ENGG103 – Materials in Design

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Week 4: Lecture 4 - Failure

OUTLINE

- Fundamentals of fracture.
- Ductile and brittle fractures.
- Linear elastic fracture mechanics.
  - Stress concentrations
  - Critical stress for crack propagation
  - Fracture toughness
- Design using fracture mechanics.

San Bruno pipeline failure

FRACTURE

- Fracture is the separation of a component into two or more parts under the influence of an applied stress.

- Fracture takes place in two stages:

  (i) crack initiation, and

  (ii) crack propagation.


Fractured shaft

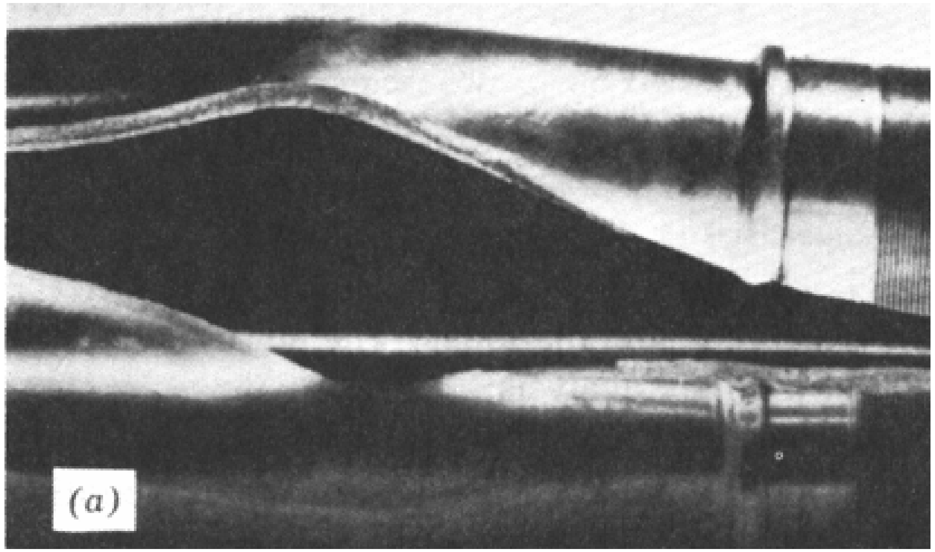
- Depending on the type of material, the applied load, the stress state, the strain rate and the temperature, metals can display different fracture modes:

  - Brittle fracture
  - Ductile fracture
  - Fatigue fracture
  - Creep fracture

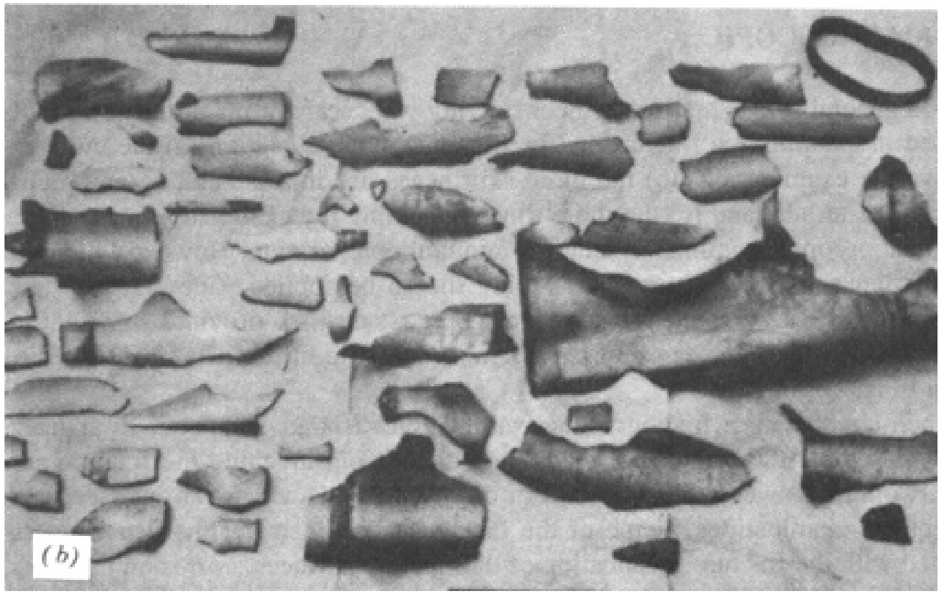
Fracture mechanisms

Example: Pipe Failures

- **Ductile failure:**
  -- one piece
  -- large deformation
  -- Accompanied by significant plastic deformation



- **Brittle failure:**
  -- many pieces
  -- small deformations
  - Little or no plastic deformation
  - Catastrophic



Figures from V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 4.1(a) and (b), p. 66 John Wiley and Sons, Inc., 1987. Used with permission.

Fracture mechanisms

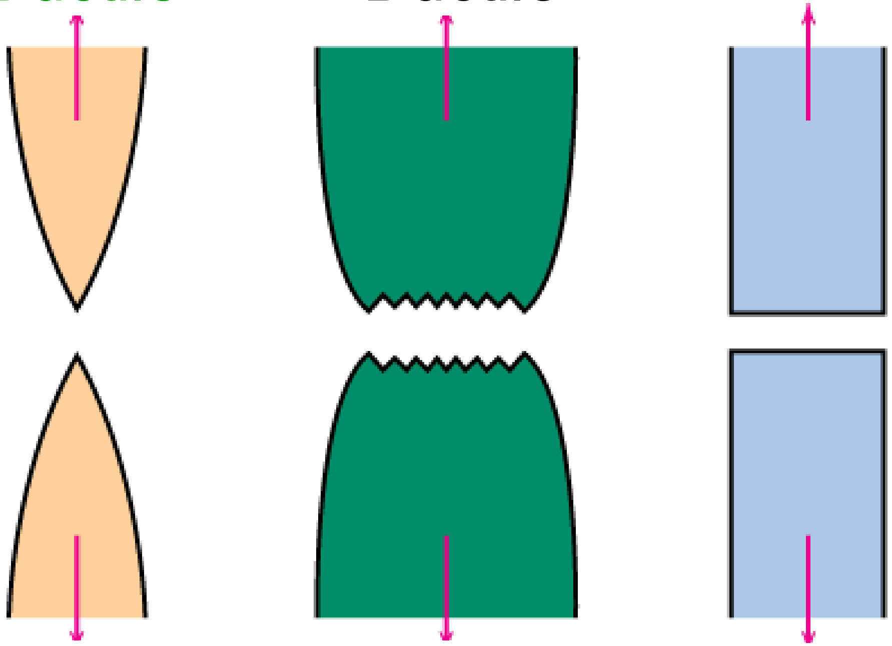
Ductility: Capacity of a material, such as copper, to be drawn or stretched under tension loading and permanently deformed without rupture or fracture. A form of plasticity deformation.

Brittleness: When subjected to stress, material breaks without significant plastic deformation. Brittle materials absorb relatively little energy prior to fracture

Ductile vs Brittle Failure

- Classification:

Fracture behavior:	Very Ductile	Moderately Ductile	Brittle



Adapted from Fig. 8.1, *Callister & Rethwisch 8e.*

%*AR* or %*EL*	Large	Moderate	Small

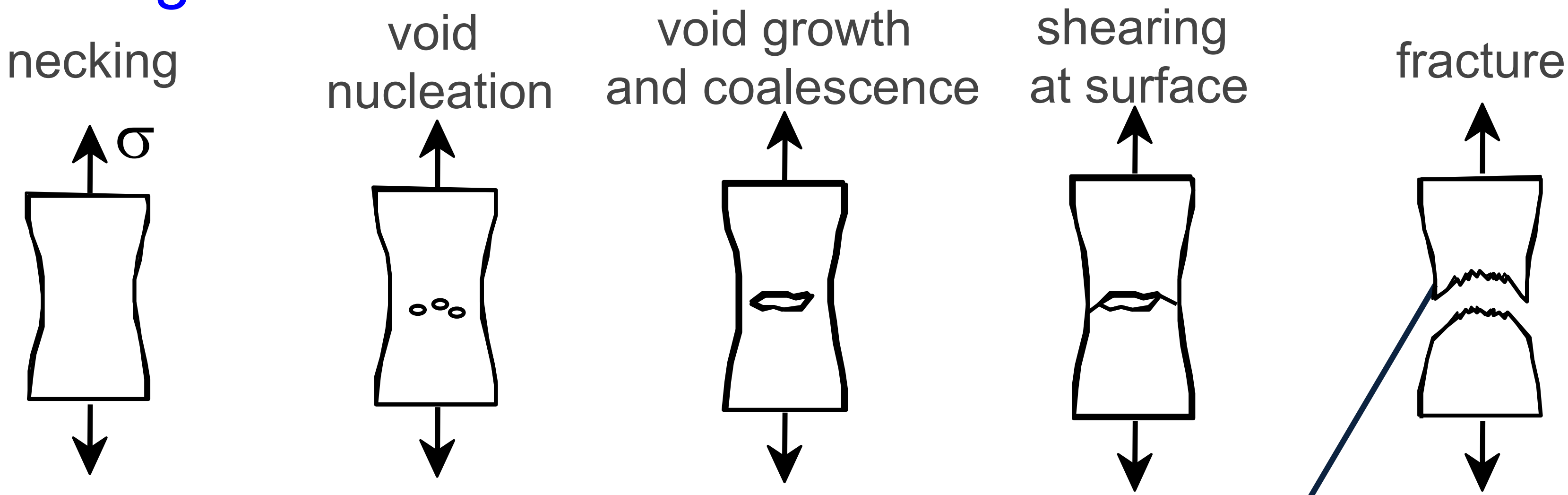
- Ductile fracture is usually more desirable than brittle fracture!

Ductile:
Warning before fracture

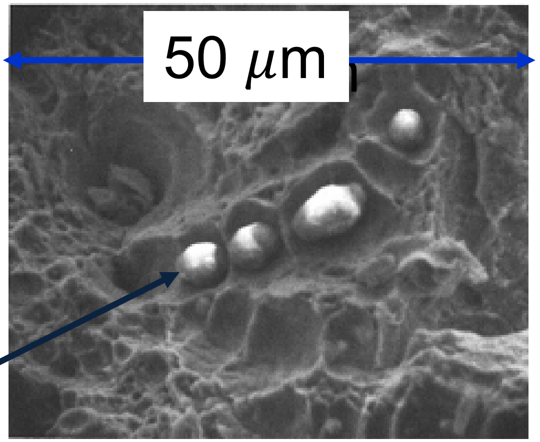
Brittle:
No warning

Moderately Ductile Failure

- Failure Stages:

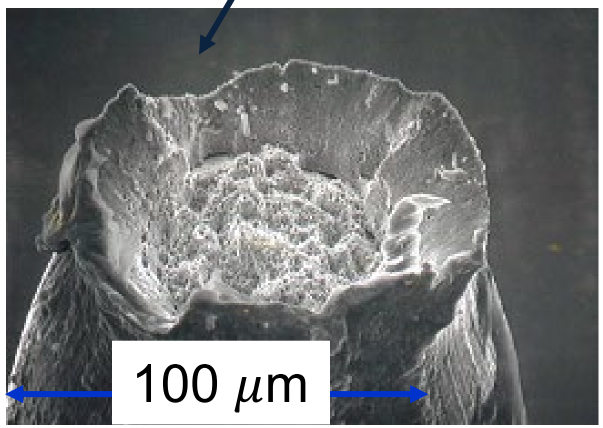


- Resulting fracture surfaces (steel)



particles serve as void nucleation sites.

