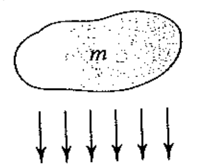
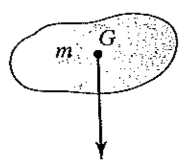
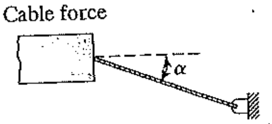
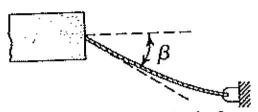
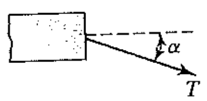
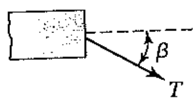
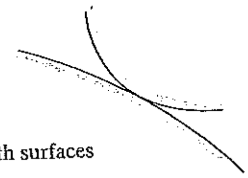
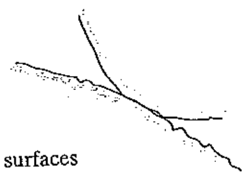
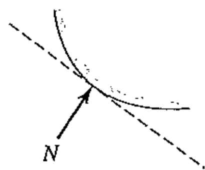
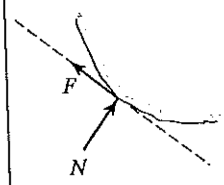
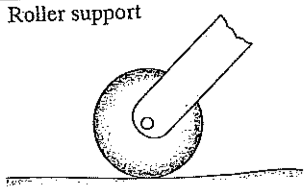
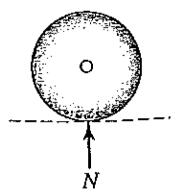
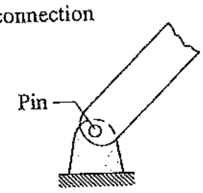
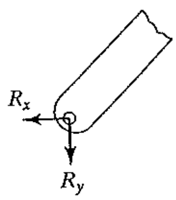
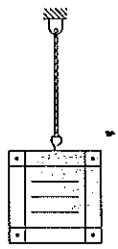
Free-body diagrams for some of the most common force configurations are illustrated in Figure 4.17.

Figure 4.17 Free-body diagrams for some common force configurations.

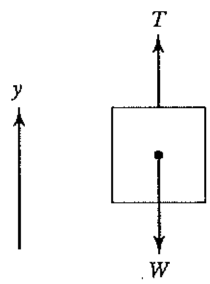
Configuration	Free-body diagram	Comments
Gravitational force 	 $W = mg$	The gravitational force acts through the center of gravity G.
Cable force  Weight of cable neglected  Weight of cable included	 	The tension force T in a cable is always directed along the axis of the cable.
Contact force  Smooth surfaces  Rough surfaces	 	For smooth surfaces, the contact force N is toward the body, normal to the tangent drawn through the point of contact. For rough surfaces, there are two forces, a normal force N and a friction force, F. These two forces are perpendicular to each other. The friction force F acts in the direction opposing the impeding motion.
Roller support 		A roller supports a normal force but no friction force because a friction force would cause the roller to rotate.
Pin connection 		A pin connection can support a reaction force in any direction in the plane normal to the pin's axis. This force may be resolved into its x and y components, R_x and R_y.

PRACTICE!

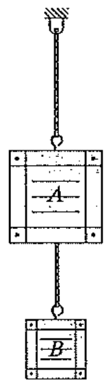
1. A crate hangs by a rope as shown. Construct a free-body diagram of the crate.



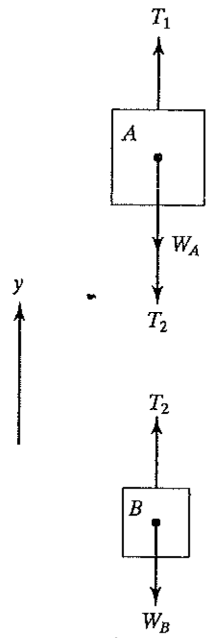
Answer:



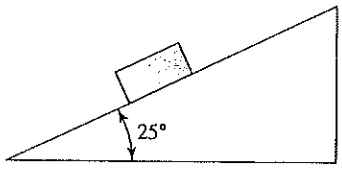
2. Two crates hang by ropes from a ceiling as shown. Construct a free-body diagram of (a) crate A and (b) crate B.



