Dynamics 1, Rectilinear Motion

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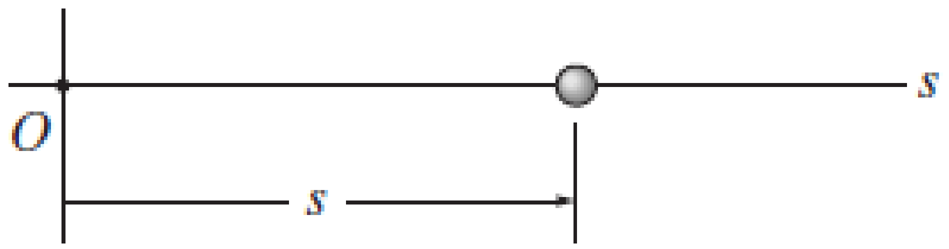
Objectives

- Refers to only the **position**(displacement), **velocity** and **acceleration** of particles
- Remember –**no influence of forces**
- Reference frames or **coordinate systems** are needed
  - Cartesian coordinate system (x, y, z)
  - Normal and tangential coordinate system (n, t, b)
  - Cylindrical coordinate system (r, θ, z)
- **Scalars**–magnitude only
- **Vectors**–magnitude and direction

Rectilinear Motion of Particles

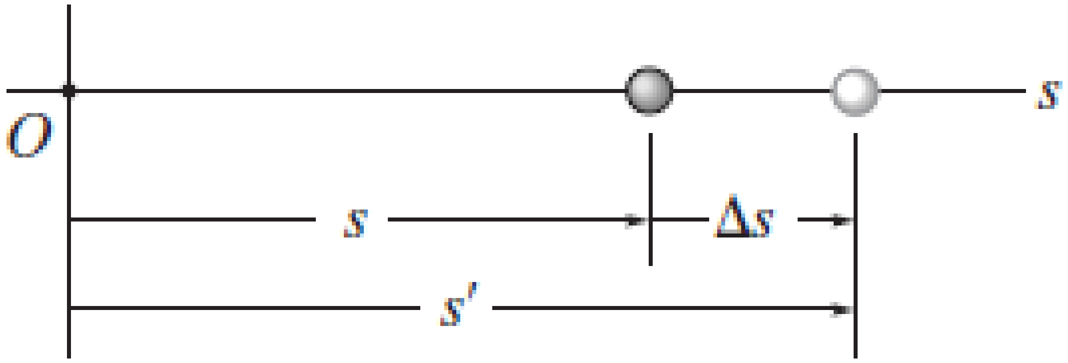
- Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle’s position, velocity, and acceleration.
- Position. The straight-line path of a particle will be defined using a single coordinate axis *s*.
- The origin *O* on the path is a fixed point, and from this point the *position coordinate s* is used to specify the location of the particle at any given instant. The magnitude of *s* is the distance from *O* to the particle, usually measured in meters (m), and the sense of direction is defined by the algebraic sign on *s*.



Position

Rectilinear Motion of Particles

- Displacement. The *displacement* of the particle is defined as the *change* in its *position* . For example, if the particle moves from one point to another, the displacement is
- In this case s is *positive* since the particle's final position is to the *right* of its initial position



