

Application Of Queuing Theory On A Food Chain

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Abstract: In their daily life people go to many places such as hotels, hospitals, banks etc. to avail some kinds of services and the biggest problem that they face there is the formation of long queues and high waiting time. It affects customer in a negative way and harms the growth of business. To grow in today's competitive market a firm always need to improve its services and should focus on satisfying customer's needs in best possible way and apply a strategy that suits the firm. To pursue quality education students living out of station depends on food services for their food requirement and as the population of students is very high it causes load on food chains and they always run out of capacity and often fails to provide a quality service. This paper studies and evaluates queuing system and operating characteristics of a food chain by applying queuing theory. It focuses on queuing modeling of the system and finding ways to improve service and reduce waiting time by calculating arrival, departure, queue length etc. It also aims at finding a balance between cost of providing service and loss due to high waiting time, so that system can operate at an optimized minimum possible cost. The study shows that the food chain needs to increase seating capacity and instead of the current m/m/1 queuing model, a multi-server model will best suit the food chain. The study also consists of a survey to find potential customers that can join the food chain if waiting time of the system decreases. The survey also aims to find problems in the food chain that are the reasons behind high waiting time and eliminate them.

Index Terms: arrival rate, customer satisfaction, m/m/1 queuing model, multi-server queuing model, queuing theory, service time, waiting time.

1 Introduction

A queue is a line or sequence of people or job awaiting their turn to be attended to or proceed to. Formation of queue takes place due to limited capacity of servers. A short queue, if not taken care can transform into a large one which causes big problems. As the world is progressing, awareness about education and its importance is increasing, forcing small town students to shift into developed cities. As Indore is the most developed city of Madhya Pradesh and Bhawarkuan region is the education hub of Indore, thousands of students shift to Indore to study in best coachings. Student depends on food stores for food and this gives small businessmen an opportunity to open food stores and flourish their business. A food chain is a system that serves food, runs for a fixed period of time and the hours of operation are always peak hours, for the rest of the time it is empty since all students come to it nearly at the same time. And for the rest of the time staff stays busy in the preparation of next meal. The system under study, because of its good location, food quality and cheaper price rates attracts a lot of students but at the same time this population of students becomes the cause of formation of long queues and high waiting time, but if handled properly this can increase the profit of the food chain. It lacks in seating capacity and service timing due to which availing service becomes tedious, a long queue can always be seen outside the food chain. Thus, it is needed to consider this problem of queue formation, waiting time, service rate, and arrival rate, serving discipline and calling population. High waiting time is not only frustrating, but it also harms the growth of business. It reduces service quality and hence customer satisfaction which is most essential to the growth of a company since market is turning into a service dominating market. High waiting time discourages customer from entering into system and they show different behavior in queue like chalking, balking, reneging, collusion or they can even leave the line. To expand business a company does not only need to make new customer but also retain the old customers.

So a service providing firm must avoid condition of formation of long queue. One more problem is of staff. A company must have sufficient number of staff as queue time depends on how much time it takes to get the service. Optimum use of all resources is necessary. To overcome such problems, one must look at it through the lens of an industrial engineer. All these problems can be solved by mathematical modeling of the system. Based on its characteristics a system can be fit into some previously defined models. Depending upon the type of arrival and departure (which type of distribution it follows), whether the system has a single or multiple no of servers, calling population and the discipline in which customers are being served a system can be of following type:
 Single server finite capacity model
 Single server infinite capacity model
 Multi server finite capacity model
 Multi server infinite capacity model

By fitting the system under consideration into any suitable model, the problem can be solved. This theory enables mathematical analysis of several related processes, including arrivals in the queue, waiting in the queue, and provision of service at the front of the queue. This theory enables us to do the derivation and calculation of the key operating characteristics including the average waiting time in the queue, the expected number customers or job queuing for service and the probability of the system to be in certain states such as empty or full. It explores the relationship between demand on a service system and the delay of services based upon the concept of random behaviour of customers. Mess, restaurants, cafe frequently face the problem of long waiting lines, especially during peak hours or during promotions, lunch and dinner time. If the system is unable to serve customers immediately, customers have to wait for their turn, often impatiently. Due to which the service provider loses its reputation and customers and bears loss.

