ENGG103 – Materials in Design

- **Tutorial 3**

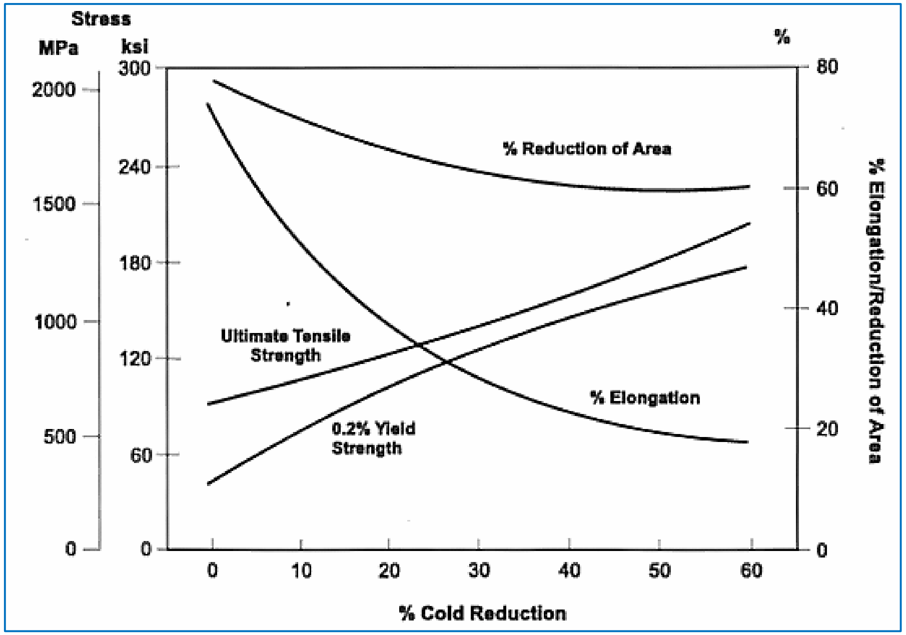
- *Strengthening Mechanisms*

Exercise 3.1 – Cold Work

The figure below shows the mechanical properties of type 204-Cu stainless steel as a function of cold work (%CW). The material is to be cold drawn in a single step into a circular wire of diameter 2.4 mm with yield strength in excess of 600 MPa.

a) What is the minimum diameter of the circular bar used as feedstock for the drawing process? Assume the bar feedstock is in the fully annealed condition.

b) What will be the ductility (%EL) of the final wire after this drawing process?



