ENGG103 –Materials in Design

- **Tutorial 4**

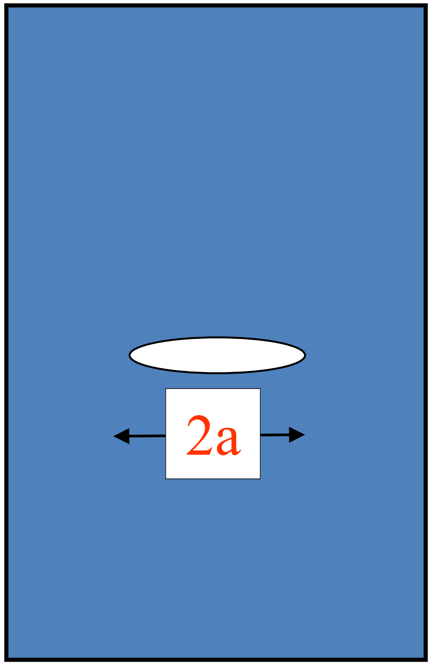
- *Fracture*
- *Fatigue Failure*

Additional Crack characteristic information

Characteristics of Cracks

Cracks can be characterized looking into the following aspects.

- Its connection with the external free surface:
  (i) completely internal,
  (ii) internal cracks with connections to the outer surfaces,
  (iii) Surface cracks.
-
  • Cracks with some contact with external surfaces are exposed to outer media and hence may be prone to oxidation and corrosion (cracking).
- Crack length
- Crack tip radius Crack tip radius is dependent of the type of loading and the ductility of the material.
- Crack orientation with respect to geometry and loading.



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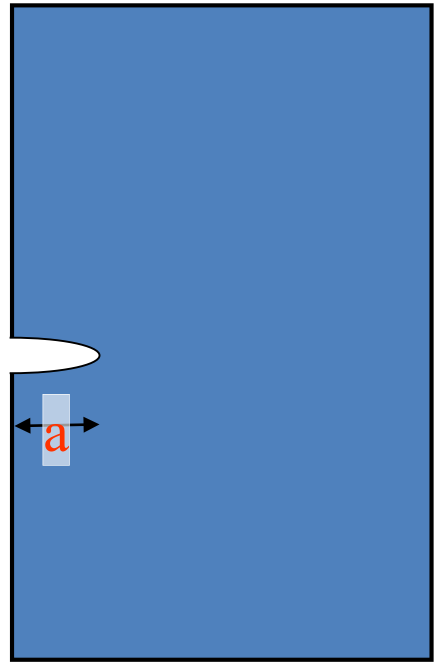
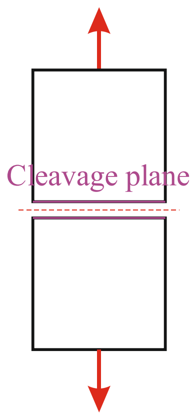
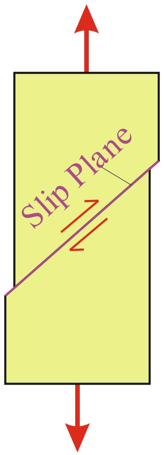
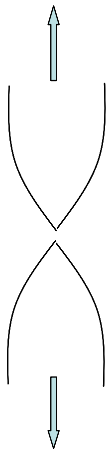
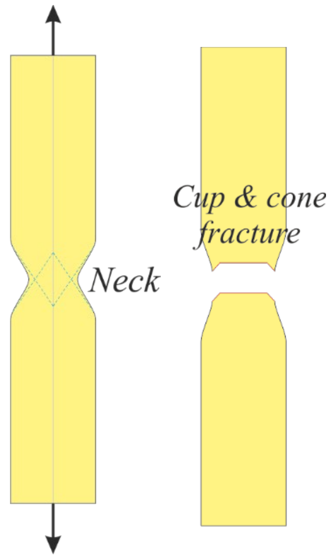


Types of failure in an uniaxial tension test









Brittle	Shear	Rupture	Ductile fracture
Little or no deformation	Shear fracture of ductile single crystals	Completely ductile fracture of polycrystals	Ductile fracture of usual polycrystals
Observed in single crystals and polycrystals	Not observed in polycrystals	Very ductile metals like gold and lead neck down to a point and fail	Cup and cone fracture
Have been observed in BCC and HCP metals but not in FCC metals		*Here technically there is no fracture (there is not enough material left to support the load)*	Cracks may nucleate at second phase particles (void formation at the matrix-particle interface)

Exercise 4.1 – Fracture Toughness

A 50mm wide sample plate of 7074-T8 aluminium alloy contains a central through-crack of length 2a. For 7074-T8: Kc = 22.2 MN m-3/2; σ y = 520MPa

a) an applied stress of 200 MPa, determine if the plate will fail by fracture with a crack half-length a of: 1 mm; 5 mm; 10 mm

b) Determine the limiting crack size 2a below which the plate will fail by yielding (assume Y = 1)

Brittle fracture occurs when $K_{Ic} = Y\sigma\sqrt{\pi a}$

4.8) ~~crack length = 2a~~

$\boxed{K_c = 22.2\,MN\,m^{-3/2}}$

$\sigma_y = 520\,MPa$

$K = Y\sigma\sqrt{\pi a}$

when $K_{IC} = Y\sigma\sqrt{\pi a}$

fracture occurs.

a) @ $\sigma = 200\,MPa$

① @ 1mm = a

$K = (1)(200\times10^6)\sqrt{\pi(.001)}$

$K = 11.2\,MN/m^{3/2}$

$K < K_{IC}$ Safe

22.2MN/m^{3/2}

② @ 5mm = a

$K = (1)(200\times10^6)\sqrt{\pi(.005)}$

$K = 25.07\,MN/m^{3/2}$

$K > K_{IC}$ Not safe.

