UNIVERSITY OF WOLLONGONG IN DUBAI

ENGG103 Midterm: Additional study problems

Question 1.1 Show that the atomic packing factor for the FCC crystal structure is 0.74.

Question 1.2 Copper has an atomic radius of 0.128 nm, an FCC crystal structure, and an atomic weight of 63.5 g/mol. Compute its theoretical density, and compare the answer with its measured density (8.94 g/cm^3).

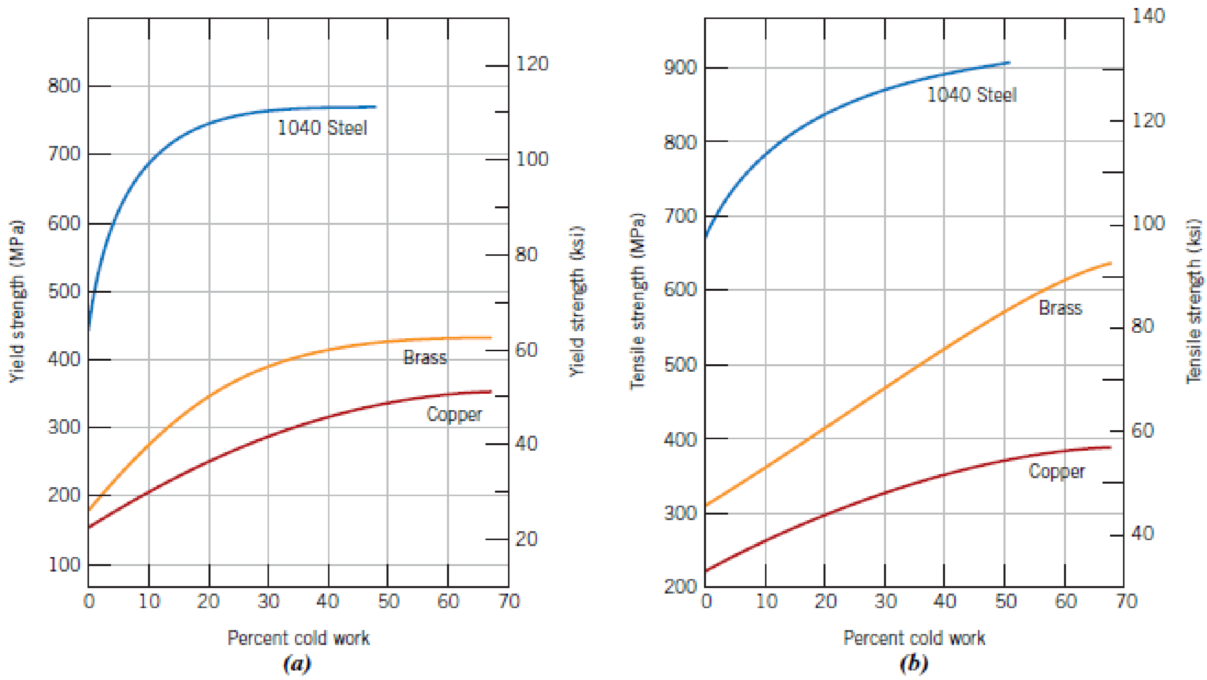
Question 1.3 Calculate the equilibrium number of vacancies per cubic meter for copper at 1000°C. The energy for vacancy formation is 0.9 eV/atom; the atomic weight and density (at 1000°C) for copper are 63.5 g/mol and 8.4 g/cm^3, respectively.

Question 1.4 A piece of copper originally 305 mm long is pulled in tension with a stress of 276 MPa. If the deformation is entirely elastic, what will be the resultant elongation?

Question 1.5 Compute the tensile strength and ductility (%EL) of a cylindrical copper rod if it is cold worked such that the diameter is reduced from 15.2 mm to 12.2 mm.

Question 1.6 A cylindrical rod of noncold-worked brass having an initial diameter of 6.4 mm is to be cold worked by drawing such that the cross-sectional area is reduced. It is required to have a cold-worked yield strength of at least 345 MPa and a ductility in excess of 20%EL; in addition, a final diameter of 5.1 mm is necessary.

Describe the manner in which this procedure may be carried out.



