ENGG102 Fundamentals of Engineering Mechanics

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Outline

- Characteristics of Dry Friction
- Problems Involving Dry Friction

1 Characteristics of Dry Friction

Friction: <u>resistive force against movement</u>

- or retards slipping of the body relative to a second body or surface which it is in contact
- Acts tangent to the surfaces at points of contact with other body
- Opposing possible or existing motion of the body relative to points of contact
- <u>Two types of friction – Fluid and Coulomb Friction</u>

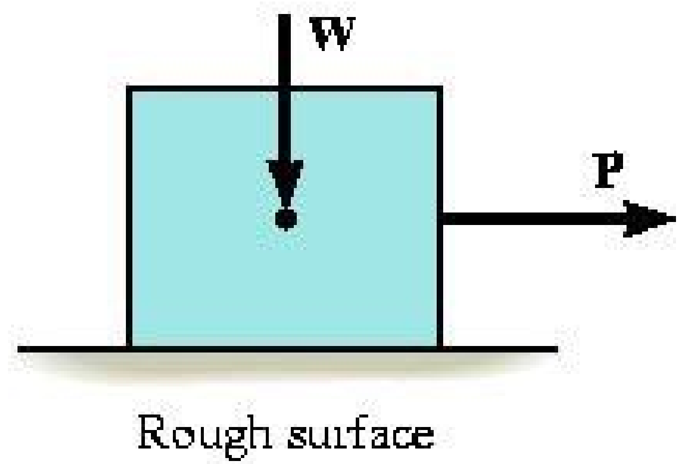
1 Characteristics of Dry Friction

- Fluid friction exist when the contacting surfaces are separated by a film of fluid (gas or liquid); parallel plates with fluid in between
- Depends on velocity of the fluid and its ability to resist shear force
- Coulomb friction, also known as dry friction, occurs between contacting surfaces of bodies in the absence of a lubricating fluid

1 Characteristics of Dry Friction

Theory of Dry Friction

- Consider the effects caused by pulling horizontally on a block of uniform weight **W** which is resting on a rough horizontal surface
- Consider the surfaces of contact to be nonrigid or deformable and other parts of the block to be rigid



(a)

1 Characteristics of Dry Friction

Theory of Dry Friction

- FBD of the block
- The floor exerts a distribution of the normal force $\Delta \mathbf{N}_n$ and frictional force $\Delta \mathbf{F}_n$ along the contact surface
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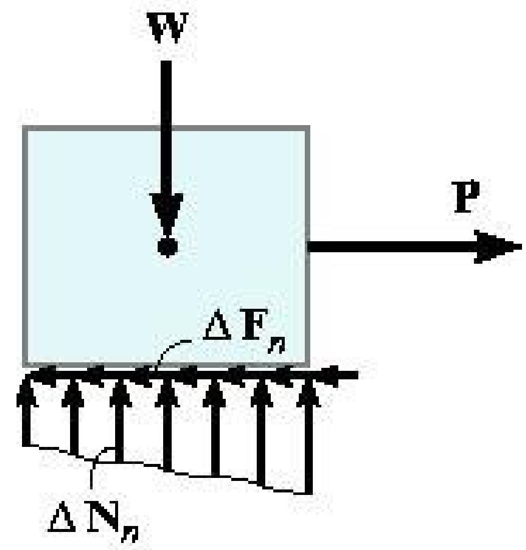
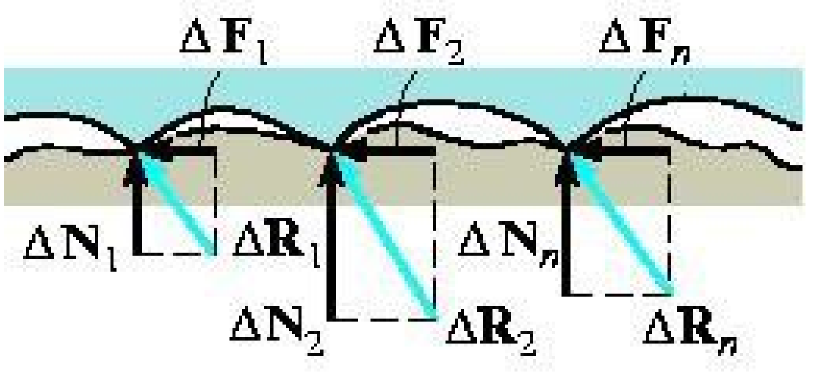


(b)

1 Characteristics of Dry Friction

Theory of Dry Friction

- Examining the contacting surfaces between the floor and the block, it can seen that many <u>microscopic irregularities </u>exist between the two surfaces
- Reactive forces $\Delta\mathbf{R}_n$ developed at each of the protuberances

