ENGG102 Fundamentals of Engineering Mechanics

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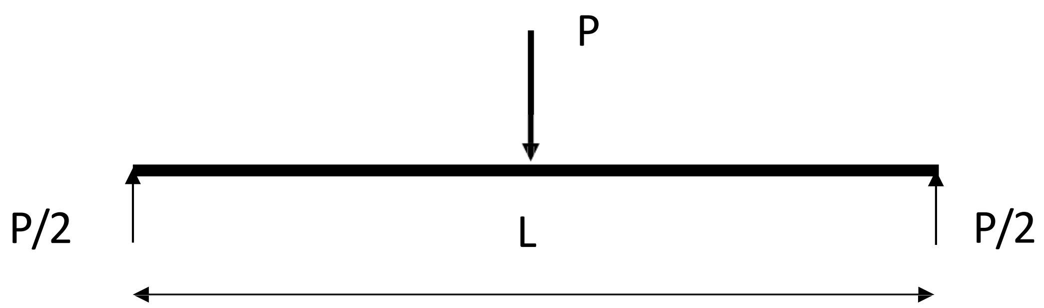
Outline:

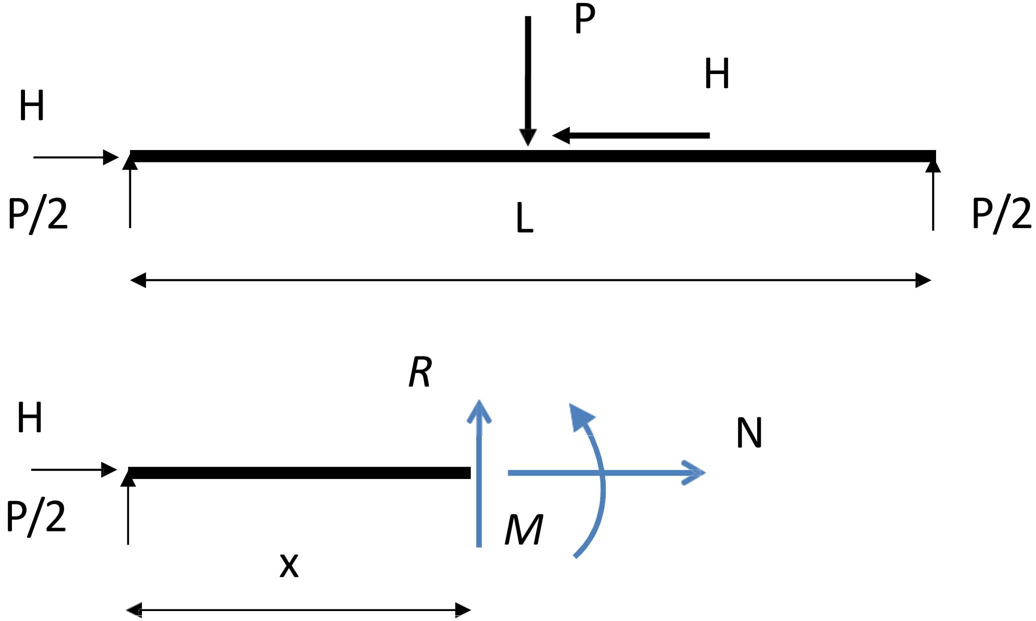
1. Recap on last lecture
2. Revision of Internal Forces in Structural Members
3. Shear and Bending Moment Equations and Diagrams
4. Relationship between Shear and Bending Moment

Recap on Lecture 3

- Making FBD of part of the beam
- Allows us to determine INTERNAL forces and moments

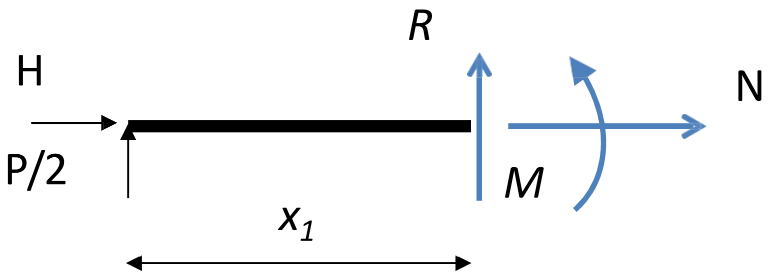


General Case



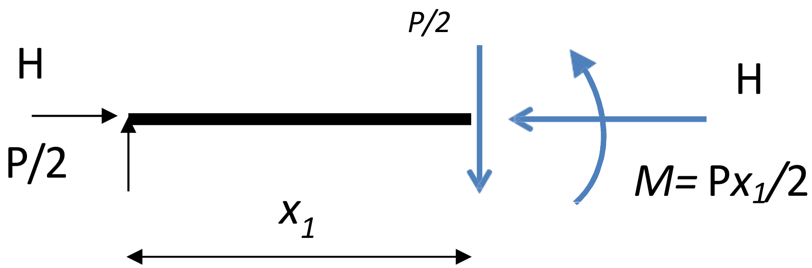
- When we cut a solid beam element we replace the right hand part by THREE actions
- Axial Force, Moment and a transverse force R

Solving FBD

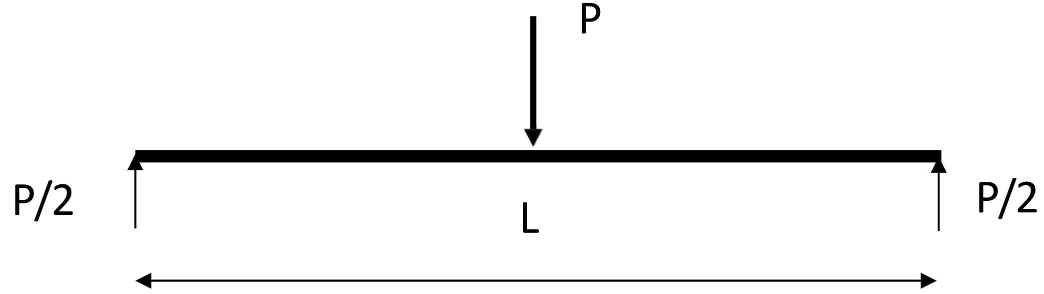


- $\Sigma\ F_x = 0$: H + N = 0 implies N = -H
- $\Sigma\ F_y = 0$: P/2 + R = 0 implies R = -P/2
- $\Sigma\ M_{x1} = 0$: $-(P/2).x_1 + M = 0$ implies $M = Px_1/2$

- The N and R arrows are in the incorrect sense
- M is correct
- Final FBD

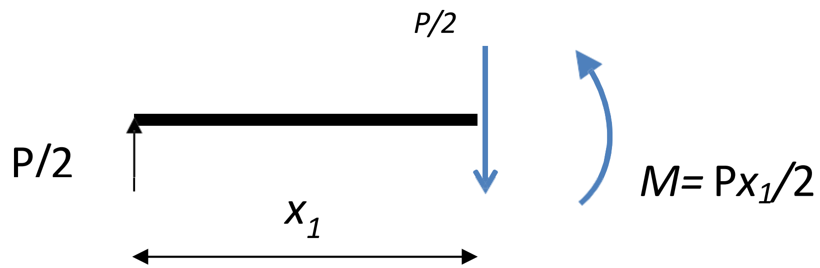


Our Balsa Beam

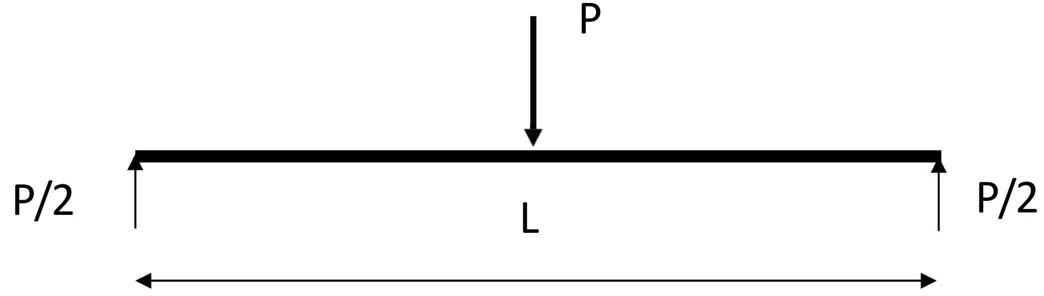


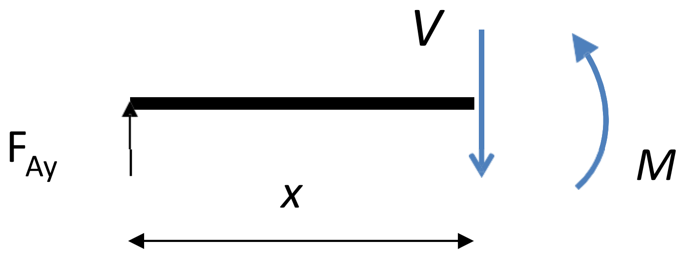
- $\Sigma\ F_x = 0$: $H = N = 0$
- $\Sigma\ F_y = 0$: $P/2 + R = 0$ implies $R = -P/2$
- $\Sigma\ M_{x1} = 0$: $-(P/2).x_1 + M = 0$ implies $M = Px_1/2$

