Kinetic Energy and Work

7.2 What is energy?

Energy is a scalar quantity associated with the state (or condition) of one or more objects.

Some characteristics:

1. Energy can be transformed from one type to another and transferred from one object to another,
2. The total amount of energy is always the same (energy is ***conserved)***.

7.3 Kinetic energy

Kinetic energy *K* is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy.

For an object of mass *m* whose speed *v* is well below the speed of light,

$$K = \tfrac{1}{2}mv^2 \qquad \text{(kinetic energy).}$$

The SI unit of kinetic energy (and every other type of energy) is the **joule (J),**

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kgm}^2/\text{s}^2.$$

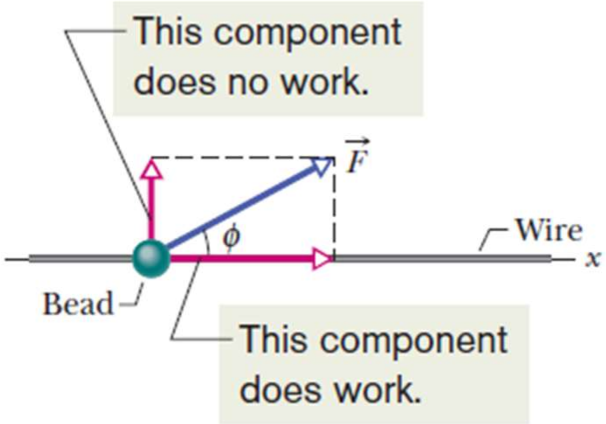
7.4: Work

Work W is energy transferred to or from an object by means of a force acting on the object.

Energy transferred to the object is positive work, and energy transferred from the object is negative work.

7.5: Work and kinetic energy

To calculate the work a force ***F*** does on an object as the object moves through some displacement ***d***, we use only the force component along the object's displacement. The force component perpendicular to the displacement direction does zero work.



A constant force directed at angle ϕ to the displacement (in the x-direction) of a bead does work on the bead. The only component of force taken into account here is the x-component.

For a constant force ***F***, the work done W is:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \phi$$

When two or more forces act on an object, the net work done on the object is the sum of the works done by the individual forces.

Work-kinetic energy theorem

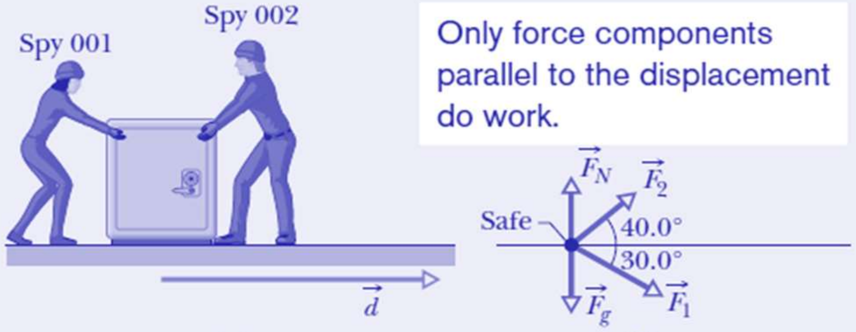
The theorem says that the change in kinetic energy of a particle is the net work done on the particle.

$$\begin{pmatrix}\text{change in the kinetic} \\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on} \\ \text{the particle}\end{pmatrix}$$

It holds for both positive and negative work: If the net work done on a particle is positive, then the particle's kinetic energy increases by the amount of the work, and the converse is also true.