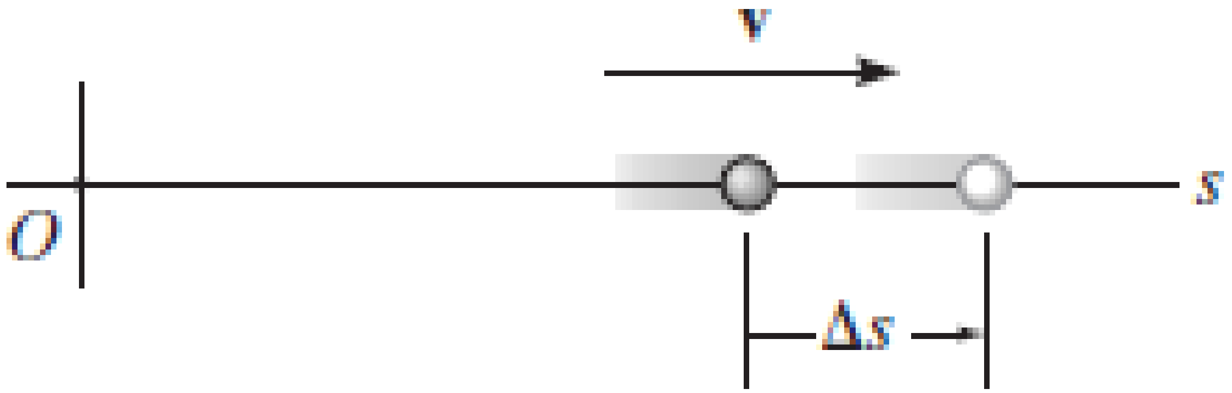
Displacement

$$\Delta s = s' - s$$

Rectilinear Motion of Particles

- Velocity. If the particle moves through a displacement s during the time interval Δt, the *average velocity* of the particle during this time interval is

$$v_{avg} = \frac{\Delta s}{\Delta t}$$



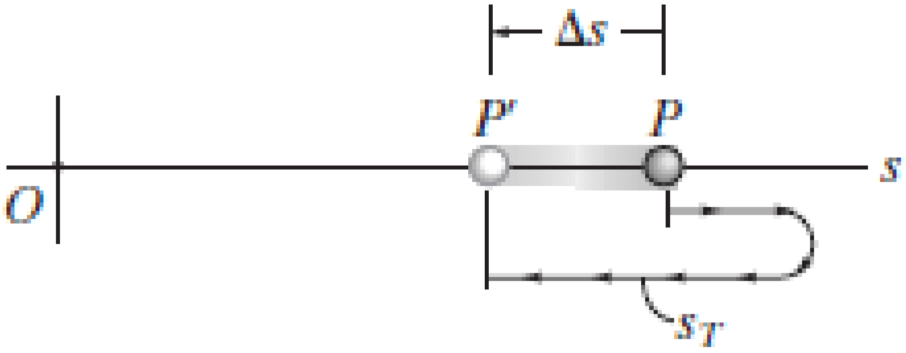
Velocity

Rectilinear Motion of Particles

- The speed:

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

$$v_{(sp)avg} = \frac{S_T}{\Delta t}$$



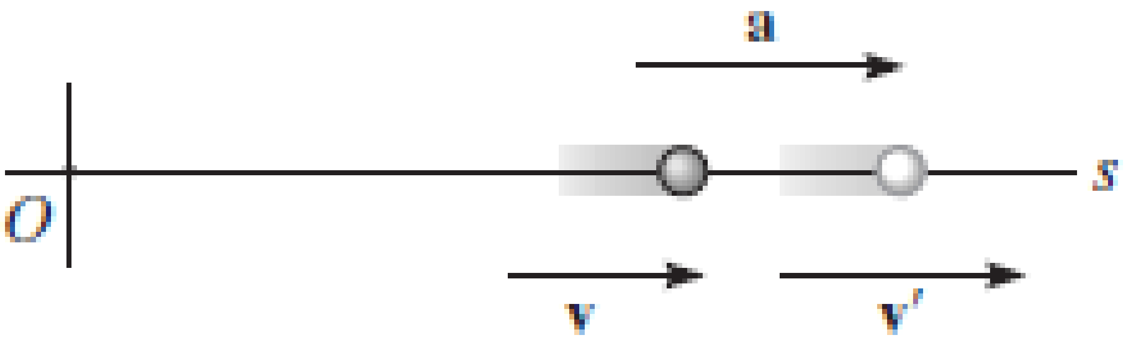
Average velocity and
Average speed

Rectilinear Motion of Particles

- Acceleration. Provided the velocity of the particle is known at two points, the *average acceleration* of the particle during the time interval Δt is defined as

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \qquad a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$$

