Dynamics 1, Rectilinear Motion

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Objectives

- Curvilinear Motion
- Cartesian Equations
- Projectile Motion
- Normal and Tangential Components

- Sections 12.5, 12.6 and 12.7 of

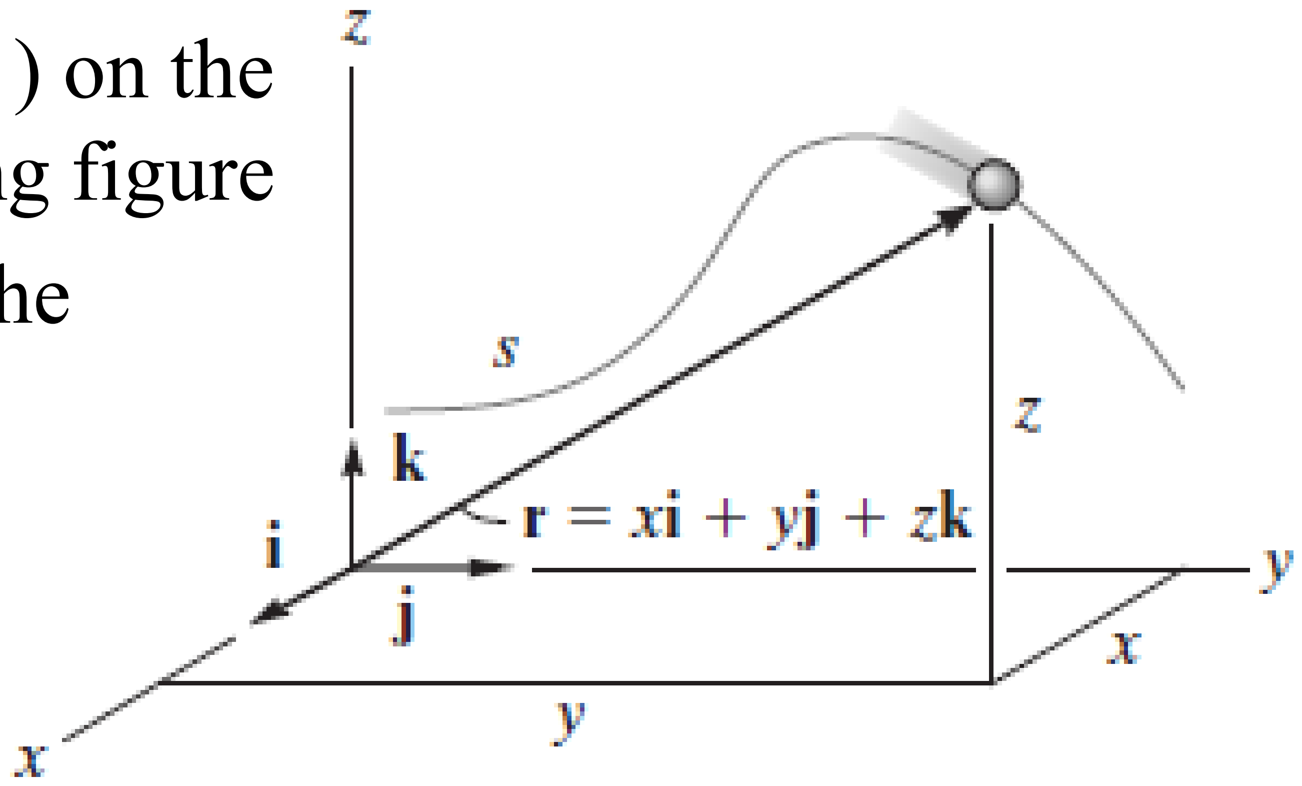
Tim McCarthy, Fundamentals of engineering mechanics for ENGG102 and ENGG100, Frenchs Forest, Sydney, NSW: Pearson Australia, 2015. ISBN: 9781488610431

Rectangular Components

- If the particle is at point (*x, y, z*) on the curved path s shown in following figure
- Then its location is defined by the *position vector*

