UNIVERSITY OF WOLLONGONG IN DUBAI

Tutorial 2 – Week 2

Example1: Example 12.1 Text Book

The car on the left in the photo and in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by v = (3t2 + 2t) >s, where *t* is in seconds. Determine its position and acceleration when *t* = 3 s. When *t* = 0, *s* = 0.

Solution:

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ m} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ m/s}^2 \rightarrow \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

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Example 2: Example 12.2 Text Book

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m>s. Due to the drag resistance of the fluid the projectile experiences a deceleration of *a* = (-0.4v3) m>s2, where v is in m>s. Determine the projectile's velocity and position 4 s after it is fired.

Solution

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v, a, and t. (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

$$(+\downarrow) \qquad a = \frac{dv}{dt} = -0.4v^3$$

$$\int_{60\text{ m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\bigg|_{60}^{v} = t - 0$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\} \text{m/s}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559 \text{ m/s}\downarrow \qquad \textit{Ans.}$$

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s, v, and t. Using the initial condition $s = 0$, when $t = 0$, we have

$$(+\downarrow) \qquad v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\bigg|_0^t$$

$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\} \text{m}$$

When $t = 4$ s,

$$s = 4.43 \text{ m} \qquad \textit{Ans.}$$

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Example 3: Example 12.3 Text Book

During a test a rocket travels upward at 75 m>s, and when it is 40 m from the ground its engine fails. Determine the maximum height *sB* reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m>s2 due to gravity. Neglect the effect of air resistance.

Solution:

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$ m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81\ \mathrm{m/s^2}$ (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$
$$0 = (75\ \mathrm{m/s})^2 + 2(-9.81\ \mathrm{m/s^2})(s_B - 40\ \mathrm{m})$$
$$s_B = 327\ \mathrm{m} \qquad \textit{Ans.}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$
$$= 0 + 2(-9.81\ \mathrm{m/s^2})(0 - 327\ \mathrm{m})$$
$$v_C = -80.1\ \mathrm{m/s} = 80.1\ \mathrm{m/s}\downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward.
Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$
$$= (75\ \mathrm{m/s})^2 + 2(-9.81\ \mathrm{m/s^2})(0 - 40\ \mathrm{m})$$
$$v_C = -80.1\ \mathrm{m/s} = 80.1\ \mathrm{m/s}\downarrow \qquad \textit{Ans.}$$

Example 4:

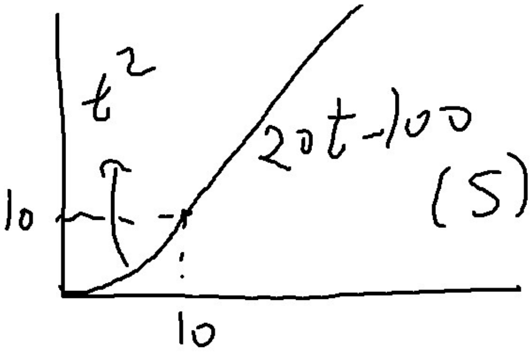
A bicycle moves along a road with a displacement which can be described by:

$$a = \begin{cases} t^2 \; for \; 0 \leq t \leq 10 \\ 20t - 100 \quad for \; t > 10 \end{cases}$$

Where s is in metres and t is in seconds. Manually sketch the graphs for displacement, velocity and acceleration versus time.

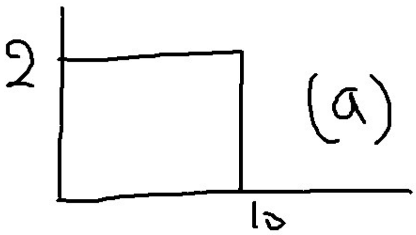
Solution

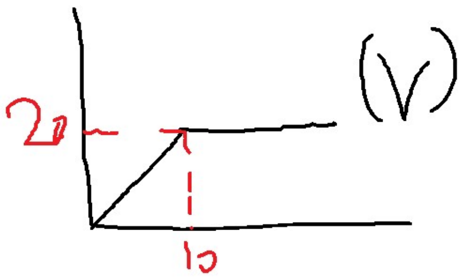
$$S = \begin{cases} t^2 & 0 \leq t \leq 10 \\ 20t - 100 & t > 10 \end{cases}$$



$$V = \begin{cases} 2t & 0 \leq t \leq 10 \\ 20 & t > 0 \end{cases}$$

$$a \begin{cases} 2 & 0 \leq t \leq 10 \\ 0 & t > 0 \end{cases}$$





Faculty of Engineering and Information Science
ENGG100 Engineering Computing and Analysis
Tutorial problem sheet

Section 2: For you to do:

Q 2.1

Starting from rest, a particle moving in a straight line has an acceleration of $a=(2t-6)$ m/s^2, where t is in seconds. What is the particle's velocity when t=6 seconds and its position when t=11 seconds.

Manually sketch the graphs for displacement, velocity and acceleration versus time, over [0 12].