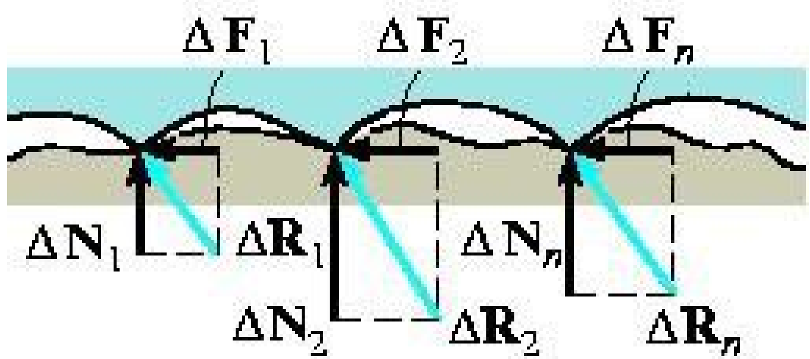


(c)

1 Characteristics of Dry Friction

Theory of Dry Friction

- These forces act at all points of contact and each reactive force consist of both a frictional component $\Delta \mathbf{F}_n$ and a normal component $\Delta \mathbf{N}_n$



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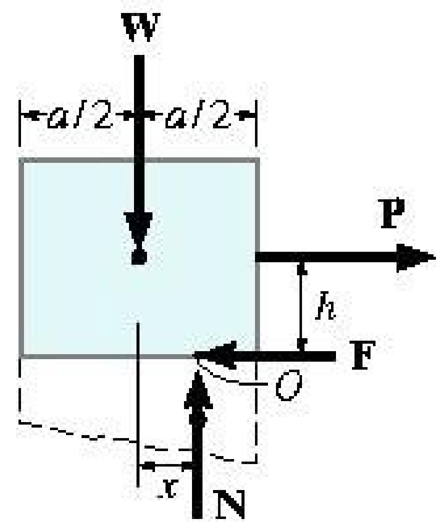
1 Characteristics of Dry Friction

Theory of Dry Friction

Equilibrium

- Effect of normal and frictional loadings are indicated by their resultant **N** and **F**
- Distribution of $\Delta \mathbf{F}_n$ indicates that **F** is tangent to the contacting surface, opposite to the direction of **P**
- Normal force **N** is determined from the distribution of $\Delta \mathbf{N}_n$



Resultant Normal and Frictional Forces

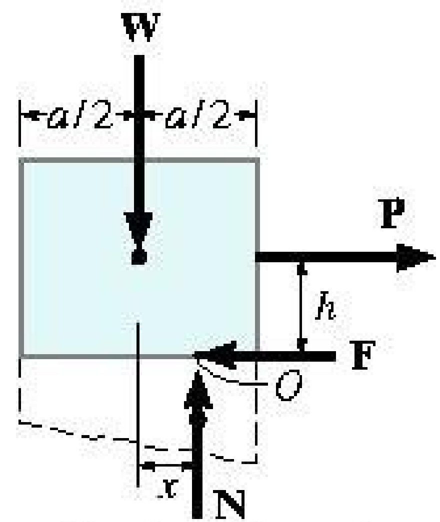
(d)

1 Characteristics of Dry Friction

Theory of Dry Friction

Example

- **P** is applied at a height h from the surface
- Moment equilibrium about point O is satisfied if W x = Ph or x = Ph/W
- The block is on the verge of tipping if N acts at the right corner of the block, x = a/2



Resultant Normal and Frictional Forces

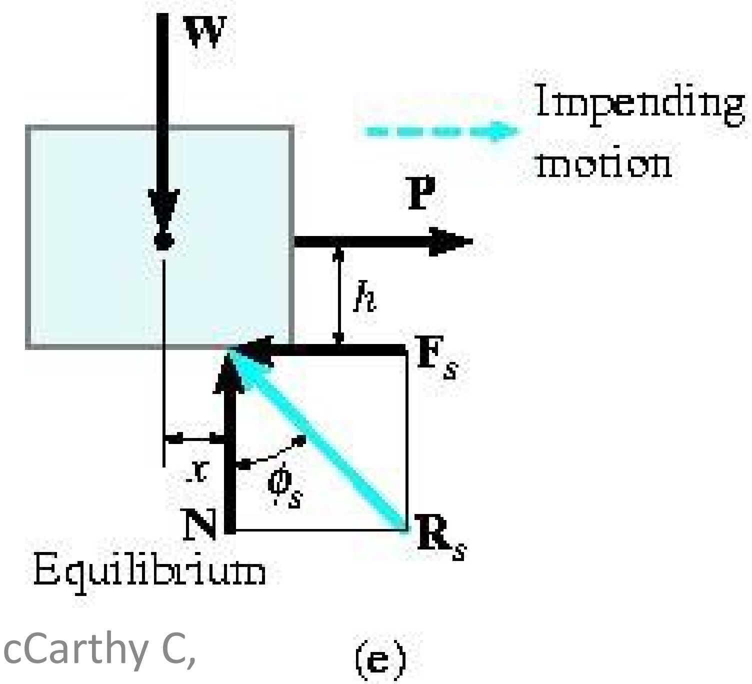
(d)