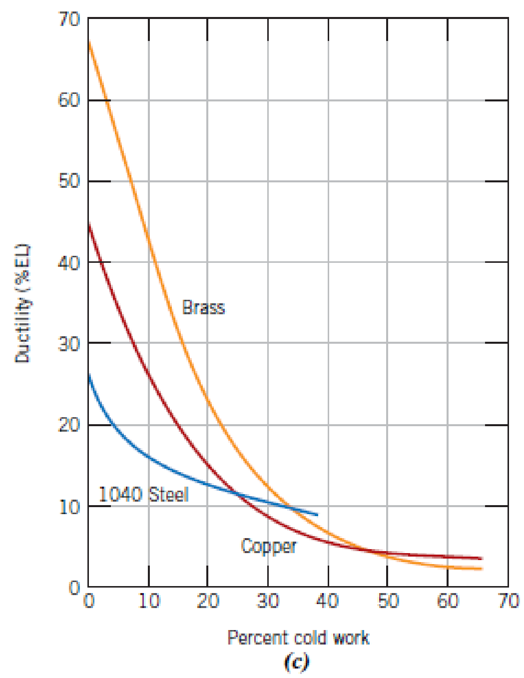


Figure 7.19 For 1040 steel, brass, and copper, (*a*) the increase in yield strength, (*b*) the increase in tensile strength, and (*c*) the decrease in ductility (%EL) with percent cold work.
[Adapted from *Metals Handbook: Properties and Selection: Irons and Steels*, Vol. 1, 9th edition, B. Bardes (Editor), 1978; and *Metals Handbook: Properties and Selection: Nonferrous Alloys and Pure Metals*, Vol. 2, 9th edition, H. Baker (Managing Editor), 1979. Reproduced by permission of ASM International, Materials Park, OH.]

Question 1.7

Mechanical Property Determinations from Stress–Strain Plot

From the tensile stress–strain behavior for the brass specimen shown in Figure 6.12, determine the following:

(a) The modulus of elasticity
(b) The yield strength at a strain offset of 0.002
(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm (0.505 in.)
(d) The change in length of a specimen originally 250 mm (10 in.) long that is subjected to a tensile stress of 345 MPa (50,000 psi)

