ENGG102 Fundamentals of Engineering Mechanics

Dr. Sana Amir

VISIONARY / PASSIONATE / DYNAMIC
CONNECT:
UNIVERSITY OF WOLLONGONG

UOWD
University of Wollongong in Dubai

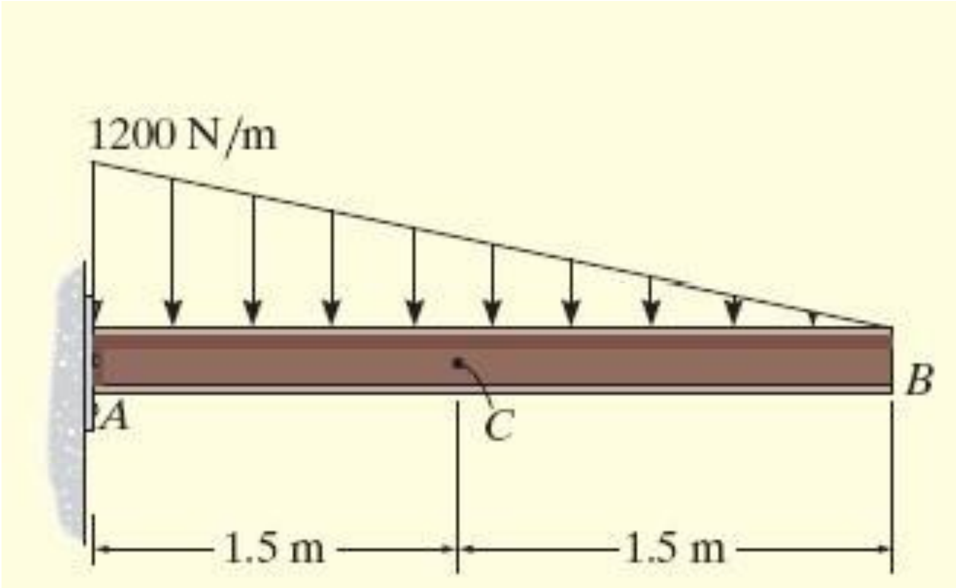
Outline:

1. Internal Forces in Structural Members

2. Relationship between Shear and Bending Moment

Internal Forces in Structural Members

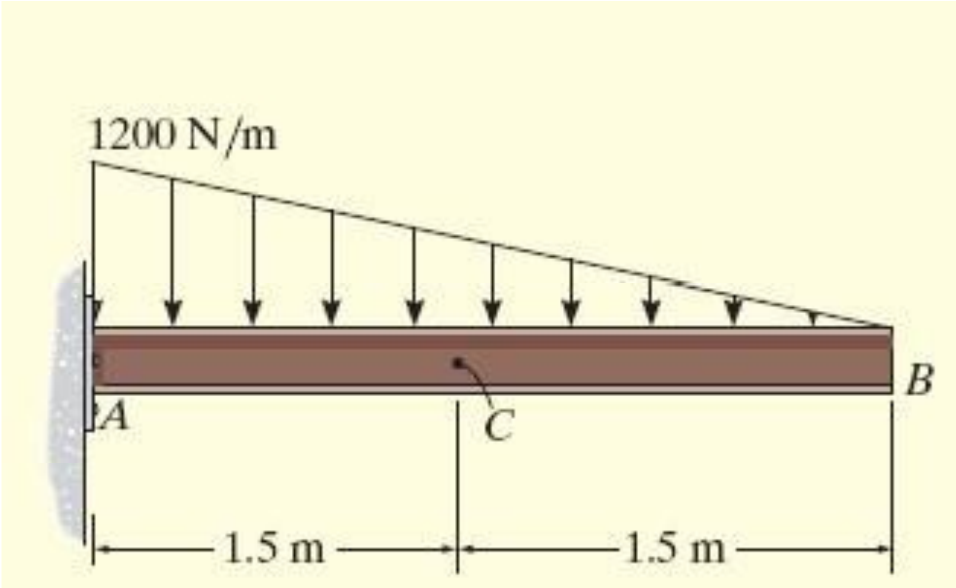
Line of action of triangular load through centroid of triangle



Calculate reactions first:

$$\sum F_y = 0 \;\rightarrow\; F_{Ay} = \left(\frac{1}{2}\right)(1200kN/m)(3m)$$
$$= 1800kN$$
$$\sum M_A = 0 \;\rightarrow\; M_A = -\left(\frac{1}{2}\right)(1200kN/m)(3m)(1m)$$
$$= -1800kN.m$$

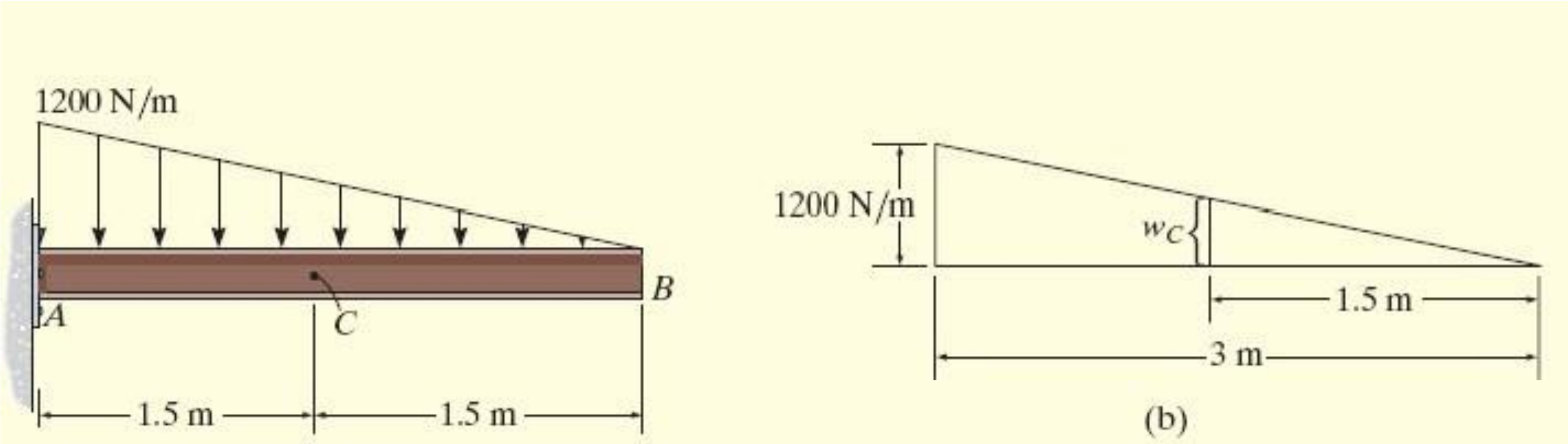
Internal Forces in Structural Members



Determine the internal forces at C.

Note: For this problem, it is not necessary to determine the support reactions first since segment BC can be used to find the internal forces at C.

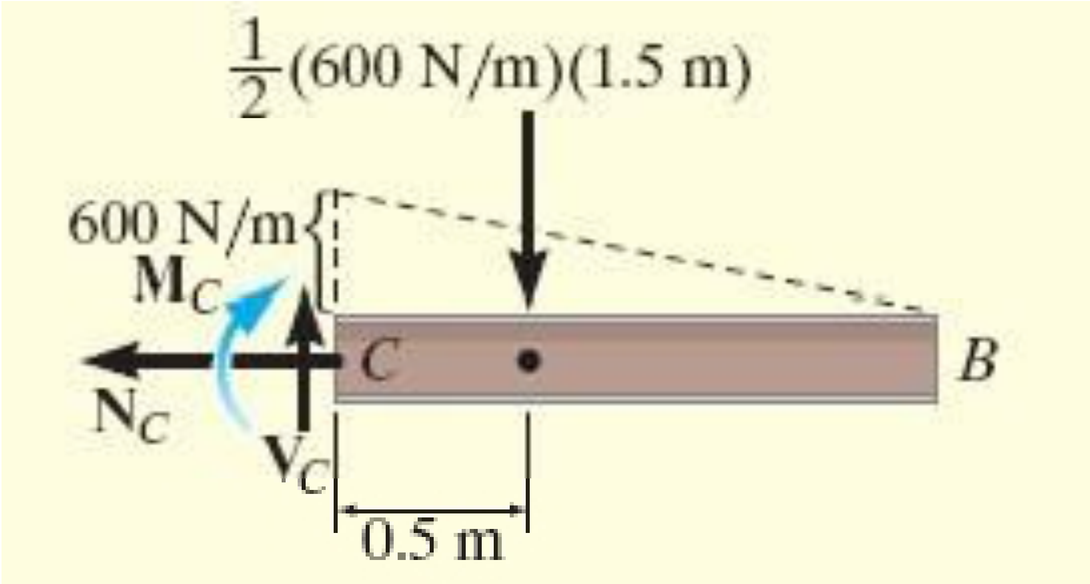
Internal Forces in Structural Members



The intensity of the load at C is determined using similar triangles:

$$w_c = 1200\text{N/m}\left(\frac{1.5\text{m}}{3\text{m}}\right) = 600\text{N/m}.$$

