Thermal Properties

Exercise 7.1: A length of lead piping is 50.0 m long at a temperature of 16° C. When hot water flows through it the temperature of the pipe rises to 80° C. Determine the length of the hot pipe if the coefficient of linear expansion of lead is 29×10^{-6} K^{-1}.

Solution:

Length L_1= 50.0 m, temperature t_1= 16°C, t_2 = 80°C and $\alpha = 29 \times 10^{-6}$ K^{-1}

Length of pipe at 80°C is given by:

$$L_2 = L_1 [1 + \alpha(t_2 - t_1)]$$

$= 50.0[1 + (29 \times 10^{-6})(80 - 16)]$

$= 50.0[1 + 0.001856]$

$= 50.0[1.001856]$

$= 50.0928$ m

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

where:

$\frac{\Delta L}{L_0}$ is the fractional change in length

α is the coefficient of linear expansion

ΔT is the change in temperature

i.e. an increase in length of 0.0928 m or 92.28 mm

Kelvin scale uses the same temperature interval as the Celsius scale, a **change** of temperature of, say, 50°C, is the same as a change of temperature of 50 K).

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Exercise 7.2: A rod of metal is measured at 285K and is 3.521m long. At 373K the rod is 3.523m long.

Determine the value of the coefficient of linear expansion (α) of the metal.

Solution:

Length L_1= 3.521 m, L_2= 3.523 m, temperature t_1= 285K, t_2 = 373K

Length of pipe at 373K is given by:

$L_2 = L_1 [1 + \alpha(t_2 - t_1)]$

3.523 = 3.521[1 + α (373 - 285)]
3.523 = 3.521+(3.521)(α)(88)
3.523 - 3.521= (3.521)(α)(88)

coefficient of linear expansion, $\alpha = \frac{.002}{(3.521)(88)} = 6.45 \times 10^{-6} K^{-1}$

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Exercise 7.3: A block of cast iron has dimensions of 50 mm by 30 mm by 10 mm at 15°C. Determine the increase in volume when the temperature of the block is raised to 75°C. Assume the coefficient of linear expansion of cast iron to be $11 \times 10^{-6}\ K^{-1}$.

Coefficient of volume expansion (γ) is 3 times the Coefficient of linear expansion (α)

$$\gamma = 3\alpha$$

Solution:

Length $V_1 = (50)(30)(10) mm^3$, temperature $t_1 = 15°C$, $t_2 = 75°C$

Volume of pipe at 75°C is given by:

$$V_2 = V_1 [1 + \gamma(t_2 - t_1)] = V_2 = V_1 [1 + 3\alpha (t_2 - t_1)]$$

$$V_2 = 15000 [1 + 3\alpha (75 - 15)] = 15000[1 + 3(11 \times 10^{-6}) (75 - 15)]$$

$$V_2 = 15000 [1 + 0.00198] = 15000 + 15000(0.00198)$$

Hence, the increase in volume = 15000 (0.00198) = <u>29.7 mm^3</u>

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$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

Additional Question

A zinc sphere has a radius of 30.0 mm at a temperature of 20°C. If the temperature of the sphere is raised to 420°C, determine the increase in: (a) the radius, (b) the surface area, (c) the volume of the sphere. Assume the coefficient of linear expansion for zinc to be 31×10^{-6} K^{-1}.

a) Solution:

Initial radius L_1= 30.0mm, temperature t_1= 20+273 = 293K, t_2 = 420 + 273 = 693K and $\alpha = 31 \times 10^{-6}$ K^{-1}

Length of pipe at 373K is given by:

$$L_2 = L_1 [1 + \alpha(t_2 - t_1)]$$

$$L_2 = 30[1 + (31 \times 10^{-6})(693-293)]$$
$$L_2 = 30 (1 + 0.0124)$$
$$L_2 = 30.372 \text{ mm}$$

Hence the increase in the radius is **0.372 mm**

Hence the increase in the radius is 0.372 mm

(b) Initial surface area of sphere, $A_1 = 4\pi r^2 = 4\pi(30.0)^2 = 3600\pi \text{ mm}^2$