1 Characteristics of Dry Friction

Theory of Dry Friction

- Limiting static frictional force F_s is directly proportional to the resultant normal force N

$$F_s = \mu_s N$$



1 Characteristics of Dry Friction

Theory of Dry Friction

- Constant of proportionality μ_s is known as the coefficient of static friction
- When the block is on the verge of sliding, the normal force **N** and the frictional force $\mathbf{F}_s$ combine to form a resultant $\mathbf{R}_s$
- Angle Φ_s that $\mathbf{R}_s$ makes with **N** is called the angle of static friction

$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$

1 Characteristics of Dry Friction

Theory of Dry Friction
(Project 2)

Typical Values of μ_s

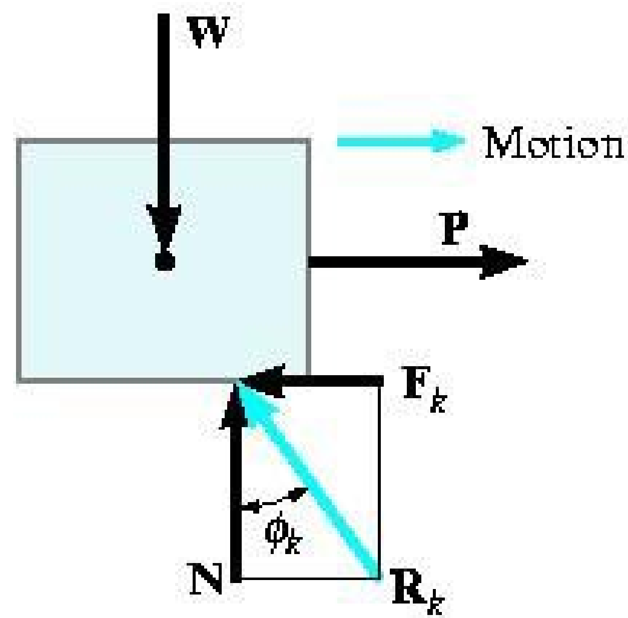
Contact Materials	Coefficient of Static Friction μ_s
Metal on ice	0.03 – 0.05
Wood on wood	0.30 – 0.70
Leather on wood	0.20 – 0.50
Leather on metal	0.30 – 0.60
Aluminum on aluminum	1.10 – 1.70

1 Characteristics of Dry Friction

Theory of Dry Friction

Motion

- If the magnitude of **P** acting on the block is increased so that it is greater than F_s, the frictional force at the contacting surfaces drops slightly to a smaller value F_s, called **kinetic** frictional force
- The block will not be held in equilibrium ($P > F_s$) but slide with increasing speed

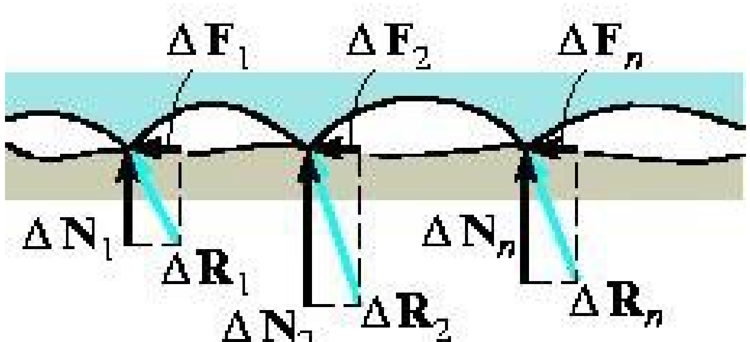


(a)

1 Characteristics of Dry Friction

Theory of Dry Friction: why $F_k < F_s$?

- The reduction made in the frictional force magnitude, from F_s (static) to F_k (kinetic), can by explained by examining the contacting surfaces
- When $P > F_s$, P has the capacity to shear off the peaks at the contact surfaces, causing the blocks to lift and ride on top of these peaks



(b)

1 Characteristics of Dry Friction

Theory of Dry Friction

- Resultant frictional force $\mathbf{F}_k$ is directly proportional to the magnitude of the resultant normal force **N**

$$F_k = \mu_k N$$

- Constant of proportionality μ_k is called the coefficient of kinetic friction
- μ_k are typically 25% smaller than μ_s

