TUTORIAL SESSION 3

PROBLEMS ON:

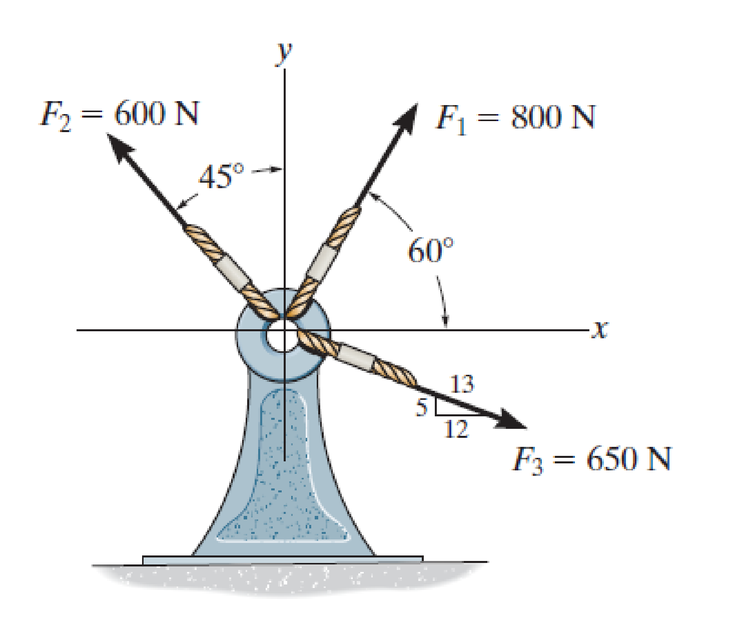
- VECTORS: FORCES AND RESULTANTS
- EQUILIBRIUM : FORCES AND MOMENTS

USING FREE BODY DIAGRAMS AND PRINCIPLES OF STATICS AND MECHANICS

The following problems have been taken from Engineering Mechanics: Statics – R. C. Hibbeler 13th edition

VECTORS – FORCES AND RESULTANTS

1- Write down each force in cartesian vector form.



SOLUTION

$$\mathbf{F}_1 = \{800 \cos 60°(+\mathbf{i}) + 800 \sin 60°(+\mathbf{j})\} \text{ N}$$

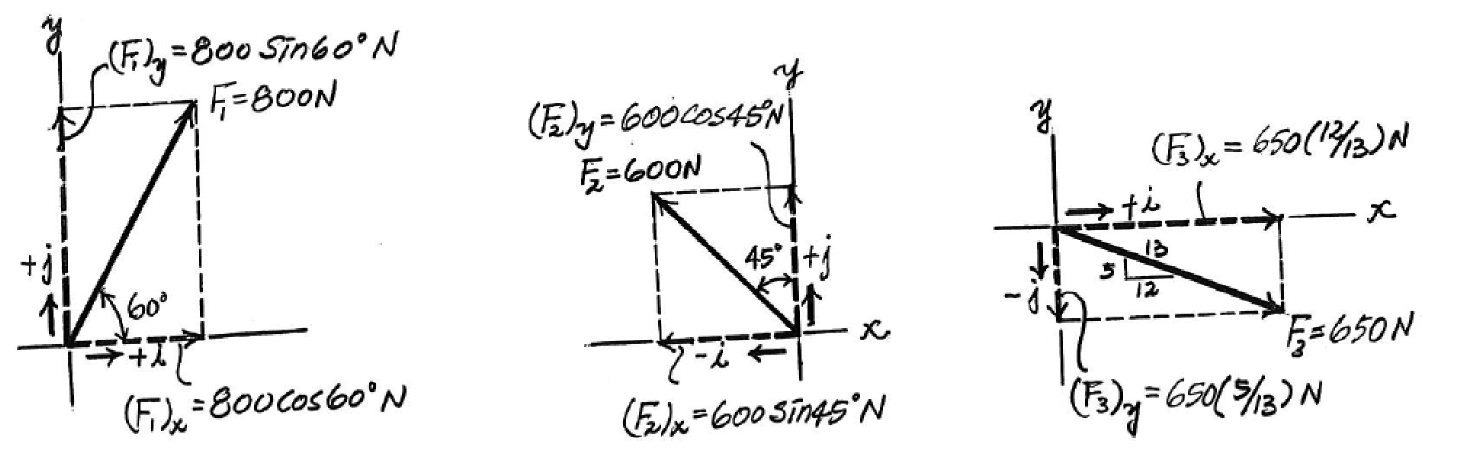
$$= \{400\mathbf{i} + 693\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = \{600 \sin 45°(-\mathbf{i}) + 600 \cos 45°(+\mathbf{j})\} \text{ N} \qquad \textbf{Ans.}$$

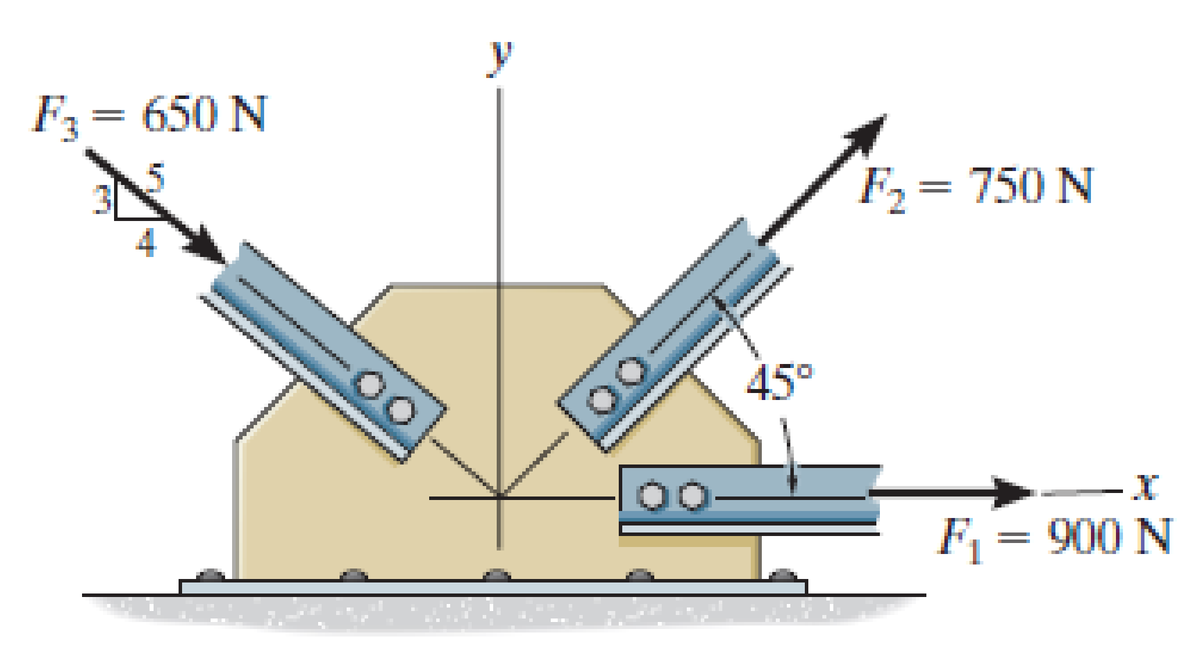
$$= \{-424\mathbf{i} + 424\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_3 = \left\{650\left(\frac{12}{13}\right)(+\mathbf{i}) + 650\left(\frac{5}{13}\right)(-\mathbf{j})\right\} \text{ N} \qquad \textbf{Ans.}$$

