3. $xy''+2y'=6x$

$U=y'$ $\quad U'=y''$

$xU'+2U=6x$

$U'=\frac{dU}{dx}$ $\quad U''=\frac{d}{dx}\left(\frac{dU}{dx}\right)$

$$x\frac{dU}{dx}+2U=6x$$

$$\frac{dU}{dx}+\frac{2}{x}U=6$$

$$x^2\frac{dU}{dx}+2xU=6x^2$$

$$\frac{d}{dx}(x^2U)=6x^2$$

$$x^2U=\int 6x^2\,dx=2x^3$$

$$e^{\int \frac{2}{x}dx} \rightarrow 2\int\frac{1}{x}dx$$

$$=e^{2\ln|x|}=x^2$$

$$U=y'=2x+\frac{C}{x^2}$$

$$y=\int 2x+\frac{C}{x^2}\,dx$$

$$x^2-\frac{C}{x}+C_2$$

$$y(x)=x^2-\frac{C}{x}+C_2$$

$\frac{dU}{dx}=6-\frac{2}{x}U$ —estándar→ $U'=f(x,U)=6-\frac{2}{x}U$ continua, excepto cuando $x=0$

No cumple en regiones donde $x=0$ (eje y)

Ecuación no definida en $x=0$ ⟹ No hay soluciones donde $(0,y)$
Solo para $x>0$ y $x<0$

$y(1)=2$

$y(1)=(1)^2-\frac{C}{(1)}+C^2$
$2=1-C+C_2 \rightarrow C-C_2=1$

$y(1)=1$

$y(1)=(1)^2-\frac{C}{(1)}+C_2$
$1=1-C+C_2 \rightarrow C-C_2=0$

$y(1)=-2$

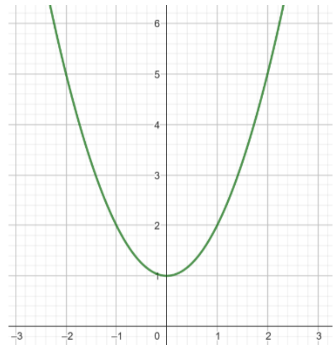
$y(1)=(1)^2-\frac{C}{(1)}+C_2$
$-2=1-C+C_2 \rightarrow C-C_2=-3$

$y(0)=-3$

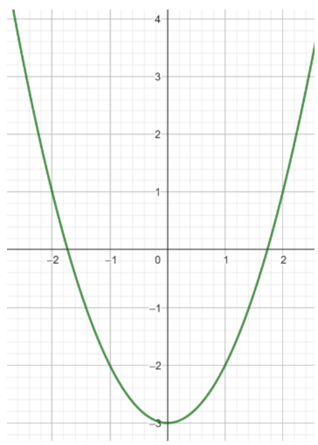
$y(0)$ No definido para $x=0$

Graficas $\quad C=0 \Rightarrow y(x) = x^2 - C_2$

$y(1)=2 \quad 2=(1)^2 - C_2 \Rightarrow C_2=-1 \longrightarrow y(x)=x^2-(-1)=x^2+1$



$y(1)=-2 \quad -2=(1)^2-C_2 \Rightarrow C_2=3 \longrightarrow y(x)=x^2-(3)=x^2-3$



$y(1)=1 \quad 1=(1)^2-C_2 \Rightarrow C_2=0 \longrightarrow y(x)=x^2-(0)=x^2$