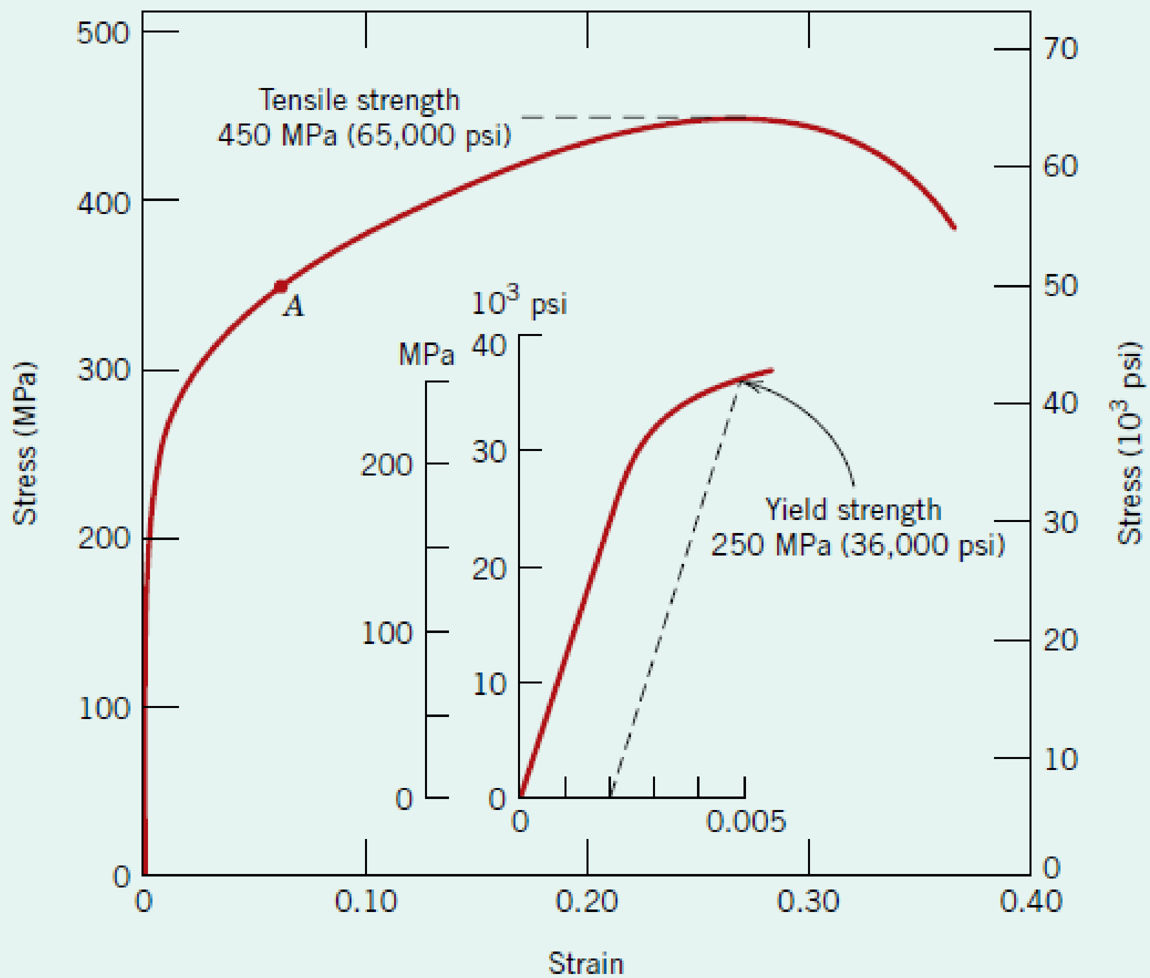


Figure 6.12 The stress–strain behavior for the brass specimen discussed in Example Problem 6.3.

Question 1.8 -

13.2 A large thick plate of steel is examined by X-ray methods, and found to contain no detectable cracks. The equipment can detect a single edge-crack of depth $a =$ 5 mm or greater. The steel has a fracture toughness K_c of 40 MN m$^{-3/2}$ m and a yield strength of 500 MN m^{-2}. Assuming that the plate contains cracks on the limit of detection, determine whether the plate will undergo general yield or will fail by fast fracture before general yielding occurs. What is the stress at which fast fracture would occur? ($Y = 1.12$ for the edge crack.)

Answer

Failure by fast fracture at 285 MN m^{-2}

Question 1.9 -

13.4 A large furnace flue operating at 440°C was made from a low-alloy steel. After 2 years in service, specimens were removed from the flue and fracture toughness tests were carried out at room temperature. The average value of K_c was only about 30 MN m$^{-3/2}$ compared to a value when new of about 80. The loss in toughness was due to temper embrittlement caused by the impurity phosphorus.

The skin of the flue was made from plate 10 mm thick. Owing to the self-weight of the flue the plate has to withstand primary membrane stresses of up to 60 MN m^{-2}. Estimate the length of through-thickness crack that will lead to the fast fracture of the flue when the plant is shut down.

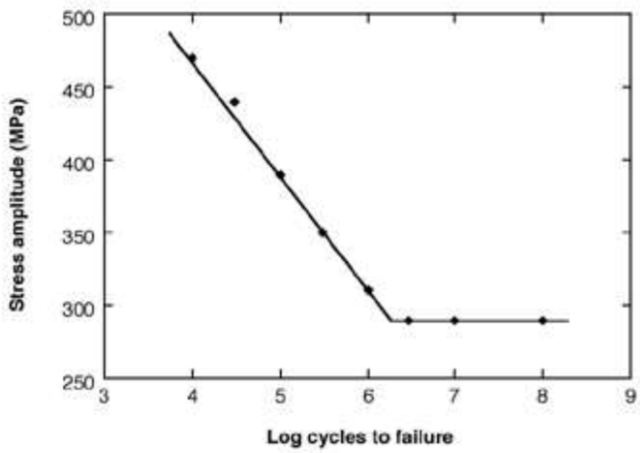
Answer

160 mm

Question 1.10 –

Question 6

A steel shaft operates at continuously with rotational speed of 600 rpm. Using the *S*-*N* curve provided determine the maximum continuous life for a stress amplitude of 400 MPa



Question 1.11 -

Question 7

For a low alloy steel the strength is $\sigma_{ts} = 950\ MPa$, when subjected to cyclic tensile stress it will have

$$\sigma_{min} = 210\ MPa$$

$$\sigma_{max} = 900\ MPa$$

Calculate the stress amplitude with zero mean stress that gives equivalent fatigue behavior.