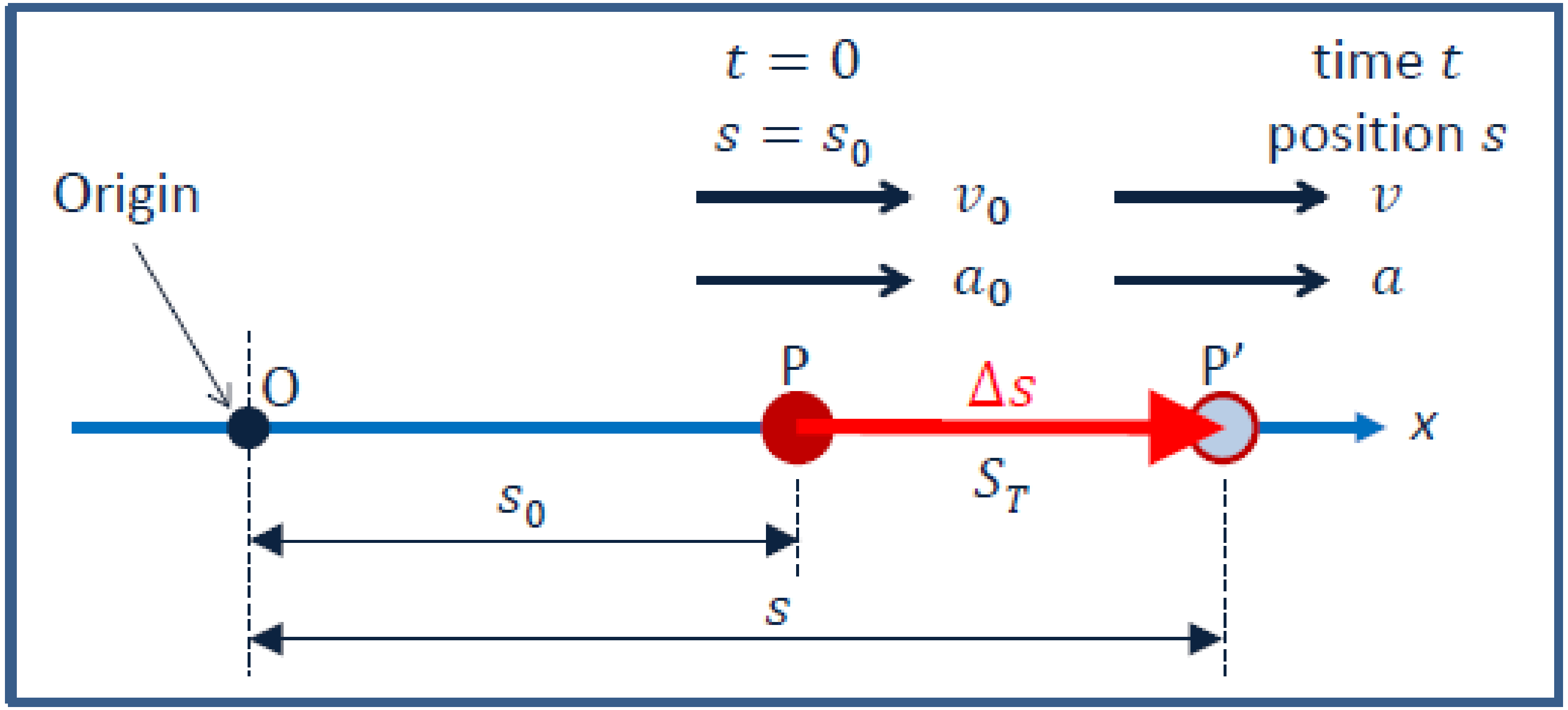
$$\boxed{a ds = v dv}$$



Acceleration

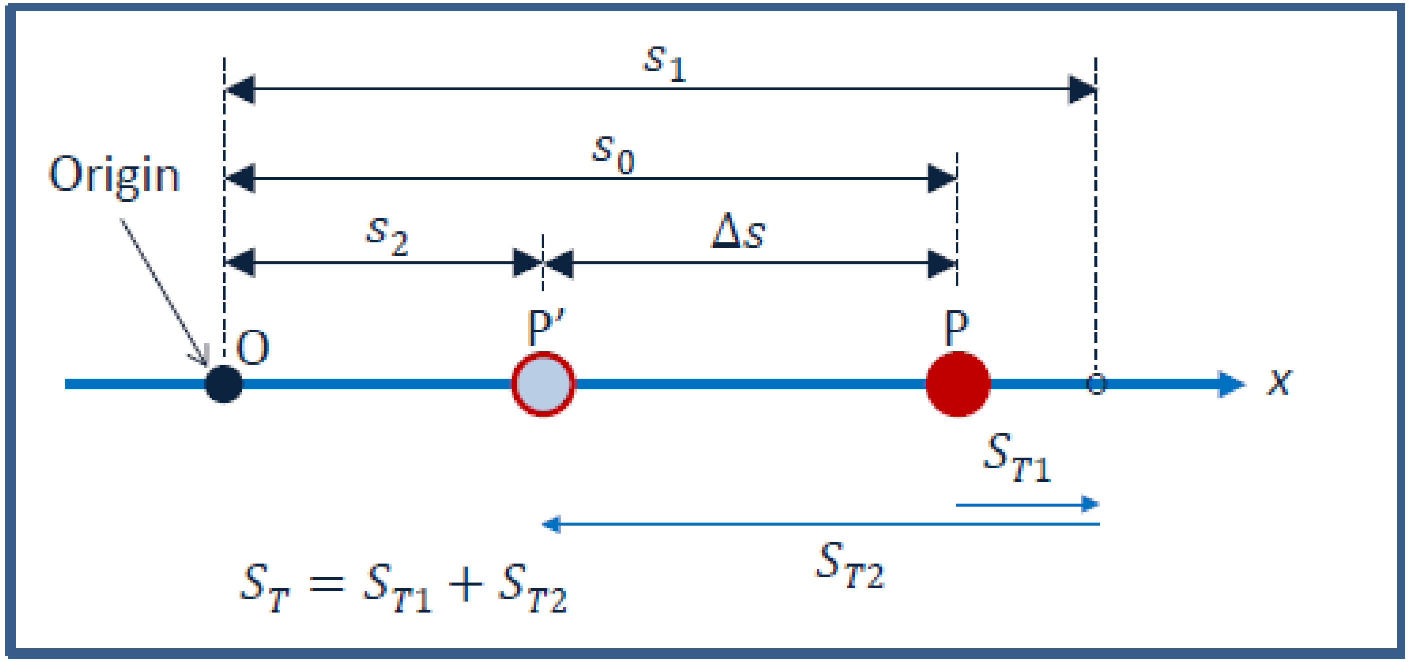
Rectilinear Motion of Particles

- Now let's consider the same particle, P, in its initial position, s_0, and then moving to another location, s



Rectilinear Motion of Particles

- Now let's look at what happens when a particle, P, changes direction during its motion
- Displacement: $\Delta s = s_2 - s_0$



- Average velocity: $\frac{\Delta s}{\Delta t} = \frac{s_2 - s_0}{\Delta t}$
- Distanced travelled: $S_T = |s_1 - s_0| + |s_2 - s_1|$
- Average Speed: $\frac{S_T}{\Delta t} = \frac{|s_1 - s_0| + |s_2 - s_1|}{\Delta t}$

Example

- Starting from rest, a particle moving in a straight line has an acceleration of a = (2t -6) m/s2, where t is in seconds.
- What is the particle's velocity when t = 6 s, and what is its position when t = 1 s?
- Solution:

Example

- The position of a particle travelling along a straight path is given by the equations $s = 0.5t3 – 4t2 + 6t$ m, where t is measured in seconds.
- Determine the maximum acceleration and the maximum velocity of the particle during the time interval $0 \leq t \leq 10$ seconds.

Rectilinear Motion of Particles