- Final FBD
- At $x_1 = L/2$
- $M = PL/4$



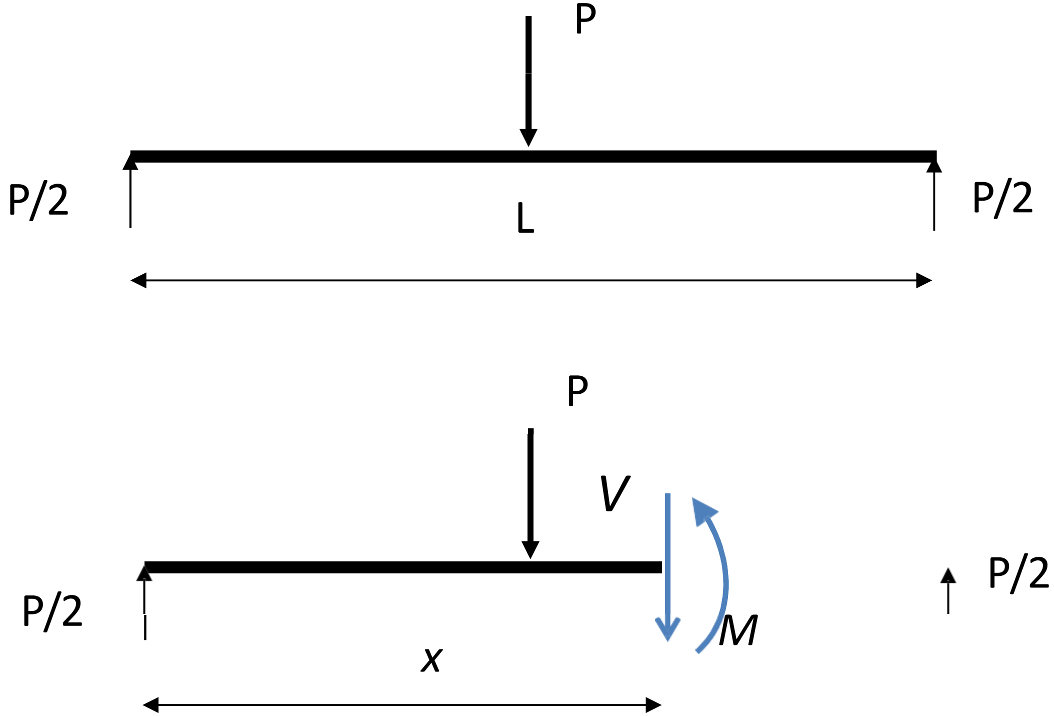
General FBD for Beam with point load



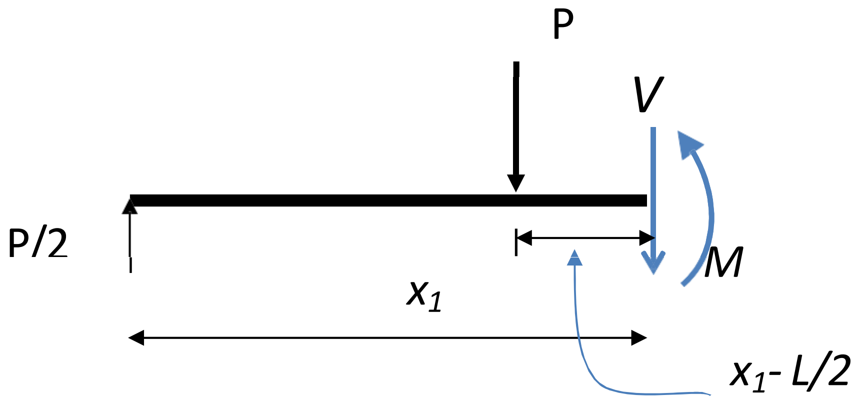


We have a special name for the internal moment: We call it the BENDING MOMENT because it makes the beam bend

What if we made out cut to right of P?



What if we made out cut to right of P?



- $\Sigma F_x = 0$: N = 0
- $\Sigma F_y = 0$: P/2 – P - V = 0 implies V = -P/2 (direction wrong)

- $\Sigma M_{x1} = 0$: $-(P/2).x_1 + P(x_1 - L/2) + M = 0$

implies $M = PL/2 - Px_1/2$ Note When $x_1 = L/2$ M = PL/2-PL/4 = PL/4

- These equations are true $\forall x \in [L/2 < x \leq L]$

Shear and Bending Moment Equations and Diagrams

Bending Moment Diagram is **<u>important</u>** because:

1. Failure occurs where the bending moment is maximum
2. Deflection can be predicted by means of bending moment diagrams

Beam failure from bending under snow overload.

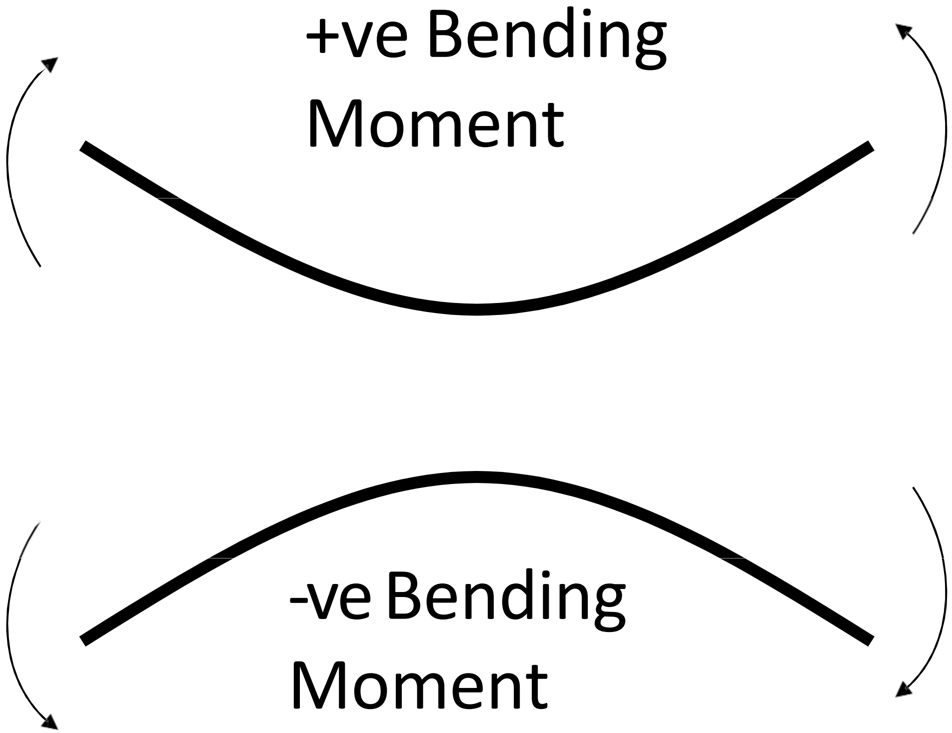
www.structurearchives.org

Beam failure – (in middle again)

www.frpdistributors.com

Shear and Bending Moment Equations and Diagrams

Sign convention for **Bending Moment Diagram**:



Sagging moment

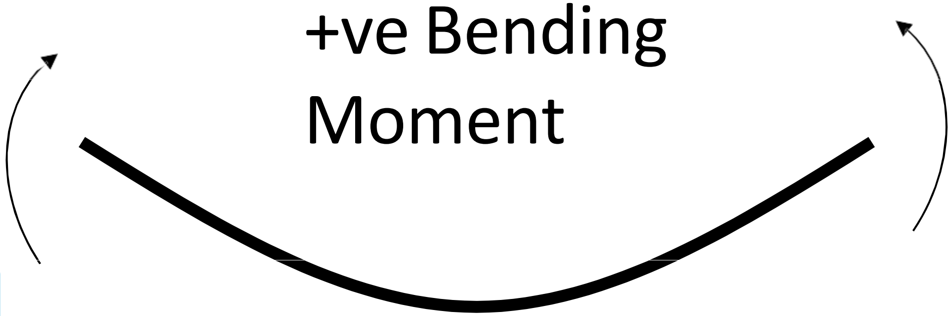
BMD is always a pair of equal but opposite moments

Hogging moment

Shear and Bending Moment Equations and Diagrams

You should not confuse the sign convention for Bending Moment Diagram with the sign convention in equilibrium equations.

+ve Bending Moment



Sagging moment

In equilibrium equations, the left end moment is normally considered negative as it has a clockwise direction.

Shear and Bending Moment Equations and Diagrams

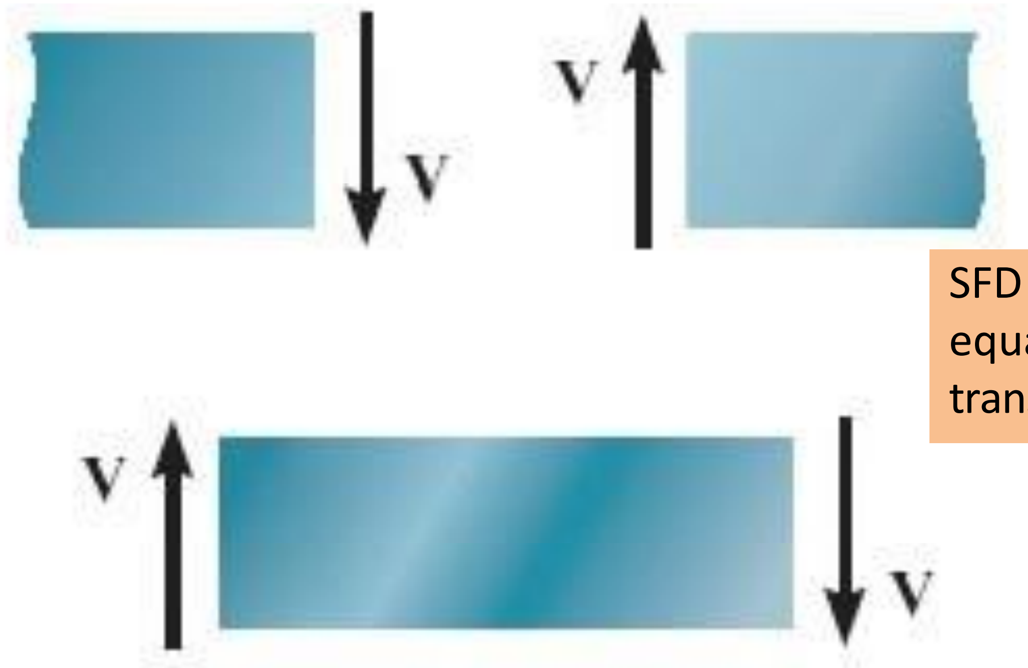
In the textbook:

Positive moment

Sagging moment

7.2 Shear and Bending Moment Equations and Diagrams

Sign convention for **Shear Force Diagram**:



