A1 $\Rightarrow T(n) = 2T\left(\frac{n}{4}\right) + 3$

$T\left(\frac{n}{4^k}\right) = T(1)$

$\frac{n}{4^k} = 1$

$\log_4 n = k$

$T(n) = \cancel{2T\left(\frac{n}{4}\right)} + 3$ step 1

$\cancel{2T\left(\frac{n}{4}\right)} = \cancel{2^2 T\left(\frac{n}{4^2}\right)} + 2 \cdot 3$ step 2

$\cancel{2^2 T\left(\frac{n}{4^2}\right)} = \cancel{2^3 T\left(\frac{n}{4^3}\right)} + 2^2 \cdot 3$ step 3

$\cancel{2^3 T\left(\frac{n}{4^3}\right)} = \cancel{2^4 T\left(\frac{n}{4^4}\right)} + 2^3 \cdot 3$ step 4

$\cancel{2^{k-1} T\left(\frac{n}{4^{k-1}}\right)} = \cancel{2^k T\left(\frac{n}{4^k}\right)} + 2^{k-1} \cdot 3$

$\cancel{2^k T\left(\frac{n}{4^k}\right)} = 2^k \cdot 1$

$T(n) = 3(2^0 + 2^1 + 2^2 + \dots 2^{k-1}) + 2^k$

$= 3(2^k - 1) + 2^k = 3(n^{\log_4 2} - 1) + n^{\log_4 2}$

$O(\sqrt{n})$

master T. Case 1: $\Theta(n^{\log_b a})$ $f(n) = O(n^{\log_b a - \epsilon})$

$3 \le c n^{\log_4 2 - \epsilon}$ yes. Choose $\epsilon = \frac{\log_4 2}{2}$

$3 \le c n^{1/4}$

so $T(n)$ is $\Theta(n^{\log_4 2})$ or $\Theta(n^{1/2})$

A2) $T(n) = 3T\left(\frac{n}{4}\right) + 2n$ $\qquad \frac{n}{4^k} = 1$

$T(n) = 3\cancel{T\left(\frac{n}{4}\right)} + 2n$ step 1 $\qquad \log_4 n$

$3\cancel{T\left(\frac{n}{4}\right)} = 3^2 \cancel{T\left(\frac{n}{4^2}\right)} + 3\cdot 2\cdot \frac{n}{4}$ step 2

$3^2\cancel{T\left(\frac{n}{4^2}\right)} = 3^3 \cancel{T\left(\frac{n}{4^3}\right)} + 3^2\cdot 2\cdot \frac{n}{4^2}$ step 3

$3^{k-1}\cancel{T\left(\frac{n}{4^{k-1}}\right)} = \cancel{3^k T\left(\frac{n}{4^k}\right)} + 2n\cdot\left(\frac{3}{4}\right)^{k-1}$ step k

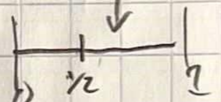
$\cancel{3^k T\left(\frac{n}{4^k}\right)} = 3^k \cdot 1$ $\qquad O(n)$

$$T(n) = 2n\left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \cdots + \left(\frac{3}{4}\right)^{k-1}\right] + 3^k$$

$$= 2n\left[\frac{1 - \frac{3^k}{4^k}}{1/4}\right] + 3^k = 2n\left[\left(1 - \frac{3^{\log_4 n}}{n}\right)4\right] + 3^{\log_4 n}$$

$\log_4 3$

$$= 2n\left[\left(1 - n^{\log_4 3 - 1}\right)4\right] + n^{\log_4 3} \rightarrow$$



MT. Case 1: $\theta(n^{\log_4 3})$ $\qquad 2n \leq cn^{\log_4 3 - \epsilon}$ NO

Case 2: $\theta(n^{\log_4 3}\log n)$ $\qquad 2n = \theta(n^{\log_4 3})$? NO

Case 3: $\theta(2n)$ $\qquad 2n \geq cn^{\log_4 3 + \epsilon}$? yes choose $\epsilon = \frac{\log_4 3 - 1}{2}$ & $c = 1$ for $n \geq 2$

And $3\cdot \cancel{2}\left(\frac{n}{\cancel{4}}\right) = 3\cdot\frac{n}{2} < c2n \rightarrow 3n < c4n$ yes choose $c = 1$ for $n \geq 1$