Internal Forces in Structural Members



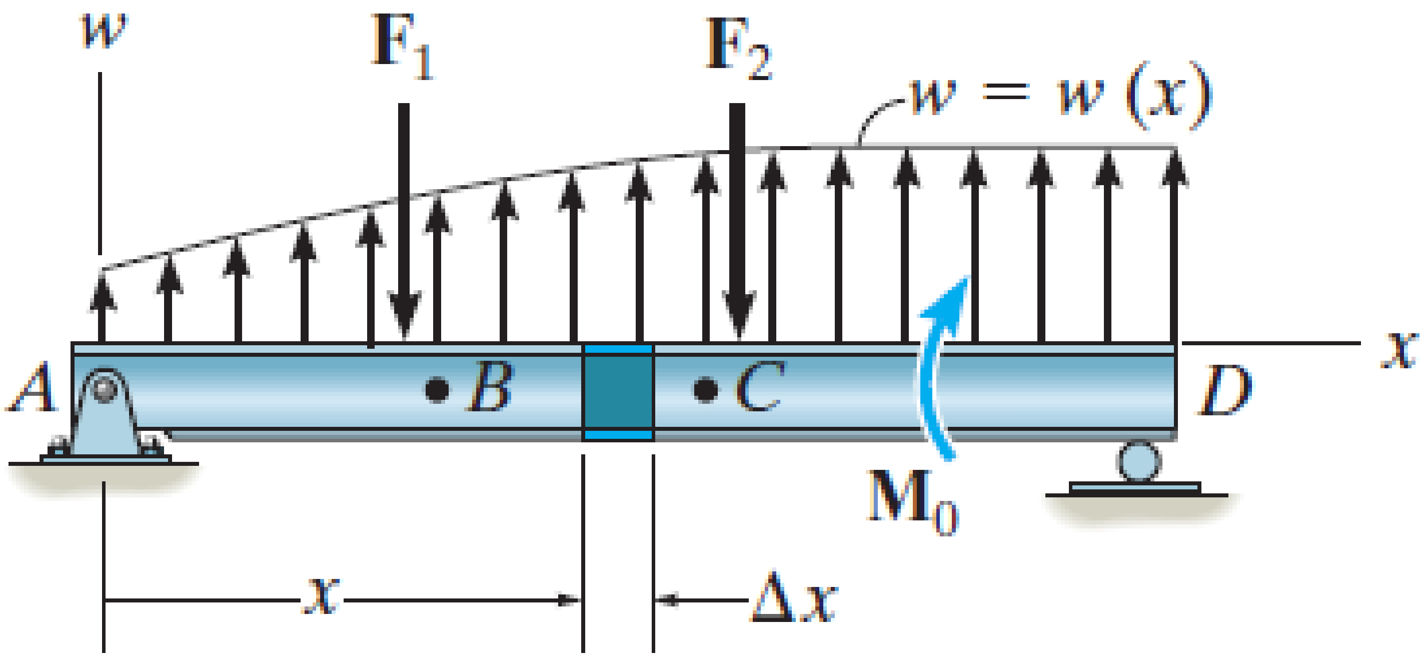
$\Sigma F_x = N_C = 0$

$\Sigma F_y = V_c - ½\,(600\ \text{N/m})(1.5\ \text{m}) = 0 : \quad V_c = 450\ \text{N}\ (\uparrow)$

$\Sigma M_C = -M_C - ½\,(600\ \text{N/m})(1.5\ \text{m})(0.5\ \text{m}) = 0$

$M_C = -225\ \text{Nm}$ (CCW)

7.3 Relationship between Shear and Bending Moment



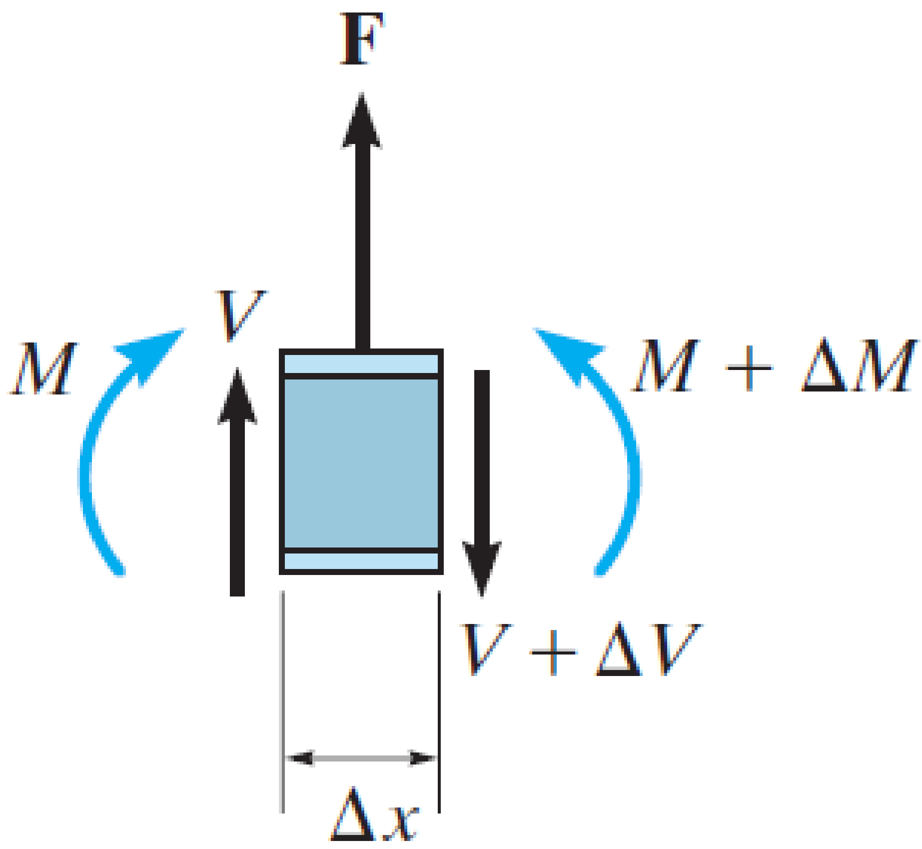
(a)

07_013a

Relationship between Shear and Bending Moment

$$\Rightarrow \frac{dV}{dx} = w(x)$$

$$\frac{dM}{dx} = V$$



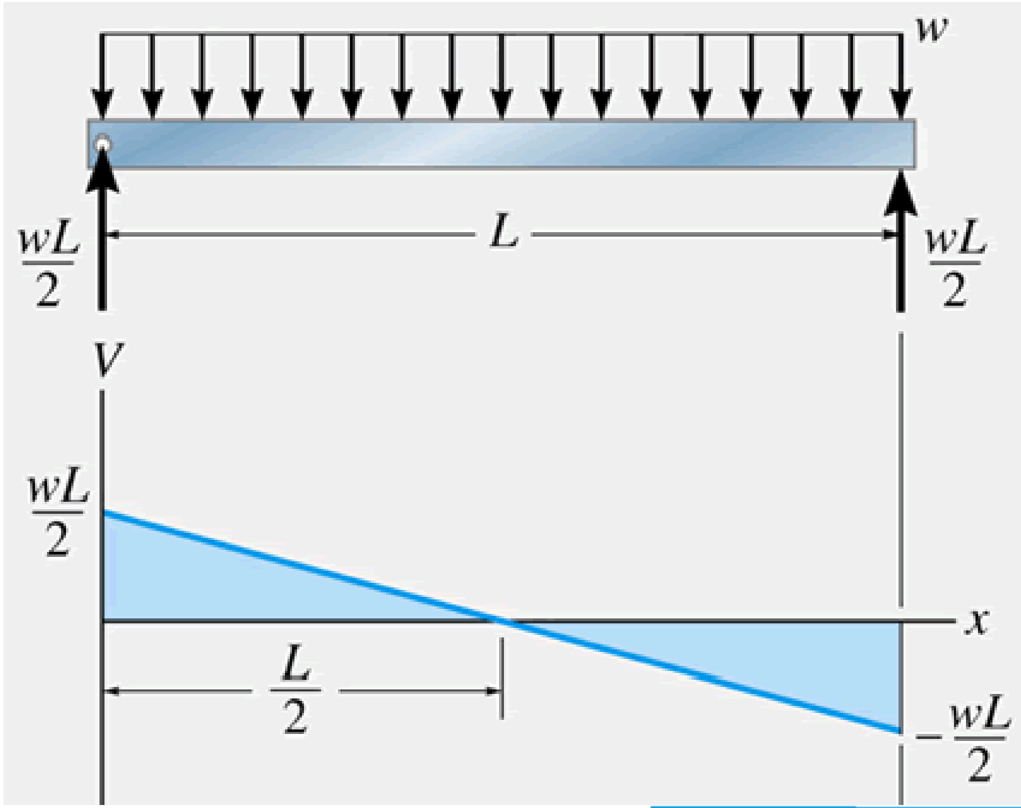
(a)

