ENGG102 Fundamentals of Engineering Mechanics

Week 2 - Lecture

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E-readings: Fundamentals of Engineering Mechanics. Compiled by Tim McCarthy

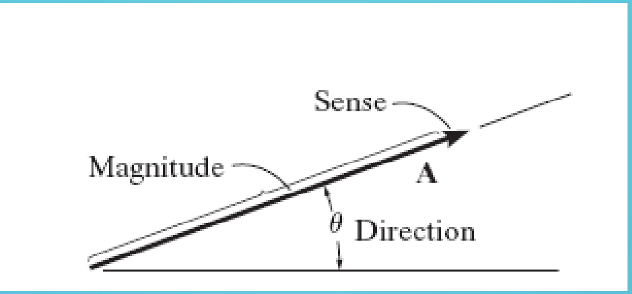
- **Chapter 4: Mechanics**
- **Chapter 5: Equilibrium of a Rigid Body**

1. Scalars and Vectors
2. Unit vector
3. Vector Addition of Forces
4. Cartesian Vectors in 3D
5. Position Vectors
6. Force Vector Directed along a Line
7. Moment of a force
8. Static Equilibrium

1 - SCALARS AND VECTORS

- **Scalar**
- A scalar is any positive or negative physical quantity that can be completely specified by its magnitude only.
- e.g. Mass, volume and length

- Vector



– A quantity that is specified by its magnitude as well as direction

e.g. Position, Velocity, force and moment

– Represent by a letter with an arrow over it, $\vec{A}$

– Length of vector is its Magnitude $\left|\vec{A}\right|$

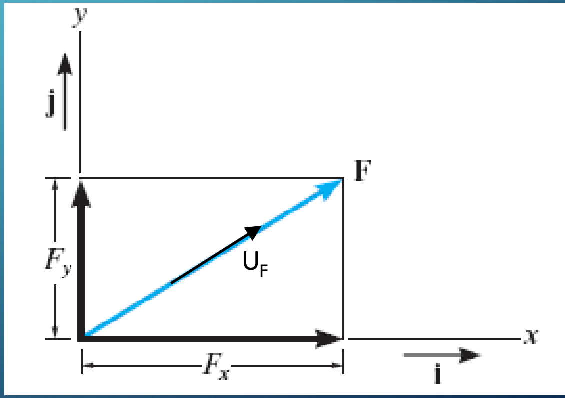
– In this figure, vector **A** is shown by an arrow, and its magnitude is represented by *A* and direction by θ.

2 - UNIT VECTOR

- A vector **F** may be expressed by multiplying its magnitude F by a unit vector $\mathbf{u}_F$ whose magnitude is one and whose direction coincides with that of **F**. The vector u_F is called a unit vector.

$$\mathbf{F} = F\, u_F$$

$$u_F = \mathbf{F} \,/\, F$$



3- VECTOR ADDITION OF FORCES

- A force has both magnitude and direction, therefore:

Force is a vector quantity; its units are newtons, N.

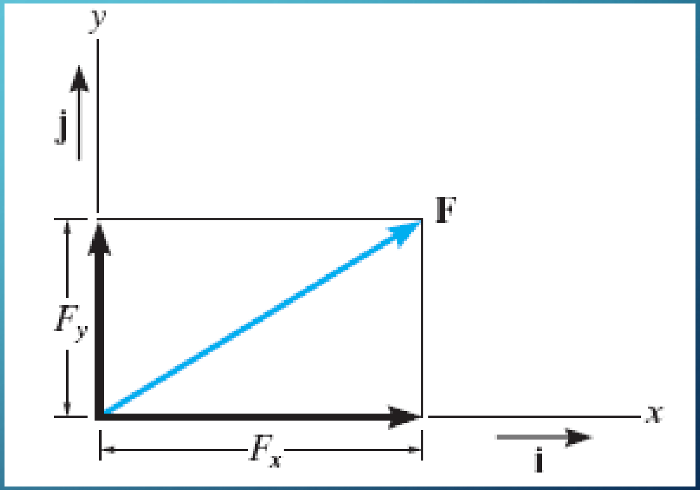
1) Rectangular components
2) Coplanar force resultants

Rectangular components

- When the vector components are mutually perpendicular to each other, they are called rectangular components of that vector.

