Potential Energy and Conservation of Energy

8.1 Potential energy

Technically, potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

Some forms of potential energy:

1. Gravitational Potential Energy,
2. Elastic Potential Energy

8.2 Work and potential energy

The change ΔU in potential energy (gravitational, elastic, etc) is defined as being equal to the negative of the work done on the object by the force (gravitational, elastic, etc)

$$\Delta U = -W.$$

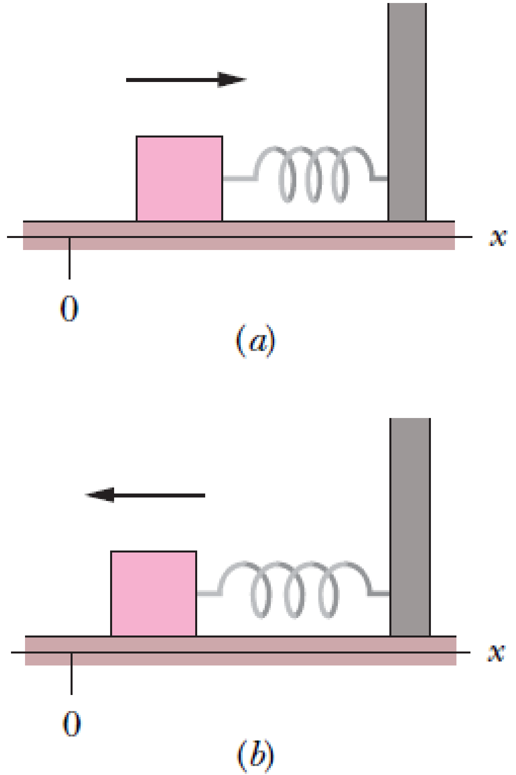


Fig. 8-3 A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (*a*) As the block moves rightward (as indicated by the arrow), the spring force does negative work on it. (*b*) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

8.2 Conservative and non-conservative forces

Suppose:

1. A system consists of two or more objects.

2. A force acts between a particle-like object in the system and the rest of the system.

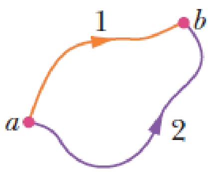
3. When the system configuration changes, the force does work (call it W_1) on the object, transferring energy between the kinetic energy of the object, K, and some other type of energy of the system.

4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation in which $W_1 = -W_2$ *is always true, the other type of energy is* a potential energy and the force is said to be a conservative force.

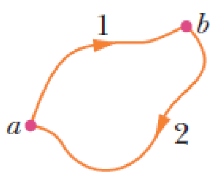
A force that is not conservative is called a non-conservative force. The kinetic frictional force and drag force are non-conservative.

The net work done by a conservative force on a particle moving around any closed path is zero.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

$$W_{ab,1} = W_{ab,2},$$

If the work done from a to b along path 1 as $W_{ab,1}$ and the work done from b back to a along path 2 as $W_{ba,2}$. If the force is conservative, then the net work done during the round trip must be zero

$$W_{ab,1} + W_{ba,2} = 0,$$

$$W_{ab,1} = -W_{ba,2}.$$

If the force is conservative,

$$W_{ab,2} = -W_{ba,2}.$$

$$\longrightarrow W_{ab,1} = W_{ab,2}$$

8.4: Determining Potential Energy values:

For the most general case, in which the force may vary with position, we may write the work W:

$$W = \int_{x_i}^{x_f} F(x)\, dx.$$

$$\Downarrow$$

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.$$

8.4: Determining Potential Energy values:

Gravitational Potential Energy

A particle with mass *m* moving vertically along a *y* axis (the positive direction is upward). As the particle moves from point y_i to point y_f, the gravitational force does work on it. The corresponding change in the gravitational potential energy of the particle–Earth system is:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\, dy = mg \int_{y_i}^{y_f} dy = mg\Big[\, y \,\Big]_{y_i}^{y_f},$$

$$\Big\downarrow$$

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y.$$

$$\Big\downarrow$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position y =0, not on the horizontal position.

8.4: Determining Potential Energy values: Elastic Potential Energy

In a block–spring system, the block is moving on the end of a spring of spring constant k. As the block moves from point x_i to point x_f , the spring force F_x =- kx does work on the block. The corresponding change in the elastic potential energy of the block–spring system is

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k\Big[x^2\Big]_{x_i}^{x_f},$$

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$

If the reference configuration is when the spring is at its relaxed length, and the block is at $x_i = 0$.

$$U - 0 = \tfrac{1}{2}kx^2 - 0,$$

$$\Downarrow$$

$$\boxed{U(x) = \tfrac{1}{2}kx^2}$$

8.5: Conservation of Mechanical Energy

Principle of conservation of energy:

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

The mechanical energy E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it: $E_{\text{mec}} = K + U \quad \text{(mechanical energy).}$

We have $\Delta U = -W.$

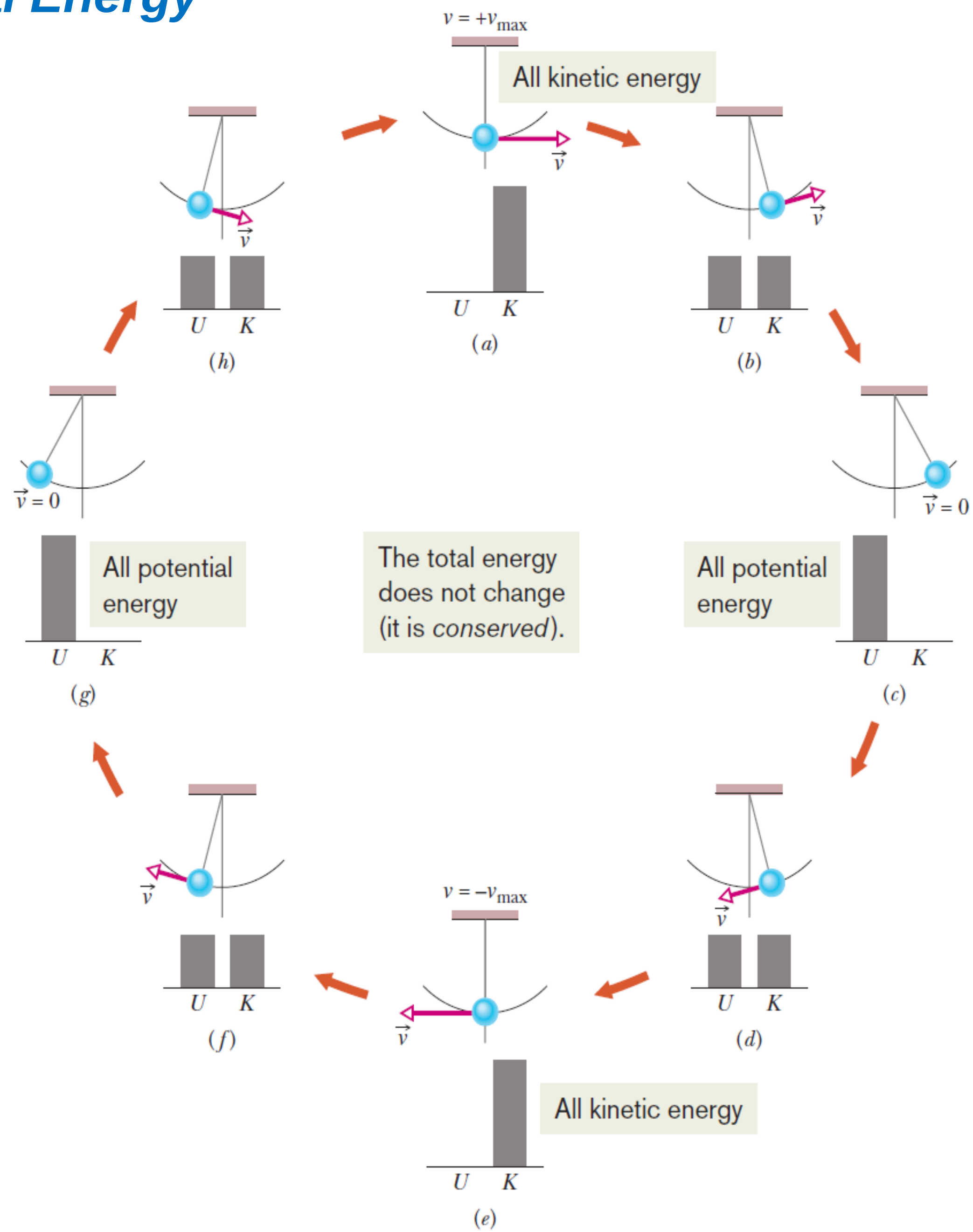
We have: $\Delta K = W$ ➡ $\begin{pmatrix} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system} \end{pmatrix} = \begin{pmatrix} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system} \end{pmatrix}$