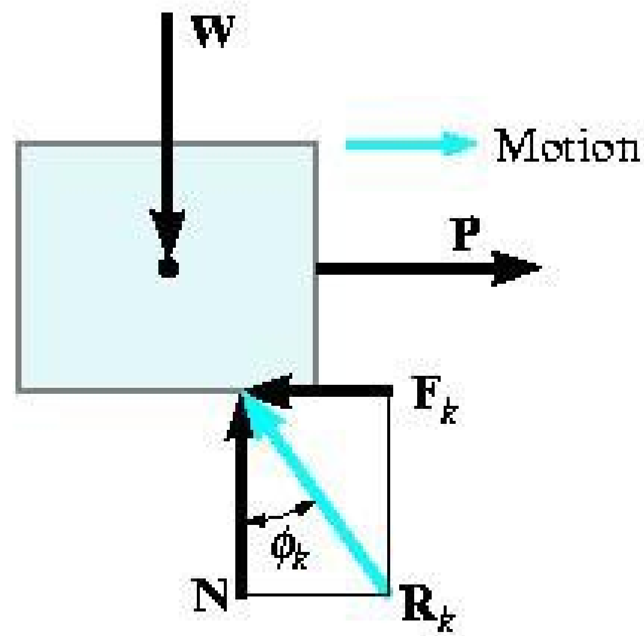
1 Characteristics of Dry Friction

Theory of Dry Friction

- Resultant $\mathbf{R}_k$ has a line of action defined by Φ_k, angle of kinetic friction

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}\mu_k$$

- $\Phi_s \geq \Phi_k$

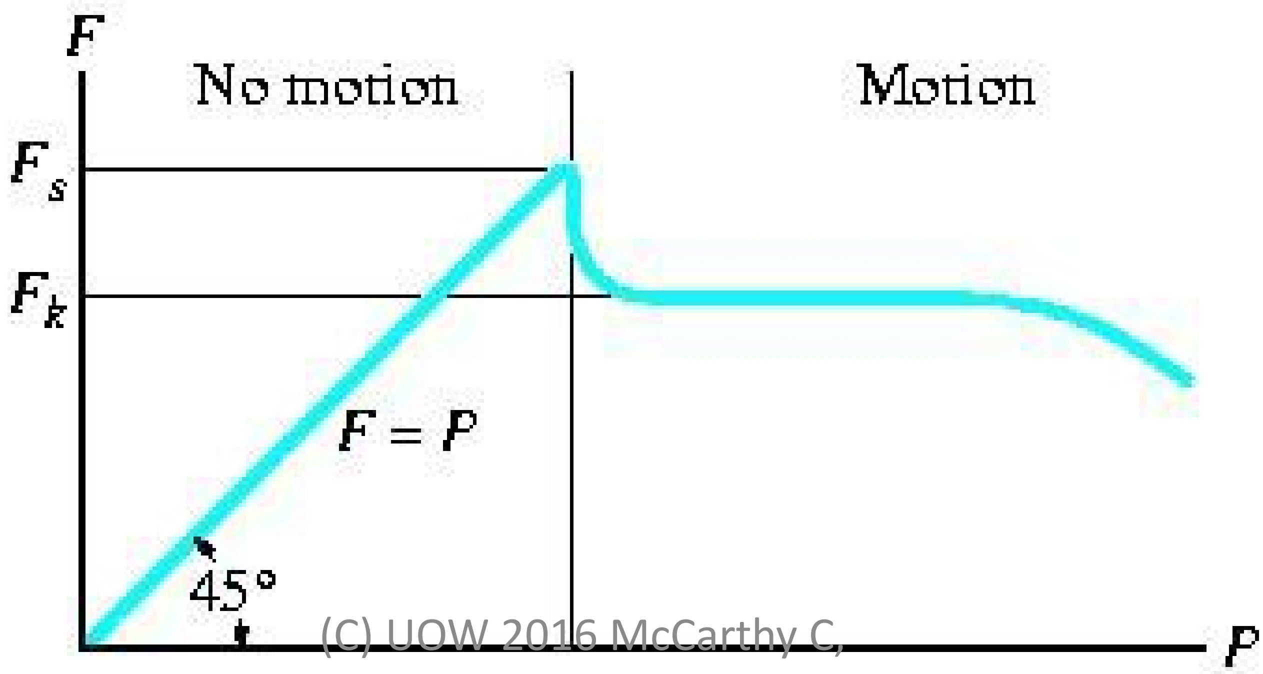


(a)

1 Characteristics of Dry Friction

Theory of Dry Friction

- The graph summarizes the effects regarding friction and shows the variation of frictional force F versus applied load P



