ENGG102 Fundamentals of Engineering Mechanics

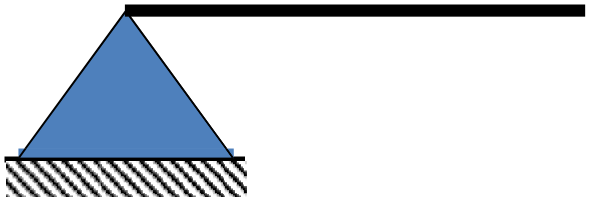
Dr. Sana Amir

Introduction to Internal Forces
Free Body Diagrams
Stress and Material Strength

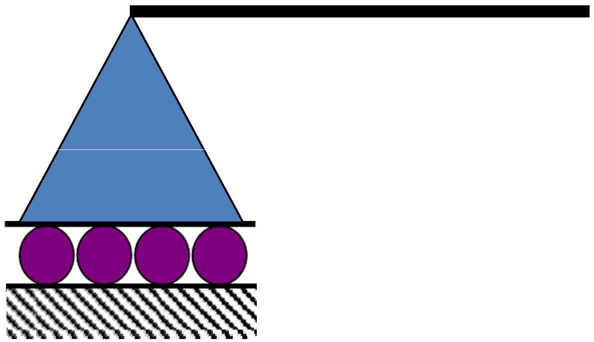
Part 1: Introduction to internal forces

1. FBD of sub-parts of structures
2. Internal forces
3. Predicting deflected shapes from FBD
4. Predicting failure location from the FBD

Idealised End connections (Degrees of freedom)



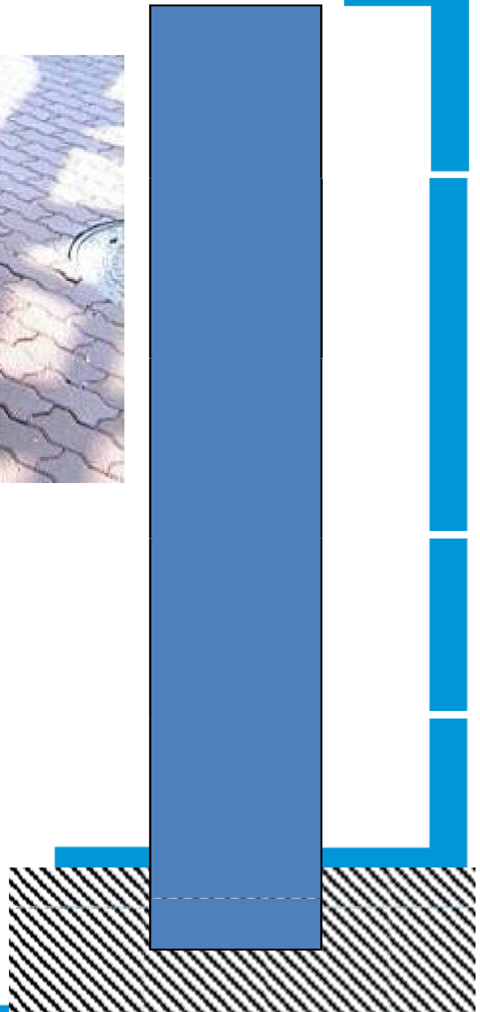
End connections
(Degrees of freedom)



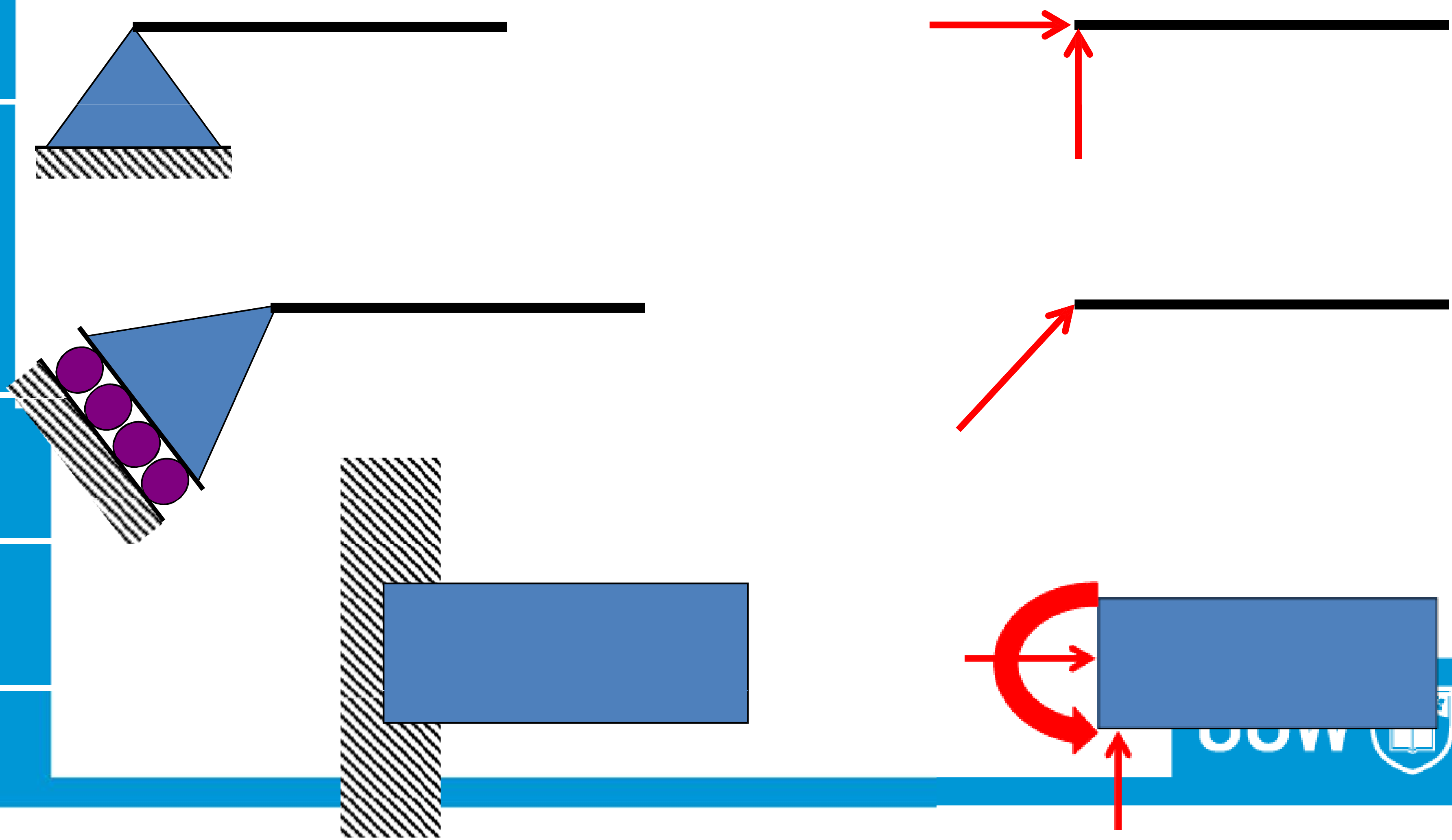
(a)

End connections (Degrees of freedom)

- *Joint transmits forces and moments*

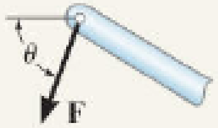
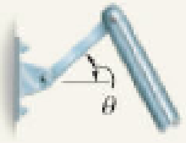
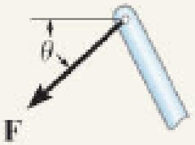
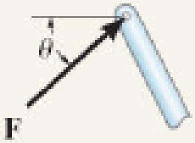
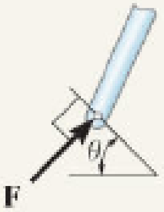


Supports and equivalent reactions



FREE BODY DIAGRAMS

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

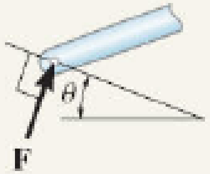
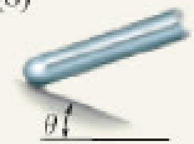
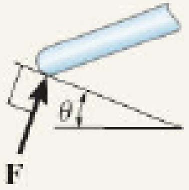
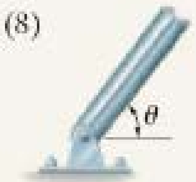
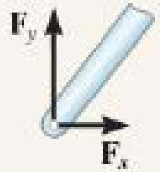
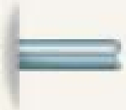
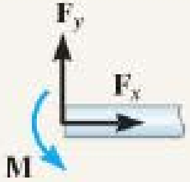
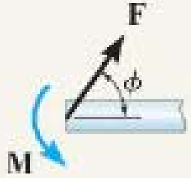
(4)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

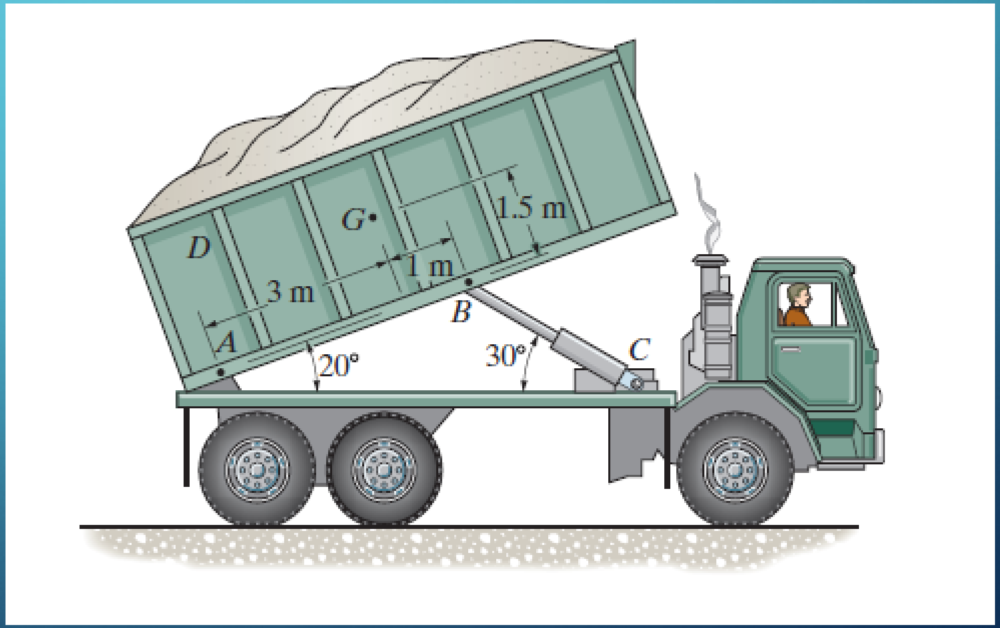
TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Q-1

5–1.

Draw the free-body diagram of the dumpster D of the truck, which has a weight of 5000 lb and a center of gravity at G. It is supported by a pin at A and a pin-connected hydraulic cylinder BC (short link). Explain the significance of each force on the diagram. (See Fig. 5–7b.)



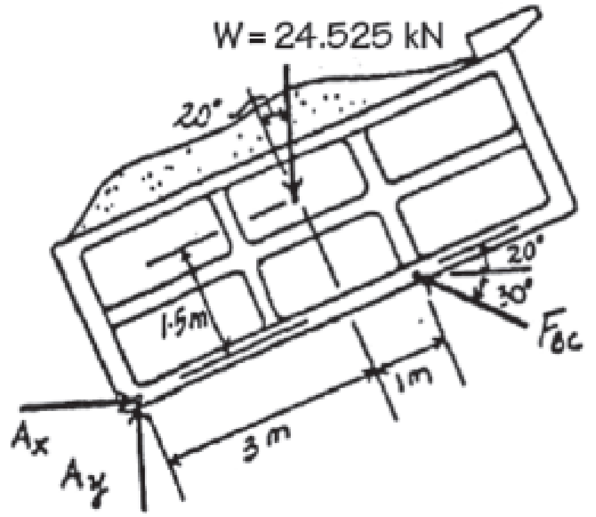
SOLUTION

The Significance of Each Force:

W is the effect of gravity (weight) on the dumpster.