A - Scalar Notation

B - Cartesian Vector Notation

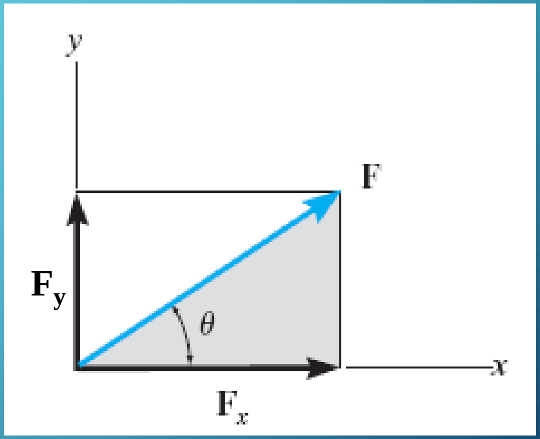


A - Scalar notation

$$F_x = F\cos\theta \quad \text{and} \quad F_y = F\sin\theta$$

$$F = \sqrt{F_x{}^2 + F_y{}^2}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

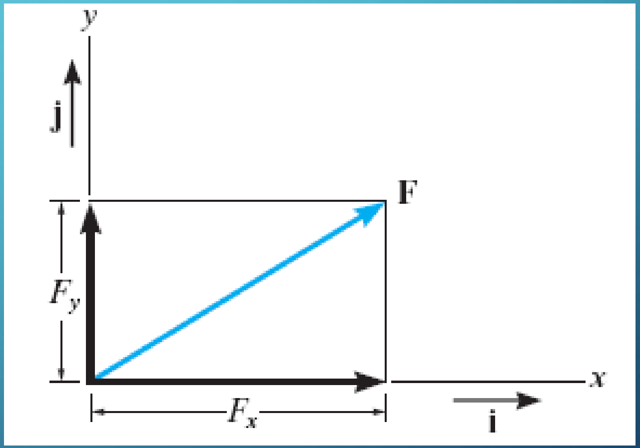


B - Cartesian Vector notation

- Unit vectors **i** and **j** have dimensionless magnitude of unity.
- Magnitude is always a positive quantity, represented by scalars F_x and F_y

$$\overrightarrow{\mathbf{F}} = \overrightarrow{\mathbf{F_x}} + \overrightarrow{\mathbf{F_y}}$$

$$\overrightarrow{F} = F_x i + F_y j$$



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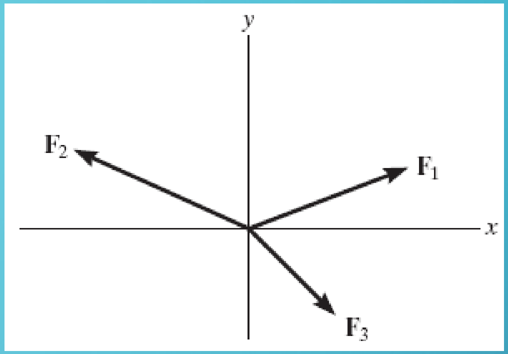
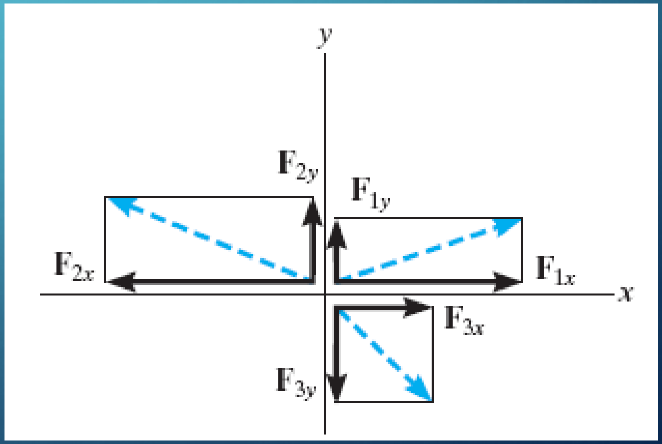
Coplanar Force Resultants:



Using scalar notation

- Resolve force into x and y components
- Addition of the respective components using scalar algebra
- Resultant force is found using the parallelogram law

$$\sum F_x = F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$
$$\sum F_y = F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$



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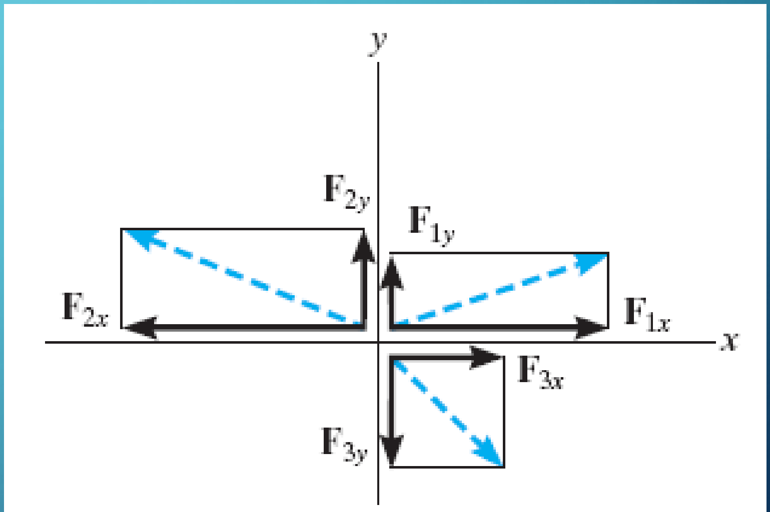
Coplanar Force Resultants

