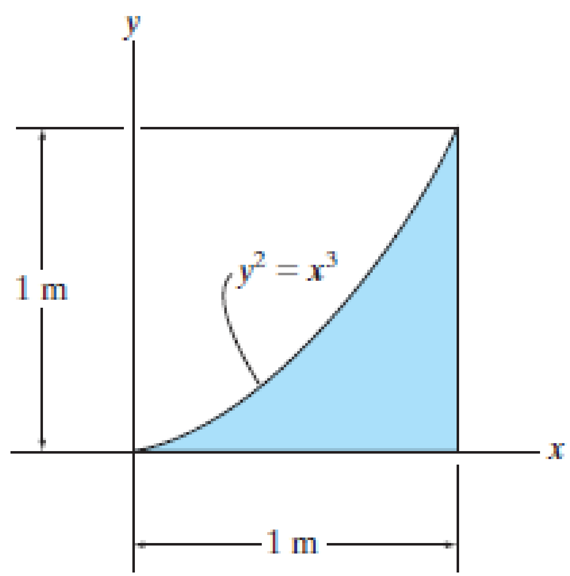
CENTROID

1. Determine the area and the centroid $(\bar{x}, \bar{y})$ of the area.



Differential Element: The area element parallel to the y axis shown shaded in Fig. a will be considered. The area of the element is

$$dA = y\,dx = x^{3/2}\,dx$$

Centroid: The centroid of the element is located at $\tilde{x} = x$ and $\tilde{y} = y/2 = \frac{x^{3/2}}{2}$.

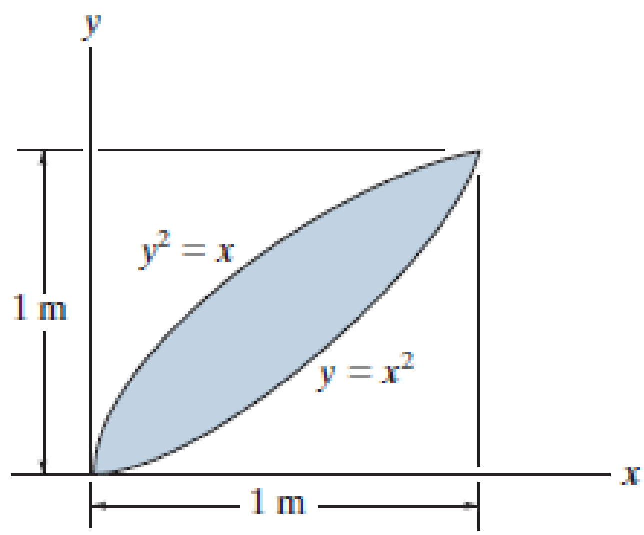
Area: Integrating,

$$A = \int_A dA = \int_0^{1\text{ m}} x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^{1\text{ m}} = \frac{2}{5}\text{ m}^2 = 0.4\text{ m}^2 \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}} x\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}} x^{5/2}\,dx}{2/5} = \frac{\left(\frac{2}{7}x^{7/2}\right)\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{7}\text{ m} = 0.714\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ m}}\left(\frac{x^{3/2}}{2}\right)\left(x^{3/2}\,dx\right)}{2/5} = \frac{\int_0^{1\text{ m}}\frac{x^3}{2}\,dx}{2/5} = \frac{\frac{x^4}{8}\Big|_0^{1\text{ m}}}{2/5} = \frac{5}{16}\text{ m} = 0.3125\text{ m} \qquad \textbf{Ans.}$$

2. Locate the centroid $\bar{x}$ and $\bar{y}$ of the shaded area shown.

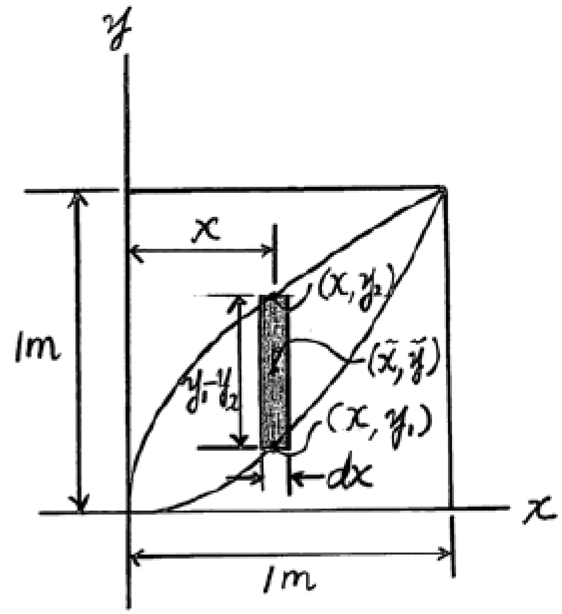


Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{x} = x$.

