Tutorial 3 – Week 3- Solution

Example 1: Example 12.9 Text Book

At any instant the horizontal position of the weather balloon in Fig. 1 is defined by $x = (8t)$ m, where t is in seconds. If the equation of the path is $y = x2>10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

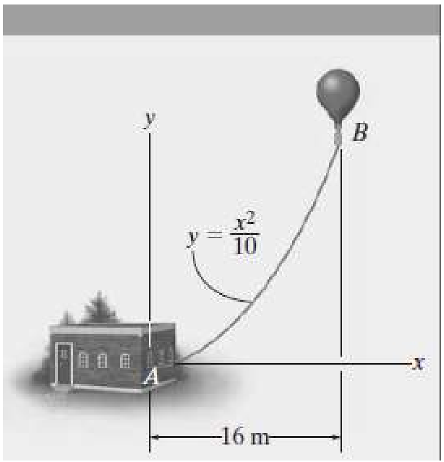

Figure 1

Solution

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ m/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. When $t = 2$ s, $x = 8(2) = 16$ m, Fig. 12–18a, and so

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ m/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ m/s})^2 + (25.6 \text{ m/s})^2} = 26.8 \text{ m/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18b, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6^\circ \qquad \textit{Ans.}$$

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ m/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ m/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90^\circ \qquad \textit{Ans.}$$

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (8t)^2/10 = 6.4t^2$ and then taking successive time derivatives.

Example 2: Example 12.10 Text Book

For a short time, an airplane follows a trajectory defined by $y=0.001x^2$ m. If the plane is rising with a constant velocity 10 m/s, determine the magnitudes of the velocity and acceleration when the plane reaches an altitude of $y=100$ m.

Solution

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, due to constant velocity $v_y = 10$ m/s, so

$$y = v_y t; \qquad 100\ \text{m} = (10\ \text{m/s})\, t \qquad t = 10\ \text{s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$y = 0.001x^2$$

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \qquad (1)$$

Thus

$$10\ \text{m/s} = 0.002(316.2\ \text{m})(v_x)$$
$$v_x = 15.81\ \text{m/s}$$

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\ \text{m/s})^2 + (10\ \text{m/s})^2} = 18.7\ \text{m/s} \qquad \textit{Ans.}$$

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = (0.002\dot{x})\dot{x} + 0.002x(\ddot{x}) = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002\left[(15.81\ \text{m/s})^2 + 316.2\ \text{m}(a_x)\right]$$
$$a_x = -0.791\ \text{m/s}^2$$

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\ \text{m/s}^2)^2 + (0\ \text{m/s}^2)^2}$$
$$= 0.791\ \text{m/s}^2 \qquad \textit{Ans.}$$

These results are shown in Fig. 12–19*b*.

Example 3: Example 12.12 Text Book

A chipping machine ejects wood chips at 7.5 m/s at an angle of 30º above the horizontal axis. The chips are ejected at a height of 1.2 m. Given that the wood chips land on a pile that is located 6 metres across from the wood chipper, how tall is the pile?

Solution

Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (7.5 \cos 30^\circ) \text{ m/s} = 6.50 \text{ m/s} \rightarrow$$

$$(v_O)_y = (7.5 \sin 30^\circ) \text{ m/s} = 3.75 \text{ m/s}\uparrow$$

Also, $(v_A)_x = (v_O)_x = 6.50$ m/s and $a_y = -9.81$ m/s^2. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$(\overset{+}{\rightarrow})$

$$x_A = x_O + (v_O)_x t_{OA}$$

$$6 \text{ m} = 0 + (6.50 \text{ m/s}) t_{OA}$$

$$t_{OA} = 0.923 \text{ s}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \qquad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2} a_c t_{OA}^2$$

$$(h - 1.2 \text{ m}) = 0 + (3.75 \text{ m/s})(0.923 \text{ s}) + \tfrac{1}{2}(-9.81 \text{ m/s}^2)(0.923 \text{ s})^2$$

$$h = 0.483 \text{ m} \qquad \textit{Ans.}$$

NOTE: We can determine $(v_A)_y$ by using $(v_A)_y = (v_O)_y + a_c t_{OA}$.

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Example 4: Example 12.13 Text Book

Motorbike riders jump off a ramp that is 1 m high at an angle of 30°. Given that the rider is airborne for 1.5 seconds, calculate the speed of the motorbike when the bike left the ramp.

Solution

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-1\text{ m} = 0 + v_A \sin 30°(1.5\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(1.5\text{ s})^2$$

$$v_A = 13.38\text{ m/s} = 13.4\text{ m/s} \qquad \textit{Ans.}$$

Horizontal Motion. The range R can now be determined.