$$\boxed{\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$



Rectangular Components

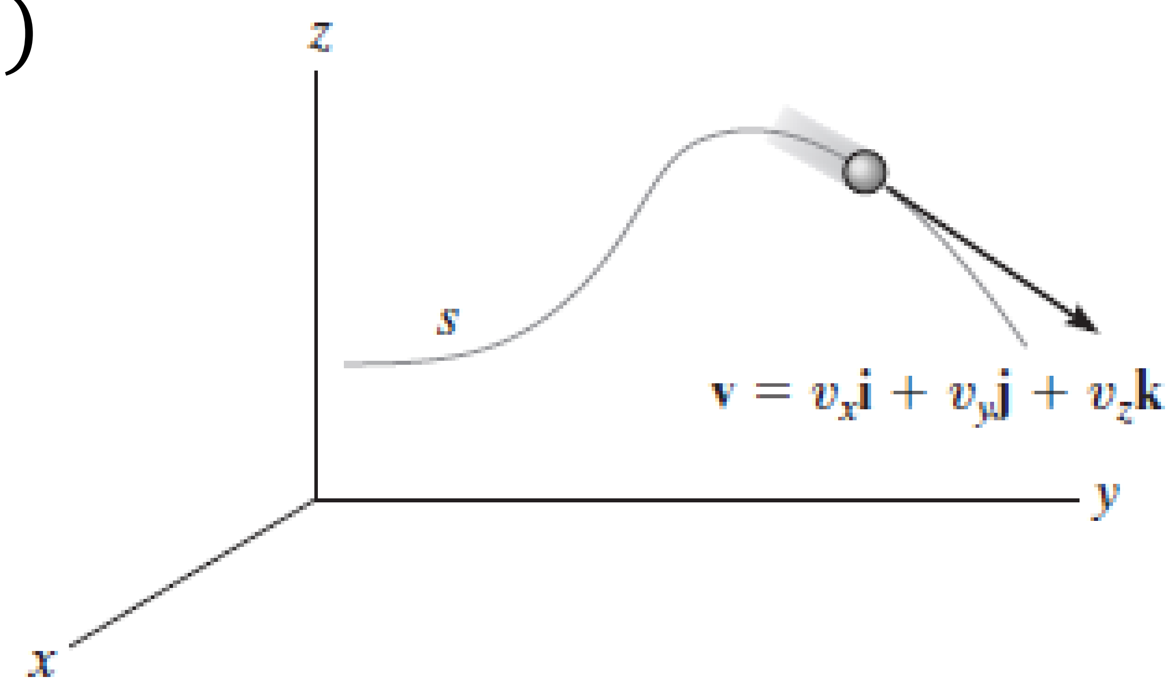
- For the velocity
- The first time derivative of **r** yields the velocity of the particle

$$v = \frac{dr}{dt} = \frac{d}{dt}(xi) + \frac{d}{dt}(yj) + \frac{d}{dt}(zk)$$

$$v = v_x i + v_y j + v_z k$$

- The magnitude of the velocity

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



Velocity

Rectangular Components

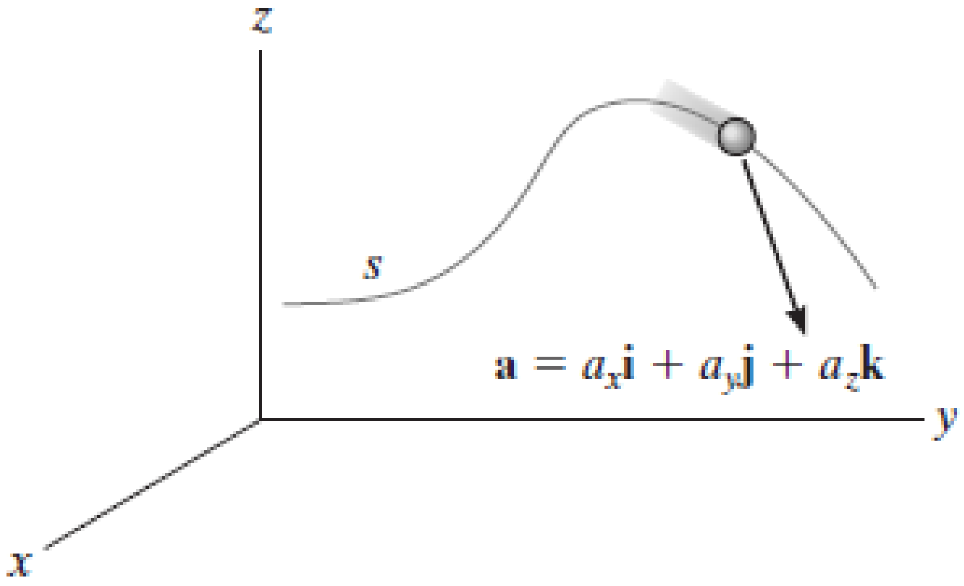
- For the acceleration
- The first time derivative of **v** yields the acceleration of the particle
- The second time derivative of r yields the acceleration of the particle