Centroid : Applying Eq. 9 – 4 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\,\text{m}} x\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\,\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$

$$= \frac{\left(\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^4\right)\Big|_0^{1\,\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\,\text{m}}} = \frac{9}{20}\text{ m} = 0.45\text{ m} \qquad \textbf{Ans}$$

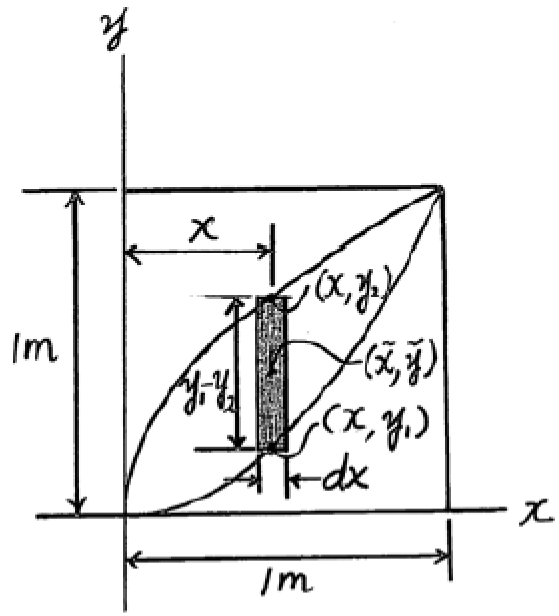


Area and Moment Arm : Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{y} = y_2 + \frac{y_1 - y_2}{2} = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)$.

Centroid : Applying Eq. 9 – 4 and performing the integration, we have

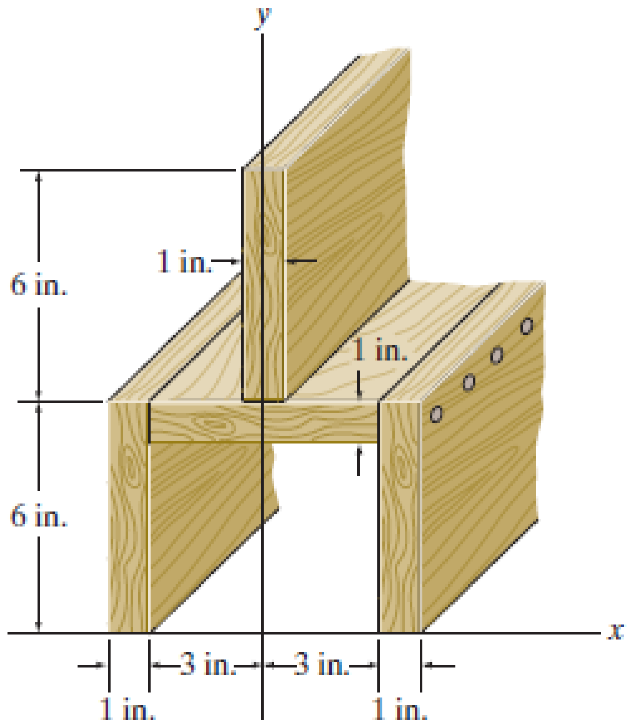
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\,\text{m}} \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\,\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$

$$= \frac{\frac{1}{2}\left(\frac{1}{2}x^2 - \frac{1}{5}x^5\right)\Big|_0^{1\,\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\,\text{m}}} = \frac{9}{20}\ \text{m} = 0.45\ \text{m} \qquad \textbf{Ans}$$



3.

•9–53. Locate the centroid $\bar{y}$ of the cross-sectional area of the built-up beam.

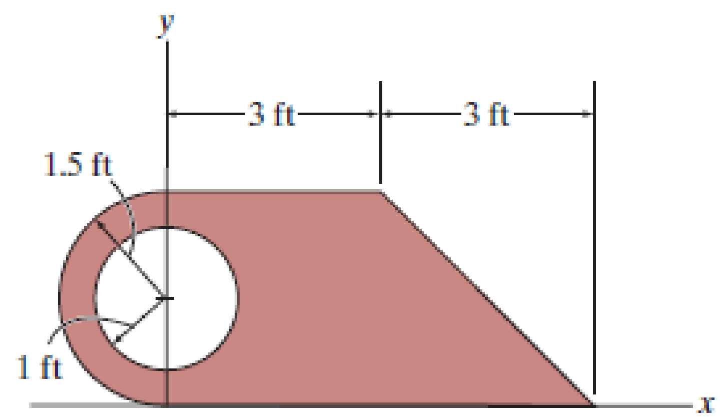


Centroid: The centroid of each composite segment is shown in Fig. *a*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{3[2(6)(1)] + 5.5(6)(1) + 9(6)(1)}{2(6)(1) + 6(1) + 6(1)} = 5.125 \text{ in.}$$