Relevant Midterm Exam Formulae

$\rho = \frac{nA}{V_C N_A}$ Avogadro number $\mathbf{N_A}$ = 6.022 × 10^{23} atoms/mol	$APF = \frac{Volume\ of\ atoms\ in\ a\ unit\ cell}{Volume\ of\ unit\ cell}$
$\frac{N_V}{N} = exp^{\left(-\frac{Q_V}{kT}\right)}$ Boltzmann's constant k_B= 8.62x10^{-5} eV/K = 1.38x 10^{-23} J/atom-K	$N_s = \rho\left(\frac{N_A}{A}\right)$ Avogadro number $\mathbf{N_A}$ = 6.022 × 10^{23} atoms/mol
$E = \frac{\sigma}{\varepsilon}$	$\sigma = \frac{F}{A}$
$\varepsilon = \frac{\Delta l}{l}$	$\delta = \frac{FL^3}{48EI}$ I=second moment of area (this will be provided in the question)
Strain hardening (cold work) $\%CW = \frac{A_o - A_f}{A_o} \times 100$	Hall-Petch Equation (grain boundary reduction) $\sigma_y = \sigma_0 + k_y d^{-0.5}$
	$K_{IC} = Y\sigma\sqrt{\pi a}$
$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{TS}}\right)$	$\Delta\sigma = \sigma_{max} - \sigma_{min}$ $\sigma_a = \frac{\Delta\sigma}{2}$ $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
1 N/m^2 = 1 Pa 1 N/mm^2 = 1 MPa	1mm = 0.001m

Solutions

Q1.1

EXAMPLE PROBLEM 3.2

Computation of the Atomic Packing Factor for FCC

Show that the atomic packing factor for the FCC crystal structure is 0.74.

Solution

The APF is defined as the fraction of solid sphere volume in a unit cell, or

$$\text{APF} = \frac{\text{volume of atoms in a unit cell}}{\text{total unit cell volume}} = \frac{V_S}{V_C}$$

Both the total atom and unit cell volumes may be calculated in terms of the atomic radius R. The volume for a sphere is $\frac{4}{3}\pi R^3$, and because there are four atoms per FCC unit cell, the total FCC atom (or sphere) volume is

$$V_S = (4)\tfrac{4}{3}\pi R^3 = \tfrac{16}{3}\pi R^3$$

From Example Problem 3.1, the total unit cell volume is

$$V_C = 16R^3\sqrt{2}$$

Therefore, the atomic packing factor is

$$\text{APF} = \frac{V_S}{V_C} = \frac{\left(\frac{16}{3}\right)\pi R^3}{16R^3\sqrt{2}} = 0.74$$

Q1.2

EXAMPLE PROBLEM 3.4

Theoretical Density Computation for Copper

Copper has an atomic radius of 0.128 nm, an FCC crystal structure, and an atomic weight of 63.5 g/mol. Compute its theoretical density, and compare the answer with its measured density.

Solution

Equation 3.8 is employed in the solution of this problem. Because the crystal structure is FCC, n, the number of atoms per unit cell, is 4. Furthermore, the atomic weight A_{Cu} is given as 63.5 g/mol. The unit cell volume V_C for FCC was determined in Example Problem 3.1 as $16R^3\sqrt{2}$, where R, the atomic radius, is 0.128 nm.

Substitution for the various parameters into Equation 3.8 yields

$$\begin{aligned}
\rho &= \frac{nA_{Cu}}{V_C N_A} = \frac{nA_{Cu}}{(16R^3\sqrt{2})N_A} \\
&= \frac{(4 \text{ atoms/unit cell})(63.5 \text{ g/mol})}{[16\sqrt{2}(1.28 \times 10^{-8} \text{ cm})^3/\text{unit cell}](6.022 \times 10^{23} \text{ atoms/mol})} \\
&= 8.89 \text{ g/cm}^3
\end{aligned}$$

The literature value for the density of copper is 8.94 g/cm^3, which is in very close agreement with the foregoing result.

