ENGG102

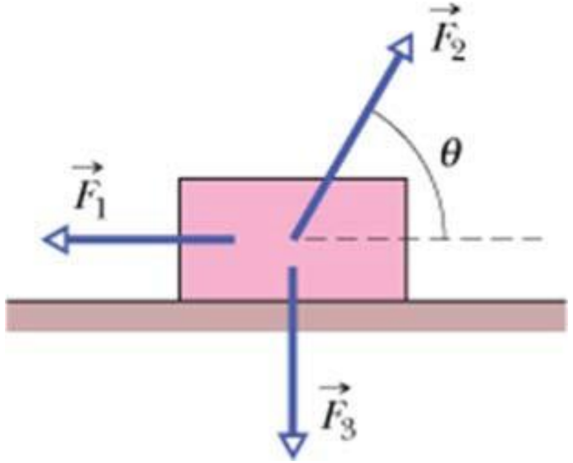
Dr. Umar Asghar

Tutorial

Example 1

Figure below shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are F1 = 5.00 N, F2 = 9.00 N, and F3 = 3.00 N, and the indicated angle is θ = 60.0°. During the displacement,

(a) what is the net work done on the trunk by the three forces and
(b) does the kinetic energy of the trunk increase or decrease?



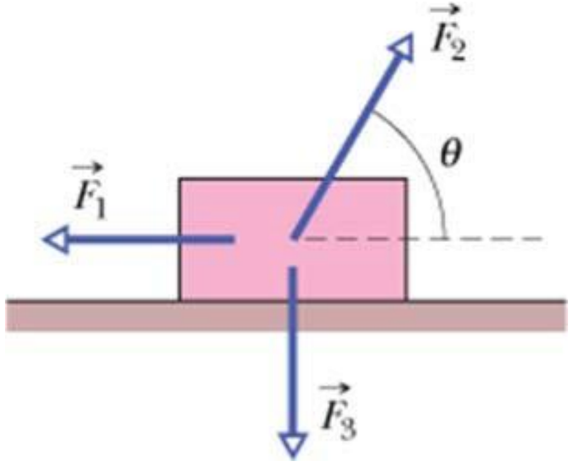
Example 1

(a) The forces are constant, so the work done by any one of them is given by $W = \vec{F}\cdot\vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d\cos\phi_1 = (5.00\,\mathrm{N})(3.00\,\mathrm{m})\cos 0° = 15.0\ \mathrm{J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d\cos\phi_2 = (9.00\,\mathrm{N})(3.00\,\mathrm{m})\cos 120° = -13.5\ \mathrm{J}.$$



Force $\vec{F}_3$ is perpendicular to the displacement, so

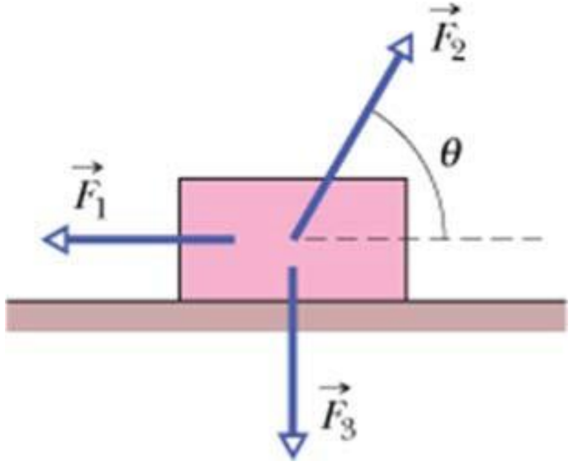
$$W_3 = F_3 d\cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\ \mathrm{J} - 13.5\ \mathrm{J} + 0 = +1.50\ \mathrm{J}.$$

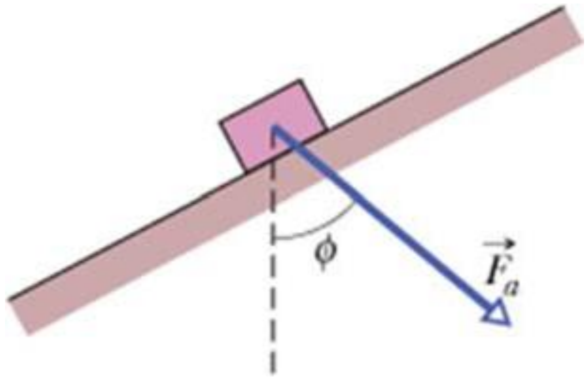
Example 1

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.