2) To express in Cartesian vector notation:

$$F_1 = F_{1x} i + F_{1y} j$$

$$F_2 = -F_{2x} i + F_{2y} j$$

$$F_3 = F_{3x} i - F_{3y} j$$



Vector resultant is therefore

$$F_R = F_1 + F_2 + F_3 = (F_{Rx})i + (F_{Ry})j$$

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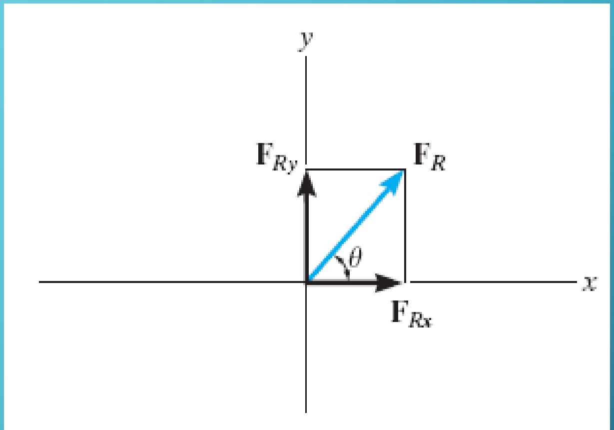
Coplanar Force Resultants

- In all cases we have

$$F_{Rx} = \sum F_x$$
$$F_{Ry} = \sum F_y$$



- Magnitude of $\mathbf{F}_R$ can be found by Pythagorean Theorem

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad \text{and } \theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

* Take note of sign conventions

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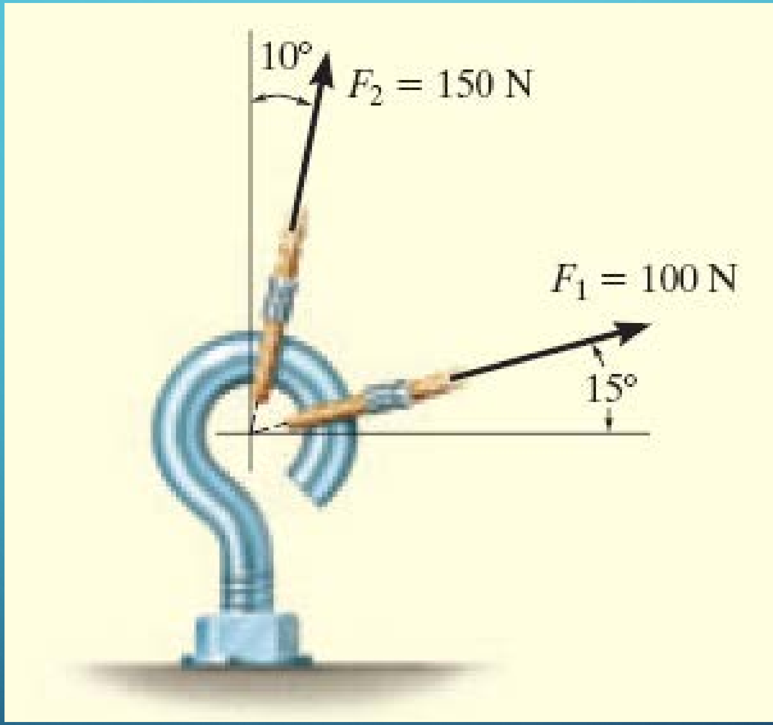
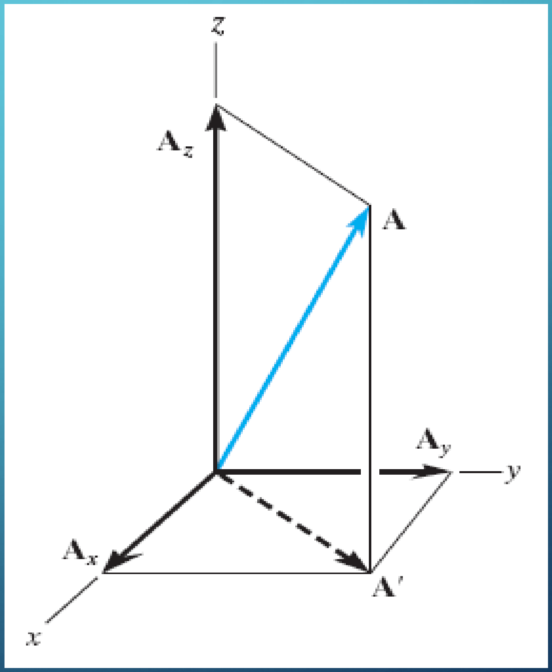
The screw eye is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