Relationship between Shear and Bending Moment

Slope of the shear diagram:

$$\frac{dV}{dx} = w(x)$$

$$\Delta V = \int w(x)dx$$



Relationship between Shear and Bending Moment

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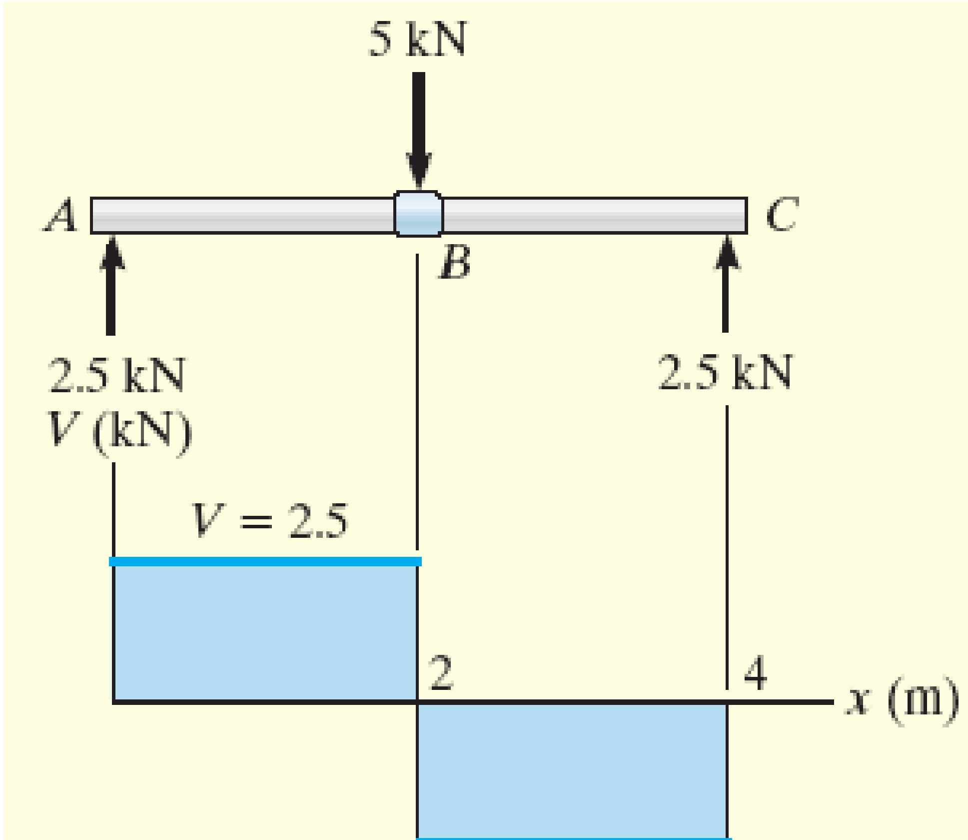

Relationship between Shear and Bending Moment

Slope of the shear diagram:

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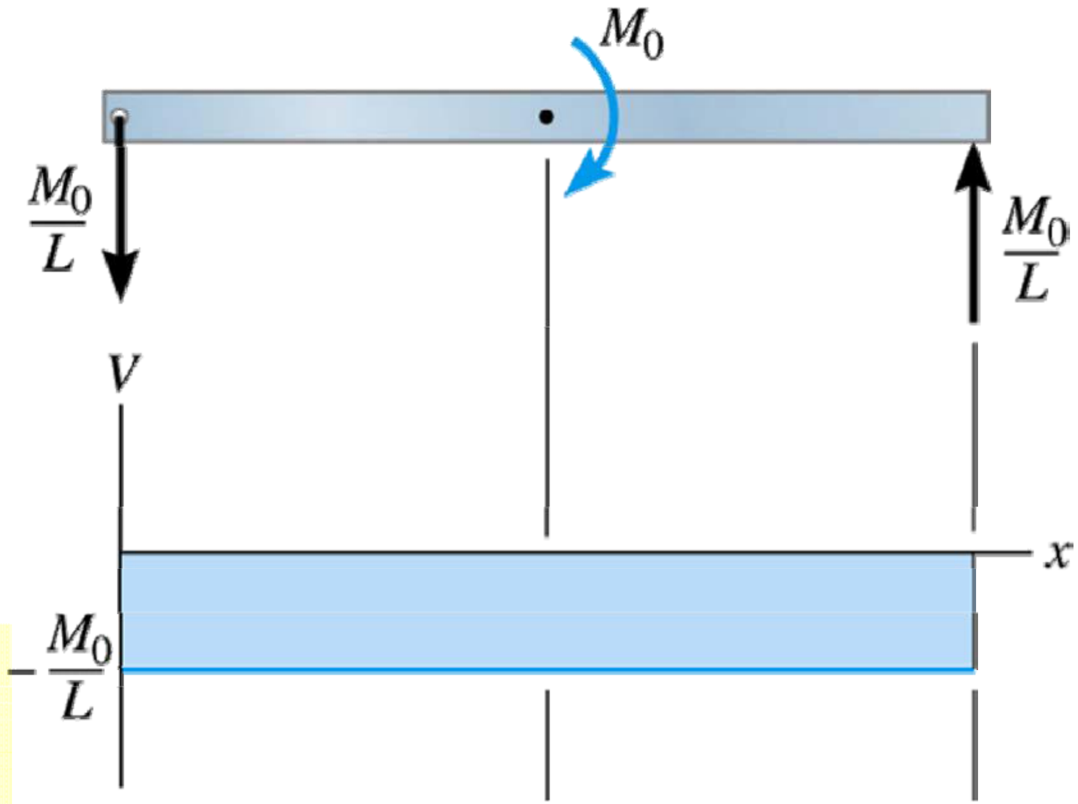
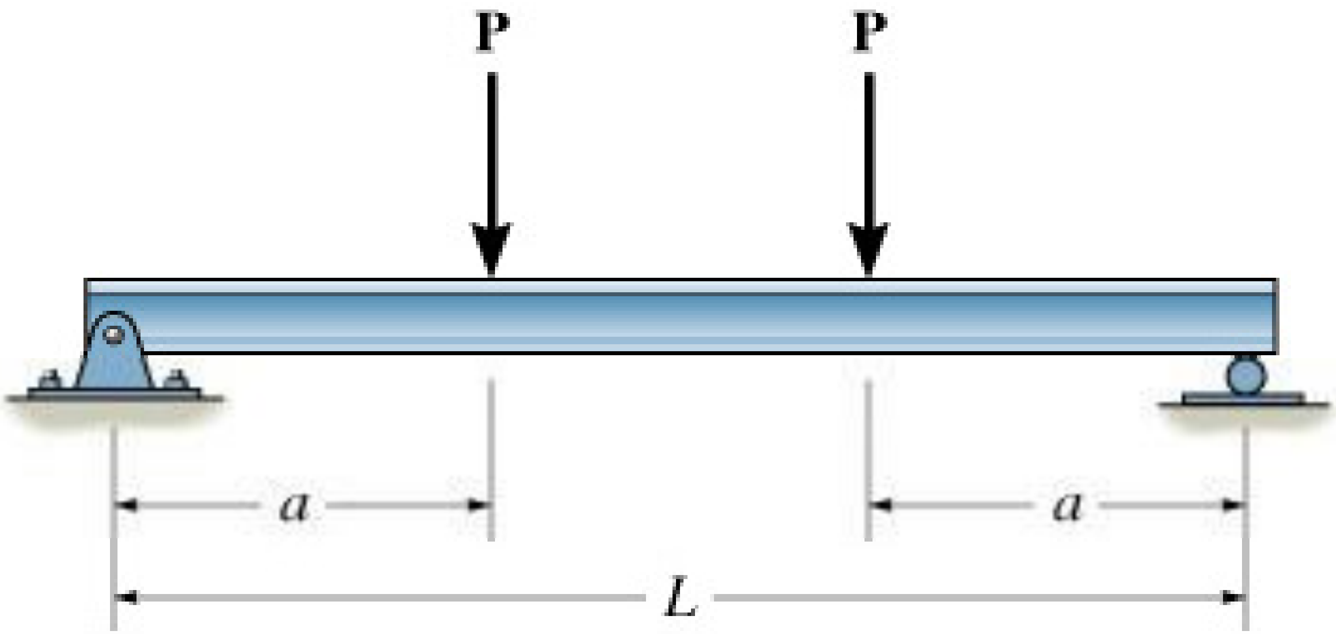


What does the corresponding Bending Moment diagram look like?