A_y and A_x are the pin A reactions on the dumpster.

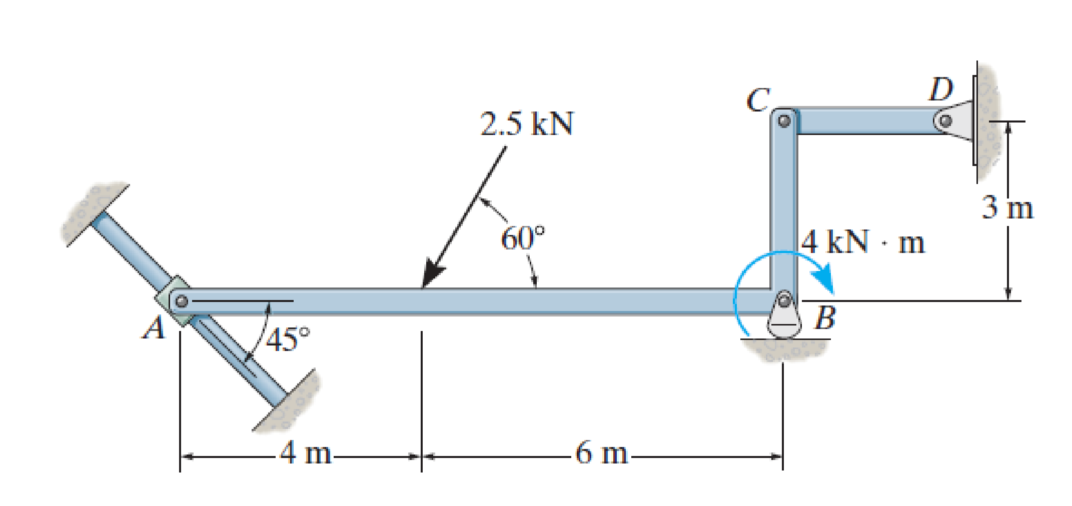
F_{BC} is the hydraulic cylinder BC reaction on the dumpster.



Q-2

5–2.

Draw the free-body diagram of member *ABC* which is supported by a smooth collar at *A*, rocker at *B*, and short link *CD*. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



SOLUTION

The Significance of Each Force:

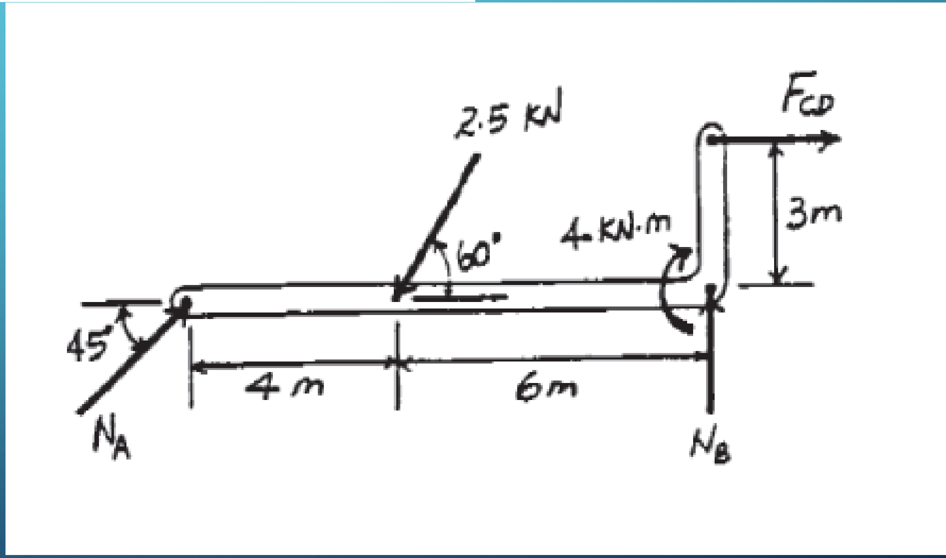
N_A is the smooth collar reaction on member ABC.

N_B is the rocker support B reaction on member ABC.

F_{CD} is the short link reaction on member ABC.

2.5 kN is the effect of external applied force on member ABC.

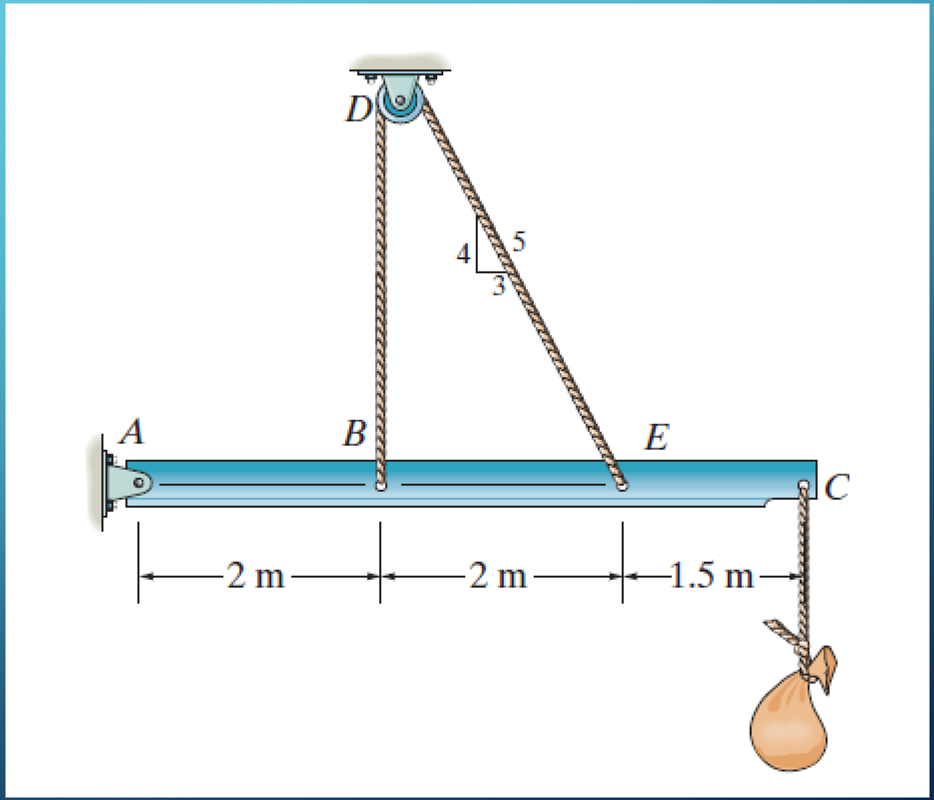
4 kN · m is the effect of external applied couple moment on member ABC.



Q-3

5–3.

Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at *A* and a cable which wraps around the pulley at *D*. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)

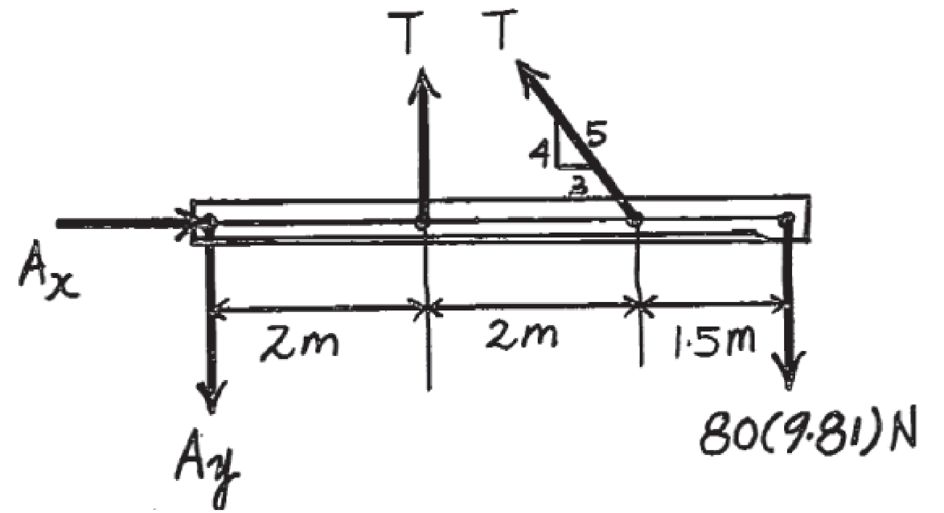


SOLUTION

T force of cable on beam.

A_x, A_y force of pin on beam.

80(9.81)N force of load on beam.



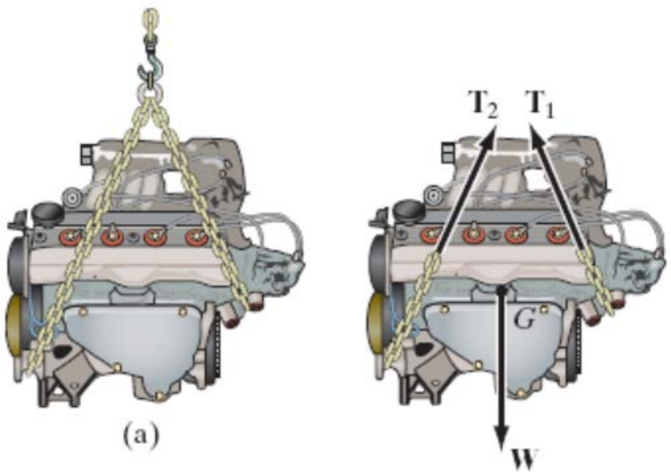
Free Body Diagrams (FBD)

Internal Forces

External and internal forces can act on a rigid body

For FBD, internal forces act between particles which are contained within the boundary of the FBD, are not represented

Particles outside this boundary exert external forces on the system



Part 2: Force, Stress and Material Strength

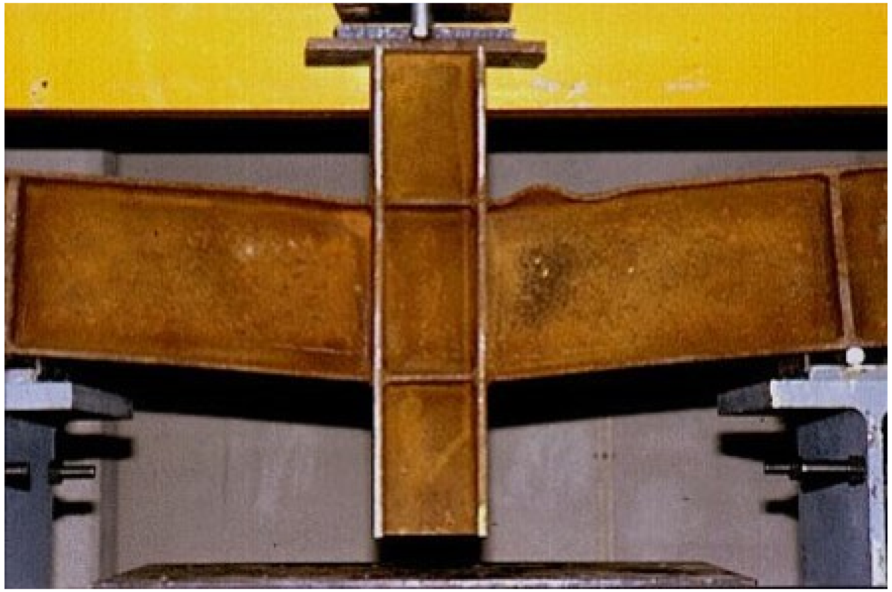
1. Determining forces on bodies
2. Case Study of Failure
3. Material strength concepts