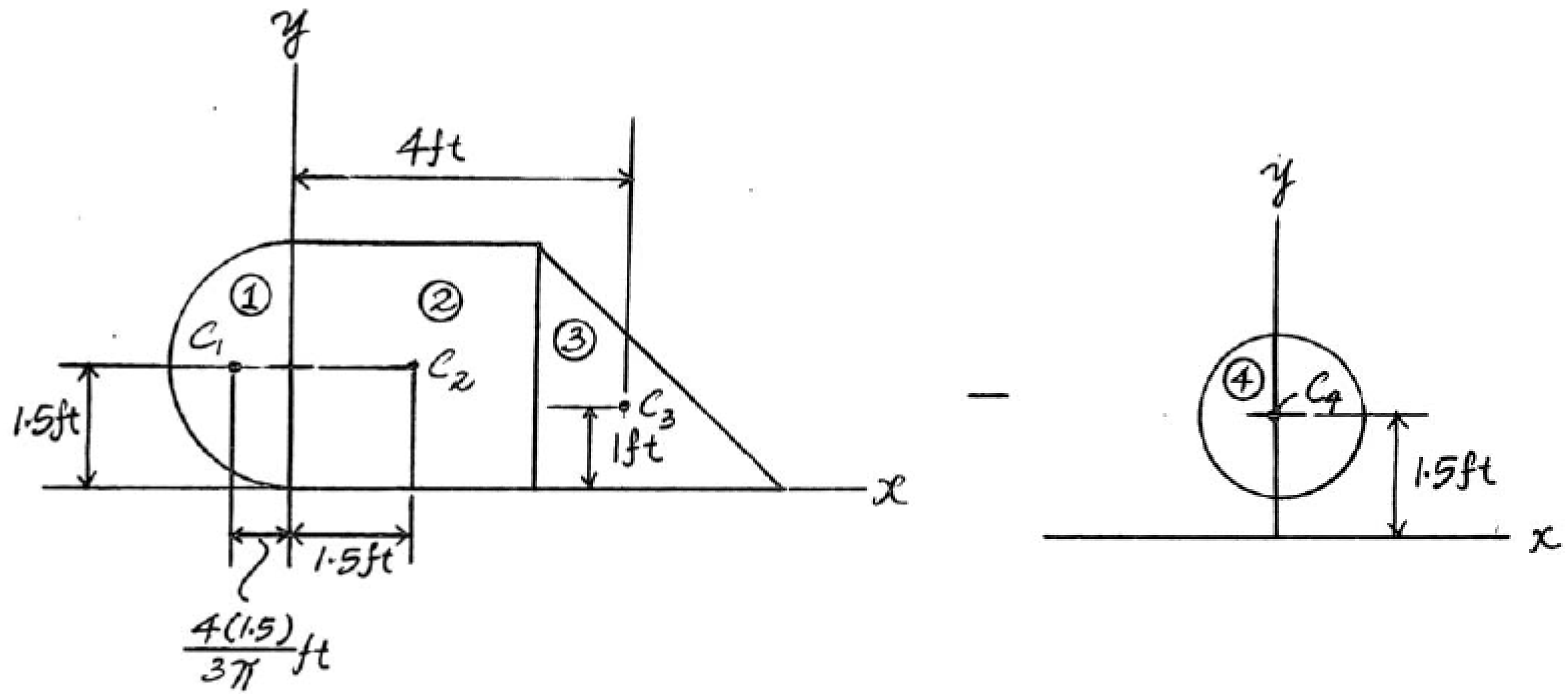
4.

*9–60. Locate the centroid $(\bar{x}, \bar{y})$ of the composite area.



$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\left(-\frac{4(1.5)}{3\pi}\right)\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 4\left(\frac{1}{2}(3)(3)\right) + 0\left(-\frac{\pi(1^2)}{4}\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\frac{\pi(1^2)}{4}\right)} = \frac{29.25}{13.89} = 2.11\text{ ft}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5\left(\frac{\pi(1.5^2)}{2}\right) + 1.5(3(3)) + 1\left(\frac{1}{2}(3)(3)\right) + 1.5\left(-\frac{\pi(1^2)}{4}\right)}{\frac{\pi(1.5^2)}{2} + 3(3) + \frac{1}{2}(3)(3) + \left(-\frac{\pi(1^2)}{4}\right)} = \frac{18.59}{13.89} = 1.34\text{ ft}$$

