Engineering Mechanics: Statics in SI Units, 12e

6

Structural Analysis

Chapter Objectives

- Determine the forces in the members of a truss using the method of joints and the method of sections

Chapter Outline

1. Simple Trusses
2. The Method of Joints
3. Zero-Force Members
4. The Method of Sections

6.1 Simple Trusses

- A truss composed of slender members joined together at their end points

Planar Trusses

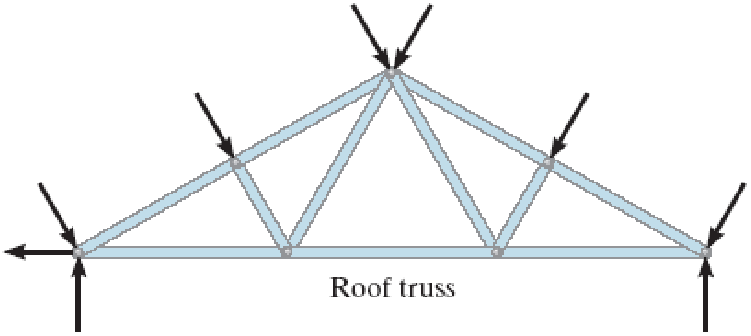
- Planar trusses used to support roofs and bridges
- Roof load is transmitted to the truss at joints by means of a series of purlins

(a)

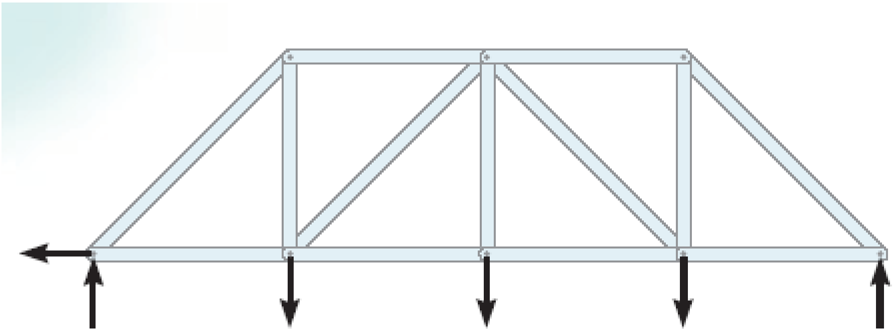
6.1 Simple Trusses

Planar Trusses

- The analysis of the forces developed in the truss members is 2D
- Similar to roof truss, the bridge truss loading is also coplanar



Roof truss



Bridge truss

6.1 Simple Trusses

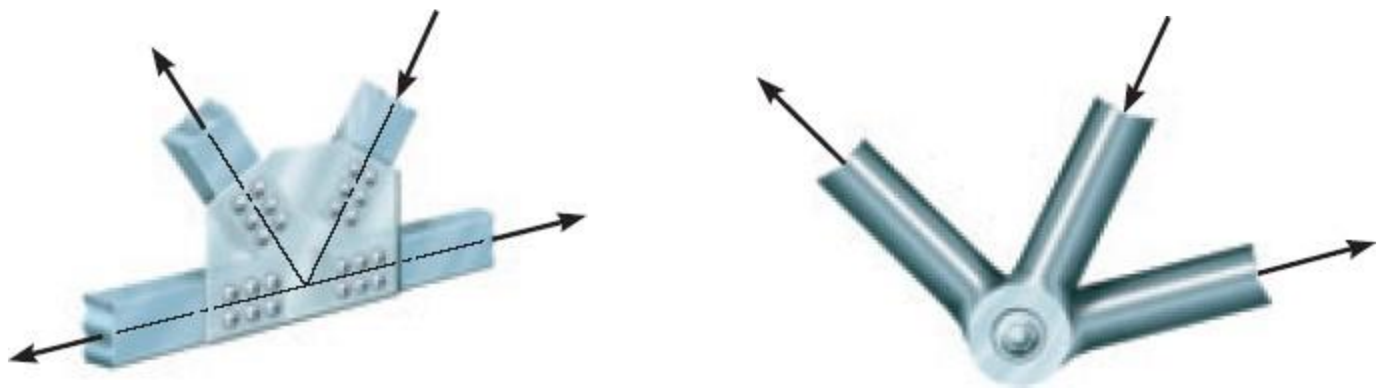
Assumptions for Design

1. "All loadings are applied at the joint"

   - Weight of the members neglected

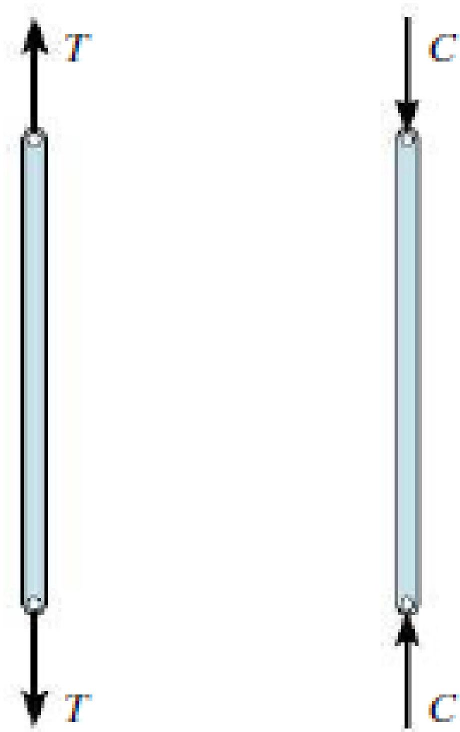
2. "The members are joined together by smooth pins"

   - Assume connections provided the center lines of the joining members are *concurrent*



6.1 Simple Trusses

- Each member in a truss is a two-force member.
- The end forces must be equal in magnitude and opposite each other
- When loaded, the member is either in tension or compression.