1 Characteristics of Dry Friction

Theory of Dry Friction

- Frictional force is categorized into three ways

  -F is a static-frictional force if equilibrium is maintained

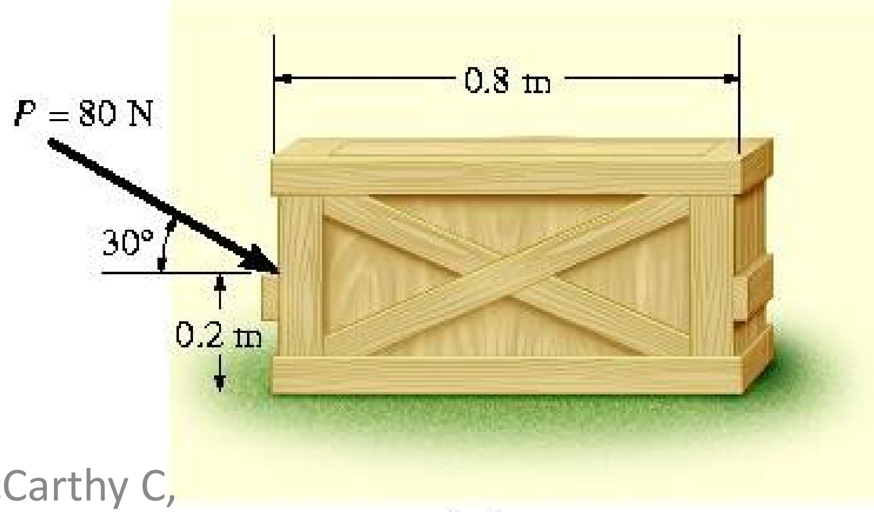
  -F is a limiting static-frictional force F_s when it reaches the maximum value needed to maintain equilibrium

  - F is a kinetic-frictional force F_k when sliding occurs at the contact surface

2 Problems Involving Dry Friction

Example 1

The uniform crate has a mass of 20kg. If a force P = 80N is applied on to the crate, <u>determine if it remains in equilibrium</u>. The coefficient of static friction is $\mu = 0.3$.

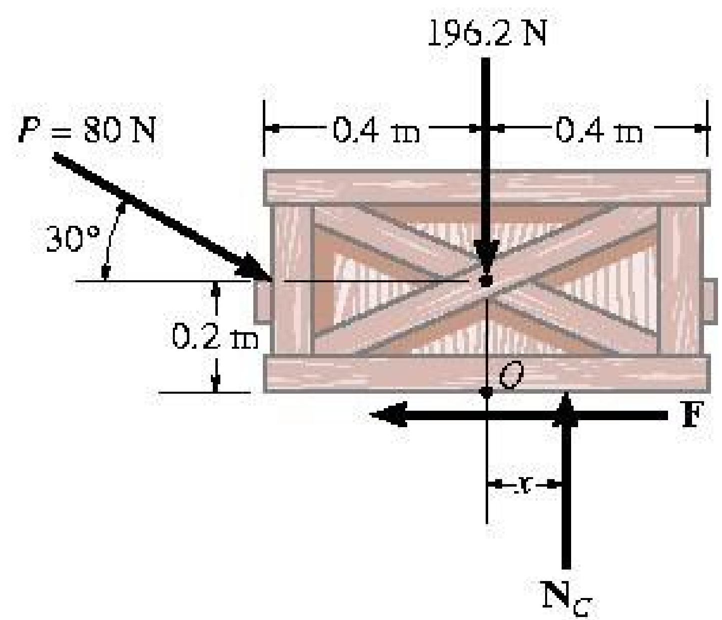


(a)

2 Problems Involving Dry Friction

Solution

- Resultant normal force **N**c act a distance x from the crate's center line in order to counteract the tipping effect caused by **P**
- 3 unknowns to be determined by 3 equations of equilibrium



2 Problems Involving Dry Friction

Solution

$$+\rightarrow \sum F_x = 0;$$

$$80\cos 30^\circ - F = 0$$

$$+\uparrow \sum F_y = 0;$$

$$-80\sin 30^\circ + N_C - 196.2 = 0$$

$$\sum M_O = 0;$$

$$80\sin 30^\circ (0.4m) - 80\cos 30^\circ (0.2m) + N_C (x) = 0$$

Solving

$$F = 69.3N, Nc = 236N,\ x = -0.00908 = -9.08mm$$

2 Problems Involving Dry Friction

Solution

- Since x is negative, the resultant force acts (slightly) to the left of the crate's center line
- No tipping will occur since $x \leq 0.4m$
- Maximum frictional force which can be developed at the surface of contact

$$F_{max} = \mu_s N_C = 0.3(236N) = 70.8N$$

- Since $F = 69.3N < 70.8N$, the crate will not slip although it is close to doing so