From V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 11.28, p. 294, John Wiley and Sons, Inc., 1987. (Orig. source: P. Thornton, *J. Mater. Sci*., Vol. 6, 1971, pp. 347-56.)



Fracture surface of tire cord wire loaded in tension. Courtesy of F. Roehrig, CC Technologies, Dublin, OH. Used with permission.

Moderately Ductile vs. Brittle Failure

- Extensive **plastic deformation** (necking) takes place before fracture.
- Shear failure at an angle of 45° during the initial stages of fracture (45° shear lips).

cup-and-cone fracture

- Flat fracture surface with a coarse, granular appearance.
- Little or no evidence of plastic deformation at the fracture surface.
- Fracture surface is perpendicular to the applied stress.

brittle fracture

Adapted from Fig. 8.3, *Callister & Rethwisch 8e.*

LINEAR-ELASTIC FRACTURE MECHANICS

- If a small incision is made into an edge of plastic packaging, it is much easier to tear the package.
- One of the basic tenets of fracture mechanics is that **an applied stress is amplified at the tip of a small incision or notch**.

LINEAR-ELASTIC FRACTURE MECHANICS

Fracture mechanics was developed during World War I by the English aeronautical engineer A.A. Griffith to explain the failure of brittle materials. Griffith's work was motivated by two contradictory facts:

- the stress needed to fracture glass is around 100 MPa, whereas
- the theoretical stress needed for breaking the atomic bonds in glass is approximately 10,000 MPa.

A theory was needed to reconcile these conflicting observations. Experiments conducted by Griffith on glass fibres suggested that the fracture stress increases as the fibre diameter decreases. Griffith suggested that the low fracture strength observed experimentally, as well as the size-dependence of strength, was due to the presence of microscopic **flaws** in the material.

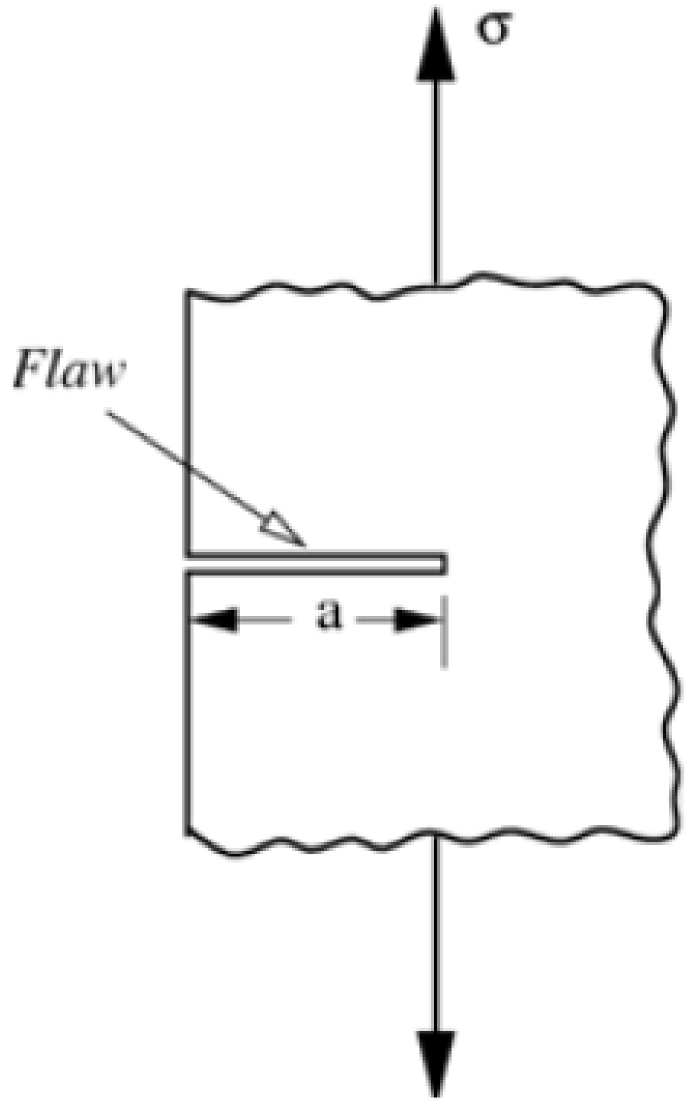
LINEAR ELASTIC FRACTURE MECHANICS

To verify the flaw hypothesis, Griffith introduced an **artificial flaw** in his experimental glass specimens. The artificial flaw was in the form of a surface crack that was much larger than other flaws in a specimen.

The experiments showed that the product of the square root of the flaw length (a) and the stress at fracture (σ_f) was nearly constant, which can be expressed by the following equation:

$$\boxed{\sigma_f\sqrt{a} \approx C}$$

where: σ_f is the fracture stress,
a is the flaw length, and
C is a constant.



THE FRACTURE PROCESS

How does it happen?

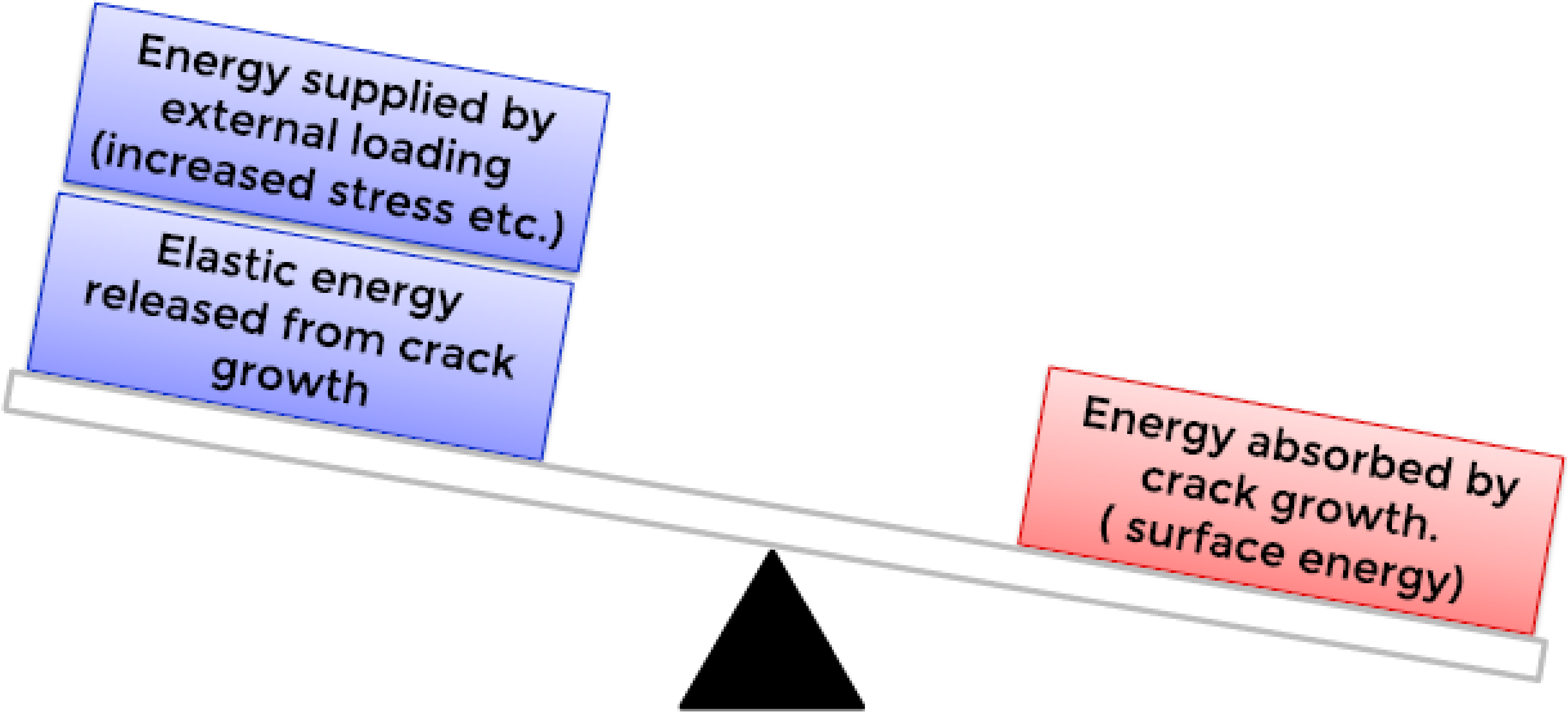
- Under a high enough load, a crack can propagate through the material causing separation of atomic layers.
- Crack growth requires energy (breaking bonds and creation of new surfaces) – the so-called surface energy γ [J/m^2] .
- Crack growth also releases energy (elastic energy).

Energy balance:

Energy supplied by external loading (increased stress etc.)	vs.	Energy absorbed by crack growth This is the energy required to create two new surfaces
Elastic energy released from crack growth		

THE FRACTURE PROCESS

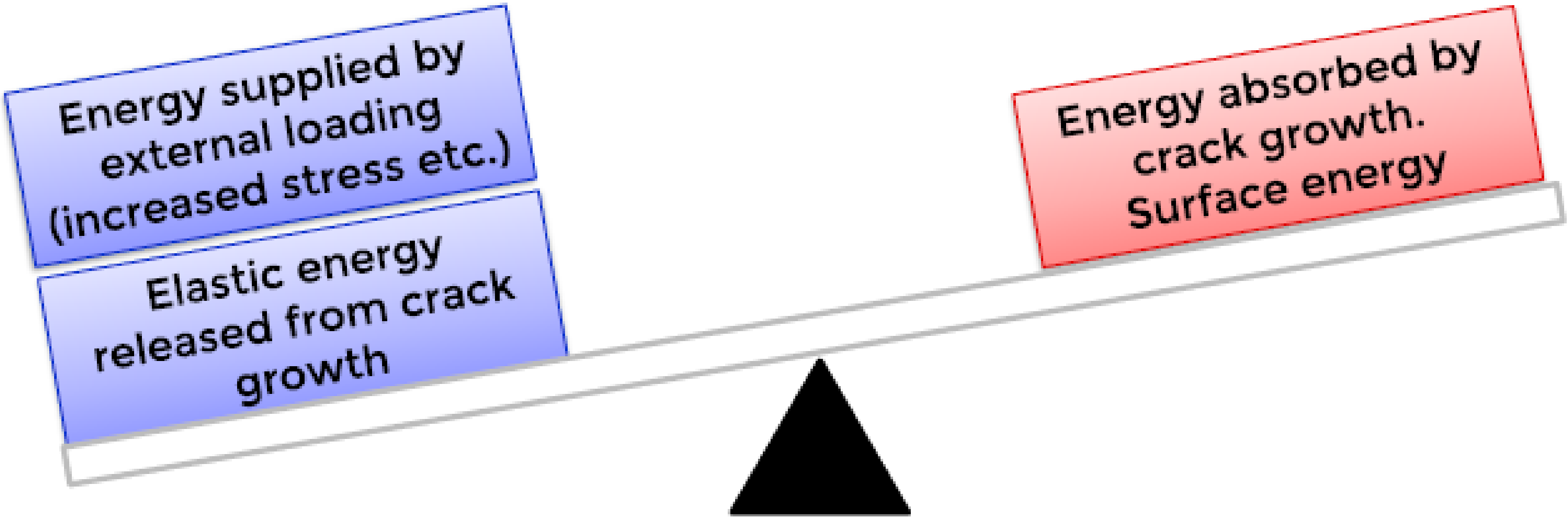
Energy balance:



Crack cannot grow

THE FRACTURE PROCESS

Energy balance:

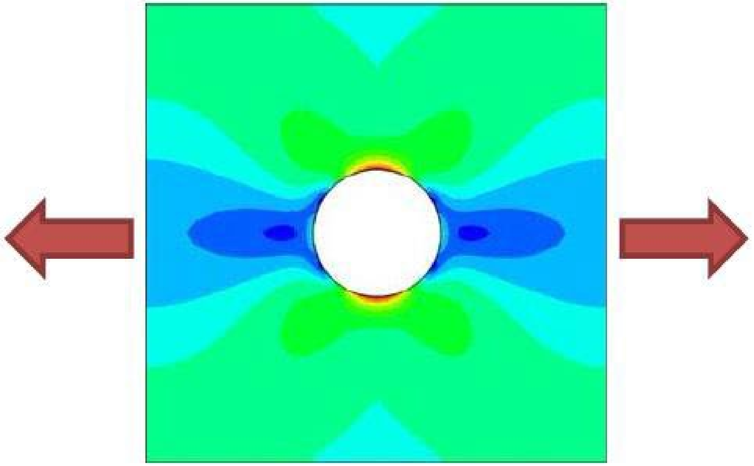


Crack is unstable and will grow rapidly

(even if the applied stress is below the yield strength of the material)

Stress Concentrations

- Holes, slots, threads and general changes in geometry cause localised changes in stress distributions.
  - Local stresses exceed the nominal ‘background’ stress.
  - Higher local stresses promotes failure from these locations.
- These features are stress concentrators.

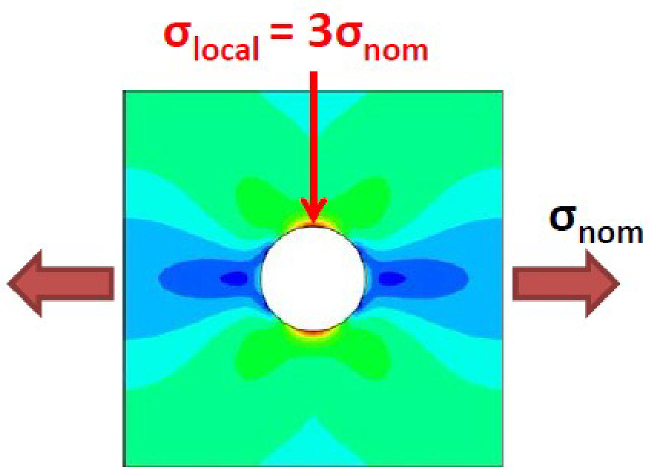


Stress Concentrations

The ratio of the maximum local stress to the background stress is called the <u>stress concentration factor</u>

- Stress Concentration Factor (SCF):

$$SCF = \frac{\sigma_{local}}{\sigma_{nom}}$$



Two common changes in a section

1 Hole in an infinite plate:

- Uniaxial applied stress (σ_{nom}).
- *SCF* = 3.
- Bolted connections in bridges, buildings etc.

2 Rotating shaft with diameter change:

- Bending gives tensile and compressive stresses.
- *SCF* depends on shaft diameters and fillet radius.
- Axles in vehicles, gearboxes, machinery etc.

Refer to page 267

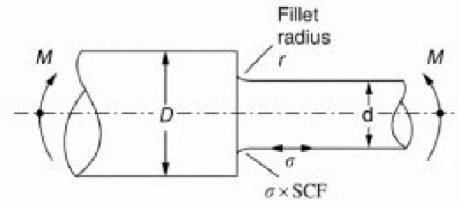


Table of SCF values

$\frac{D}{d}$ \ $\frac{r}{d}$	0.025	0.05	0.10	0.15	0.20	0.30
1.02	1.90	1.64	1.43	1.34	1.24	1.20
1.05	2.13	1.79	1.54	1.40	1.31	1.23
1.10	2.25	1.86	1.59	1.43	1.37	1.26
1.50	2.59	2.06	1.67	1.50	1.40	1.29
3.00	2.85	2.30	1.80	1.58	1.43	1.32

Stress Concentrations

- Stress Concentration Factor (SCF) must be considered in design.
- To avoid fatigue failure:

a) Design to minimise stress concentrations as much as practically possible:

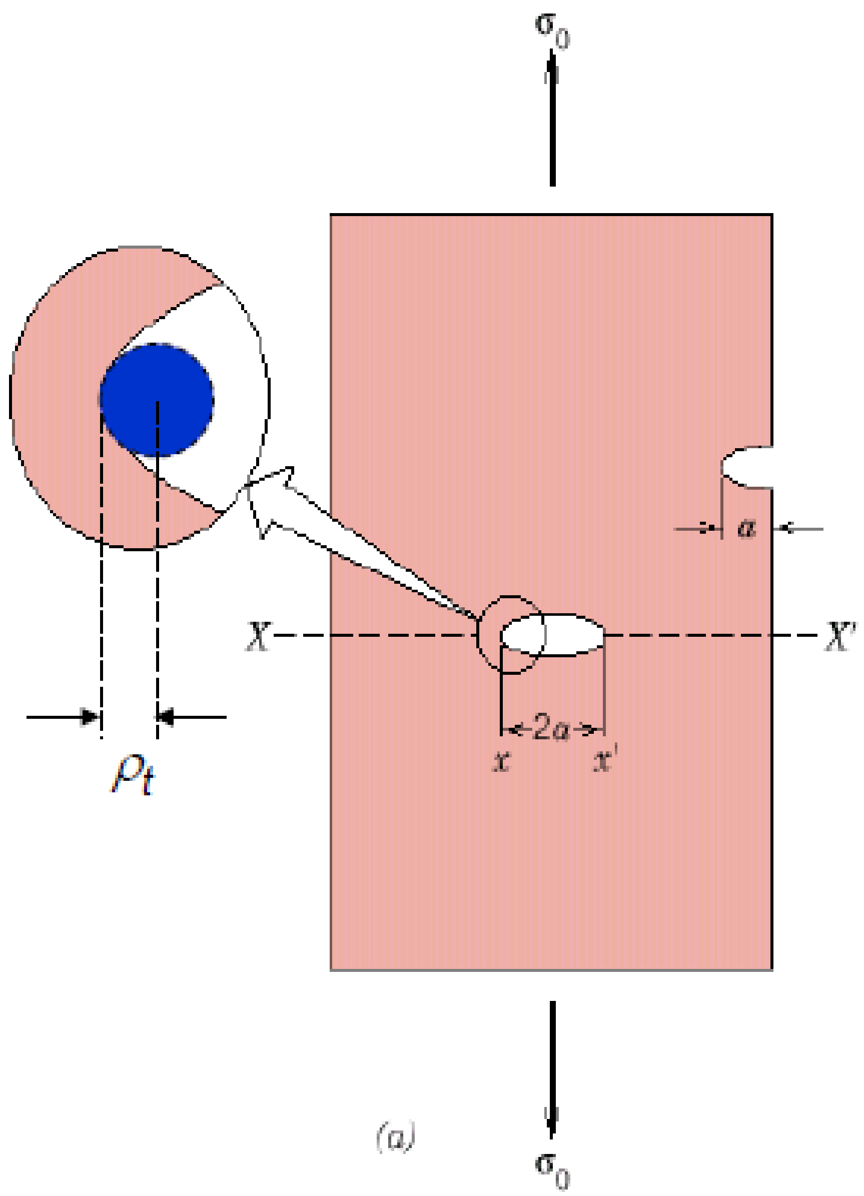
- Avoid abrupt changes in profile.
- Use chamfers or fillets.
- Use largest fillet radius possible.

b) Design stresses:

- Keep maximum stress below the required fatigue stress $\boldsymbol{\sigma_f}$
- $\boldsymbol{\sigma_{local}}$ is the critical stress, not $\boldsymbol{\sigma_{nom}}$

STRESS CONCENTRATION

Flaws are stress concentrators:



Griffith crack criterion:

$$\sigma_m = 2\sigma_0 \left(\sqrt{\frac{a}{\rho_t}} \right) = K_t \sigma_0$$

where: ρ_t = radius of curvature of the flaw

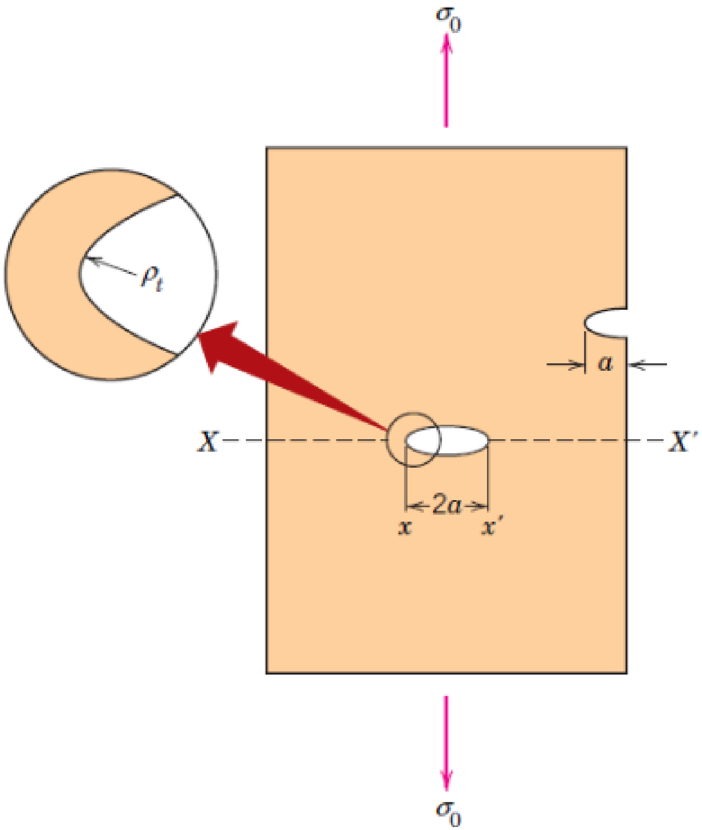
σ_0 = applied stress

σ_m = peak stress at the crack tip

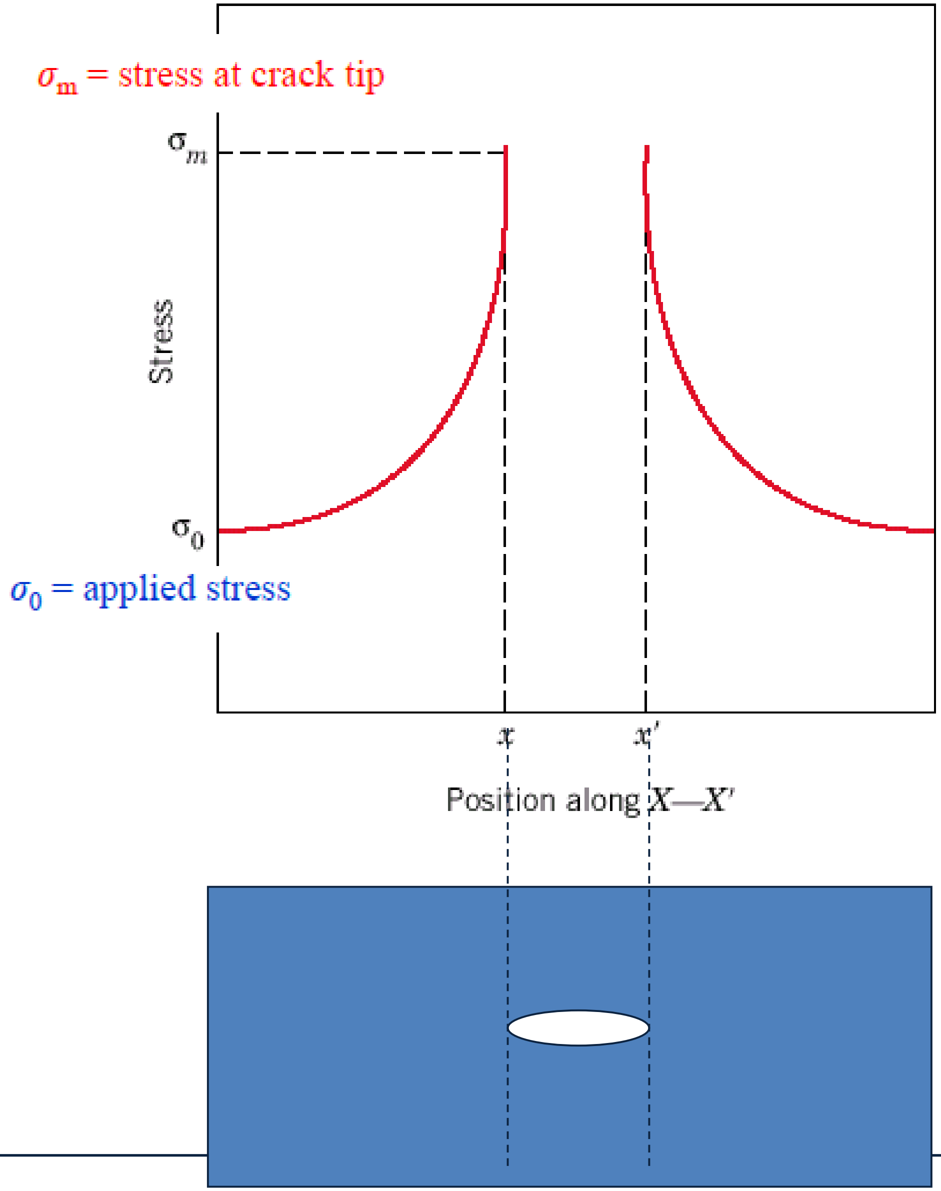
a = half crack length

K_t = stress concentration factor

Concentration of Stress at Crack Tip



(a)



Schematic stress profile along the line X–X in (a), demonstrating stress amplification at crack tip positions.

$$\sigma_m = 2\sigma_0\left(\sqrt{\frac{a}{\rho_t}}\right) = K_t\sigma_0$$

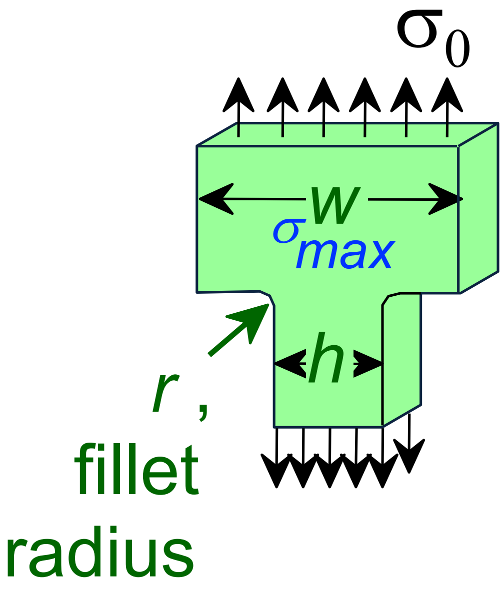
Adapted from Fig. 8.8(b),
Callister & Rethwisch 8e.

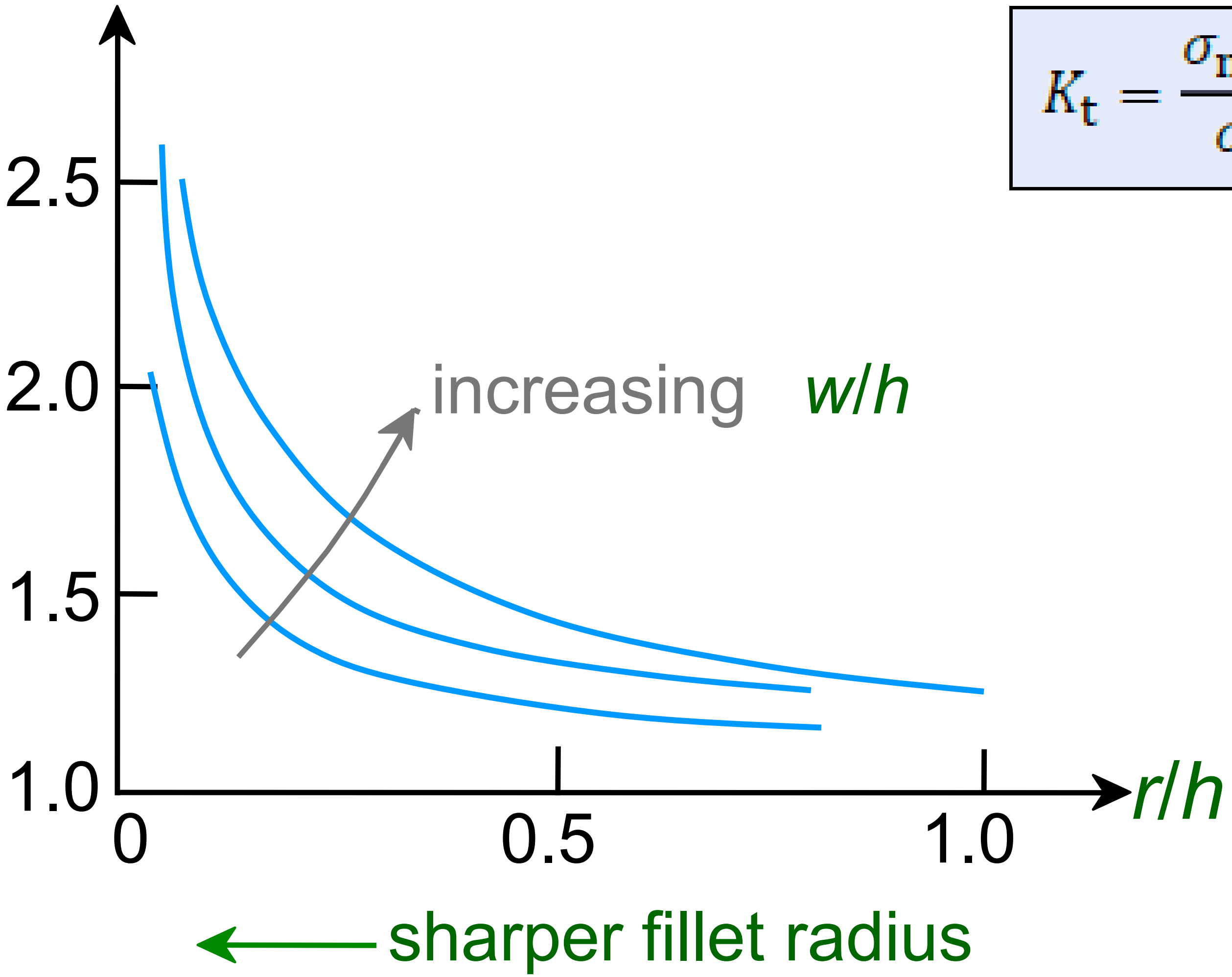
Engineering Fracture Design

- **Avoid sharp corners!**

Stress Conc. Factor, *K*

$$K_t = \frac{\sigma_{max}}{\sigma_0}$$





Adapted from Fig. 8.2W(c), *Callister 6e.* (Fig. 8.2W(c) is from G.H. Neugebauer, *Prod. Eng.* (NY), Vol. 14, pp. 82-87 1943.)

Crack Propagation

Cracks having sharp tips propagate easier than cracks having blunt tips

- A plastic material deforms at a crack tip, which "blunts" the crack.

Energy balance on the crack

- Elastic strain energy-
  - energy stored in material as it is elastically deformed
  - this energy is released when the crack propagates
  - creation of new surfaces requires energy

MATHEMATICAL EXPRESSION OF THE FAILURE CRITERION

$$\boxed{K \geq K_c = Y\sigma\sqrt{\pi a}}$$

Stress intensity (K)

Fracture toughness (K_c)
A material property
Measured experimentally

Geometrical factor (Y)