$$= \{600\mathbf{i} - 250\mathbf{j}\} \text{ N}$$



2 - Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45^\circ = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45^\circ = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

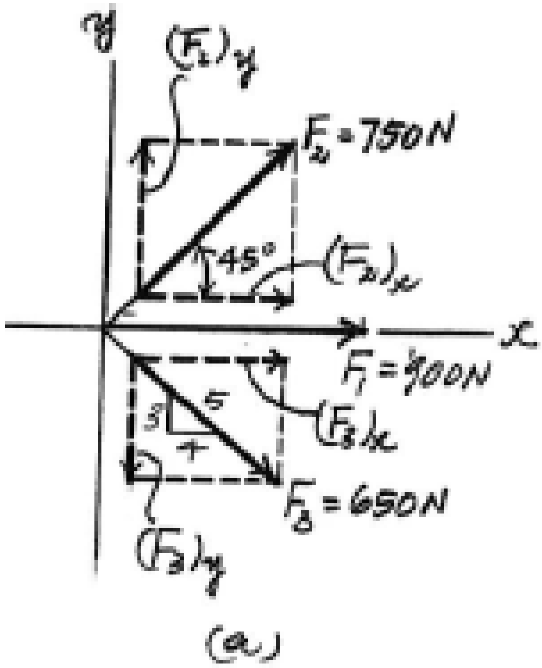
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12^\circ \quad \textbf{Ans.}$$



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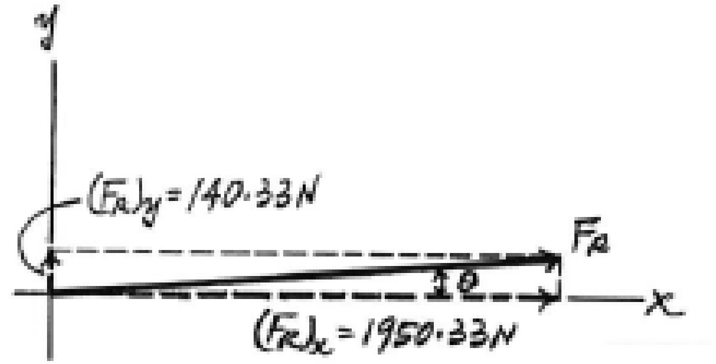
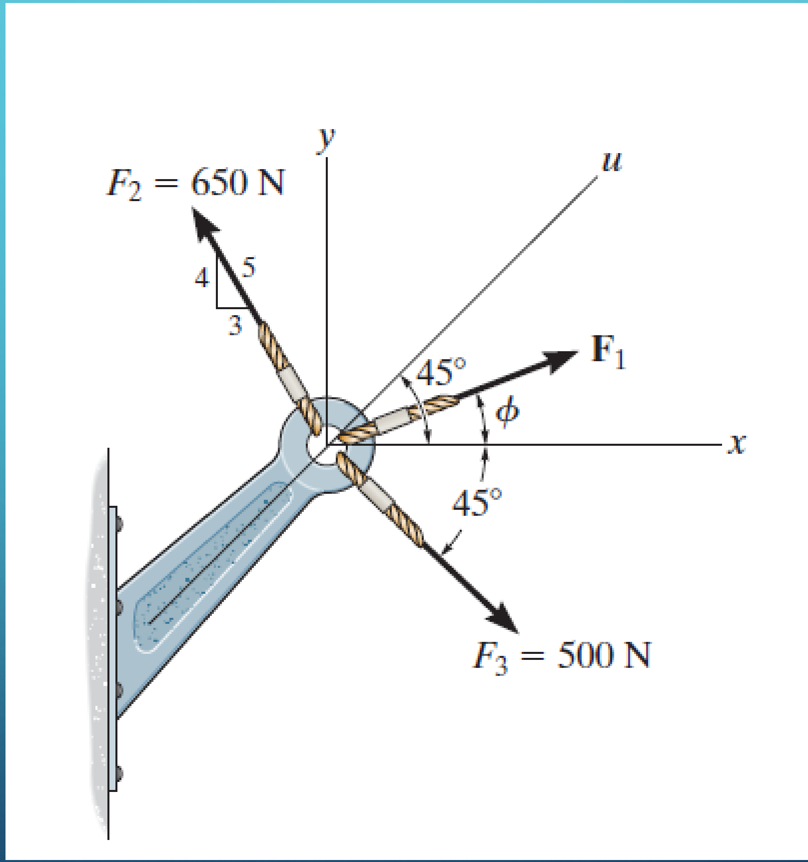


TUTORIAL ASSIGNMENT

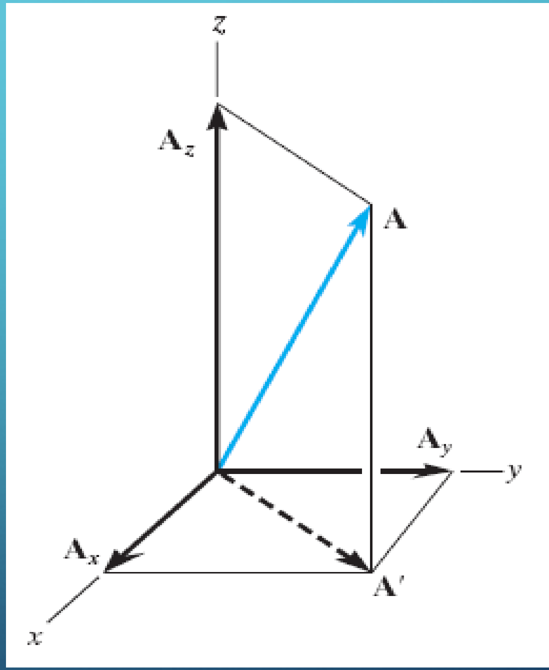
3 - If the magnitude of the resultant force acting on the bracket is 600 N, directed along the positive u axis, determine the magnitude of $\mathbf{F}$ and its direction ϕ.