4 - CARTESIAN VECTORS (IN 3D)

- A vector **A** may have one, two or three rectangular components along the *x*, *y* and *z* axes, depending on orientation

$$\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y + \vec{\mathbf{A}}_z$$



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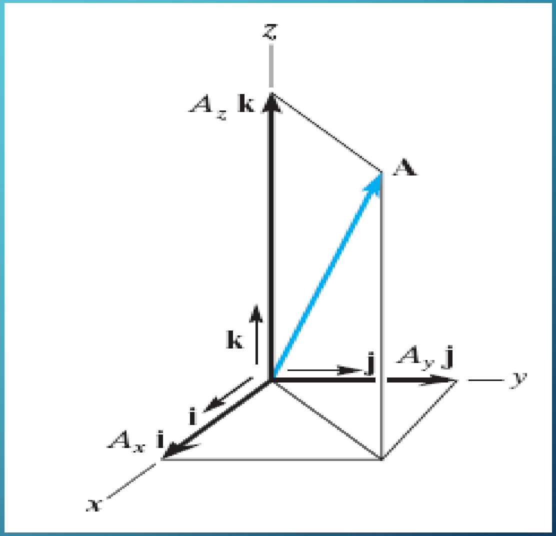
Cartesian Vector Representation

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

*Note the magnitude and direction of each components are separated, easing vector algebraic operations.



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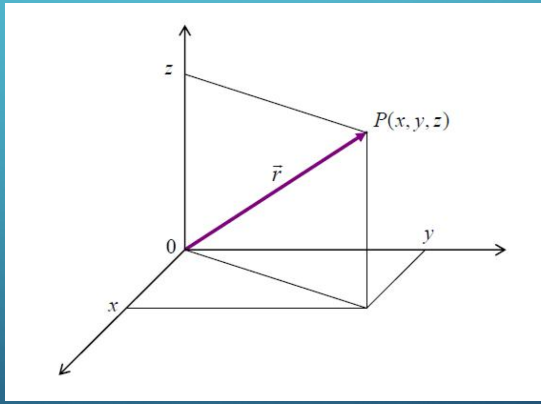
5 - POSITION VECTORS

Position Vector

- Position vector **r** is defined as a fixed vector which locates a point in space relative to another point.
- E.g. $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ Unit vector, $\mathbf{u} = \mathbf{r}/r$



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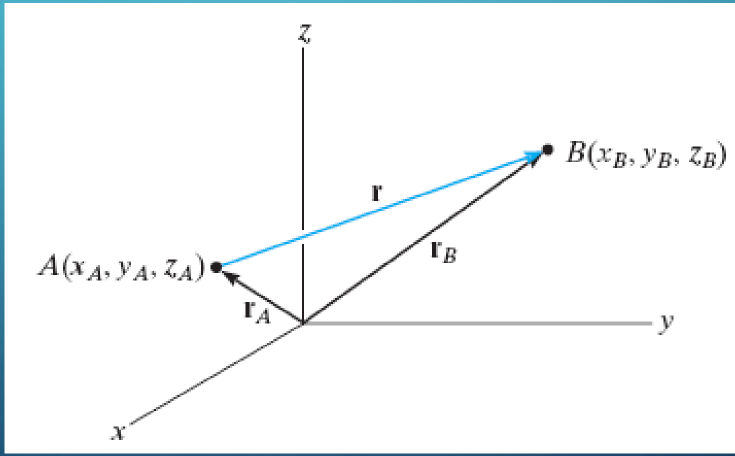
Position Vectors

Position Vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

- Between two points

$\mathbf{r} = r_B - r_A$

$= (x_B - x_A)\, i + (y_B - y_A)\, j + (z_B - z_A)\, k$



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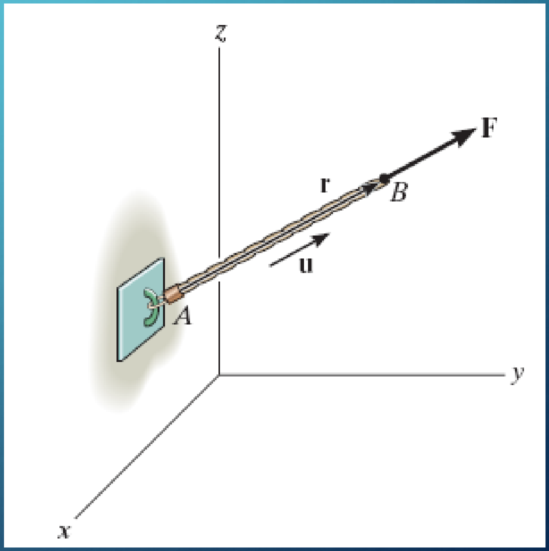
6 - FORCE VECTOR DIRECTED ALONG A LINE