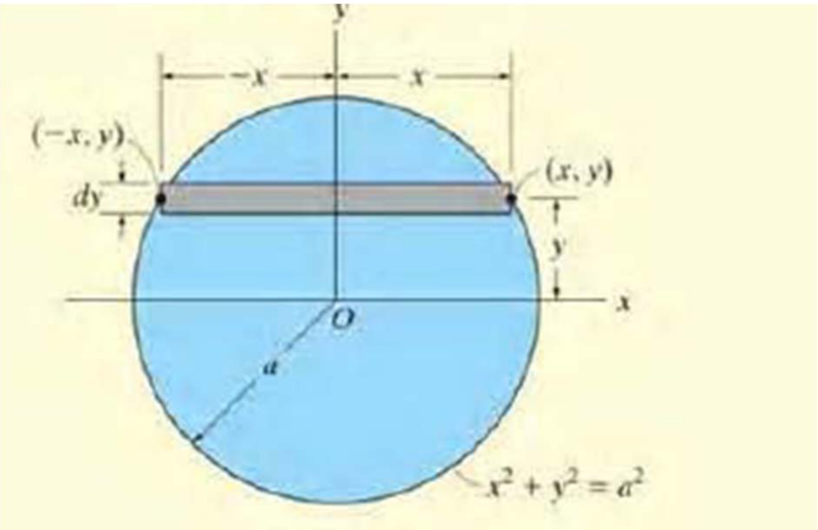


Moment of Inertia

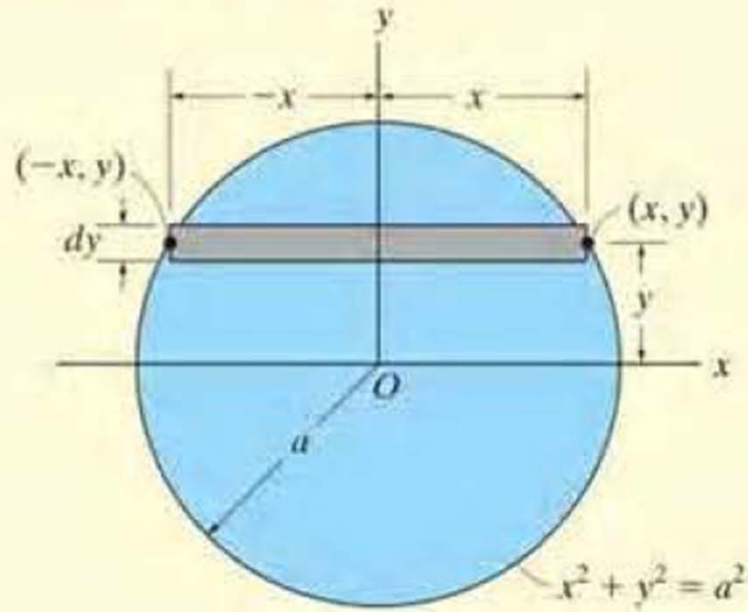
Moment of Inertia, Moment of Inertia of composite areas

5

- Determine the moment of inertia with respect to the *x-axis* for the circular area



(a)



(a)

SOLUTION I (CASE 1)

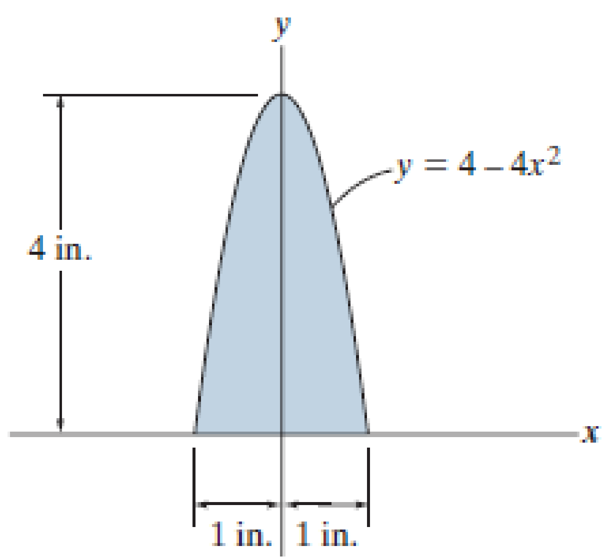
Using the differential element shown in Fig. 10–7*a*, since $dA = 2x\,dy$, we have

$$I_x = \int_A y^2\,dA = \int_A y^2(2x)\,dy$$

$$= \int_{-a}^{a} y^2\left(2\sqrt{a^2 - y^2}\right)dy = \frac{\pi a^4}{4} \qquad \textit{Ans.}$$

6

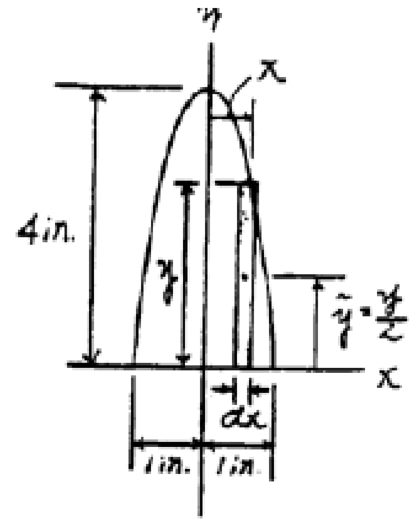
10–15. Determine the moment of inertia of the area about the y axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of dx, and (b) having a thickness of dy.