Positive shear

SFD is always a pair of equal but opposite transverse forces

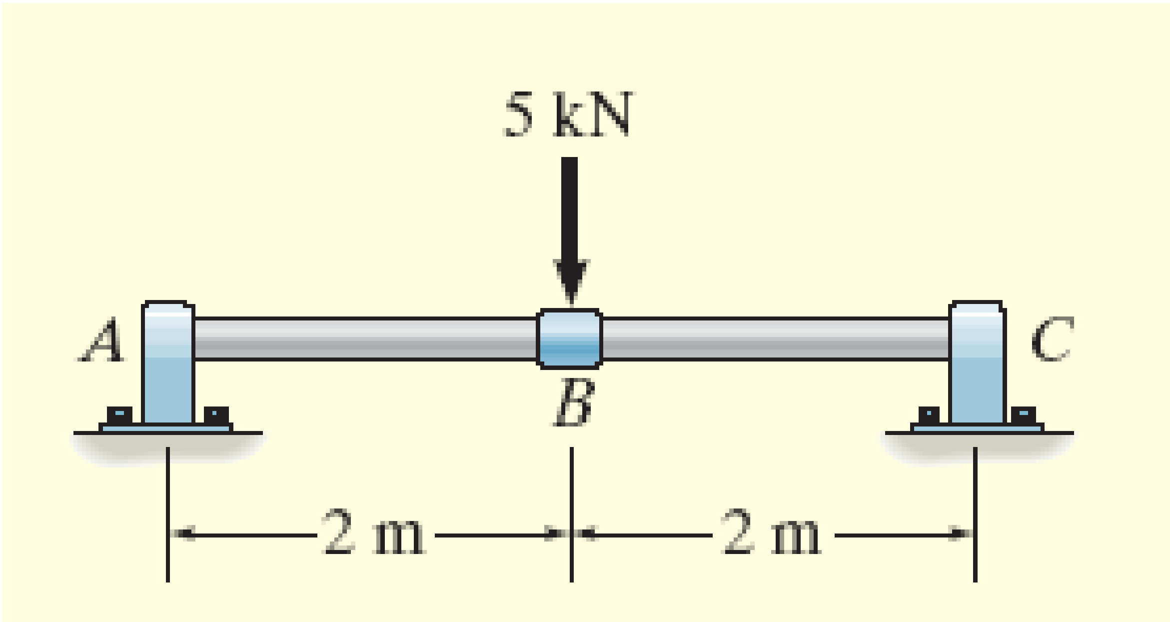
Shear and Bending Moment Equations and Diagrams

Positive shear

In equilibrium equations, the right end shear force is normally considered negative as it has a downward direction.

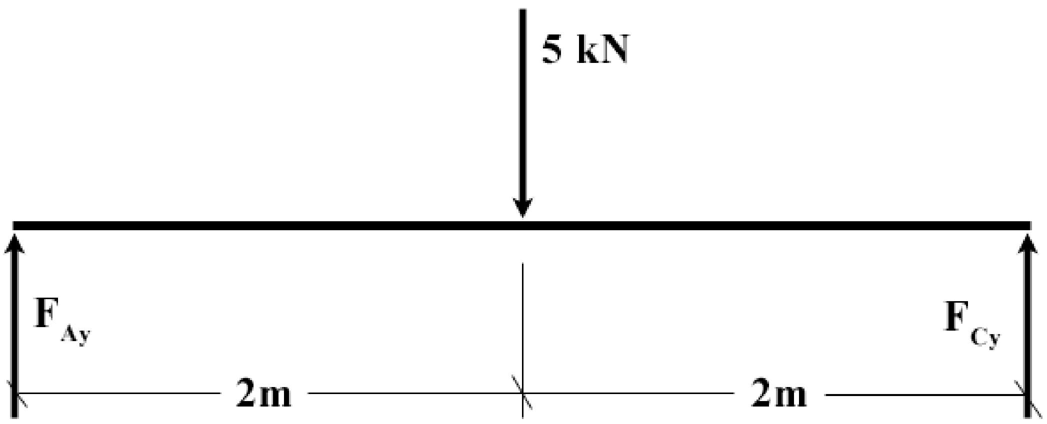
Shear and Bending Moment Equations and Diagrams

Draw the shear and bending moments diagrams for the shaft. The supports at A and C do not resist any moment.



Shear and Bending Moment Equations and Diagrams

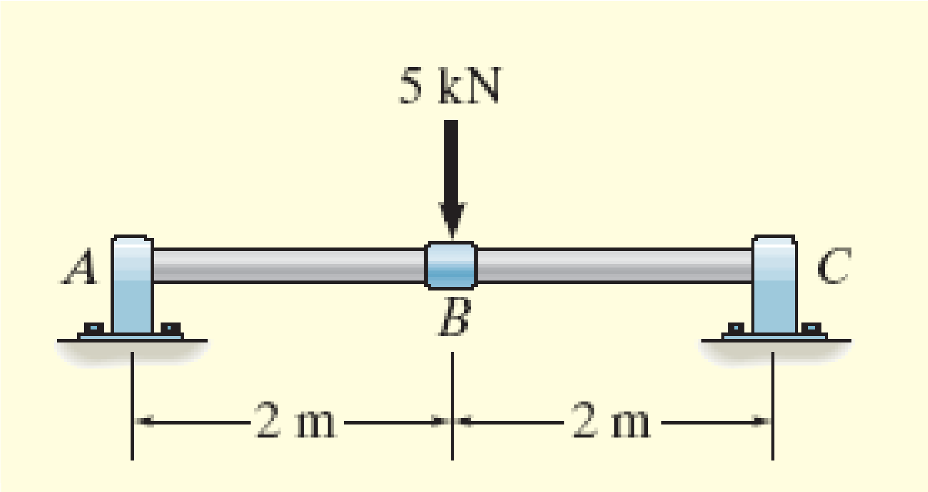
Draw free body diagram and calculate reactions:

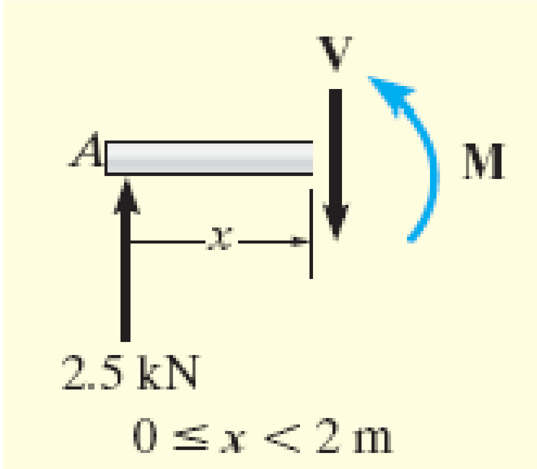


$\Sigma M_A = -(5\,kN)(2\text{ m}) + F_{Cy}(4\text{ m}) = 0$: $F_{Cy} = 2.5\text{ kN } (\uparrow)$

$\Sigma F_y = F_{Ay} - 5\text{ kN} + F_{Cy} = 0$: $F_{Ay} = 2.5\text{ kN } (\uparrow)$

Shear and Bending Moment Equations and Diagrams





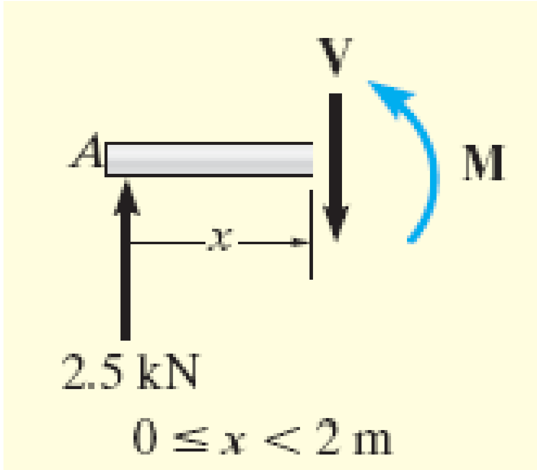
We can also see that due to symmetry, the vertical reaction at A must be half the applied load, i.e. 2.5 kN upwards.

Looking at the FBD above for $0 \leq x < 2$ m:

$$\sum F_y = 2.5\text{ kN} - V = 0 \Rightarrow V = 2.5\text{ kN}(\downarrow)$$

Shear and Bending Moment Equations and Diagrams

$$\sum F_y = 2.5\text{ kN} - V = 0 \Rightarrow V = 2.5\text{kN}(\downarrow)$$

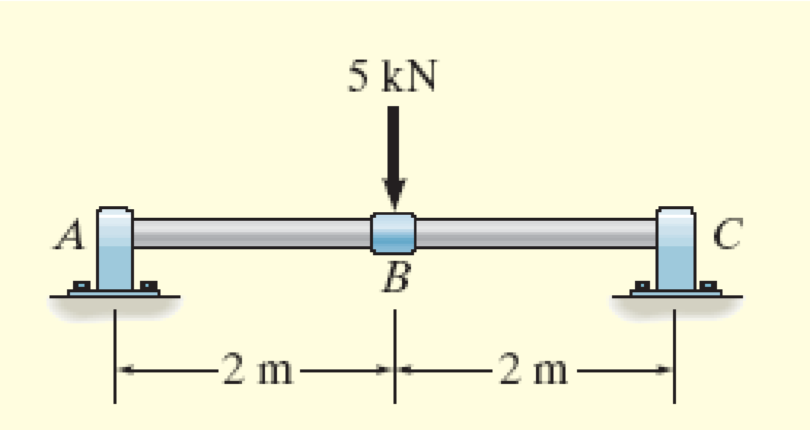


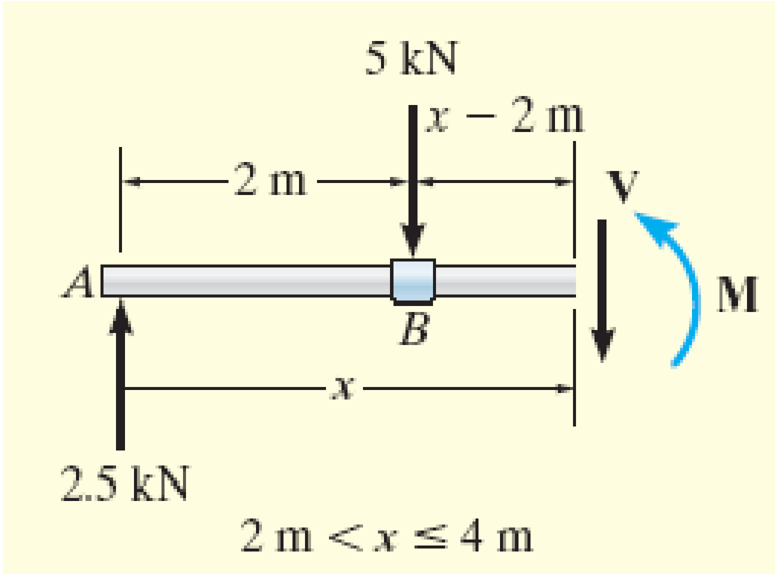
Positive shear

Note: When adding all the forces we do not include M – because it is not a force!