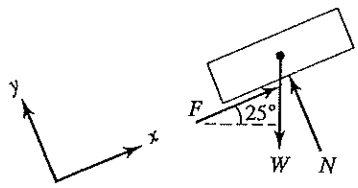
Answer:



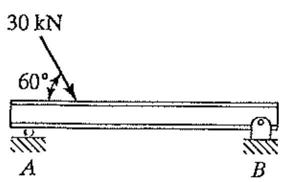
3. A wooden block rests on a rough inclined plane as shown. Construct a free-body diagram of the block.



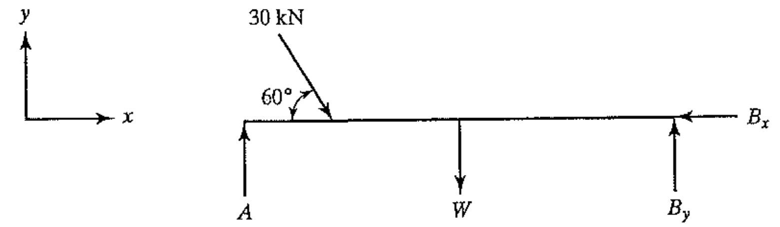
Answer:



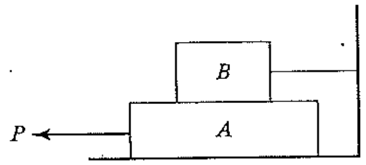
4. An obliquely loaded I-beam is supported by a roller at A and a pin at B as shown. Construct a free-body diagram of the beam. Include the weight of the beam.



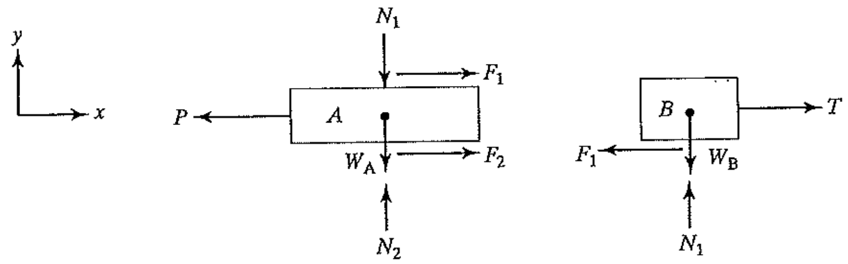
Answer:



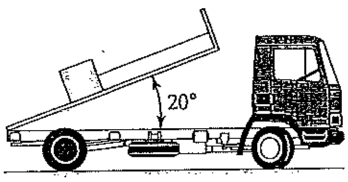
5. A horizontal pulling force P acts on block A as shown. Block B, which rests on block A, is tied to a rigid wall by a cable. The force P is not sufficient to cause block A to move. If all surfaces are rough, construct a free-body diagram of each block.



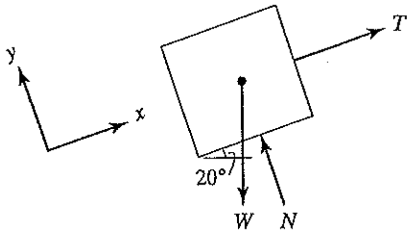
Answer:



6. A box is held in position on the bed of a truck by a cable as shown. The surface of the truck bed is smooth. Construct a free-body diagram of the box.



Answer:



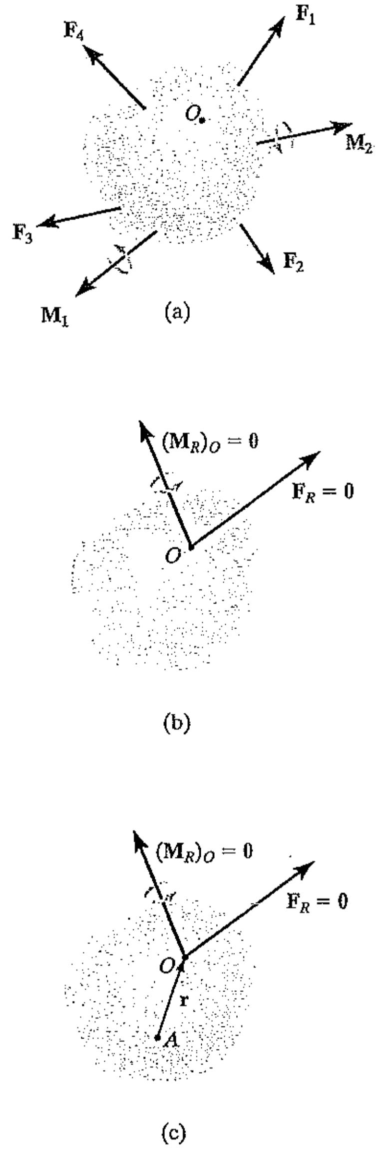


Fig. 5–1

Using the methods of the previous chapter, the force and couple moment system acting on a body can be reduced to an equivalent resultant force and resultant couple moment at any arbitrary point O on or off the body, Fig. 5–1*b*. If this resultant force and couple moment are both equal to zero, then the body is said to be in *equilibrium*. Mathematically, the equilibrium of a body is expressed as

$$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O = \mathbf{0} \tag{5-1}$$

The first of these equations states that the sum of the forces acting on the body is equal to *zero*. The second equation states that the sum of the moments of all the forces in the system about point O, added to all the couple moments, is equal to *zero*. These two equations are not only necessary for equilibrium, they are also sufficient. To show this, consider summing moments about some other point, such as point A in Fig. 5–1c. We require

$$\Sigma \mathbf{M}_A = \mathbf{r} \times \mathbf{F}_R + (\mathbf{M}_R)_O = \mathbf{0}$$

Since $\mathbf{r} \neq \mathbf{0}$, this equation is satisfied if Eqs. 5–1 are satisfied, namely $\mathbf{F}_R = \mathbf{0}$ and $(\mathbf{M}_R)_O = \mathbf{0}$.

When applying the equations of equilibrium, we will assume that the body remains rigid. In reality, however, all bodies deform when subjected to loads. Although this is the case, most engineering materials such as steel and concrete are very rigid and so their deformation is usually very small. Therefore, when applying the equations of equilibrium, we can generally assume that the body will remain *rigid* and *not deform* under the applied load without introducing any significant error. This way the direction of the applied forces and their moment arms with respect to a fixed reference remain the same both before and after the body is loaded.

EQUILIBRIUM IN TWO DIMENSIONS

In the first part of the chapter, we will consider the case where the force system acting on a rigid body lies in or may be projected onto a *single* plane and, furthermore, any couple moments acting on the body are directed perpendicular to this plane. This type of force and couple system is often referred to as a two-dimensional or *coplanar* force system. For example, the airplane in Fig. 5–2 has a plane of symmetry through its center axis, and so the loads acting on the airplane are symmetrical with respect to this plane. Thus, each of the two wing tires will support the same load $\mathbf{T}$, which is represented on the side (two-dimensional) view of the plane as $2\mathbf{T}$.

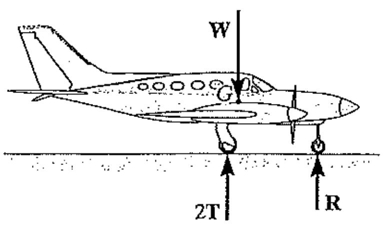


Fig. 5–2

5.2 Free-Body Diagrams

Successful application of the equations of equilibrium requires a complete specification of *all* the known and unknown external forces that act *on* the body. The best way to account for these forces is to draw a free-body diagram. This diagram is a sketch of the outlined shape of the body, which represents it as being *isolated* or "free" from its surroundings, i.e., a "free body." On this sketch it is necessary to show *all* the forces and couple moments that the surroundings exert *on the body* so that these effects can be accounted for when the equations of equilibrium are applied. *A thorough understanding of how to draw a free-body diagram is of primary importance for solving problems in mechanics.*

Support Reactions. Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of contact between bodies subjected to coplanar force systems. As a general rule,

- If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.
- If rotation is prevented, a couple moment is exerted on the body.

For example, let us consider three ways in which a horizontal member, such as a beam, is supported at its end. One method consists of a *roller* or cylinder, Fig. 5–3*a*. Since this support only prevents the beam from *translating* in the vertical direction, the roller will only exert a *force* on the beam in this direction, Fig. 5–3*b*.