③ @ 10mm = a

$K = (1)(200\times10^6)\sqrt{\pi(.01)}$

$K = 35.5\,MN/m^{3/2}$

$K > K_{IC}$ not safe.

b) Determine Limiting crack size a

$Y = 1$

$\sigma = 200\,MPa$

$K_{IC} = 22.2\,MN/m^{3/2}$

$22.2\times10^6 = (1)(520\times10^6)\sqrt{\pi a}$

0.58 mm = a

Limiting Crack length = 2a

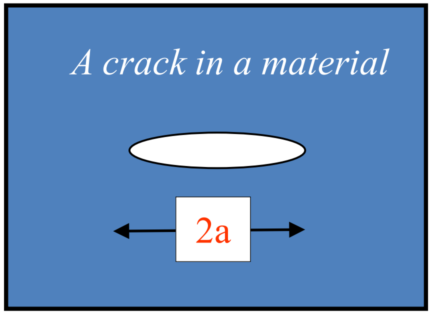
= 1.16 mm.

Exercise 4.2 – Fracture Toughness

In a component σ_y =800 MPa, K_{1C}=85 Mpa.m$^{0.5}$. What is the maximum allowable size of a fully internal crack (Assume Y=0.95)?

Brittle fracture occurs when

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$



$$K_{Ic} = Y\sigma\sqrt{\pi a}$$

85 Mpa.m$^{0.5}$ = 0.95 (800MPa) $\sqrt{\pi a}$

$$\frac{85}{760} = \sqrt{\pi a}$$

$(0.111842)^2 = \pi a$

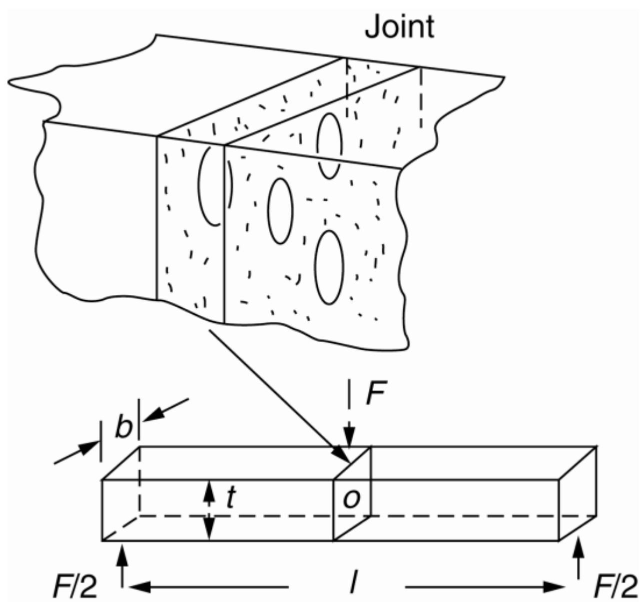
0.003981 m = a

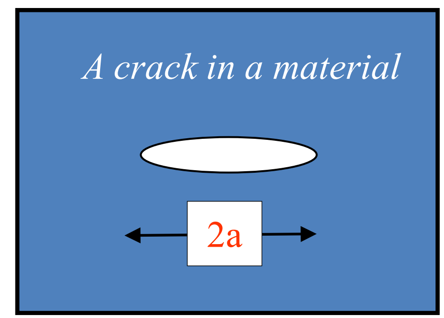
Maximum length of internal crack = 7.96mm

Exercise 4.3 – Fracture Toughness

Two pieces of timber are glued together end-to-end with an epoxy adhesive and used as a beam in a 3-point bending arrangement as shown in the figure below. The beam width, thickness and span are $b = 250\ mm$, $t = 50\ mm$ and $l = 2.4\ m$ respectively. The glued joint failed by fast fracture when the applied load, *F*, reached $2.6\ kN$. Close inspection of the fracture revealed disc shaped bubbles up to 2.0 mm in diameter trapped in the epoxy.

If the maximum bending stress in the beam is given by $\sigma_{max} = \frac{3Fl}{2bd^2}$, what is the fracture toughness for the epoxy adhesive. (Assume $Y = 0.64$).





Exercise 4.6 – Fracture Toughness

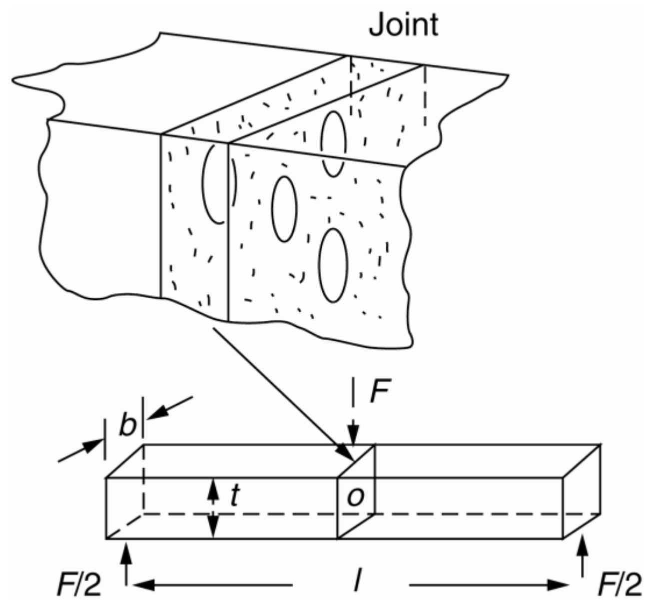
Step 1 – Define

Fracture toughness K_{Ic} (read K one C not K I C) of the epoxy

Step 2 – Data

Glued timber beam in 3-point bending

Beam span	*l=2.4 m*
Beam width	*b=250 mm*
Beam thickness	*t=50 mm*
Fracture load	*F=2.6 kN*
Disc bubbles	*2a=2.0 mm (diameter)*
Geometry factor	*Y=0.64*