Q1.3

EXAMPLE PROBLEM 4.1

Number-of-Vacancies Computation at a Specified Temperature

Calculate the equilibrium number of vacancies per cubic meter for copper at 1000°C. The energy for vacancy formation is 0.9 eV/atom; the atomic weight and density (at 1000°C) for copper are 63.5 g/mol and 8.4 g/cm^3, respectively.

Solution

This problem may be solved by using Equation 4.1; it is first necessary, however, to determine the value of N—the number of atomic sites per cubic meter for copper, from its atomic weight A_{Cu}, its density ρ, and Avogadro's number N_{A}, according to

$$N = \frac{N_{\mathrm{A}}\rho}{A_{\mathrm{Cu}}} \tag{4.2}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(8.4 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{63.5 \text{ g/mol}}$$

$$= 8.0 \times 10^{28} \text{ atoms/m}^3$$

Thus, the number of vacancies at 1000°C (1273 K) is equal to

$$N_v = N \exp\left(-\frac{Q_v}{kT}\right)$$

$$= (8.0 \times 10^{28} \text{ atoms/m}^3) \exp\left[-\frac{(0.9 \text{ eV})}{(8.62 \times 10^{-5} \text{ eV/K})(1273 \text{ K})}\right]$$

$$= 2.2 \times 10^{25} \text{ vacancies/m}^3$$

Q1.4

EXAMPLE PROBLEM 6.1

Elongation (Elastic) Computation

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

Solution

Because the deformation is elastic, strain is dependent on stress according to Equation 6.5. Furthermore, the elongation Δl is related to the original length l_0 through Equation 6.2. Combining these two expressions and solving for Δl yields

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right)E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

The values of σ and l_0 are given as 276 MPa and 305 mm, respectively, and the magnitude of E for copper from Table 6.1 is 110 GPa (16×10^6 psi). Elongation is obtained by substitution into the preceding expression as

$$\Delta l = \frac{(276\ \text{MPa})(305\ \text{mm})}{110 \times 10^3\ \text{MPa}} = 0.77\ \text{mm (0.03 in.)}$$

Q1.5

Tensile Strength and Ductility Determinations for Cold-Worked Copper

Compute the tensile strength and ductility (%EL) of a cylindrical copper rod if it is cold worked such that the diameter is reduced from 15.2 mm to 12.2 mm (0.60 in. to 0.48 in.).

Solution

It is first necessary to determine the percent cold work resulting from the deformation. This is possible using Equation 7.8:

$$\%\mathrm{CW} = \frac{\left(\frac{15.2\text{ mm}}{2}\right)^2 \pi - \left(\frac{12.2\text{ mm}}{2}\right)^2 \pi}{\left(\frac{15.2\text{ mm}}{2}\right)^2 \pi} \times 100 = 35.6\%$$

The tensile strength is read directly from the curve for copper (Figure 7.19*b*) as 340 MPa (50,000 psi). From Figure 7.19*c*, the ductility at 35.6%CW is about 7%EL.

Q1.6

Solution

Let us first consider the consequences (in terms of yield strength and ductility) of cold working in which the brass specimen diameter is reduced from 6.4 mm (designated by d_0) to 5.1 mm (d_i). The %CW may be computed from Equation 7.8 as

$$\%\mathrm{CW} = \frac{\left(\frac{d_0}{2}\right)^2 \pi - \left(\frac{d_i}{2}\right)^2 \pi}{\left(\frac{d_0}{2}\right)^2 \pi} \times 100$$

$$= \frac{\left(\frac{6.4\text{ mm}}{2}\right)^2 \pi - \left(\frac{5.1\text{ mm}}{2}\right)^2 \pi}{\left(\frac{6.4\text{ mm}}{2}\right)^2 \pi} \times 100 = 36.5\%\mathrm{CW}$$

From Figures 7.19*a* and 7.19*c*, a yield strength of 410 MPa (60,000 psi) and a ductility of 8%EL are attained from this deformation. According to the stipulated criteria, the yield strength is satisfactory; however, the ductility is too low.

Another processing alternative is a partial diameter reduction, followed by a recrystallization heat treatment in which the effects of the cold work are nullified. The required yield strength, ductility, and diameter are achieved through a second drawing step.