Example 2

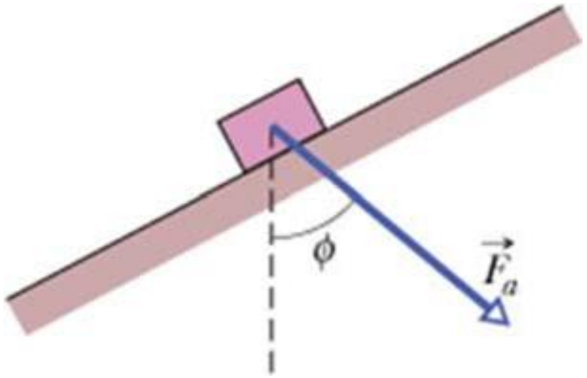
In Figure below, a constant force of magnitude 82.0 N is applied to a 3.00 kg shoe box at angle ϕ = 53.0°, causing the box to move up a frictionless ramp at constant speed. How much work is done on the box by when the box has moved through vertical distance h = 0.150 m?



Example 2

The fact that the applied force F_a causes the box to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy: $\Delta K = 0$. Thus, the work done by $\vec{F}_a$ must be equal to the negative work done by gravity: $W_a = -W_g$. Since the box is displaced vertically upward by $h = 0.150\ \text{m}$, we have

$$W_a = +mgh = (3.00\ \text{kg})(9.80\ \text{m/s}^2)(0.150\ \text{m}) = 4.41\ \text{J}$$



Example 3

You drop a 2.00 kg book to a friend who stands on the ground at distance D = 10.0 m below. If your friends outstretched hands are at distance d = 1.50 m above the ground,

(a) how much work Wg does the gravitational force do on the book as it drops to her hands?

(b) What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop?

(c) If the gravitational potential energy U of that system is taken to be zero at ground level, what is U(c) when the book is released

(d) when it reaches her hands?

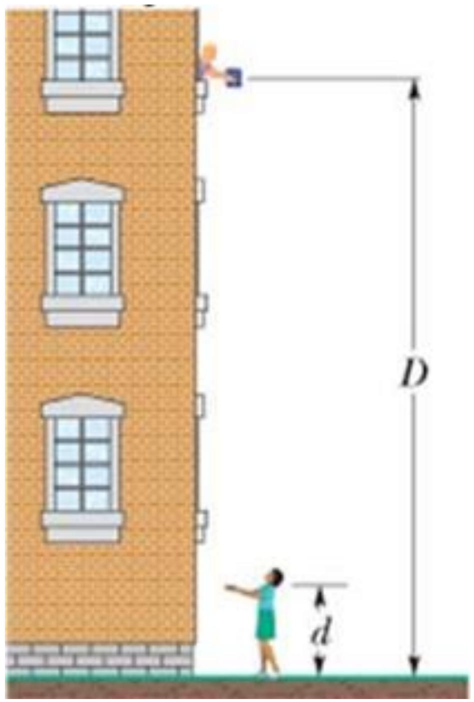
Now take U to be 100 J at ground level and again find

(e) Wg,

(f) ΔU,

(g) U at the release point,

(h) U at her hands



Example 3

(a) Noting that the vertical displacement is 10.0 m – 1.50 m = 8.50 m downward (same direction as F_g), yields

$$W_g = mgd\cos\phi = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(8.50\ \text{m})\cos 0° = 167\ \text{J}.$$

(b) ΔU where $U = mgy$ (with upward understood to be the +y direction). The result is

$$\Delta U = mg(y_f - y_i) = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m} - 10.0\ \text{m}) = -167\ \text{J}.$$

(c) In part (b) we used the fact that $U_i = mgy_i$ =196 J.

(d) In part (b), we also used the fact $U_f = mgy_f$ = 29 J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

Example 3

(f) we must again find $\Delta U = -W_g = -167$ J.