Step 3 – Theory

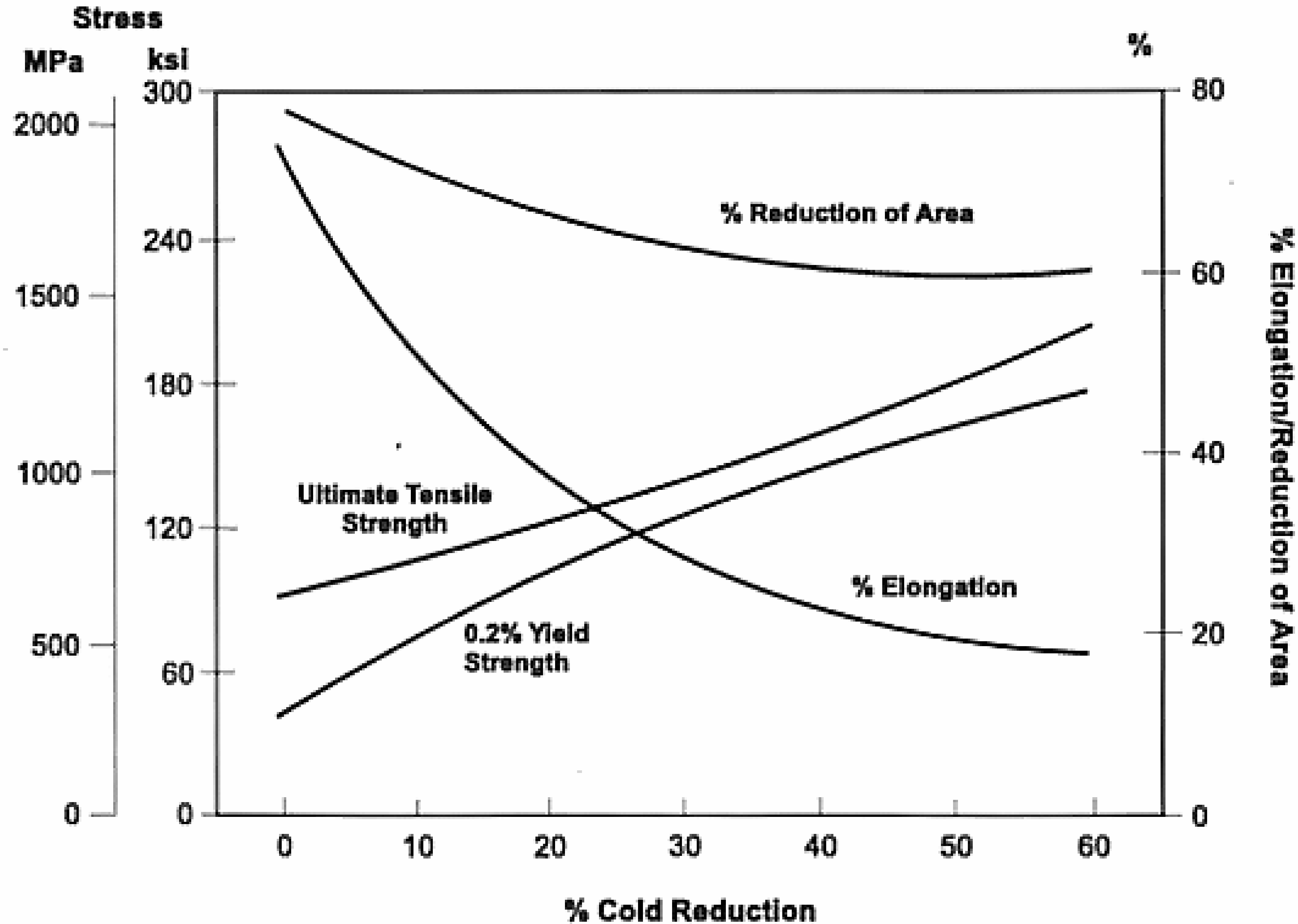
Cold work %CW from area change

$$\%CW = \frac{A_o - A_d}{A_o} \times 100\%$$

A_o = original cross-sectional area (mm^2)

A_d = deformed cross-sectional area (mm^2)