- Special Cases: Constant Speed
- Under this condition, the acceleration is zero

$$a = \frac{dv}{dt} = 0$$

- So if we take vdt=ds and we integrate, we have

$$\int ds = \int vdt \leftrightarrow \boxed{s = s_0 + vt}$$

- If S_0=0, we have s=vt

Rectilinear Motion of Particles

- Special Cases: Acceleration is Constant

$$a = \frac{dv}{dt} \leftrightarrow adt = dv$$

$$\int dv = \int adt \leftrightarrow \boxed{v = v_0 + at}$$

$$\int_{s_0}^{s} ds = \int_0^t vdt \leftrightarrow s - s_0 = \int_0^t (v_0 + at)dt$$

$$s - s_0 = v_0 t + \frac{1}{2}at^2$$

$$\boxed{s = s_0 + v_0 t + \frac{1}{2}at^2}$$

Rectilinear Motion of Particles

- Special Cases: Acceleration is Constant
- Recall $\boxed{ads = vdv}$

$$a\int_{s_0}^{s} ds = \int_{v_0}^{v} vdv = \frac{1}{2}(v^2 - v_0^2)$$

$$\boxed{v^2 = v_0^2 + 2a(s - s_0)}$$

- **Note:** these equations are only for the situation where constant acceleration exists and cannot be used when acceleration is varying with respect to time

Example

- The position of the particle is given by $s = (2t^2 - 8t + 6)$ m, where t is in seconds
- Determine the time when the velocity of the particle is zero, and the total distance travelled by the particle when $t = 3$ s.