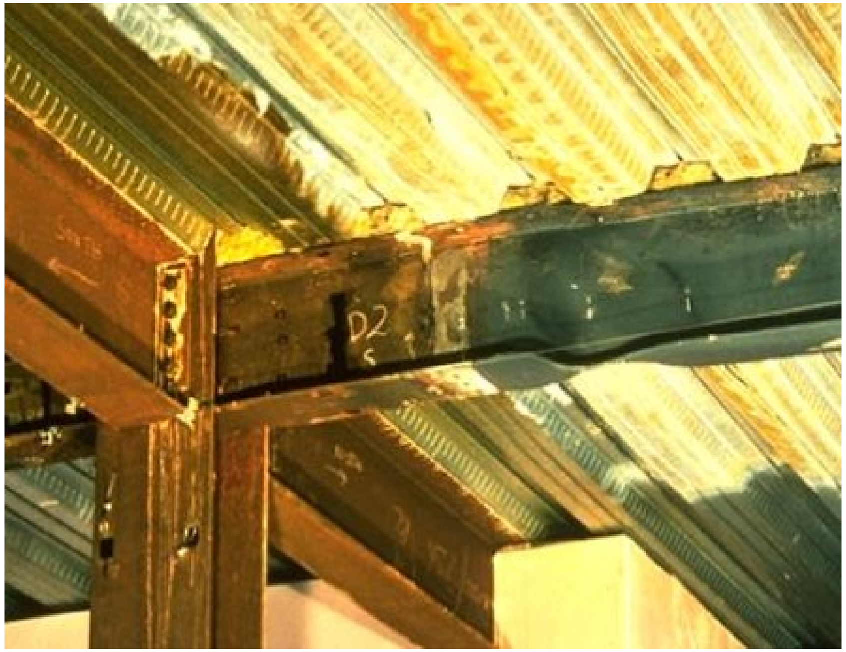
Failure of Structures

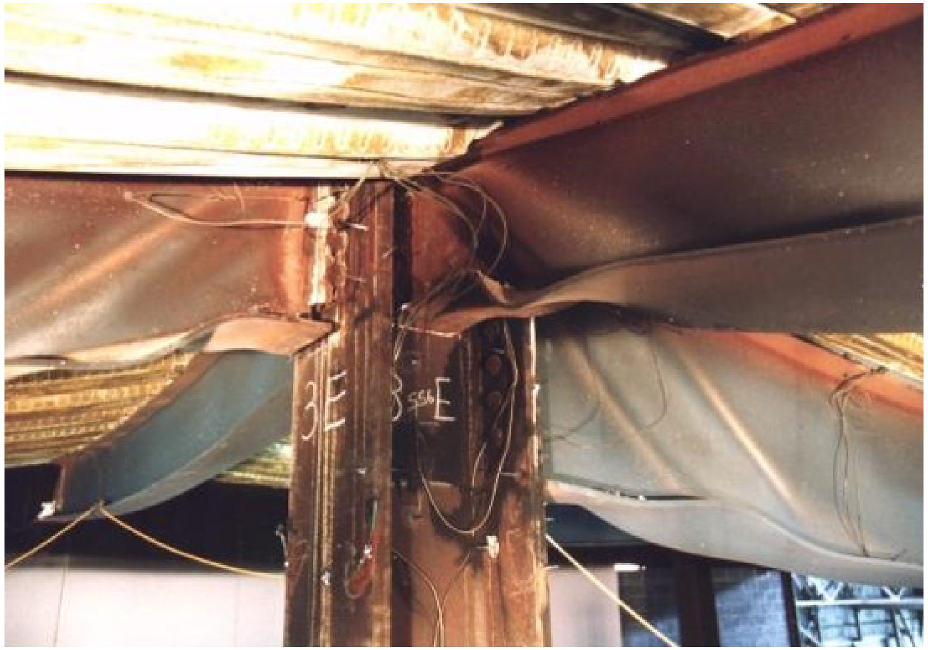
- Global buckling of strut
  - often preceded by noticeable distortion and shortening of strut due to curving
- Local buckling of thin plates in compression
  - compression flange of beams
  - once “kinked” plate rapidly loses strength
- Bending
  - preceded by large deflections
  - second order effects may increase strength

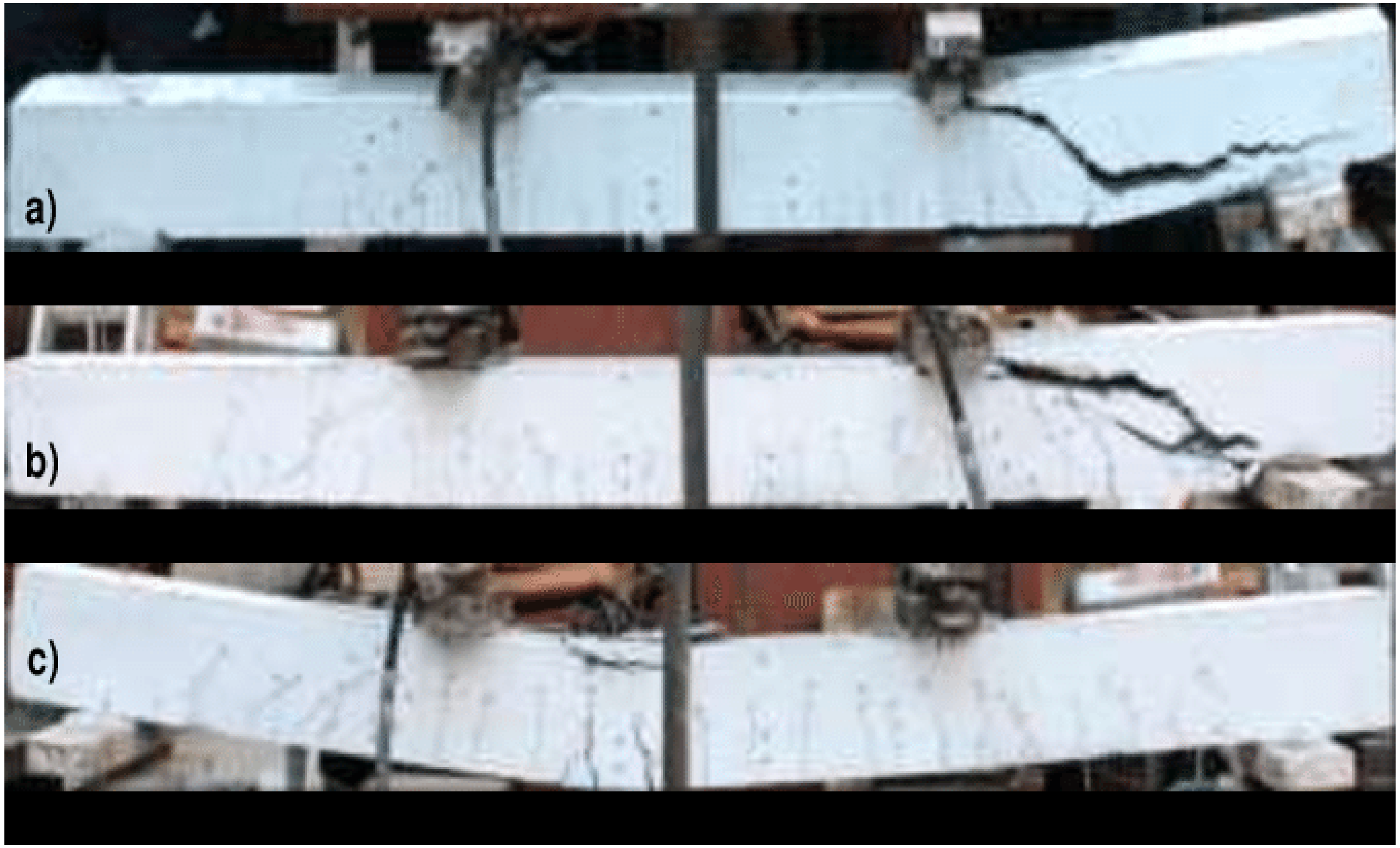
Failure of structures

- Tension failure
  - member elongates - largish deflection
  - little chance of recovery
- Connection failure
  - usually sudden rupture




D2


3E


a)
b)
c)

Go to video

Bending Beam Test...watch beam failure in slow-motion!
https://www.youtube.com/watch?v=6ycbDCnoO8M

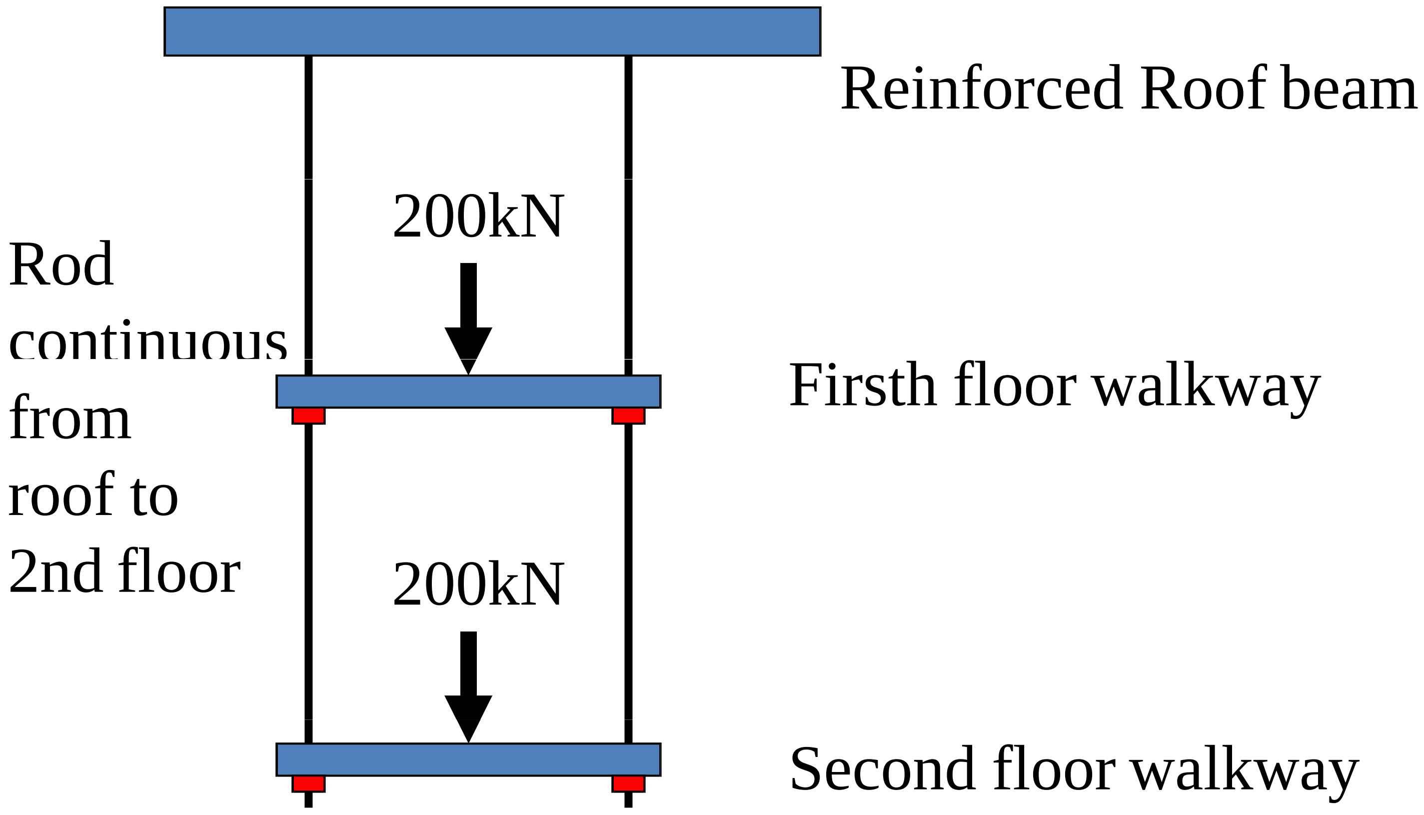
Tensile Test

https://www.youtube.com/watch?v=CXdJSEb_DLc

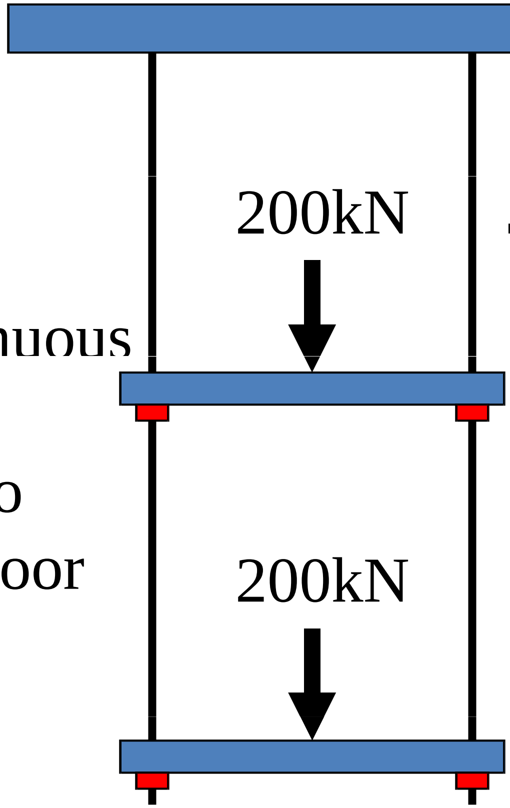
Record Truss Bridge testing-2012 - University of Auckland Engineering
https://www.youtube.com/watch?v=Z_h6gMlRf7k