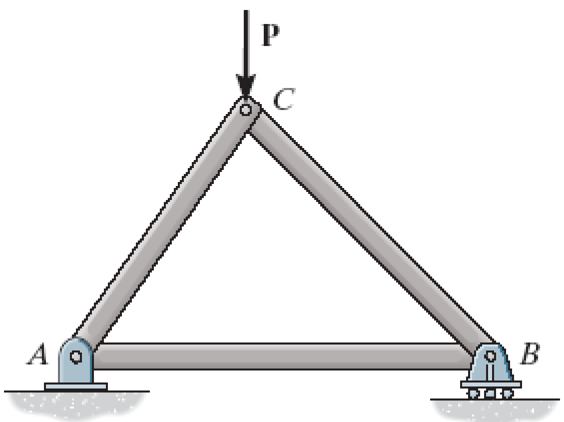
Tension
(a)

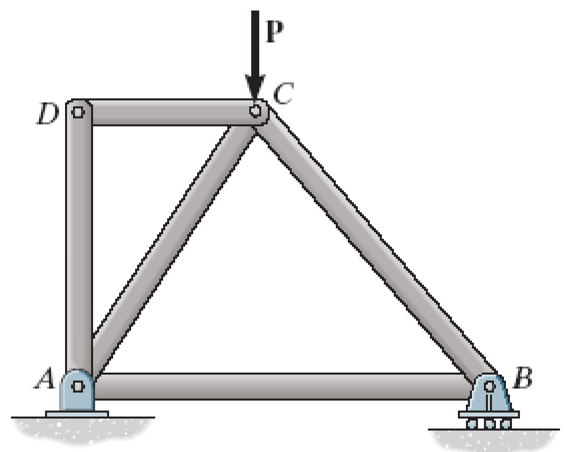
Compression
(b)

6.1 Simple Trusses

Simple Truss

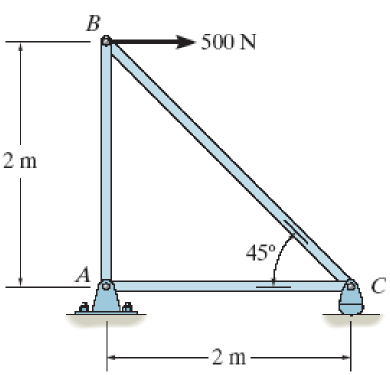
- Form of a truss must be rigid to prevent collapse
- The simplest form that is rigid or stable is a triangle

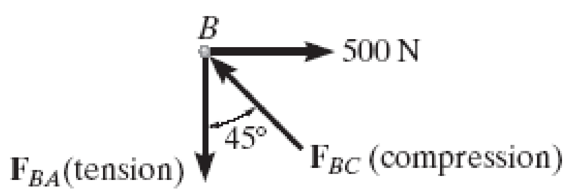


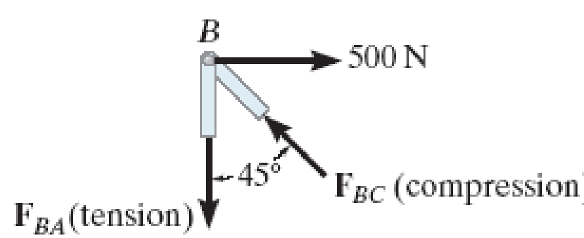


6.2 The Method of Joints

- For truss, we need to know the force in each members
- Forces in the members are internal forces
- For external force members, equations of equilibrium can be applied
- Force system acting at each joint is coplanar and concurrent
- $\sum F_x = 0$ and $\sum F_y = 0$ must be satisfied for equilibrium







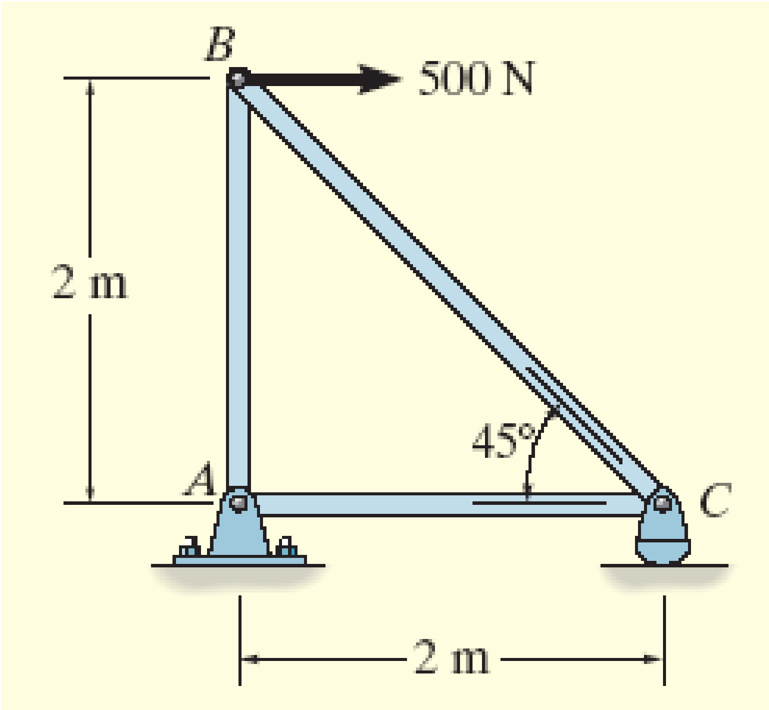
6.2 The Method of Joints

Procedure for Analysis

- Find the external reactions at the truss support
- Draw the FBD with at least 1 known and 2 unknown forces
- Determine the correct direction of the member
- Orient the x and y axes
- Apply $\sum F_x = 0$ and $\sum F_y = 0$
- Use known force to analyze the unknown forces

Example 6.1

Determine the force in each member of the truss and indicate whether the members are in tension or compression.



Solution

- 2 unknown member forces at joint B
- 1 unknown reaction force at joint C
- 2 unknown member forces and 2 unknown reaction forces at point A