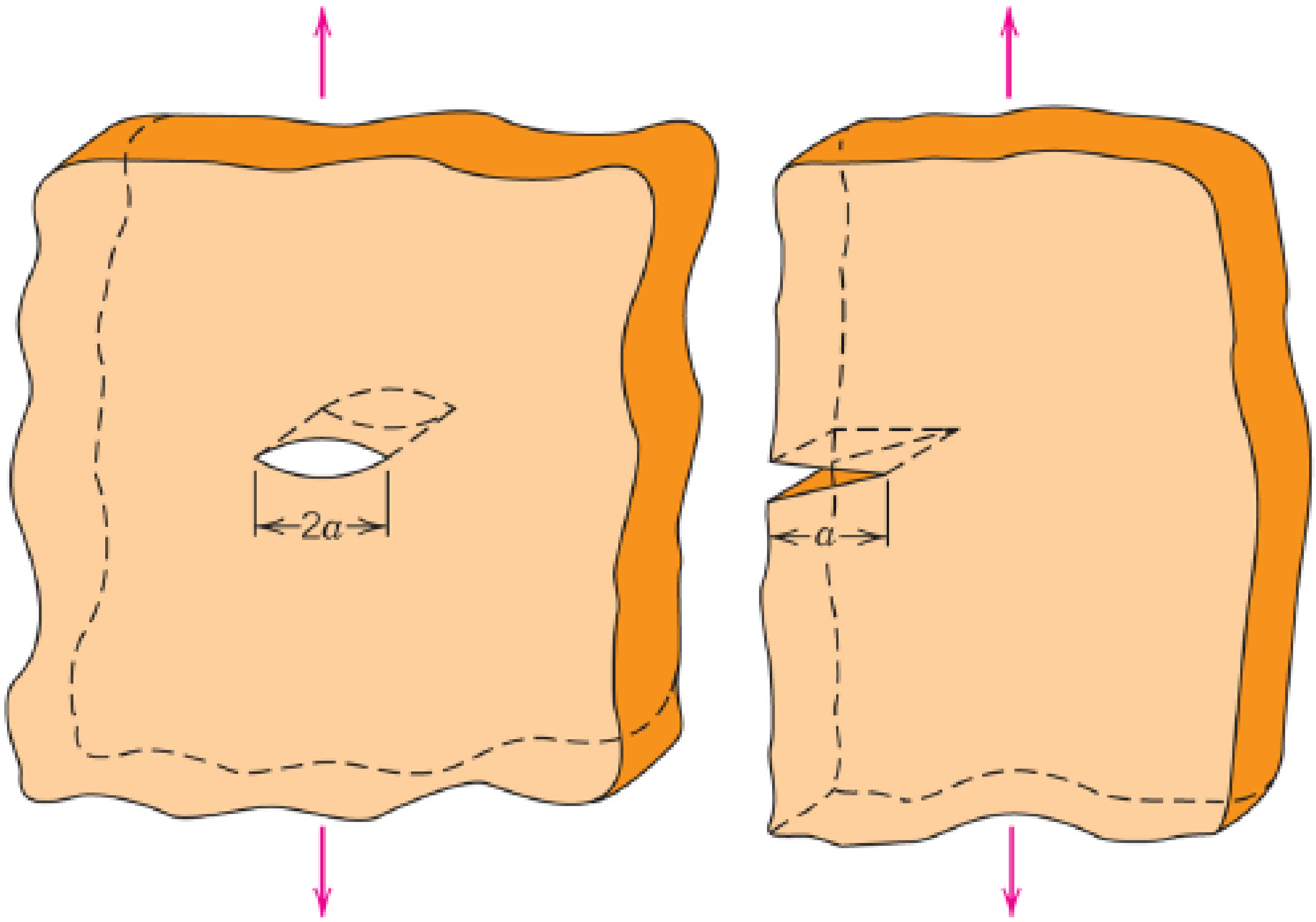
Critical stress (σ)
Design stress
The maximum stress that can be sustained in a material with fracture toughness K_c containing a crack of size $2a$

Half critical crack length (a)

Cracks will propagate when the fracture toughness of the material is exceeded for a combination of applied stress and crack length.

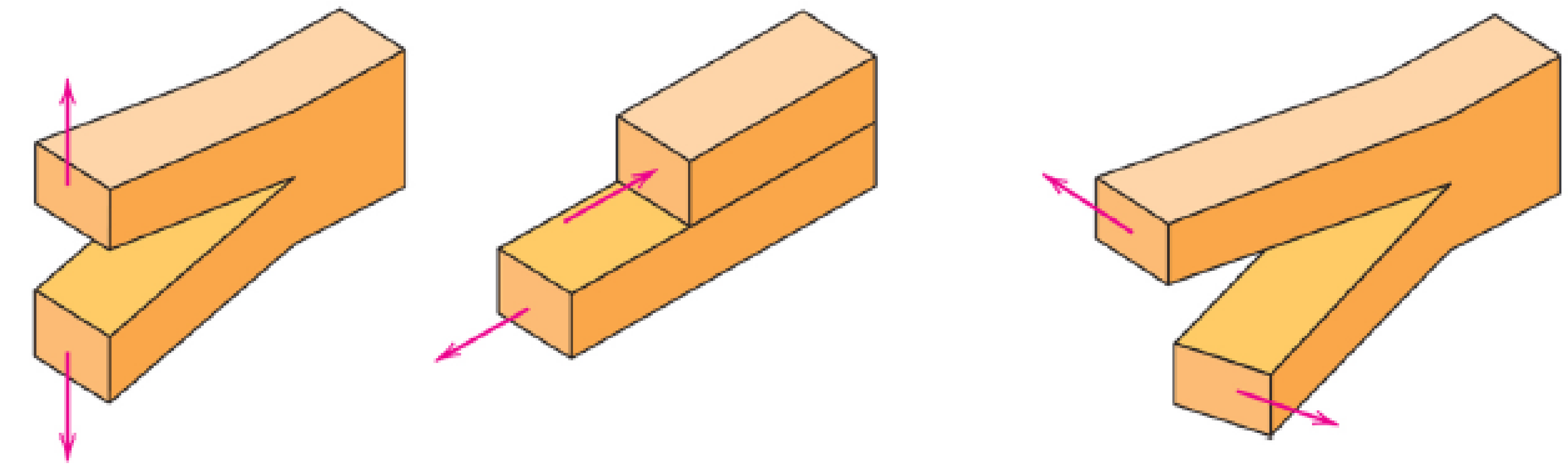
STRESS CONCENTRATION

Interior and edge cracks in a plate of infinite width:



(a) *(b)*

THREE MODES OF CRACK SURFACE DISPLACEMENT



Mode I Tensile loading

Mode II Shear loading

Mode III Tearing

Fracture mechanics calculations are usually concerned with Mode I and the fracture criterion is then expressed as:

$$\boxed{K_{Ic} = Y\sigma\sqrt{\pi a}}$$

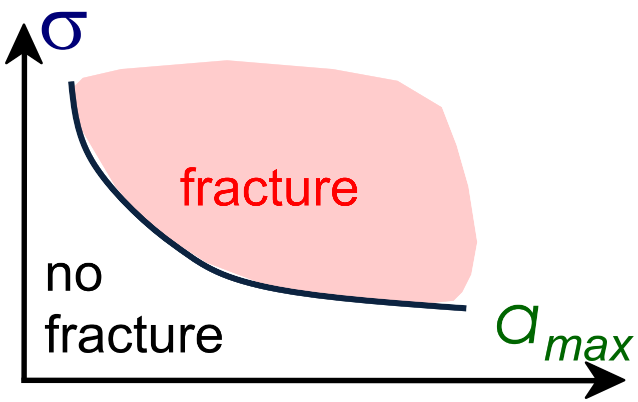
- Crack growth condition:

$$K \geq K_c = Y\sigma\sqrt{\pi a}$$

- Largest, most highly stressed cracks grow first!

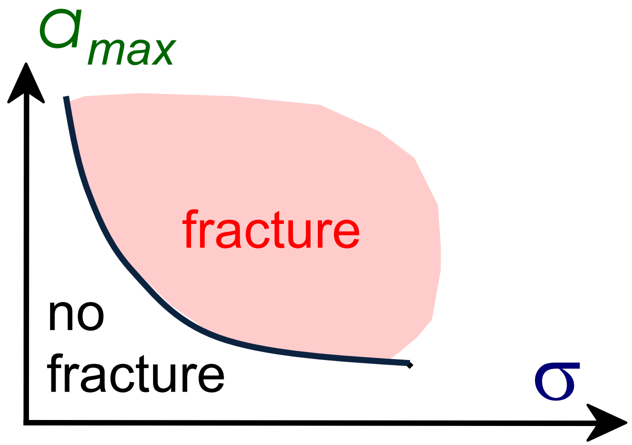
--Scenario 1: Max. flaw size dictates design stress.

$$\sigma_{design} < \frac{K_c}{Y\sqrt{\pi a_{max}}}$$

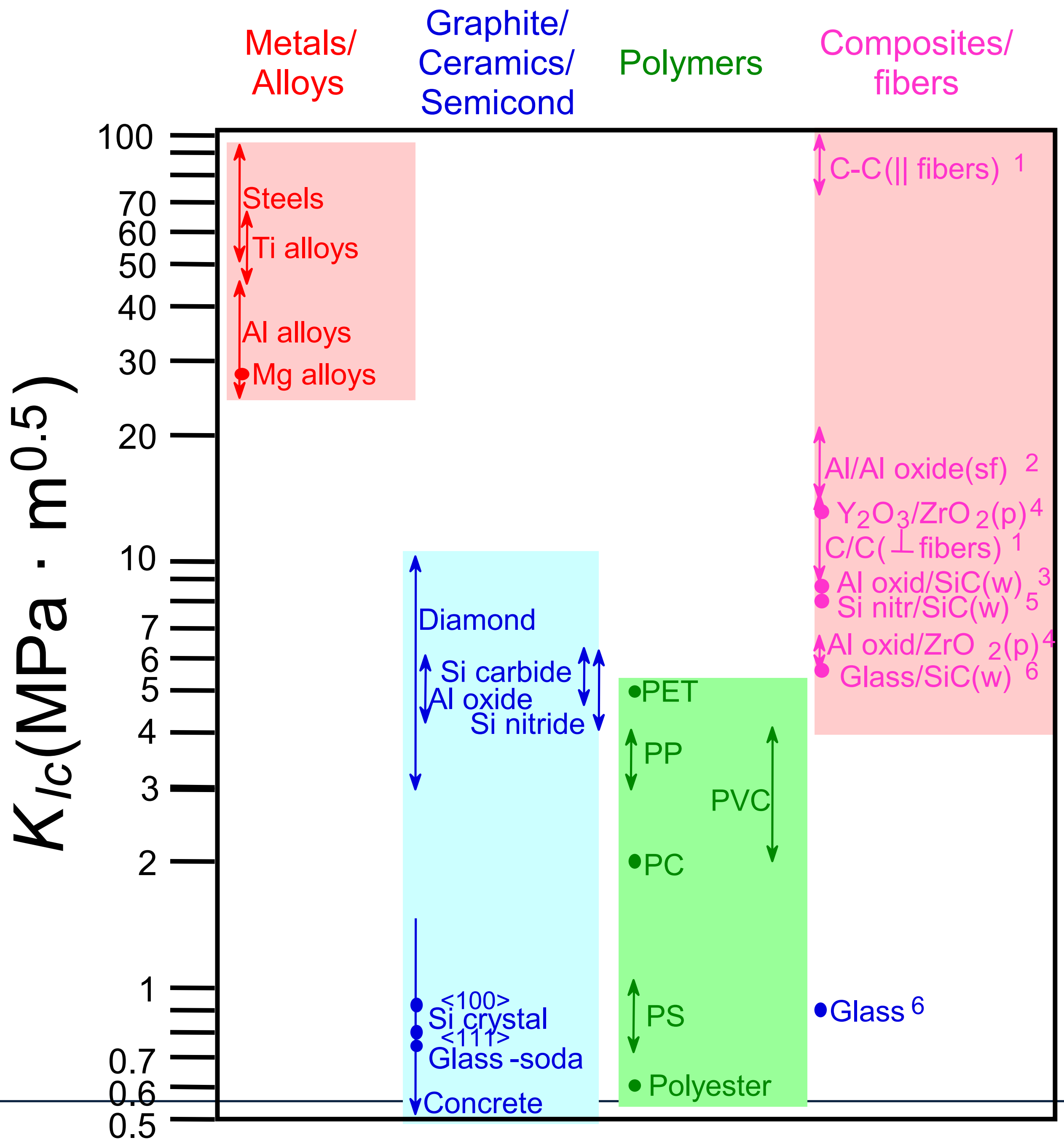


--Scenario 2: Design stress dictates max. flaw size.

$$a_{max} < \frac{1}{\pi}\left(\frac{K_c}{Y\sigma_{design}}\right)^2$$



Fracture Toughness Ranges



Based on data in Table B.5, *Callister & Rethwisch 8e.*

Composite reinforcement geometry is: f = fibers; sf = short fibers; w = whiskers; p = particles. Addition data as noted (vol. fraction of reinforcement):

1. (55vol%) *ASM Handbook*, Vol. 21, ASM Int., Materials Park, OH (2001) p. 606.
2. (55 vol%) Courtesy J. Cornie, MMC, Inc., Waltham, MA.
3. (30 vol%) P.F. Becher et al., *Fracture Mechanics of Ceramics*, Vol. 7, Plenum Press (1986). pp. 61-73.
4. Courtesy CoorsTek, Golden, CO.
5. (30 vol%) S.T. Buljan et al., "Development of Ceramic Matrix Composites for Application in Technology for Advanced Engines Program", ORNL/Sub/85-22011/2, ORNL, 1992.
6. (20vol%) F.D. Gace et al., *Ceram. Eng. Sci. Proc.*, Vol. 7 (1986) pp. 978-82.