$$a = a_x i + a_y j + a_z k$$

- The magnitude of the acceleration

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

- The direction is not tangent to the path



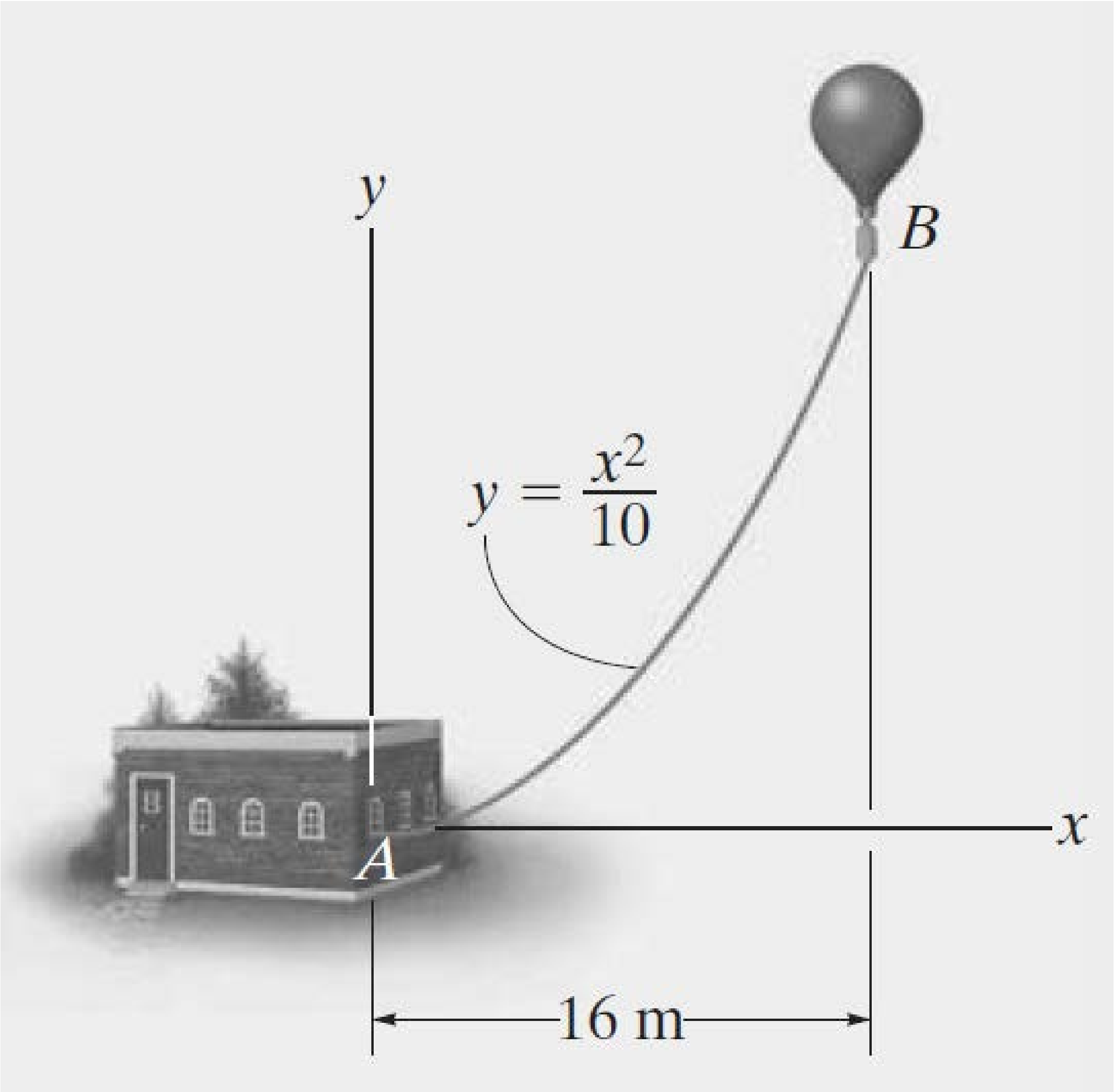
Acceleration

Curvilinear Motion

- Curvilinear motion can cause changes in *both* the magnitude and direction of the position, velocity, and acceleration vectors.
- The velocity vector is always directed *tangent* to the path.
- In general, the acceleration vector is *not* tangent to the path, but rather, it is tangent to the hodograph.
- If the motion is described using rectangular coordinates, then the components along each of the axes do not change direction, only their magnitude and sense (algebraic sign) will change.
- By considering the component motions, the change in magnitude and direction of the particle’s position and velocity are automatically taken into account.