2 LITERATURE REVIEW

Concept of queue modeling was first introduced by a Danish telephone engineer A.K. Erlang in 1908. He applied it on the problem of congestion and waiting time in Telephone calls. The queue modeling of RAMBAM Medical Centre, Israel by Dr Mor Armony et al. (2015) studies patient flow in Hospital and considers patients as customers and internal wards emergency departments, beds, nurses etc as servers. The

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study helped them remove inefficient processes that were increasing service time and queue length. Hospital systems are very complex, they have vital societal benefits but the processes are associated with huge mounting costs. These costs are further increased by inefficiencies in hospital processes, which are often the result of congestion and long delays in patient care. Thus, a queuing network view of patient flow in hospitals is best for studying and improving its performance. The goal of the research was to explore patient flow data through the lens of a queuing scientist. It was done with the help of exploratory data analysis (EDA) in a large Israeli hospital, which revealed important features that are not easily explainable by existing models. Questions raised by EDA included: Can a simple queuing model explain the complex operational reality of the Emergency Department (ED)? What time scales and operational procedures are required for the modeling of patient's length of stay in the Internal Wards (IWs)? How do protocols of patient transfer between the ED and the IWs influence patient delay, workload division and fairness? EDA also emphasize the importance of an integrative view of hospital units by, for example, relating ED bottlenecks to IW physician protocols" [1]. Modeling of urban transportation system was done by Avigdor Gal (2015) for an Israeli city to predict the real time variation in time required to complete a journey with the help of queuing theory and machine learning. To enhance urban mobility, much research was invested in traveling time prediction. That is by providing the origin and destination; a passenger can know an accurate estimation of how long the journey would last. The ability to predict traveling time in scheduled transportation, e.g., buses, was shown to be feasible. In their work, they investigated a novel use of methods from Queuing Theory and Machine Learning in the prediction process and proposed a prediction engine that, given a scheduled bus journey or a route and a pair of source and destination, provides an estimate of the traveling time, by considering both historical data of past journeys and real-time streams of information that are transmitted by buses" [2]. Dr Ugwa Magnus et al. (2015) applied queuing theory on zenith bank to find a cost optimized model. It enables business executive determine and install the optimum service facility so that overall service cost is minimized" [3]. Dr Bolanle et al. (2011) carried out a study emphasizing the importance of queuing theory to the problem of port congestion to enhance sustainable development of Nigeria ports. Nigeria Ports were facing the problem of a never ending congestion in the past and this had resulted in diversion of ships scheduled for Nigeria Ports to the other neighboring country's ports which has caused the country a lot of revenue loss. The effectiveness of a Port depends upon loading and unloading of ships. Because of the random nature of the arrival and service time of the ships, the movement of traffic through a port becomes a complex phenomenon and requires a systematic approach for port planning and management. Queuing model was applied to the arrival and services pattern which were causing the problems of congestion and solutions to the problem areas were put forward. It was also used to predict the average arrival rate of ships to Nigerian Ports and the average service time taken per ship. The study found out that the number of berth in Nigeria port is adequate for the traffic intensity of vessels but other factors leading to port congestion were also identified through the analysis of the question asked in the interviews conducted with stakeholders at the port" [4].

3 RESEARCH OBJECTIVES

- To find balance between cost of providing service and loss due to high waiting time.
- To analyze the food chain and fit it into suitable queue model.

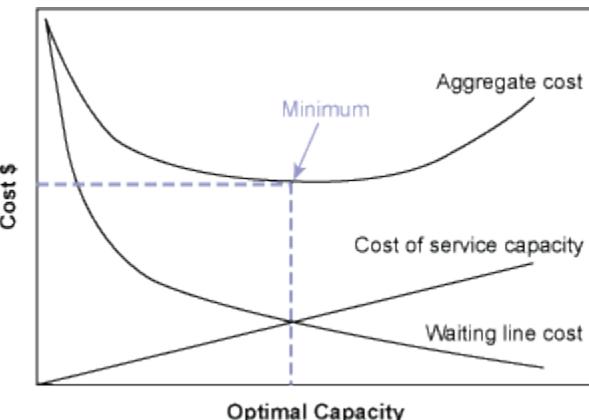


Figure-1" [42]"

- To design a model that best suits the food chain.
- To gather feedback from customer and evaluate system based on findings.
- To find causes of mismanagement in the system.
- To find the potential customers.

4 METHODOLOGY ADOPTED

First step in the research was to collect data. Data was collected by observation and from the manager. The system serves lunch from 12:00 pm to 3:00 pm and dinner from 7:00 pm to 11:00 pm i.e. it runs 7 hours a day for customers. The arrival and departure were observed taking 1 hour windows and data was collected for a period of one month. The system has a seating capacity of 50. According to the data there is only one cook and waiter hence; it can be assumed as m/m/1 model. Based on this, calculations were made to find various operating characteristics such as waiting time in queue as well as in system, length of queue and system etc.