Exercise 4.3– Fracture Toughness

Step 3 – Theory

Maximum stress due to bending is

$$\sigma_{max} = \frac{3Fl}{2bt^2}$$

Brittle fracture occurs when

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$

Step 4 – Estimate

Units for fracture toughness should be MPa.m$^{1/2}$

Epoxies typically show low fracture toughness

We might expect 0.2<K_{Ic}<2 MPa.m$^{1/2}$

Exercise 4.3 – Fracture Toughness

Step 5 – Solve

Maximum stress due to bending is:

$$\sigma_{max} = \frac{3Fl}{2bt^2}$$

$$\sigma_{max} = \frac{3\times 2600\ N\times 2.4\ m}{2\times 0.25\ m\times (0.05\ m)^2}$$

$$\sigma_{max} = 14.976\ MPa$$

Assuming that the largest disc shaped flaw was present in this region of highest stress then the fracture toughness is:

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$

$$K_{Ic} = 0.64 \times 14.976\ MPa \times \sqrt{\pi \times 0.001\ m}$$

$$K_{Ic} = 0.54\ MPa.m^{1/2}$$

Step 6 – Verify

The calculated value agrees with our initial estimate and so is considered a reasonable solution

Exercise 4.4 – Fatigue

A rotating shaft in a gearbox is to be made from AISI 4340 quenched and tempered steel which has a tensile strength of $\sigma_{TS} = 1820\ MPa$. Using the information in the following figure, determine:

a) What is the fatigue strength σ_f at 10^7 cycles?

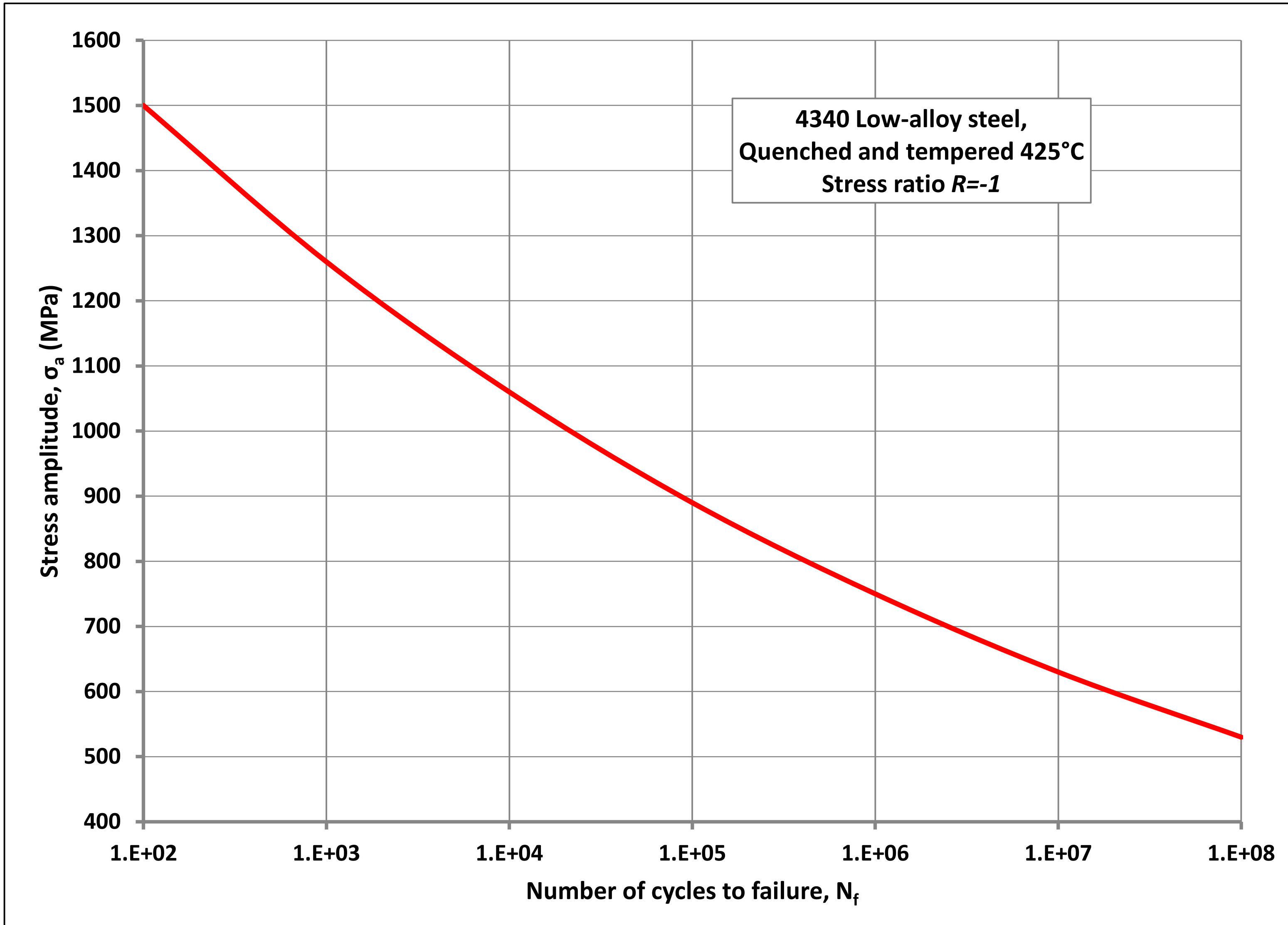
b) Will the shaft fail by fatigue if it is subjected to 100 cycles with amplitude of 1200 MPa and zero mean stress?

c) Will the shaft fail by fatigue if it is subjected to 100 000 cycles with amplitude of 900 MPa and zero mean stress?

d) Will the shaft fail by fatigue if it is subjected to 100 000 cycles with amplitude of 800 MPa and mean stress of 300 MPa?

e) If cycled between -100 MPa and 1100 MPa, how many cycles will the shaft survive? If the rotation frequency is 8 Hz, how long (in hours) will the shaft survive?