Buckling of Axially Loaded Struts in Compression
https://www.youtube.com/watch?v=_Ga6mCHc7JE

Walkway structure - original design



Walkway structure - original design

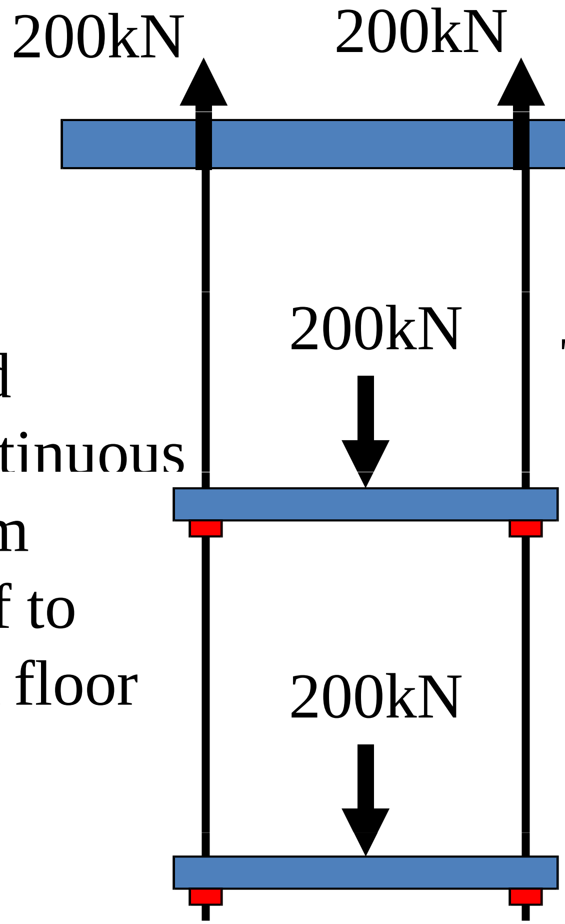


Rod continuous from roof to 2nd floor

Tension in top part of rod = ?

Tension in bottom part of rod = ?

Walkway structure - original design



Rod continuous from roof to 2nd floor

Tension in top part of rod = 100+100 =200kN

Tension in bottom part of rod = 100 kN

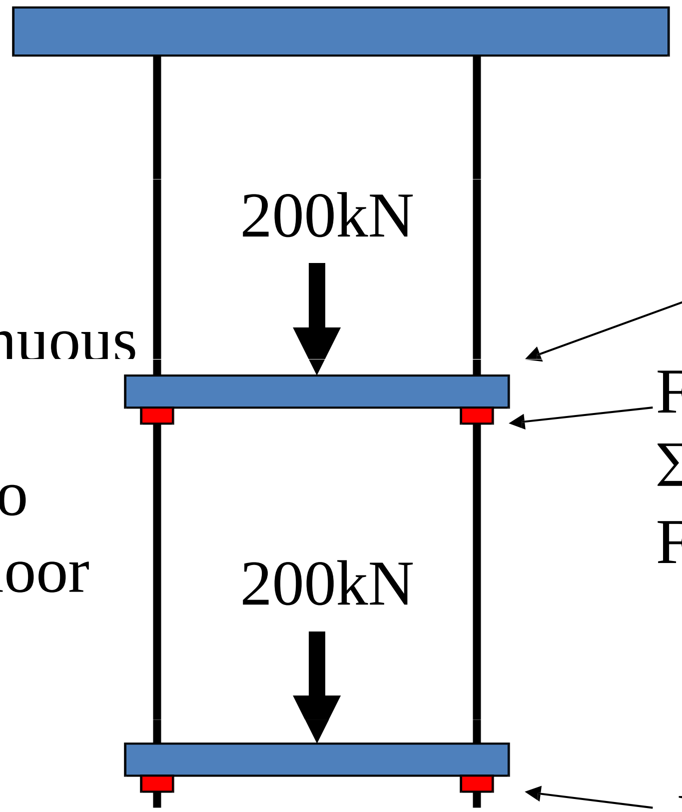
Rod diameter = 44.5mm

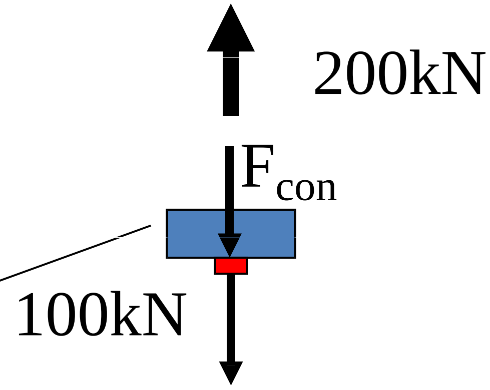
Yield strength f_y= 275 N/mm²

Tension capacity = ?

Walkway structure - original design

Rod continuous from roof to 2nd floor





Force on this connection

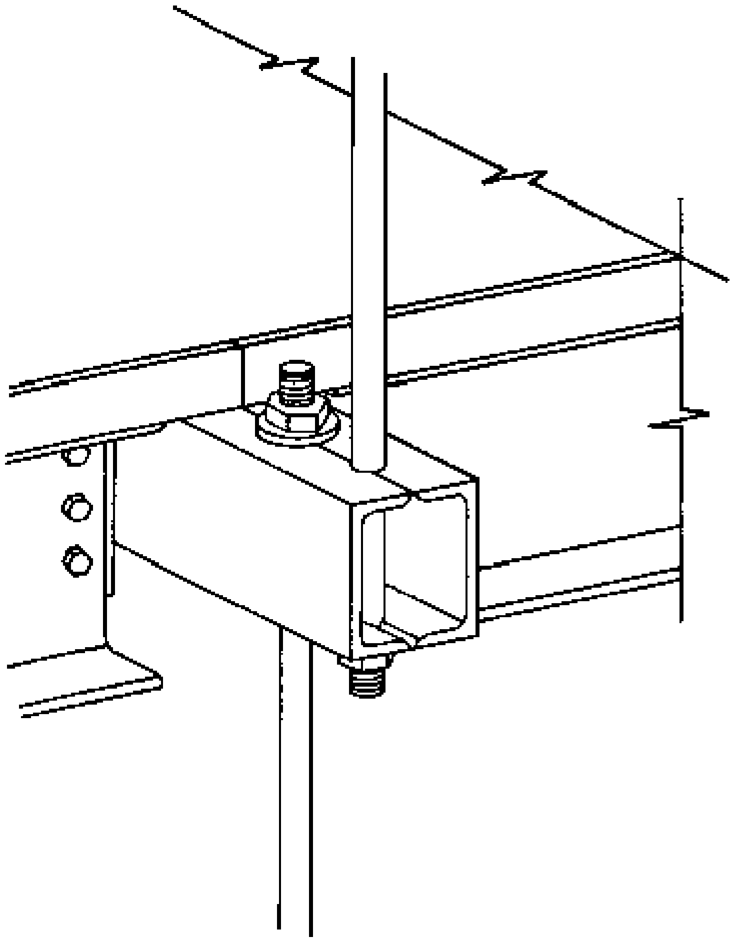
$\Sigma Fy=0 = 200 – 100 - F_{con}$

F_{con}= 100 kN downwards

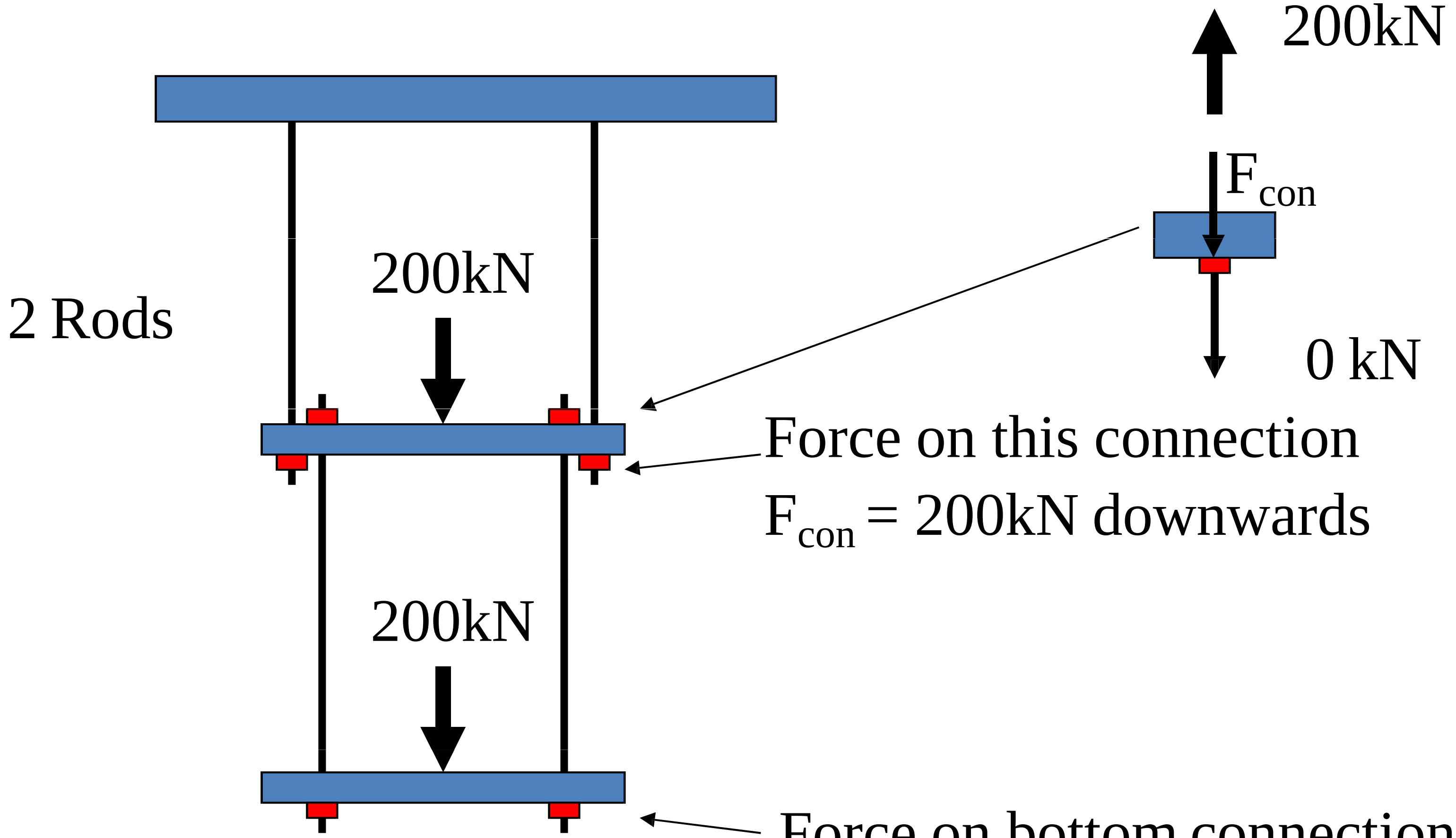
Force on bottom connection

= 100 kN

Actual design



Walkway structure - actual design



Force on this connection
F_{con} = 200kN downwards

Force on bottom connection
= 100 kN

Checking part suitability for applied loads

1. Determine the loads

2. Determine the effect of loads …esp. stresses induced; deflections…

3. Check the material can cope with these stresses, deliver the deflections…

Stress and Strength

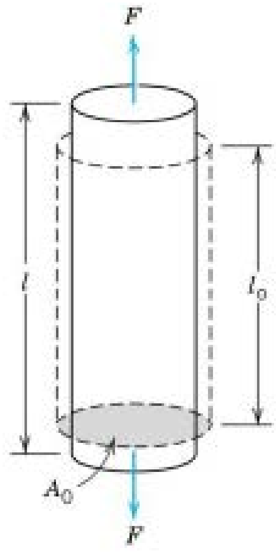
- Hagen, KD Introduction to Engineering Analysis
  - Chapter 4 Section 4.6

- Stress - The force applied divided by the area over which it acts (pressure)
- Strain – **the change of length** divided by the original length

The problem

'The design' will put 'the material' under all sorts (types and magnitudes) of STRESS

How can we find out whether the material will cope?



