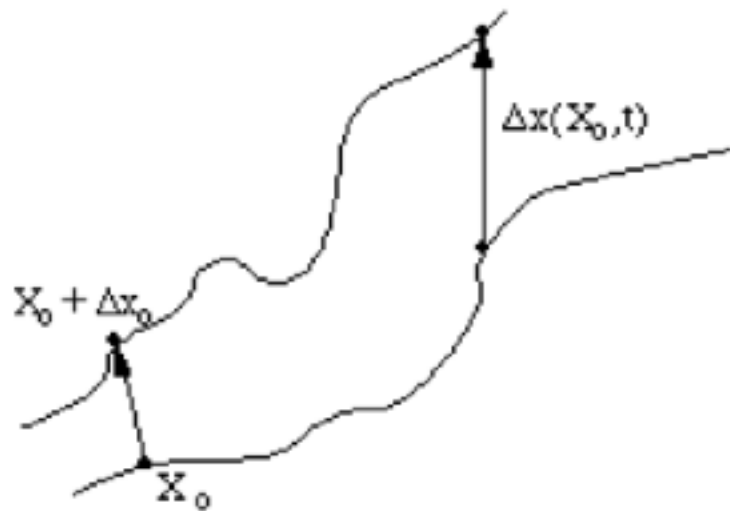
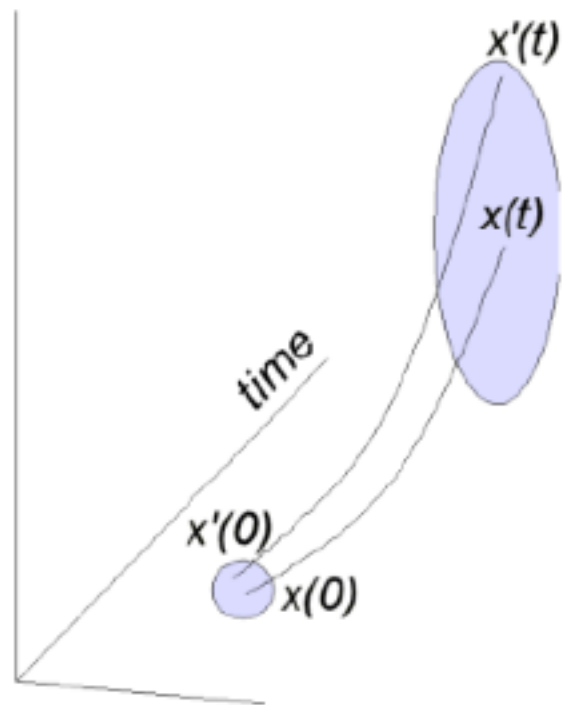
**CHAOS, TURBULENCE  
and other unsolved mysteries**

What can we say about chaos?

4. sensitive dependence on initial conditions



The *Lyapunov Exponent* characterises (quantifies) the rate of separation of two infinitesimally close trajectories in state space.