New surface area at 693 K is given by:

$$A_2 = A_1[1 + \beta(t_2 - t_1)]$$

i.e. $A_2 = A_1[1 + 2\alpha(t_2 - t_1)]$ since $\beta = 2\alpha$, to a very close approximation

Thus $A_2 = 3600\pi[1 + 2(31 \times 10^{-6})(400)]$

$$= 3600\pi[1 + 0.0248] = 3600\pi + 3600\pi(0.0248)$$

Hence increase in surface area $= 3600\pi(0.0248) =$ **280.5 mm²**

(c) Initial volume of sphere, $V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(30.0)^3 \text{ mm}^3$

New volume at 693 K is given by:

$$V_2 = V_1[1 + \gamma(t_2 - t_1)]$$

i.e. $V_2 = V_1[1 + 3\alpha(t_2 - t_1)]$ since $\gamma = 3\alpha$, to a very close approximation

7.4

We are asked to determine the temperature to which 11 kg of steel initially at 25°C would be raised if 130 kJ of heat is supplied. This is accomplished by utilization of a modified form of Equation 19.1 as

$$\Delta T = \frac{\Delta Q}{m c_p}$$

in which ΔQ is the amount of heat supplied, m is the mass of the specimen, and c_p is the specific heat. From Table 19.1, $c_p = 486 \text{ J/kg} \cdot \text{K}$ for steel.

Thus

$$\Delta T = \frac{130 \times 10^3 \text{ Joules}}{(11 \text{ kg})(486 \text{ J/kg} \cdot \text{K})} = 24 \text{ K}$$

and

$$T_f = T_0 + \Delta T = 298\text{ K} + 24\text{ K} = 322\text{ K}\ (49°\text{C})$$

7.5

(a) If a rod of 1025 steel 0.5 m long is heated from 20 to 80°C (293 to 353 K) while its ends are maintained rigid, determine the type and magnitude of stress that develops. Assume that at 20°C the rod is stress free. (b) What will be the stress magnitude if a rod 1 m long is used? (c) If the rod in part (a) is cooled from 20 to –10°C (293 K to 263 K), what type and magnitude of stress will result?

Solution

(a) We are asked to compute the magnitude of the stress within a 1025 steel rod that is heated while its ends are maintained rigid. To do this we employ Equation 19.8, using a value of 207 GPa for the modulus of elasticity of steel (Table 6.1), and a value of 12.0×10^{-6} $(°C)^{-1}$ for α_l (Table 19.1). Therefore

$$\sigma = E\alpha_l(T_0 - T_f)$$

$$= (207 \times 10^3 \text{ MPa})\left[12.0 \times 10^{-6} (°C)^{-1}\right](20°C - 80°C)$$

$$= -150 \text{ MPa}$$

The stress will be compressive since its sign is negative.

(b) The stress will be the same [–150 MPa], since stress is independent of bar length.

(c) Upon cooling the indicated amount, the stress becomes

$$\sigma = E\alpha_l (T_0 - T_f)$$

$$= (207 \times 10^3 \text{ MPa})\left[12.0 \times 10^{-6} (^\circ\text{C})^{-1}\right]\left[(20^\circ\text{C} - (-10^\circ\text{C})\right]$$

$$= +74.5 \text{ MPa}$$

This stress will be tensile since its sign is positive.

7.6

A copper wire is stretched with a stress of 70 MPa at 20°C (293 K). If the length is held constant, to what temperature must the wire be heated to reduce the stress to 35 MPa.

Solution

We want to heat the copper wire in order to reduce the stress level from 70 MPa to 35 MPa; in doing so, we reduce the stress in the wire by 70 MPa – 35 MPa = 35 MPa, which stress will be a compressive one (i.e., $\sigma = -35$ MPa). Solving for T_f from Equation 19.8 [and using values for E and α_l of 110 GPa (Table 6.1) and $17.0 \times 10^{-6}\ (°C)^{-1}$ (Table 19.1), respectively] yields

$$T_f = T_0 - \frac{\sigma}{E\alpha_l}$$

$$= 20°C - \frac{-35 \text{ MPa}}{(110 \times 10^3 \text{ MPa})[17.0 \times 10^{-6}\ (°C)^{-1}]}$$

$$= 20°C + 19°C = 39°C \ \ (312 \text{ K})$$