QUICK REMINDER: CARTESIAN VECTORS (IN 3D)

- A vector **A** may have one, two or three rectangular components along the *x*, *y* and *z* axes, depending on orientation

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y + \vec{\mathbf{A}}_z$$

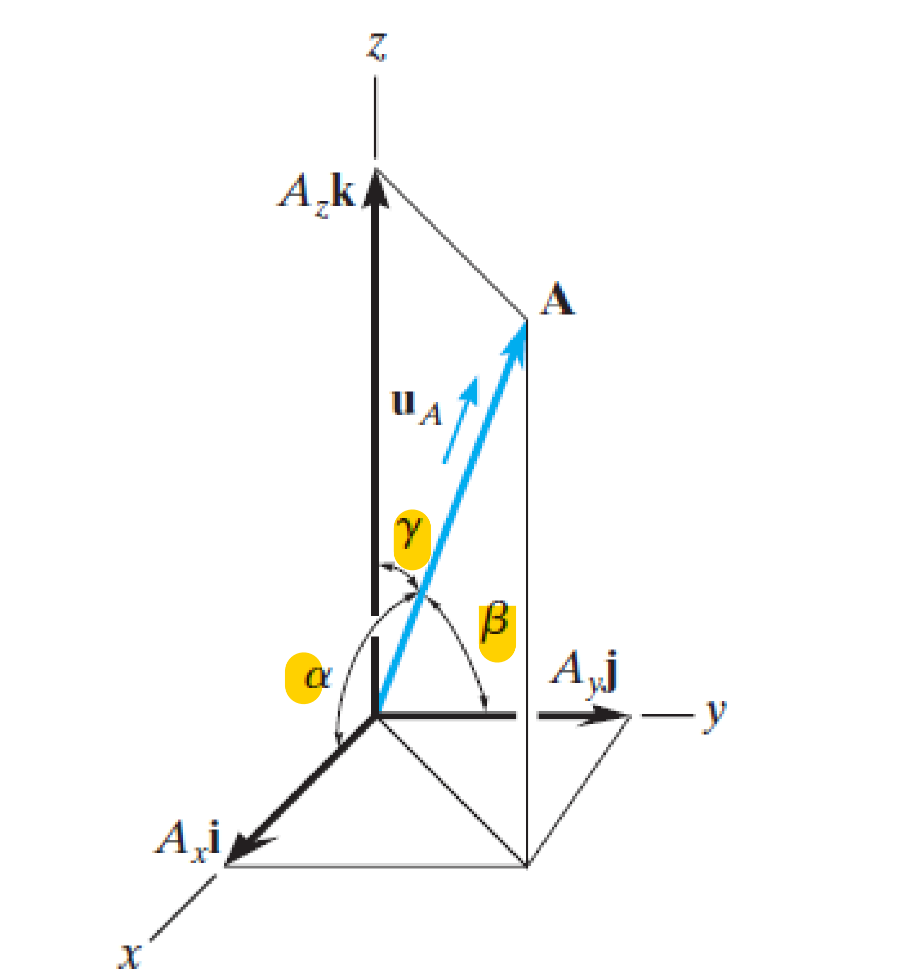


Direction

$$\cos\alpha = \frac{A_x}{A}$$

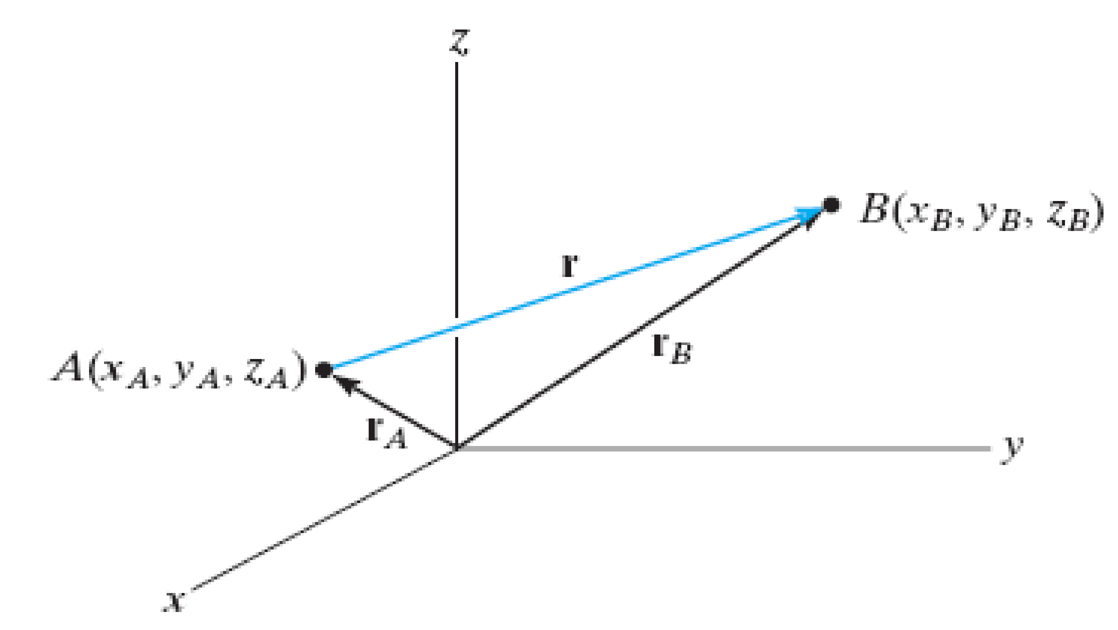
$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$