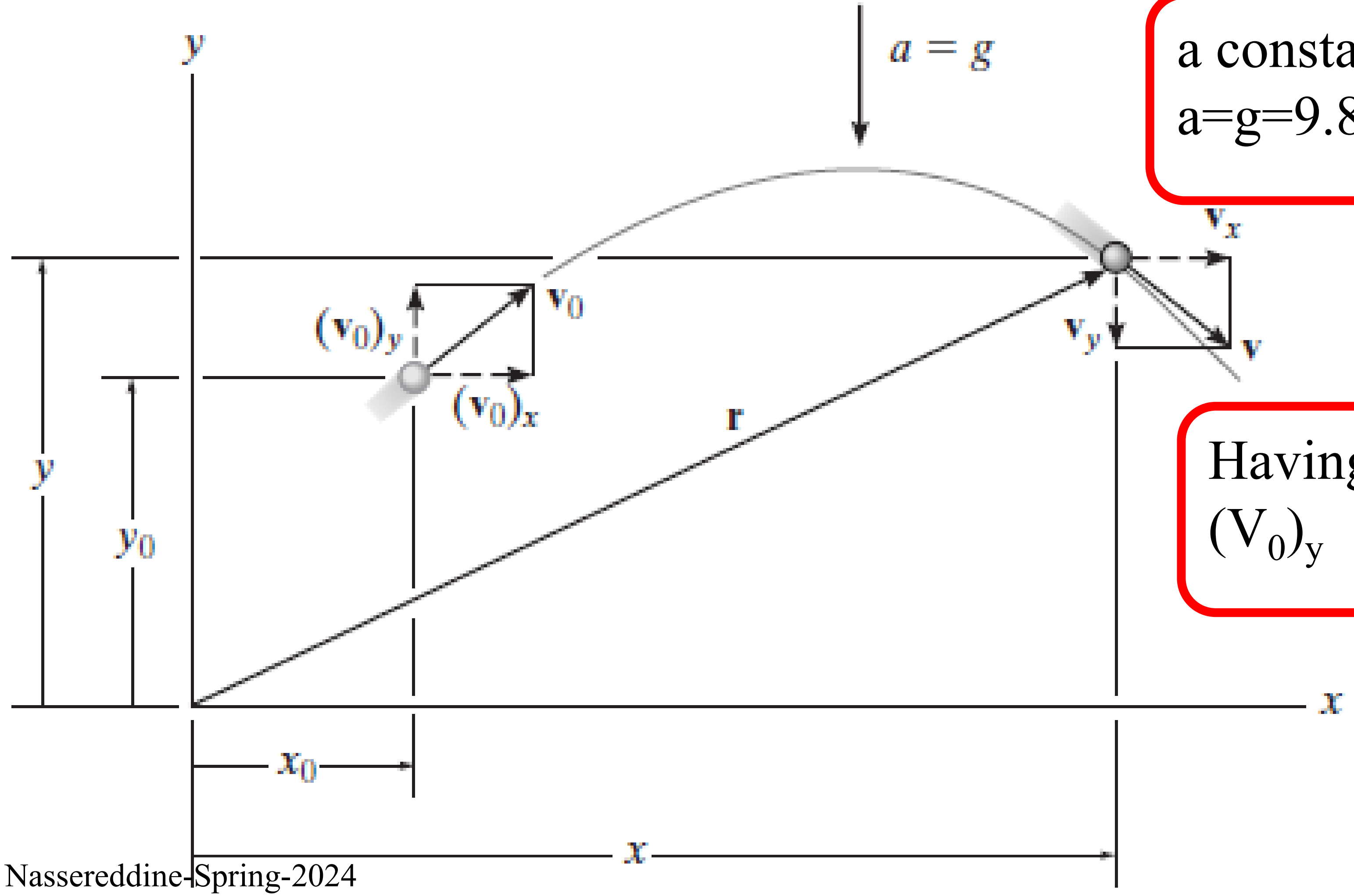
Example 1

- (12.9 in textbook)
- A hot air balloon follows a x axis path defined by $x=8t$m where t is in seconds.
- The y axis path follows: $y=x^2/10$m.
- Determine the velocity and acceleration when t = 2 seconds.



Projectile Motion

- Consider a projectile is launched at point (x_0, y_0)



a constant down acceleration $a=g=9.81m/s^2$

Having a constant $(V_0)_x$ and $(V_0)_y$

a_x is zero

Projectile Motion

- We have established that in the horizontal direction there is no acceleration (i.e. a_x=0).
- •This means that the velocity in the horizontal direction remains constant ($v_x = (v_0)_x$) and to determine the position of the particle in the x direction the following equation is used

$$x = x_0 + (V_0)_x\, t$$

- which is one of the constant acceleration equations we've seen previously

Projectile Motion

- Applying the equations from constant acceleration

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

In this case we will use the y axis

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

Acceleration direction down

$$v_y = v_{0y} - gt$$

Projectile Motion- Procedure for Analysis

- Coordinate System.
  - Establish the fixed *x, y* coordinate axes and sketch the trajectory of the particle. Between any *two points* on the path specify the given problem data and identify the *three unknowns* . In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2

  The particle's initial and final velocities should be represented in terms of their *x* and *y* components.
  - Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions.