a) ***Differential Element :*** The area of the differential element parallel to yaxis is $dA = y\,dx = \left(4 - 4x^2\right) dx$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_{-1\text{ in.}}^{1\text{ in.}} x^2\left(4 - 4x^2\right) dx$$
$$= \left[\frac{4}{3}x^3 - \frac{4}{5}x^5\right]\Bigg|_{-1\text{ in.}}^{1\text{ in.}}$$
$$= 1.07\text{ in}^4 \qquad \textbf{Ans}$$



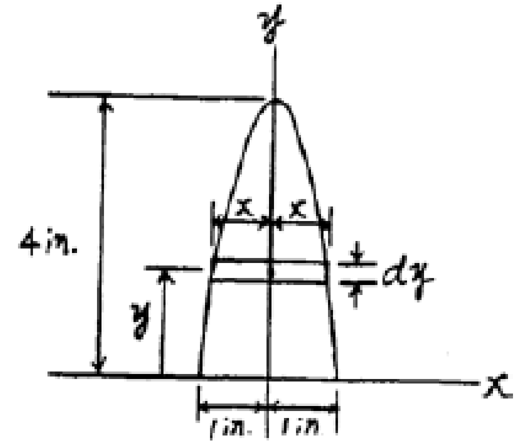
(a)

b) ***Differential Element :*** Here, $x = \frac{1}{2}\sqrt{4-y}$. The moment of inertia of the differential element about y axis is

$$dI_y = \frac{1}{12}(dy)\left(2x\right)^3 = \frac{2}{3}x^3 dy = \frac{1}{12}(4-y)^{\frac{3}{2}} dy$$

Moment of Inertia : Performing the integration, we have

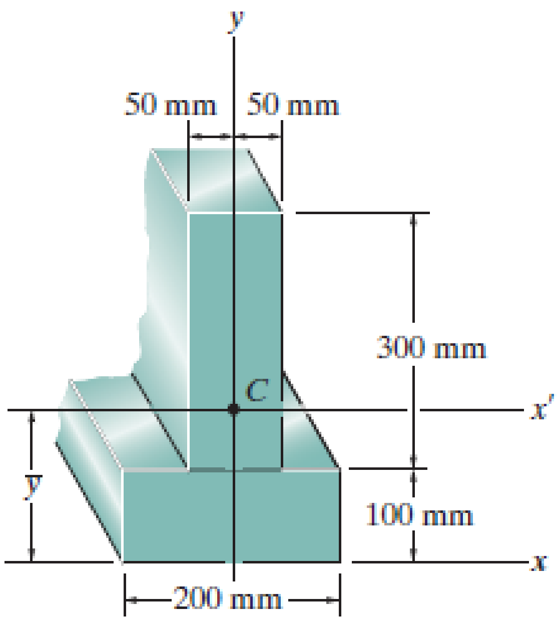
$$I_y = \int dI_y = \frac{1}{12}\int_0^{4\text{ in.}} (4-y)^{\frac{3}{2}} dy$$
$$= \frac{1}{12}\left[-\frac{2}{5}(4-y)^{\frac{5}{2}}\right]\Bigg|_0^{4\text{ in.}}$$
$$= 1.07\text{ in}^4 \qquad \textbf{Ans}$$

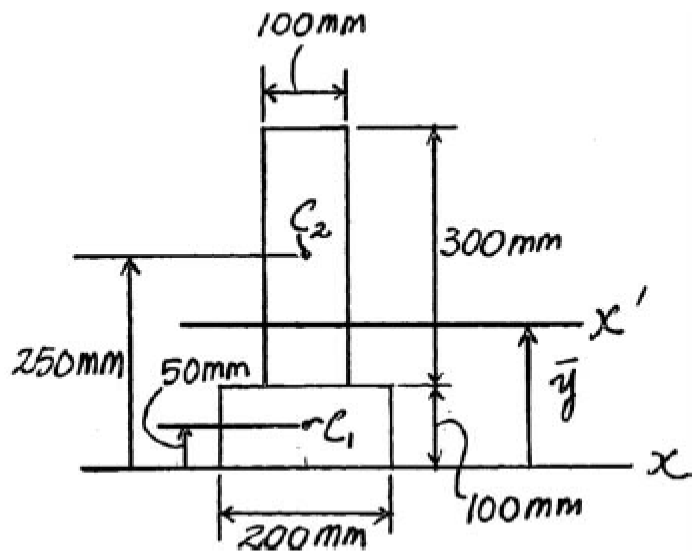


(b)

7

10–38. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x' axis.