Sample problem, industrial spies

Figure 7-4*a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



(*a*) (*b*)

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4*b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos\phi_1 = (12.0\ \text{N})(8.50\ \text{m})(\cos 30.0°) = 88.33\ \text{J},$$

and the work done by $\vec{F}_2$ is

$$W_2 = F_2 d \cos\phi_2 = (10.0\ \text{N})(8.50\ \text{m})(\cos 40.0°) = 65.11\ \text{J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33\ \text{J} + 65.11\ \text{J} = 153.4\ \text{J} \approx 153\ \text{J}. \qquad \text{(Answer)}$$

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd\cos 90° = mgd(0) = 0 \qquad \text{(Answer)}$$

and

$$W_N = F_N d\cos 90° = F_N d(0) = 0. \qquad \text{(Answer)}$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

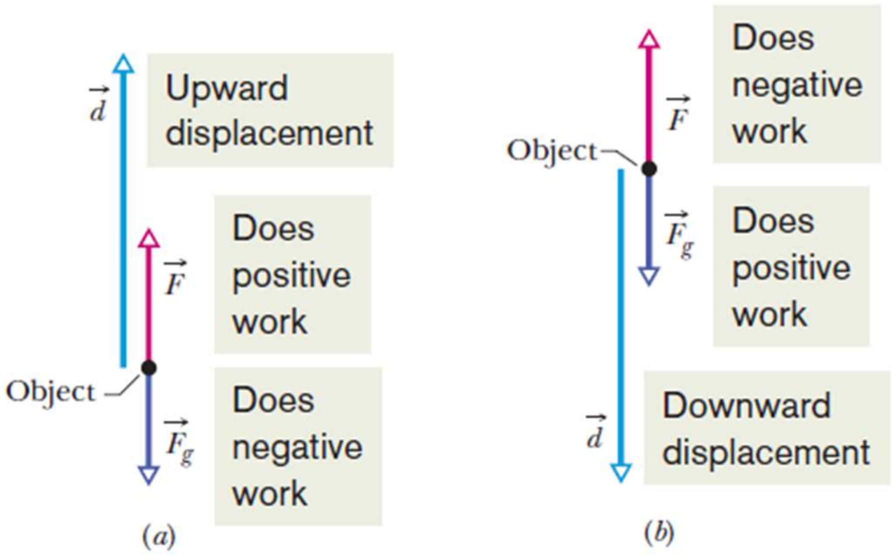
$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\ \text{J})}{225\ \text{kg}}} = 1.17\ \text{m/s}. \qquad \text{(Answer)}$$

7.6: Work done by gravitational force

$$W_g = mgd\cos\phi \qquad \text{(work done by gravitational force).}$$



(*a*) *An applied force lifts* an object. The object's displacement makes an angle ϕ=180° with the gravitational force on the object. The applied force does positive work on the object.

(*b*) *An applied force* lowers an object. The displacement of the object makes an angle with the gravitational force .The applied force does negative work on the object.

7.7: Work done by a spring force

The *spring force is given by* $F_s = -kx$

The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant *k is called the **spring constant (or force constant) and is a measure*** of the stiffness of the spring.

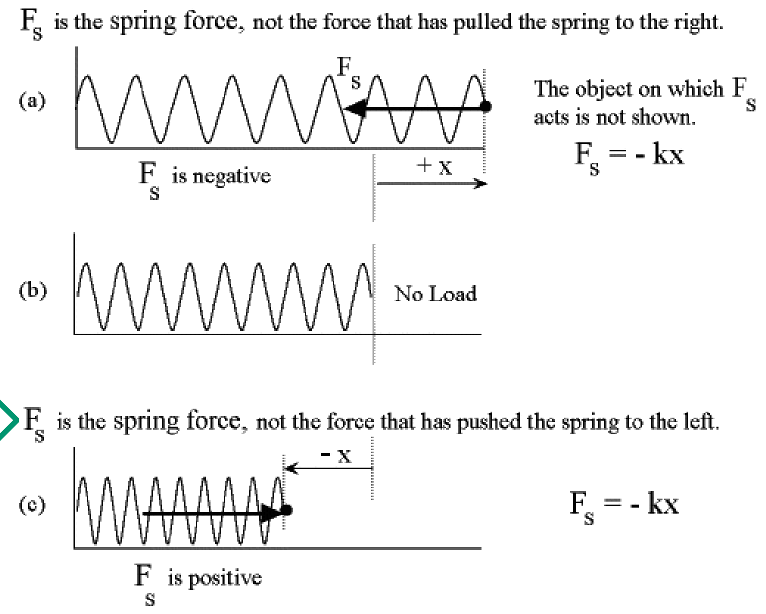
The net work W_s *done by a spring, when it has a distortion from* x_i *to* x_f*, is:*