Exercise 4.4 – Fatigue



Exercise 4.4 – Fatigue

Step 1 – Define

a) fatigue strength σ_f at 10^7 cycles

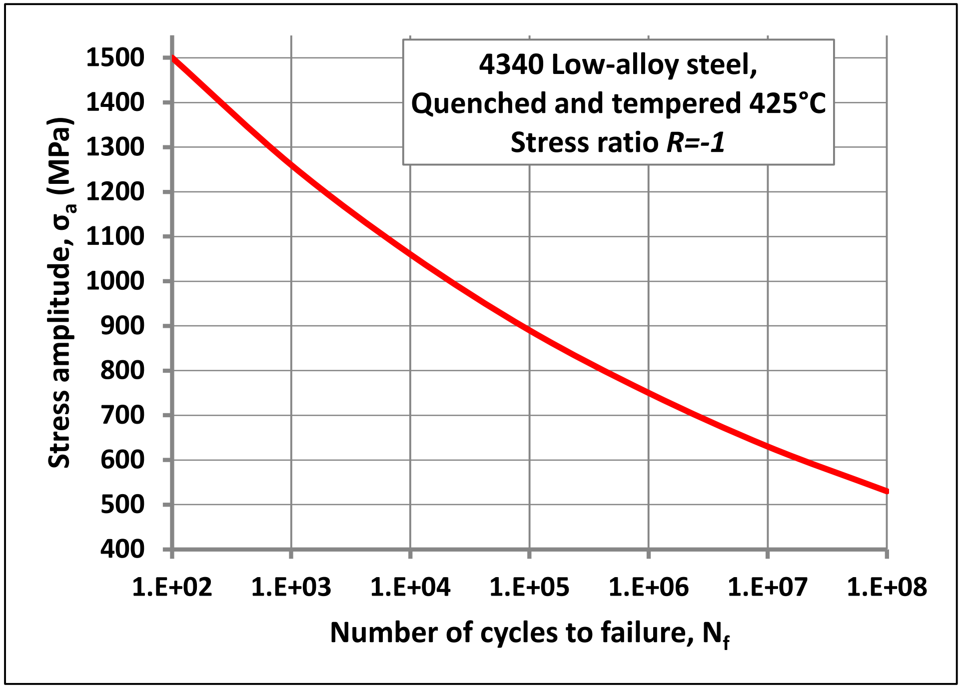
b) – d) will shaft fail by fatigue

e) fatigue life in cycles and time

Step 2 – Data

Graph (for σ_m = 0) provided

Stress and cycles in each case



Step 3 – Theory

Goodman Rule for $\sigma_m \neq 0$:

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{TS}}\right)$$

Hertz = cycles per second

Exercise 4.4 – Fatigue

Step 4 – Estimate

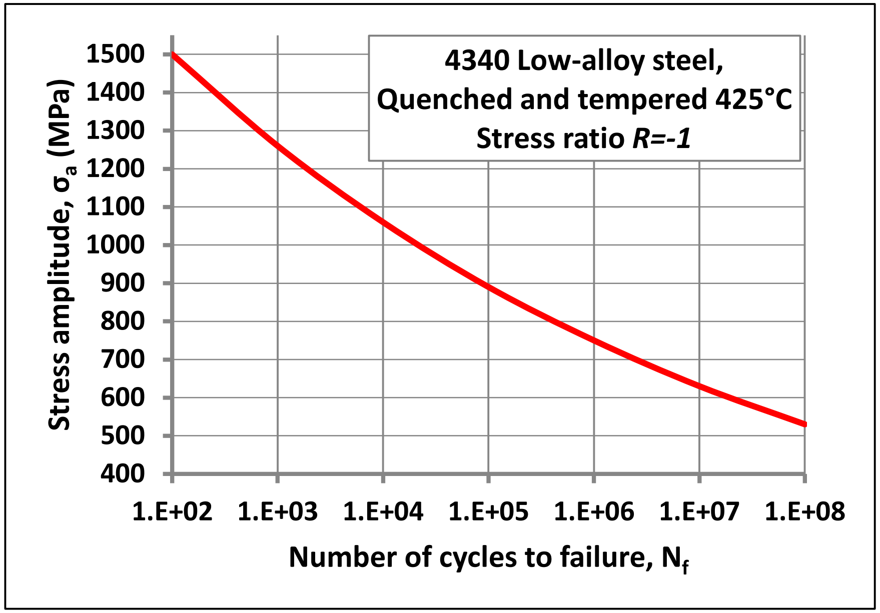
a) From the graph we can see $600\ MPa < \sigma_f < 700\ MPa$

b) – d) no need to estimate (answer is yes or no)

e) for σ_a=600 MPa → $N_f \approx 10^7$ cycles (rough estimate)
then for 8 Hz we get $t_f \approx 10^7$ cycles / 8 cycles s^{-1} = $1.25\text{x}10^6$ s ≈ 350 hrs