Example

- The position of a particle along a straight-line path is defined by $s = (t^3 - 6t^2 - 15t + 7)$ m, where t is in seconds.
- Determine the total distance travelled when $t = 10$ s.

Special Case: Erratic Motion

- This term is assigned to problems where it is hard to obtain a mathematical function for s, v and/or a.
- This is most commonly because the **motion of the particle is changing** and consists of several different functions to describe the motion at different intervals.
- In this case, it is often better to describe the motion graphically.
- If any two of the variables a, v, s or t are graphed, then other relations can be found

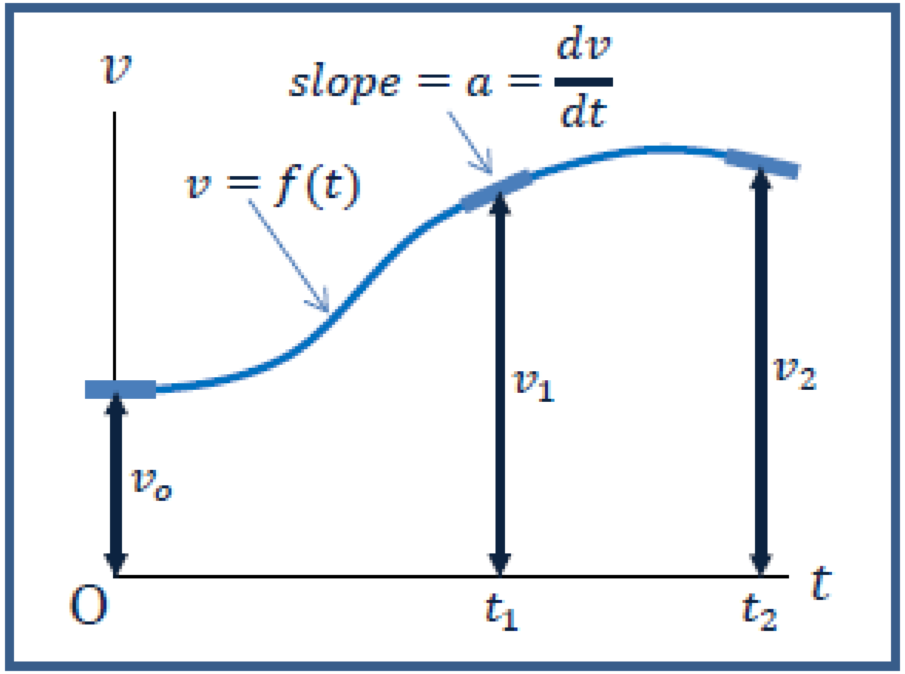
$$a = \frac{dv}{dt} \qquad\qquad v = \frac{ds}{dt}$$