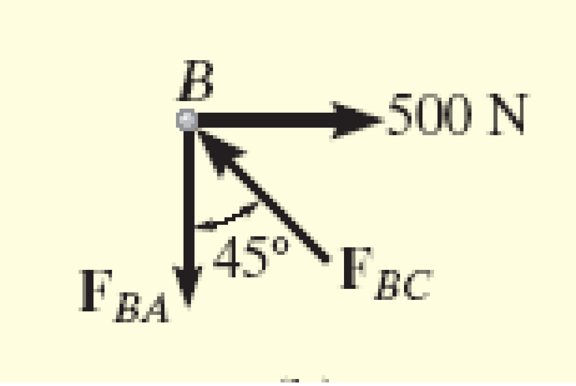
For Joint B,

$$+\rightarrow \sum F_x = 0;$$

$$500N - F_{BC} \sin 45^\circ N = 0 \Rightarrow F_{BC} = 707.1N(C)$$

$$+\uparrow \sum F_y = 0;$$

$$F_{BC} \cos 45^\circ N - F_{BA} = 0 \Rightarrow F_{BA} = 500N(T)$$



Solution

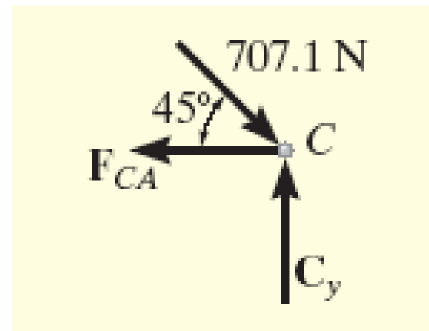
For Joint C,

$$+\rightarrow \sum F_x = 0;$$

$$-F_{CA} + 707.1\cos 45^\circ N = 0 \Rightarrow F_{CA} = 500N(T)$$

$$+\uparrow \sum F_y = 0;$$

$$C_y - 707.1\sin 45^\circ N = 0 \Rightarrow C_y = 500N$$



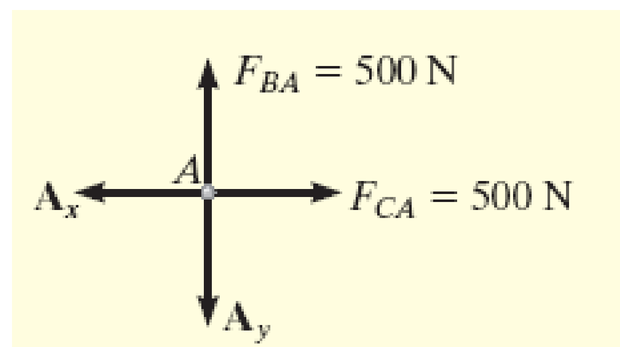
For Joint A,

$$+\rightarrow \sum F_x = 0;$$

$$500N - A_x = 0 \Rightarrow A_x = 500N$$

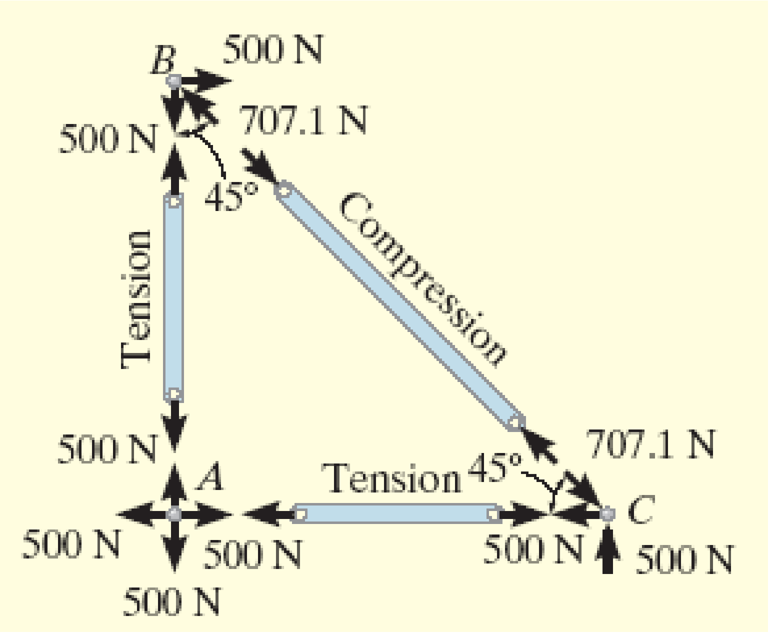
$$+\uparrow \sum F_y = 0;$$

$$500N - A_y = 0 \Rightarrow A_y = 500N$$



Solution

- FBD of each pin shows the effect of all the connected members and external forces applied to the pin
- FBD of each member shows only the effect of the end pins on the member

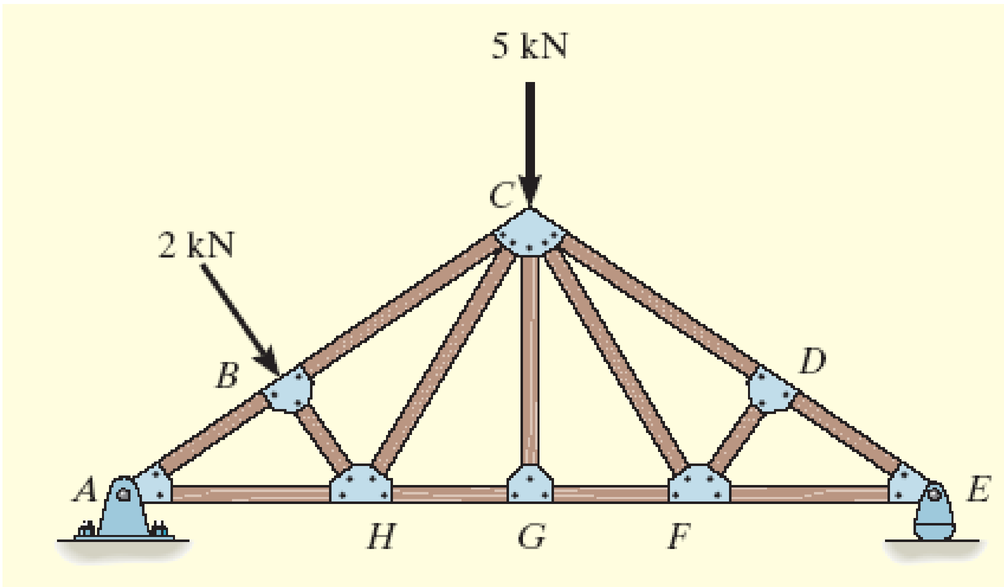


6.3 Zero-Force Members

- Method of joints is simplified using zero-force members
- Zero-force members is supports with no loading
- In general, when 3 members form a truss joint, the 3rd member is a zero-force member provided no external force or support reaction is applied to the joint

Example 6.4

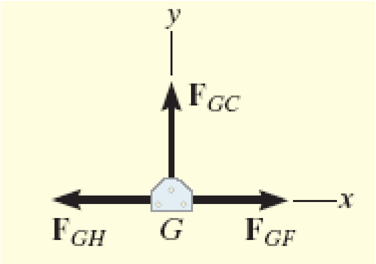
Using the method of joints, determine all the zero-force members of the Fink roof truss. Assume all joints are pin connected.



Solution

For Joint G,

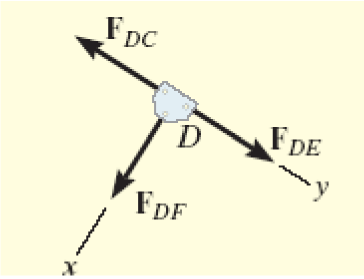
$$+\uparrow \sum F_y = 0 \Rightarrow F_{GC} = 0$$



GC is a zero-force member.