Again, reference to Figure 7.19*a* indicates that 20%CW is required to give a yield strength of 345 MPa. However, from Figure 7.19*c*, ductilities greater than 20%EL are possible only for deformations of 23%CW or less. Thus during the final drawing operation, deformation must be between 20%CW and 23%CW. Let's take the average of these extremes, 21.5%CW, and then calculate the final diameter for the first drawing d_0', which becomes the original diameter for the second drawing. Again, using Equation 7.8,

$$21.5\%\text{CW} = \frac{\left(\dfrac{d_0'}{2}\right)^2 \pi - \left(\dfrac{5.1\text{ mm}}{2}\right)^2 \pi}{\left(\dfrac{d_0'}{2}\right)^2 \pi} \times 100$$

Now, solving for d_0' from the preceding expression gives

$$d_0' = 5.8\text{ mm (0.226 in.)}$$

Q1.7

$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \tag{6.10}$$

Inasmuch as the line segment passes through the origin, it is convenient to take both σ_1 and ϵ_1 as zero. If σ_2 is arbitrarily taken as 150 MPa, then ϵ_2 will have a value of 0.0016. Therefore,

$$E = \frac{(150 - 0)\text{ MPa}}{0.0016 - 0} = 93.8\text{ GPa } (13.6 \times 10^6\text{ psi})$$

which is very close to the value of 97 GPa (14×10^6 psi) given for brass in Table 6.1.

(b) The 0.002 strain offset line is constructed as shown in the inset; its inter-section with the stress–strain curve is at approximately 250 MPa (36,000 psi), which is the yield strength of the brass.

(c) The maximum load that can be sustained by the specimen is calculated by using Equation 6.1, in which σ is taken to be the tensile strength, from Figure 6.12, 450 MPa (65,000 psi). Solving for F, the maximum load, yields

$$F = \sigma A_0 = \sigma\left(\frac{d_0}{2}\right)^2 \pi$$

$$= (450 \times 10^6\text{ N/m}^2)\left(\frac{12.8 \times 10^{-3}\text{ m}}{2}\right)^2 \pi = 57{,}900\text{ N } (13{,}000\text{ lb}_\text{f})$$

(d) To compute the change in length, Δl, in Equation 6.2, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accomplished by locating the stress point on the stress–strain curve, point A, and reading the corresponding strain from the strain axis, which is approximately 0.06. Inasmuch as $l_0 = 250$ mm, we have

$$\Delta l = \epsilon l_0 = (0.06)(250\text{ mm}) = 15\text{ mm } (0.6\text{ in.})$$

Q1.8

13.2. Calculate the stress for failure by (a) general yield and (b) fast fracture.

(a) $\sigma = 500\text{ MN m}^{-2}$ for general yield.

(b) $\sigma = \dfrac{K_c}{Y\sqrt{\pi a}} = \dfrac{40}{1.12\sqrt{\pi \times 0.005}} = \underline{285\text{ MN m}^{-2}}$,

for fast fracture, assuming that a crack on the limit of detection is present. The plate will fail by fast fracture before it fails by general yield.

Q1.9

13.4.

$$K_c = Y\sigma\sqrt{\pi a},$$
$$Y = 1,$$
$$K_c = \sigma\sqrt{\pi a},$$
$$a = \frac{1}{\pi}\left(\frac{K_c}{\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{30\,\text{MPa}\sqrt{\text{m}}}{60\,\text{MPa}}\right)^2 = 0.080\text{ m} = 80\text{ mm.}$$
$$2a = \underline{\underline{160\text{ mm}}}$$

Q1.10

Solution:

$$For\ \sigma_a = \boldsymbol{400\ MPa} \rightarrow N_f \approx 10^{4.8}\ cycles$$

$$N_f \approx 63100\ cycles$$

$$t_f = \frac{63100\ cycles}{600\ cycles/min} \approx 1\ hr\ 45\ min$$

Q1.10

Solution:

Stress amplitude (= $\Delta\sigma/2$)

$$\sigma_m = 555\ MPa$$
$$\sigma_a = 345\ MPa$$

$$\left(\frac{\Delta\sigma_{\sigma_0}}{2}\right) = \frac{\left(\frac{\Delta\sigma_{\sigma_m}}{2}\right)}{\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)} = \frac{345MPa}{\left(1 - \frac{555MPa}{950MPa}\right)} = 830\text{MPa}$$