- In 3D problems, direction of **F** is specified by 2 points, through which its line of action lies
- **F** can be formulated as a Cartesian vector

Unit vector, $\mathbf{u} = \mathbf{r}/r$

$\mathbf{F} = F\,\mathbf{u} = F\,(\mathbf{r}/r)$

- Note that **F** has units of forces (N) unlike **r**, with units of length (m)



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7 - MOMENT OF A FORCE – SCALAR FORMATION

Magnitude

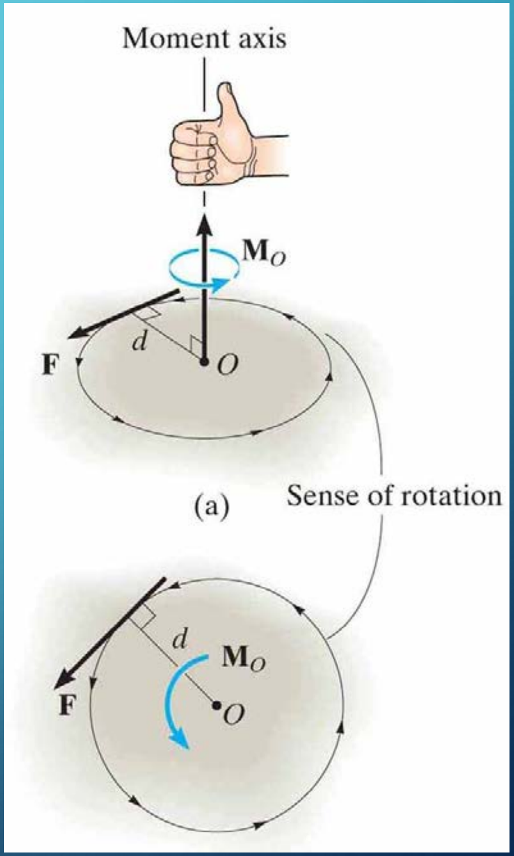
- For magnitude of $\mathbf{M}_O$,

$$\mathbf{M}_O = Fd\ (Nm)$$

where d = perpendicular distance from O to its line of action of force

Direction

- Direction using "right hand rule"



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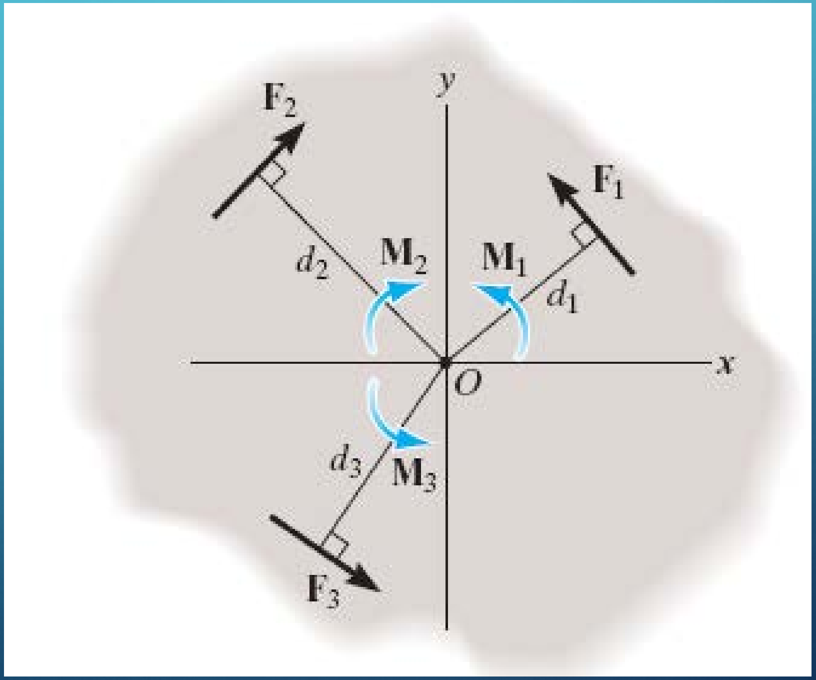
MOMENT OF A FORCE – SCALAR FORMATION