For Joint D,

$$\sum F_x = 0 \Rightarrow F_{DF} = 0$$

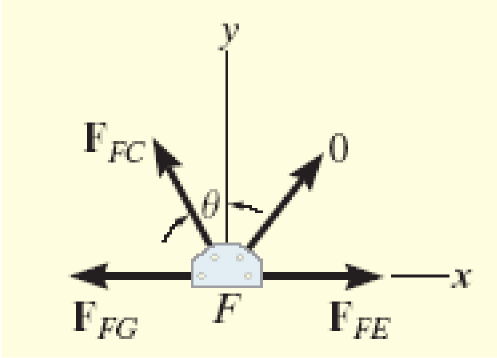


Solution

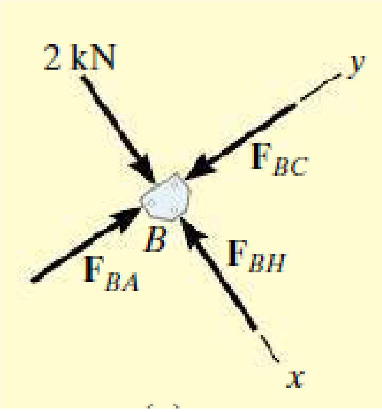
For Joint F,

$$+\uparrow \sum F_y = 0 \Rightarrow F_{FC} \cos\theta = 0$$

$$\theta \neq 90^\circ, F_{FC} = 0$$

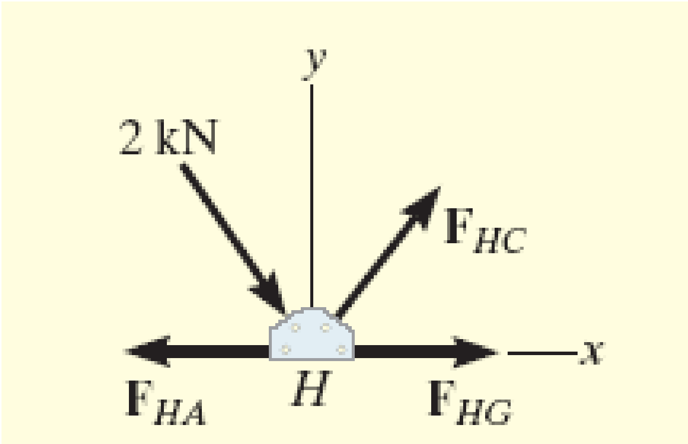


$+\searrow\Sigma F_x = 0;$ $2\text{ kN} - F_{BH} = 0 \quad F_{BH} = 2\text{ kN} \quad \text{(C)}$



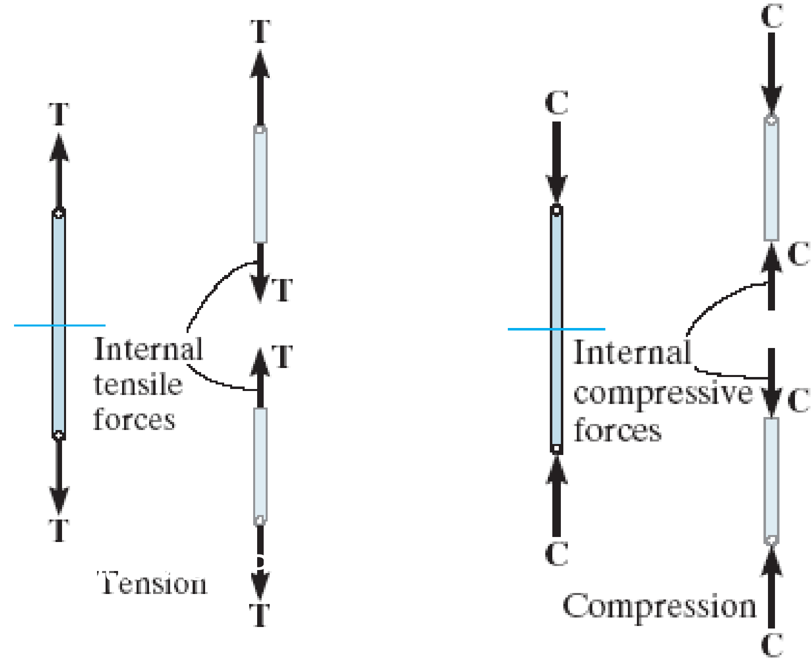
Solution

F_{HC} satisfy $\sum F_y = 0$ and therefore HC is not a zero-force member.



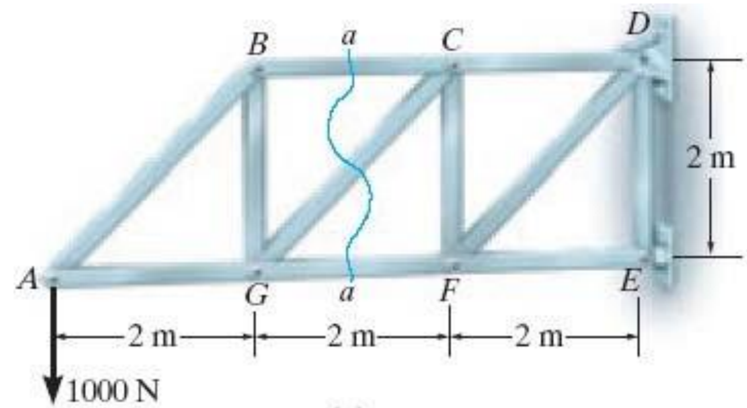
6.4 The Method of Sections

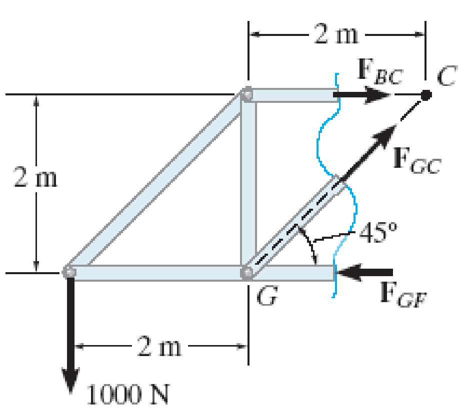
- Used to determine the loadings within a body
- If a body is in equilibrium, any part of the body is in equilibrium
- To find forces within members, an imaginary section is used to cut each member into 2 and expose each internal force as external



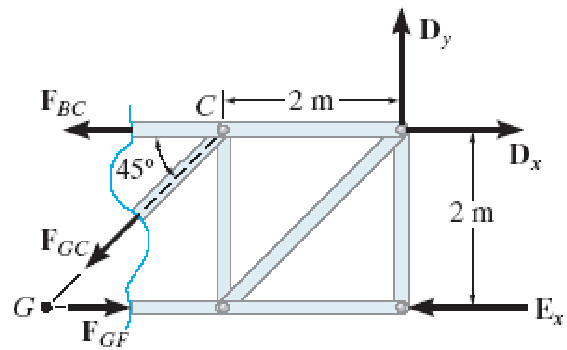
6.4 The Method of Sections

- Consider the truss and section a-a as shown
- Member forces are equal and opposite to those acting on the other part – Newton's Law





(b)



(c)

6.4 The Method of Sections

Procedure for Analysis

Free-Body Diagram

- Decide the section of the truss
- Determine the truss’s external reactions
- Use equilibrium equations to solve member forces at the cut session
- Draw FBD of the sectioned truss which has the least number of forces acting on it
- Find the sense of an unknown member force