Calculate if you have a model

May be experimentally accessible

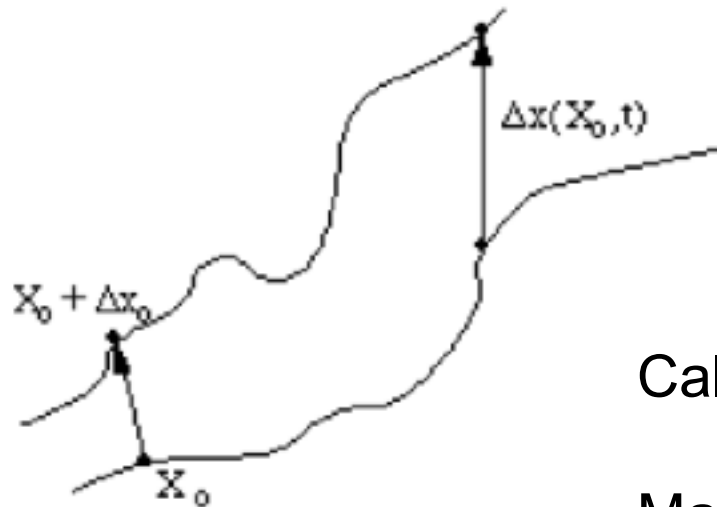
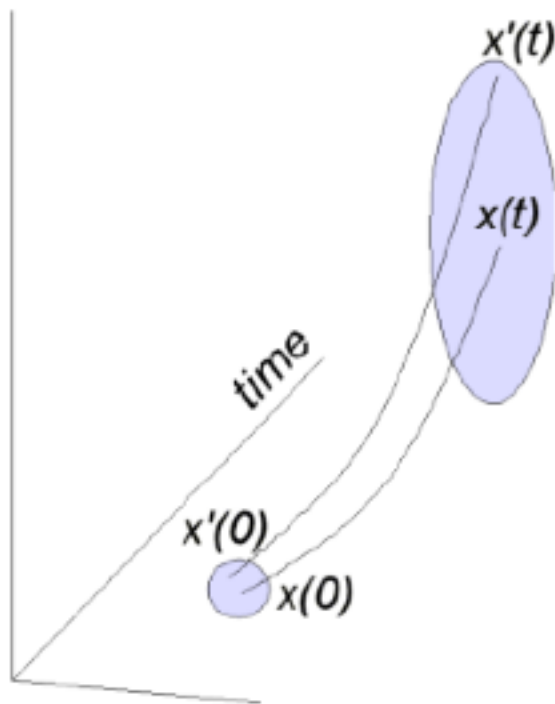
Analytic techniques (in R) are available

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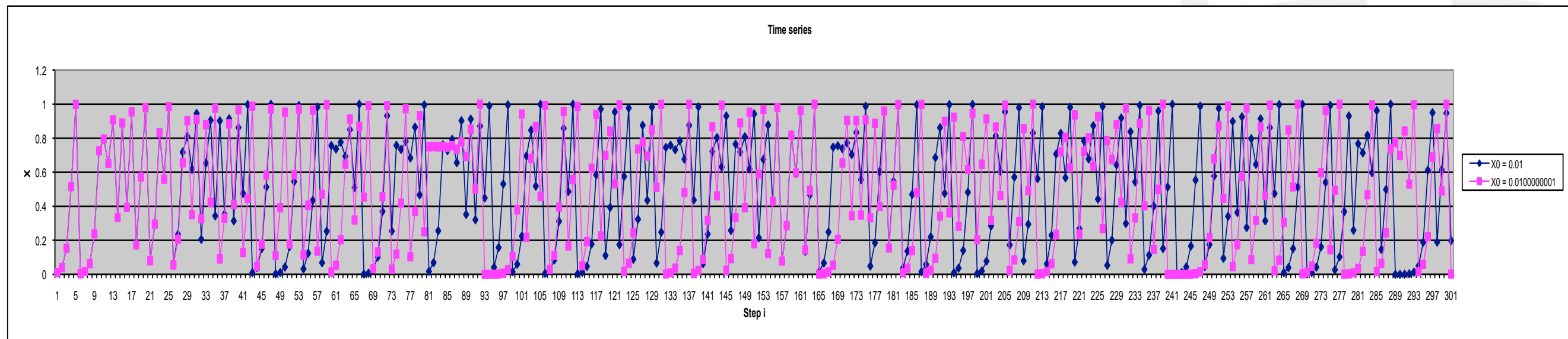
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# Sensitive Dependence on Initial Conditions

What *can* we say about deterministic chaos and complexity?

$$X_0 = 0.01$$

$$X_0 = 0.01000000001$$



Tiny differences in initial conditions can yield diverging time-evolutions of system states

Lorenz observed this in his models of the upper atmosphere:

The divergence was so extreme it resembled a butterfly flapping its wings -or not- could be the difference between weather developing as a hurricane or a summer breeze