For $0 \leq x < 2$ m, i.e. between A and B, the internal shear force diagram is *positive*.

Shear and Bending Moment Equations and Diagrams





For 2 m < x ≤ 4 m, i.e. between B and C:

$$\sum F_y = 2.5\text{ kN} - 5\text{ kN} - V = 0 \Rightarrow V = -2.5\text{ kN}(\uparrow)$$

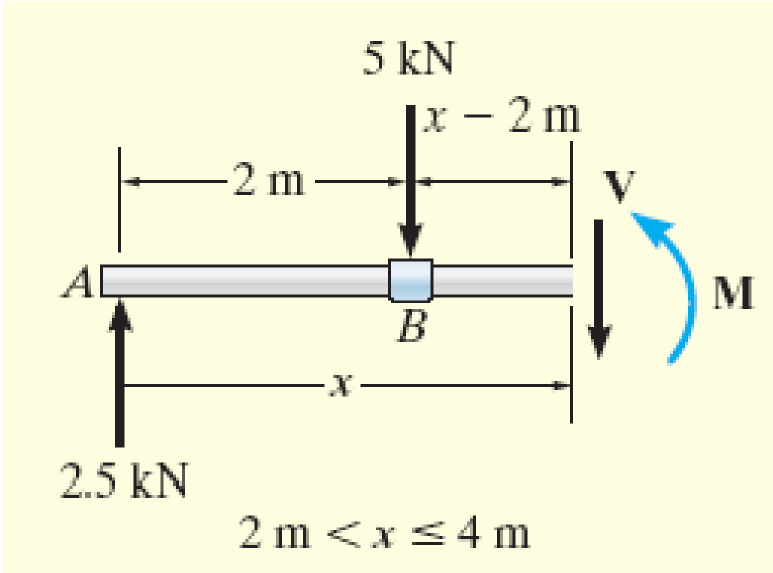
Shear and Bending Moment Equations and Diagrams

For 2 m < x ≤ 4 m:

$$\sum F_y = 2.5\text{ kN} - 5\text{ kN} - V = 0$$

$$V = -2.5\text{ kN}(\uparrow)$$

Positive shear

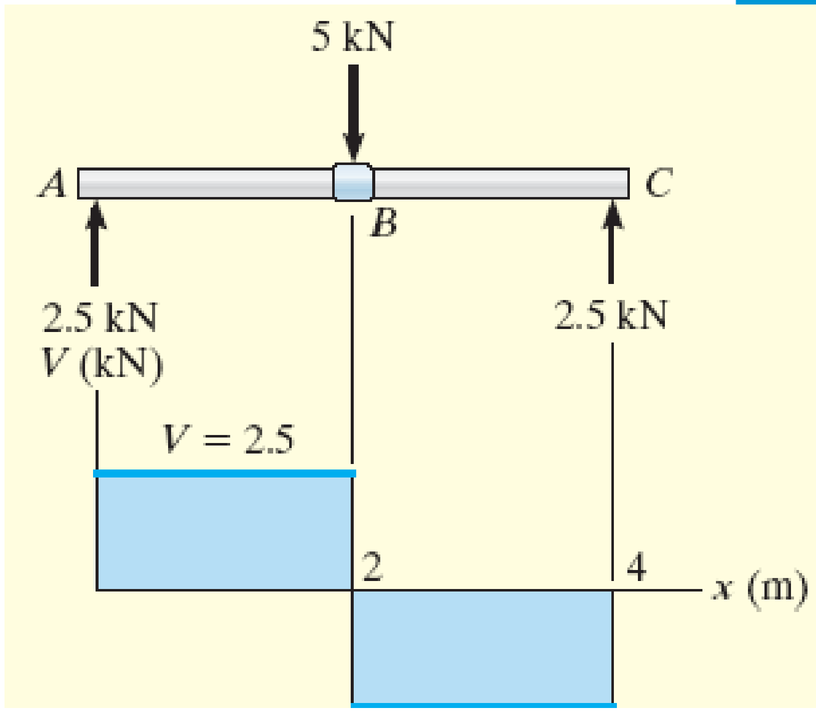


For 2 m < x ≤ 4 m, i.e. between B and C, the internal shear force diagram is *negative*.

Shear and Bending Moment Equations and Diagrams

For $0 \leq x < 2$ m, i.e. between A and B, the internal shear force diagram is positive.

For 2 m $< x \leq 4$ m, i.e. between B and C, the internal shear force diagram is negative.



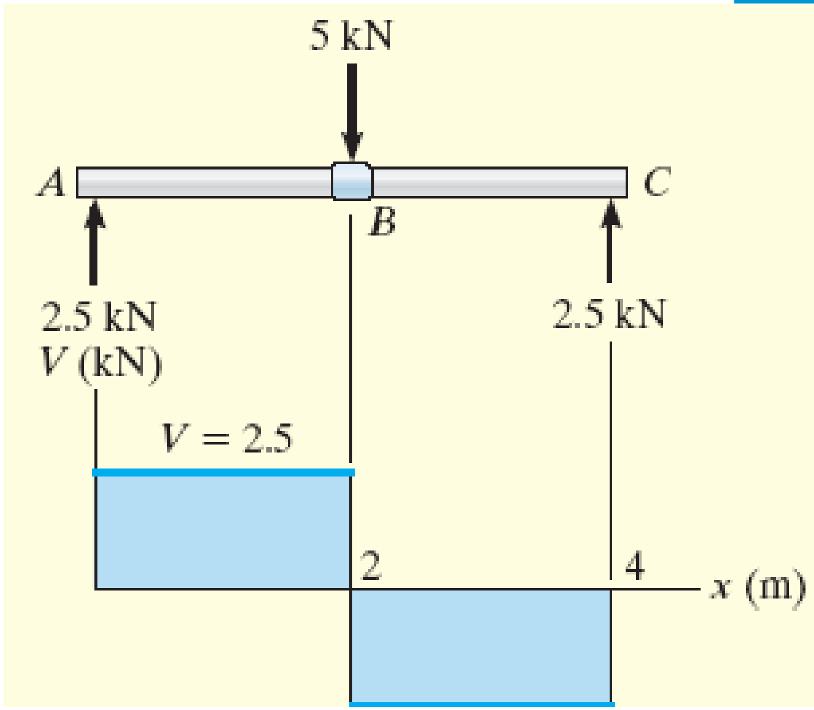
Shear Force Diagram

Shear and Bending Moment Equations and Diagrams

Note that, at B, the internal shear force abruptly changes

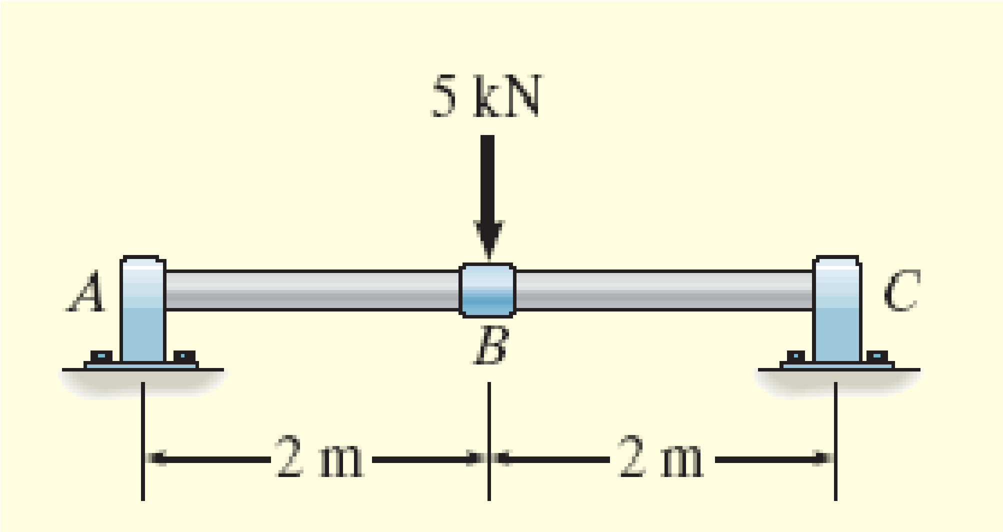
from 2.5 k to -2.5 k ,

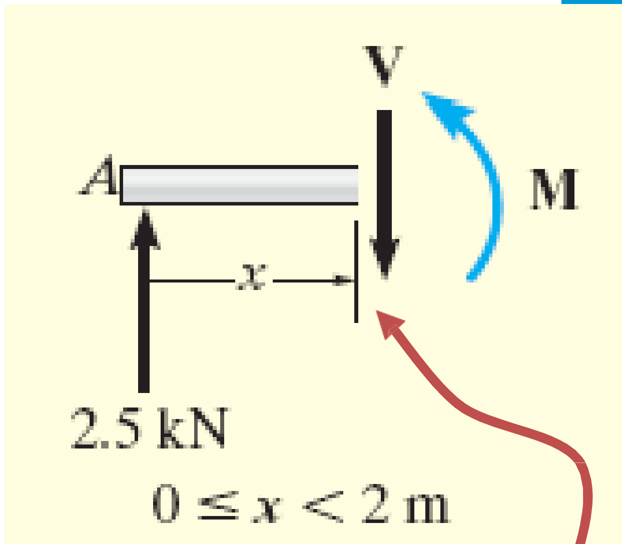
i.e. by the amount of the applied load of -5.0 kN.