Position Vectors

Position Vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$



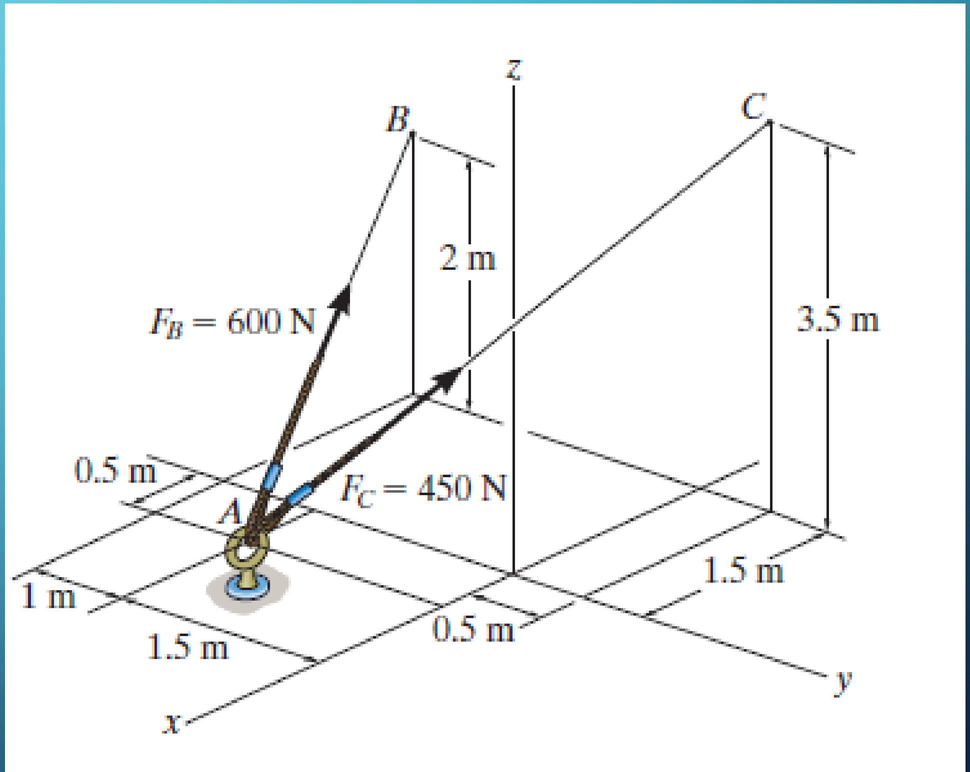
- Between two points

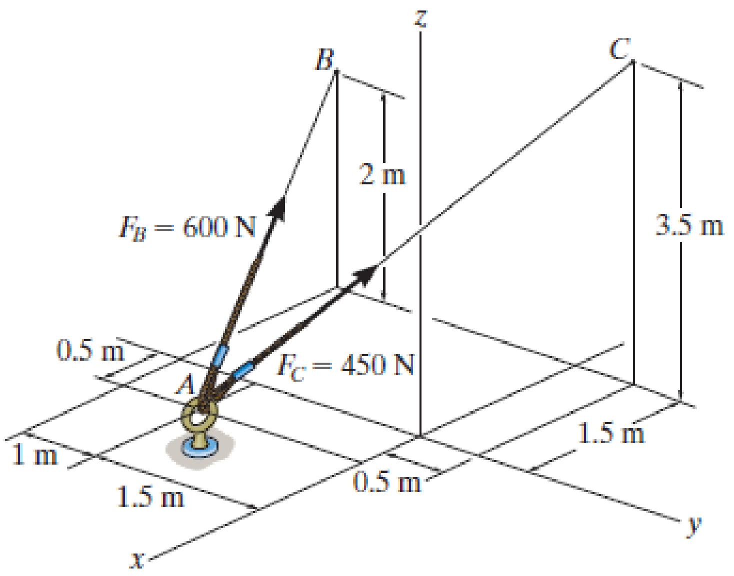
$$\mathbf{r} = r_B - r_A$$

$$= (x_B - x_A)\, i + (y_B - y_A)\, j + (z_B - z_A)\, k$$

4 – Write the two forces in cartesian vector form and find out the resultant.

- Write coordinates of all points A, B, C
- Write the position/distance vectors r_B, r_C
- Write unit vectors u_B, u_C
- Write force vectors F_B, F_C.





SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(-1.5 - 0.5)\mathbf{i} + [-2.5 - (-1.5)]\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [-2.5 - (-1.5)]^2 + (2 - 0)^2}}$$

$$= -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-1.5 - 0.5)\mathbf{i} + [0.5 - (-1.5)]\mathbf{j} + (3.5 - 0)\mathbf{k}}{\sqrt{(-1.5 - 0.5)^2 + [0.5 - (-1.5)]^2 + (3.5 - 0)^2}}$$

$$= -\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}$$

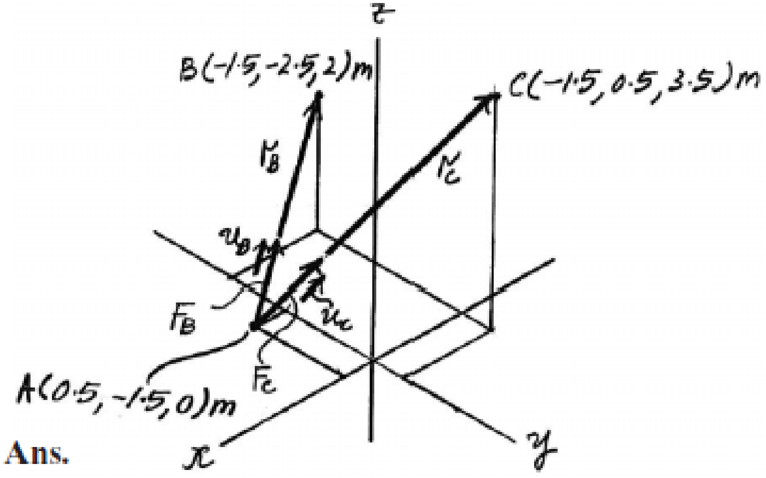
Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 600\left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}\}\text{ N}$$

Ans.

$$\mathbf{F}_C = F_C\mathbf{u}_C = 450\left(-\frac{4}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} + \frac{7}{9}\mathbf{k}\right) = \{-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k}\}\text{ N}$$

Ans.