So $T(n)$ is $\theta(n)$

A3) $T(n) = \cancel{T(n-2)} + 3$ step 1

$\cancel{T(n-2)} = \cancel{T(n-4)} + 3$ $\quad 2 \to 2(2-1) = 2$

$\cancel{T(n-4)} = \cancel{T(n-6)} + 3$ $\quad 3 \to 2(3-1) = 2 \cdot 2 = 4$

$\cancel{T(n-6)} = \cancel{T(n-8)} + 3$ $\quad 4 \to 2(4-1) = 2 \cdot 3 = 6$

$\cancel{T(n-2(k-1))} = \cancel{T(n-2k)} + 3$ step k

$\cancel{T(n-2(\frac{n}{2}-1))} = T(n-2(\frac{n}{2})) + 3$ $\quad 2^{nd}$ last step

$T(n-2(\frac{n}{2})) = T(0) = 1$

$T(n) = 3 + 3 + \dots + 3 + 1 = 3(\frac{n}{2}) + 1$ $\quad O(n)$

A4) $T(n) = \cancel{2T(n-1)} + 1$ step 1

$\cancel{2T(n-1)} = \cancel{2^2 T(n-2)} + 2 \cdot 1$ $\quad 2$

$\cancel{2^2 T(n-2)} = \cancel{2^3 T(n-3)} + 2^2 \cdot 1$ $\quad 3$

$\cancel{2^{k-1} T(n-(k-1))} = \cancel{2^k T(n-k)} + 2^{k-1} \cdot 1$ $\quad k$

$\cancel{2^{n-2} T(n-(n-2))} = \cancel{2^{n-1} T(n-(n-1))} + 2^{n-2} \cdot 1$ $\quad 2^{nd}$ last

$\cancel{2^{n-1} T(n-(n-1))} = 2^{n-1} \cdot 1$

$T(n) = 2^0 + 2^1 + 2^2 + \dots + 2^{n-1} = \boxed{2^n - 1}$

$O(2^n)$

(A5) $T(n) = 4T(\frac{n}{2}) + n\log(n)$ step 1

$4T(\frac{n}{2}) = 4^2T(\frac{n}{2^2}) + 4\frac{n}{2}\log(\frac{n}{2})$ step 2

$= \frac{4}{2}n(\log(n)-1)$

$4^2T(\frac{n}{2^2}) = 4^3T(\frac{n}{2^3}) + \frac{4^2}{2^2}n(\log(\frac{n}{2^2}))$ step 3

$\hookrightarrow (\frac{4}{2})^2 n(\log n - 2)$

$4^{k-1}T(\frac{n}{2^{k-1}}) = 4^kT(\frac{n}{2^k}) + n(\frac{4}{2})^{k-1}(\log n - (k-1))$

$4^kT(\frac{n}{2^k}) = 4^k \cdot 1 \rightarrow T(\frac{n}{2^k}) \equiv T(1) \rightarrow \frac{n}{2^k} = 1 \rightarrow \log n = k$

$T(n) = n(\frac{4}{2})^0(\log n - 0) + n(\frac{4}{2})^1(\log n - 1) + \ldots + n(\frac{4}{2})^{k-1}(\log n - (k-1))$
$+ 4^k$

$= n(2)^0\log n - 0 + n(2)^1\log n - n(2)^1\log n + \ldots +$
$n(2)^{k-1}\log n - kn(2)^{k-1} + n(2)^{k-1} + 4^k$

$= n\log n[(2)^0 - 0 + (2)^1 - (2)^1 n + \ldots + (2)^{k-1}n - (k-1)n] + 4^k$

$= n\log n[2^0 + 2^1 + 2^2 + \ldots + 2^{k-1}] - n[0\cdot 2^0 + 1\cdot 2^1 + \ldots (k-1)\cdot 2^{k-1}]$
$+ 4^k$

$= n\log n[n^{k}-1] - n[0\cdot 2^0 + 1\cdot 2^1 + \ldots + (\log n - 1)\cdot n] + \underbrace{n^{\log_2 4}}_{n^2} =$
$\Theta(n^2)$

