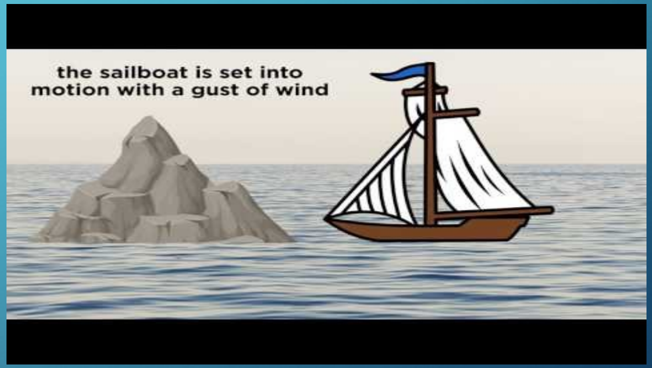
# MOMENT OF INERTIA

This concept is useful for structural design, mechanical design etc. For example in design of beams, columns, pipes, rotating shafts, and other elements in engineering.

# What is Inertia:

An object at rest will remain at rest and an object in motion will remain in motion unless an unbalanced force acts upon it.

- https://www.youtube.com/watch?v=1XSyyjcEHo0



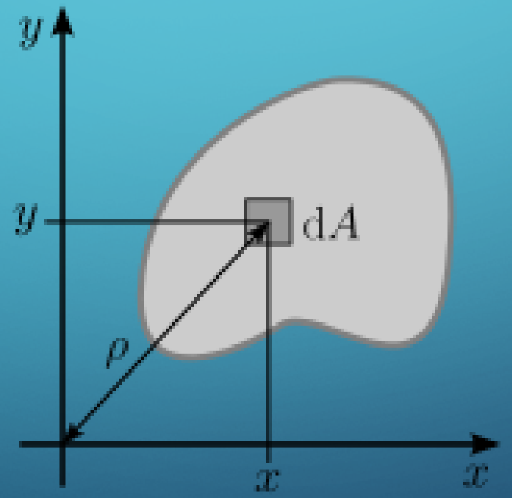
*In general, the moment of inertia of an object is its resistance to rotational acceleration.*

- Mass moment of inertia
- Area moment of inertia
- Polar moment of inertia

- In simple terms, moment of inertia symbolizes strength or stiffness.

# Second moment of area or moment of inertia

- The 2nd moment of area, also known as the area moment of inertia, or second area moment, is a geometrical property of an area which reflects how its points are distributed with regard to an arbitrary axis.



## Moment of Inertia

Consider area A lying in the x-y plane

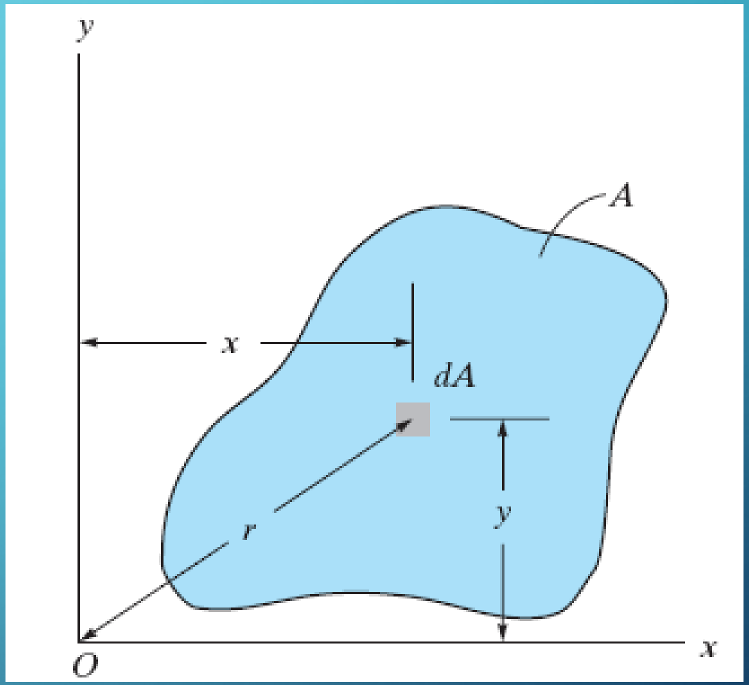
- By definition, moments of inertia of the differential plane area dA about the x and y axes