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.$$

8.5: Conservation of Mechanical Energy

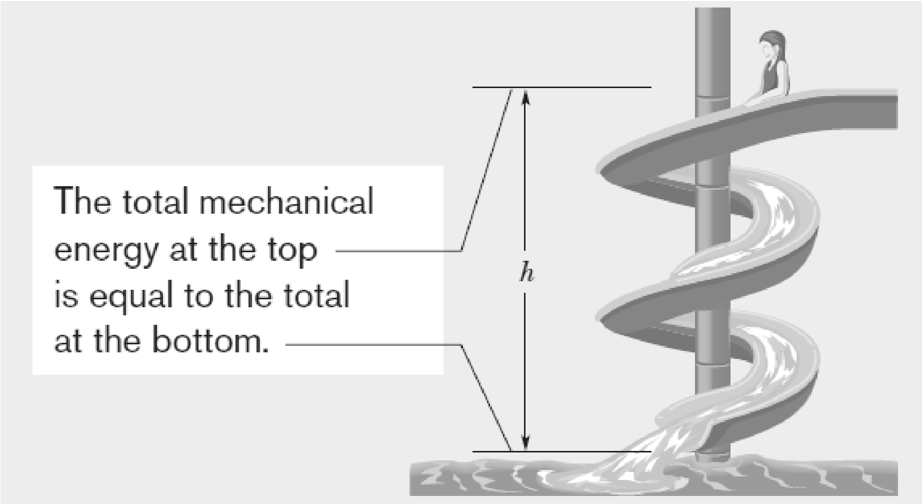
A pendulum swings back and forth. During one full cycle the values of the potential and kinetic energies of the pendulum– Earth system vary as the bob rises and falls, but the mechanical energy E_{mec} of the system remains constant. The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms. In stages (a) and (e), all the energy is kinetic energy. In stages (c) and (g), all the energy is potential energy. In stages (b), (d), (f), and (h), half the energy is kinetic energy and half is potential energy.

If the swinging involved a frictional force then E_{mec} would not be conserved, and eventually the pendulum would stop.



Sample problem: water slide

In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



Calculations: Let the mechanical energy be $E_{\text{mec},t}$ when the child is at the top of the slide and $E_{\text{mec},b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{\text{mec},b} = E_{\text{mec},t}. \tag{8-19}$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \tag{8-20}$$

or $$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}. \qquad \text{(Answer)}$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

8.7: Work done on a System by an External Force

FRICTION NOT INVOLVED

$$W = \Delta K + \Delta U,$$

$$\boxed{W = \Delta E_{\text{mec}}}$$

Your lifting force transfers energy to kinetic energy and potential energy.

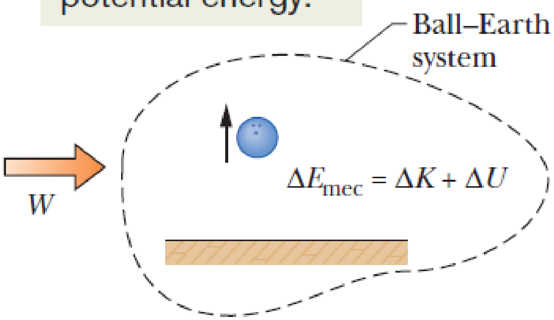


Fig. 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

FRICTION INVOLVED

$$F - f_k = ma.$$

$$Fd = \Delta K + f_k d.$$

$$Fd = \Delta E_{\text{mec}} + f_k d.$$

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

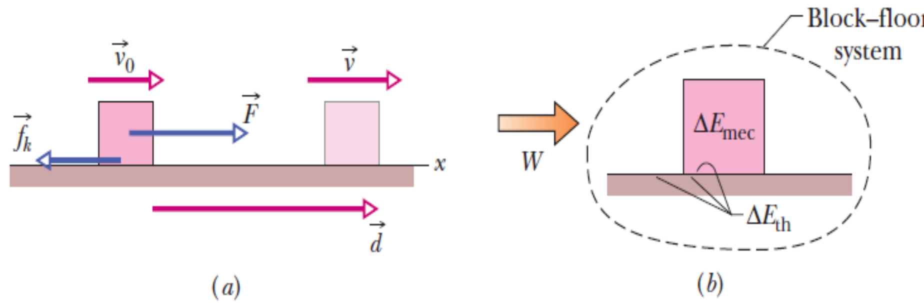


Fig. 8-13 (*a*) A block is pulled across a floor by force $\vec{F}$ while a kinetic frictional force $\vec{f}_k$ opposes the motion. The block has velocity $\vec{v}_0$ at the start of a displacement $\vec{d}$ and velocity $\vec{v}$ at the end of the displacement. (*b*) Positive work W is done on the block–floor system by force $\vec{F}$, resulting in a change ΔE_{mec} in the block's mechanical energy and a change ΔE_{th} in the thermal energy of the block and floor.

Law of Conservation of Energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$$

where E_{mec} *is any change in the mechanical energy of the system*, E_{th} *is any* change in the thermal energy of the system, and E_{int} *is any change in any* other type of internal energy of the system.

The total energy E of an *isolated system* cannot change.

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \qquad \text{(isolated system)}$$

In Class Activity

A 2 kg stone is dropped h = 40 cm onto a spring of spring constant k = 2000 N/m. Find the maximum distance the spring is compressed.