

$$\text{Centroid } (c) = (\bar{x}, \bar{y})$$

$$\left. \begin{aligned} \bar{x} &= \frac{\int_A \tilde{x} dA}{\int_A dA} & \bar{y} &= \frac{\int_A \tilde{y} dA}{\int_A dA} \end{aligned} \right\} \text{General Equations}$$

$$\begin{aligned} dA &= y dx \\ &= x^{3/2} dx \end{aligned}$$

$$\begin{aligned} A &= \int_A dA = \int_0^1 x^{3/2} dx = \left. \frac{2}{5} x^{5/2} \right|_0^1 \\ &= \frac{2}{5} = \underline{0.4 \text{ m}^2} \end{aligned}$$

$$\tilde{x} = x \quad \tilde{y} = \frac{y}{2} = \frac{x^{3/2}}{2}$$

$$\begin{aligned} \bar{x} &= \frac{\int_0^1 \tilde{x} dA}{\int_0^1 dA} = \frac{\int_0^1 x^{5/2} dx}{\int_0^1 x^{3/2} dx} = \frac{\left. \frac{2}{7} x^{7/2} \right|_0^1}{\left. \frac{2}{5} x^{5/2} \right|_0^1} \\ &= \frac{2/7}{2/5} \\ &= \underline{\frac{5}{7}} \end{aligned}$$

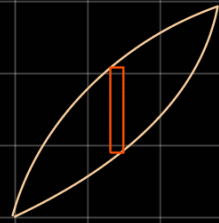
$$\begin{aligned} \bar{y} &= \frac{\int_A \tilde{y} dA}{\int_A dA} = \frac{\int_0^1 \frac{x^{3/2}}{2} x^{3/2} dx}{\int_0^1 x^{3/2} dx} = \frac{\left. \frac{1}{2} \cdot \frac{1}{4} x^4 \right|_0^1}{\left. \frac{2}{5} x^{5/2} \right|_0^1} \\ &= \frac{1/8}{2/5} \\ &= \underline{\frac{5}{16}} \end{aligned}$$

$$\text{Centroid} = \left( \frac{5}{7}, \frac{5}{16} \right) = (0.7143, 0.3125)$$

$$2. \quad y^2 = x \Rightarrow y = x^{1/2}$$

$$y = x^2$$

$$dA = (x^{1/2} - x^2) dx$$



$$\tilde{x} = x$$

$$\tilde{y} = \frac{1}{2}(x^{1/2} - x^2) + x^2 = \frac{x^2}{2} + \frac{x^{1/2}}{2}$$

$$\begin{aligned} \bar{x} &= \frac{\int_A \tilde{x} dA}{\int_A dA} = \frac{\int_0^1 x(x^{1/2} - x^2) dx}{\int_0^1 (x^{1/2} - x^2) dx} \\ &= \frac{\int_0^1 x^{3/2} - x^3 dx}{\int_0^1 x^{1/2} - x^2 dx} \\ &= \frac{\left[ \frac{2}{5} x^{5/2} - \frac{1}{4} x^4 \right]_0^1}{\left[ \frac{2}{3} x^{3/2} - \frac{1}{3} x^3 \right]_0^1} \\ &= \frac{0.4 - 0.25}{1/3} \\ &= 3(0.15) \\ &= \underline{0.45} \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{\int_A \tilde{y} dA}{\int_A dA} = \frac{\frac{1}{2} \int_0^1 (x^2 + x^{1/2})(x^{1/2} - x^2) dx}{\int_0^1 (x^{1/2} - x^2) dx} \\ &= \frac{1}{2} \frac{\int_0^1 (x - x^4) dx}{\int_0^1 (x^{1/2} - x^2) dx} \\ &= \frac{1}{2} \left[ \frac{\frac{1}{2} x^2 - \frac{1}{5} x^5}{\frac{2}{3} x^{3/2} - \frac{1}{3} x^{3/3}} \right]_0^1 \\ &= \frac{1}{2} \left[ \frac{0.5 - 0.2}{1/3} \right] \\ &= \frac{3}{2} \times \frac{3}{10} \end{aligned}$$

$$= \frac{9}{20}$$

$$= \underline{0.45}$$

$$\therefore \text{Centroid} = (0.45, 0.45)$$

$$3. \quad I_x = \int_A y^2 dA \quad I_y = \int_A x^2 dA$$

$$\begin{aligned} dA &= 2x dy \\ &= 2(a^2 - y^2)^{1/2} dy \end{aligned}$$

$$I_x = \int_{-a}^a y^2 2(a^2 - y^2)^{1/2} dy$$

$$= \frac{\pi}{4} a^4 \text{ (not easily solvable)}$$