$$dI_x = y^2 dA \quad dI_y = x^2 dA$$

- For entire area, moments of inertia are given by

$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$



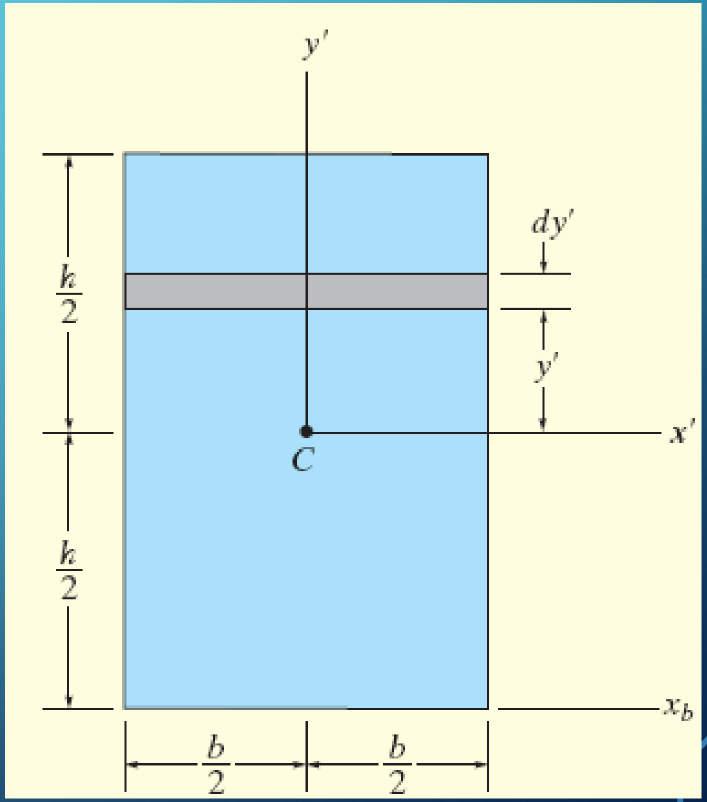
Q- Determine the moment of inertia for the rectangular area with respect to (a) the centroidal x' axis, (b) centroidal y' axis.

Solution - Differential element chosen, distance y' from x' axis.

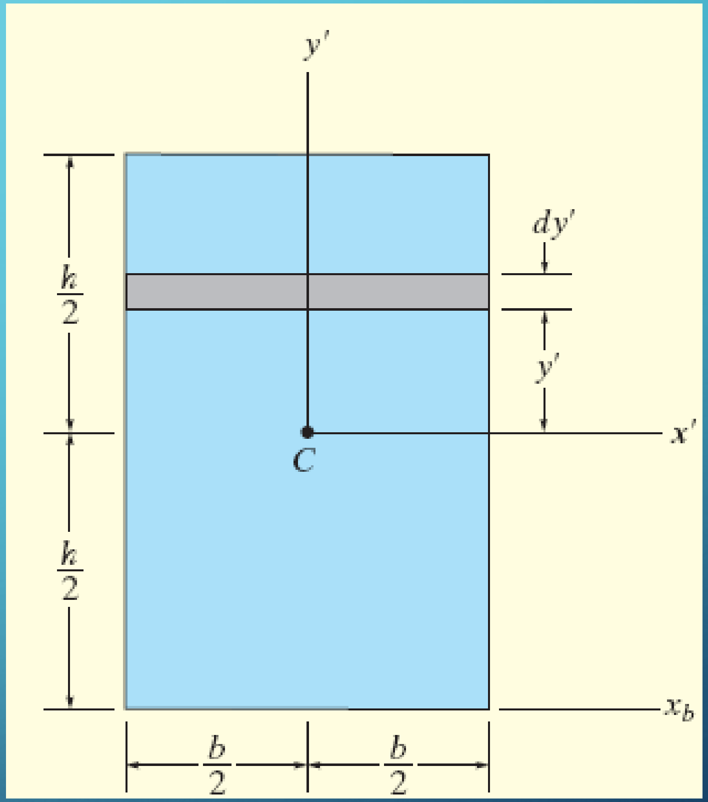
Since,

**dA = b dy'**

$$\bar{I}_x = \int_A y'^2 \, dA = \int_{-h/2}^{h/2} y'^2 (b\,dy') = b\int_{-h/2}^{h/2} y'^2 \, dy' = \frac{1}{12} b h^3$$