(a)

$\sigma = F/A$

Can the material cope?

Developing analogies:

We cannot test materials under all possible loading situations we might put on them.

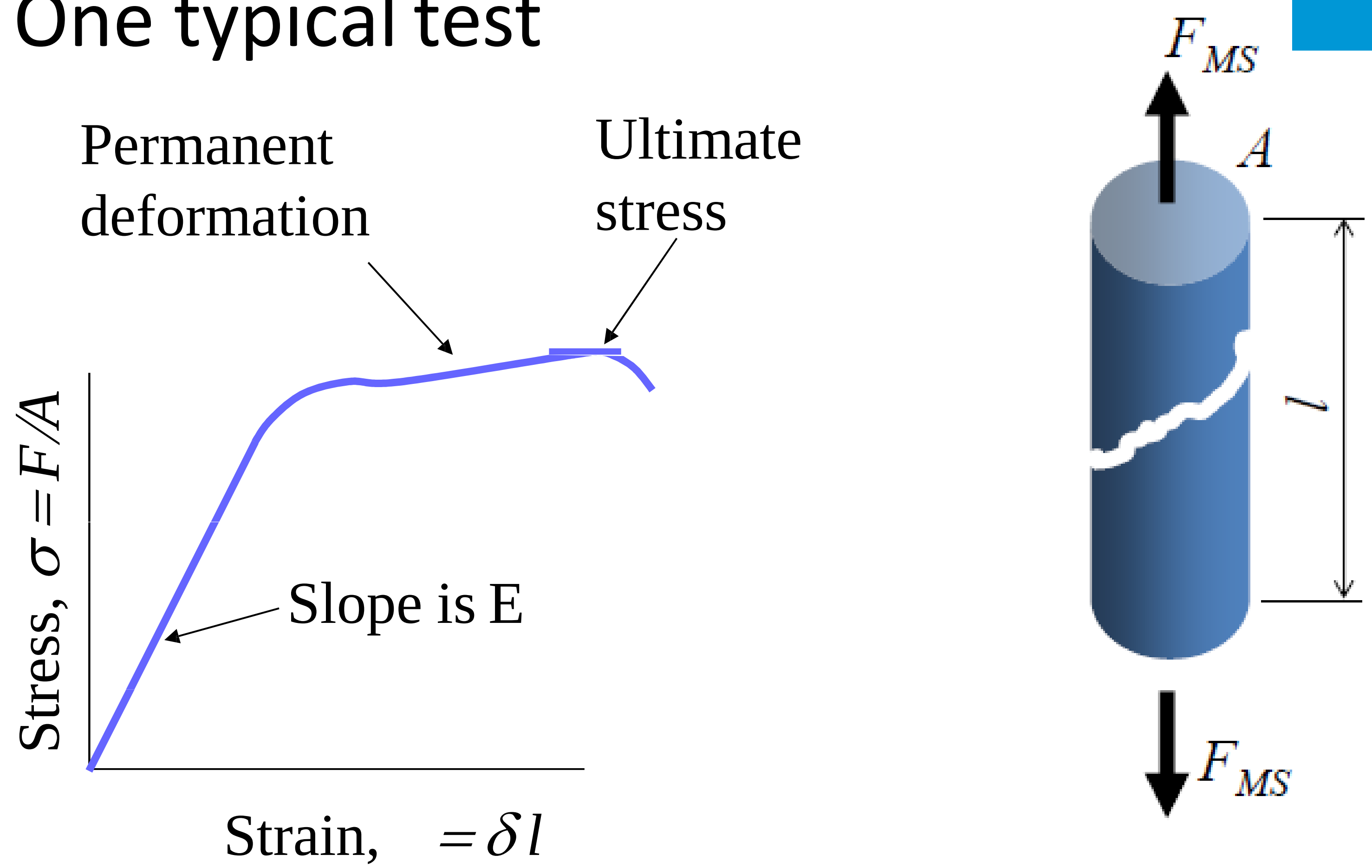
BUT

We can perform some standard tests.

AND

We can try to equate the STRESSES we plan to put on a material to the STRESSES we put on it during a laboratory test.

One typical test



What caused it to fail?...i.e what is the ***'theory' of failure***

For Now, know that:

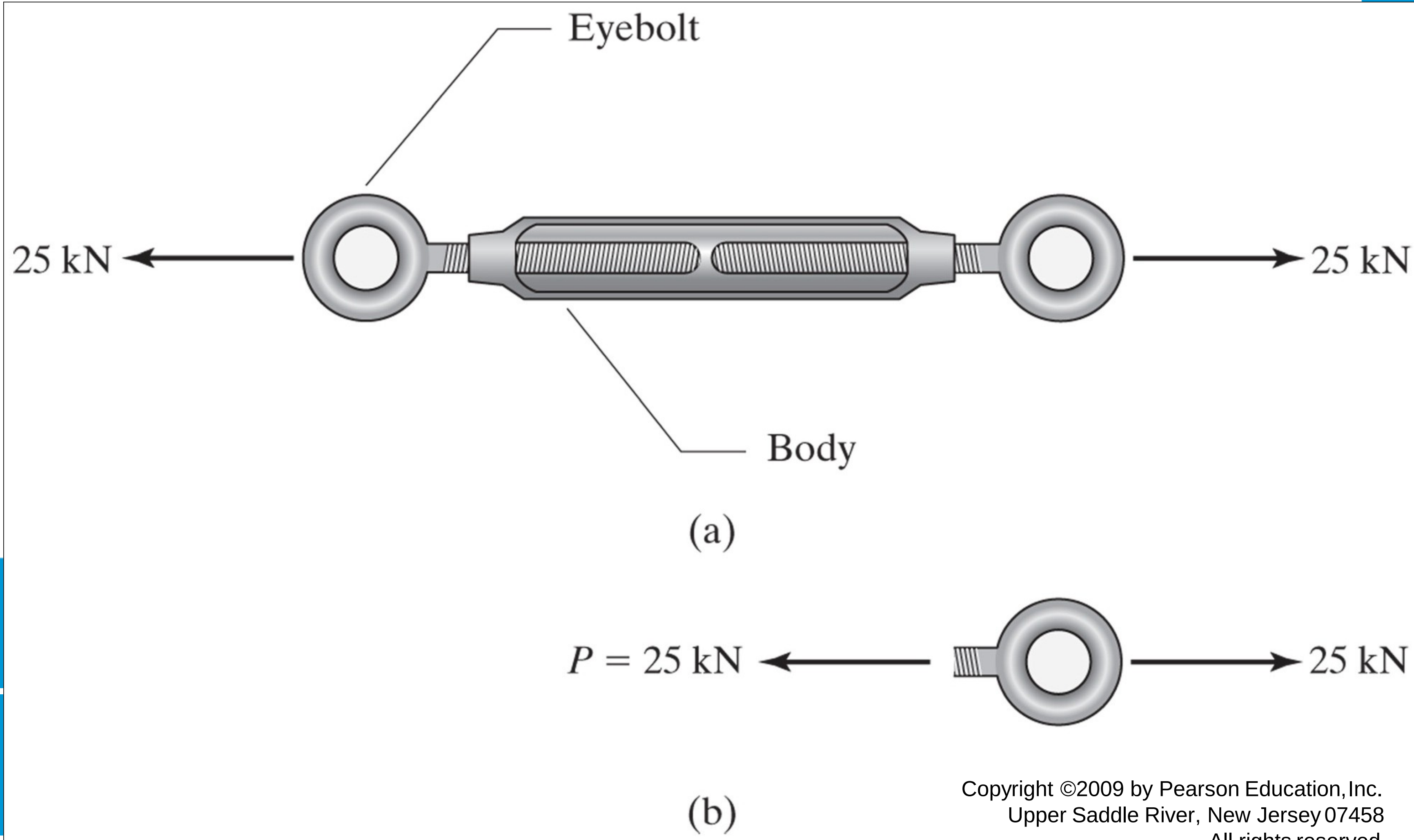
$$\sigma = \frac{F}{A}$$ Direct stress in tension or compression

- Different materials fail through different mechanisms of failure.

3 Step Process for "Solve"

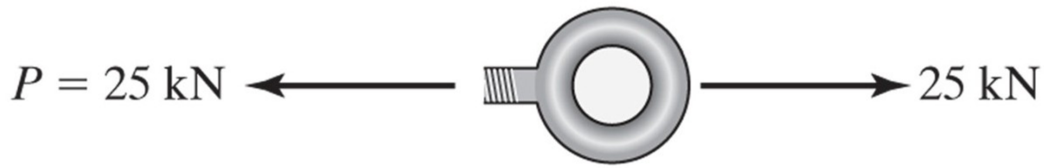
1. Determine external loads (Free-Body Diagrams, Solve for equilibrium using the 6 step method!)
2. Determine the effect of loads …especially stresses induced; strains, deflections…
3. Sort out whether the material can cope with these stresses.
    1. Yield stress of material = σ_y
    2. Factor of safety = FoS

Designing a turnbuckle Hagen p132



Designing a turnbuckle

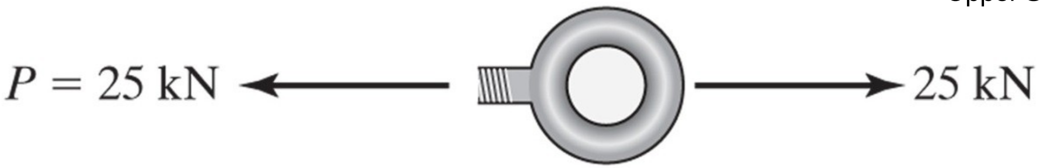
- Min diameter for eye-bolt
- F=25kN
- FoS = 2.0
- Yield stress
  - σ_y = 760MPa



(b)

What is the minimum diameter of the turnbuckle to withstand the load?

Designing a turnbuckle



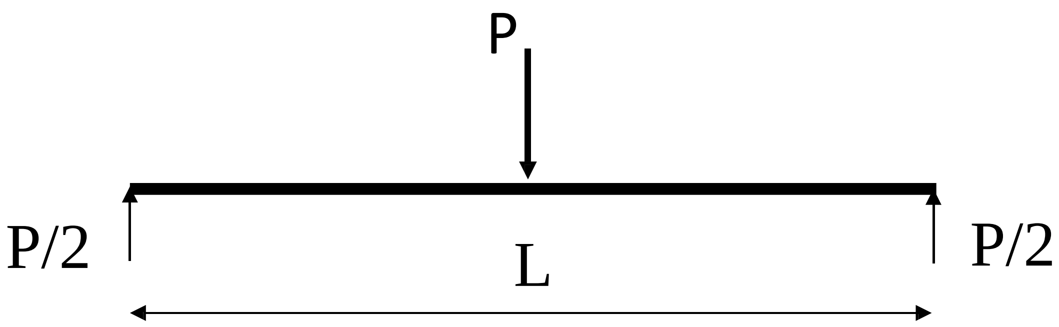
- Allowable stress = Yield / FoS
- σ_{allow} = 760 MPa /2 = 380 MPa
- Convert MPa to N/mm^2
- F=25kN = 25000 N
- Stress = F/A so A = F/Stress
- A = 65.79 mm^2
- But A = π d^2/4 so d = √(4A/π)
- d = 9.15mm For practical design this would be rounded to 10mm

Balsa beam test

- Balsa beam with central point load
- Why does it break there?
- What does that tell us about the internal forces.