Projectile Motion- Procedure for Analysis

- Kinematic Equations.
  - Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem.
    - Horizon movement

$$x = x_0 + (v_0)_x \, t$$

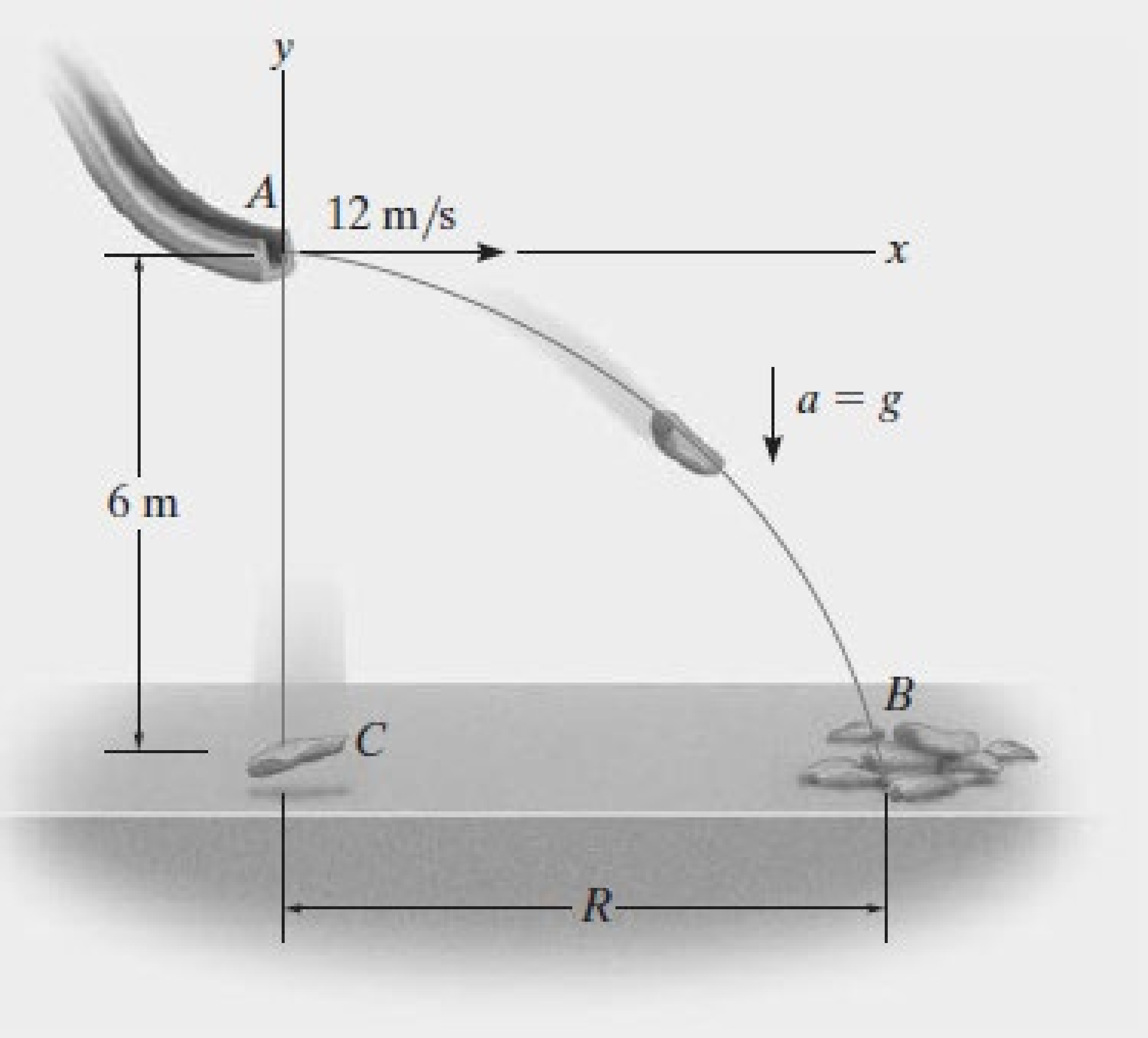
    - Vertical movement

$$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 \qquad v_y = v_{0y} - gt \qquad v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

