Centroid & Moment of Inertia

Center of Area (Geometric Center) ≠ Center of Gravity / Mass

it is only same if body has uniform density

Inertia - ability to resist change in forces

Material Strength > Applied Stress

$$\left.\begin{aligned} A_T &= \int dA = \int y \, dx \\ \bar{y} \int_A dA &= \int_A \tilde{y} \, dA \end{aligned}\right\} \text{About } x\text{-axis}$$

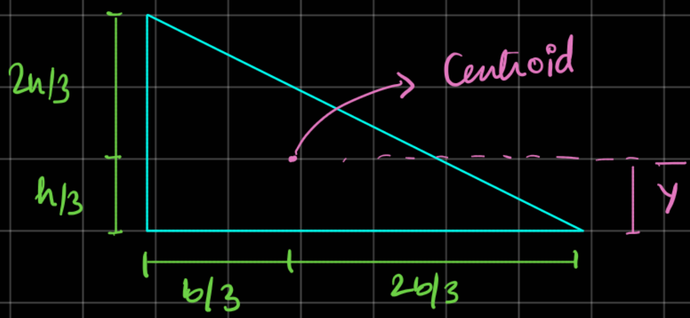
$$A_T = \int dA = \int_A x \, dy$$

Example

$$\bar{y} \int dA = \int \tilde{y} \, dA$$

$$dA = x \, dy$$

$$\int_A dA = \int_0^h x \, dy$$



$$y = \frac{h}{b}(b - x)$$

$$y = h - \frac{x}{b}$$

$$x = \frac{b(h-y)}{h}$$

$$dA = x\,dy = \frac{b(h-y)}{h}\,dy$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(\frac{h-y}{h}\right) b\,dy}{\int_0^h \frac{h-y}{h}\, b\,dy} = \frac{\cancel{\frac{b}{h}}\int_0^h yh - y^2\,dy}{\cancel{\frac{b}{h}}\int_0^h h-y\,dy} = \frac{\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h}{\left[hy - \frac{y^2}{2}\right]_0^h}$$

$$= \frac{\left[\frac{h^3}{2} - \frac{h^3}{3}\right]}{h^2 - \frac{h^2}{2}} = \frac{\frac{h^3}{6}}{\frac{h^2}{2}} = \frac{h^3}{h^2} \times \frac{2}{6} = \frac{h}{3}$$

$$dA = y\,dx$$

$$\int_A dA = \int_0^b y\,dx$$

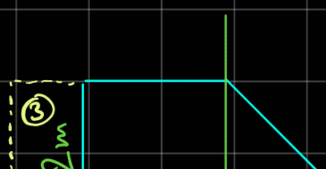
$$\int_A dA = \int_0^b \frac{h}{b}(b-x)\,dx$$

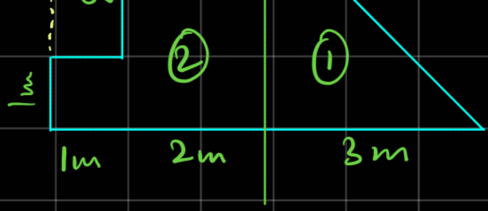
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\cancel{\frac{h}{b}}\int_0^b xb - x^2\,dx}{\cancel{\frac{h}{b}}\int_0^b b-x\,dx} = \frac{\left[\frac{x^2 b}{2} - \frac{x^3}{3}\right]_0^b}{\left[bx - \frac{x^2}{2}\right]_0^b} = \frac{\frac{b^3}{2} - \frac{b^3}{3}}{b^2 - \frac{b^2}{2}} = \frac{b}{3}$$