6.4 The Method of Sections

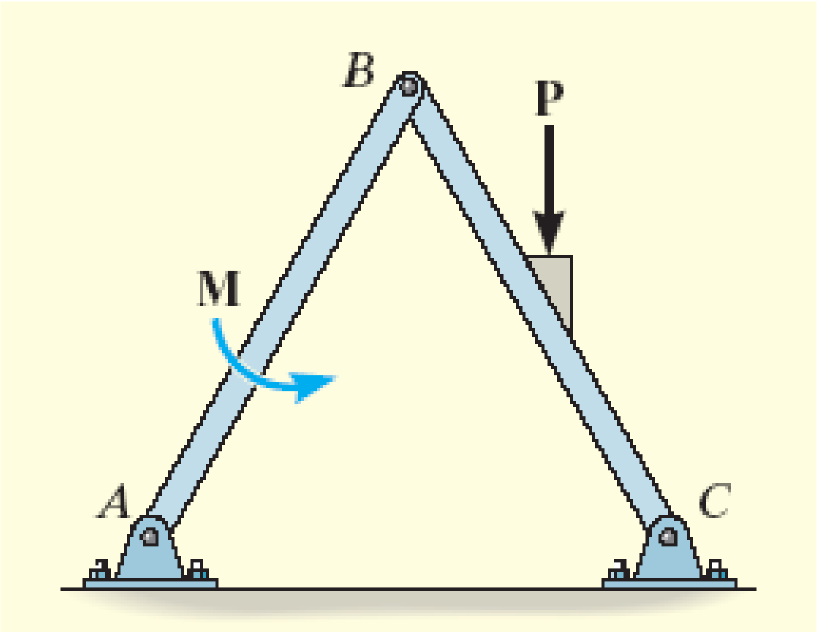
Procedure for Analysis

Equations of Equilibrium

- Summed moments about a point
- Find the 3rd unknown force from moment equation

Example 6.9

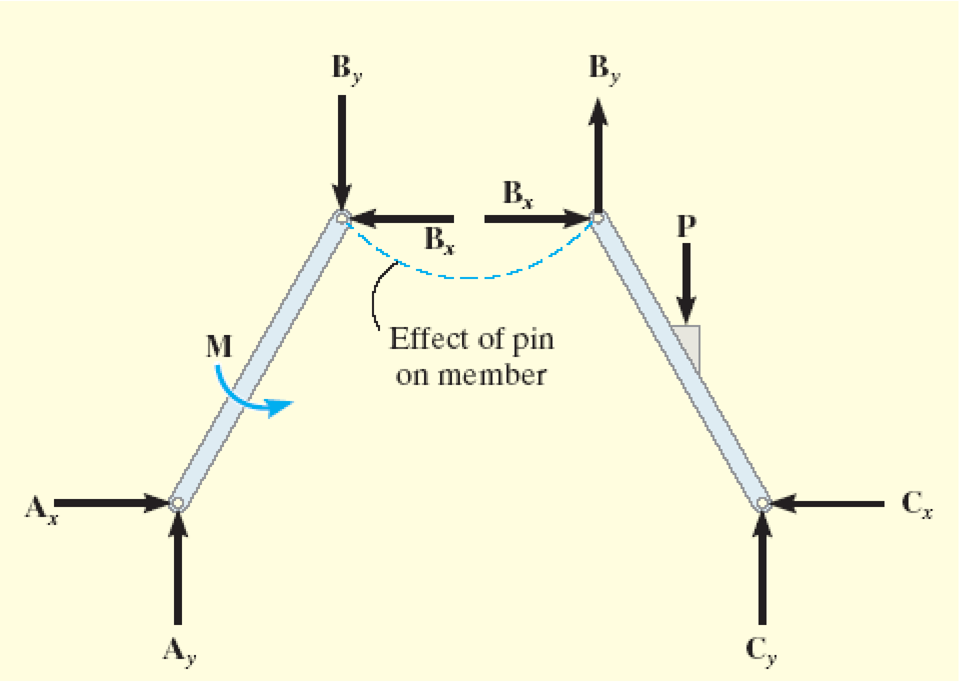
For the frame, draw the free-body diagram of (a) each member, (b) the pin at B and (c) the two members connected together.



Solution

Part (a)

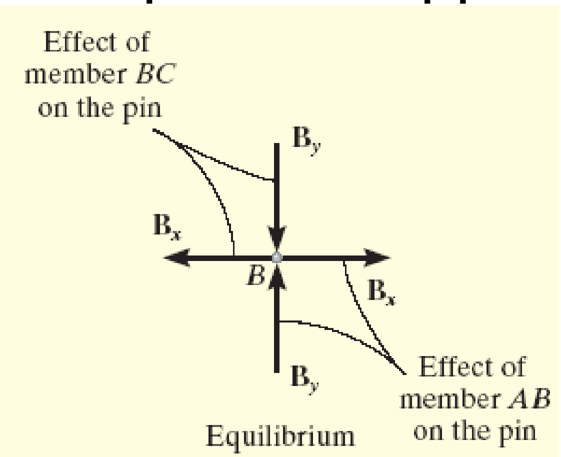
- BA and BC are not two-force
- AB is subjected to the resultant forces from the pins



Solution

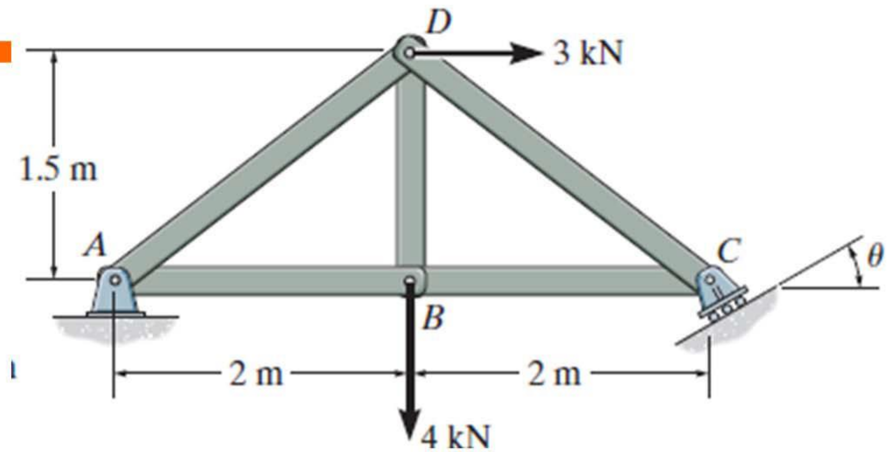
Part (b)