- Derive $I_{y'}$ in class



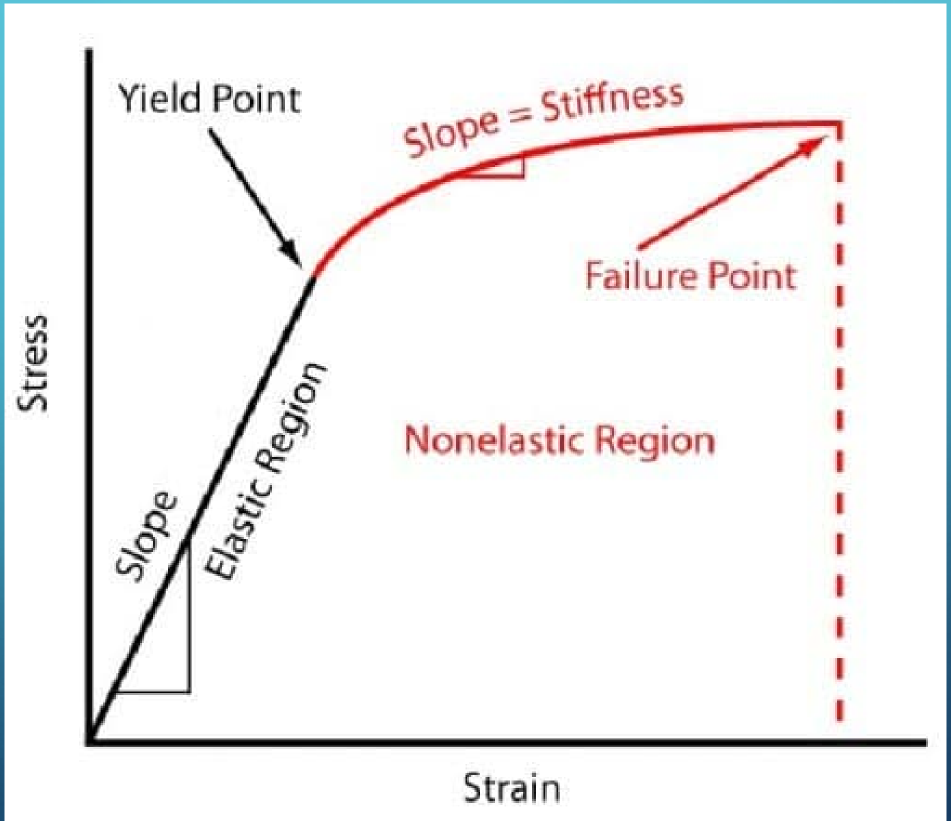
# YOUNG'S MODULUS OF ELASTICITY

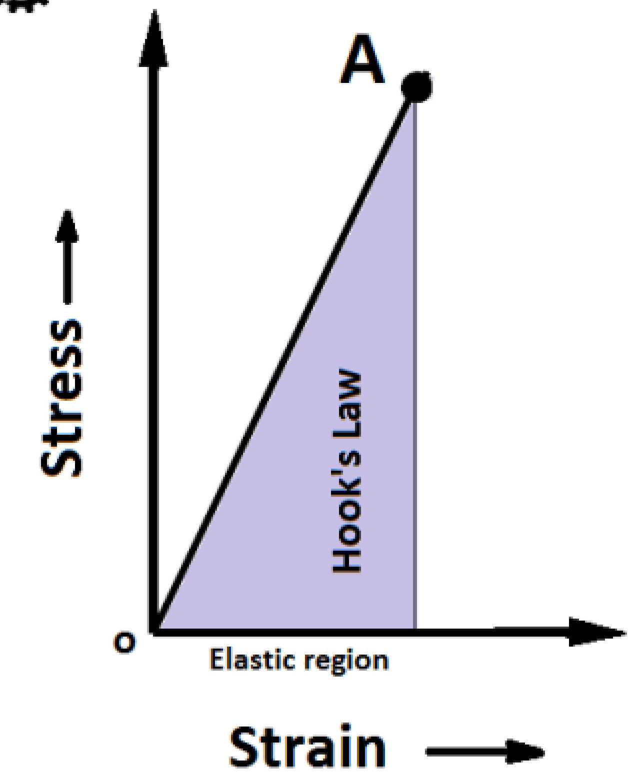
- A measure of how stiff a material is.
- This is a constant property for any material.
- An elastic modulus (also known as modulus of elasticity) is a quantity that measures an object or substance's resistance to being deformed elastically (i.e., non-permanently) when a stress is applied to it.

- A stiffer material will have a higher elastic modulus. Most material values are available in mechanics and statics books.

Generally, the elastic modulus of an object is defined as the slope of its stress–strain or load – deflection curve in the **elastic deformation region**.

It can also be determined from a deflection – load graph for a beam.