Resultant Moment

- Resultant moment, $\mathbf{M}_{R_o}$ = moments of all the forces

$$\mathbf{M}_{R_o} = \sum Fd = F_1 d_1 - F_2 d_2 + F_3 d_3$$



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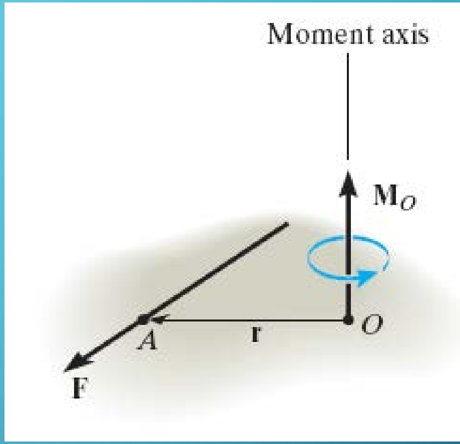
7 - MOMENT OF FORCE - VECTOR FORMATION

Moment of force **F** about point O can be expressed using cross product

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

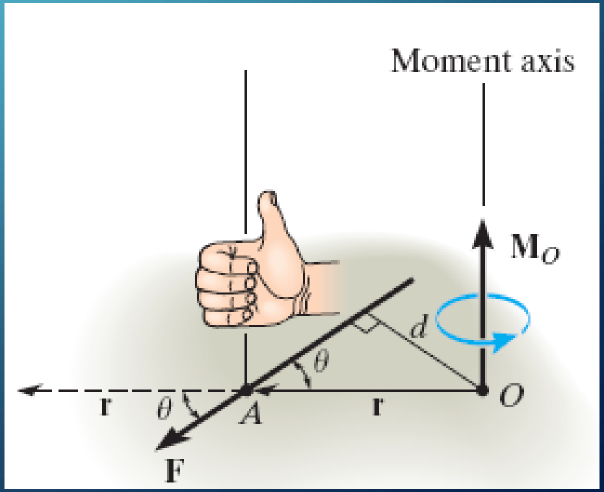


Magnitude

For magnitude, $d = r \sin\theta$,

$$M_O = Fd$$

$$= F\,(r\sin\theta)$$



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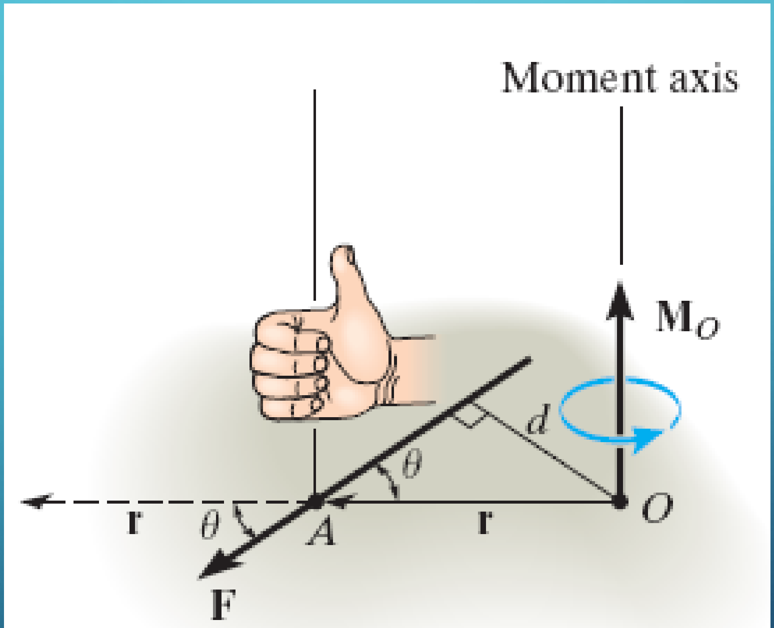
Direction

Direction and sense of $\mathbf{M}_O$ are determined by right- hand rule



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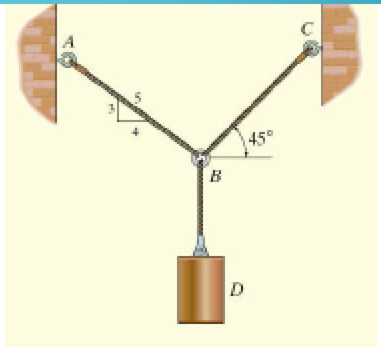
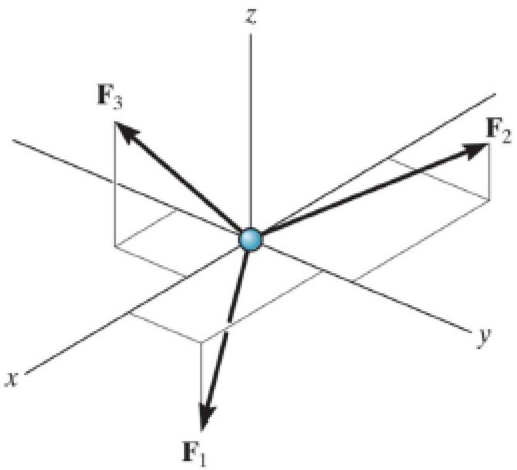
8 – STATIC EQUILIBRIUM