LINEAR ELASTIC FRACTURE MECHANICS

Example 1:

A material used in an aerospace application has a fracture toughness of $\boldsymbol{K_{Ic}} = \mathbf{26\ MPa.m^{1/2}}$. The material is observed to have a maximum flaw size of **9 mm**, and fails at a fracture stress of **112 MPa**. At what stress will fracture occur if the same material is used with a maximum flaw size of 4 mm?

$$\boxed{K_{Ic} = Y\sigma\sqrt{\pi a}}$$

K_{Ic} is a material constant and therefore remains the same in this example. Unless the design changes, the geometrical factor Y will also remain constant.

$$\frac{K_{Ic}}{Y} = \sigma\sqrt{\pi a}$$

Constant

$$\therefore\ 112\ \text{MPa}\sqrt{\pi(9\text{x}10^{-3}\ \text{m})} = \sigma\sqrt{\pi(4\text{x}10^{-3})\text{m}}$$

$$\therefore\ \boldsymbol{\sigma = 168\ \text{MPa}}$$

LINEAR ELASTIC FRACTURE MECHANICS

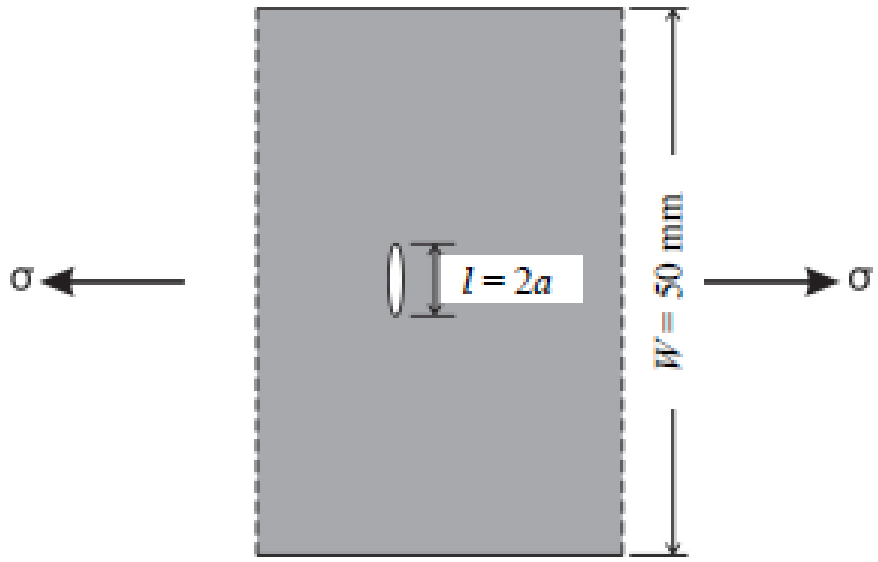
Example 2:

A tensile sample of aluminium alloy 7075-T6 with a width of 50 mm contains an internal crack with a length $l = 3.0$ mm. When loaded under tension, the crack propagates suddenly when the stress reaches $\boldsymbol{\sigma}$ **= 364 MPa**. What is the fracture toughness of the aluminium alloy? (Assume that $Y = 1$).

For an internal crack, $l = 2a$, therefore $a = 1.5$ mm.

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$
$$= (1)(364\text{ MPa})(\sqrt{\pi(1.5\text{x}10^{-3})\text{ m}}$$

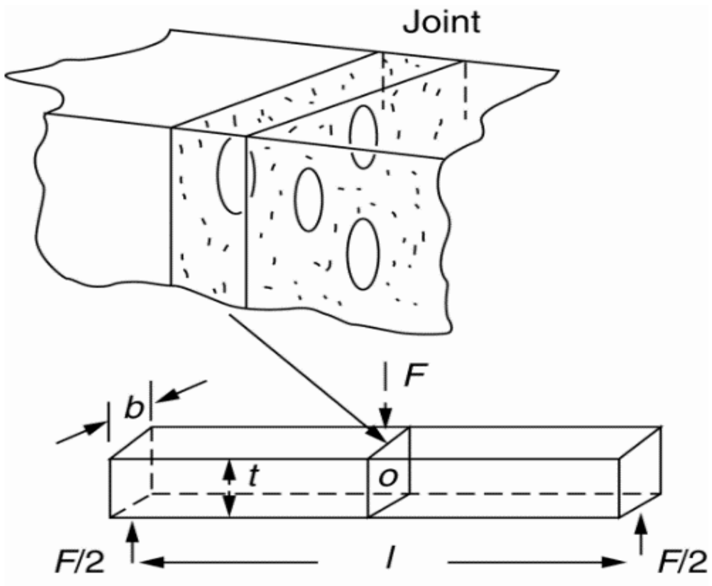
$\therefore$ $\boldsymbol{K_{Ic} = 25\ \mathrm{MPa.m^{1/2}}}$



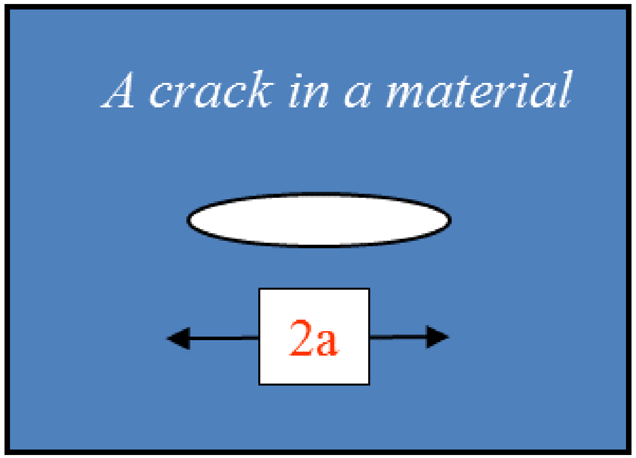
Fracture Mechanics
Exam Example

1. Two pieces of timber are glued together end-to-end with an epoxy adhesive and used as a beam in a 3-point bending arrangement as shown in the figure below. The beam width, thickness and span are $b = 250\ mm$, $t = 50\ mm$ and $l = 2.4\ m$ respectively. The glued joint failed by fast fracture when the applied load, F, reached 2.6 kN. Close inspection of the fracture revealed disc shaped bubbles up to 2.0 mm in diameter trapped in the epoxy.

If the maximum bending stress in the beam is given by $\sigma_{max} = \frac{3Fl}{2bd^2}$, what is the fracture toughness for the epoxy adhesive. (Assume $Y = 0.64$).



$$K_{Ic} = Y\sigma\sqrt{\pi a}$$



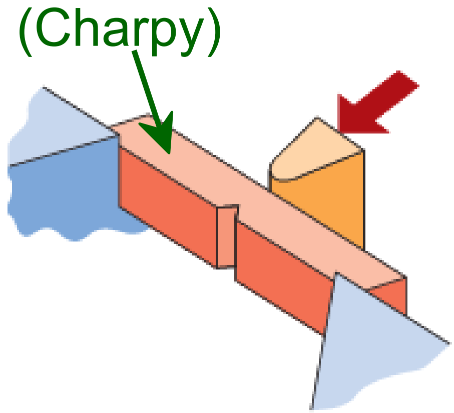
Stop and check video on moodle

Lecture 4: Stop and Check videos

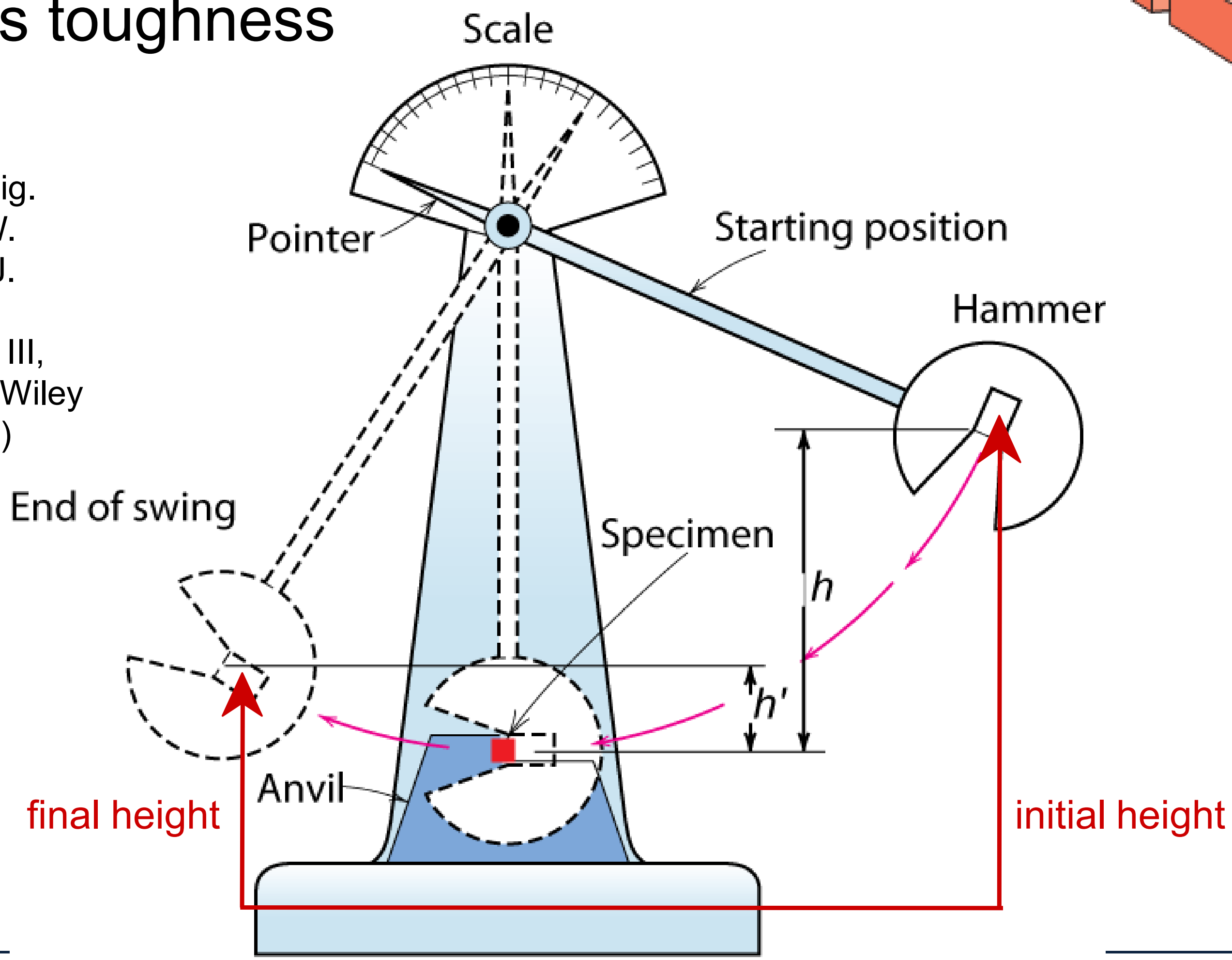
+ Distance Learning Resources folder - Fracture Mechanics video