2 Problems Involving Dry Friction

Example 2

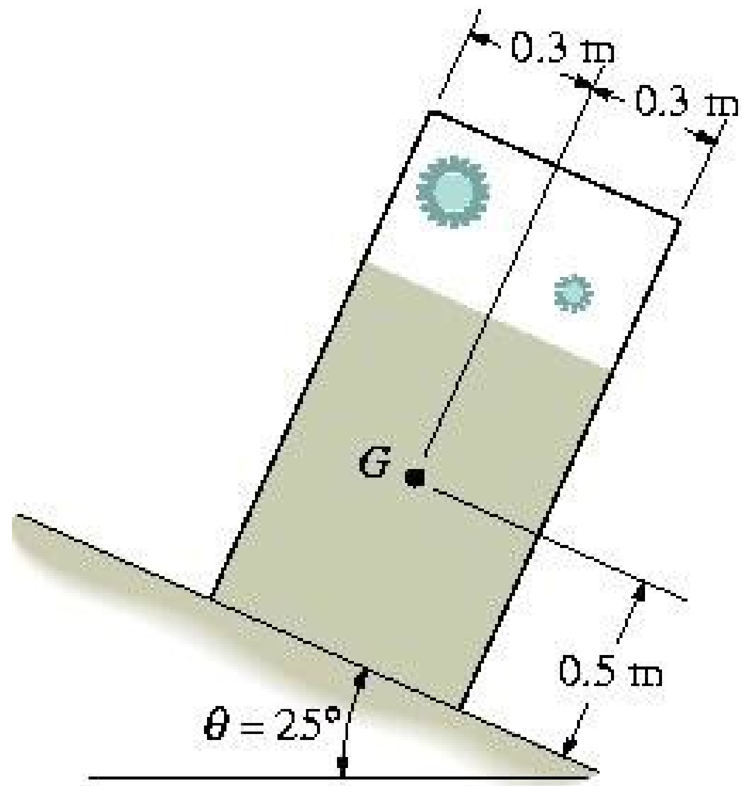
It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25°$ the vending machines *begin* to slide off the bed. Determine the static of coefficient of friction between them and the surface of the truck

(a)

2 Problems Involving Dry Friction

Solution

- Idealized model of a vending machine lying on the bed of the truck
- Dimensions measured and center of gravity located
- Assume machine weighs W

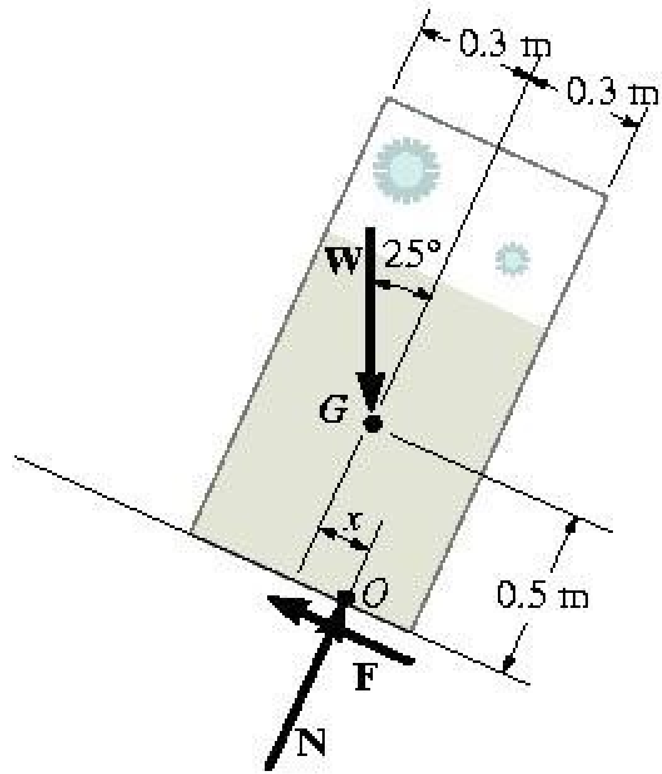


(b)

2 Problems Involving Dry Friction

Solution

- Dimension x used to locate position of the resultant normal force **N**
- 4 unknowns



(c)

2 Problems Involving Dry Friction

Solution

$$\sum F_x = 0;$$

$$W \sin 25^\circ - F = 0$$

$$\sum F_y = 0;$$

$$N - W \cos 25^\circ = 0$$

$$\sum M_O = 0; \text{ (if checking for tipping)}$$

$$-W \sin\theta (0.5m) + W \cos\theta (x) = 0$$

Slipping occurs at θ = 25°

$$F_s = \mu_s N; \quad W \sin 25^\circ = \mu_s (W \cos 25^\circ) \quad ; \quad \mu_s = \tan 25^\circ = 0.466$$