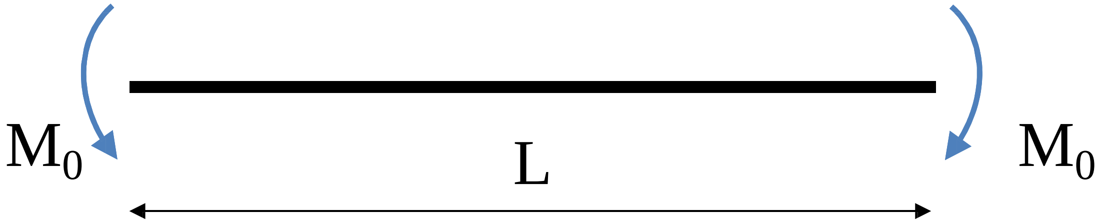
Bending Beam Test...watch beam failure in slow-motion!

https://www.youtube.com/watch?v=6ycbDCnoO8M



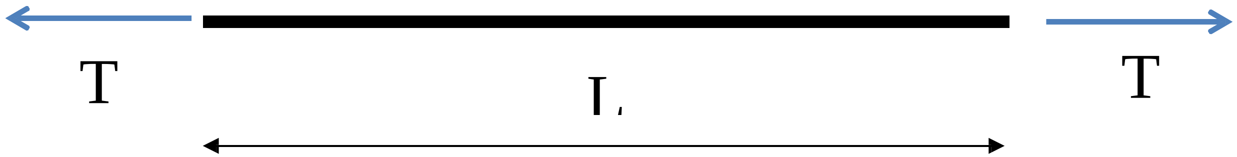
Beam with two end moments

- Where will this one break?

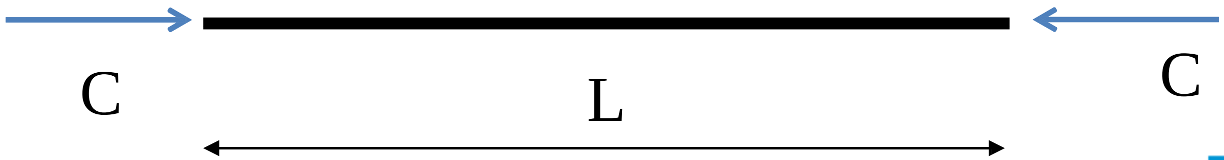


Axial load

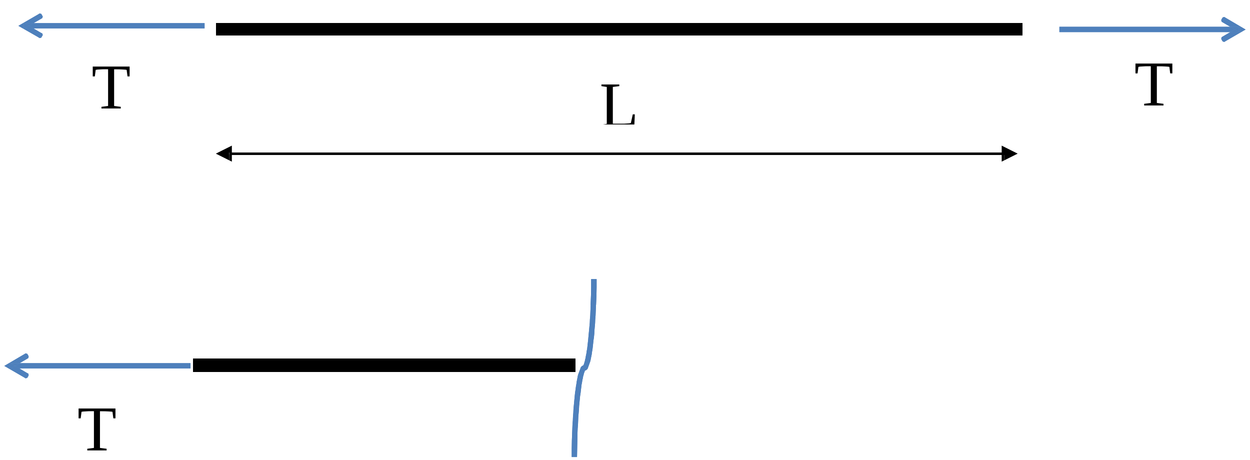
- Where will this one break?



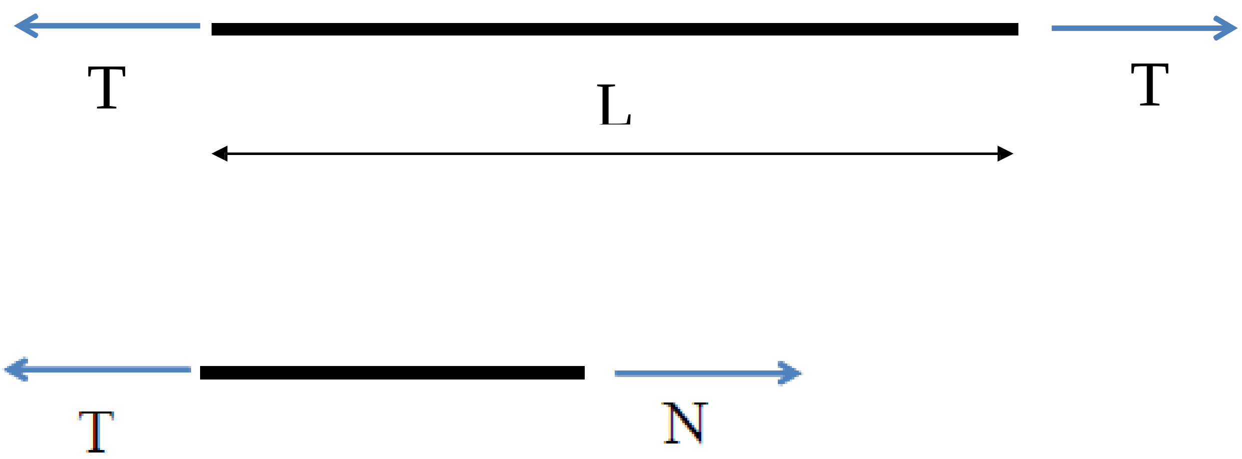
- And this one



FBD's for part of the object



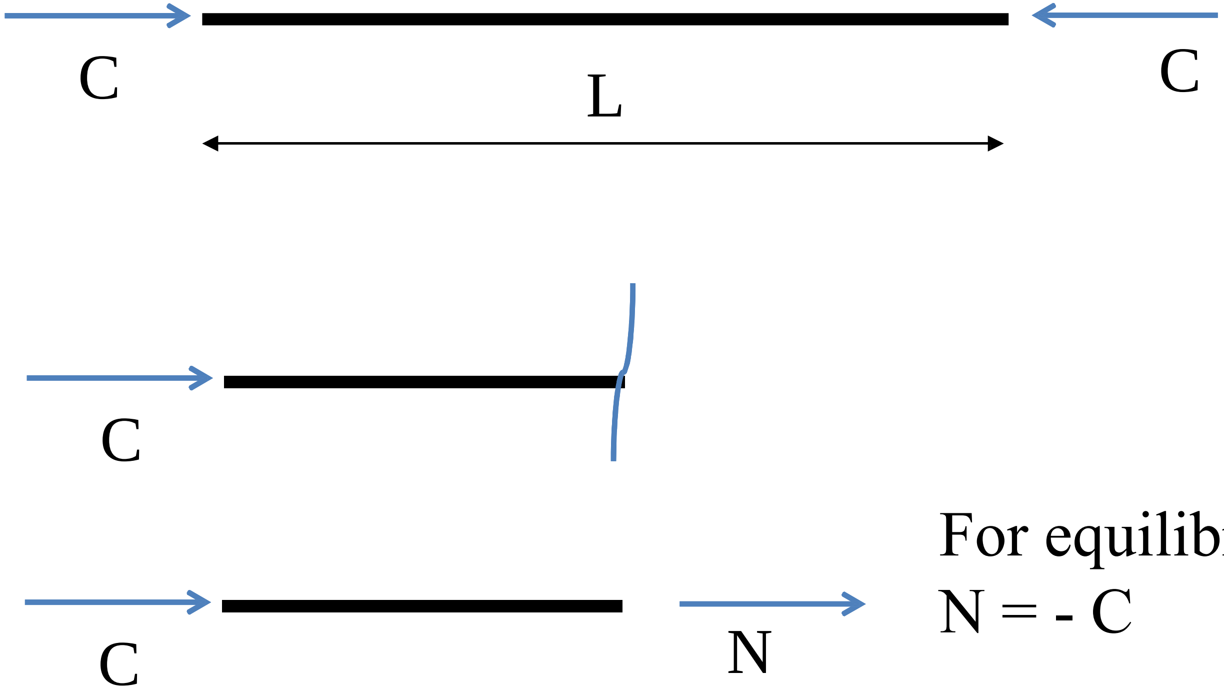
FBD's for part of the object



Tension member

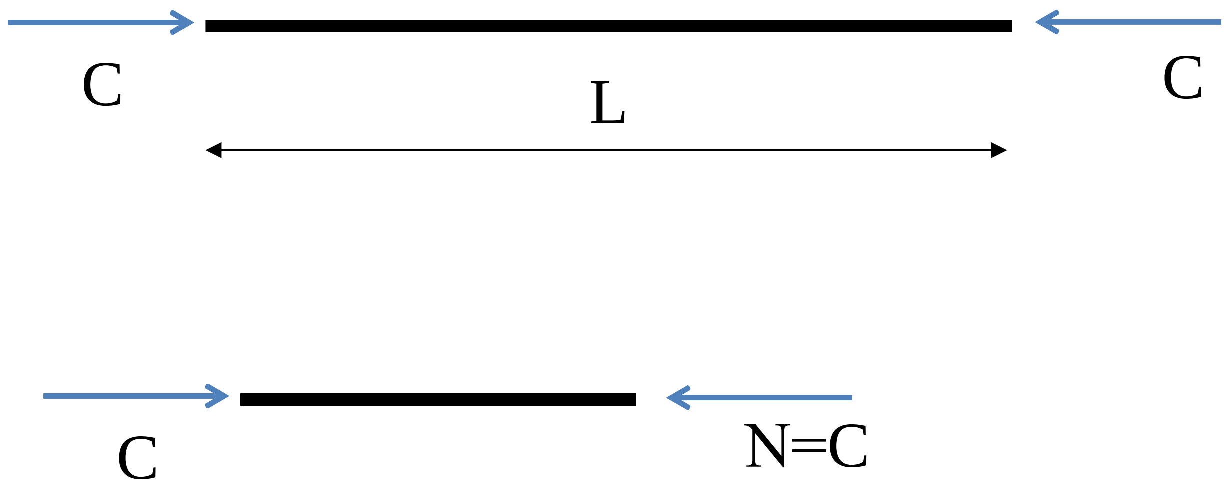
- The FBD of the left hand side shows internal force
- The right hand side is replaced by an axial force, N
- Sum of forces in the direction of the member
- $-T + N = 0$
- $N = T$