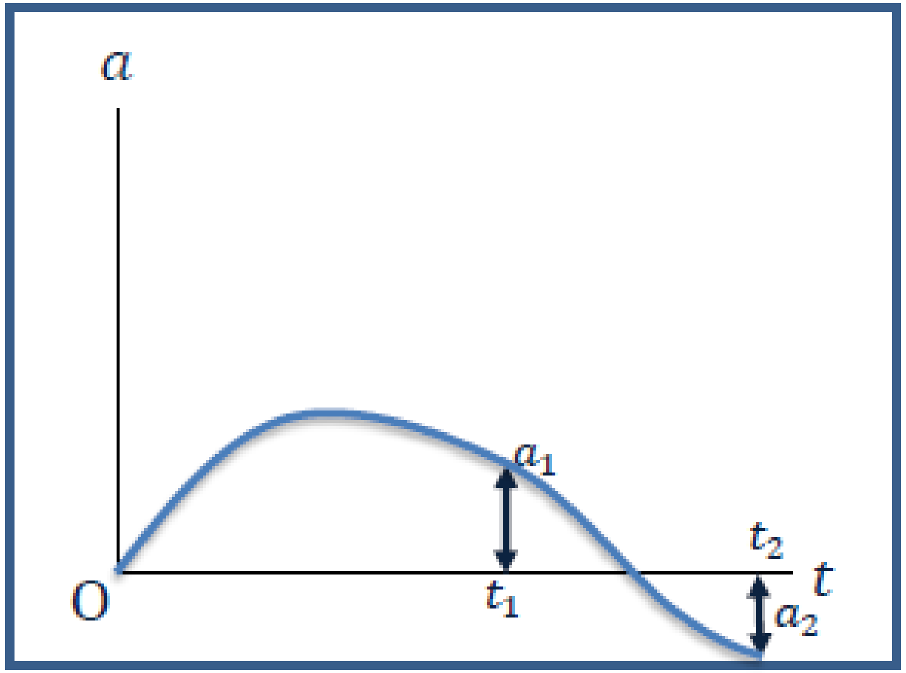
Graphical Method

- A range of two dimensional graphs can be generated with the **four variables $\boldsymbol{a}$, $\boldsymbol{v}$, $\boldsymbol{s}$ or $\boldsymbol{t}$**
  - displacement –time, $s–t$
  - velocity –time, $v–t$
  - acceleration –time, $a–t$
  - velocity –displacement, $v–s$
  - acceleration –displacement, $a–s$
- As with the previous sections, differentiation or integration is required to move from one graph to the next