Since the mean stress isn't zero this is our upper limit

We should expect the life to be significantly reduced compared to this value



Exercise 4.4 – Fatigue

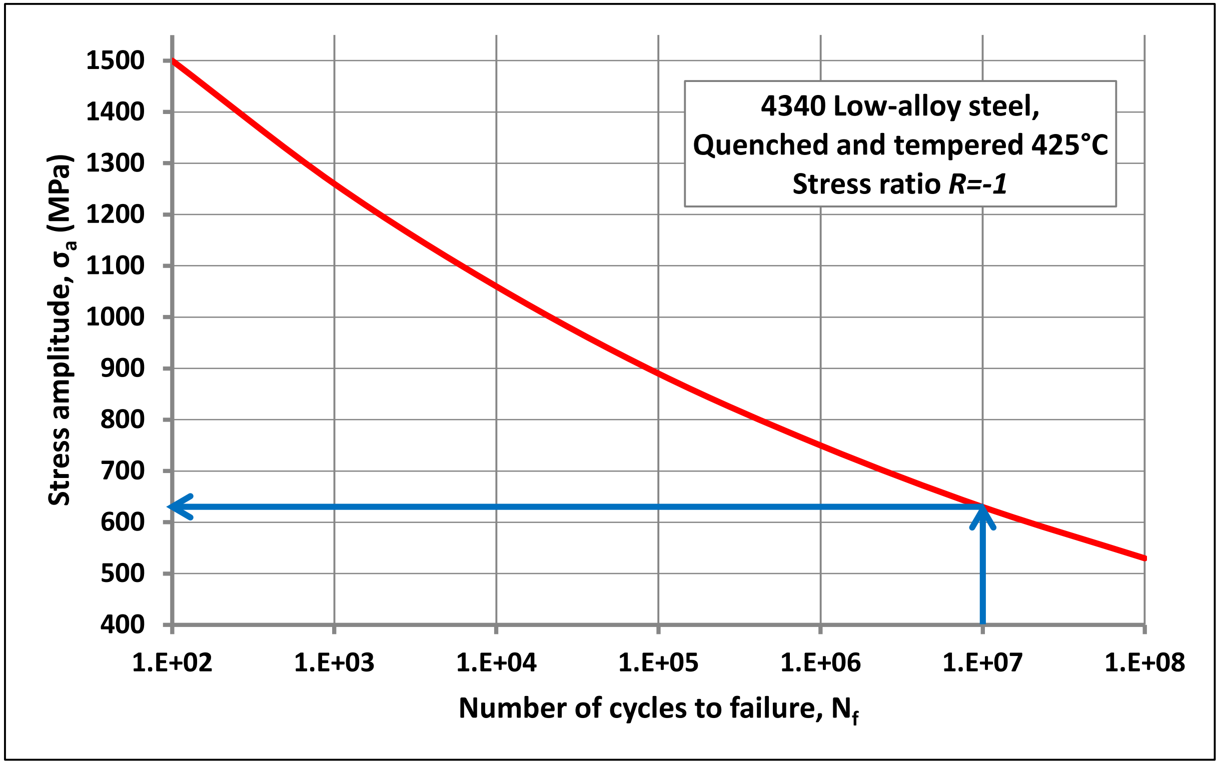
Step 5 – Solve

Part (a) *As there is no distinct plateau in the S-N curve we can't define an endurance limit* $\boldsymbol{\sigma_e}$

Instead we consider the fatigue strength $\boldsymbol{\sigma_f}$ *at* 10^7 *cycles*

Reading from the graph:

$$\sigma_f \approx 630\ MPa$$



Exercise 4.4 – Fatigue

Step 5 – Solve

Part (a) continued

In some cases the endurance limit $\boldsymbol{\sigma_e}$ *can be estimated from the tensile strength*

$$\sigma_e \approx 0.33\sigma_{TS}$$

$$\sigma_e \approx 0.33 \times 1820\ MPa$$

$$\sigma_e \approx 607\ MPa$$

Comparing to the value of fatigue strength from Part (a)