Compression member



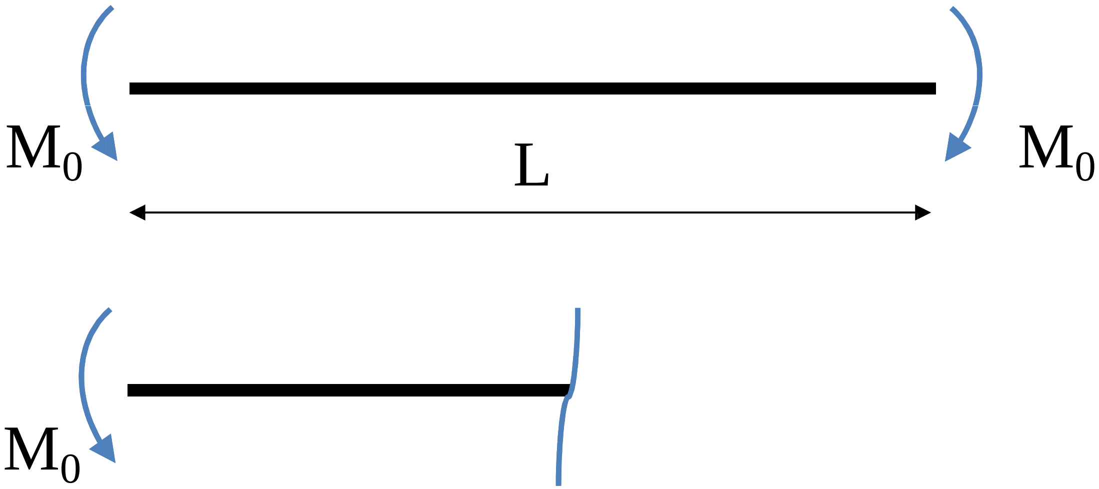
For equilibrium,
$N = -C$

Compression member Final FBD



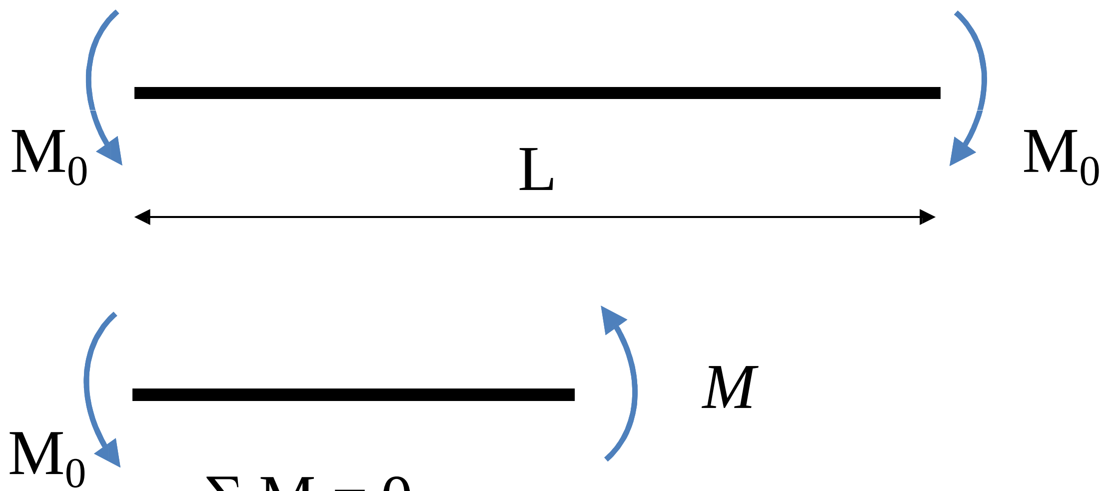
Beam with two end moments

- Cut the tension member near the middle – replace the right hand side by the force it provided



Beam with two end moments

- Cut the tension member near the middle – replace the right hand side by the force it provided



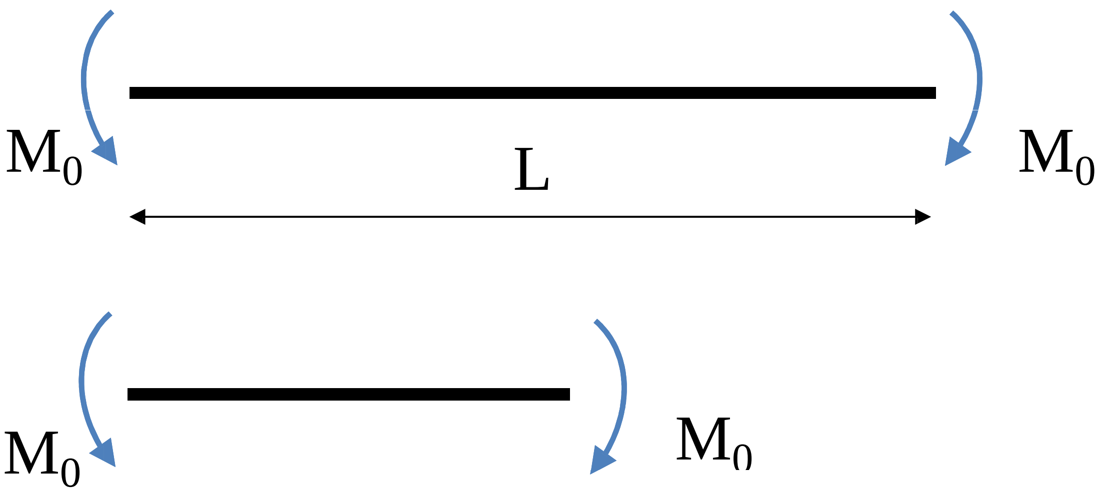
$$\Sigma\ \mathrm{M} = 0$$

$$\mathrm{M}_0 + M = 0$$

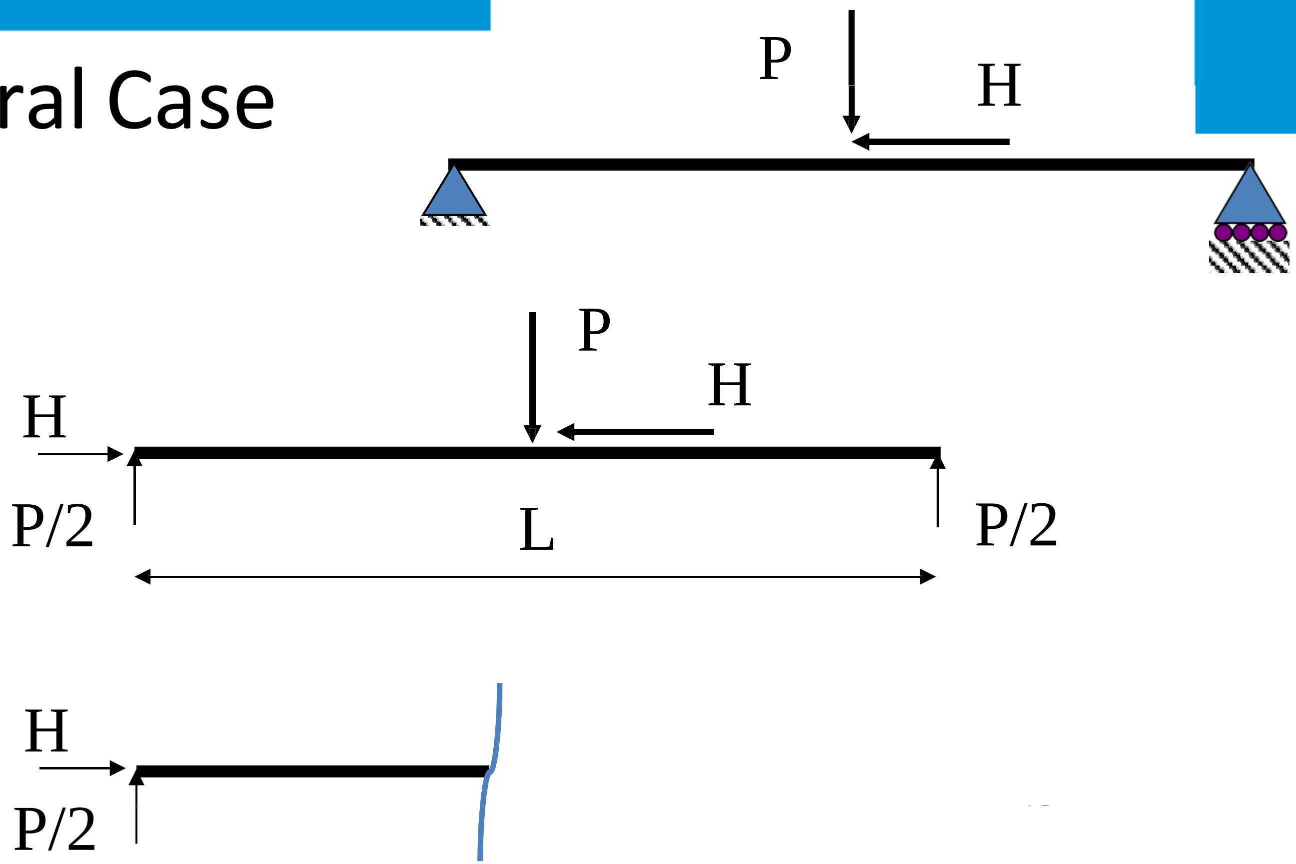
$$M = -\mathrm{M}_0$$

Beam with two end moments

- Final FBD

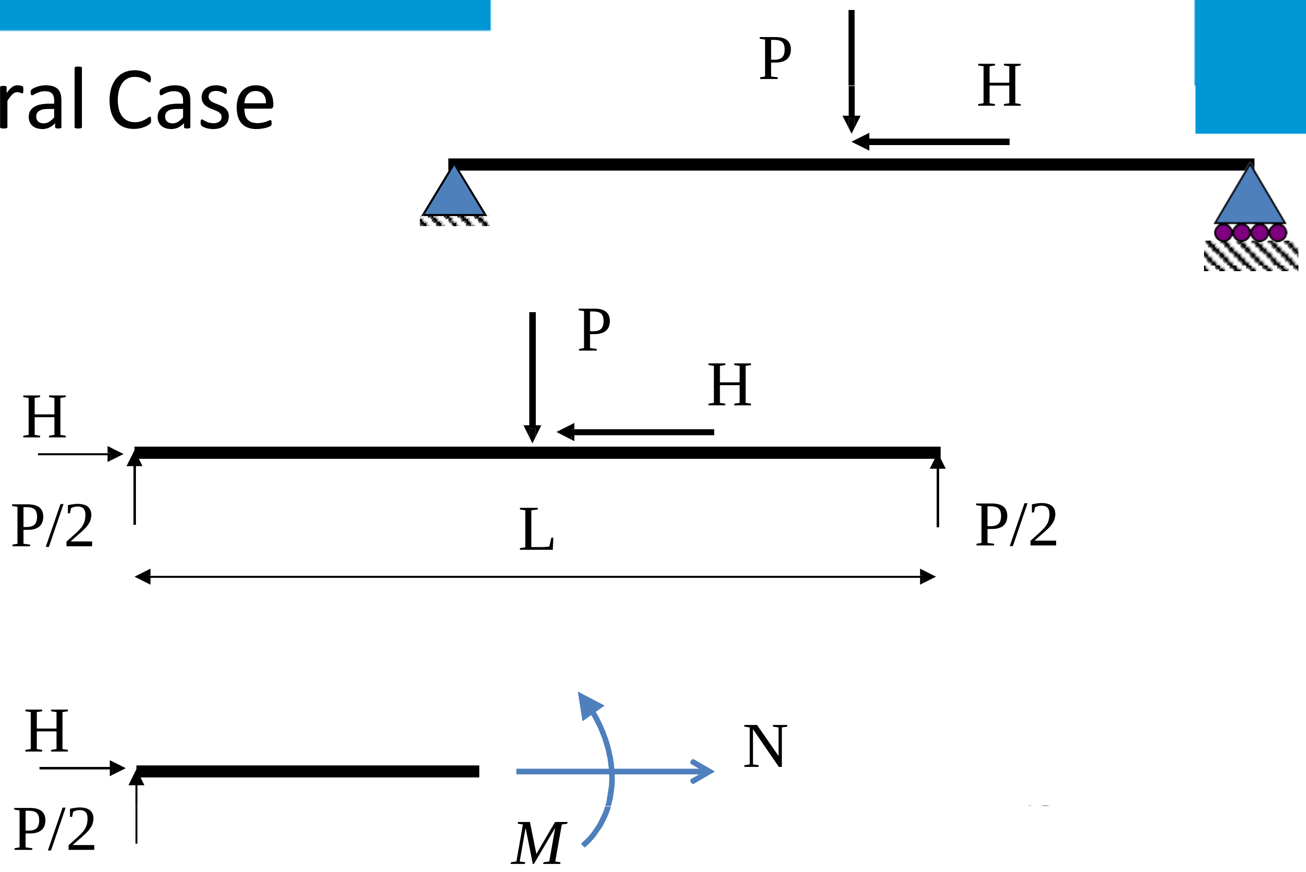


General Case



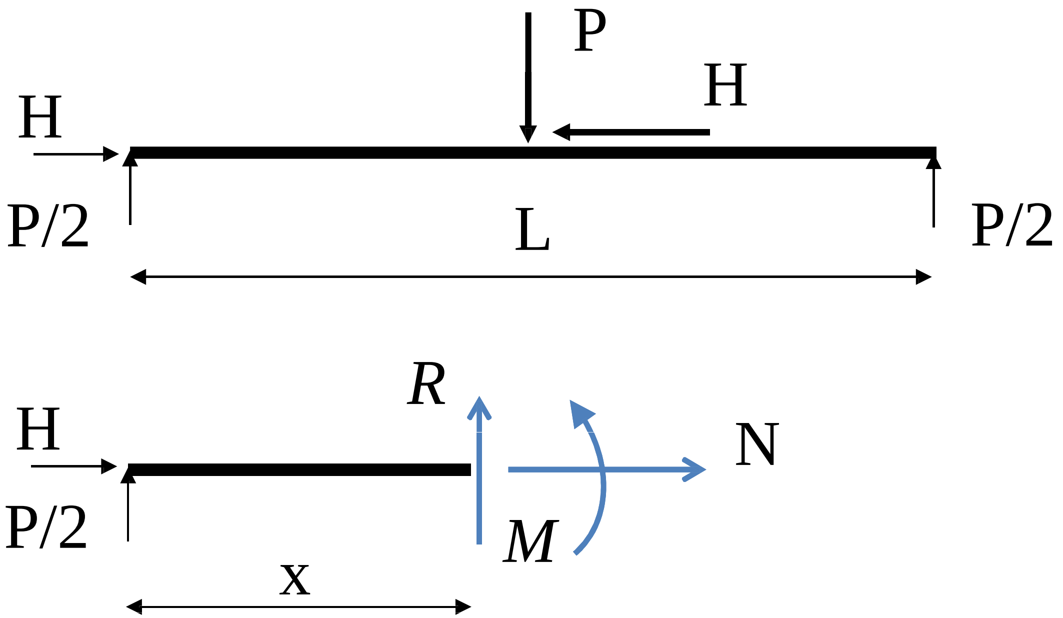
- Cut the beam just to the left of the central loads