2 Problems Involving Dry Friction

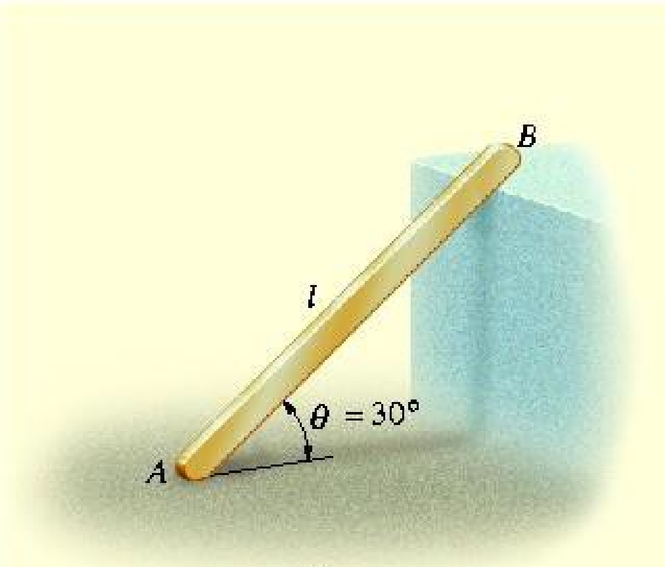
Solution

- Angle $\theta = 25°$ is referred as the angle of repose
- By comparison, $\theta = \Phi_s$
- θ is independent of the weight of the vending machine so knowing θ provides a method for finding coefficient of static friction
- $\theta = 25°$, $x = 0.233m$
- Since $0.233m < 0.3m$ the vending machine will slip before it can tip as observed

2 Problems Involving Dry Friction

Example 3

The uniform rod having a weight of W and length l is supported at its ends against the surfaces A and B. If the rod is on the verge of slipping when $\theta = 30°$, determine the coefficient of static friction μ_s at A and B. Neglect the thickness of the rod for calculation.

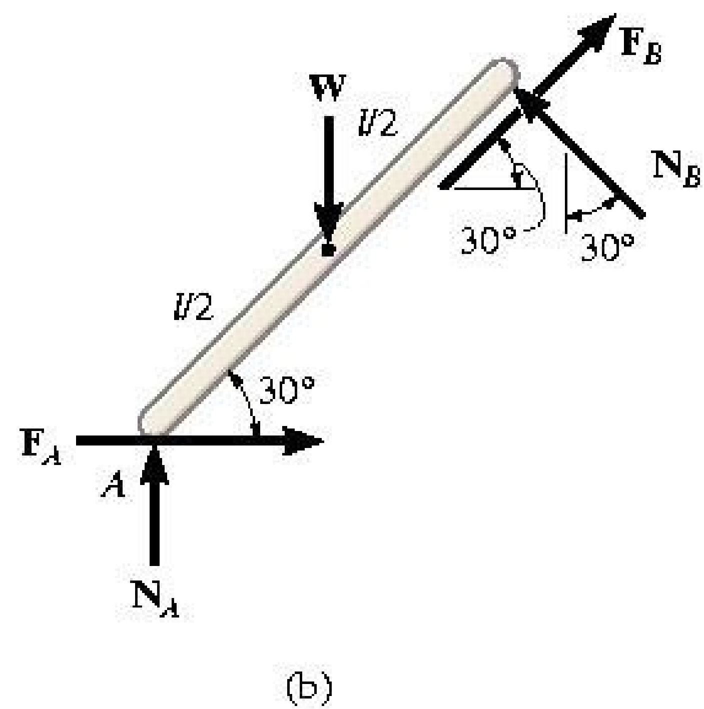


(a)

2 Problems Involving Dry Friction

Solution

- 5 unknowns
- 3 equilibrium equations and 2 frictional equations applied at A and B
- Frictional forces must be drawn with their correct sense so that they oppose the tendency for motion of the rod