- Free-body diagram for a particle
- Forces acting on a specified point
- Resultant force and force equilibrium



$$\boxed{\begin{aligned} \sum \mathbf{F} &= \mathbf{0} \\ \sum F_x\mathbf{i} + \sum F_y\mathbf{j} + \sum F_z\mathbf{k} &= \mathbf{0} \end{aligned}}$$

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned}$$



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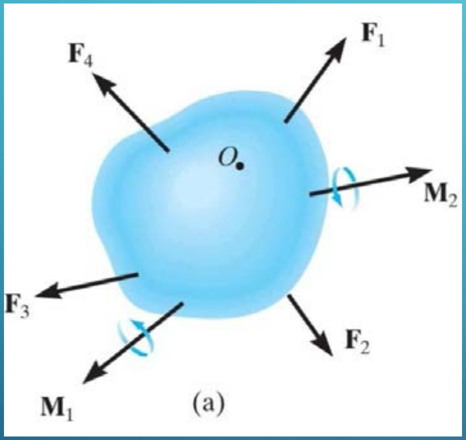
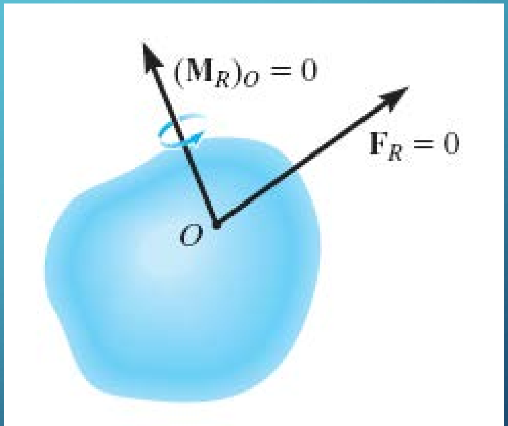
- What about moment equilibrium?
- Is only force equilibrium important?

CONDITIONS FOR RIGID-BODY EQUILIBRIUM

$$F_R = \sum F = 0$$

$$(M_R)_O = \sum M_O = 0$$





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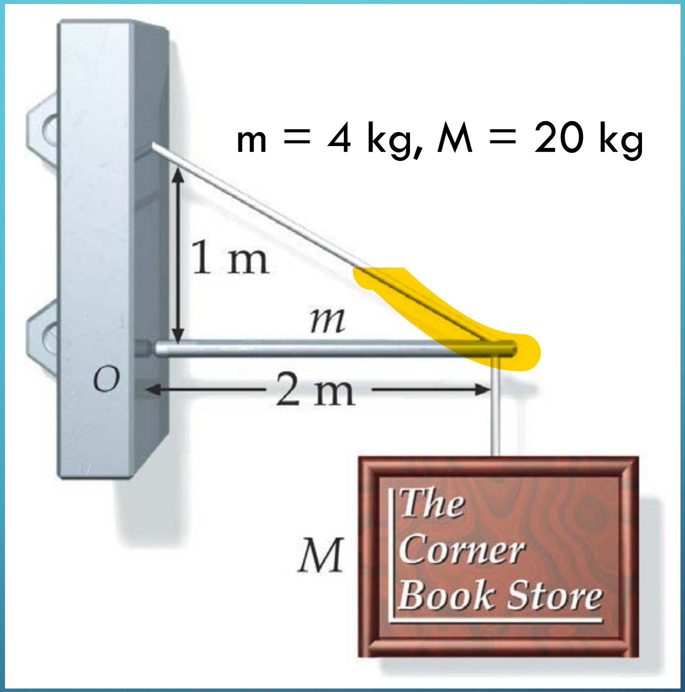
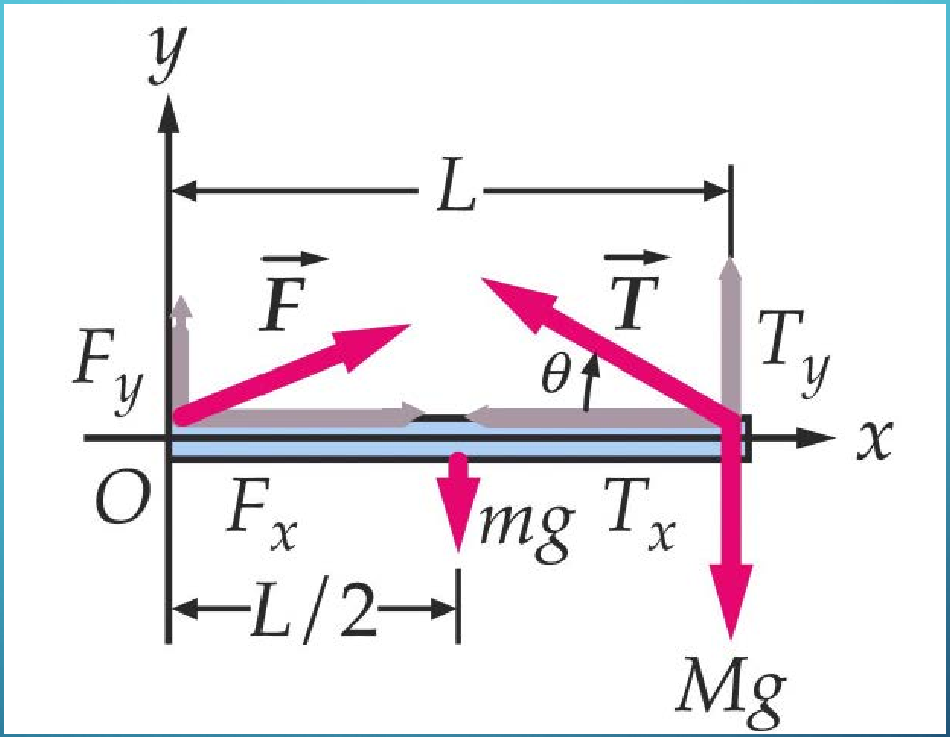
1. How strong does the diagonal wire need to be?
2. How much force does the rod put on the wall?





