

Locate the centroid of the rod bent into the shape of a parabolic arc.

$$y = x^3$$

$$x = y^{1/3}$$

$$\tilde{x} = \frac{1}{2} x = \frac{y^{1/3}}{2}$$

$$dA = x \, dy = y^{1/3} \, dy$$

$$\bar{x} = \frac{\int \tilde{x} \, dA}{\int dA} = \frac{\frac{1}{2} \int_0^1 y^{2/3} \, dy}{\int_0^1 y^{1/3} \, dy}$$

$$= \frac{\frac{1}{2} \times \frac{3}{5} y^{5/3} \Big|_0^1}{\frac{3}{4} y^{4/3} \Big|_0^1}$$

$$= \frac{3/10}{3/4}$$

$$= 4/10$$

$$= \underline{0.4}$$

$$y = x^3$$

$$\tilde{y} = x^3$$

$$dA = y \, dx = x^3 \, dx$$

$$\bar{y} = \frac{\int \tilde{y} \, dA}{\int dA} = \frac{\int_0^1 x^6 \, dx}{\int_0^1 x^3 \, dx}$$

$$= \left[\frac{x^7/7}{x^4/4} \right]_0^1$$