The beam can be supported in a more restrictive manner by using a *pin*, Fig. 5–3*c*. The pin passes through a hole in the beam and two leaves which are fixed to the ground. Here the pin can prevent *translation* of the beam in *any direction* ϕ, Fig. 5–3*d*, and so the pin must exert a *force* $\mathbf{F}$ on the beam in this direction. For purposes of analysis, it is generally easier to represent this resultant force $\mathbf{F}$ by its two rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$, Fig. 5-3*e*. If F_x and F_y are known, then F and ϕ can be calculated.

The most restrictive way to support the beam would be to use a *fixed support* as shown in Fig. 5–3*f*. This support will prevent both *translation and rotation* of the beam. To do this a *force and couple moment* must be developed on the beam at its point of connection, Fig. 5–3*g*. As in the case of the pin, the force is usually represented by its rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$.

Table 5–1 lists other common types of supports for bodies subjected to coplanar force systems. (In all cases the angle θ is assumed to be known.) Carefully study each of the symbols used to represent these supports and the types of reactions they exert on their contacting members.

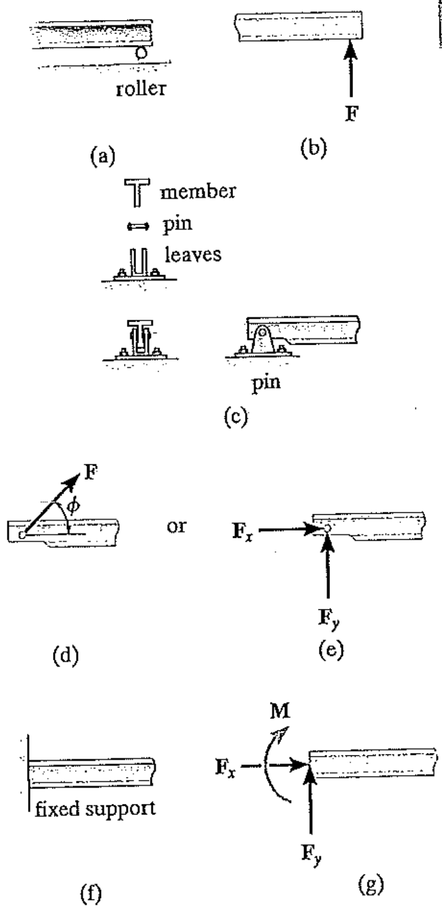
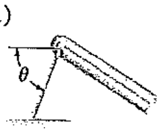
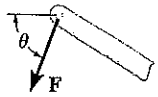
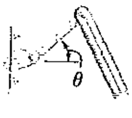
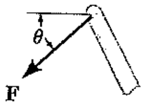
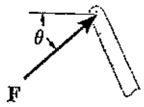
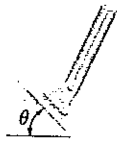
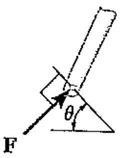
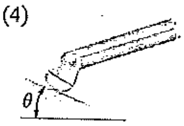
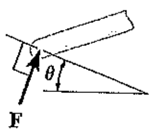
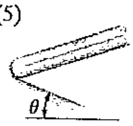
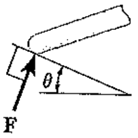
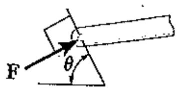
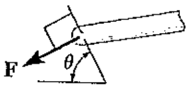
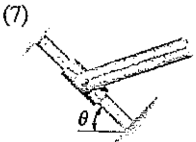
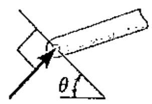
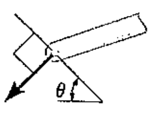


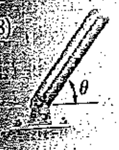
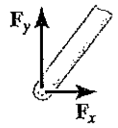
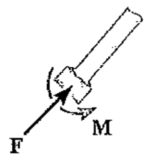
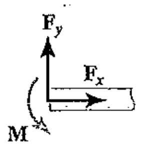
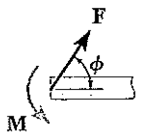
Fig. 5-3

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

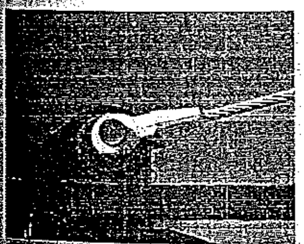
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(5)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

continued

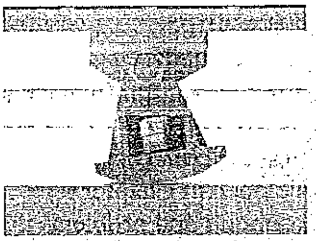
TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Typical examples of actual supports are shown in the following sequence of photos. The numbers refer to the connection types in Table 5–1.