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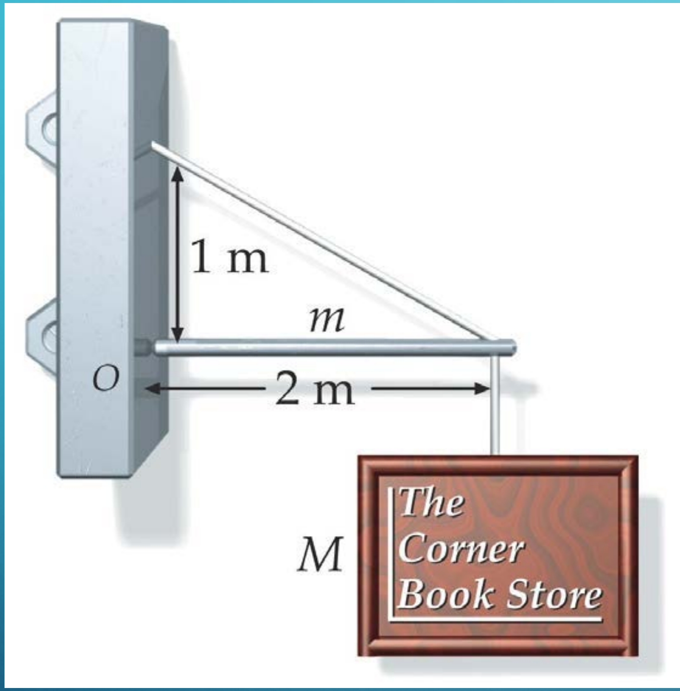
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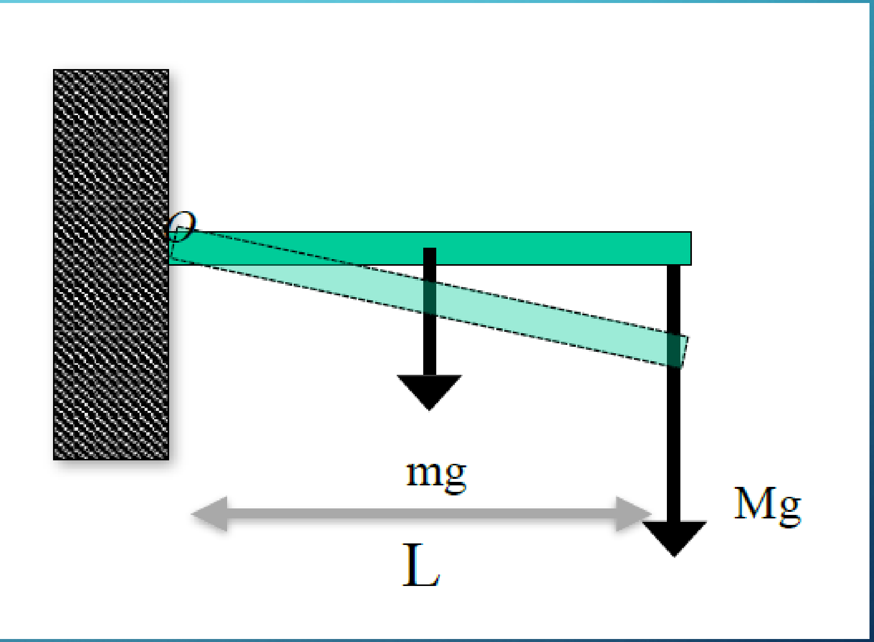
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