MATLAB Extension Question:

Draw the graphs for displacement, velocity and acceleration versus time, over [0 12].

Solution:

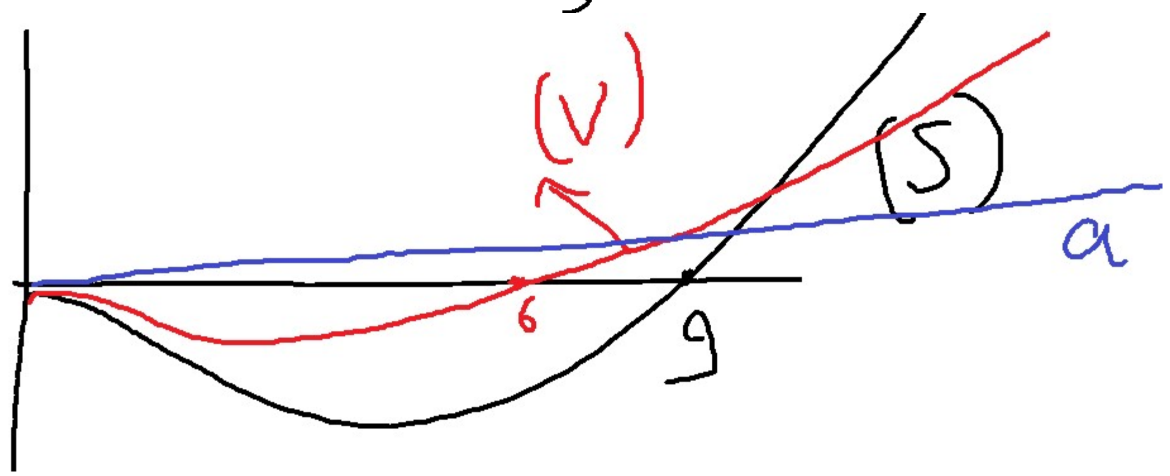
$$a = 2t - 6 \text{ m/s}^2$$

$$V = \frac{2t^2}{2} - 6t = t^2 - 6t$$

$$S = \frac{t^3}{3} - \frac{6t^2}{2} = \frac{t^3}{3} - 3t^2$$

$$\text{at } t = 6 \longrightarrow V = 6^2 - 6(6) = 0$$

$$\text{at } t = 11 \longrightarrow S = \frac{11^3}{3} - 3(11)^2 = 80.66\text{m}$$



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Q 2.2

A car starting from rest moves along a straight track with an acceleration defined by:

$$a = \begin{cases} 0.8t \; for \; 0 \leq t \leq 10 \\ 8 \quad for \; t > 10 \end{cases}$$

Determine the time T required for the car to reach speed 50 m/s.

MATLAB Extension Question:

Construct the velocity versus time graph until time T.

Solution

$$a = \begin{cases} 0.8t & 0 \leq t \leq 10 \\ 8 & t > 10 \end{cases}$$

$$V = \begin{cases} \frac{0.8t^2}{2} & 0 \leq t \leq 10 \\ 8t + k & t > 10 \end{cases}$$

at $t=10$ $V=40$, $40 = 80 + k$ ⟹ $k = -40$

WHEN $V = 50$ m/s

$50 = 8t - 40 \Leftrightarrow t = \frac{10}{8} = 11.25$ s

Q 2.3 (2018 Exam paper)

A motorcycle moves along a straight line such that its position is defined by s = 2t3 - 18t2 + 48t where s is measured in meters and t is measured in seconds. Determine

1. Write an equation for velocity with respect to time and sketch the velocity-time graph
2. The minimum and maximum velocity of the particle over the time interval [0, 5]
3. The minimum and maximum acceleration of the particle over the time interval [0, 5]

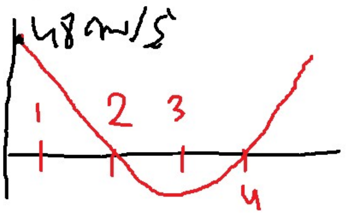
Solution:

$S = 2t^3 - 18t^2 + 48t$

$V = 6t^2 - 36t + 48$

$a = 12t - 36$

i)



ii) MIN VELOCITY @ $t = 3$ s

MAX " @ $t = 0$

iii) MIN ACC @ $t = 0$

MAX " @ $t = 5$

Q 2.4 (2018 Exam paper)

A truck is traveling along a straight road at 6 m/s is at t=0 seconds. It increases its speed to 35 m/s over the following 15 seconds. If its acceleration is constant, determine the distance travelled from t=0 seconds until t=15 seconds.

Solution:

acc is constant

$V = V_0 + a_c t$

$35 = 6 + a_c(15)$

$a_c = 1.93 \text{ m/s}^2$

$s = s_0 + V_0 t + \frac{1}{2} a t^2$

$= 0 + 6(15) + \frac{1}{2}(1.93)(15)^2$

$= 307.125 \text{ m}$