Tutorial 7&8

Week 8

Question 1

A continuous and aligned carbon fibre reinforced polymer (CFRP) composite consists of **60 vol%** carbon fibres having a modulus of elasticity of **265 GPa** and **40 vol%** polyester resin that, when hardened, has an elastic modulus of **3.8 GPa**.

a) Calculate the modulus of elasticity of this composite where loading is applied in the longitudinal direction of the fibres

b) If a component with cross sectional area **180 mm²** is subjected to a normal tensile stress of **90 MPa** in the longitudinal direction of the fibres, determine the force carried by each of the fibre and matrix phases

c) Calculate the strain sustained by each phase under the loading described in part (b)

Theory

Elastic Modulus of aligned composite loaded longitudinally:

$$E_{comp,long} = E_{matrix}V_{matrix} + E_{fibre}V_{fibre}$$

Distribution of load in aligned composite loaded longitudinally:

$$\frac{F_{fibre}}{F_{matrix}} = \frac{E_{fibre}V_{fibre}}{E_{matrix}V_{matrix}}$$

Strains (Hooke's Law)

$$\varepsilon_{fibre} = \frac{\sigma_{fibre}}{E_{fibre}}$$

$$\varepsilon_{matrix} = \frac{\sigma_{matrix}}{E_{matrix}}$$

Q1– FRP

Step 1 – Define

a) Elastic Modulus, $\boldsymbol{E_{CFRP}}$

b) Forces, $\boldsymbol{F_{fibre}}$ and $\boldsymbol{F_{matrix}}$

c) Strains, $\boldsymbol{\varepsilon_{fibre}}$ and $\boldsymbol{\varepsilon_{matrix}}$

Step 2 – Data

Volume fractions: $V_{fibre} = 0.60$ $V_{matrix} = 0.40$

Elastic Modulii: $E_{fibre} = 265\ GPa$ $E_{matrix} = 3.80\ GPa$

Fibres are aligned and loaded longitudinally

Component Cross-sectional Area: $A = 180\ mm^2$

Applied Stress: $\sigma = 90\ MPa$

Q1– FRP

Step 3 – Theory

Elastic Modulus of aligned composite loaded longitudinally:

$$E_{comp,long} = E_{matrix}V_{matrix} + E_{fibre}V_{fibre}$$

Distribution of load in aligned composite loaded longitudinally:

$$\frac{F_{fibre}}{F_{matrix}} = \frac{E_{fibre}V_{fibre}}{E_{matrix}V_{matrix}}$$

Strains (Hooke's Law)

$$\varepsilon_{fibre} = \frac{\sigma_{fibre}}{E_{fibre}}$$

$$\varepsilon_{matrix} = \frac{\sigma_{matrix}}{E_{matrix}}$$

Q1– FRP

Step 5 – Solve

a) Elastic modulus:

$$E_{comp,long} = E_{matrix}V_{matrix} + E_{fibre}V_{fibre}$$

$$E_{comp,long} = 3.80\ GPa \times 0.40 + 265\ GPa \times 0.60$$

$$E_{comp,long} = 160.5\ GPa$$

Q1– FRP

Step 5 – Solve

b) Force distribution:

$$\frac{F_{fibre}}{F_{matrix}} = \frac{E_{fibre}V_{fibre}}{E_{matrix}V_{matrix}}$$

$$\frac{F_{fibre}}{F_{matrix}} = \frac{265\ GPa \times 0.60}{3.80\ GPa \times 0.40}$$

$$\frac{F_{fibre}}{F_{matrix}} \approx 104.6$$

$$F_{fibre} = 104.6 \times F_{matrix}$$

Q1– FRP

Step 5 – Solve

b) Force distribution (continued):

$$F_{fibre} = 104.6 \times F_{matrix}$$

$$F_{CFRP} = S \times A = 90\ MPa \times 180\ mm^2 = 16200\ N$$

$$F_{CFRP} = F_{fibre} + F_{matrix} = 105.6 \times F_{matrix}$$

$$\therefore F_{matrix} = \frac{16200\ N}{105.6} = 153\ N$$

$$\therefore F_{fibre} = 16200\ N\ - 153.4\ N = 16047\ N = 16.0\ kN$$

Q1– FRP

Step 5 – Solve

c) Strains:

Start with area fractions

$$A_{fibre} = V_{fibre}A = 0.60 \times 180\,mm^2 = 108\,mm^2$$

$$A_{matrix} = V_{matrix}A = 0.40 \times 180\,mm^2 = 72\,mm^2$$

Then calculate stresses

$$\sigma_{fibre} = \frac{F_{fibre}}{A_{fibre}} = \frac{16047\,N}{108\,mm^2} = 148.6\,MPa$$

$$\sigma_{matrix} = \frac{F_{matrix}}{A_{matrix}} = \frac{153\,N}{72\,mm^2} = 2.13\,MPa$$

Finally, calculate strains

$$\varepsilon_{fibre} = \frac{S_{fibre}}{E_{fibre}} = \frac{148.6\,MPa}{265\,000\,MPa} = 5.61 \times 10^{-4}$$

$$\varepsilon_{matrix} = \frac{S_{matrix}}{E_{matrix}} = \frac{2.13\,MPa}{3800\,MPa} = 5.61 \times 10^{-4}$$

Question 2

a) How would the elastic modulus of the CFRP calculated in Exercise 5.1 differ if the loading was not applied along the longitudinal axis of the fibres?