Relationship between Shear and Bending Moment

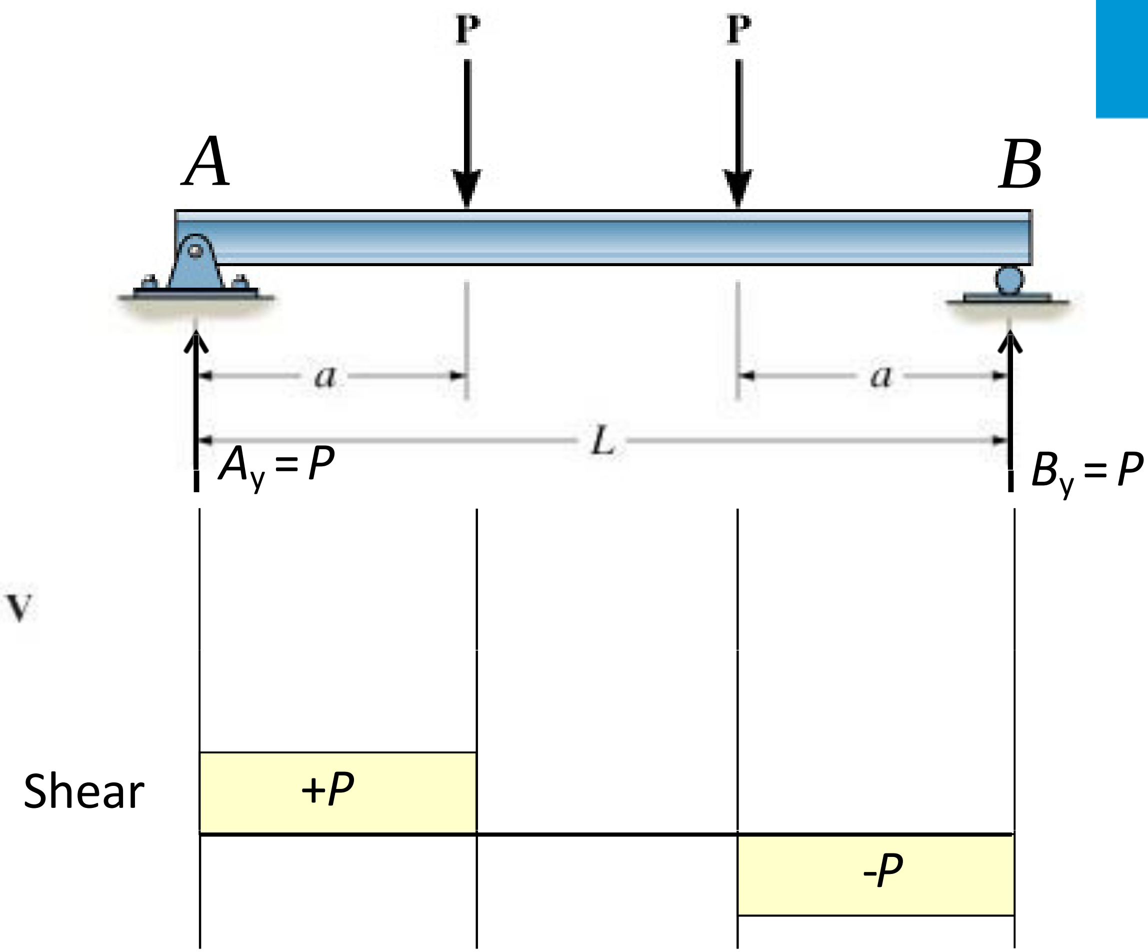
Draw the shear and bending moments diagrams for the simply supported beam.



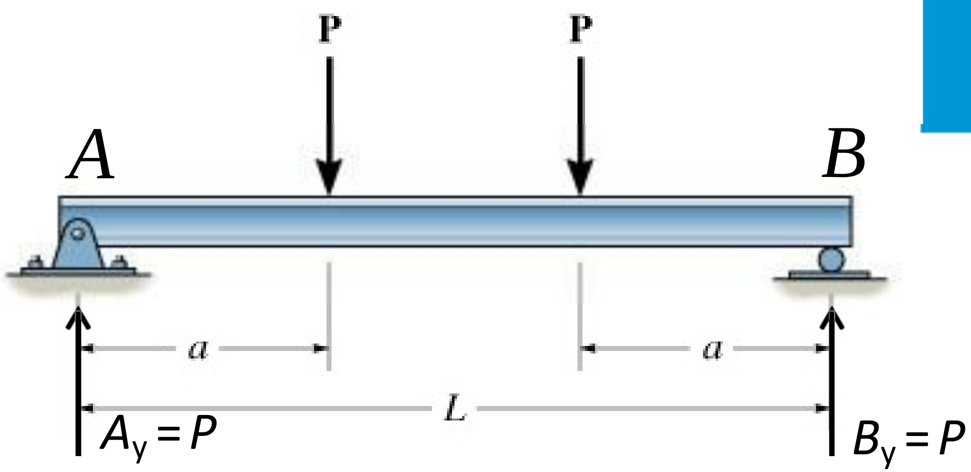
Relationship between Shear and Bending Moment