Impact Testing – Student self study

- Impact loading:
  - -- severe testing case
  - -- makes material more brittle
  - -- decreases toughness

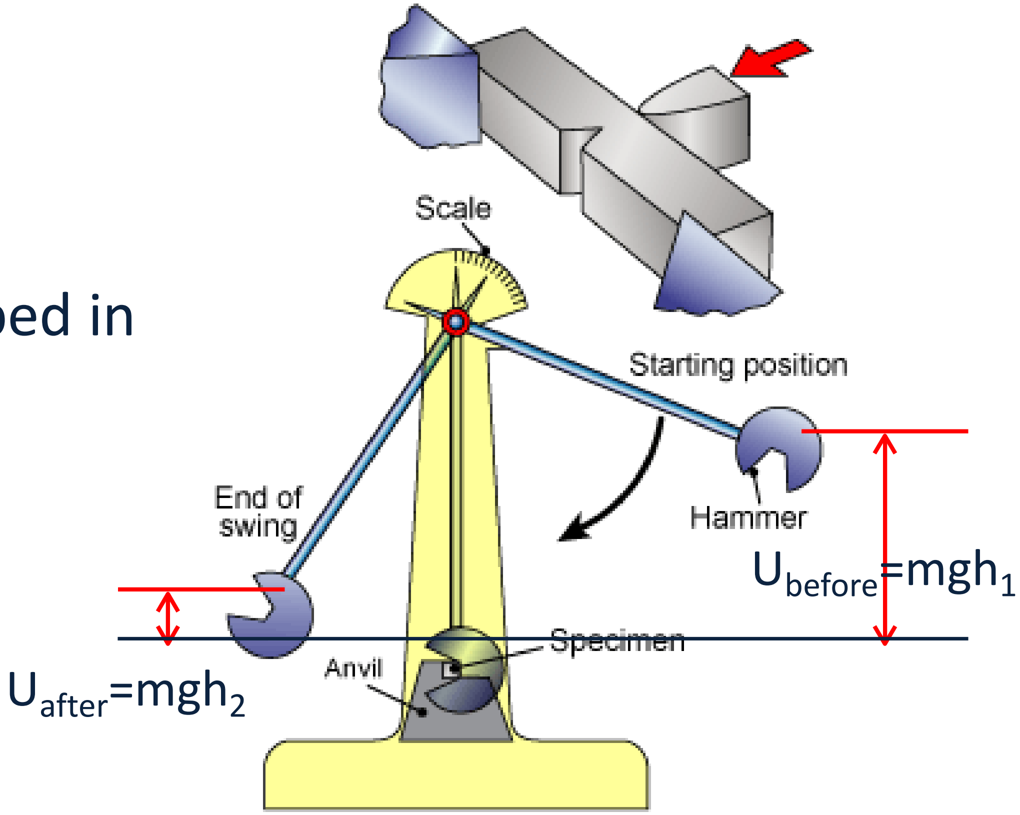


Adapted from Fig. 8.12(b), *Callister & Rethwisch 8e.* (Fig. 8.12(b) is adapted from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, John Wiley and Sons, Inc. (1965) p. 13.)



Charpy Impact Testing – Student self study

- The standard test for measuring impact energy is the Charpy test.
- This gives an indication of the characteristics of the material during fracture.
- An arm is swung down in a pendulum motion to impact with the test material.
- The energy required to fracture the sample is recorded.



Qualitative measure of toughness

- Can't be used to measure $\boldsymbol{K_{Ic}}$ or $\boldsymbol{G_c}$
- Relative measure of energy absorbed in impact fracture

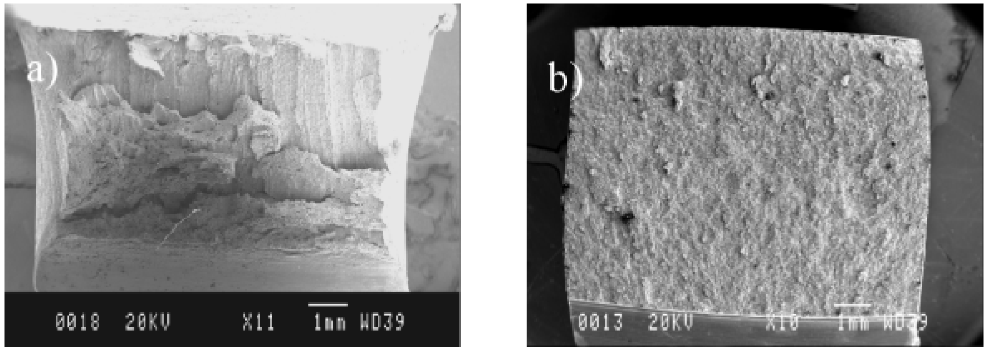
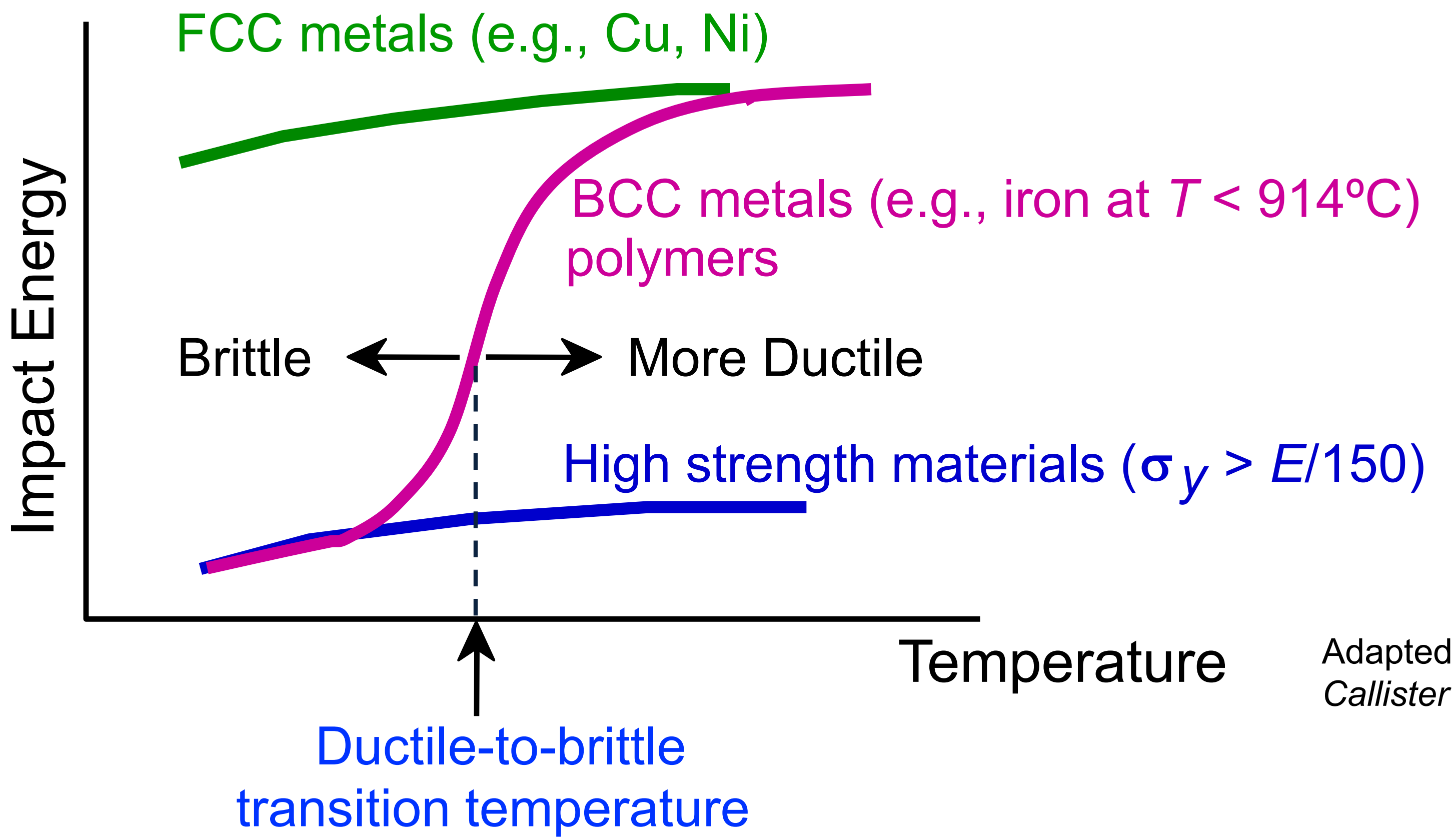


Image from: http://www.twi-global.com/technical-knowledge/faqs/material-faqs/faq-what-is-charpy-testing/

Influence of Temperature on Impact Energy

– Student self study

- **Ductile-to-Brittle Transition Temperature (DBTT)...**



Adapted from Fig. 8.15, *Callister & Rethwisch 8e.*

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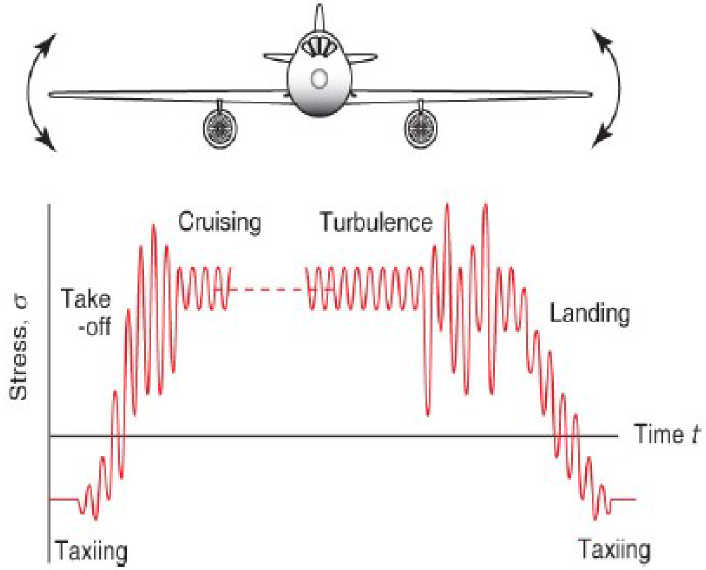
Stop and check video on moodle

Distance Learning Resources folder - Charpy Impact Test video

What is Fatigue?