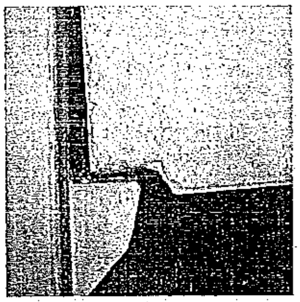


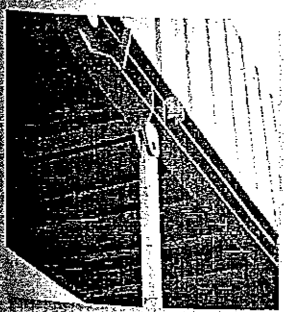
The cable exerts a force on the bracket in the direction of the cable. (1)



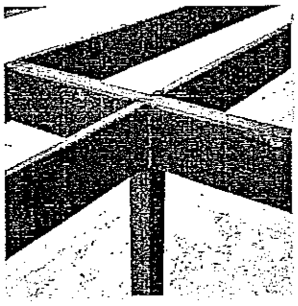
The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature. (4)

This concrete girder rests on the ledge that is assumed to act as a smooth contacting surface. (5)



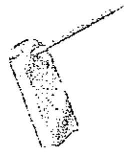
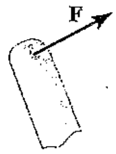
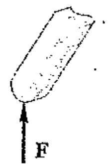
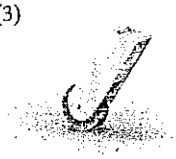
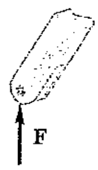
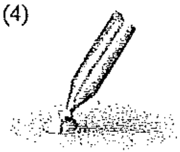
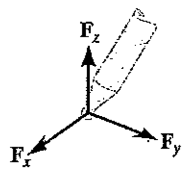
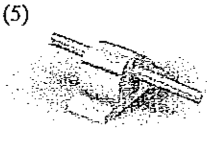
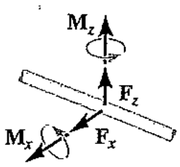


This utility building is pin supported at the top of the column. (8)



The floor beams of this building are welded together and thus form fixed connections. (10)

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts awa from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and twc couple-moment components which act perpendicular tc the shaft. Note: The couple moments are *generally noi applied* if the body is supported elsewhere. See the examples.

continu

of each block, and the tension T_2. Using the free-body diagram in Figure 4.20(b), we have:

$$\Sigma F_y = 0 = T_2 - W_A - W_B$$
$$= T_2 - (25 \text{ kg} + 40 \text{ kg})(9.81 \text{ m/s}^2)$$

which yields:

$$T_2 = 637.65 \text{ N}.$$

EXAMPLE 4.5

Problem statement

A 200 kg engine block hangs from a system of cables as shown in Figure 4.21. Find the tension in cables AB and AC. Cable AB is horizontal.

Diagram

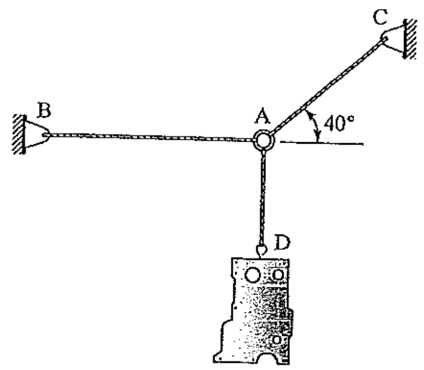


Figure 4.21
Suspended engine block for Example 4.5.

We have a coplanar force system in which the force in each cable acts concurrently at A, so we construct a free-body diagram for a "particle" at A. (See Figure 4.22.) The tension force in cable AB acts to the left along the x-axis, and the tension force in cable AC acts along a line 40° with respect to the x-axis. The tension force in cable AD, which is equivalent to the engine block's weight, acts straight down.