$$\frac{dV}{dx} = w(x)$$

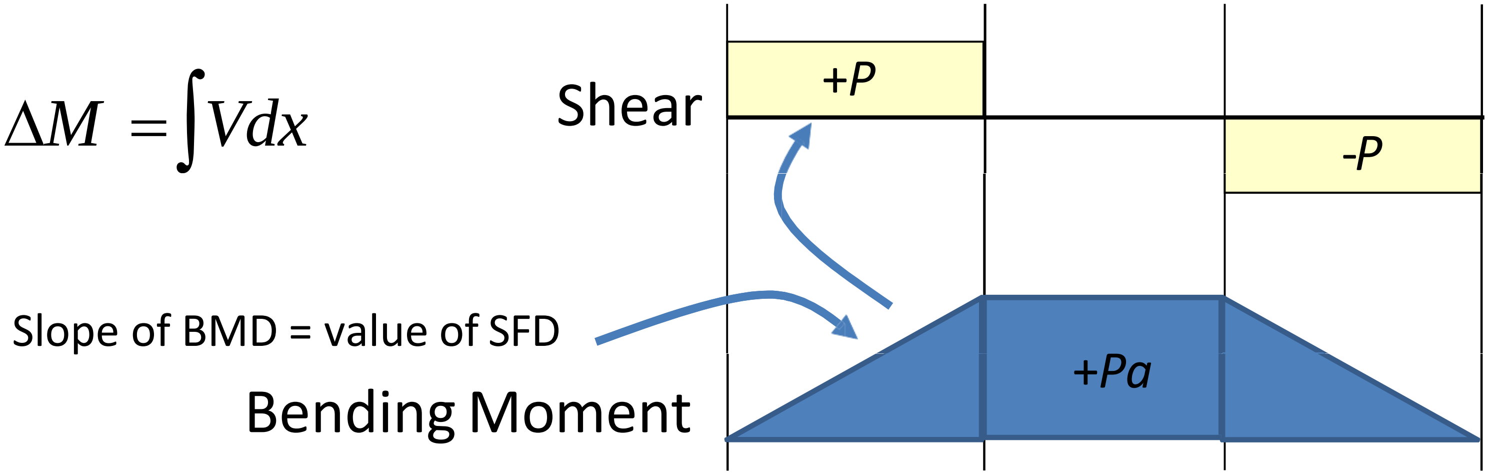
Positive shear



Relationship between Shear and Bending Moment

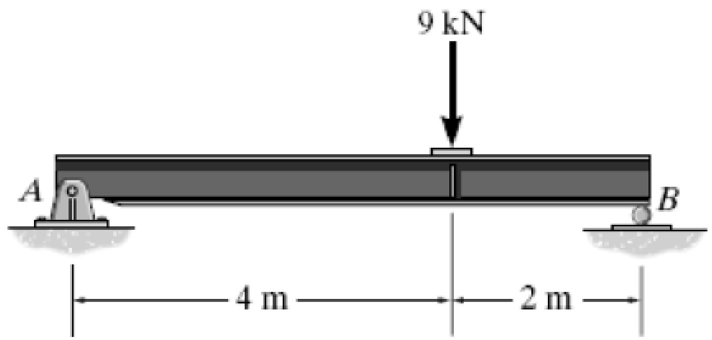


$$\Delta M = \int V dx$$

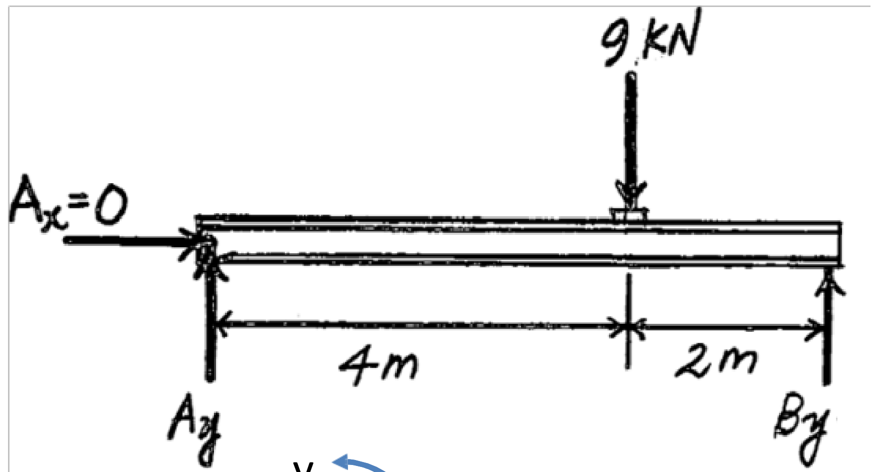


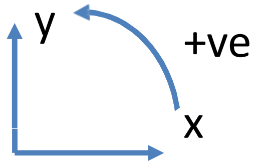
Relationship between Shear and Bending Moment

Example 1:



<u>1. Define</u>: To derive the shear and moment equations to draw the shear force and bending moment diagrams for the simply supported beam.





<u>2. Data</u>: Use FBD (Right).
Assumption - Beam does not have mass

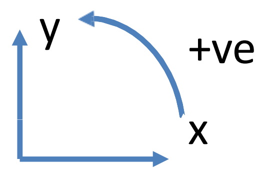
<u>3. Theory</u>: Internal forces in a structural member; Method of Sections for beam analysis.

Relationship between Shear and Bending Moment

4. <u>Estimate</u>:

- No distributed load so the shear force between either support and the point load must be constant.
- Beam sags under the point load, therefore the bending moment diagram is positive throughout.
- The bending moment must be zero at both supports, and maximum under the point load.
- Since the shear force is constant between either support and the point load, the bending moment varies linearly between them.

<u>5. Solve</u>: The first step is to determine the support reactions:

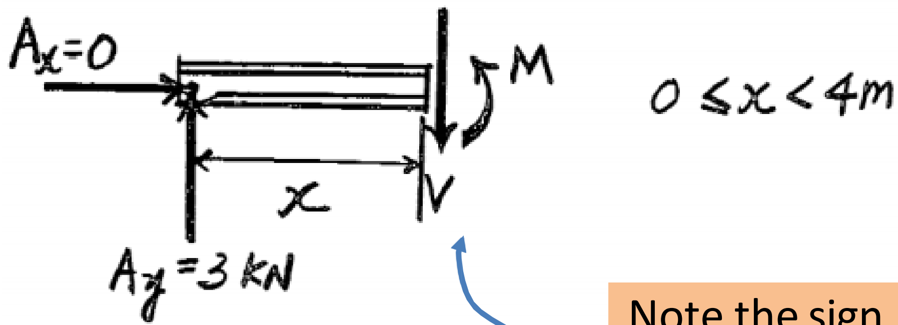


$$\Sigma M_B = -A_y(6\text{m}) + 9\text{ kN}(2\text{m}) = 0 \rightarrow A_y = +3\text{ kN}$$

$$\Sigma F_y = 3\text{ kN} - 9\text{ kN} + B_y = 0 \rightarrow By = +6\text{ kN}$$

Relationship between Shear and Bending Moment

For $0 \le x < 4$ m:

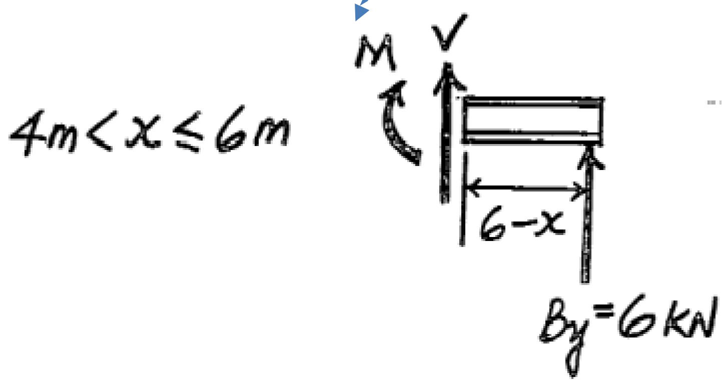


$\Sigma F_y = 3 \text{ kN} - V = 0 \rightarrow V = 3 \text{ kN} (\downarrow)$

$\Sigma M_A = M - (3 \text{ kN})x = 0 \rightarrow M = 3x \text{ kNm}$

Note the sign convention

For $4 < x \le 6$ m: Use the right hand FBD!

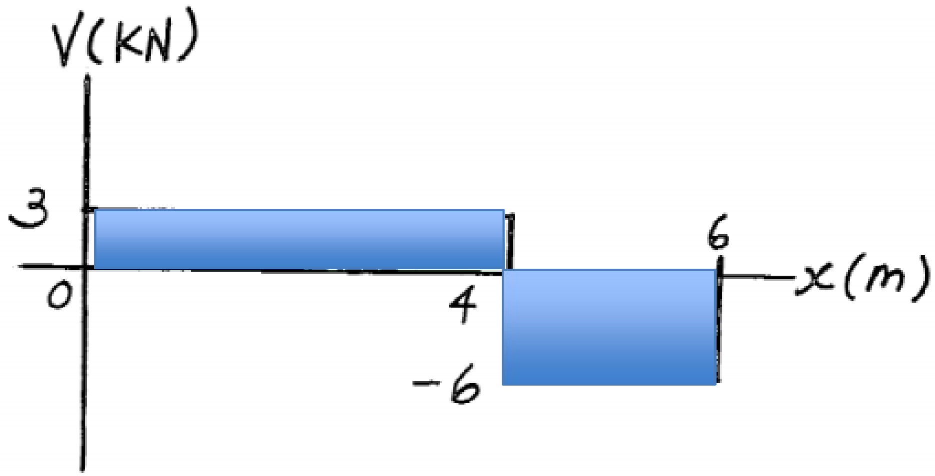


$\Sigma F_y = V + 6 \text{ kN} = 0 \rightarrow V = -6 \text{ kN} (\downarrow)$

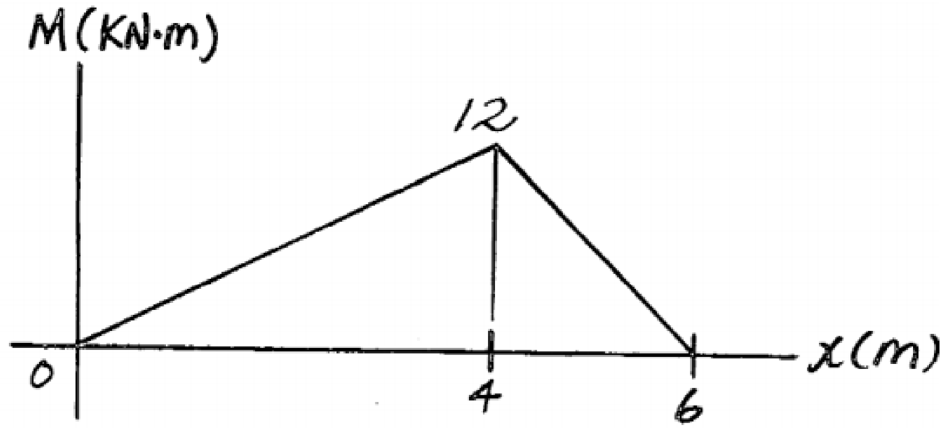
$\Sigma M_B = -M - (-6 \text{ kN})(6 - x) = 0 \rightarrow M = (36 - 6x) \text{ kNm}$