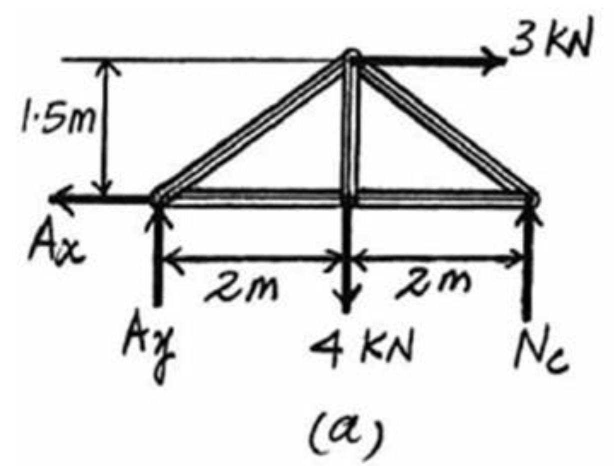
- Pin at B is subjected to two forces, force of the member BC and AB on the pin
- For equilibrium, forces and respective components must be equal but opposite
- $\mathbf{B}_x$ and $\mathbf{B}_y$ shown equal and opposite on members AB



Problems

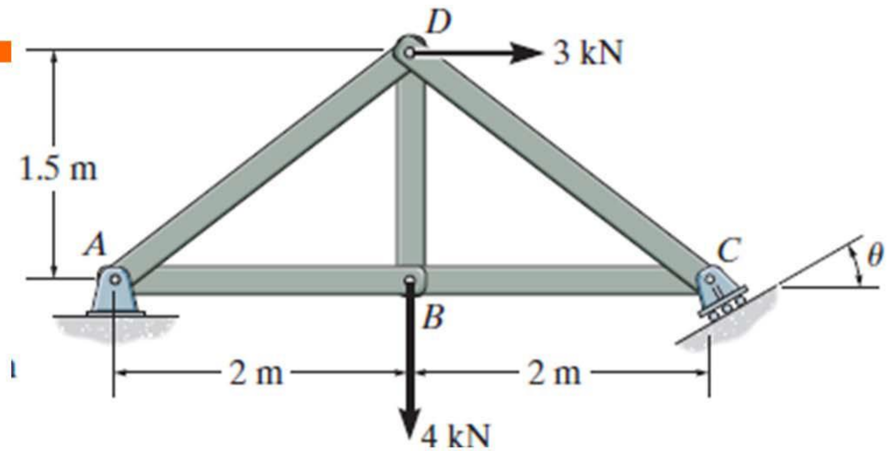
1. Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 0°$.





Ans.

1. Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 0°$.



SOLUTION

Support Reactions: Applying the equations of equilibrium to the free-body diagram of the entire truss, Fig. *a*, we have

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_C(2+2) - 4(2) - 3(1.5) = 0$

$N_C = 3.125 \text{ kN}$

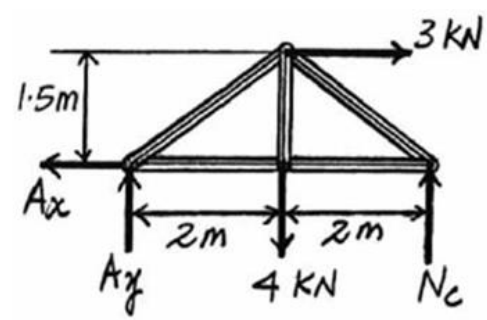
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 3 - A_x = 0$

$A_x = 3 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 3.125 - 4 = 0$

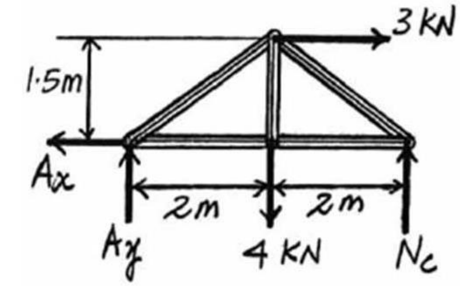
$A_y = 0.875 \text{ kN}$

Ans.



(a)

Method of Joints: We will use the above result to analyze the equilibrium of joints C and A, and then proceed to analyze of joint B.



(a)

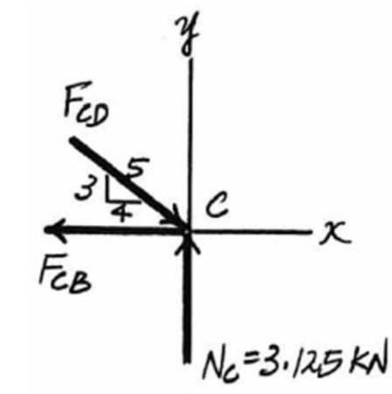
Joint C: From the free-body diagram in Fig. *b*, we can write

$$+\uparrow \Sigma F_y = 0; \qquad 3.125 - F_{CD}\left(\frac{3}{5}\right) = 0$$

$$F_{CD} = 5.208 \text{ kN} = 5.21 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 5.208\left(\frac{4}{5}\right) - F_{CB} = 0$$

$$F_{CB} = 4.167 \text{ kN} = 4.17 \text{ kN (T)} \qquad \textbf{Ans.}$$



(b)

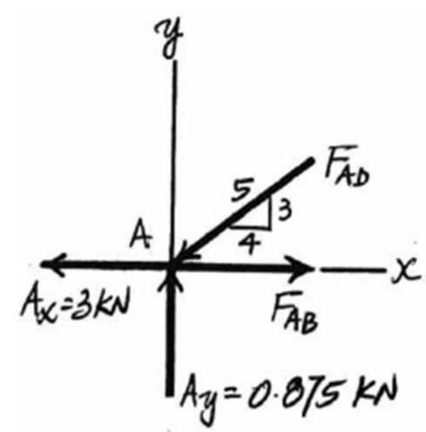
Joint A: From the free-body diagram in Fig. *c*, we can write

$$+\uparrow \Sigma F_y = 0; \qquad 0.875 - F_{AD}\left(\frac{3}{5}\right) = 0$$

$$F_{AD} = 1.458 \text{ kN} = 1.46 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} - 3 - 1.458\left(\frac{4}{5}\right) = 0$$

$$F_{AB} = 4.167 \text{ kN} = 4.17 \text{ kN (T)} \qquad \textbf{Ans.}$$



(c)