$$W_s = \int_{x_i}^{x_f} -F_x \, dx.$$

$$W_s = \int_{x_i}^{x_f} -kx \, dx = -k \int_{x_i}^{x_f} x \, dx$$

$$= (-\tfrac{1}{2}k)[x^2]_{x_i}^{x_f} = (-\tfrac{1}{2}k)(x_f^2 - x_i^2).$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).}$$



Sample problem: work done by spring

In Fig. 7-10, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

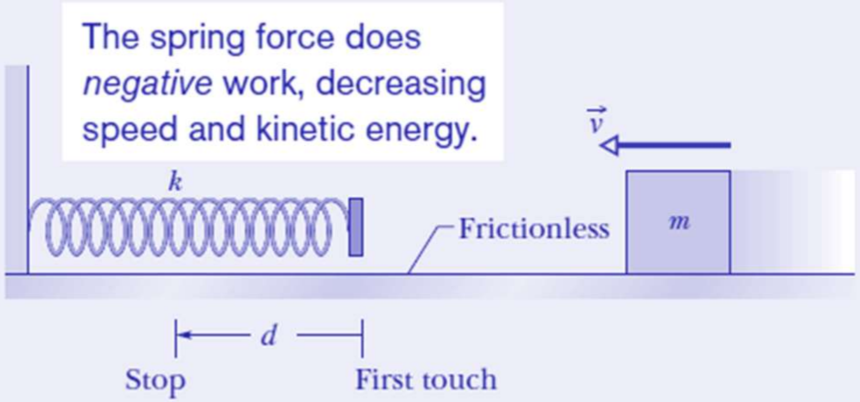


Fig. 7-10 A canister of mass m moves at velocity $\vec{v}$ toward a spring that has spring constant k.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

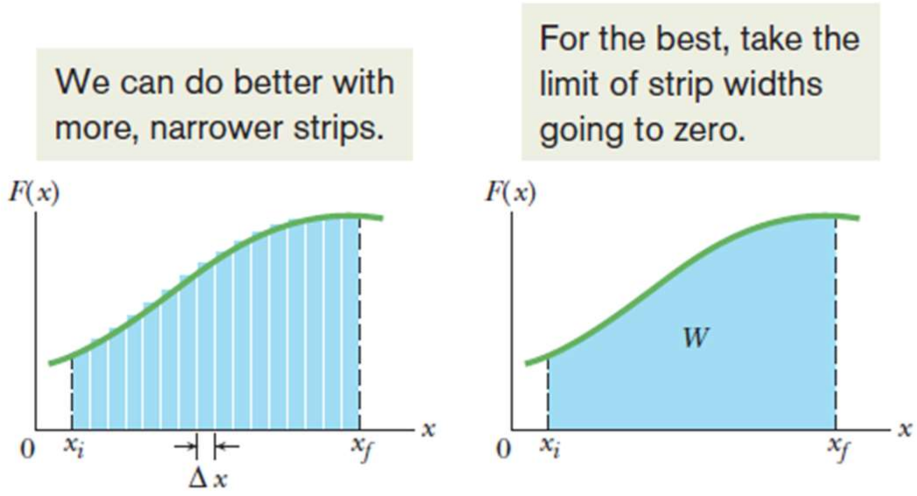
Simplifying, solving for d, and substituting known data then give us

$$d = v\sqrt{\frac{m}{k}} = (0.50 \text{ m/s})\sqrt{\frac{0.40 \text{ kg}}{750 \text{ N/m}}}$$
$$= 1.2 \times 10^{-2} \text{ m} = 1.2 \text{ cm}. \qquad \text{(Answer)}$$