Example 12.11

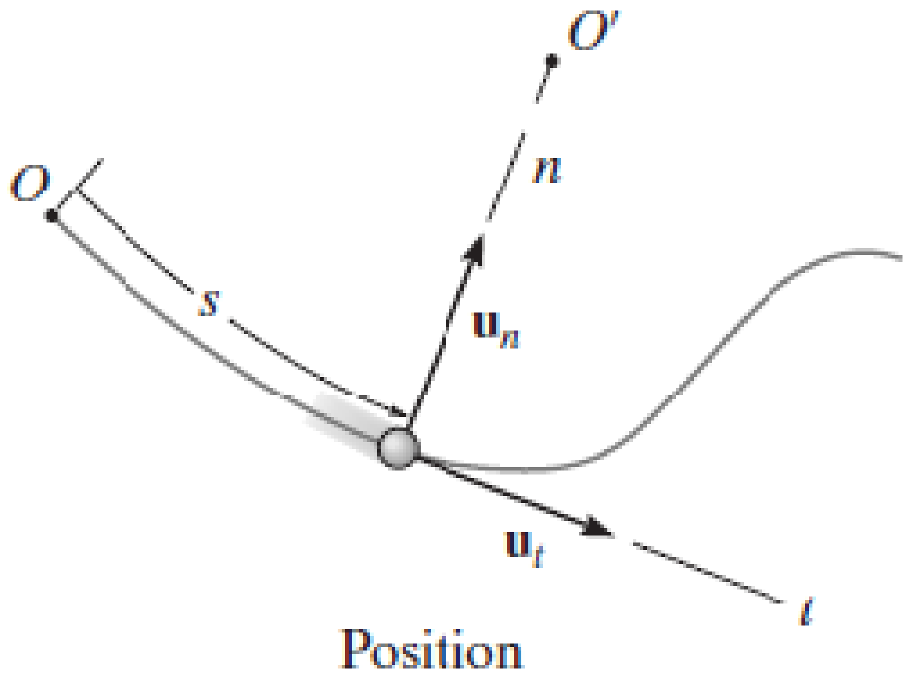
- A sack slides off the ramp, shown in Fig. 12–21 , with a horizontal

velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range *R* where sacks begin to pile up.



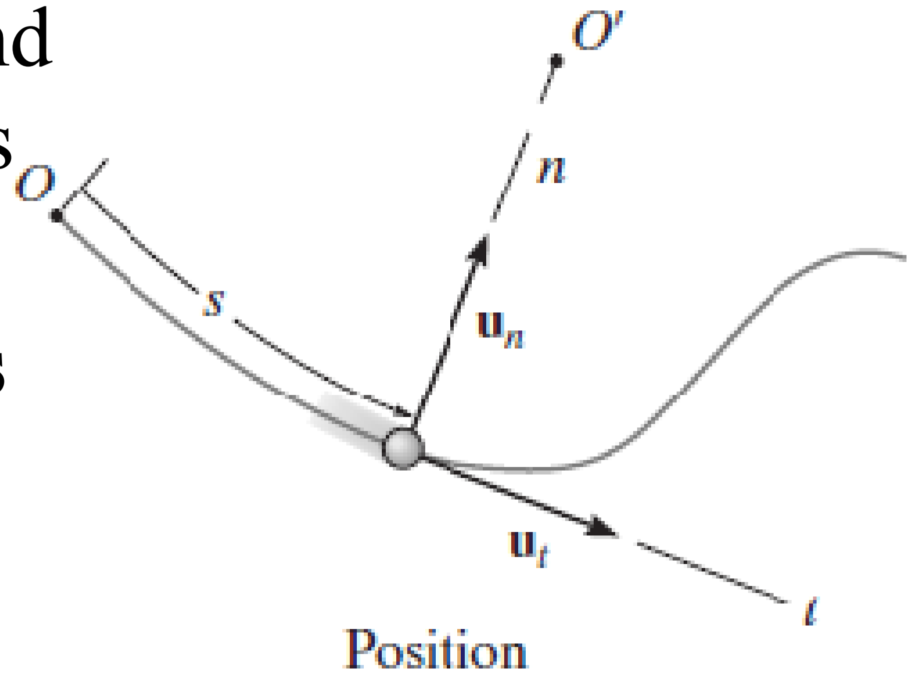
Normal and Tangential Components

- When a particle is moving along a curved path, it is sometimes more convenient to describe its motion using a coordinate system other than Cartesian.
- When the path of motion is known, **normal**(n) and **tangential**(t) coordinates are often used.
- In the $n\text{−}t$coordinate system, the origin is located on the particle, thus is continually moving with the particle.