Shear Force Diagram

Shear and Bending Moment Equations and Diagrams





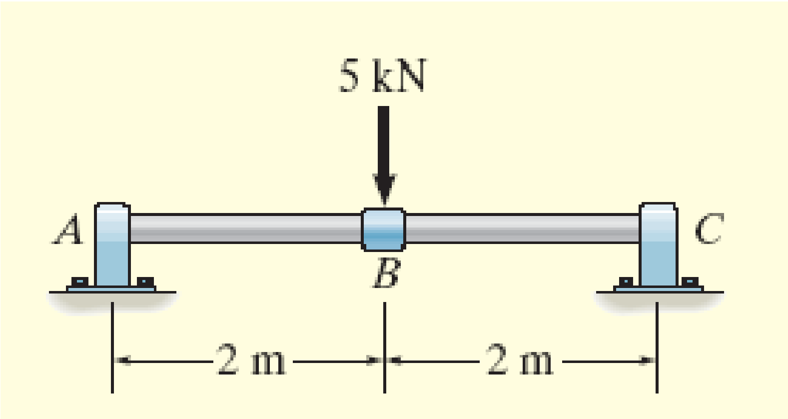
Take moments about here

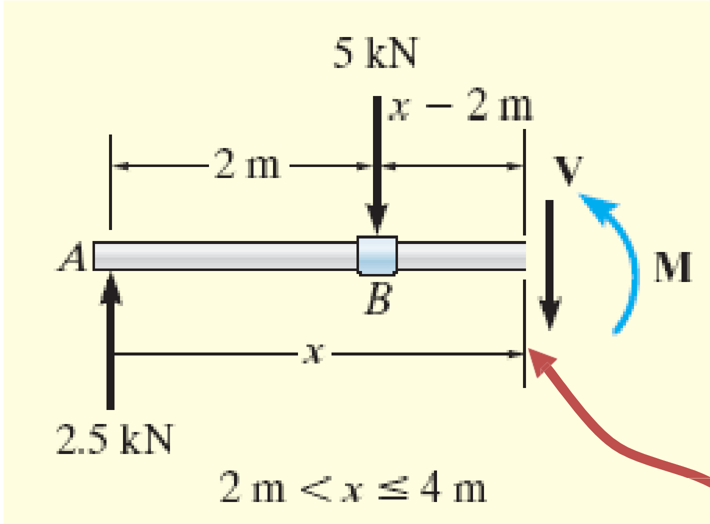
For 0 ≤ *x* < 2 m, i.e. between A and B:

$$\sum M_x = -2.5\ \text{kN}(x) + M = 0 \Rightarrow M = 2.5x\ \text{kN.m}$$

Note: When adding up the moments, forces times distance = moment. We must add in M at the cut because it is one of the moments in the diagram

Shear and Bending Moment Equations and Diagrams





Take moments about here

For 2 m < x ≤ 4 m:

$$\sum M_x = M + 5\text{ kN}(x - 2\text{m}) - 2.5\text{ kN}(x) = 0$$

$$M = (10 - 2.5x)\text{ kN.m}$$

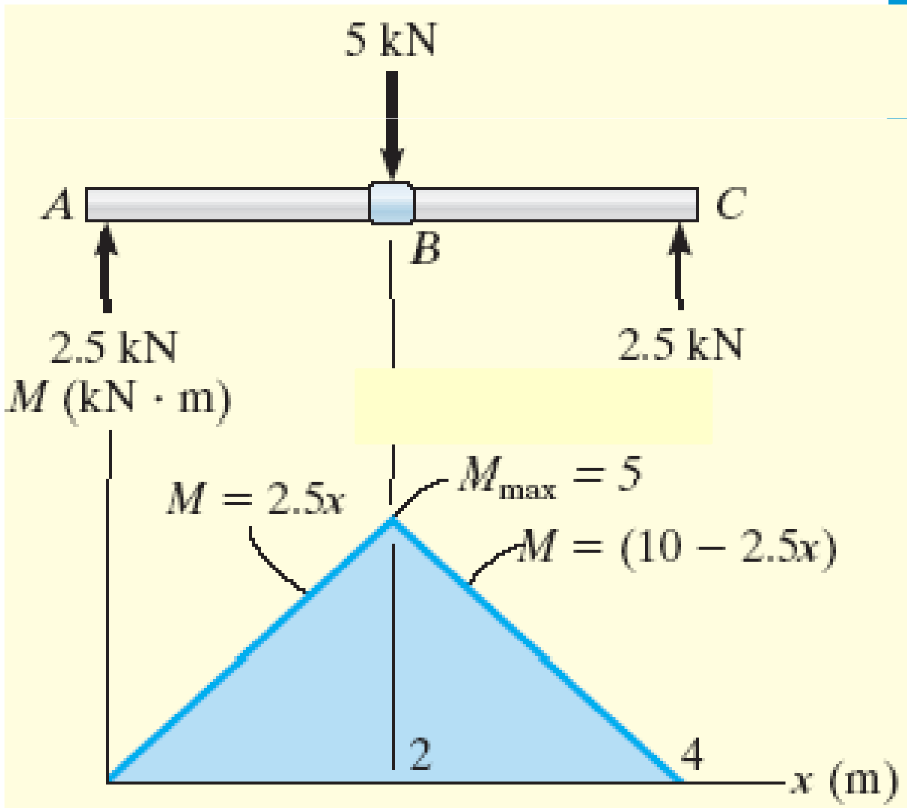
Shear and Bending Moment Equations and Diagrams

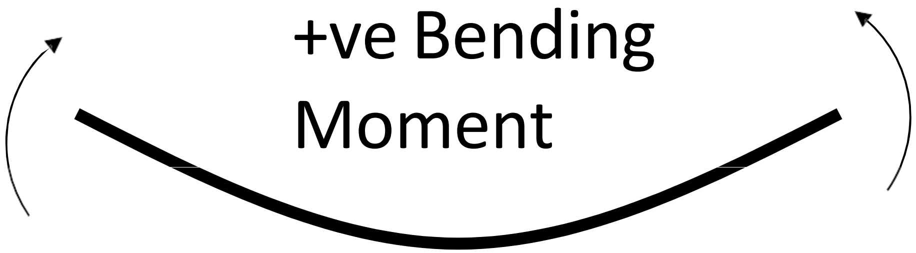
For $0 \le x < 2$ m, i.e. between A and B:

. .

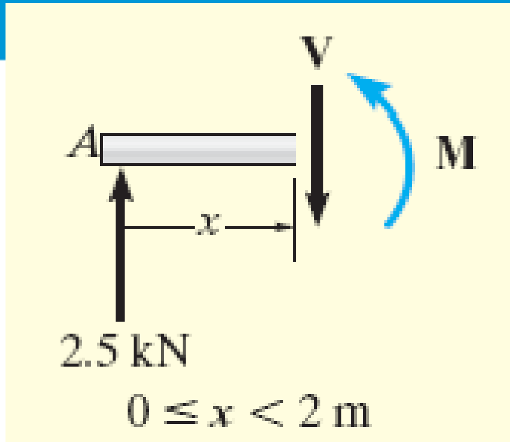
For $2 \text{ m} < x \le 4$ m, i.e. between B and C:

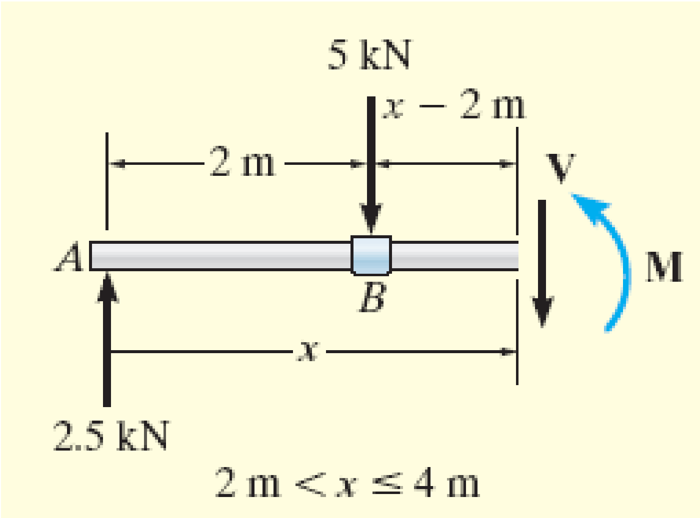
$$M = (10 - 2.5x) \text{ kN.m}$$

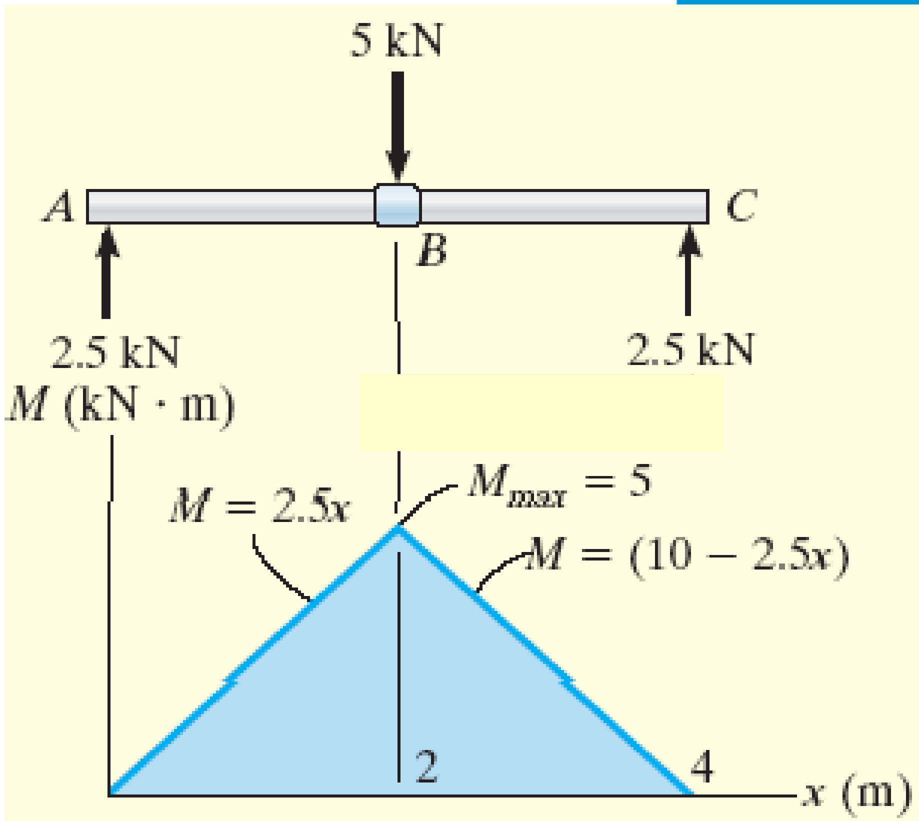


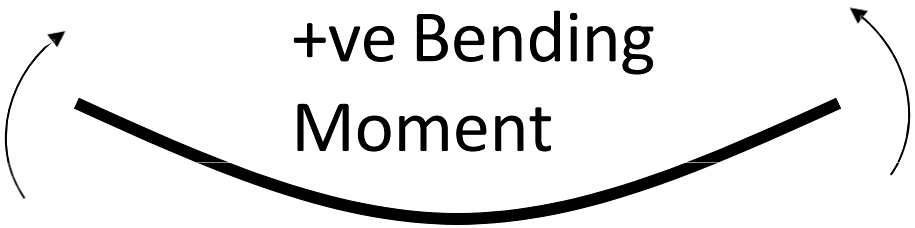


Sagging Bending moment









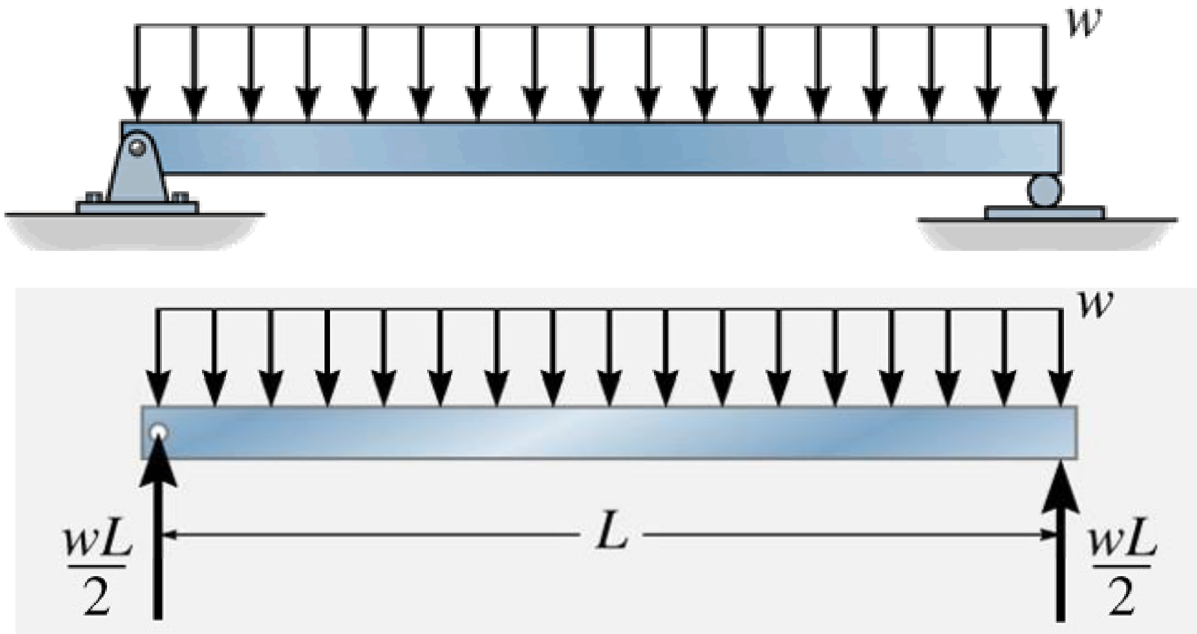
+ve Bending Moment

Sagging Bending moment

Shear and Bending Moment Equations and Diagrams

In practice, beams are commonly subject to distributed loads.

Shear and Bending Moment Equations and Diagrams

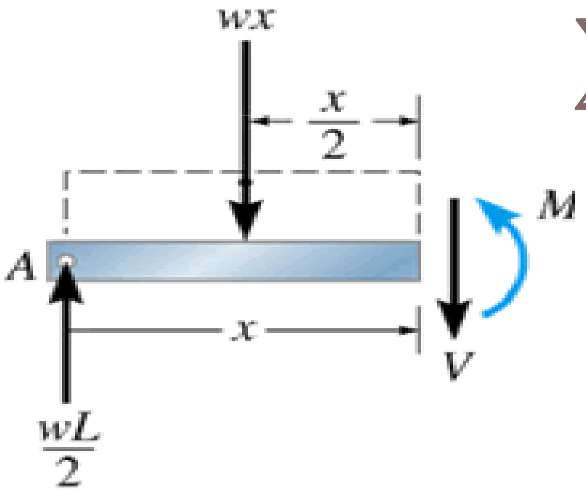


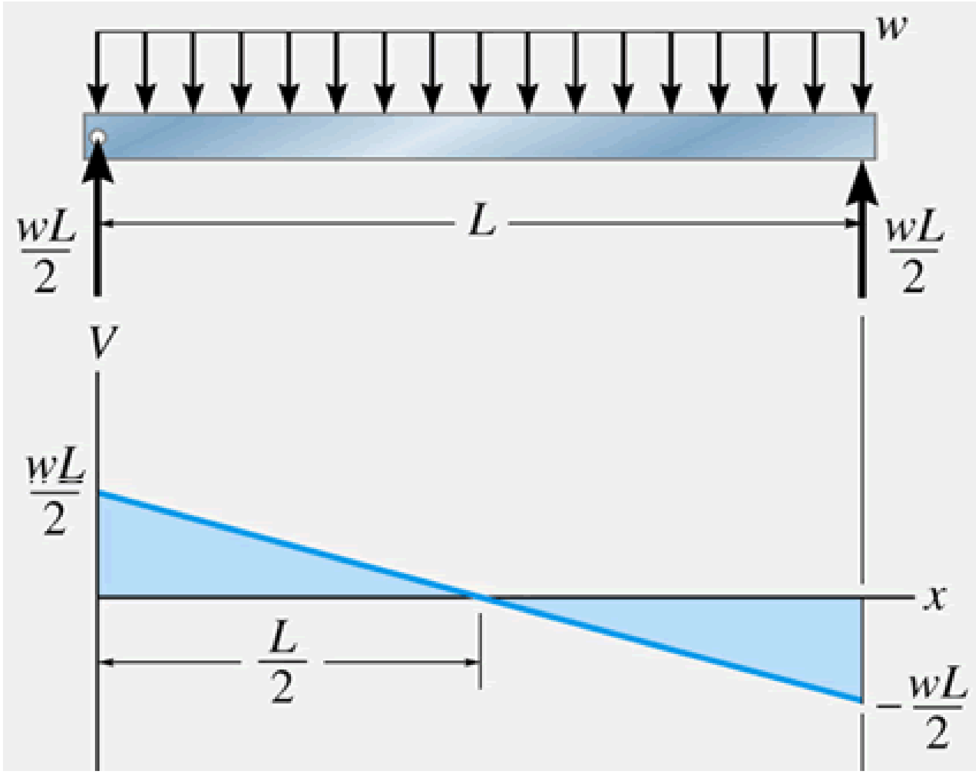
Units for w are usually N/m or kN/m

Due to symmetry, we know that the vertical reaction at A must be half the applied load, i.e. ${}^{wL}/_{2}$ upwards.

Shear and Bending Moment Equations and Diagrams

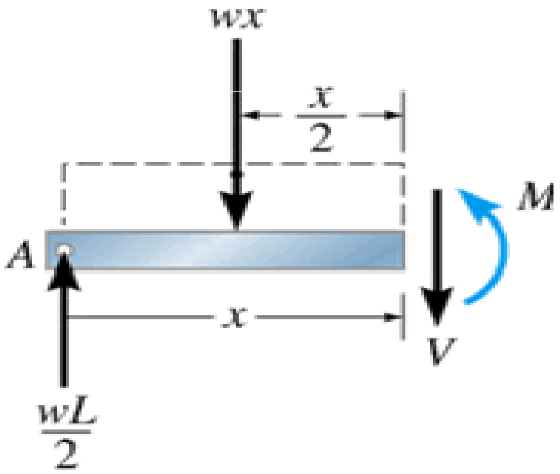
$$\sum F_y = {}^{wL}/_2 - wx - V = 0 \Rightarrow V = w\left({}^{L}/_2 - x\right)$$