(a)

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (-400\mathbf{i} - 200\mathbf{j} + 400\mathbf{k}) + (-200\mathbf{i} + 200\mathbf{j} + 350\mathbf{k})$$

$$= \{-600\mathbf{i} + 750\mathbf{k}\}\ \text{N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2} + (F_R)_z^{\,2}}$$

$$= \sqrt{(-600)^2 + 0^2 + 750^2} = 960.47\ \text{N} = 960\ \text{N}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{-600}{960.47}\right) = 129^\circ \qquad \textbf{Ans.}$$

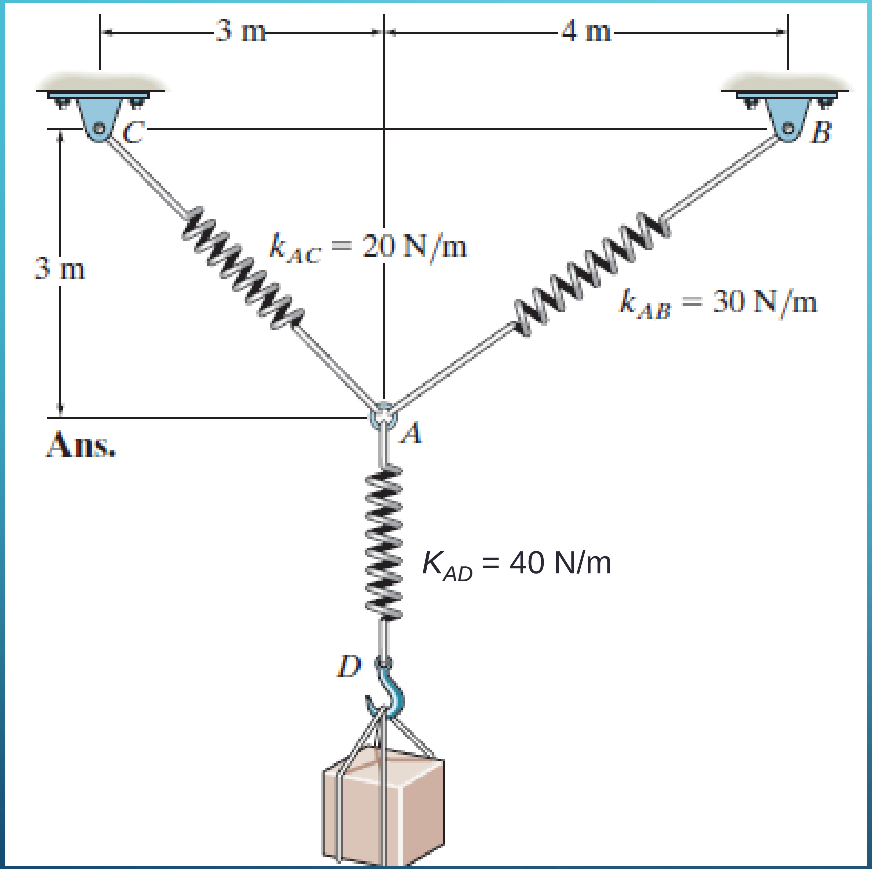
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{960.47}\right) = 90^\circ \qquad \textbf{Ans.}$$

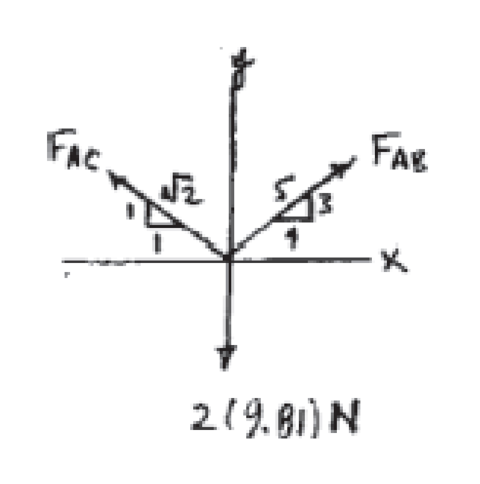
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{760}{960.47}\right) = 38.7^\circ \qquad \textbf{Ans.}$$

EQUILIBRIUM

5

Determine the stretch in springs AC and AB for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.





SOLUTION

$F_{AD} = 2(9.81) = x_{AD}(40) \quad x_{AD} = 0.4905 \text{ m}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{4}{5}\right) - F_{AC}\left(\frac{1}{\sqrt{2}}\right) = 0$

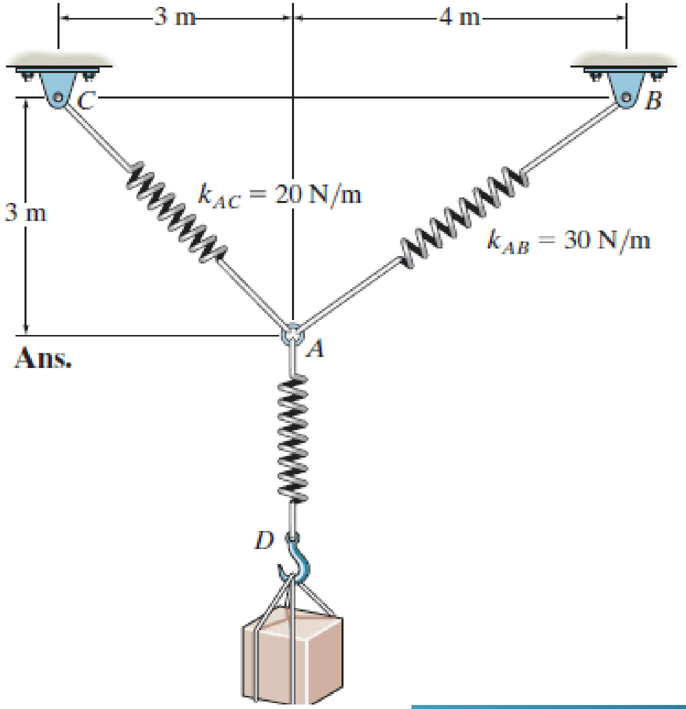
$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{1}{\sqrt{2}}\right) + F_{AB}\left(\frac{3}{5}\right) - 2(9.81) = 0$