Differentiation

- Given the ***v*–*t* graph**, construct the ***a*–*t* graph**

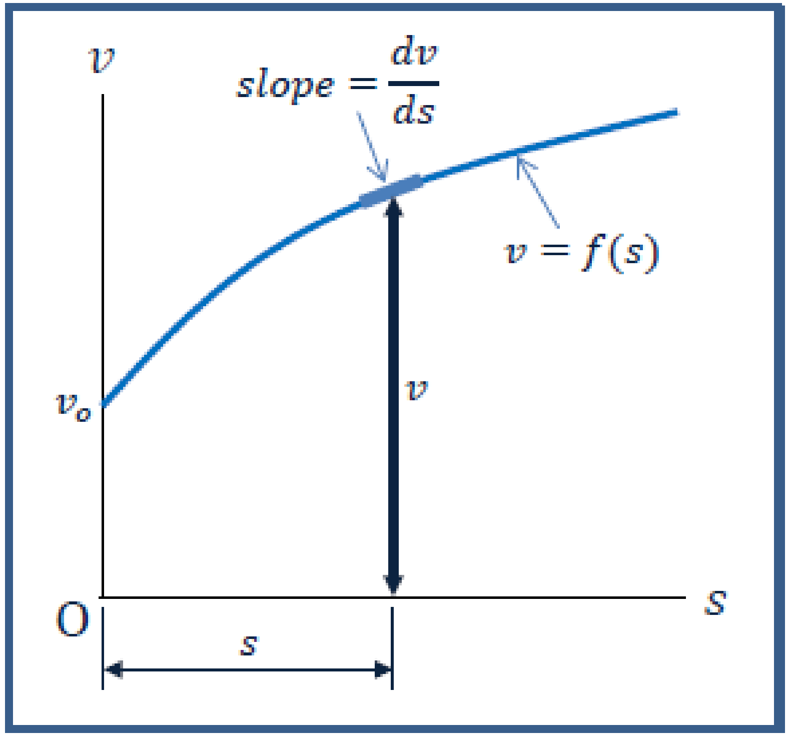


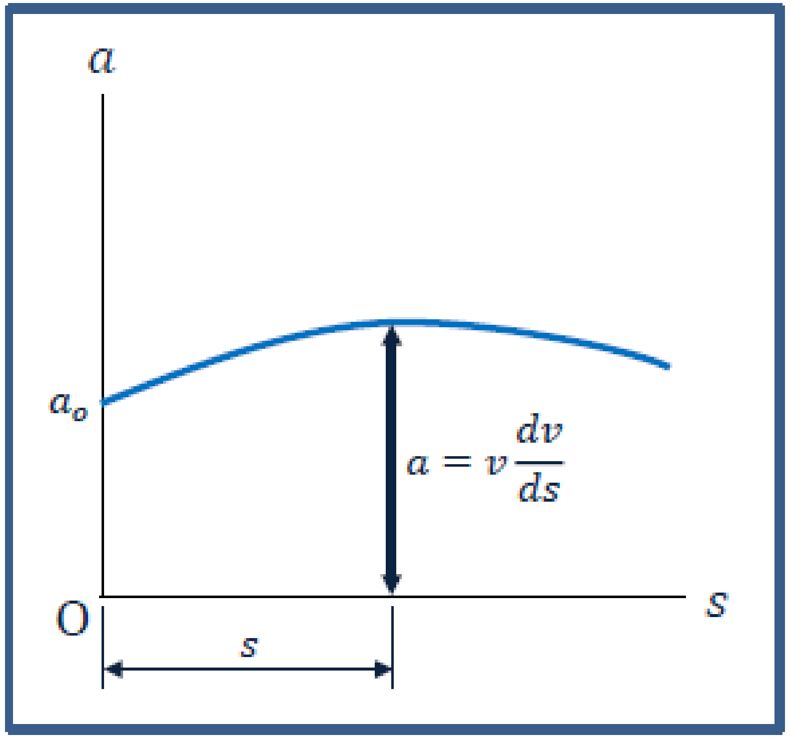


- The velocity curve is a function of time
- The slope of the curve at any time, t, is equal to the acceleration, a, at that time