Relationship between Shear and Bending Moment

The shear force diagram:



The bending moment diagram:



Relationship between Shear and Bending Moment

6. <u>Verify</u>:

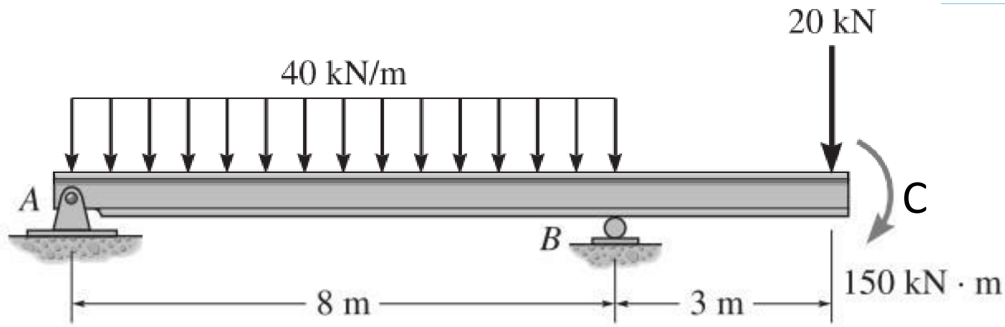
- The shear force between either support and the point load is constant as expected.
- As also expected, the bending moment is positive throughout, zero at both supports, and maximum under the point load.
- Consistent with the constant shear force, the bending moment varies linearly between either support and the point load.

How else could we verify the result?

Relationship between Shear and Bending Moment

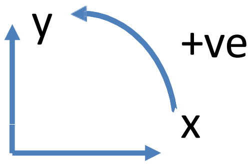
Example 2:

1. **<u>Define</u>**:
To use the relationships between distributed load, shear force and bending moment to draw the shear force and bending moment diagrams for the beam.

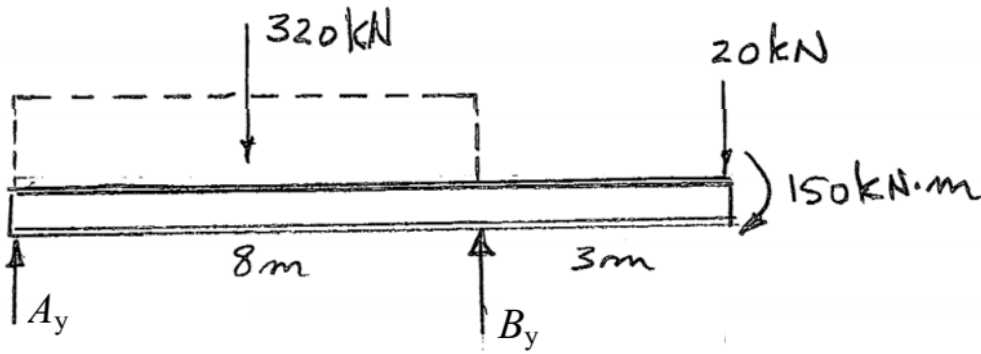


2. <u>Data</u>:

- Best described using the FBD.
- There are no horizontal components of forces.
- Beam has no mass.



(40 kN/m)(8 m) = 320 kN
Acting at centroid



Relationship between Shear and Bending Moment

Theory:

- Internal forces in a structural member;
- Relationship between distributed load, shear force and bending moment;
- Static equilibrium.

Estimate:

Due to the distributed load between A and B, the shear force varies linearly between supports A and B. However, the shear force is constant in the cantilevered span as there is no member loading other than at the tip, and must be 20 kN in magnitude.

It follows that the bending moment varies parabolically between supports A and B, and linearly in the cantilevered span as the shear force is the variation of the bending moment along the beam length.

At the tip of the cantilevered span, the bending moment is equal to the applied moment in magnitude, 150 kNm.

Relationship between Shear and Bending Moment

Solve:

Support Reactions:

$\Sigma M_B = -A_y(8m) + \{(40\ kN/m)(8m)\}(4m) - 20\ kN(3\ m) - 150\ kNm = 0$

$A_y = 133.75\ kN\ (\uparrow)$

$\Sigma F_y = 133.75\ kN - (40\ kN/m)(8m) - 20\ kN + B_y = 0$

$B_y = 206.25\ (\uparrow)$

Shear:

From an earlier example $V(x) = A_y - wx$

The shear force starts at support A at a magnitude of 133.75 kN, and decreases linearly with the distributed load of 40 kN/m. To the left of support B, the shear force reduces to:

$V_{B,\ left} = 133.75 - 40(8) = -186.25\ kN$

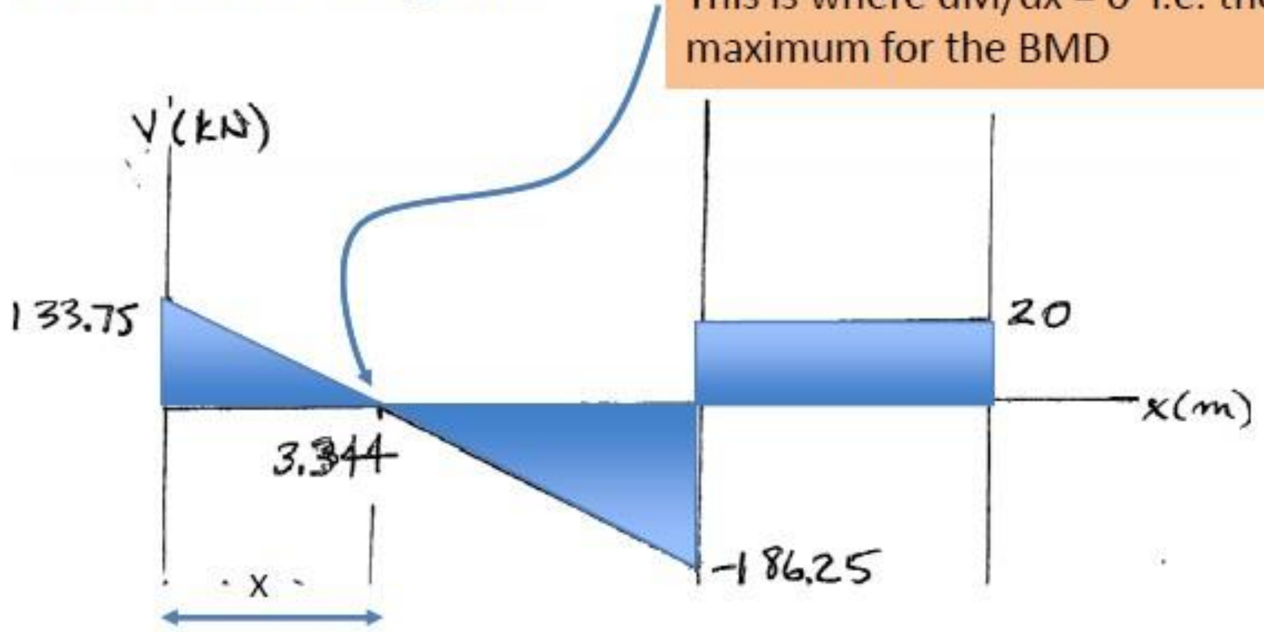
To the right of support B, the shear force increases to:

$V_{B,\ right} = -186.25 + 206.25 = 20\ kN$

Relationship between Shear and Bending Moment

Shear Force Diagram:

The point where V = 0 is very important.
This is where dM/dx = 0 i.e. the maximum for the BMD