$F_{AC} = 15.86 \text{ N}$

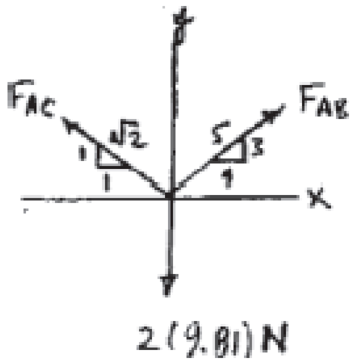
$x_{AC} = \frac{15.86}{20} = 0.793 \text{ m}$

$F_{AB} = 14.01 \text{ N}$

$x_{AB} = \frac{14.01}{30} = 0.467 \text{ m}$



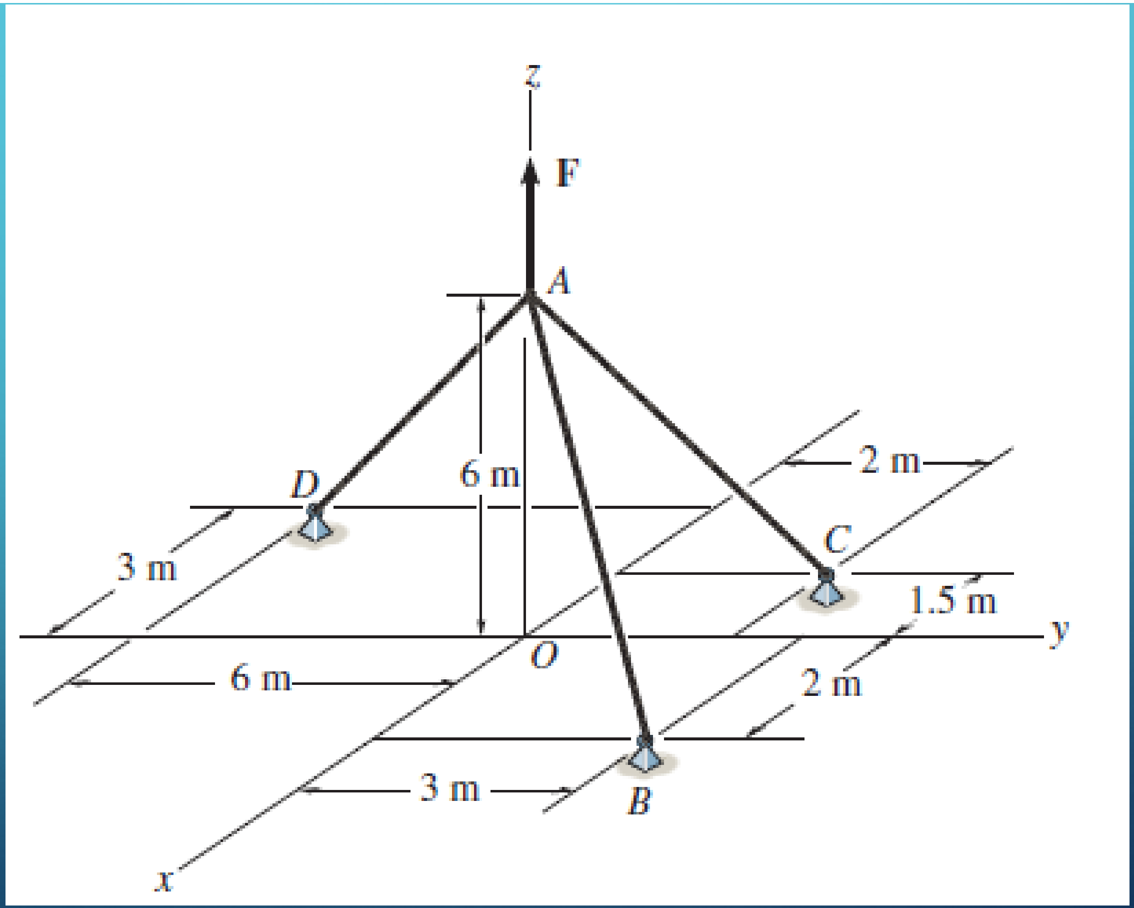
Ans.



TUTORIAL ASSIGNMENT

6 -

If cable AB is subjected to a tension of 700 N, determine the tension in cables AC and AD and the magnitude of the vertical force $\mathbf{F}$.

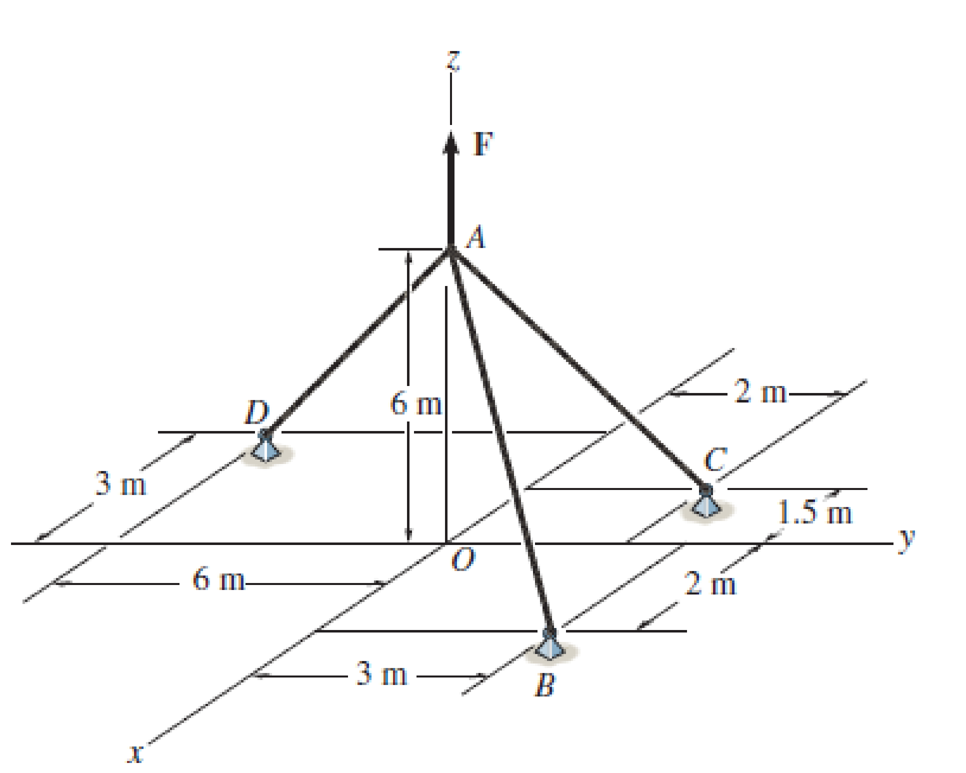


- Write coordinates of all points A, B, C and D
- Write the position/distance vectors AB, AC and AD.
- Write unit vectors AB, AC and AD.