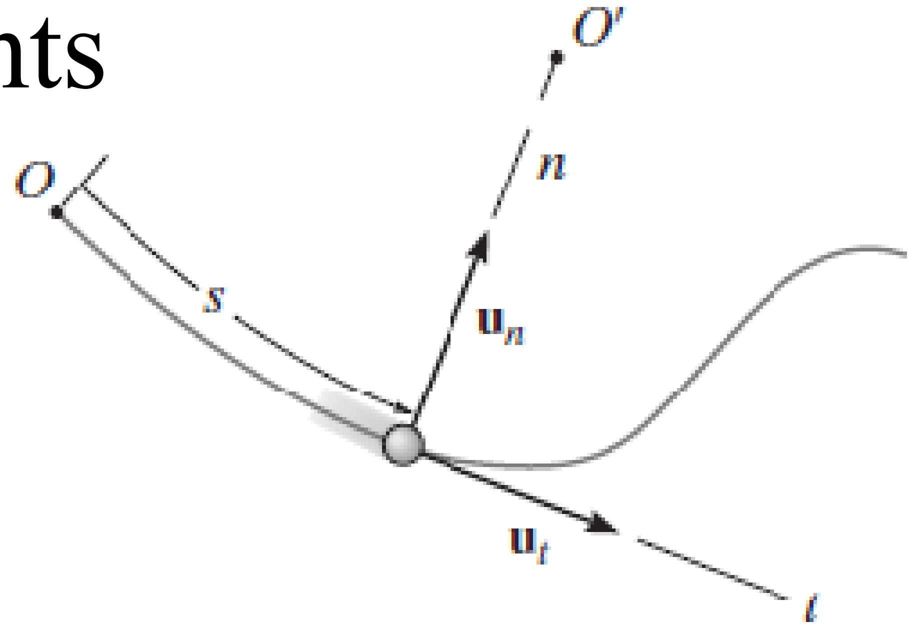
Normal and Tangential Components

- The ***t*** **axis** is **tangent to the path** (usually a curve) at the instant being considered and is positive in the direction of the particle's motion.
- The ***n*** **axis** is **perpendicular to the** ***t*** **axis** with the positive direction towards the centre of curvature of the curved path.

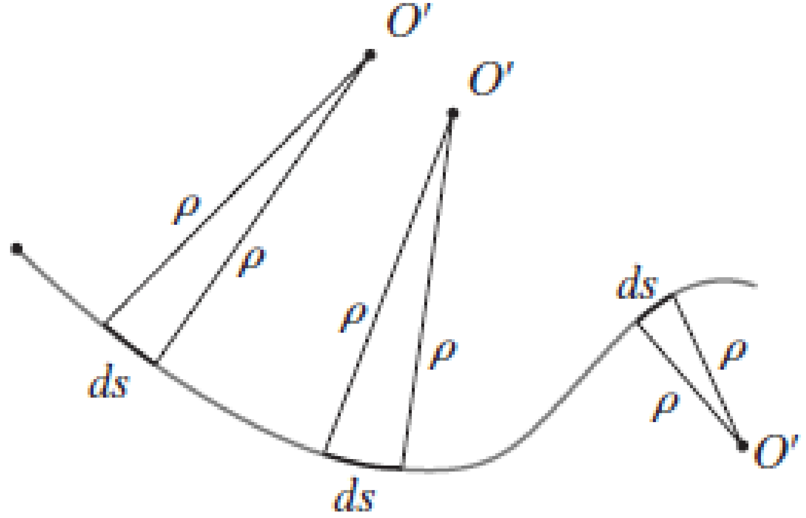


Normal and Tangential Components

- Additionally, the positive n and t **directions** are defined by the unit vectors $\boldsymbol{u_n}$and $\boldsymbol{u_t}$
- The **centre of curvature**, O', always lies on the concave side of the curve (where the centre is)
- The **position** of the particle at any particular instant is defined by the distance, **s**, along the curved path from a fixed reference point on the curve