Centroid :

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{50(100)(200) + 250(100)(300)}{100(200) + 100(300)} = 170 \text{ mm} \qquad \textbf{Ans}$$

Moment of inertia :

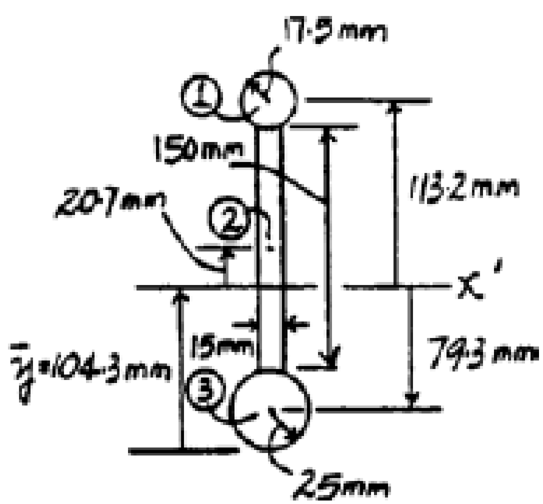
$$I_{x'} = \frac{1}{12}(200)(100)^3 + 200(100)(170 - 50)^2$$

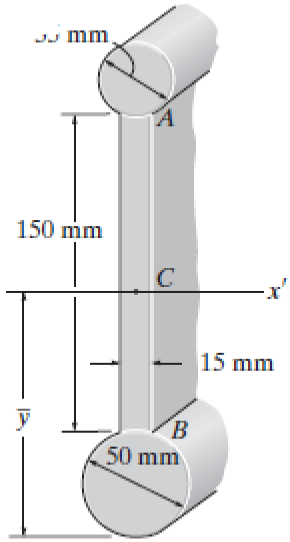
$$+ \frac{1}{12}(100)(300)^3 + 100(300)(250 - 170)^2$$

$$= 722(10)^6 \text{ mm}^4 \qquad \textbf{Ans}$$

8

10–59. Determine the moment of inertia of the beam's cross-sectional area with respect to the x' axis passing through the centroid C of the cross section. $\overline{y} = 104.3$ mm.





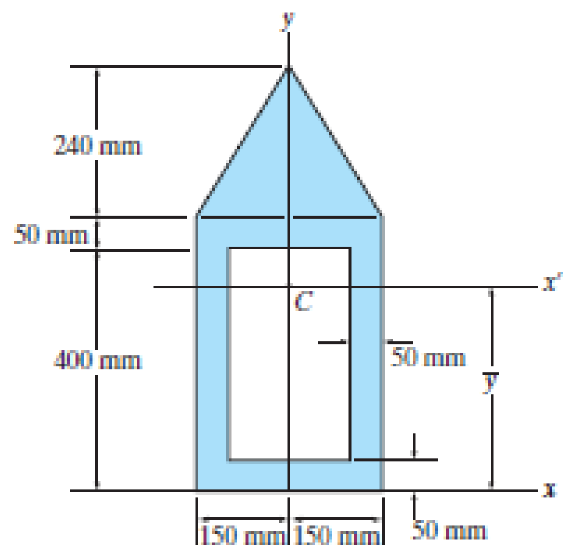
Moment of Inertia : The moment of inertia about the x' axis for each segment can be determined using the parallel − axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