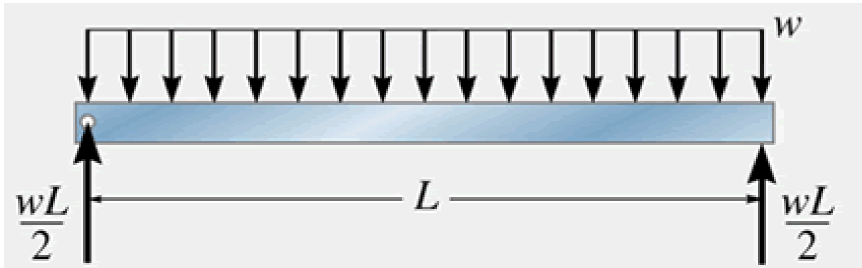
Shear Force Diagram

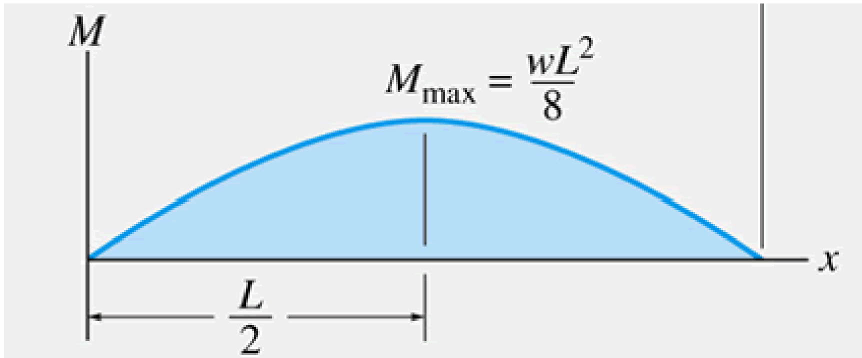
Shear and Bending Moment Equations and Diagrams



$$\sum M_x = -\frac{wL}{2}(x) + wx\left(\frac{x}{2}\right) + M = 0$$

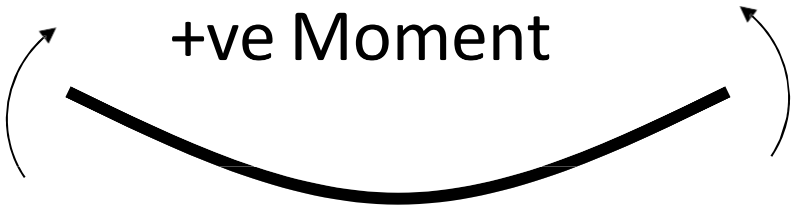
$$M = \frac{w}{2}\left(Lx - x^2\right)$$





Sagging moment

+ve Moment

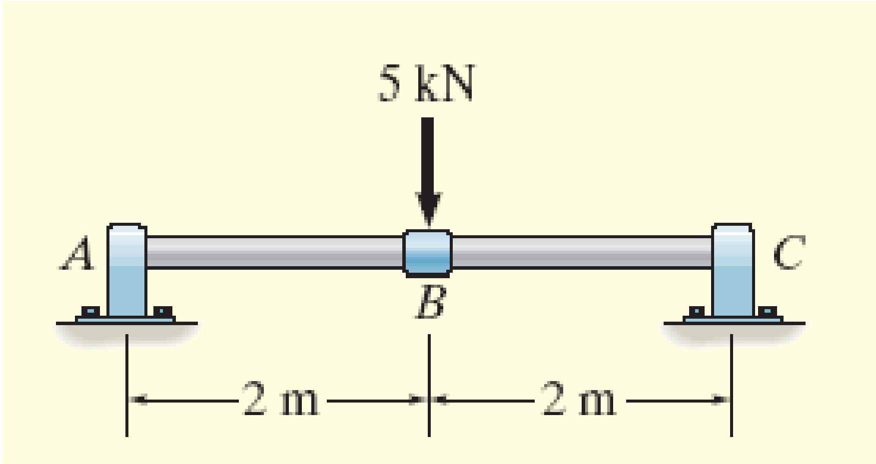


Bending moment diagram calculators

http://beamguru.com/online/beam-calculator/
https://bendingmomentdiagram.com/free-calculator/

NOT allowed in exams/quizzes.

Relationship between Shear and Bending Moment



For $0 \le x < 2$ m (between A and B):

$$M = 2.5x \text{ kN.m}$$

$$V = 2.5 \text{ kN}$$

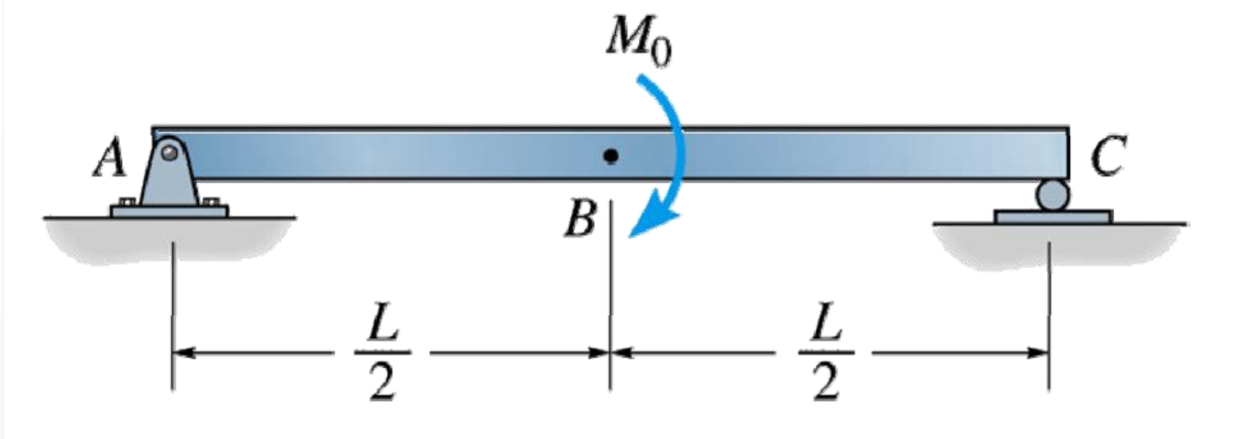
For $2 \text{ m} < x \le 4$ m (between B and C):

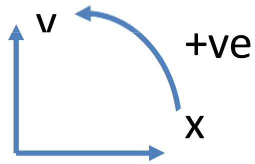
$$M = (10 - 2.5x)\,\text{kN.m}$$

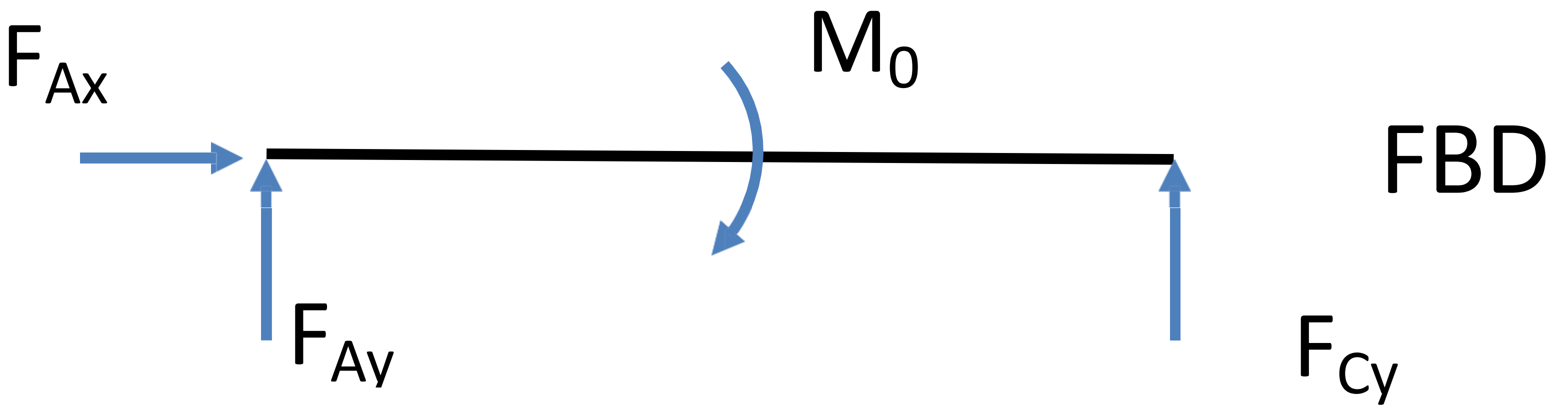
$$V = -2.5 \text{ kN}$$

Notice any pattern between V and M equations?

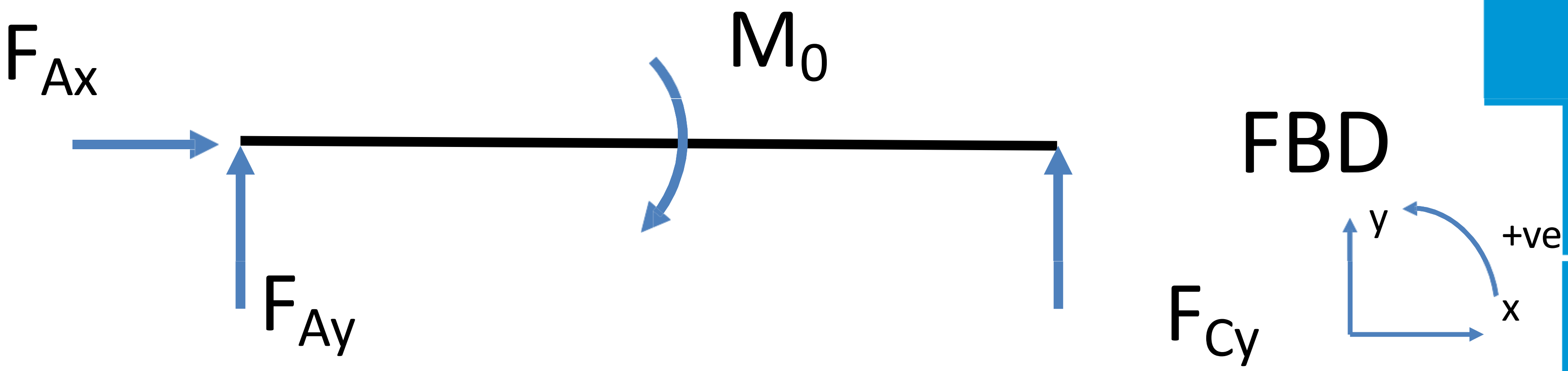
Relationship between Shear and Bending Moment