MT: case 1: $\Theta(n^{\log_2 4})$ $n\log(n) = O(n^{2-\epsilon})$
$\Rightarrow n\log n \leq cn^{2-\epsilon} \Rightarrow \log n \leq cn^{1-\epsilon}$ yes choose $\epsilon = \frac{1}{2}$

So $T(n)$ is $\Theta(n^2)$

A(6) $T(n) = 3\cancel{T(\frac{n}{5})} + n\log n$ 1

$\cancel{3T(\frac{n}{5})} = \cancel{3^2T(\frac{n}{5^2})} + 3\cdot\frac{n}{5}\cdot\log\frac{n}{5}$ 2

$(\frac{3}{5})n(\log n - \log 5)$

$\cancel{3^2T(\frac{n}{5^2})} = \cancel{3^3T(\frac{n}{5^3})} + (\frac{3}{5})^2 n\cdot(\log n - 2\log 5)$ 3

$\cancel{3^{k-1}T(\frac{n}{5^{k-1}})} = \cancel{3^kT(\frac{n}{5^k})} + (\frac{3}{5})^{k-1} n\cdot(\log n - (k-1)\log 5)$

$\cancel{3^kT(\frac{n}{3^k})} = 3^k\cdot 1 \Rightarrow \frac{n}{5^k} = 1 \to k = \log_5 n$

$T(n) = (\frac{3}{5})^0 n(\log n - 0\log 5) + (\frac{3}{5})^1 n(\log n - 1\log 5) + \dots + (\frac{3}{5})^{k-1} n(\log n - (k-1)\log 5) + 3^k$

$= (\frac{3}{5})^0 n\log n - (\frac{3}{5})^0 n\cdot 0\log 5 + (\frac{3}{5})^1 n\log n - 1(\frac{3}{5})\log 5 + \dots + (\frac{3}{5})^{k-1} n\log n - (\frac{3}{5})^{k-1}(k-1)\log 5 + 3^k$

$= n\log n\left[(\frac{3}{5})^0 + (\frac{3}{5})^1 + \dots + (\frac{3}{5})^{k-1}\right] - n\log 5\left[1\cdot(\frac{3}{5})^1 + 2(\frac{3}{5})^2 + \dots + (k-1)(\frac{3}{5})^{k-1}\right] + 3^k$

$= n\log n\left[(\frac{3^k}{5^k} - 1)\cdot\frac{5}{2}\right] - n\log 5\left[(\frac{3}{2})^1 + 2(\frac{3}{5})^2 + \dots + (k-1)(\frac{3}{5})^{k-1}\right] + 3^k$

$$\boxed{= n\log n\left[(n^{\log_5 3} - 1)\cdot\frac{5}{2}\right] - n\log 5\left[(\frac{3}{2})^1 + 2(\frac{3}{5})^2 + \dots + (\log_5 n - 1)(n^{\log_5 3 - 1})\right] + n^{\log_5 3}}$$

$O(n\log n)$

MT: case 1: $\Theta(n^{\log_5 3})$ $n\log n \le cn^{\log_5 3 - \epsilon} \to \log n \le cn^{\log_5 3 - 1 - \epsilon}$ NO.

case 2: $\Theta(n^{\log_5 3}\log n)$ $n\log n \stackrel{?}{=} \Theta(n^{\log_5 3})$ NO

case 3: $\Theta(n\log n)$ $n\log n \ge cn^{\log_5 3 + \epsilon}$ yes choose $\epsilon = 1 - \log_5 3$ & $c = \frac{1}{2}$

$3(\frac{n}{5})\log(\frac{n}{5}) < cn\log n \Rightarrow \frac{3}{5}n(\log n - \log 5) < cn\log n$

$\to \frac{3}{5}(\log n - \log 5) < c\log n$ yes choose $c > 3/5$ but < 1

so $T(n)$ is $\Theta(n\log n)$

A7) $T(n) = 2T\left(\frac{n}{3}\right) + n^2$

$2T\left(\frac{n}{3}\right) = 2^2 T\left(\frac{n}{3^2}\right) + 2\frac{n^2}{3} \to \frac{2}{3}n^2$

$2^2 T\left(\frac{n}{3^2}\right) = 2^3 T\left(\frac{n}{3^3}\right) + 2^2\frac{n^2}{3^2} \to \frac{2^2}{3^2}n^2$