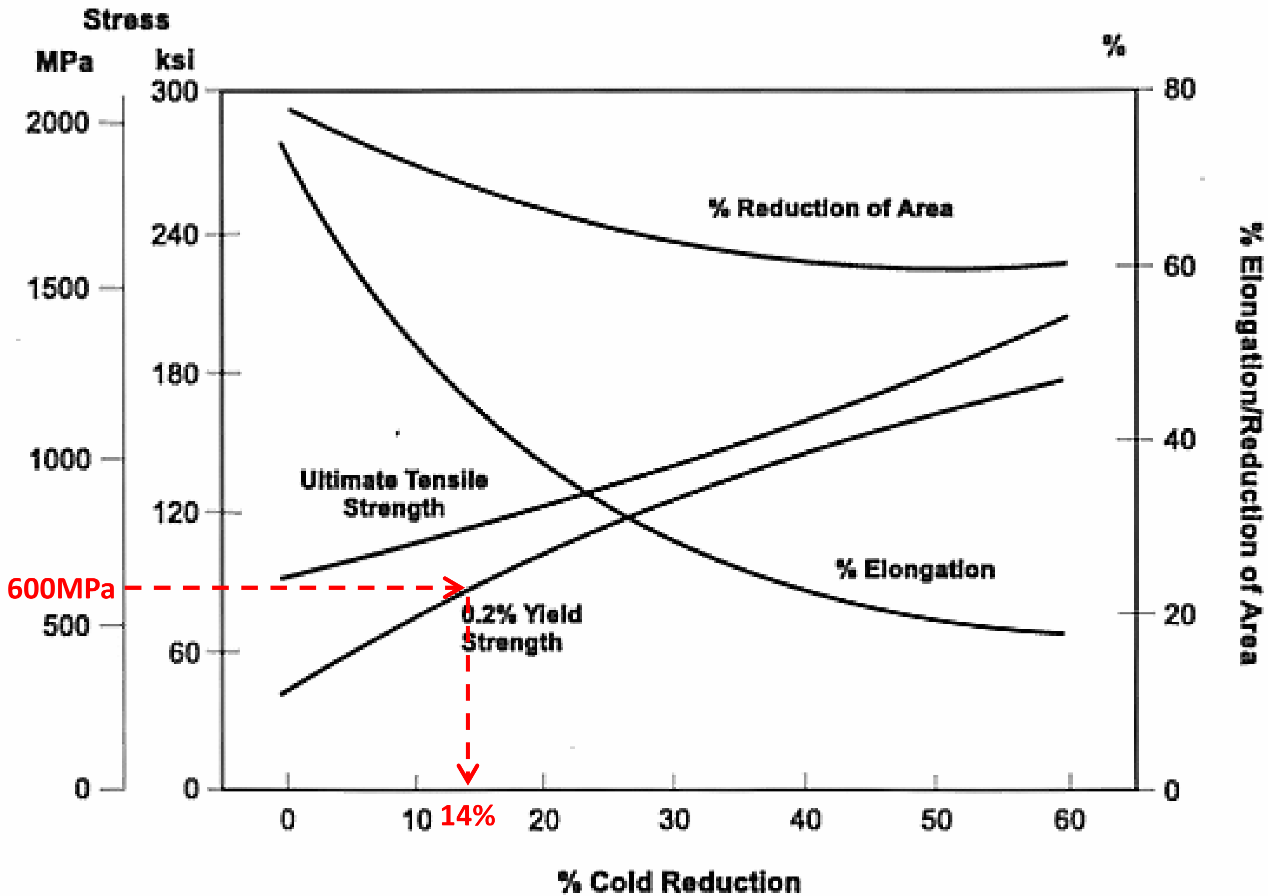
Exercise 3.1 – Cold Work



Exercise 3.1 – Cold Work

Solve

a) Firstly find the required %CW for $\sigma_y > 600$ MPa



Exercise 3.1 – Cold Work

(continued)

A minimum of 14 % cold work is required for $\sigma_y > 600\ MPa$

$$\%CW = \frac{A_o - A_d}{A_o} \times 100\ \%$$

$$A_o = \frac{A_d}{\left(1 - \frac{\%CW}{100\ \%}\right)}$$

$$A_d = 0.25 \times \pi \times (2.4\ mm)^2$$

$$A_d = 4.524\ mm^2$$

Exercise 3.1 – Cold Work

(continued)

Then

$$A_o = \frac{4.524\ mm^2}{\left(1 - \frac{14\ \%}{100\ \%}\right)}$$

$$A_o = \frac{4.524\ mm^2}{(1 - 0.14)}$$

$$A_o = 5.260\ mm^2$$

$$d_o = \sqrt{\frac{4A_o}{\pi}}$$

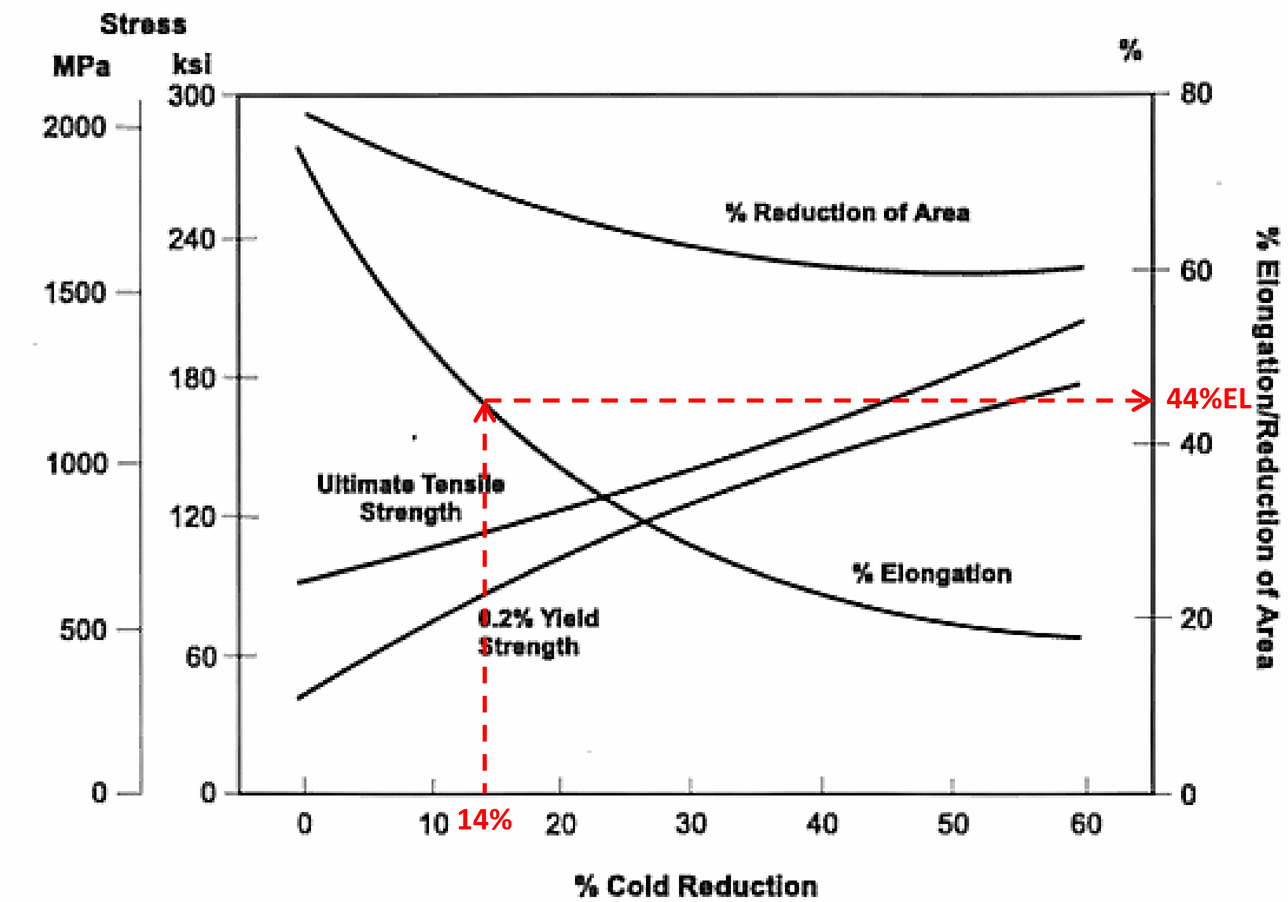
$$d_o = \sqrt{\frac{4 \times 5.260\ mm^2}{\pi}}$$