$$U_{AB} = \left(\frac{2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}}{\sqrt{2^2 + 3^2 + (-6)^2}}\right)$$

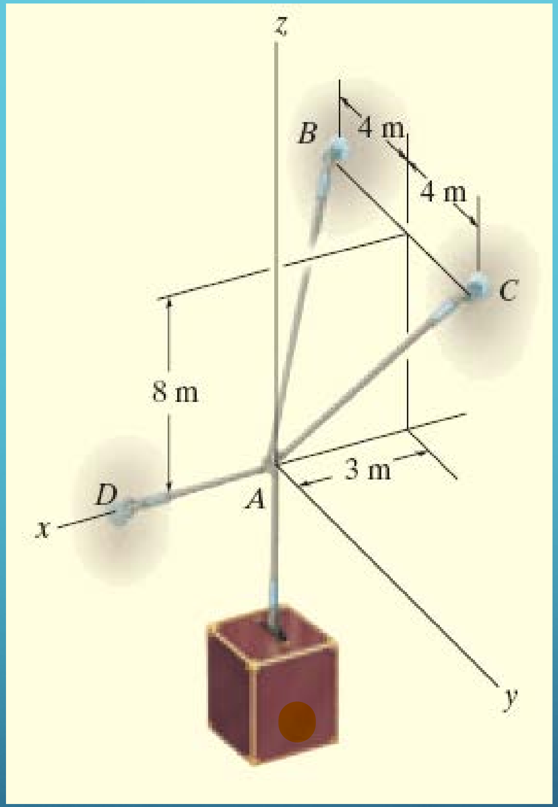
$$U_{AC} = \left(\frac{-1.5\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}}{\sqrt{(-1.5)^2 + 2^2 + (-6)^2}}\right)$$

$$U_{AD} = \left(\frac{-3\mathbf{i} - 6\mathbf{j} - 6\mathbf{k}}{\sqrt{(-3)^2 + (-6)^2 + (-6)^2}}\right)$$



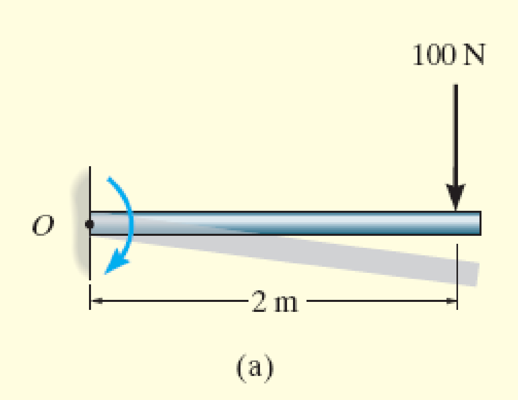
TUTORIAL ASSIGNMENT

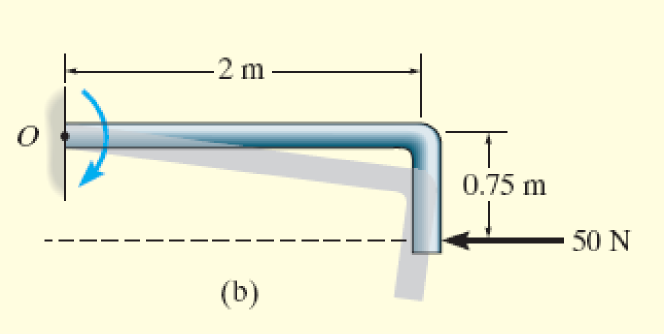
7 - Determine the force developed in each cable used to support the 40 kN crate.

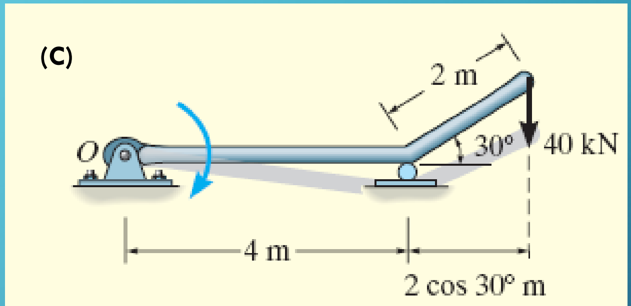


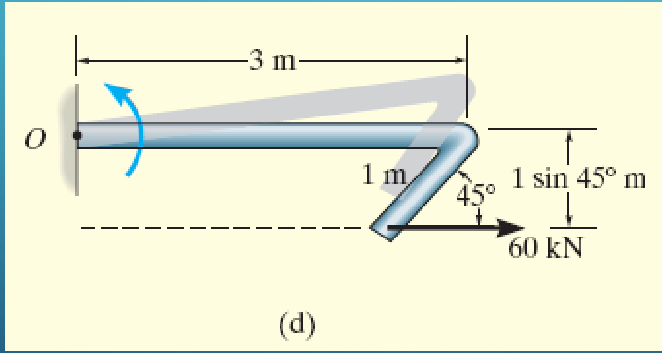
Practice on moments (not an equilibrium question):

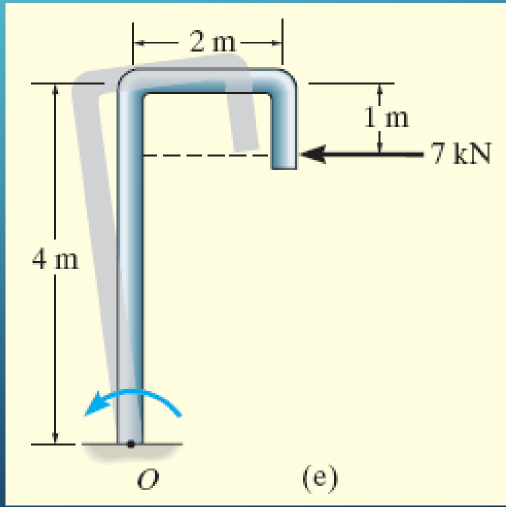
8 - For each case, determine the moment of the force about point **O**.