**Stress $\propto$ Strain**

$$\sigma \propto \varepsilon$$

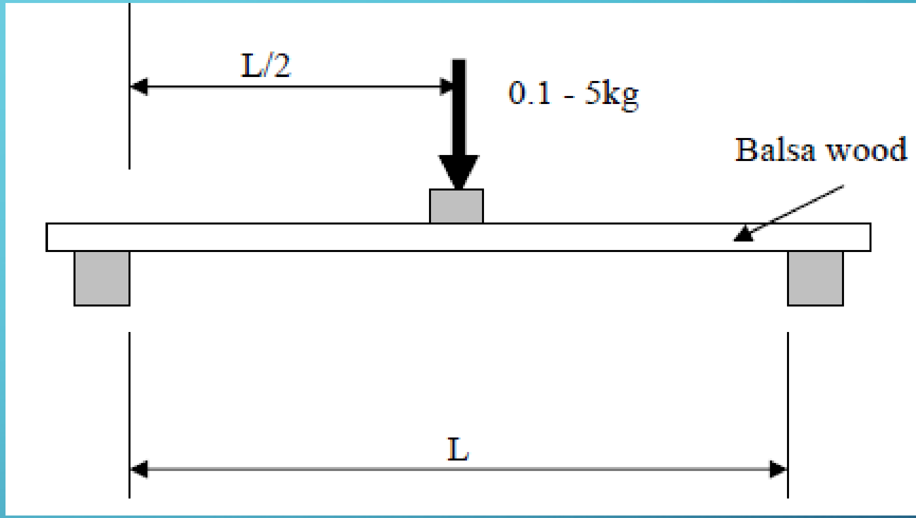
$$\sigma = E\varepsilon$$

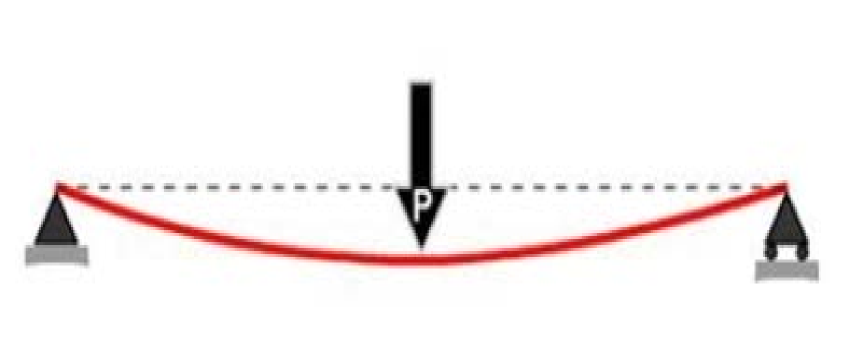
$$E = \frac{\sigma}{\varepsilon}$$

*$E$ = Young's modulus of elasticity

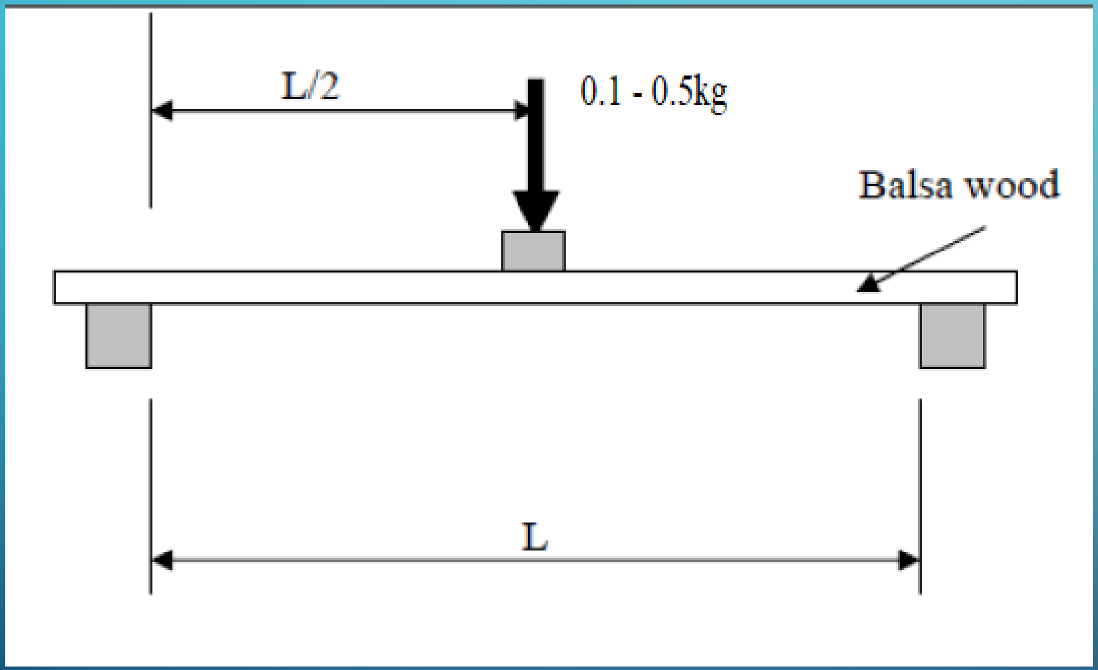
# Deflection in a beam

- You will be performing experiments in lab on such a beam.
- The design of beam requires knowledge of Statics, Moment of Inertia of the beam and material properties like Modulus of Elasticity.



- To find out Modulus of Elasticity, you need to do an experiment where you apply load and find out deflection under that load. Plot a graph between these two and viola! You have the Modulus of Elasticity.




$$\delta = PL^3/48EI$$

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