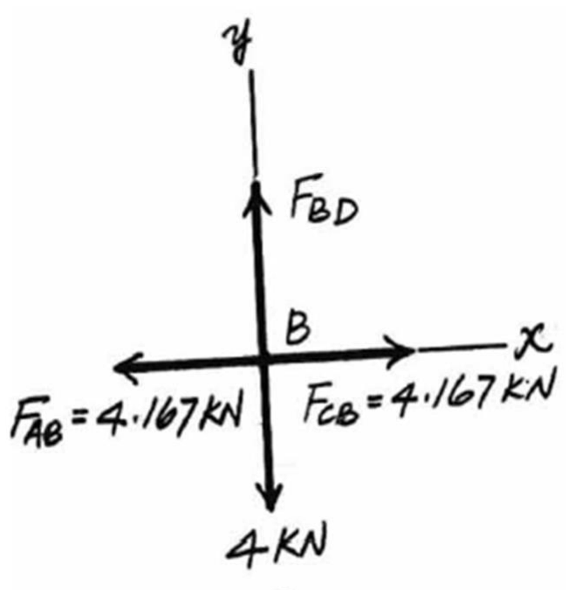
Joint B: From the free-body diagram in Fig. d, we can write

$+\uparrow \Sigma F_y = 0;$ $\qquad F_{BD} - 4 = 0$

$$F_{BD} = 4 \text{ kN (T)} \qquad \textbf{Ans.}$$

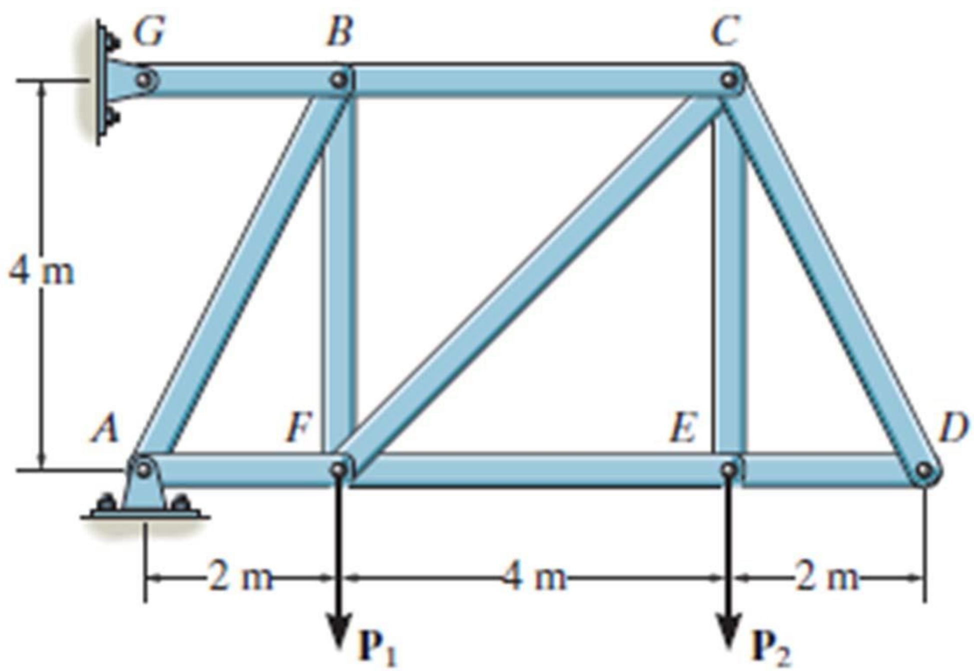
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\qquad 4.167 - 4.167 = 0 \qquad$ (check!)

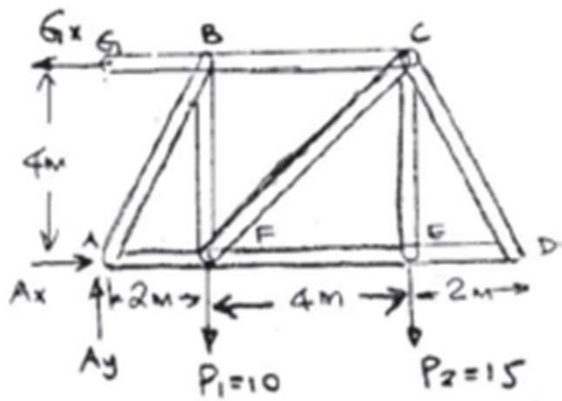
Note: The equilibrium analysis of joint D can be used to check the accuracy of the solution obtained above.

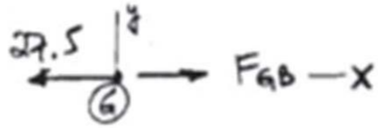


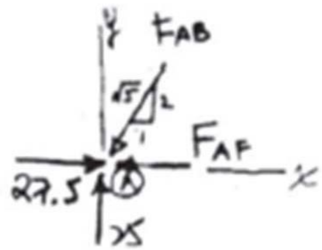
2.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 10$ kN, $P_2 = 15$ kN.

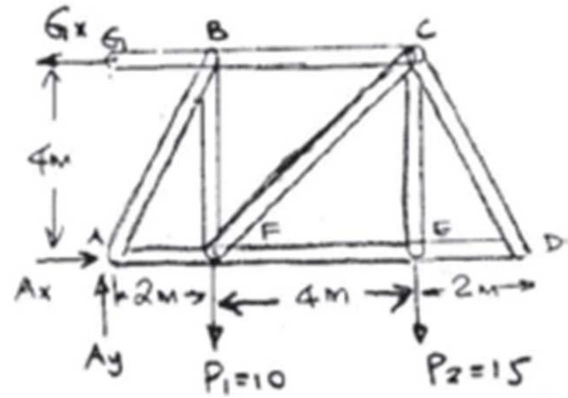



Gx
G
B
C
4m
A
F
E
D
Ax
2m
4m
2m
Ay
P1=10
P2=15


27.5
y
FGB
X
G


y
FAB
√5
2
1
FAF
x
27.5
A
25

SOLUTION



$$\circlearrowleft + \Sigma M_A = 0; \qquad G_x(4) - 10(2) - 15(6) = 0$$

$$G_x = 27.5 \text{ kN}$$

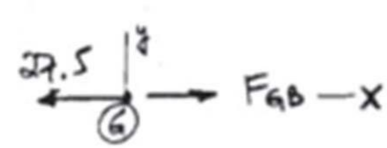
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 27.5 = 0$$

$$A_x = 27.5 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 10 - 15 = 0$$

$$A_y = 25 \text{ kN}$$

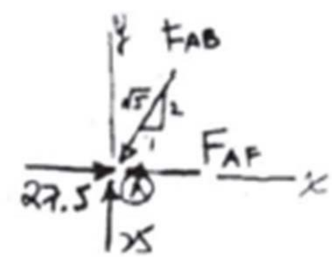
Joint G:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{GB} - 27.5 = 0$$

$$F_{GB} = 27.5 \text{ kN (T)}$$

Joint A:



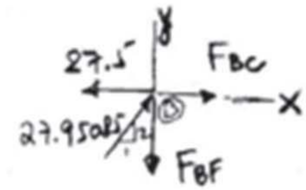
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 27.5 - F_{AF} - \frac{1}{\sqrt{5}}(F_{AB}) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 25 - F_{AB}\left(\frac{2}{\sqrt{5}}\right) = 0$$

$$F_{AF} = 15.0 \text{ kN (C)}$$

$$F_{AB} = 27.95 = 28.0 \text{ kN (C)}$$

Joint B:



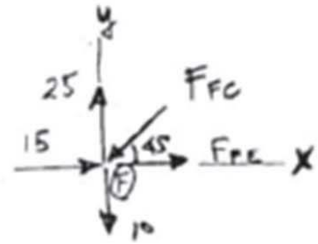
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 27.95\left(\frac{1}{\sqrt{5}}\right) + F_{BC} - 27.5 = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 27.95\left(\frac{2}{\sqrt{5}}\right) - F_{BF} = 0$$

$$F_{BF} = 25.0 \text{ kN (T)}$$

$$F_{BC} = 15.0 \text{ kN (T)}$$

Joint F:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad 15 + F_{FE} - \frac{1}{\sqrt{2}}(F_{FC}) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 25 - 10 - F_{FC}\left(\frac{1}{\sqrt{2}}\right) = 0$$

$$F_{FC} = 21.21 = 21.2 \text{ kN (C)}$$

$$F_{FE} = 0$$

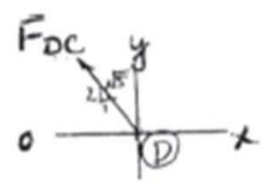
Joint E:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{ED} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{EC} - 15 = 0$$

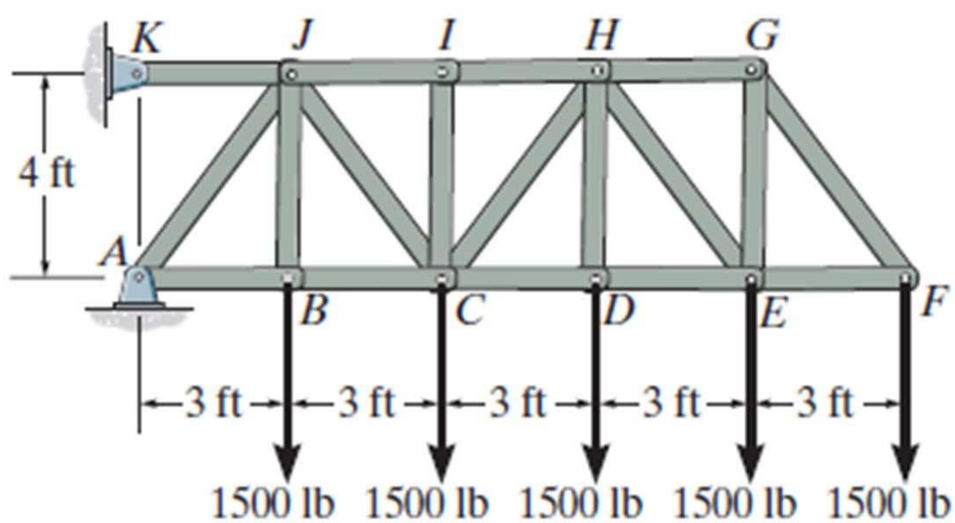
$$F_{EC} = 15.0 \text{ kN (T)}$$

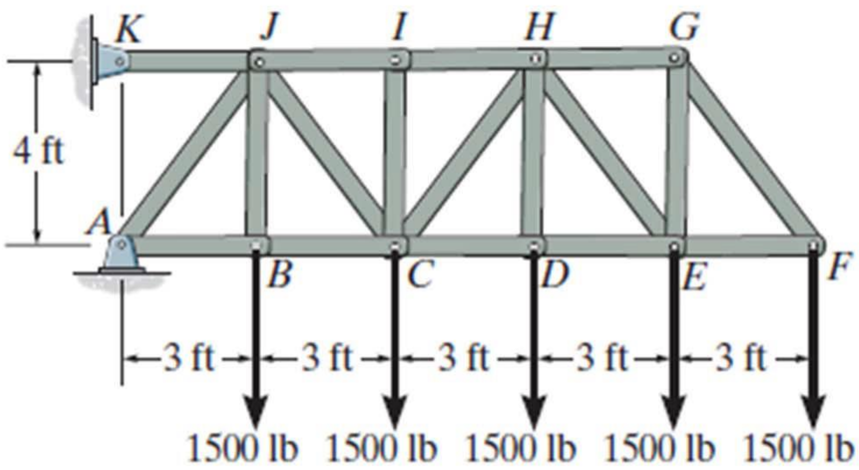
Joint D:

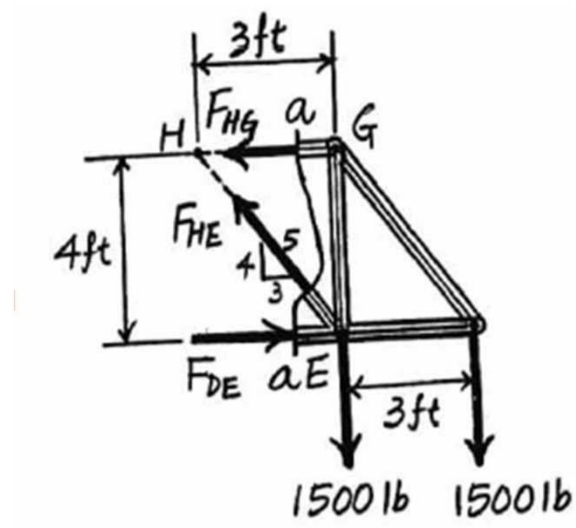


$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{DC} = 0$$

3. Determine the force in members HG, HE, and DE of the truss, and state if the members are in tension or compression.







SOLUTION

Method of Sections: The forces in members HG, HE, and DE are exposed by cutting the truss into two portions through section $a–a$ and using the upper portion of the free-body diagram, Fig. a. From this free-body diagram, F_{HG} and F_{DE} can be obtained by writing the moment equations of equilibrium about points E and H, respectively. $\mathbf{F}_{HE}$ can be obtained by writing the force equation of equilibrium along the y axis.

Joint D: From the free-body diagram in Fig. a,

$\circlearrowleft + \Sigma M_E = 0;$ $\quad F_{HG}(4) - 1500(3) = 0$

$F_{HG} = 1125 \text{ lb (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_H = 0;$ $\quad F_{DE}(4) - 1500(6) - 1500(3) = 0$

$F_{DE} = 3375 \text{ lb (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{HE}\left(\frac{4}{5}\right) - 1500 - 1500 = 0$

$F_{EH} = 3750 \text{ lb (T)}$ **Ans.**