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 13.38 \cos 30°\text{ m/s}(1.5\text{ s})$$

$$= 17.4\text{ m} \qquad \textit{Ans.}$$

In order to find the maximum height h we will consider the path AC, Fig. 12–23b. Here the three unknowns are the time of flight t_{AC}, the horizontal distance from A to C, and the height h. At the maximum height $(v_C)_y = 0$, and since v_A is known, we can determine h *directly* without considering t_{AC} using the following equation.

$$(v_C)_y^2 = (v_A)_y^2 + 2a_c[y_C - y_A]$$

$$0^2 = (13.38 \sin 30°\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)[(h - 1\text{ m}) - 0]$$

$$h = 3.28\text{ m} \qquad \textit{Ans.}$$

NOTE: Show that the bike will strike the ground at B with a velocity having components of

$$(v_B)_x = 11.6\text{ m/s} \rightarrow, \quad (v_B)_y = 8.02\text{ m/s}\downarrow$$

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Example 5: Example 12.16 Text Book

The boxes in Fig. 12– 29 a travel along the industrial conveyor. If a box as in Fig. 12– 29 b starts from rest at A and increases its speed such that a = (0.2 t) m > s 2 t , where t is in seconds, determine the magnitude of its acceleration when it arrives at point B .

Solution

Acceleration. To determine the acceleration components $a_t = \dot{v}$ and $a_n = v^2/\rho$, it is first necessary to formulate v and $\dot{v}$ so that they may be evaluated at B. Since $v_A = 0$ when $t = 0$, then

$$a_t = \dot{v} = 0.2t \qquad (1)$$

$$\int_0^v dv = \int_0^t 0.2t\,dt$$

$$v = 0.1t^2 \qquad (2)$$

The time needed for the box to reach point B can be determined by realizing that the position of B is $s_B = 3 + 2\pi(2)/4 = 6.142$ m, Fig. 12–29b, and since $s_A = 0$ when $t = 0$ we have

$$v = \frac{ds}{dt} = 0.1t^2$$

$$\int_0^{6.142\text{ m}} ds = \int_0^{t_B} 0.1t^2 dt$$

$$6.142\text{m} = 0.0333t_B^3$$

$$t_B = 5.690\text{s}$$

Substituting into Eqs. 1 and 2 yields

$$(a_B)_t = \dot{v}_B = 0.2(5.690) = 1.138\text{ m/s}^2$$

$$v_B = 0.1(5.69)^2 = 3.238\text{ m/s}$$

At B, $\rho_B = 2$ m, so that

$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(3.238\text{ m/s})^2}{2\text{ m}} = 5.242\text{ m/s}^2$$

The magnitude of $\mathbf{a}_B$, Fig. 12–29c, is therefore

$$a_B = \sqrt{(1.138\text{ m/s}^2)^2 + (5.242\text{ m/s}^2)^2} = 5.36\text{ m/s}^2 \qquad \textit{Ans.}$$

<u>**For student to do:**</u>

(Q12-74 from textbook) The velocity of a particle is $v=3\tilde{\imath}+(6-2t)\tilde{\jmath}$ m/s where t is in seconds. If the particle is originally at the origin, determine the displacement of the particle during the time interval t=1 to t=3 seconds.

<u>**Solution**</u>

$$\vec{V} = 3\vec{i} + (6-2t)\vec{j}$$

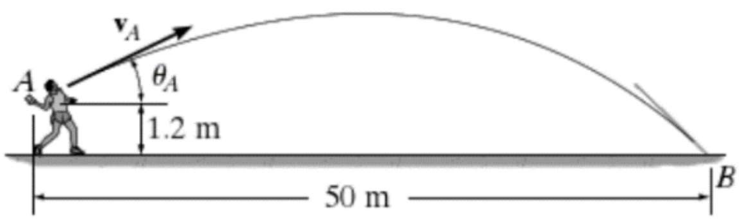
$$V = \frac{dr}{dt}$$

$$\int \frac{dr}{dt} = \int \left[3\vec{i} - (6-2t)j\right]$$

$$\int_{r_1}^{r_2} dr = \int_1^3 3\,dt\,\vec{i} + \int_1^3 (6-2t)\,dt\,\vec{j}$$

$$r = 6\vec{i} + 4\vec{j}$$

(Q12-89 from textbook) A ball is thrown 1.2 m above the ground and travels 50 m before hitting the ground at t=2.5 seconds. Determine the initial velocity magnitude and angle.



Prob. 12–89

Solution

$$(V_A)_x = V_A \cos(\theta_A)$$

$$(V_A)_y = V_A \sin(\theta_A)$$

$$(S_B)_x = (S_0)_x + (V_A)_x t$$

$$50 = 0 + V_A \cos(\theta_A)\, 2.5$$

$$20 = V_A \cos\theta$$

$$(S_B)_y = (S_0)_y + V_A \sin(\theta_A) t - \frac{1}{2} g t^2$$

$$-1.2 = 0 + V_A \sin(\theta_A)\, 2.5 - \frac{1}{2} 9.81 (2.5)^2$$

$$11.78 = V_A \sin(\theta_A)$$

$$\tan(\theta_A) = \frac{\sin(\theta_A)}{\cos(\theta_A)}$$

$$\frac{11.78}{20} = \frac{\cancel{V_A} \sin(\theta_A)}{\cancel{V_A} \cos(\theta_A)} \Leftrightarrow \theta = 30.5^\circ$$

$$(V_A)_x = \frac{20}{\cos(30.5)}$$

$$(V_A)_y = \frac{11.78}{\sin(30.5)}$$