$2^3 T\left(\frac{n}{3^3}\right) = 2^4 T\left(\frac{n}{3^4}\right) + 2^3\frac{n^2}{3^3} \to \frac{2^3}{3^3}n^2$

$2^{k-1} T\left(\frac{n}{3^{k-1}}\right) = 2^k T\left(\frac{n}{3^k}\right) + \left(\frac{2}{3}\right)^{k-1} n^2$

$2^k T\left(\frac{n}{3^k}\right) = 2^k \cdot 1 \to \frac{n}{3^k} = 1 \to k = \log_3 n$

$T(n) = \left(\frac{2}{3}\right)^0 n^2 + \left(\frac{2}{3}\right)^1 n^2 + \left(\frac{2}{3}\right)^2 n^2 + \dots + \left(\frac{2}{3}\right)^{k-1} n^2 + 2^k$

$= n^2\left[\left(\frac{2}{3}\right)^0 + \left(\frac{2}{3}\right)^1 + \dots \left(\frac{2}{3}\right)^{k-1}\right] + 2^k$

$= n^2\left[\left(1 - \frac{2^k}{3^k}\right)3\right] + 2^k = \boxed{n^2\left[\left(1 - n^{\log_3 2 - 1}\right)3\right] + n^{\log_3 2}}$

$O(n^2)$

MT: Case 1: $\Theta(n^{\log_3 2})$ $\quad n^2 = O(n^{\log_3 2 - \epsilon})$? NO

Case 2: $\Theta(n^{\log_3 2}\log n)$ $\quad n^2 = \Theta(n^{\log_3 2})$? NO

Case 3: $\Theta(n^2)$ $\quad n^2 \geq cn^{\log_3 2 + \epsilon}$ yes choose $\epsilon = \frac{\log_3 2 - 1}{2}$

$c = 1$ for $n \geq 2$

$2\left(\frac{n}{3}\right)^2 < cn^2 \to \frac{2}{9}n^2 < cn^2 \to \frac{2}{9} < c$

yes choose $c > \frac{2}{9}$

so $T(n)$ is $\Theta(n^2)$

Part B

2-3-4

$$T(n) = \begin{cases} 1 & n = 1 \\ T(n-1) + n & n > 1 \end{cases}$$

$$T(n) = \cancel{T(n-1)} + n$$

$$\cancel{T(n-1)} = \cancel{T(n-2)} + (n-1)$$

$$\cancel{T(n-2)} = \cancel{T(n-3)} + (n-2)$$

$$T(\cancel{n-(k-1)}) = \cancel{T(n-k)} + (n-(k-1))$$

$$\cancel{T(n-(n-2))} = \cancel{T(n-(n-1))} + (n-(n-2))$$

$$\cancel{T(n-(n-1))} = T(1) = 1$$

$$T(n) = n + (n-1) + (n-2) + \cdots + 2 + 1$$

$$= \frac{n(n+1)}{2} \qquad O(n^2)$$

2-1

ⓐ In worst case IS is $f(n) = (n+3)(n-1) - 2$

Using the Pseudo from Text book:

```
for j = 2 to k
    key = A[j]                        k-1
    i = j-1                           k-1
    while i > 0 AND A[i] > key
        A[i+1] = A[i]        \
        i = i-1              /  sum_{j=2}^{k} (t_j - 1)
    A[i+1] = key                      k-1
```

$$f(n) = 3(k-1) + 2\sum_{j=2}^{k}(t_j - 1) = 3(k-1) + k(k-1) - 1$$

$= (k+3)(k-1) - 2$ but this is going to Run for $\frac{n}{k}$ subarrs.

$$\text{so} \rightarrow \frac{n}{k}\left[(k+3)(k-1) - 2\right] = \frac{n(k+3)(k-1)}{k} - \frac{2n}{k}$$

$$= \frac{n(k^2+2k-3)}{k} - \frac{2n}{k} = \frac{nk^2+2kn-3n}{k} - \frac{2n}{k}$$

$$= \frac{nk^{\cancel{2}}}{\cancel{k}} + \frac{2\cancel{k}n}{\cancel{k}} - \frac{3n}{k} - \frac{2n}{k} = nk + 2n - \frac{5n}{k}$$

so $f(n)$ is $\Theta(nk)$