7.8: Work done by a general variable force

A. One-dimensional force, calculus analysis:

We can make the approximation better by reducing the strip width Δx and using more strips (Fig. c). In the limit, the strip width approaches zero, the number of strips then becomes infinitely large and we have, as an exact result,



(c) (d)

$$\Delta W = \lim_{\Delta x \to 0} \sum F_{j,avg} \Delta x = \int_{x_i}^{x_f} F(x)\,dx$$

7.8: Work kinetic energy theorem with a variable force

A particle of mass *m* is moving along an x axis and acted on by a net force *F(x)* that is directed along that axis.

The work done on the particle by this force as the particle moves from position x_i to position x_f is :

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx,$$

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

7.8: Work done by a general variable force

B. Three dimensional force:

If $$\vec{F} = F_x\hat{\mathrm{i}} + F_y\hat{\mathrm{j}} + F_z\hat{\mathrm{k}},$$

where F_x is the x-components of $\boldsymbol{F}$ and so on,

and $$d\vec{r} = dx\hat{\mathrm{i}} + dy\hat{\mathrm{j}} + dz\hat{\mathrm{k}}.$$

where dx is the x-component of the displacement vector $\boldsymbol{dr}$ and so on,

then $$dW = \vec{F}\cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz.$$

Finally, $$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz.$$

7.9: Power

The time rate at which work is done by a force is said to be the power due to the force. If a force does an amount of work W in an amount of time t, the average power due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t} \qquad \text{(average power).}$$

The instantaneous power P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \qquad \text{(instantaneous power).}$$

The SI unit of power is the joule per second, or Watt (W).

In the British system, the unit of power is the footpound per second. Often the horsepower is used.

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$$

$$1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}.$$

Sample problem: accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-8*a*).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

Calculation: From Fig. 7-8*b*, we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. Then, from Eq. 7-12, we find

$$W_g = mgd\cos 0° = (500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(1)$$
$$= 5.88 \times 10^4\text{ J} \approx 59\text{ kJ}. \qquad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

Calculations: We get

$$T - F_g = ma.$$

$$W_T = Td\cos\phi = m(a + g)d\cos\phi.$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5} + g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$
$$= -4.70 \times 10^4\text{ J} \approx -47\text{ kJ}. \qquad \text{(Answer)}$$

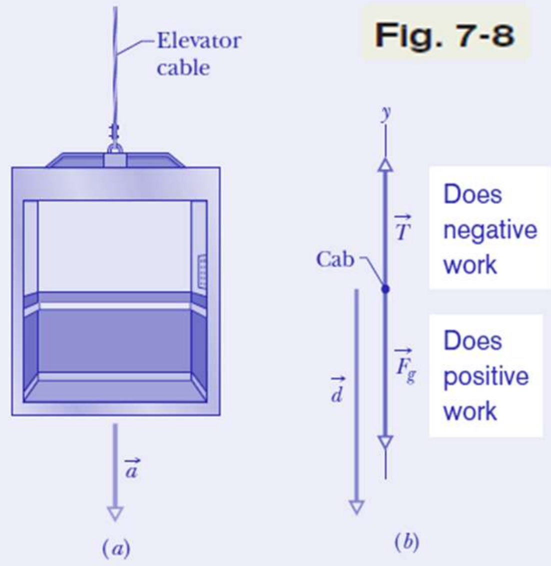


Fig. 7-8

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4\text{ J} - 4.70 \times 10^4\text{ J}$$
$$= 1.18 \times 10^4\text{ J} \approx 12\text{ kJ}. \qquad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

Calculation: From Eq. 7-1, we can write the kinetic energy at the start of the fall as $K_i = \frac{1}{2}mv_i^2$. We can then write Eq. 7-11 as

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500\text{ kg})(4.0\text{ m/s})^2 + 1.18 \times 10^4\text{ J}$$
$$= 1.58 \times 10^4\text{ J} \approx 16\text{ kJ}. \qquad \text{(Answer)}$$