The number of arrivals on a day for a month was recorded as follows:

Table 1:

Days	Week 1	Week 2	Week 3	Week 4
Monday	364	373	377	369
Tuesday	377	389	379	387
Wednesday	410	407	395	408
Thursday	403	398	385	411
Friday	389	391	390	408
Saturday	197	201	213	208
Sunday	503	501	497	501

Data regarding salaries of workers and other monthly expenses

Table 2

Functions	Cost associated (Rs/month)
Cook	16000
Roti Maker	8000
Waiter	8000
Washer	8000
Other (gas, water, electricity, vegetables etc.)	205000-295000
Rent	35000

MODEL SPECIFICATION

Model specification is the process of converting theory into regression model. It consists of selecting appropriate functional form for the model and choosing which variables to include. If the estimated model is not specified properly, it will be biased and inconsistent. The study adopted a Queueing Model in the form of: m/m/c/n/FCFS

Where;

M=Markovian (or Poisson) arrivals and exponential service time.

C= no of servers;

FCFS = First come, first served (discipline in which customers are served in a food chain);

n= finite system capacity;

∞ = Infinite source population.

For the analysis of the system M/M/1 queuing model with the following variables was investigated;

λ - Mean arrival rate (number of arrivals per unit time),

μ -Mean service rate per server,

C-Number of servers,

L-Average number of units (customers) in the system (includes both the customers waiting for service and those being served),

L_q -Average number of customers in the queue (waiting to be served),

W -Average waiting time in the system,

W_q -Average waiting time in the queue (service time not included)

Calculation of various operating characteristics of the system by considering it as multiple server model.

MODEL FORMULATION

The parameter that relates to probabilities in a single channel queue is;

1. The traffic intensity: This is the probability that an object arriving at the service points has to wait for sometime before being attended to. An object arriving for service has to wait for service if on arrival it finds one or more objects already at the service point. This probability denoted by "rho" is given by the ratio of the arrival rate to the service rate, so that we can write:

Traffic intensity: λ / μ , which in obedience to the probability theory lies between zero and one.

2. The probability that there are at least 'n' items in the system is given by ℓ^n .

It is necessary to observe that being in the system implies all the objects in the queue together with the object that is receiving service.

3. The probability of 'n' objects in the system is given by

$$P_n = (\lambda/\mu)^n * (1 - \lambda/\mu)^{n-1}$$

We now give the system parameters that are concerned with numbers.

4. The average number of objects in the system is given by

$$n_s = \lambda / (\mu - \lambda)$$

5. The average length of the queue, which is the same as the average number of object in the queue i.e. excluding the objects that is receiving attention is given by:

$$n_q = \lambda^2 / \mu (\mu - \lambda)$$

6. The average length of the queue excluding zero queuing is given by

$$n_o = \mu / (\mu - \lambda)$$

7. The average number of time an object is in the system which is also known as the average system process time; (A.S.P.T) is given by

$$W_s = \lambda / (\mu - \lambda)$$

8. Finally, the average number of time units that an object stays in the queue i.e. the average queuing time is given by

$$W_q = \lambda / \mu (\mu - \lambda)$$

MULTIPLE CHANNEL QUEUE:

In the single channel queue situation, there is only one service point, while in this case, more than one server is available. It will be assumed that these service points are approached through only one line (queue). The customer at the head of the queue has the option of going to the server who is idle. Another vital assumption here is that the service times at the service points are negative exponential and that the simple queue situation also apply. If we denote the number of service points by "C", then as in single channel queue, the following formulae apply in each case.

- i. The probability of an object having to wait for service is given by

$$P_w = [(\ell c) / c! (1 - \ell)] * P_0$$

- ii. The average number of objects in the system is

$$n_s = [(\ell c)^c / (c! (1 - \ell)^2)] * P_0 + C$$

The average number of object awaiting service i.e. in the queue is given by;

$$L_q = [\ell (\ell c)^c / (c! (1 - \ell)^2)] * P_0$$

- iii. The average time an object is in system is (A.S.P.T.)

$$W_s = [(\ell c)^c / (c! (1 - \ell)^2 C \mu)] * P_0 + 1/P$$

- iv. Finally the average time an object stays waiting for service i.e. On the queue is given by

$$W_q = [(\ell c)^c / (C! (1 - \ell)^2 C \mu)] * P_0$$

Where in all cases

$$P_0 = C! (1 - \ell) / [(P c)^c + C! (1 - \ell)]$$

ASSUMPTIONS

- The size of the calling population is assumed to be infinite that means input sources are unlimited.
- The arrival rate is approximated by Poisson's distribution.
- There is no balking. This assumption implies that arriving customers always join the queue,
- There is no reneging. This assumption implies that customers stay in line until served (i.e. patient customers).
- The queue discipline is first come, first serve (FCFS).
- The permissible length of the queue is infinite.
- The service time distribution is approximated by an exponential distribution.
- The rate of service is greater than rate of arrivals.