General Case



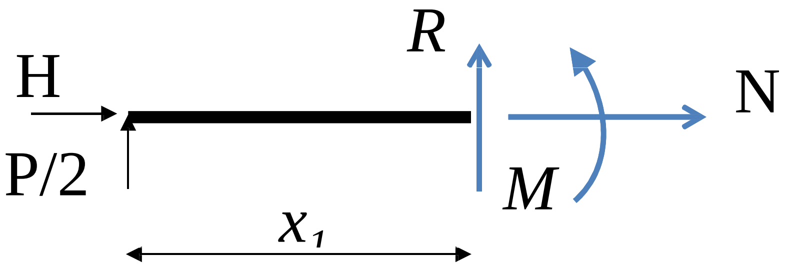
- Something is missing to give equilibrium

General Case



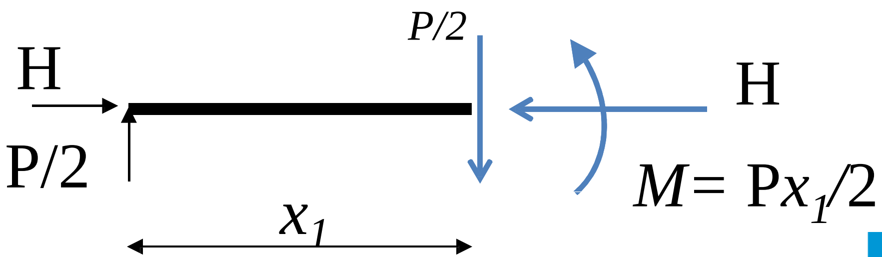
- When we cut a solid beam element we replace the right hand part by THREE actions
- Axial Force, Moment and a transverse force R

Solving FBD

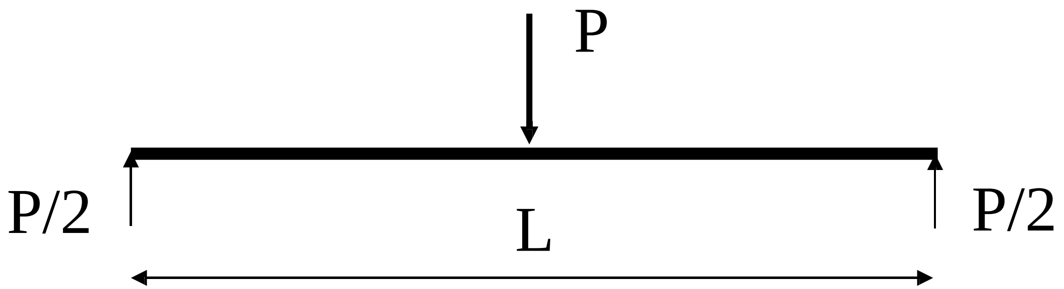


- $\Sigma\ F_x = 0$: H + N = 0 implies N = -H
- $\Sigma\ F_y = 0$: P/2 + R = 0 implies R = -P/2
- $\Sigma\ M_{x1} = 0$: $-(P/2).x_1 + M = 0$ implies $M = Px_1/2$

- The N and R arrows are in the incorrect sense
- M is correct
- Final FBD

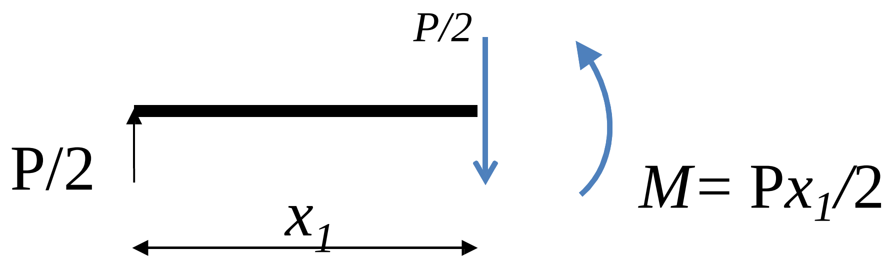


Our Balsa Beam

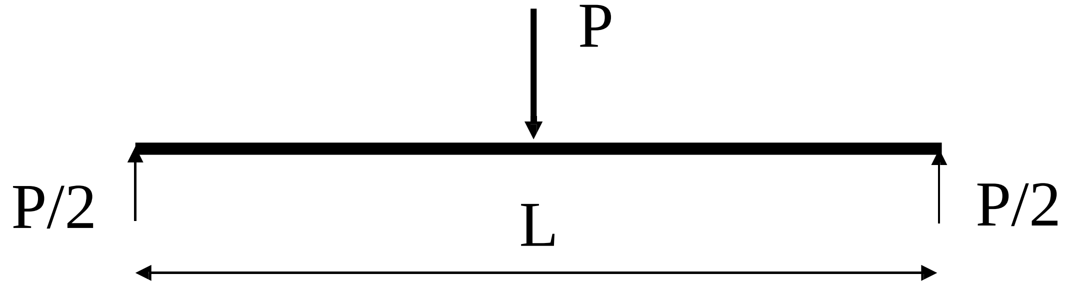


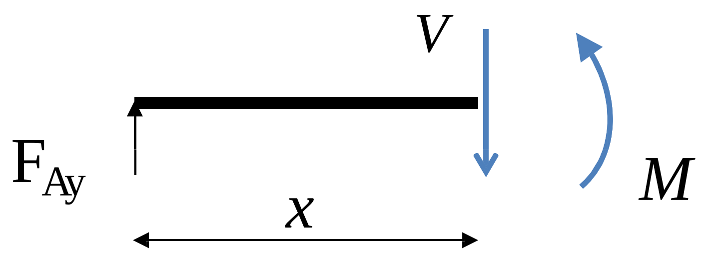
- $\Sigma\, F_x = 0$: $H = N = 0$
- $\Sigma\, F_y = 0$: $P/2 + R = 0$ implies $R = -P/2$
- $\Sigma\, M_{x1} = 0$: $-(P/2).x_1 + M = 0$ implies $M = Px_1/2$

- Final FBD
- At $x_1 = L/2$
- $M = PL/4$



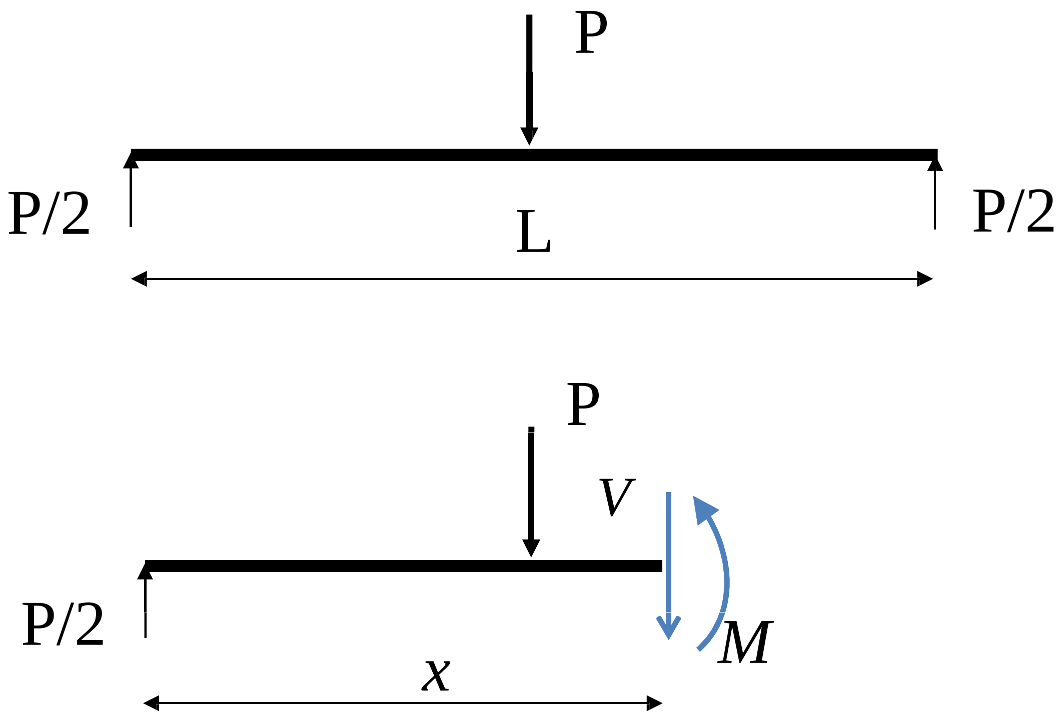
General FBD for Beam with point load



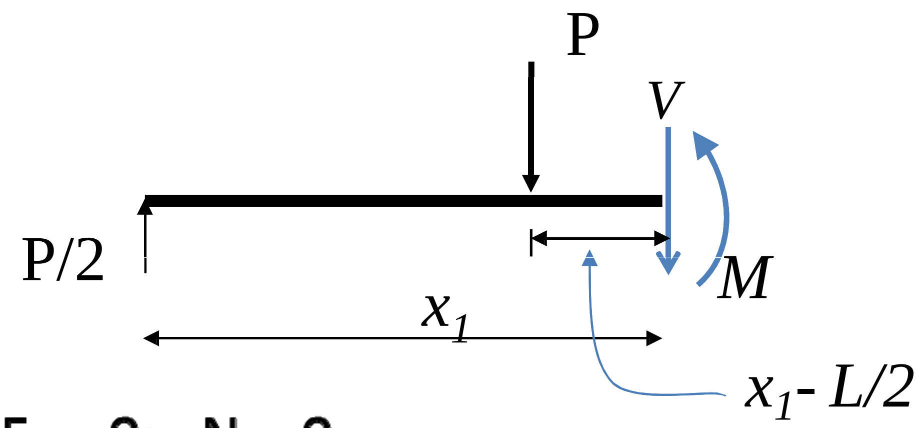


We will come across this diagram a lot

What if we made out cut to right of P?



What if we made out cut to right of P?



- $\Sigma F_x = 0$: $N = 0$
- $\Sigma F_y = 0$: $P/2 – P - V = 0$ implies $V = -P/2$ (direction wrong)

- $\Sigma M_{x1} = 0$: $-(P/2).x_1 + P(x_1 - L/2) + M = 0$

  implies $M = PL/2 - Px_1/2$
- These equations are true $\forall x \in [L/2 < x \leq L]$

Bending moment diagram calculators

http://beamguru.com/online/beam-calculator/
https://bendingmomentdiagram.com/free-calculator/

NOT allowed in exams/quizzes.