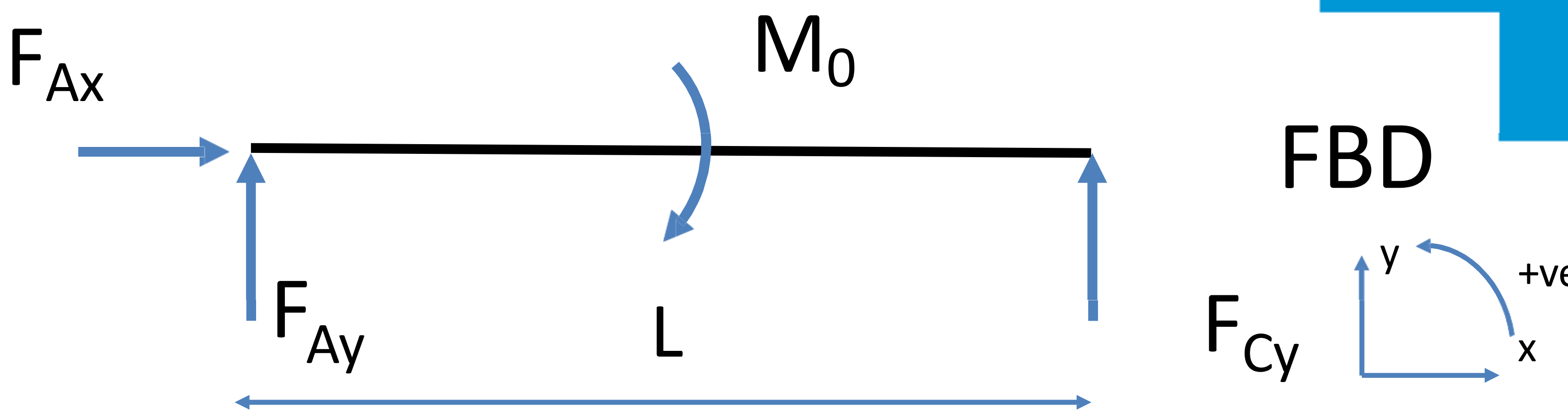
Relationship between Shear and Bending Moment



$$\sum F_x = 0 \Rightarrow F_{Ax} = 0$$

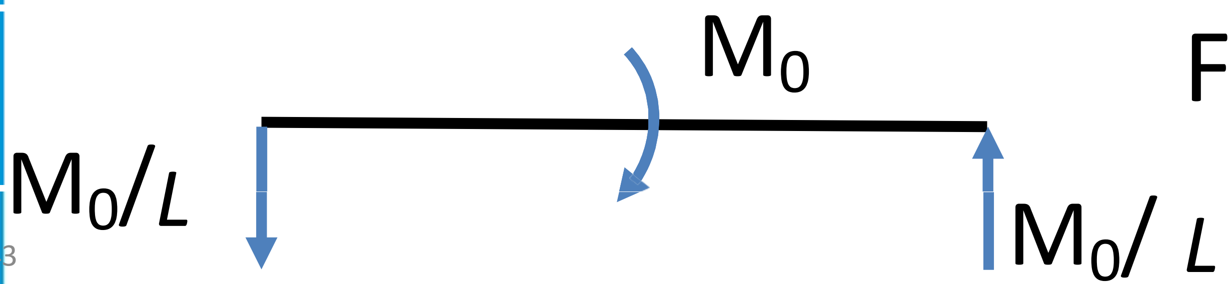
$$\sum F_y = 0 \Rightarrow F_{Ay} + F_{Cy} = 0 \Rightarrow F_{Ay} = -F_{Cy}$$

Relationship between Shear and Bending Moment



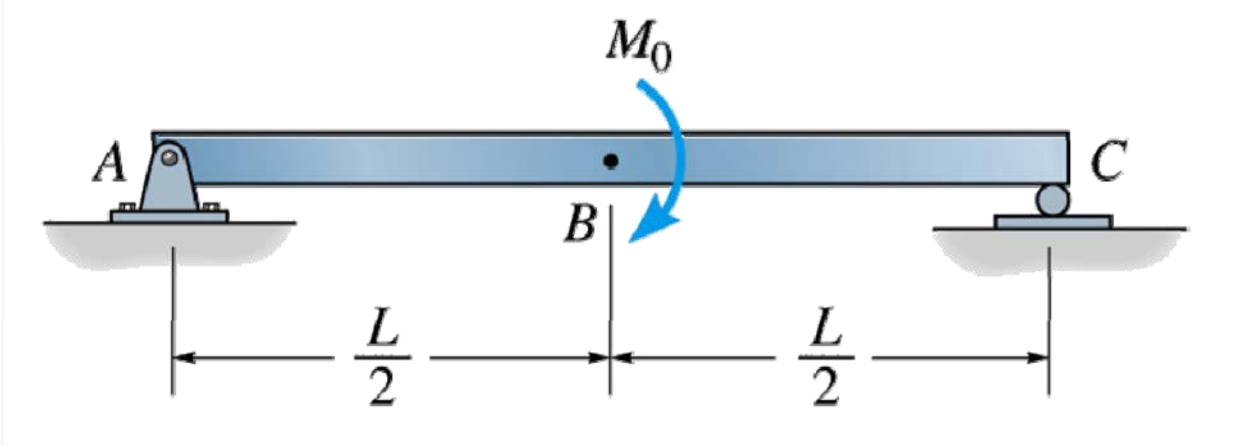
$$\sum M_A = 0 \Rightarrow F_{Cy} * L - M_0 = 0 \Rightarrow F_{Cy} = M_0/L$$

$$F_{Ay} = -F_{Cy} = -M_0/L$$



Final FBD

Relationship between Shear and Bending Moment



For $0 \le x < L/2$ (between A and B):

$$M = -M_o x/L$$

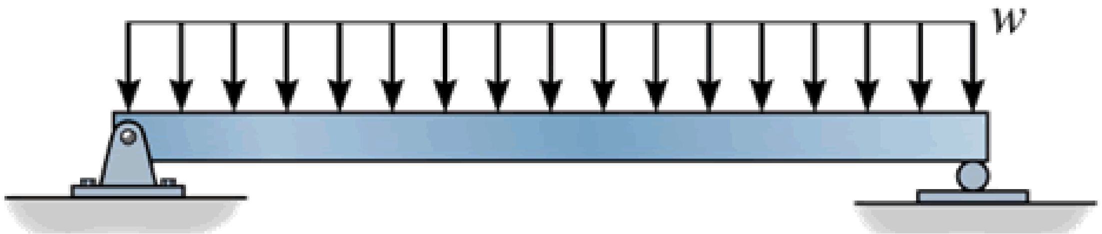
$$V = M_o/L$$

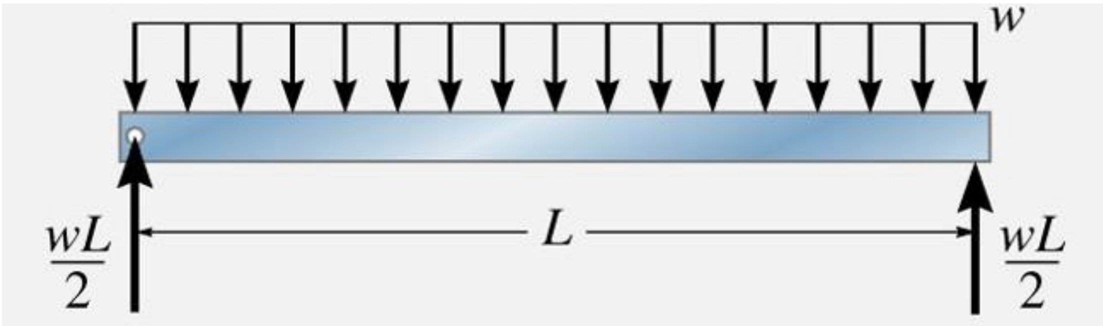
For $L/2 < x \le L$ (between B and C):

$$M = M_o\left(1 - x/L\right)$$

$$V = -M_o/L$$

Relationship between Shear and Bending Moment





$$M = \frac{w}{2}\left(Lx - x^{2}\right)$$

$$V = w\left({}^{L}/_{2} - x\right) = \frac{w}{2}(L - 2x)$$

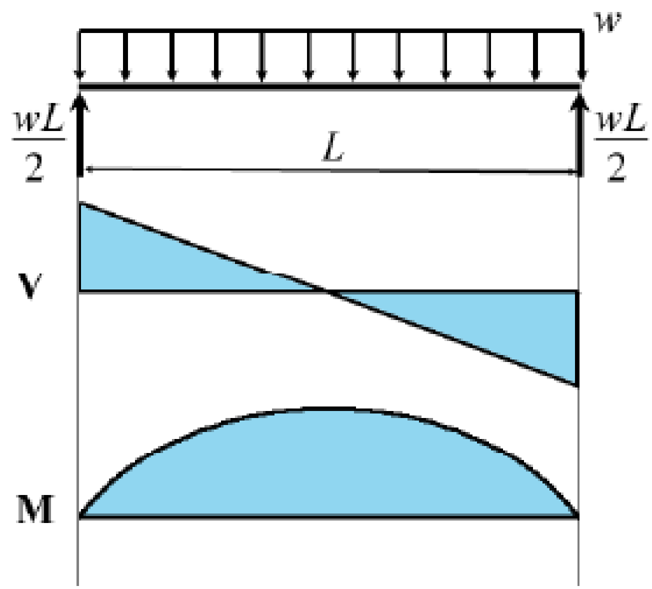
7.3 Relationship between Shear and Bending Moment

Do you notice a pattern?

$$M = 2.5x \text{ kN.m} \qquad M = (10 - 2.5x) \text{ kN.m}$$
$$V = 2.5 \text{ kN} \qquad V = -2.5 \text{ kN}$$

$$M = -\frac{M_0 x}{L} \qquad M = M_0\left(1 - \frac{x}{L}\right)$$
$$V = \frac{M_0}{L} \qquad V = -\frac{M_0}{L}$$

$$M = \frac{w}{2}\left(Lx - x^2\right)$$
$$V = w\left(\frac{L}{2} - x\right)$$



7.3 Relationship between Shear and Bending Moment

07_PH05

In order to design the beam used to support these power lines, it is important to first draw the shear and moment diagrams for the beam.

YOUR Beam: stress and strength

FAST FORWARD TO SECOND YEAR ENGG251

- Maximum tensile stress allowed in BALSA is for you to find
- In this case the beam will break (mostly) because it is bending
- In that case stress on the bottom fibres in the middle will be $\sigma_{\max} = M_{\max} h/(2I)$

$$M_{\max} = PL/4$$