ENGG100 Tutorial 5

Rectilinear motion of particles

Equations for constant acceleration:

$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$v^2 = {v_0}^2 + 2a_c(s - s_0)$$

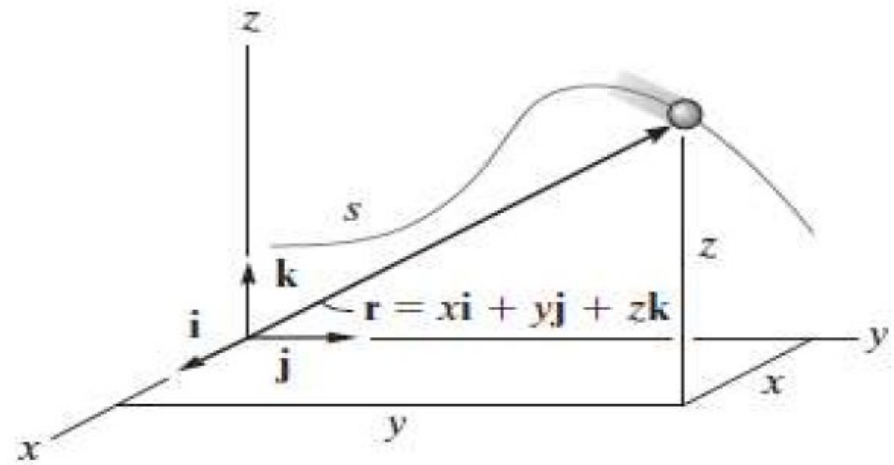
General equations:

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad a = v\frac{dv}{ds}$$

If the question is constant velocity, set a_c=0.

position vector

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$



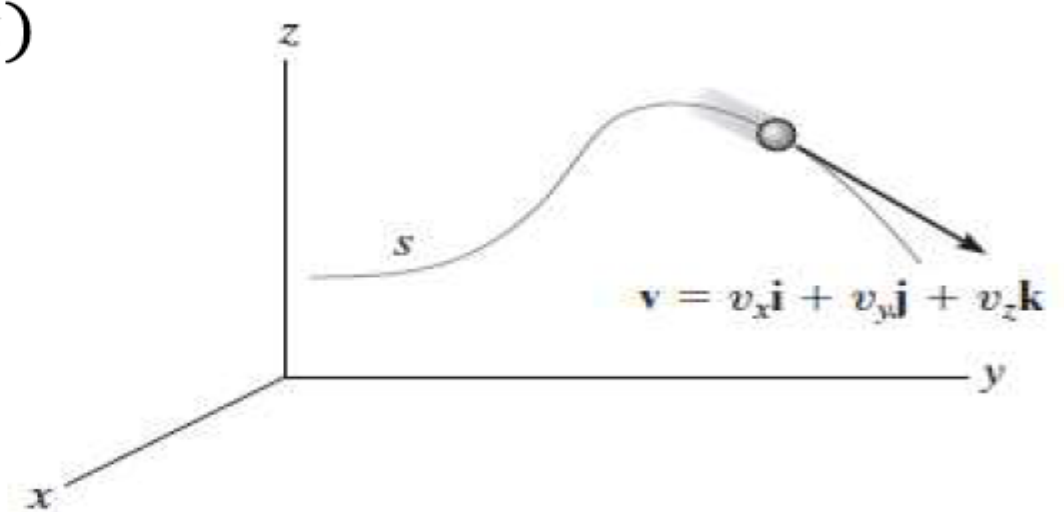
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$v = \frac{dr}{dt} = \frac{d}{dt}(xi) + \frac{d}{dt}(yj) + \frac{d}{dt}(zk)$$

$$v = v_x i + v_y j + v_z\, k$$

The magnitude of the velocity

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$



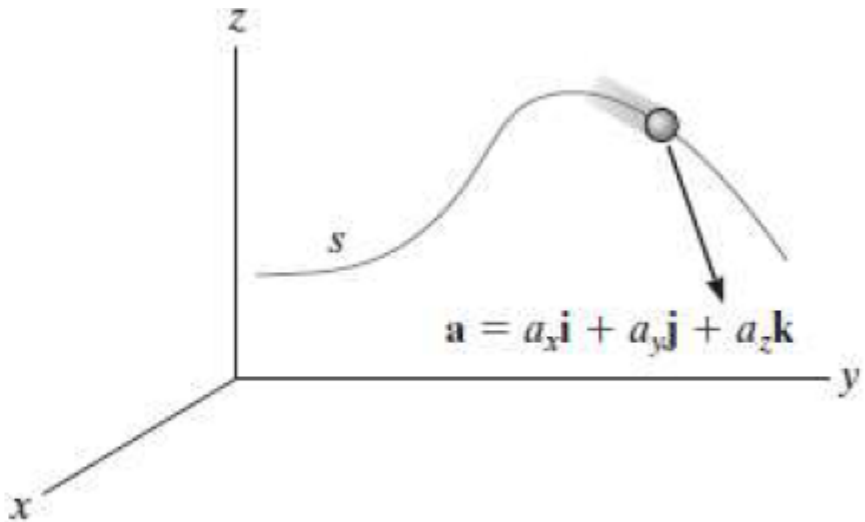
Velocity

$$a = a_x i + a_y j + a_z k$$

The magnitude of the acceleration

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

The direction is not tangent to the path



Acceleration

Horizontal Motion.

- The *velocity* in the horizontal or x direction is *constant*, i.e., $v_x = (v_0)_x$, and

$$x = x_0 + (v_0)_x t$$

Vertical Motion.

- In the vertical or y direction *only two* of the following three equations can be used for solution.

$$v_y = (v_0)_y + a_c t$$

$$y = y_0 + (v_0)_y t + \tfrac{1}{2} a_c t^2$$

$$v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$

Tangential Acceleration.

$$a_t = \dot{v} \quad a_t\,ds = v\,dv$$

$$\begin{aligned} s &= s_0 + v_0 t + \tfrac{1}{2}(a_t)_c t^2 \\ v &= v_0 + (a_t)_c t \\ v^2 &= v_0^2 + 2(a_t)_c (s - s_0) \end{aligned}$$

Normal Acceleration.

$$a_n = \frac{v^2}{\rho}$$