Relationship between Shear and Bending Moment

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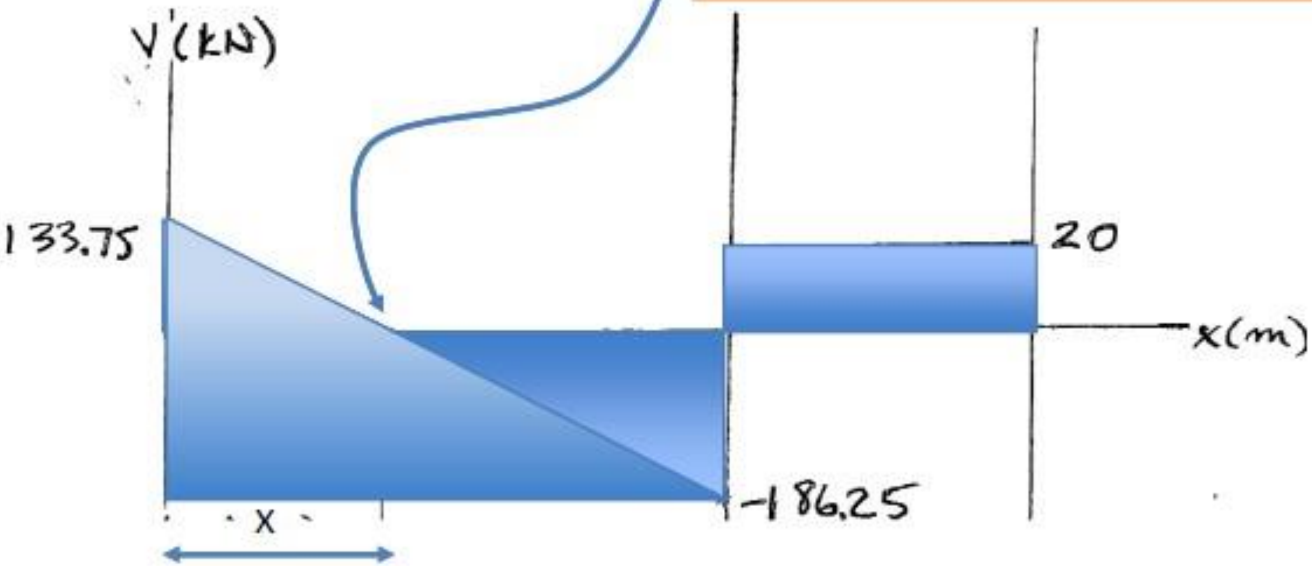


Similar triangles

$$\frac{133.75}{x} = \frac{40(8)}{8} \Rightarrow x = 3.344 \text{ m}$$

Relationship between Shear and Bending Moment

Bending Moment:

If V varies with x then M varies with x^2

The bending moment increases parabolically from zero at support A and becomes maximum where the shear force is zero,

Then the bending moment decreases parabolically until support B

The maximum positive bending moment is equal to the bending moment at support A plus the area of the shear force diagram between A and $x = 3.344$ m:

$$M_{\text{pos,max}} = 0 + 0.5(133.75)(3.344) = 223.6 \text{ kNm}$$

UOW

Relationship between Shear and Bending Moment

The bending moment at support B is equal to the maximum positive bending moment minus the area of the shear force diagram between x = 3.344 m and B:

$$M_B = 223.6 + 0.5(-186.25)(8 - 3.344) = -210 \text{ kNm}$$

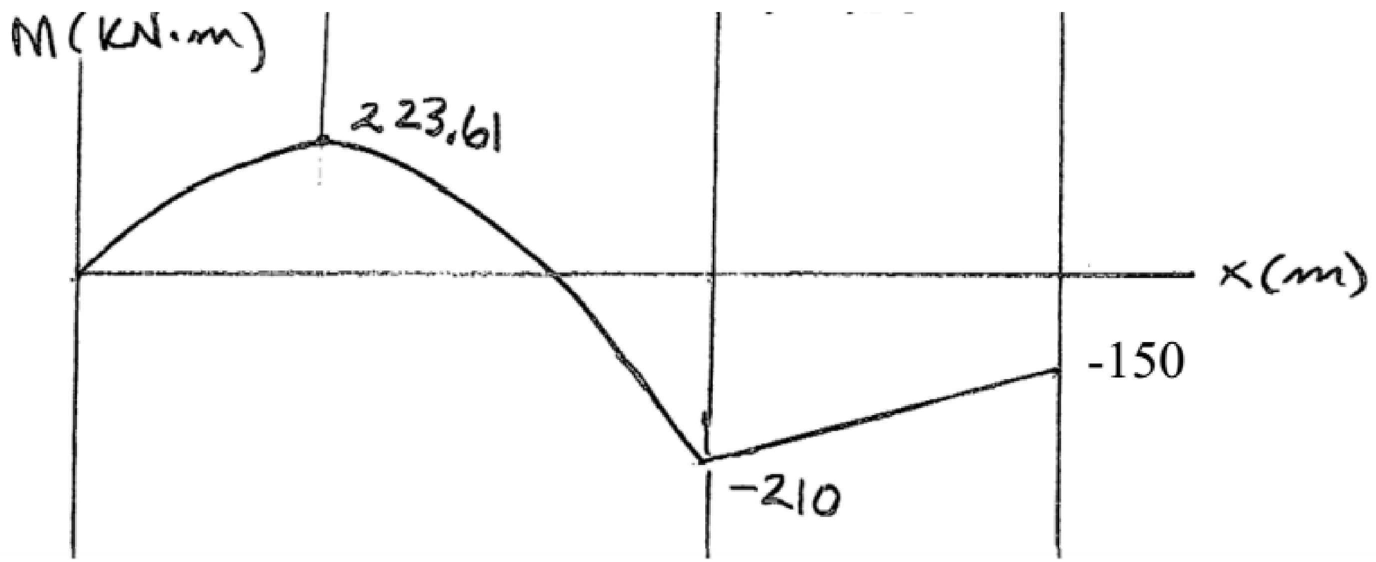
To find M at the free end we can either add the rectangle area to -210 kNm or draw a FBD for the cantilever section.

Mc = -210 +20*3 = -150 kNm

The shear force is constant and positive thus he BMD slope is constant and positive

Relationship between Shear and Bending Moment

Bending Moment Diagram:



Relationship between Shear and Bending Moment

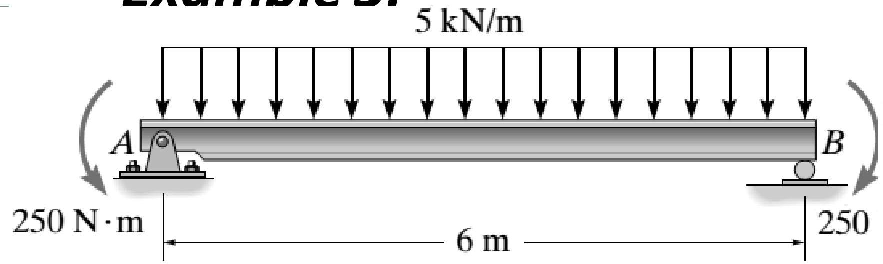
Verify:

- In the cantilevered span, the shear force is constant at 20 kN as expected.
- At the tip of the cantilevered span, the bending moment M_C is equal to the applied moment as expected.
- The maximum positive bending moment is located between supports A and B as expected.
- The bending moment at support B is the most negative as expected.

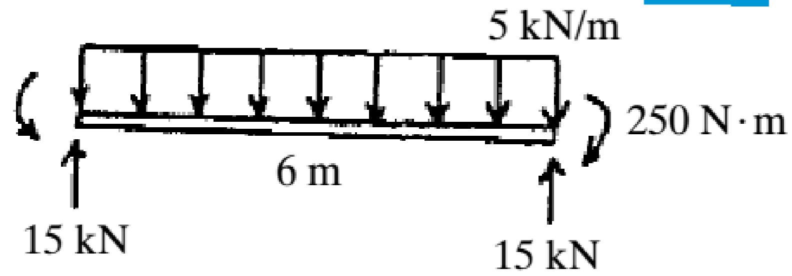
How else could we verify the result?