$$\sigma_e \approx 630\ MPa$$

While not identical, it is clear the values are of similar magnitude

Exercise 4.4 – Fatigue

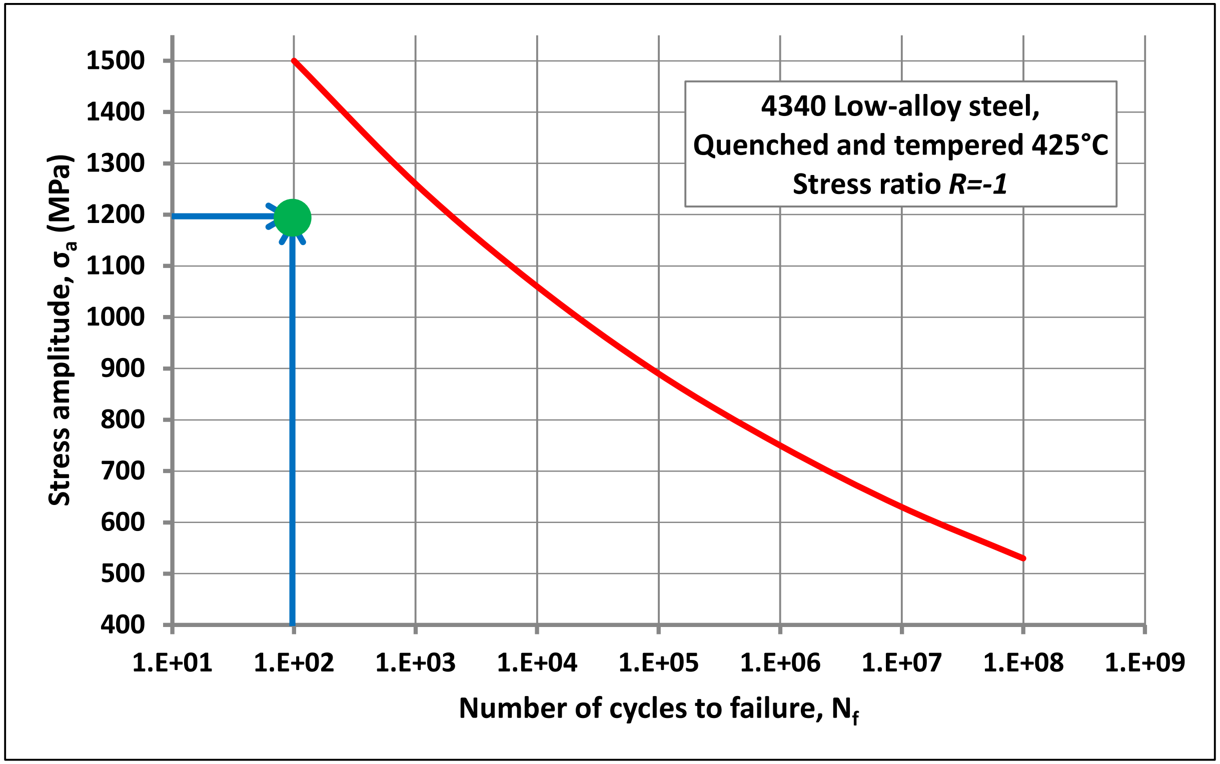
Step 5 – Solve

Part (b) $\sigma_m = 0$

$\sigma_a = 1200\ MPa$

$N = 100\ cycles$

Not expected to fail by fatigue



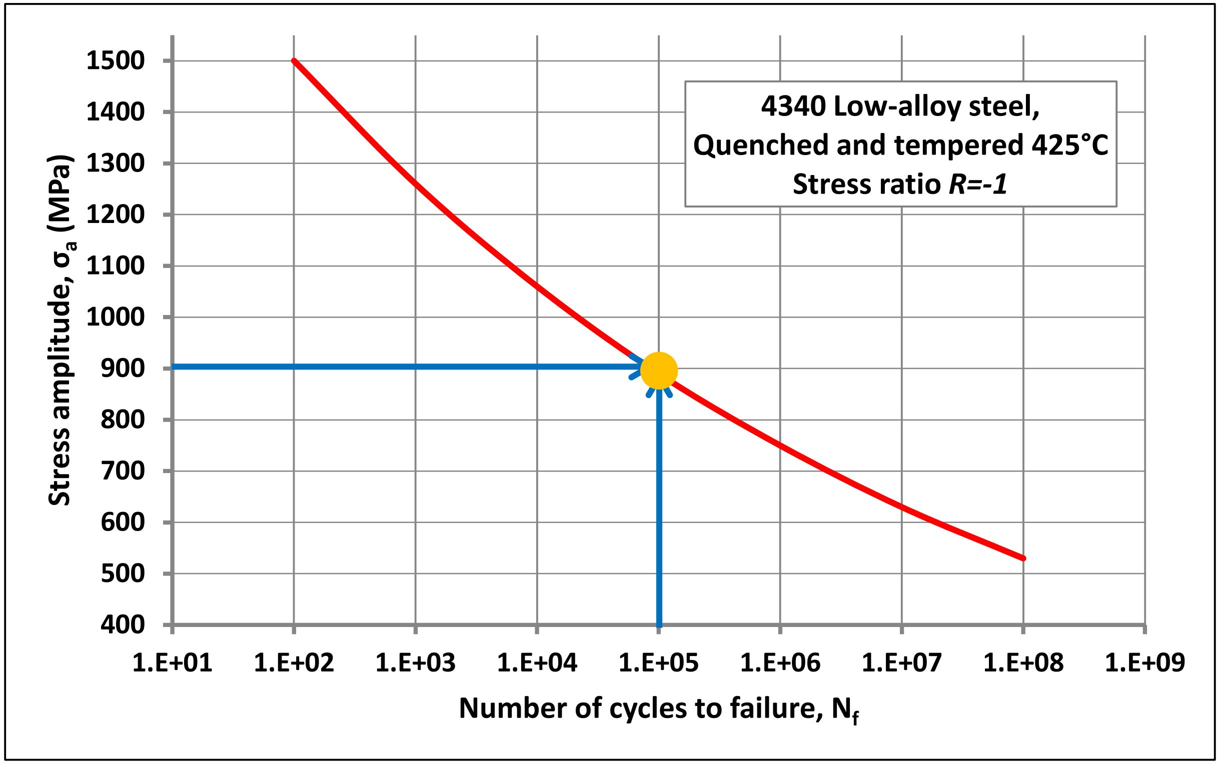
Exercise 4.4 – Fatigue

Step 5 – Solve

Part (c) $\sigma_m = 0$

$\sigma_a = 900\ MPa$

$N = 100\ 000\ cycles$ **Likely to fail by fatigue**



Exercise 4.4 – Fatigue

Step 5 – Solve

Part (d) $\sigma_m = 300\ MPa$

$\sigma_a = 800\ MPa$

$N = 100\ 000\ cycles$

Since $\sigma_m \neq 0$ *we have to use the Goodman rule to determine an equivalent stress amplitude which has* $\sigma_m = 0$

Stress range at a mean stress of $\boldsymbol{\sigma_m}$

Mean stress $\boldsymbol{\sigma_m}$

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{TS}}\right)$$

Stress range at a mean stress of ***zero***

Tensile strength of material

Exercise 4.4 – Fatigue

Step 5 – Solve

Part (d) $\sigma_m = 300\ MPa$

$\sigma_a = 800\ MPa$

$N = 100\ 000\ cycles$

Since $\sigma_m \neq 0$ we have to use the Goodman rule to determine an equivalent stress amplitude which has $\sigma_m = 0$

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{TS}}\right)$$

$$\Delta\sigma_{\sigma_0} = \frac{\Delta\sigma_{\sigma_m}}{\left(1-\frac{\sigma_m}{\sigma_{TS}}\right)}$$