Points to consider:

i. Loading perpendicular to fibres in an aligned composite:

$$E_{ct} = \frac{E_m E_f}{V_m E_f + V_f E_m} = \frac{E_m E_f}{(1 - V_f)E_f + V_f E_m}$$

ii. For other configurations of fibre orientation and loading:

By way of summary, then, aligned fibrous composites are inherently anisotropic, in that the maximum strength and reinforcement are achieved along the alignment (longitudinal) direction. In the transverse direction, fiber reinforcement is virtually nonexistent: fracture usually occurs at relatively low tensile stresses.

For other stress orientations, composite strength lies between these extremes. The efficiency of fiber reinforcement for several situations is presented in Table 16.3; this efficiency is taken to be **unity** for an **oriented fiber** composite in the alignment direction, and **zero perpendicular** to it.

Fiber Orientation	***Stress Direction***	***Reinforcement Efficiency***
All fibers parallel	Parallel to fibers	1
	Perpendicular to fibers	0
Fibers randomly and uniformly distributed within a specific plane	Any direction in the plane of the fibers	$\frac{3}{8}$
Fibers randomly and uniformly distributed within three dimensions in space	Any direction	$\frac{1}{5}$

Applications involving totally multidirectional applied stresses normally use discontinuous fibers, which are randomly oriented in the matrix material. Table 16.3 shows that the reinforcement efficiency is only one-fifth that of an aligned composite in the longitudinal direction; however, the mechanical characteristics are isotropic.

From Table 16.3, Callister *Materials Science and Engineering, an Introduction*, 9th ed. 2014, New Jersey: John Wiley & Sons, Inc.

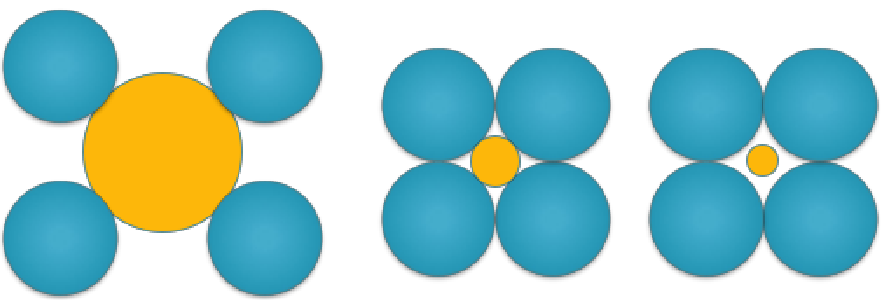
What are ceramics

A ceramic material may be defined as any inorganic crystalline material, compounded of a metal and a non-metal. It is solid and inert. Ceramic materials are brittle, hard, strong in compression, weak in shearing and tension. They withstand chemical erosion that occurs in an acidic or caustic environment. In many cases withstanding erosion from the acid and bases applied to it. Ceramics generally can withstand very high temperatures such as temperatures that range from 1,000 ° C to 1,600 ° C (1,800 ° F to 3,000 ° F). Ceramic materials that do not have oxygen such as silicon carbide can withstand 2,730 ° C.

Pauling's Rules for Ionic Crystals

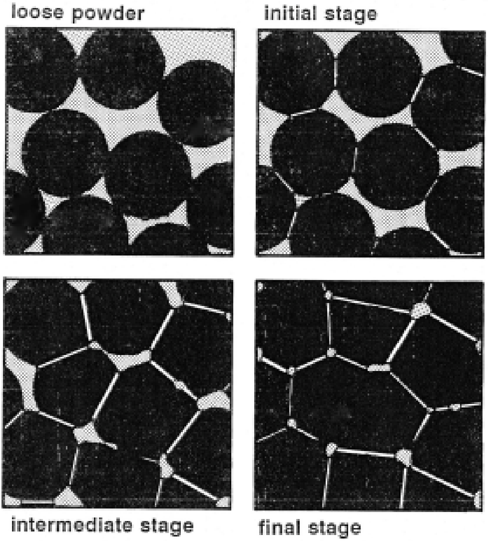
- Neighbouring ions must be touching for stable structure
  - Radius ratio of cation to anion determines geometry
- Local electroneutrality must be maintained
- Shared edges, faces reduce stability
- If several cations, small, high valency ones tend not to share edges, faces
- Number of types of ions tend to be small

Ionic Crystals



stable　　　　stable　　unstable

Stages in Solid State Sintering



R.M. German, Sintering Theory and Practice,Wiley Interscience, 1996, Fig 1.7

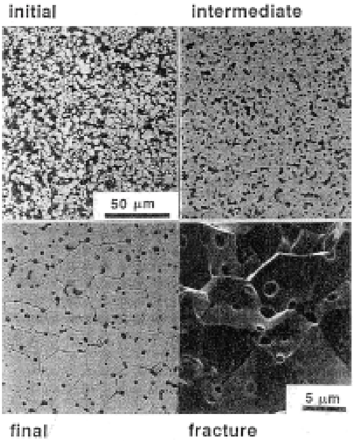
Stages of Sintering

- Adhesion
  - compacted particles
- Initial
  - neck growth, significant loss of surface area
  - some densification

Stages of Sintering

- Intermediate
  - pore rounding and elongation
  - most open pores gone
  - significant densification
  - some grain growth
- Final
  - pore closure
  - minimal densification
  - extensive grain growth

Stages in Solid State Sintering



Stages of Sintering

Stage	Process	Surface Area Loss	Densification	Coarsening
Adhesion	contact formation	minimal (unless high P is used)	none	none
Initial	neck growth	large, up to 50% loss	small	minimal
Intermediate	pore rounding, elongation	loss of most open porosity	major	grain growth and larger pores
Final	pore closure, densification	very little	very little, slow	extensive grain growth and pore growth