$$d_o = 2.59\ \mathrm{mm}$$

Exercise 3.1 – Cold Work

Solve

b) Find the %EL for 14 %CW



Exercise 3.1 – Cold Work

(continued)

To summarise:

- *A minimum of 14 % cold work is required for $\sigma_y > 600$ MPa*
- *Given the final diameter is to be 2.4 mm, the diameter before the drawing process must be at least 2.59 mm to get the required level of cold work*
- *At 14 % cold work, the ductility (as % elongation) would be expected to be equal to or less than 44 %*

Step 6 – Verify

The calculated values for cold work, initial diameter and ductility agree with our initial estimates and so are considered reasonable solutions

Exercise 3.2

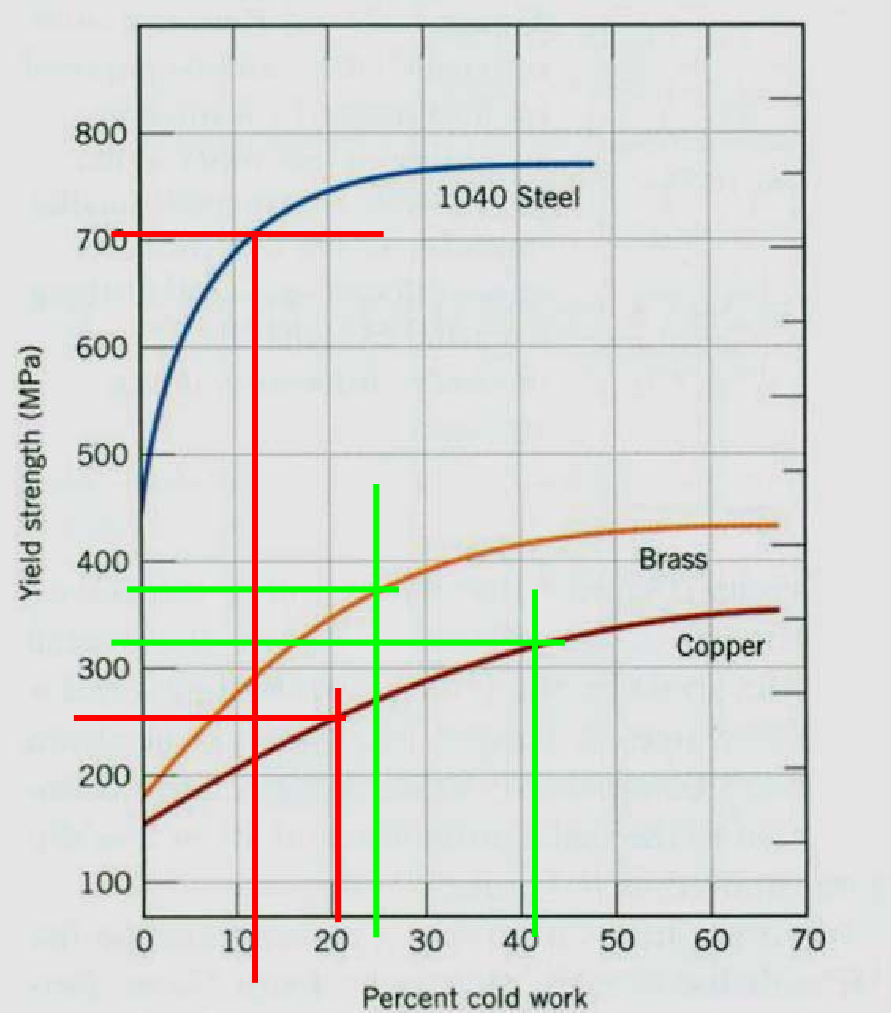
With relation to the data presented in the Figure 2:

a) What level of cold work would be required to give brass with yield strength of 370MPa?

b) What would be the yield strength of copper cold worked to 42%?

c) 1040 steel is to be rolled in a 'cold strip mill' to sheet 1.2mm in thickness with yield strength of 700MPa. Assuming the width of the strip doesn't change during the rolling process, what is the minimum thickness of the plate feedstock for the rolling process if it is initially in the fully annealed condition.

d) A square copper billet 20mm x 20mm is cold rolled into a rectangular profile 10mm x 32mm before being annealed. The rectangular section is then again cold rolled into bar section 5mm x 50mm. What would be the yield strength of this bar section?



c) 1040 steel

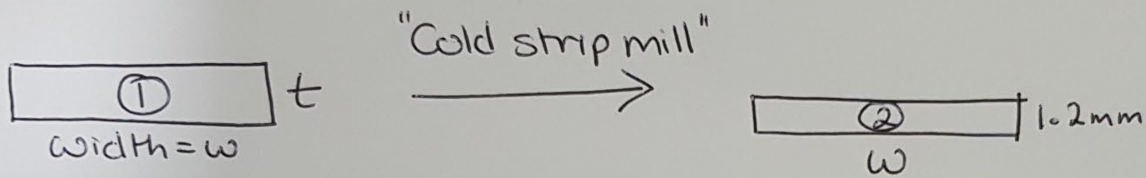
$t_f = 1.2\,mm$; σ_y 700 MPa

ω = width

$$\%CW = \frac{A_o - A_f}{A_o} \times 100$$

① $A_o = (\omega)(t)$

② $A_f = (\omega)(1.2\,mm)$



Q: Calculate minimum thickness of feedstock plate.