Values of different operating characteristics formulated above are shown below in the table:

Table 3:

Parameters	M/m/1 model	m/m/2	m/m/3
Lambda	43	43	43
Mu	48	48	48
Ro	0.89583	0.44791	0.29861
L	8.6	1.1206	0.89612
Lq	7.7	0.2247	0.00029
W(minute)	12	1.563	1.25
Wq	11.1	1.54	0.35457
P(n=0)	0.1041	0.3812	0.4052
P(t<0)	0.1041	0.7703	0.8123

Cost analysis of different models:

Cost the objective of this research is to find balance between two costs

- Cost of providing services: It includes
 - Rent of space
 - Salary of workers
 - Cost associated with facilities provided
- Loss due to high waiting time: cost of losing customer due to high waiting time.

Model M/M/1->

$$\begin{aligned} \text{Cost of providing services} &= \text{rent} + \text{salary} + \text{other} \\ &= 35000 + 100000 + 50000 \\ &= 185000 \end{aligned}$$

Revenue lost due to loss of customer

$$\begin{aligned} &= 150 * 30 * 32.5 * 2 = 292500 \\ \text{Total cost} &= 477500 \end{aligned}$$

Model M/M/2 ->

$$\begin{aligned} \text{COPS} &= 70000 + 150000 + 70000 \\ &= 290000 \\ \text{RLDLC} &= 50 * 30 * 32.5 = 97500 \\ \text{Total cost} &= 387500 \end{aligned}$$

Model M/M/3->

$$\begin{array}{lll} \text{COPS} & & = 105000 + 200000 + \\ 90000 & & = 395000 \\ \text{RLDLC} & & = 0 \\ \text{Total cost} & & = 395000 \end{array}$$

Table 4:

Model	Cost of providing services	Loss due high waiting time	Total cost
m/m/1	185000	292000	477500
m/m/2	290000	97500	387500
m/m/3	395000	0	395000

A survey was conducted and questions were asked from customers as well as from students living nearby to get the feedback about services and potential customers. There were 178 respondents to the questionnaire. They were asked about following questions:

- Which food chain do you like the most?
- Which food chain serves best quality food?
- Which food chain provides the best services?
- What are the areas of improvement of the system?
- How much time can you afford to wait for your turn?

5 RESULTS

- From table 3 it can be seen that length of queue formed outside the food chain was 7.7 earlier that decreases to 0.2279 for two server model which is good, further increase in server will reduce queue length to nearly 0.
- Waiting time in queue which was 11.1 minute initially, reduced to 1.54 minute and 0.35 minute for 2 and 3 server models respectively, both of which are affordable.
- One more thing that can be observed from the table is that as system shifts from single server to multiserver, the probability of system being empty increases from 10.41% to 38.12% to 40.52% that means system is available to receive customers.
- Table 4 suggests that single server costs the most Rs. 477500 while 2 server model costs Rs 387500/- which is least and as we progress it increases to 395000 for 3 server model and further increase in server will add cost to system rather than making profit.
- The result showed that 97% of the respondents were not happy with the waiting time and service time of the system. 78% were satisfied with the quality of food. 87% said that the food chain is the best in the area.

6 RECOMMENDATION

- Survey suggests that 95% of respondents think that the seating capacity should be increased. This can be increased by purchasing some more space on rent, which is available there.
- On the basis of length of queue and waiting time two-server and three-server model both are good.

- On the basis of probability of system being idle, both two and three server model are good.
- But cost analysis shows that minimum cost at which service can be provided with a two server model.
- As the potential customers are nearly 70% of the existing customers, increasing space and doubling the no of workers will be advantageous.

7 CONCLUSION

It can be concluded from the results that the food quality provided by the system is good enough to attract most of the students of the area, and rates of the food items are also cheaper than other nearby food services. But the problem arises due to space and number of workers. The load on staff is more than they can handle. Survey showed that many students, who find it frustrating to wait for so long at the system, still prefer to go there instead of any other place due to quality and hygiene of food. Many students want to join the system but they don't because of the high waiting time. Hence, space and number of staff must be increased in the system.

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