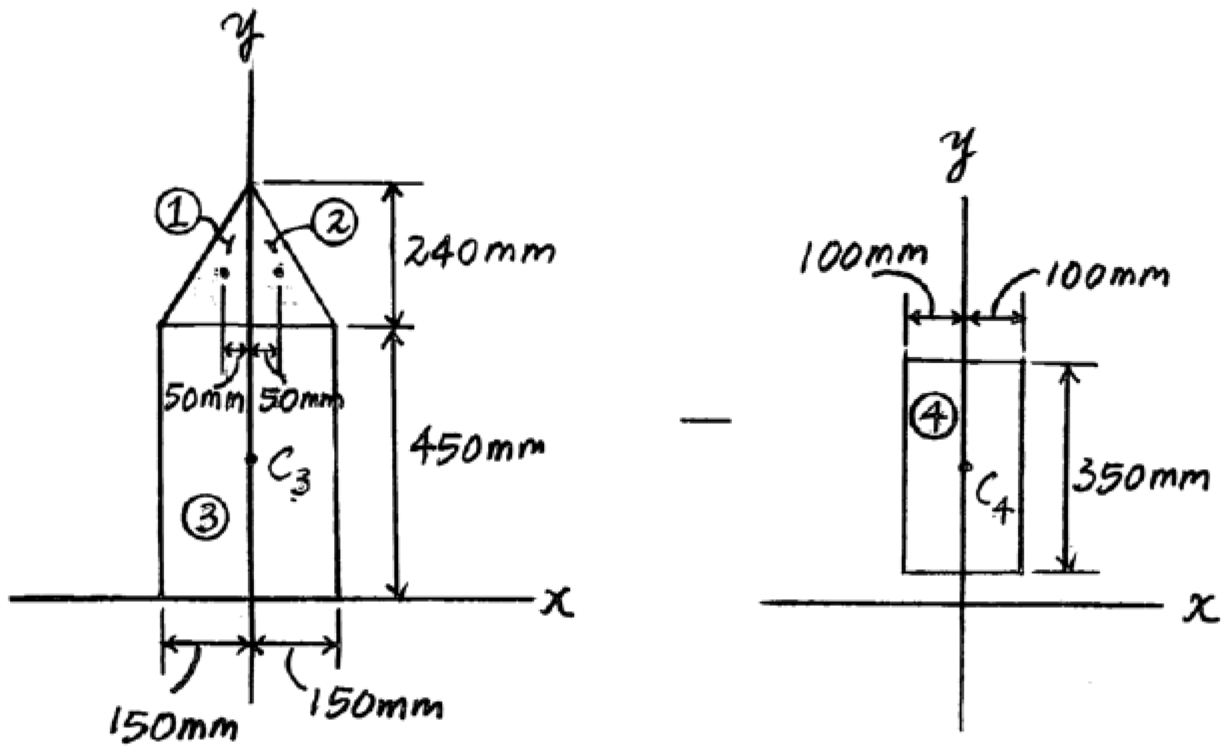
Segment	A_i (mm²)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm⁴)	$(Ad_y^2)_i$ (mm⁴)	$(I_{x'})_i$ (mm⁴)
1	$\pi(17.5^2)$	113.2	$\frac{\pi}{4}(17.5^4)$	$12.329(10^6)$	$12.402(10^6)$
2	$15(150)$	20.7	$\frac{1}{12}(15)(150^3)$	$0.964(10^6)$	$5.183(10^6)$
3	$\pi(25^2)$	79.3	$\frac{\pi}{4}(25^4)$	$12.347(10^6)$	$12.654(10^6)$

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 30.24(10^6)\ \text{mm}^4 = 30.2(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

10–47. Determine the moment of inertia of the composite area about the centroidal y axis.




y
1
2
240mm
50mm
50mm
C3
3
450mm
x
150mm
150mm
—
y
100mm
100mm
4
C4
350mm
x

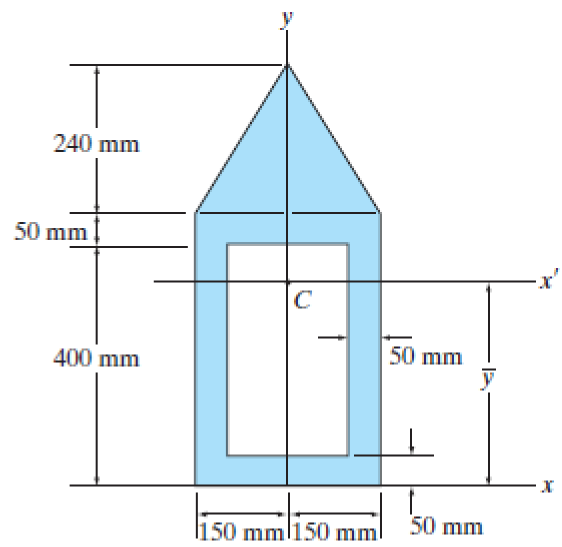
Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

$$= \left[2\left(\frac{1}{36}(240)(150^3)\right) + 2\left(\frac{1}{2}(240)(150)\right)(50)^2\right] + \left[\frac{1}{12}(450)(300^3) + 450(300)(0)^2\right] + \left[-\frac{1}{12}(350)(200^3) + (-350(200))(0)^2\right]$$

$$= 914(10^6)\ \text{mm}^4$$ **Ans.**

*10–48. Locate the centroid $\overline{y}$ of the composite area, then determine the moment of inertia of this area about the x' axis.



Centroid: The perpendicular distances measured from the centroid of each segment to the x axis are indicated in Fig. *a*.