(g) With this new information (that $U_0 = 100$ J where $y = 0$) we have

$$U_i = mgy_i + U_0 = 296 \text{ J}.$$

(h) With this new information (that $U_0 = 100$ J where $y = 0$) we have

$$U_f = mgy_f + U_0 = 129 \text{ J}.$$

We can check part (f) by subtracting the new U_i from this result.

Example 4

A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring.

(a) What is the change ΔU_g in the gravitational potential energy of the marble–Earth system during the 20 m ascent?
(b) What is the change ΔU_s in the elastic potential energy of the spring during its launch of the marble?
(c) What is the spring constant of the spring?

Example 4

We take the reference point for gravitational potential energy at the position of the marble when the spring is compressed.

(a) The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$, where *h* = 20 m is the height of the highest point. Thus,

$$U_g = 5 \times 10^{-3} \times 9.81 \times 20 = 0.98 \text{ J}$$

Example 4

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is $U_s = 0.98$ J. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{2(0.98\ \text{J})}{(0.080\ \text{m})^2} = 3.1 \times 10^2\ \text{N/m} = 3.1\ \text{N/cm}.$$

Example 5

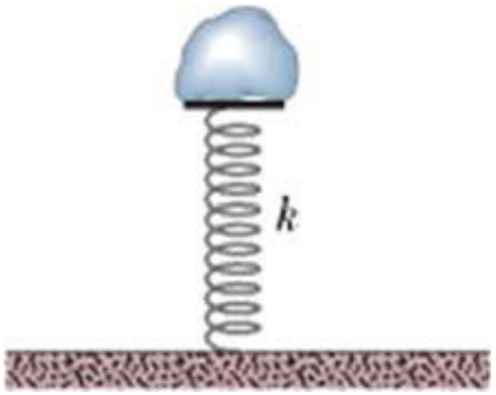
Figure shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone.

(a) What is the spring constant?

(b) The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release?

(c) What is the change in the gravitational potential energy of the stone–Earth system when the stone moves from the release point to its maximum height?

(d) What is that maximum height, measured from the release point?



Example 5

(a) When the stone is in the equilibrium ($a = 0$) position, Newton's second law becomes

$$\vec{F}_{\text{net}} = ma$$
$$F_{\text{spring}} - mg = 0$$
$$-k(-0.100) - (8.00)(9.8) = 0$$

This leads to a spring constant equal to $k = 784\ \text{N/m}$.

Example 5

(b) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upward, turning some of its elastic potential energy (stored in the spring) into kinetic energy. The amount of elastic potential energy at the moment of release is, using,

$$U = \frac{1}{2}ky_1^2 = \frac{1}{2}(784\ \text{N/m})(-0.400)^2 = 62.7\ \text{J}\,.$$

Example 5

(c) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the gravitational potential energy, then

$$K_1 + U_1 = K_2 + U_2$$

$$0 + \frac{1}{2}ky_1^2 = 0 + mgh$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the gravitational potential energy) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

(d) We find $h = ky_1^2/2mg = 0.800$ m, or 80.0 cm.

Example 6

A horizontal force of magnitude 41 N pushes a block of mass 4.00 kg across a floor where the coefficient of kinetic friction is 0.600.

(a) How much work is done by that applied force on the block–floor system when the block slides through a displacement of 2.00 m across the floor?

(b) During that displacement, the thermal energy of the block increases by 40.0 J. What is the increase in thermal energy of the floor?

(c) What is the increase in the kinetic energy of the block?

Example 6

(a) The work is $W = Fd = (41.0\text{ N})(2.00\text{ m}) = 82.0\text{ J}$.

(b) The total amount of energy that has gone to thermal form is:

$$\Delta E_{th} = \mu_k mgd = (0.600)(4.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 47.0\text{ J}.$$

If 40.0 J has gone to the block then (47.0 – 40.0) J = 7.0 J has gone to the floor.

(c) Much of the work (82.0 J) has been "wasted" due to the 47.0 J of thermal energy generated, but there still remains (82.0 – 47.0) J = 35.0 J that has gone into increasing the kinetic energy of the block. (It has not gone into increasing the potential energy of the block because the floor is presumed to be horizontal.)