Differentiation

- Given the ***v*–s graph**, construct the ***a*–s graph**

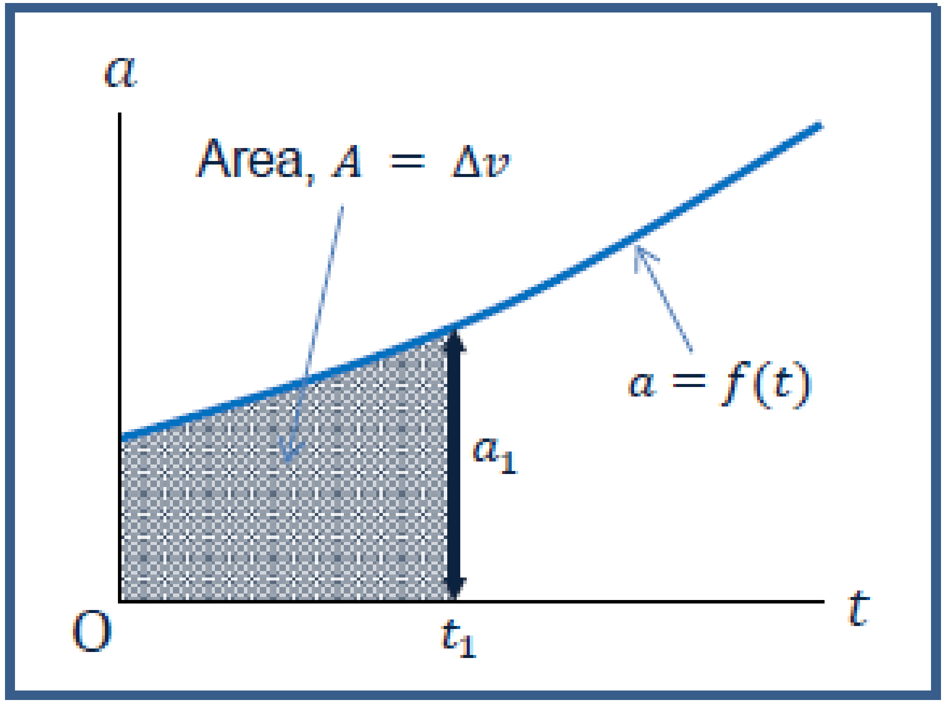


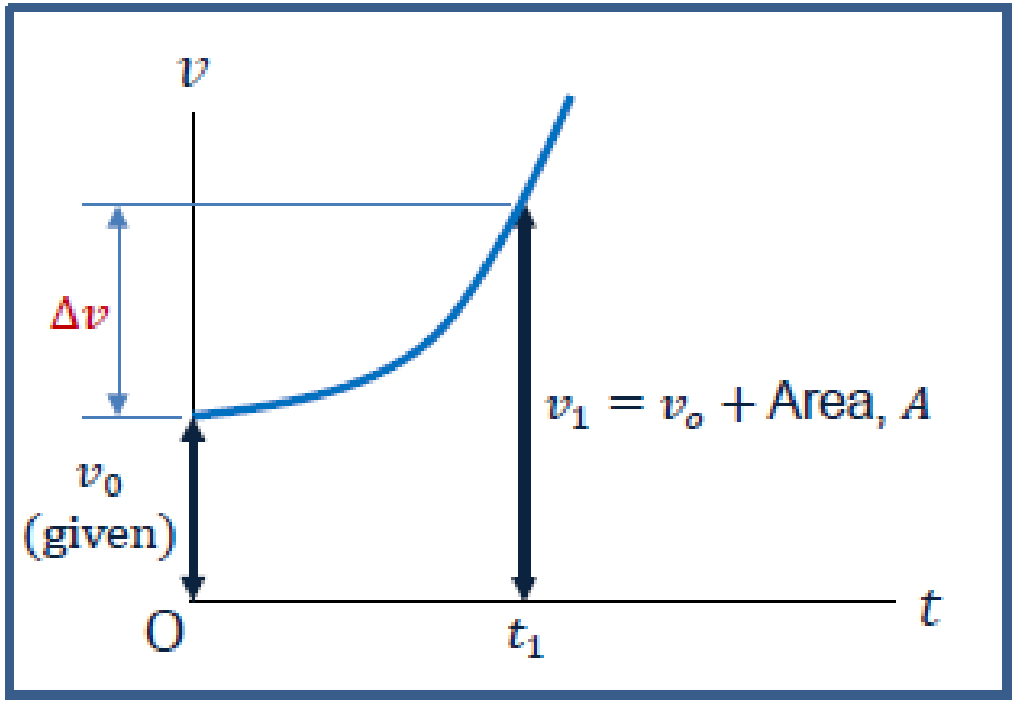


- The velocity curve is a function of displacement
- The slope of the curve at any displacement, *s*, multiplied by the velocity at that point is equal to the acceleration, *a*, at that displacement

Integration

- Given the a–t **graph**, construct the v–t **graph**



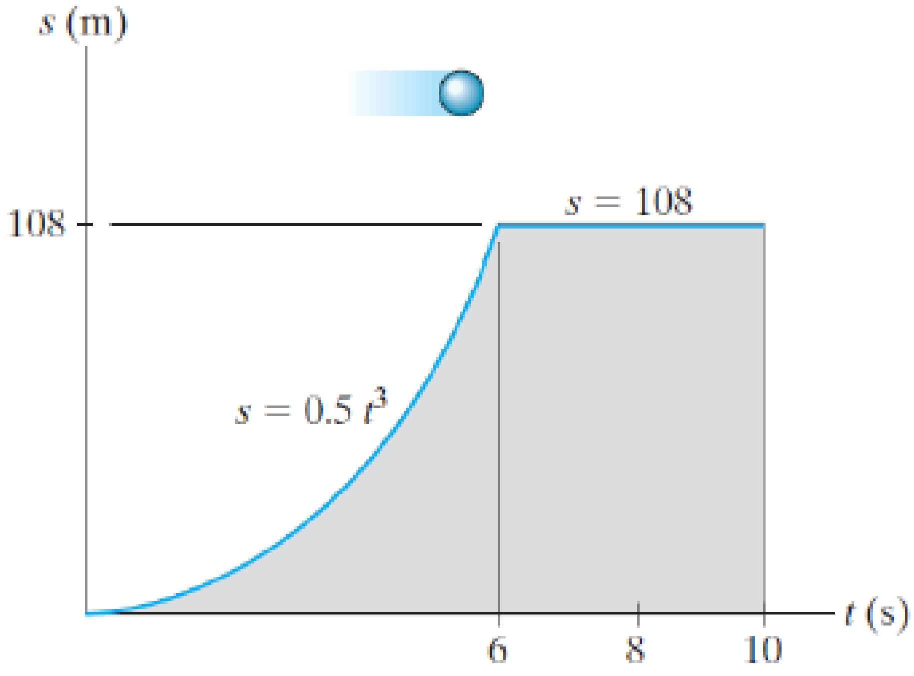


$$dv = a\,dt \quad \Longrightarrow \quad \int_{v_0}^{v_1} dv = \int_0^{t_1} a\,dt$$

$$v_1 - v_0 = \int_0^{t_1} a\,dt = \text{Area, } A \quad \Longrightarrow \quad \Delta v = \int_0^{t_1} a\,dt = \text{Area, } A$$

Example

- The particle travels along a straight track such that its position is described by the *s*-*t* graph. Construct the v-t graph for the same time interval.



Prob. F12–9