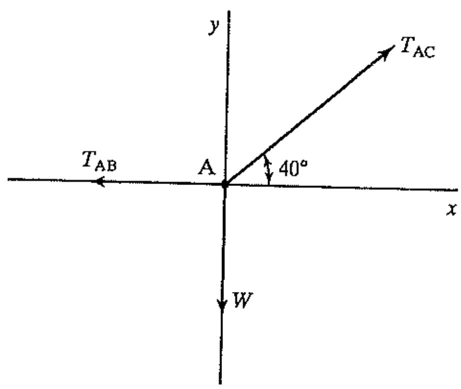


Figure 4.22
Free-body diagram for Example 4.5.

Assumptions

1. All forces are concurrent at A.
2. The weights of the cables are negligible.
3. All cables are taut.

Governing equations

The governing equations are Newton's second law and the equations of equilibrium in the x and y directions:

$$W = mg$$
$$\sum F_x = 0$$
$$\sum F_y = 0.$$

Calculations

Using the free-body diagram in Figure 4.22, the equations of equilibrium are:

$$\sum F_x = 0 = -T_{AB} + T_{AC} \cos 40°$$
$$\sum F_y = 0 = T_{AC} \sin 40° - W$$

where $W = mg = (200\text{ kg})(9.81\text{ m/s}^2) = 1962\text{ N}$. The second equation can be immediately solved for T_{AC}:

$$T_{AC} = \underline{\underline{3052\text{ N}}}.$$

Substituting this value of T_{AC} into the first equation and solving for T_{AB}, we get:

$$T_{AB} = \underline{\underline{2338\text{ N}}}.$$

Solution check

To verify that our answers are correct, we substitute them back into the equilibrium equations. If they satisfy the equations, they are correct.

$$\sum F_x = -2338\text{ N} + (3052\text{ N}) \cos 40° = -0.032 \approx 0$$
$$\sum F_y = (3052\text{ N}) \sin 40° - (200\text{ kg})(9.81\text{ m/s}^2) = -0.212 \approx 0.$$

Within the numerical precision of the calculations, the sum of the forces in the-x and y directions is zero. Our answers are therefore correct.

Discussion

Now that we know the tension forces in the cables, what do we do with them? Knowing the forces per se does not tell us how the cables perform structurally. The next step in the analysis would be to determine the stress in each cable. If the calculated stresses are less than an allowable or design stress, the cables will support the engine block without experiencing failure. In this situation, failure most likely means cable breakage, but may also mean permanent cable strain. Stress and strain would have to be calculated in order to make a full structural assessment of the cables.

EXAMPLE 5.9

Please refer to the Companion Website for the animation: *Free-Body Diagram for a Beam on Slanting Support*

Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at the roller B in Fig. 5–16*a*.

SOLUTION

Free-Body Diagram. The free-body diagram is shown in Fig. 5–16*b*. The pin at A exerts two components of reaction on the member, $\mathbf{A}_x$ and $\mathbf{A}_y$.

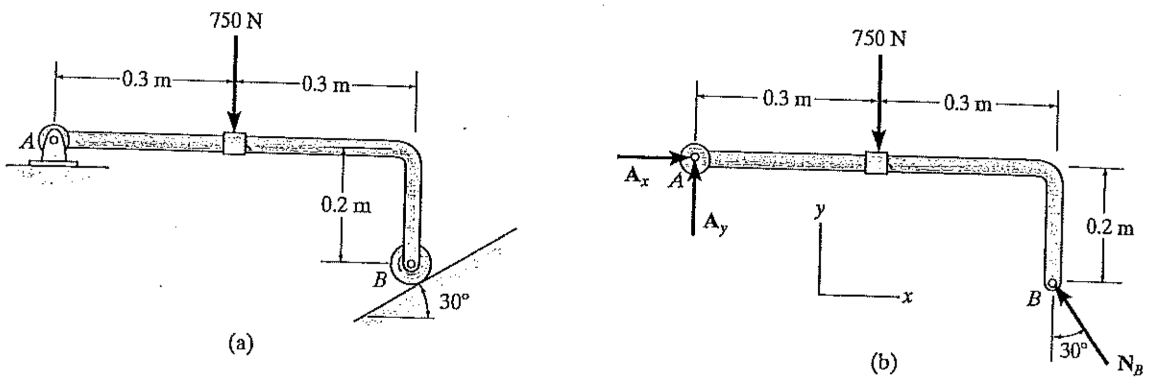
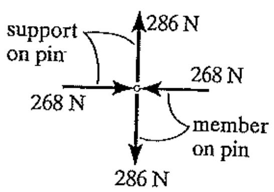