2 Problems Involving Dry Friction

Solution

Frictional equations

$$F = \mu_s N;$$

$$F_A = \mu_s N_A \ , F_B = \mu_s N_B$$

Equilibrium equations

$$+ \rightarrow \sum F_x = 0;$$

$$\mu_s N_A + \mu_s N_B \cos 30^\circ - N_B \sin 30^\circ = 0$$

$$+ \uparrow \sum F_y = 0;$$

$$N_A - W + N_B \cos 30^\circ + \mu_s N_B \sin 30^\circ = 0$$

$$\sum M_A = 0;$$

$$N_B \ell - W \left(\frac{\ell}{2} \right) \cos 30^\circ = 0$$

2 Problems Involving Dry Friction

Solution

Solving

$$N_B = 0.4330W$$

$$\mu_s N_A = 0.2165W - (0.3750W)\mu_s$$

$$N_A = 0.6250W - (0.2165W)\mu_s$$

By division

$$0.6250\mu_s - 0.2165\mu_s^2 = 0.2165 - 0.375\mu_s$$

$$\mu_s^2 - 4.619\mu_s + 1 = 0$$

Solving for the smallest root

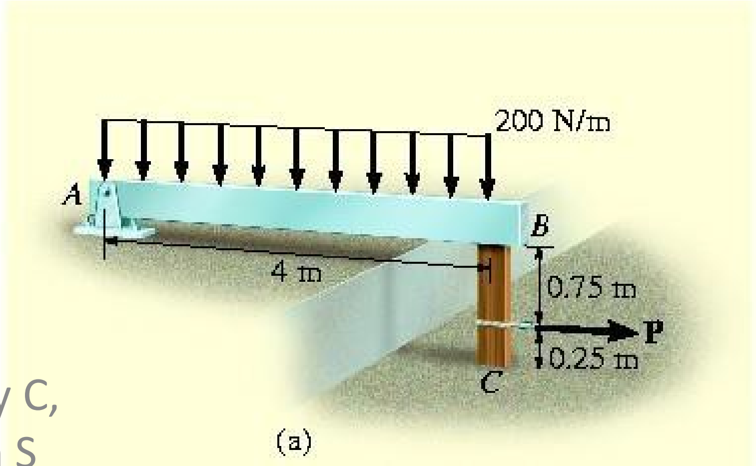
$$\mu_s = 0.228$$

2 Problems Involving Dry Friction

Example 4

Beam AB is subjected to a uniform load of 200N/m and is supported at B by post BC. If the coefficients of static friction at B and C are μ_B =0.2 and μ_C = 0.5, <u>determine the force **P** needed to pull the post out from under the beam</u>.

Neglect the weight of the members and the thickness of the post.

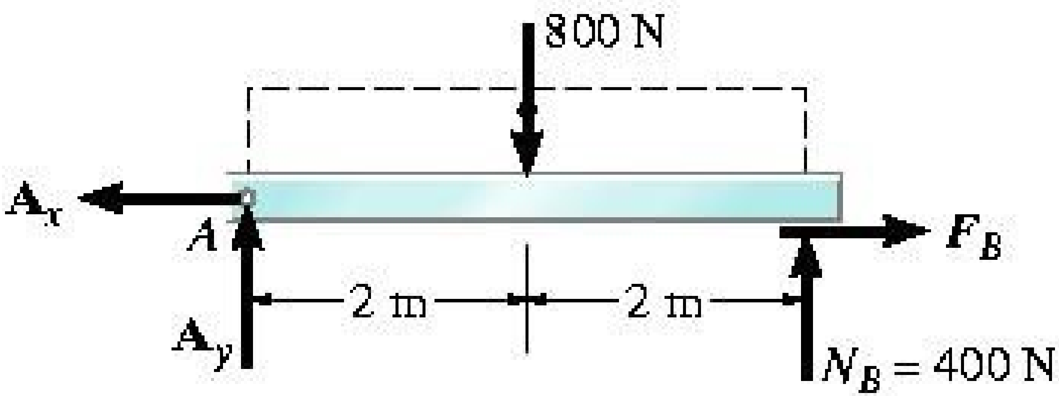


(a)

2 Problems Involving Dry Friction

Solution

- FBD of beam AB and the post
- Apply $\sum M_A = 0$, $N_B = 400N$
- 4 unknowns
- 3 equilibrium equations and 1 frictional equation applied at either B or C



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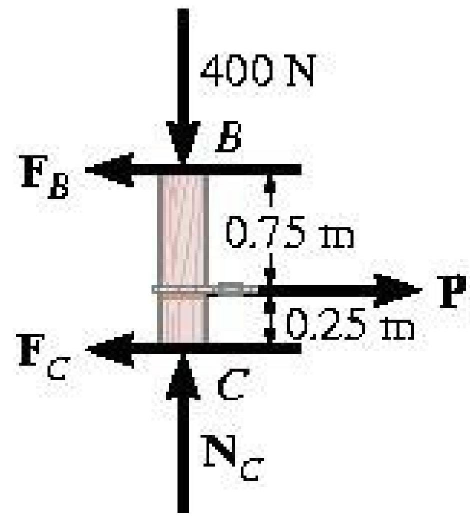

(c)

2 Problems Involving Dry Friction

Solution

$$+\rightarrow \sum F_x = 0; \qquad P - F_B - F_C = 0$$

$$+\uparrow \sum F_y = 0; \qquad N_C - 400N = 0$$

$$\sum M_c = 0; \quad -P(0.25m) + F_B(1m) = 0$$

Post slips only at B

$$F_C \le \mu_C N_C$$

$$F_B = \mu_B N_B; F_B = 0.2(400N) = 80N$$

Solving

$P = 320N, F_C = 240N, N_C = 400N$

$F_C = 240N > \mu_C N_C = 0.5(400N) = 200N$ *(Post is also slipping at C)*

2 Problems Involving Dry Friction

Solution

Post slips only at C

$$\sum M_B = 0; \quad \text{P}(0.75m) - F_C(1m) = 0$$

$$F_B \leq \mu_B N_B$$

$$F_C = \mu_C N_C \,;\, F_C = 0.5 N_C = 0.5(400) = 200\text{N}$$

Solving

$$P = 267N, N_C = 400N, F_C = 200N, F_B = 66.7N$$

Choose second case as it requires a smaller value of P