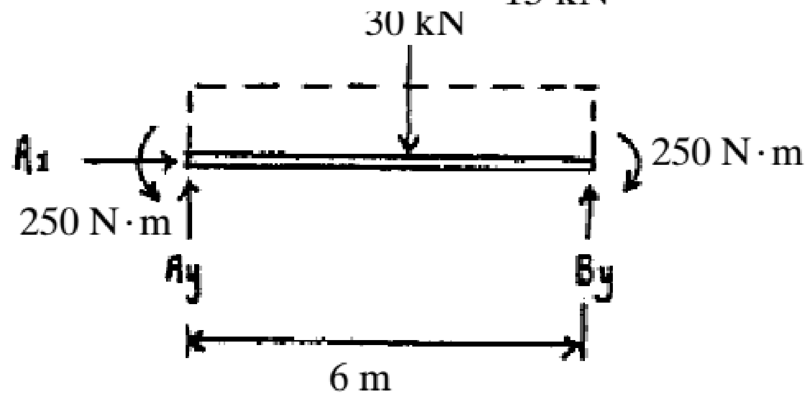
Relationship between Shear and Bending Moment

Example 3:



Find support reactions and draw FBD:





$$\sum M_A = 0 \Rightarrow +250 - 30 * 3 + 6B_y - 250 = 0$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -30\,(3) + B_y\,(6) = 0$$

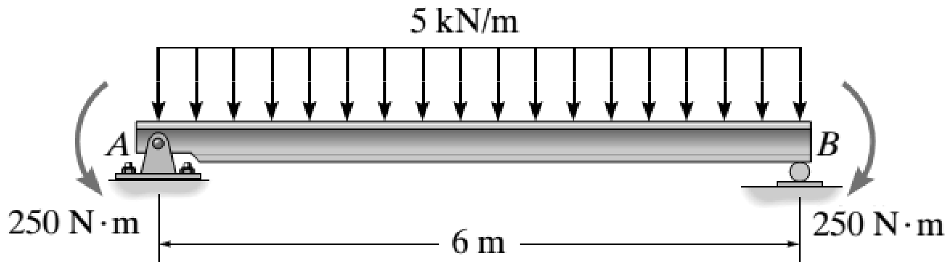
$$B_y = 15 \text{ kN}$$

Similarly Ay = +15 kN

Relationship between Shear and Bending Moment

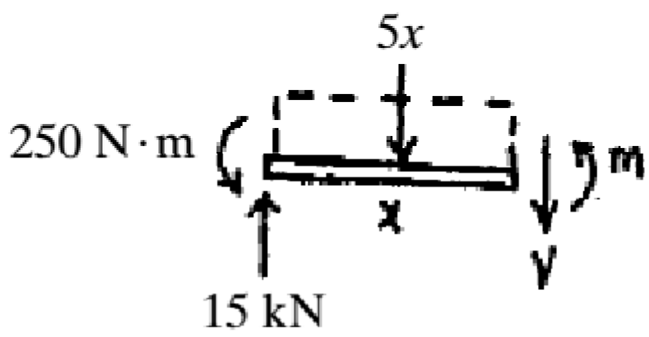
Example 3:

250 Nm = 0.250 kNm



Find shear and moment equations:

V = Ay – wx! Same as previous example!



$$\xrightarrow{+} \Sigma F_x = 0; \qquad 15 - 5x - V = 0$$

$$V = 5\,(3 - x)$$

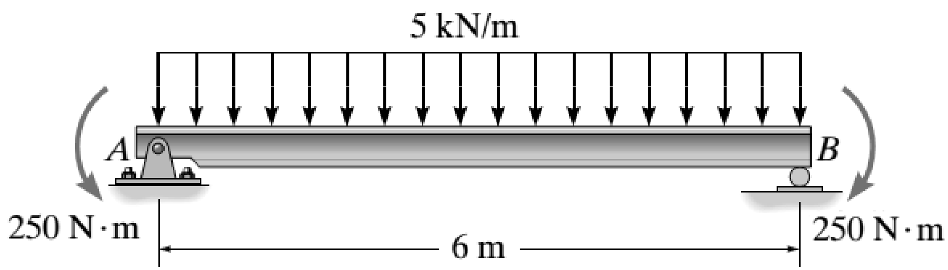
$$\circlearrowleft + \Sigma M = 0; \qquad -15\,(x) + 0.250 + 5\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{1}{2}\,(30x - 5x^2 - 0.5)$$

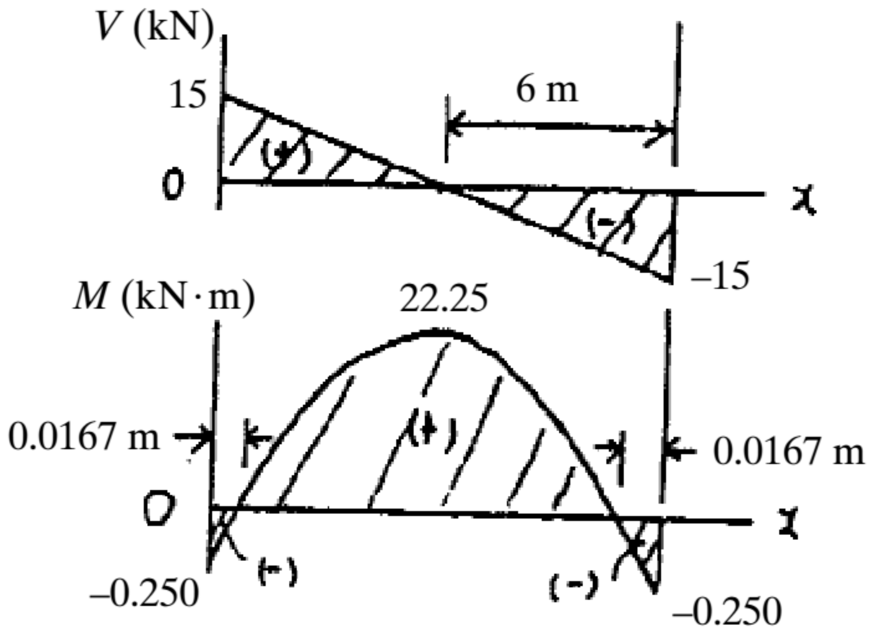
$$M = 15x - 2.5x^2 - 0.25$$

Relationship between Shear and Bending Moment

Example 3:



Draw shear force and bending moment diagrams:



The balanced applied moments have no effect on SFD and cause the BMD to be offset by -0.25 kNm

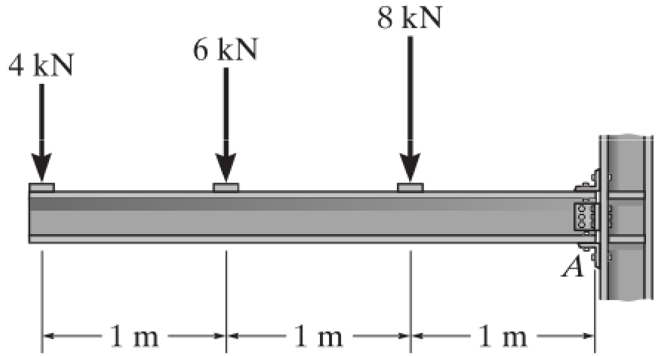
Important findings today

- Beams carrying load have internal forces and moments
- Shear force is the internal vertical force
- Bending moment in the internal moment
- Shear force and bending moment are linked
- $dM/dx = V$
- M = area of the Shear Force Diagram
- When $V = 0$ we have M_{max}
- Positive bending moment = happy face

Relationship between Shear and Bending Moment

Example 4:

Draw shear force and bending moment diagrams for the beam:

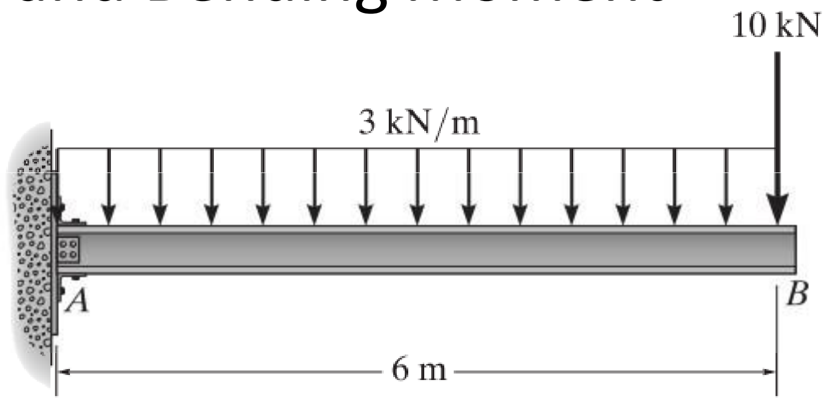


Try this at home and use bending moment calculator to check you answer

Relationship between Shear and Bending Moment

Example 5:

Draw shear force and bending moment diagrams for the beam:



Try this at home and use bending moment calculator to check you answer