Fig. 5–16



(c)

Equations of Equilibrium. The reaction N_B can be obtained *directly* by summing moments about point A, since $\mathbf{A}_x$ and $\mathbf{A}_y$ produce no moment about A.

$$\circlearrowleft + \Sigma M_A = 0;$$

$$[N_B \cos 30°](0.6 \text{ m}) - [N_B \sin 30°](0.2 \text{ m}) - 750 \text{ N}(0.3 \text{ m}) = 0$$

$$N_B = 536.2 \text{ N} = 536 \text{ N} \qquad \textit{Ans.}$$

Using this result,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - (536.2 \text{ N}) \sin 30° = 0$$

$$A_x = 268 \text{ N} \qquad \textit{Ans.}$$

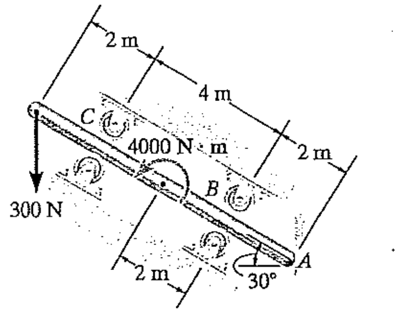
$$+\uparrow \Sigma F_y = 0; \qquad A_y + (536.2 \text{ N}) \cos 30° - 750 \text{ N} = 0$$

$$A_y = 286 \text{ N} \qquad \textit{Ans.}$$

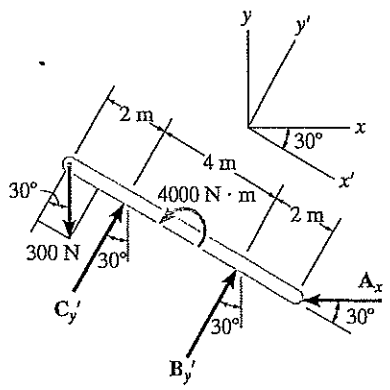
Details of the equilibrium of the pin at A are shown in Fig. 5–16*c*.

EXAMPLE 5-10

The uniform smooth rod shown in Fig. 5–17*a* is subjected to a force and couple moment. If the rod is supported at *A* by a smooth wall and at *B* and *C* either at the top or bottom by rollers, determine the reactions at these supports. Neglect the weight of the rod.



(a)



(b)

Fig. 5–17

SOLUTION

Free-Body Diagram. As shown in Fig. 5–17*b*, all the support reactions act normal to the surfaces of contact since these surfaces are smooth. The reactions at *B* and *C* are shown acting in the positive y' direction. This assumes that only the rollers located on the bottom of the rod are used for support.

Equations of Equilibrium. Using the *x*, *y* coordinate system in Fig. 5–17*b*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad C_{y'} \sin 30° + B_{y'} \sin 30° - A_x = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad -300\text{ N} + C_{y'} \cos 30° + B_{y'} \cos 30° = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -B_{y'}(2\text{ m}) + 4000\text{ N}\cdot\text{m} - C_{y'}(6\text{ m}) + (300 \cos 30°\text{ N})(8\text{ m}) = 0 \qquad (3)$$

When writing the moment equation, it should be noted that the line of action of the force component 300 sin 30° N passes through point *A*, and therefore this force is not included in the moment equation.

Solving Eqs. 2 and 3 simultaneously, we obtain

$$B_{y'} = -1000.0\text{ N} = -1\text{ kN} \qquad \textit{Ans.}$$

$$C_{y'} = 1346.4\text{ N} = 1.35\text{ kN} \qquad \textit{Ans.}$$

Since $B_{y'}$ is a negative scalar, the sense of $\mathbf{B}_{y'}$ is opposite to that shown on the free-body diagram in Fig. 5–17*b*. Therefore, the top roller at *B* serves as the support rather than the bottom one. Retaining the negative sign for $B_{y'}$ (Why?) and substituting the results into Eq. 1, we obtain

$$1346.4 \sin 30°\text{ N} + (-1000.0 \sin 30°\text{ N}) - A_x = 0$$

$$A_x = 173\text{ N} \qquad \textit{Ans.}$$