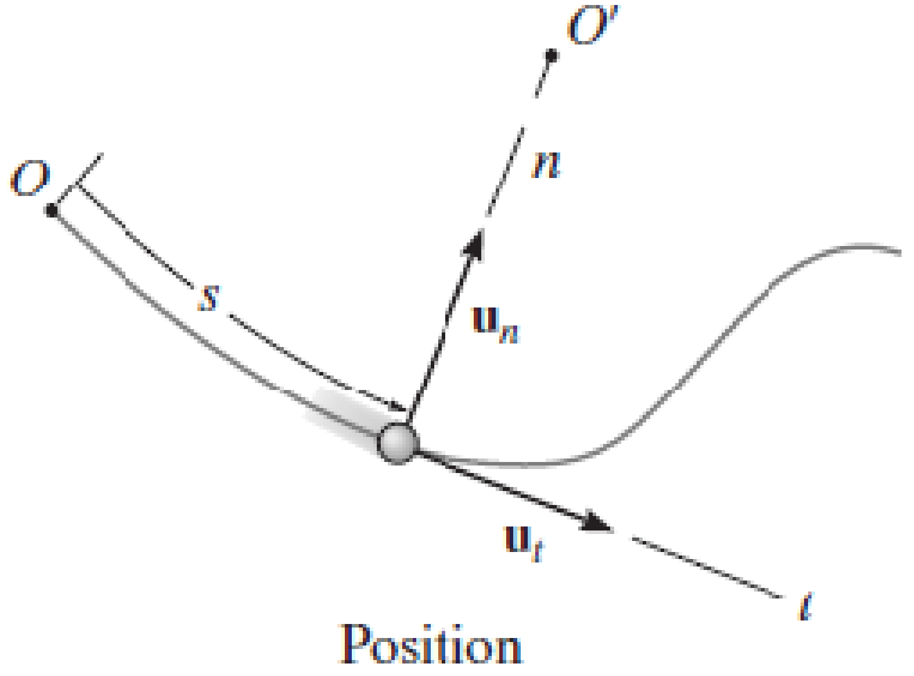


Position



Normal and Tangential Components

- The velocity vector is always at a tangent to the path of motion for every position of the particle
- The magnitude of the velocity is determined by taking the time derivative of the path function, $s = f(t)$
- $v = v\overline{u_t}$
- and, v defines the magnitude of the velocity (speed)
- u_t defines the direction of the velocity vector



Acceleration in the n-t coordinate

- The acceleration is the time rate change of velocity

$$a = \dot{v} = \frac{dv}{dt} = \frac{d(v\overline{u}_t)}{dt} = \dot{v}\overline{u}_t + v\dot{\overline{u}}_t \quad \text{(chain rule)}$$

- $\dot{v}$ represents the change in magnitude of the velocity and
- $\dot{\overline{u}}_t$ represents the rate of change in direction of $\overline{u}_t$
- After some mathematical manipulation (which we do not need to focus on, but check in textbooks if you are interested), the acceleration vector can be expressed as

$$a = \dot{v}\overline{u}_t + \left(\frac{v^2}{\rho}\right)\overline{u}_n = a_t\overline{u}_t + a_n\overline{u}_n$$

Acceleration in the n-t coordinate

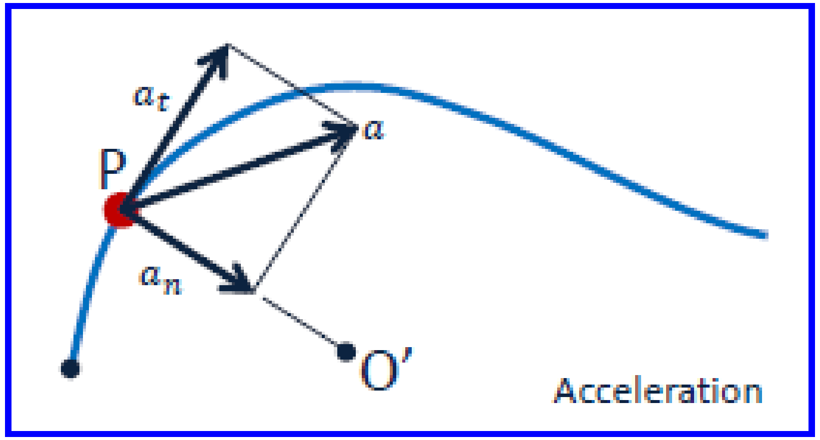
- The **tangential component** is tangent to the curve and in the direction of velocity
- The **normal**(or centripetal) **component** is always directed towards the centre of curvature
- The **magnitude** of the acceleration vector is

$$a = \sqrt{{a_n}^2 + {a_t}^2}$$

$$a_t = \dot{v} \quad \text{OR} \quad a_t ds = v dv$$

$$a_n = \left(\frac{v^2}{\rho}\right)$$

$$a = a_t \overline{u}_t + a_n \overline{u}_n$$



Acceleration in the n-t coordinate

- There are some special cases of motion that need to be considered.
  - The particle moves along a straight line

$$\rho \to \infty \quad \Rightarrow \quad a_n = \frac{v^2}{\rho} = 0 \quad a = a_t = \dot{v}$$

The tangential component of velocity represents the time rate change in the magnitude of velocity

- The particle moves along a curve with constant velocity

$$a_t = \dot{v} = 0 \quad \Rightarrow \quad a = a_n = \frac{v^2}{\rho}$$

The normal component of velocity represents the time rate change in the magnitude of velocity

Acceleration in the n-t coordinate

3. The **tangential** component of **acceleration is constant**, therefore we use the constant acceleration equations

$$a_t = (a_t)_c$$
$$v = v_0 + (a_t)_c t$$
$$s = s_0 + v_0 t + \frac{1}{2}(a_t)_c t^2$$
$$v^2 = (v_0)^2 + 2(a_t)_c(s - s_0)$$

Note that s_0 and v_0 are the position and velocity of the particle when $t = 0$.

4. The particle moves along a curve defined by y=f(x). The radius of curvature, ρ, will change throughout the curve and can be calculated at any point on the curve using the following equation

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Example

- Example 12.15

- A race car travels around a circular track of radius 300 m. The car increases its speed at rate 7 m/s2. If the car is initially at rest, determine the time needed for the car to reach 8 m/s2. What is the speed at this instant?