Remember: *$\Delta\sigma=2\sigma_a$*

$$\Delta\sigma_{\sigma_0} = \frac{1600\ MPa}{\left(1-\frac{300\ MPa}{1820\ MPa}\right)}$$

$$\Delta\sigma_{\sigma_0} = 1916\ MPa$$

The equivalent stress amplitude is then:

$$\sigma_{a_0} = 958\ MPa$$

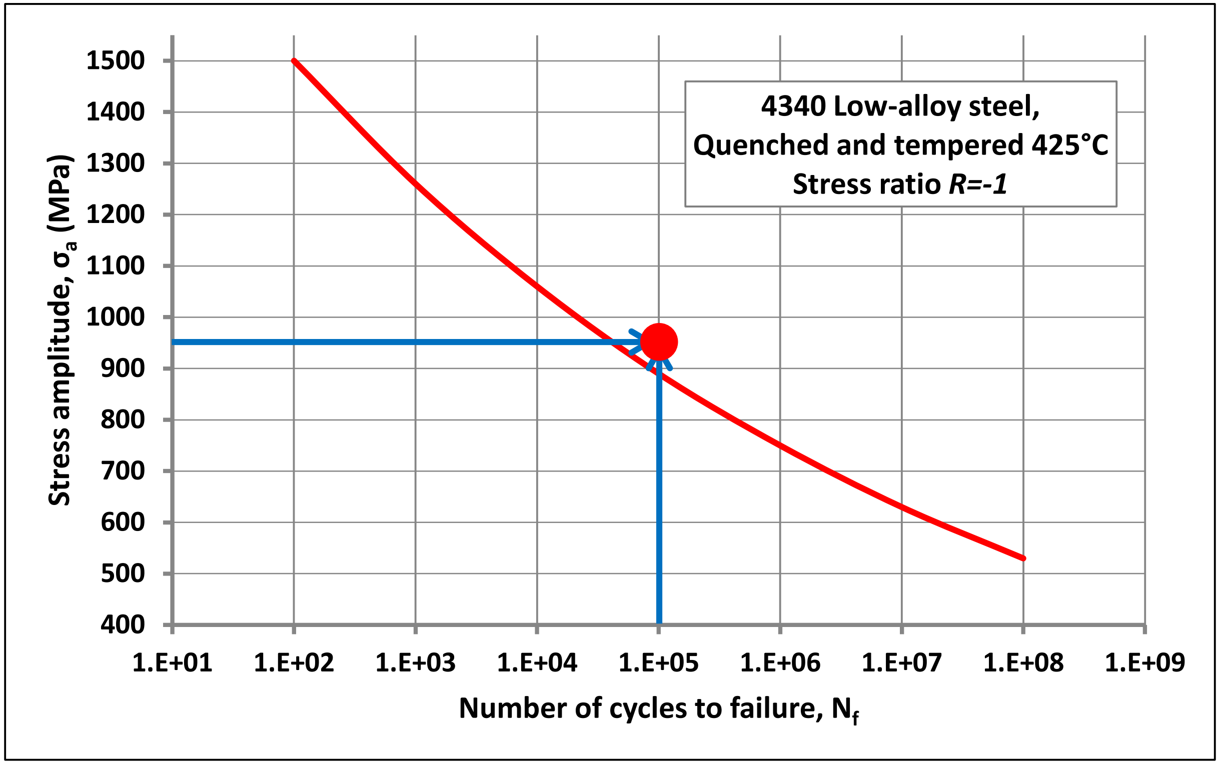
Exercise 4.1 – Fatigue

Step 5 – Solve

Part (d) $\sigma_m = 0$

$\sigma_a = 958\ MPa$

$N = 100\ 000\ cycles$ **Expected to fail by fatigue**



Exercise 4.4 – Fatigue

Step 5 – Solve

Part (e) $\sigma_{max} = 1100\ MPa$

$\sigma_{min} = -100\ MPa$

Stress range:

$$\Delta\sigma = \sigma_{max} - \sigma_{min} = 1200\ MPa$$

Stress Amplitude:

$$\sigma_a = \frac{\Delta\sigma}{2} = 600\ MPa$$

Mean stress:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 500\ MPa$$

Since $\sigma_m \neq 0$ we have to use the Goodman rule to determine an equivalent stress amplitude which has $\sigma_m = 0$

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{TS}}\right)$$

Exercise 4.4 – Fatigue

Step 5 – Solve

$$\Delta\sigma = 1000\ MPa$$

$$\sigma_a = 500\ MPa\ (= \Delta\sigma/2)$$

$$\sigma_m = 600\ MPa$$

Using Goodman's rule:

$$\Delta\sigma_{\sigma_0} = \frac{\Delta\sigma_{\sigma m}}{\left(1 - \frac{\sigma m}{\sigma_{ts}}\right)}$$

$$\Delta\sigma_{\sigma_0} = \frac{1200\ MPa}{\left(1 - \frac{500\ MPa}{1820\ MPa}\right)}$$

$$\Delta\sigma_{\sigma_0} = 1655\ MPa$$

The equivalent stress amplitude is then:

$$\sigma_{a_0} = 828\ MPa$$

Exercise 4.4 – Fatigue

Step 5 – Solve

$$\sigma_m = 0$$

$$\sigma_a = 828\ MPa$$

$$\textit{From the graph } N_f \approx 10^{5.25} cycles$$

$$N_f \approx 178000\ cycles$$

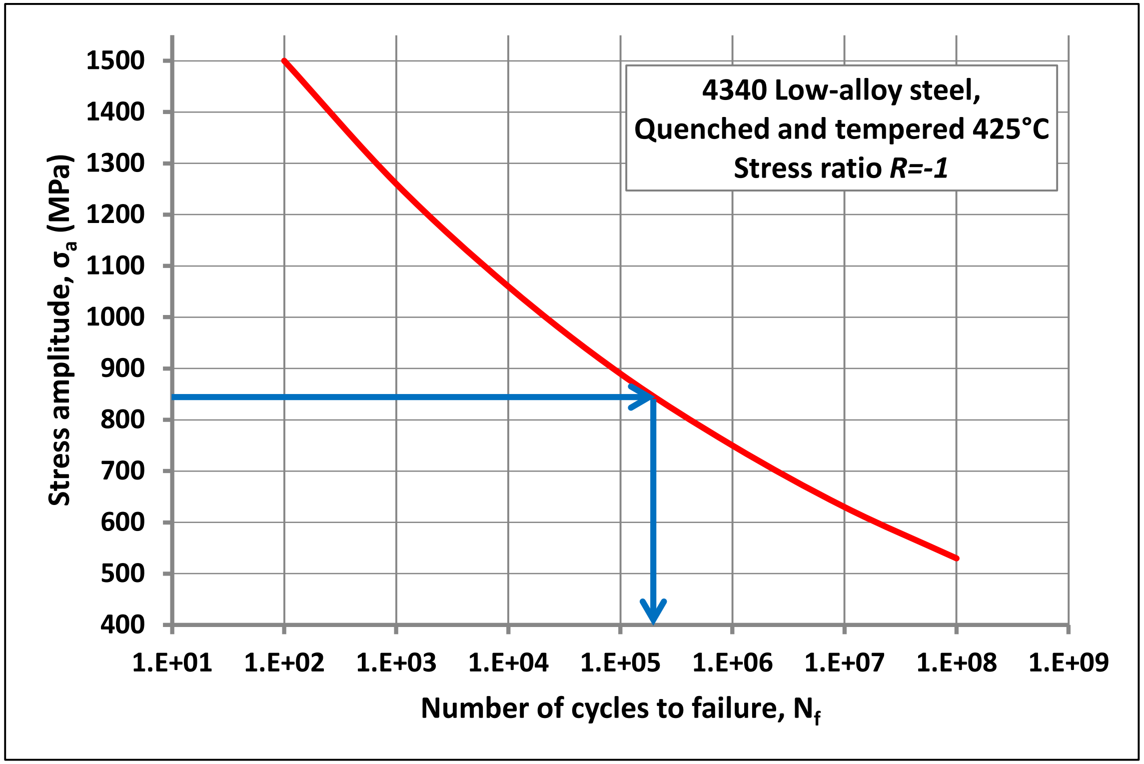
For 8Hz (= 8 cycles per second)

$$t_f = \frac{178000\ cycles}{8\ cycles/sec}$$

$$t_f = 22228\ seconds$$

$$t_f = \frac{22228\ sec}{3600\ sec/hr}$$

$$t_f = 6hrs\ 11mins$$



Exercise 4.4 – Fatigue

Step 6 – Verify

a) The value from the graph appears reasonable and agrees with theoretical estimates based on the tensile strength

b) and c) No estimate to verify against

d) The calculated equivalent stress amplitude (at $\sigma_m = 0$) is larger than the operation stress amplitude (at $\sigma_m \neq 0$) as was expected.

e) The calculated equivalent stress amplitude (at $\sigma_m = 0$) is larger than the operation stress amplitude (at $\sigma_m \neq 0$) as was expected.

The shaft life is significantly shorter than it would be for $\sigma_m = 0$ as predicted.

Exercise 4.5 – Fatigue

A small actuator is made from a polymer material with an ultimate tensile strength of σ_{ts} = 28 MPa. The loading of the actuator is fully reversed with a stress amplitude of σ_a = 11 MPa . If the loading cycles at a frequency of 3.0 Hz, how many hours would the actuator be expected to survive before failing due to fatigue? The SN curve for the polymer material is shown below

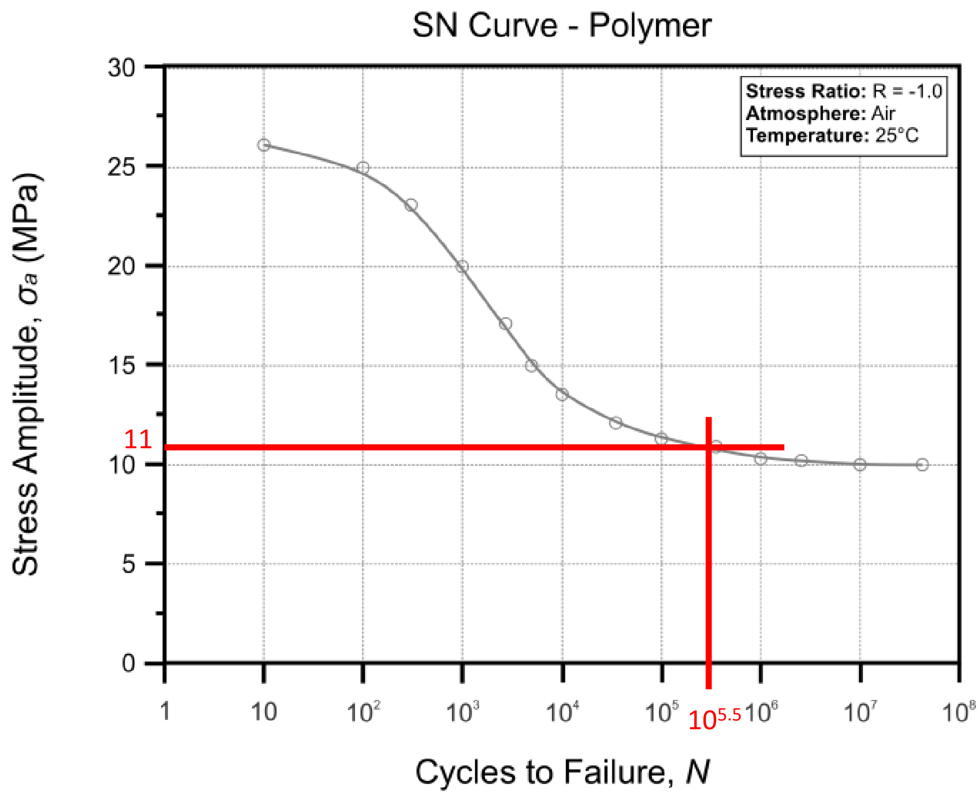


Figure 2. Fatigue behaviour of the polymer material.

1 Hz = 1 cycle per second

3600 Hz = 1 cycle per hour

$$\frac{10^{5.5}}{3(3600)} = \frac{316{,}228}{3(3600)} = 29.28 \text{ hours}$$