$$\bar{X} A_T = x_1A_1 + x_2A_2 + x_3A_3$$

$$\bar{X} = \frac{x_1A_1 + x_2A_2 + x_3A_3}{A_T}$$

Example





$$\bar{X}A = x_1A_1 + x_2A_2 - x_3A_3$$

$x_1 = 1 \quad x_2 = 1.5 \quad x_3 = 2.5$

$A_1 = 9/2 \quad A_2 = 9 \quad A_3 = 2$

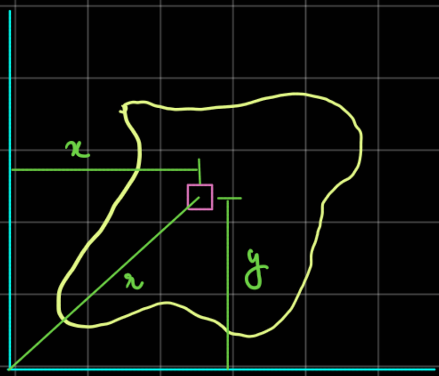
Segment	Area	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
1	9/2	1	1	9/2	9/2
2	9	−1.5	1.5	−27/2	27/2
3	(−2)	−2.5	2	−5	−4
Σ	23/2			−4	14

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{-4}{11.5} = -0.348 \text{ m}$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{14}{11.5} = 1.22 \text{ m}$$

Moment of Inertia

$dI_x = y^2\, dA \qquad dI_y = x^2\, dA$



$$I_x = \int_A y^2\, dA$$

$$I_y = \int_A x^2\, dA$$

Moment of Inertia on z-axis $\Rightarrow$ Polar Moment

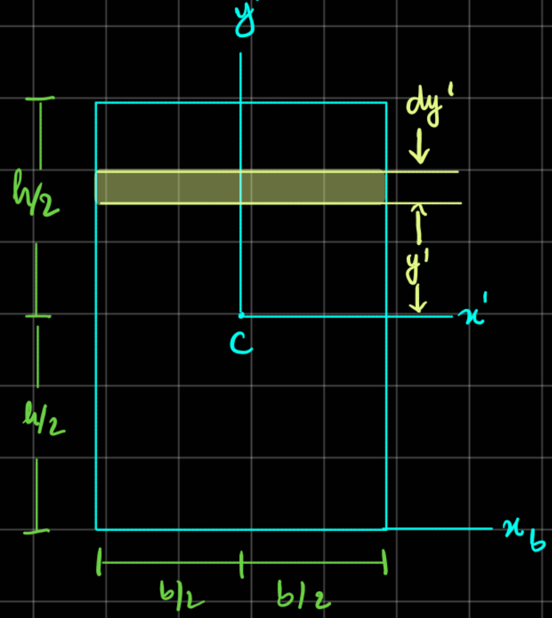
$\Downarrow$

$J_0 = J = I_p$

$J_0 = I_x + I_y$

Example

$$I_x = \int_A y'^2 \, dA = \int_{-h/2}^{h/2}$$

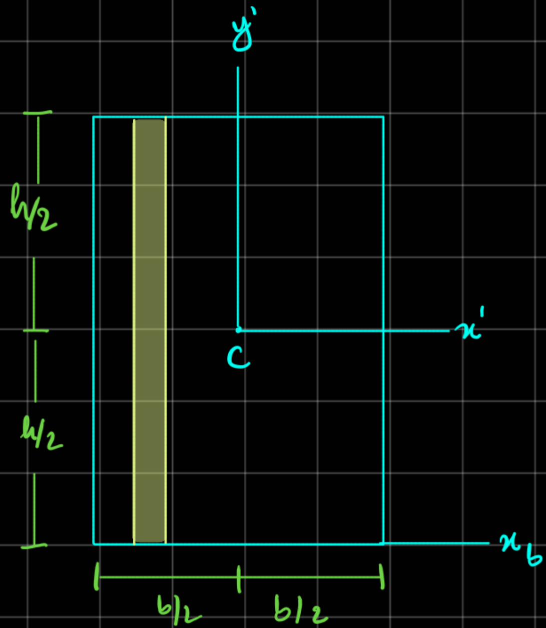


Parallel Axis Theorem

Axis MUST be parallel to centroidal axis

$$I_x = I_{x'} + Ad_y^2$$

Area of object × Transfer distance



Example

$$I = \frac{bh^3}{12} - \frac{\pi r^4}{4}$$