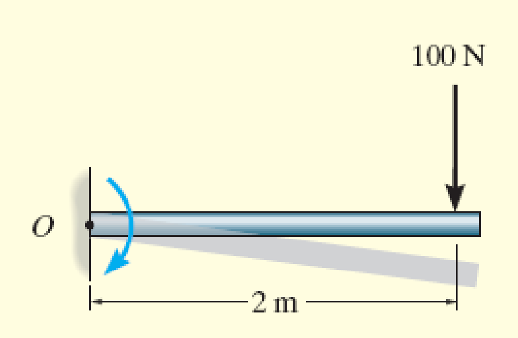


SOLUTION

Line of action is extended as a dashed line to establish moment arm **d.**

Tendency to rotate is indicated and the orbit is shown as a colored curl.

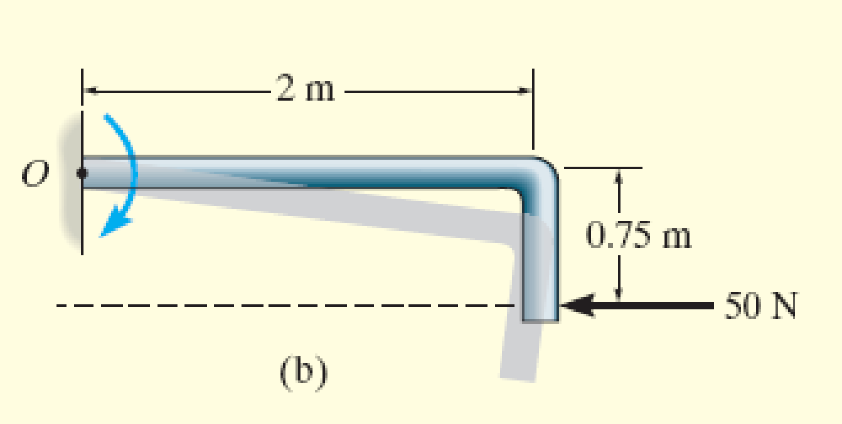
$$(a)\ \ M_o = (100N)(2m) = 200N.m(CW)$$



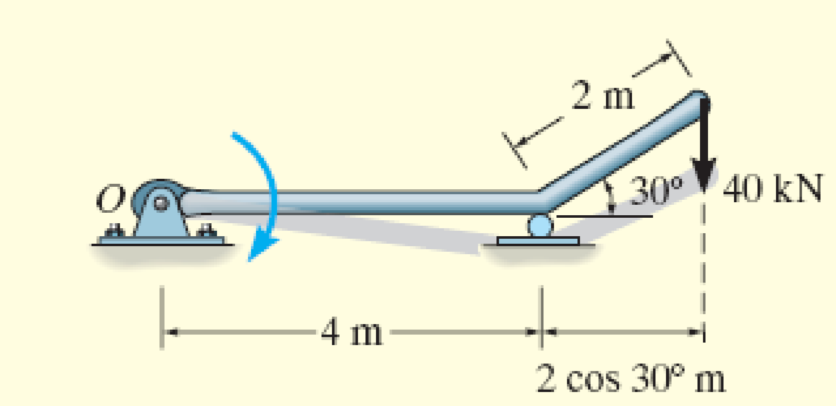
(a)

SOLUTION

$$(b)\quad M_o = (50N)(0.75m) = 37.5N.m(CW)$$

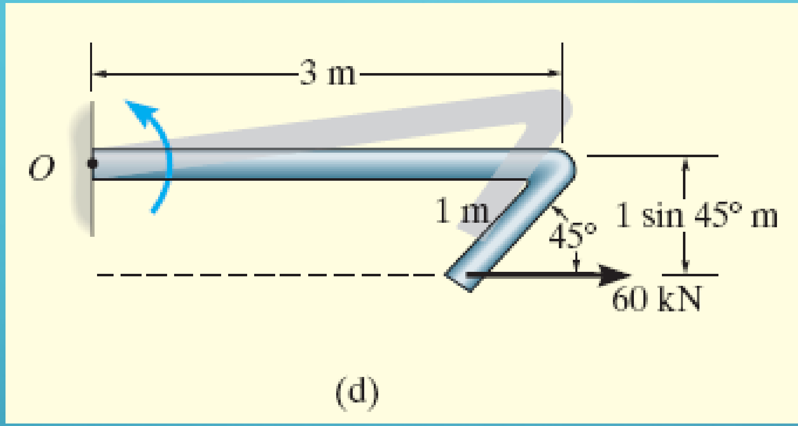


$$(c)\quad M_o = (40N)(4m + 2\cos 30^\circ m) = 229N.m(CW)$$

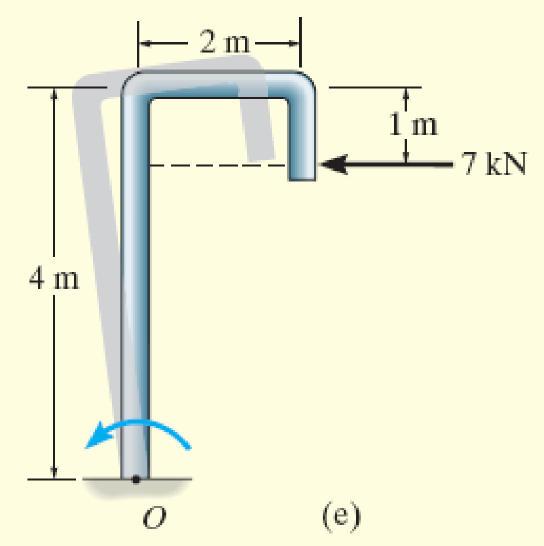


SOLUTION

$$(d)\ \ M_o = (60N)(1\sin 45^\circ m) = 42.4N.m(CCW)$$



$$(e)\ \ M_o = (7kN)(4m - 1m) = 21.0kN.m(CCW)$$



- We will continue in next tutorial with force and moment equilibrium as well as free body diagrams.