From graph σ_y @ 700 MPa ⇒ 12% CW

$$\left(\frac{12}{100}\right) = \frac{\omega t - 1.2\omega}{(\omega t)}$$

$$0.12(\omega t) = \omega t - 1.2\omega$$

$$0.12t(\cancel{\omega}) = (t - 1.2)(\cancel{\omega})$$

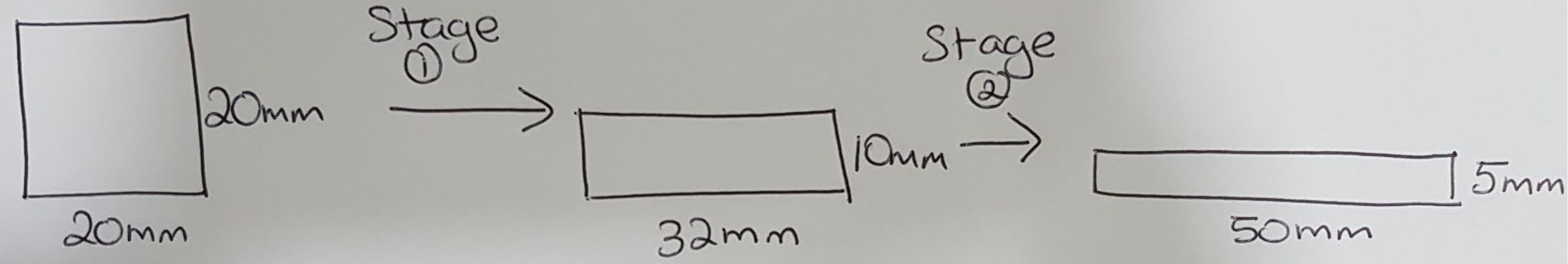
$$0.12t = t - 1.2$$

$$1.2 = t - 0.12t$$

$$1.2 = 0.88t$$

$$\Rightarrow t = \frac{1.2}{0.88} = \underline{1.363\,mm}$$

d) Square Copper Billet *Cold Rolled then annealed.*



Stage ①

$$\% CW = \frac{(20)(20) - (32)10}{(20)(20)} \times 100$$

$$= 20\% CW$$

Stage ②

$$\% CW = \frac{(32)(10) - (50)5}{32(10)} \times 100$$

$$= 21.875\% CW$$

*Note

Material is annealed after stage ① therefore the %CW will reduce to zero.

Because of this stage ② data is used for calculating yield strength.

$$\Rightarrow \sigma_y = 250 MPa$$

Exercise 3.3 – Grain size

The yield strength (σ_y) of the AZ31 magnesium alloy is 52 MPa for an average grain size (d) of 104 µm. Reducing grain size to 4 µm increases the yield strength to 253 MPa. Assuming that grain boundary refinement is the only significant strengthening mechanism at play:

a) Determine the intrinsic strength (σ_0) and grain refinement constant (k_y) of the magnesium alloy.

b) Calculate the expected yield strength of AZ31 for an average grain size of 40 µm.

Step 1 – Define

a) intrinsic strength σ_0 and constant k_y

b) σ_y for d=40 µm

Step 2 – Data

AZ31 magnesium alloy

for d=104 µm → σ_y=52 MPa

for d=4 µm → σ_y=253 MPa

Step 3 – Theory

Hall-Petch Equation

$$\sigma_y = \sigma_0 + k_y d^{-0.5}$$

σ_y = yield strength (MPa)

σ_0 = intrinsic strength of lattice (MPa)

k_y = constant (MPa.m$^{0.5}$)

d = grain size (m$^{-0.5}$)

Exercise 3.3 – Grain size

Solve

a) Solving for k_y

$$\sigma_y = \sigma_0 + k_y d^{-0.5}$$

$$\sigma_0 = \sigma_y - k_y d^{-0.5}$$

$$253\ MPa - k_y(4 \times 10^{-6}\ m)^{-0.5} = 52\ MPa - k_y(104 \times 10^{-6}\ m)^{-0.5}$$

$$k_y = \frac{253\ MPa - 52\ MPa}{(4\times10^{-6}\ m)^{-0.5} - (104\times10^{-6}\ m)^{-0.5}}$$

$$k_y = 0.5\ MPa.m^{0.5}$$

Exercise 3.3 – Grain size

(continued)

Solving for σ_0

$$\sigma_0 = 253\ MPa - 0.5\ MPa.m^{0.5} \times (4 \times 10^{-6}\ m)^{-0.5}$$

$$\sigma_0 = 253\ MPa - 250\ MPa$$

$$\sigma_0 = 3\ MPa$$

OR

$$\sigma_0 = 52\ MPa - 0.5\ MPa.m^{0.5} \times (104 \times 10^{-6}\ m)^{-0.5}$$

$$\sigma_0 = 52\ MPa - 49\ MPa$$

$$\sigma_0 = 3\ MPa$$

Exercise 3.3 – Grain size

(continued)

b)

$$\sigma_y = \sigma_0 + k_y d^{-0.5}$$

$$\sigma_y = 3\ MPa + 0.5\ MPa.m^{0.5} \times (40 \times 10^{-6}\ m)^{-0.5}$$

$$\sigma_y = 3\ MPa + 79.05\ MPa$$

$$\sigma_y = 82\ MPa$$

Exercise 3.4 – Grain size

The yield strength (σ_y) of the AZ31 magnesium alloy is 75 MPa for an average grain size (d) of 90 µm. Reducing grain size to 10 µm increases the yield strength to 200 MPa. Assuming that grain boundary refinement is the only significant strengthening mechanism at play:

a) Determine the intrinsic strength (σ_0) and grain refinement constant (k_y) of the magnesium alloy.
b) Calculate the expected yield strength of AZ31 for an average grain size of 40 µm.

Same method as above

Solutions

$$k_y = 0.592\ MPa.m^{0.5}$$

$$\sigma_0 = 12.5\ MPa$$

$$\sigma_y = 106.10\ MPa$$