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{530\left(\frac{1}{2}(300)(240)\right) + 225(300(450)) + 225(-200(350))}{\frac{1}{2}(300)(240) + 300(450) - 200(350)} = \frac{33.705(10^6)}{101(10^3)} = 333.71\ \text{mm} = 334\ \text{mm} \qquad \textbf{Ans.}$$

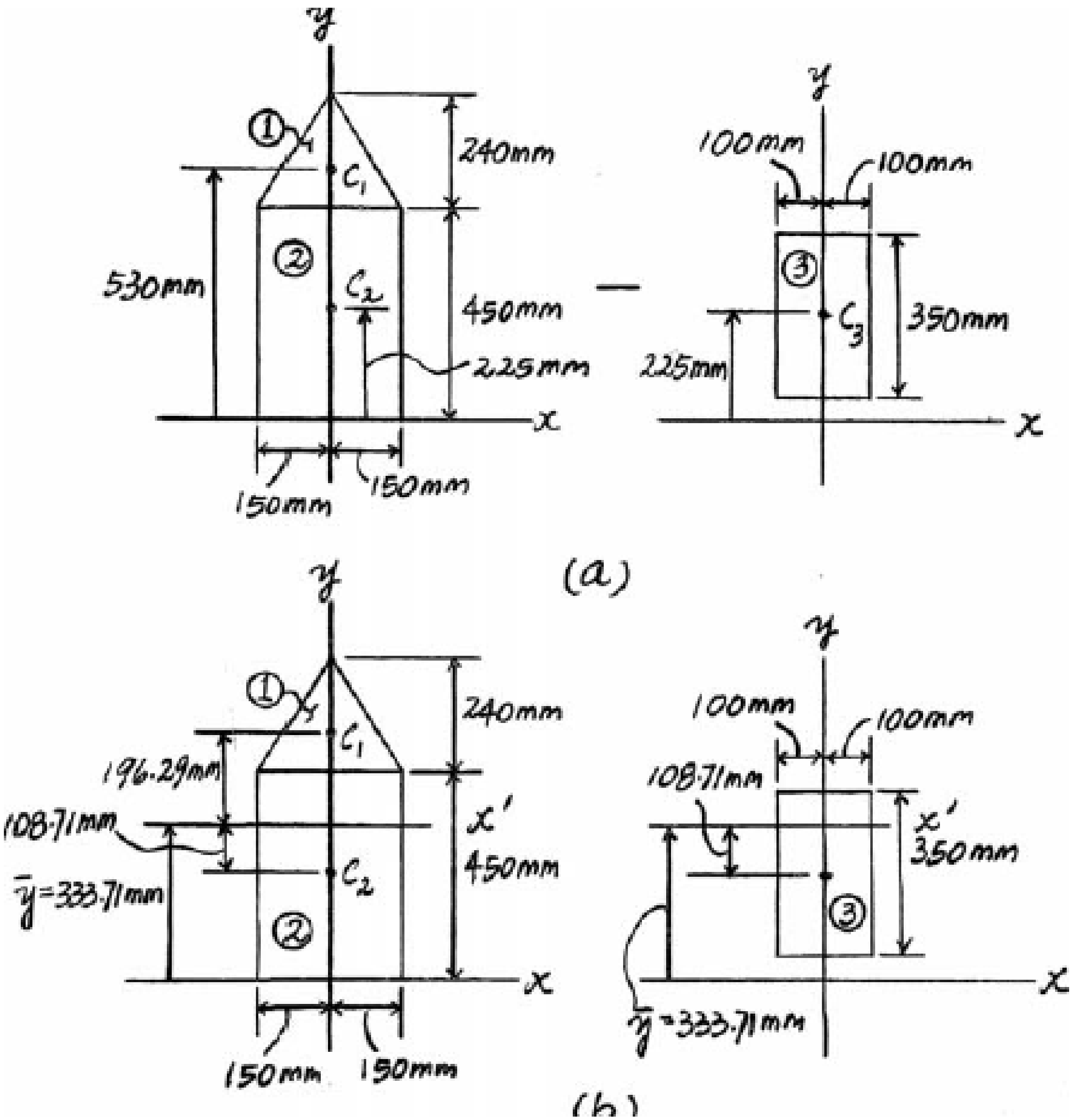
Moment of Inertia: The moment of inertia of each segment about the x' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each segment to the x' axis is indicated in Fig. *b*.

$$I_{x'} = \bar{I}_{x'} + A(d_{x'})^2$$

$$= \left[\frac{1}{36}(300)(240^3) + \frac{1}{2}(300)(240)(196.29)^2\right] + \left[\frac{1}{12}(300)(450^3) + 300(450)(108.71)^2\right]$$

$$+ \left[-\frac{1}{12}(200)(350^3) + (-200(350))(108.71)^2\right]$$

$$= 3.83(10^9)\ \text{mm}^4 \qquad \textbf{Ans.}$$


y
1
C1
240mm
2
530mm
C2
450mm
225mm
x
150mm
150mm
y
100mm
100mm
3
C3
350mm
225mm
x
(a)
y
1
C1
240mm
196.29mm
108.71mm
x'
450mm
C2
ȳ=333.71mm
2
x
150mm
150mm
y
100mm
100mm
108.71mm
x'
350mm
3
ȳ=333.71mm
x
(b)