- Many engineering components are subject to dynamic and fluctuating stresses:
  - Aircraft structures
  - Bridges
  - Pressure vessels
  - Axles, shafts …



- Is it enough to keep stress below σ_y?
  - No.

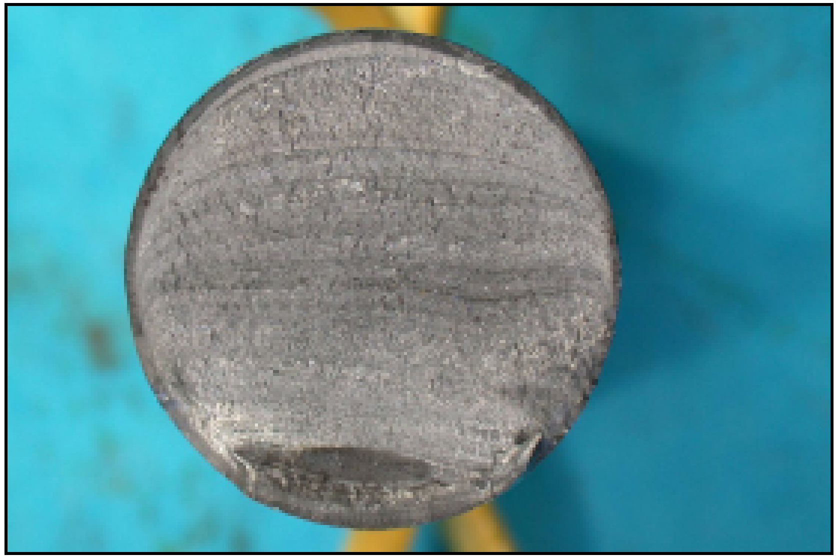
- Fatigue is failure due to these non-static loadings.

- Fatigue occurs at stress levels below σ_{TS} and below σ_y

Image from: Ashby, M., H. Shercliff, and D. Cebon, *Materials: Engineering, Science, Processing and Design*. 2nd ed. 2010, Oxford: Butterworth-Heinemann.

CHARACTERISTICS OF FATIGUE

- **Fatigue** refers to the failure of metals caused by fluctuating or dynamic loading.
- Fatigue results in brittle type fracture, even in ductile materials.
- Cracks originate from stress concentrations:
  - at the component surface, or
  - at large internal defects.
- Cracks grow with each loading cycle to leave concentric '**beach marks**' on the fracture surface.
- Crack growth continues until a critical size is reached, followed by fast, catastrophic fracture (this happens when the material's fracture toughness is exceeded).



FATIGUE LOADING

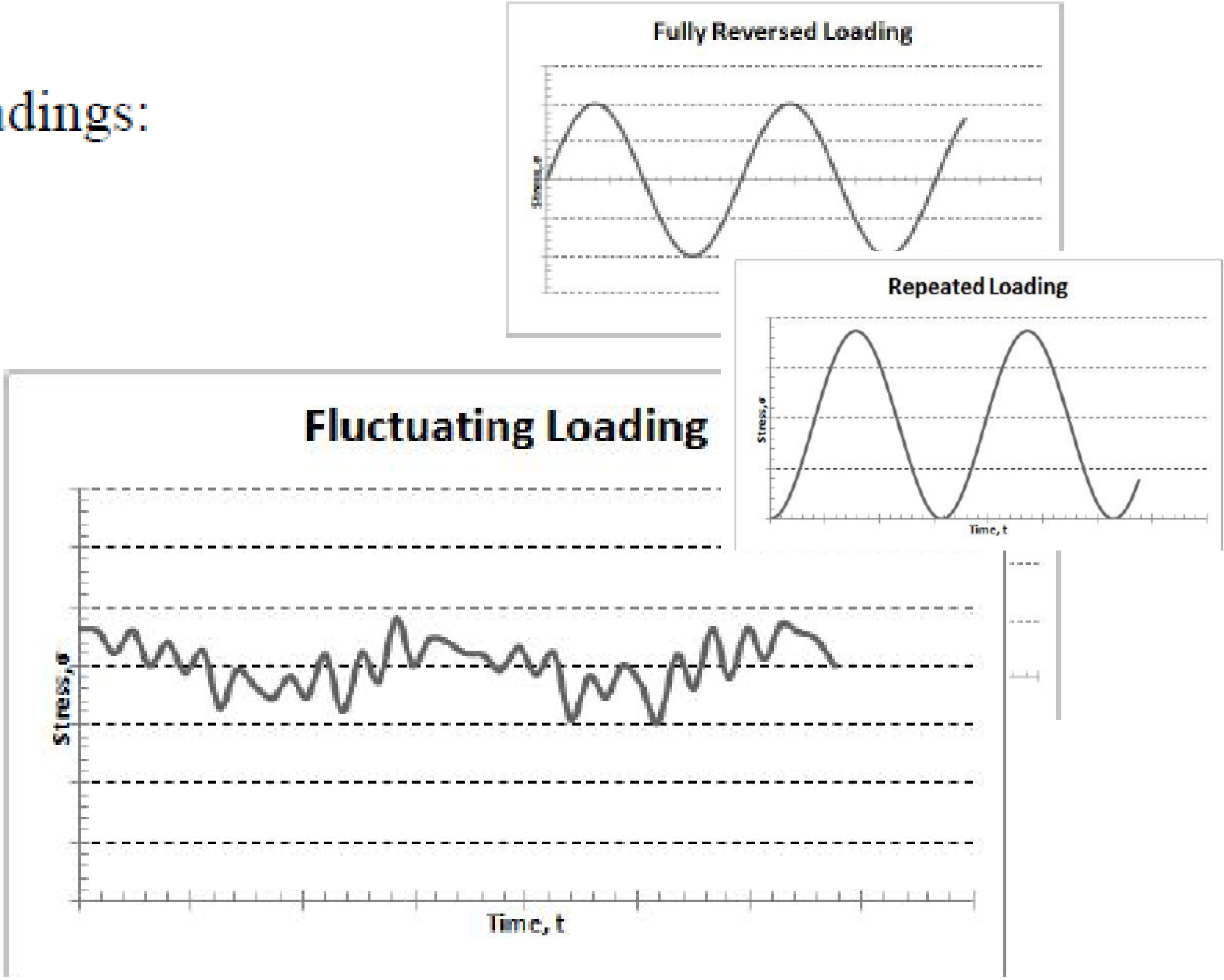
Main classifications for load cycles:

- Failure occurs due to varying loads, typically at stresses <u>lower</u> than the yield stress.
- Three main classifications for loadings:

  a) Fully reversed loading:
     - Rotating axle of rail car

  b) Repeated:
     - Batch pressure vessel

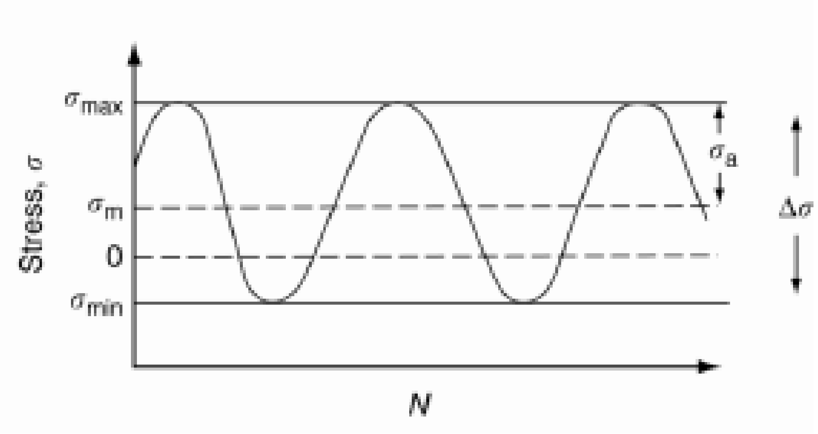
  c) Full spectrum loading:
     - Ship or oil platform

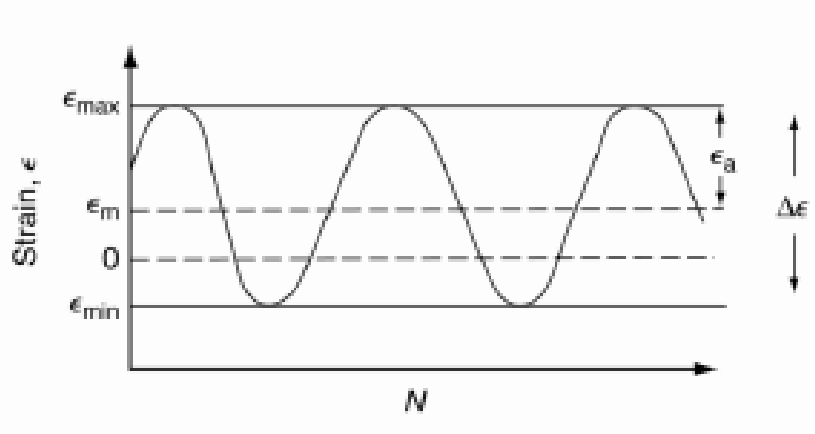


FATIGUE LOADING

Fatigue testing:

- Fatigue refers to crack initiation and propagation under fluctuating or cyclic loads.
- To characterise the fatigue behaviour of a material:
  - Subject the sample to cyclic or fluctuating loads, or
  - Subject the sample to cyclic or fluctuating displacements.
  - Observe the number of cycles to failure *N*.



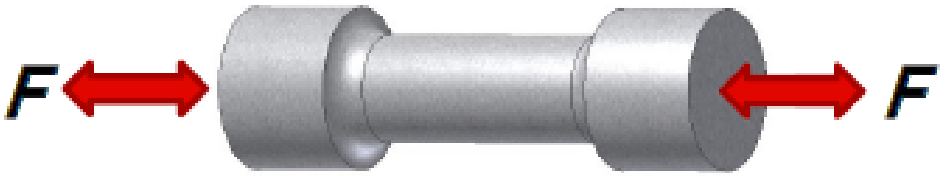


  - Repeat for various magnitudes of load/displacement.
  - Generate a relationship between stress/strain and cycles to failure.

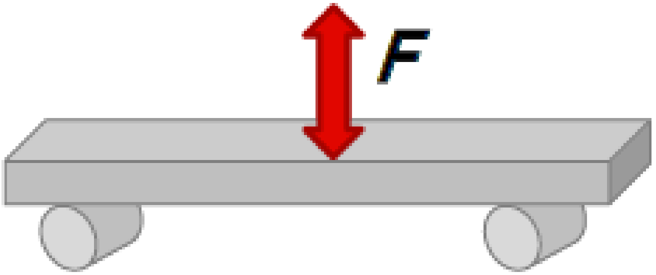
FATIGUE LOADING

Fatigue testing:

- Loading may be:
  - Axially applied loads (tension-compression or tension-tension)



  - Bending (3-point bend test)



  - Cantilever + Rotation

FATIGUE LOADING

Some definitions:

- Stress range:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

- Stress amplitude:

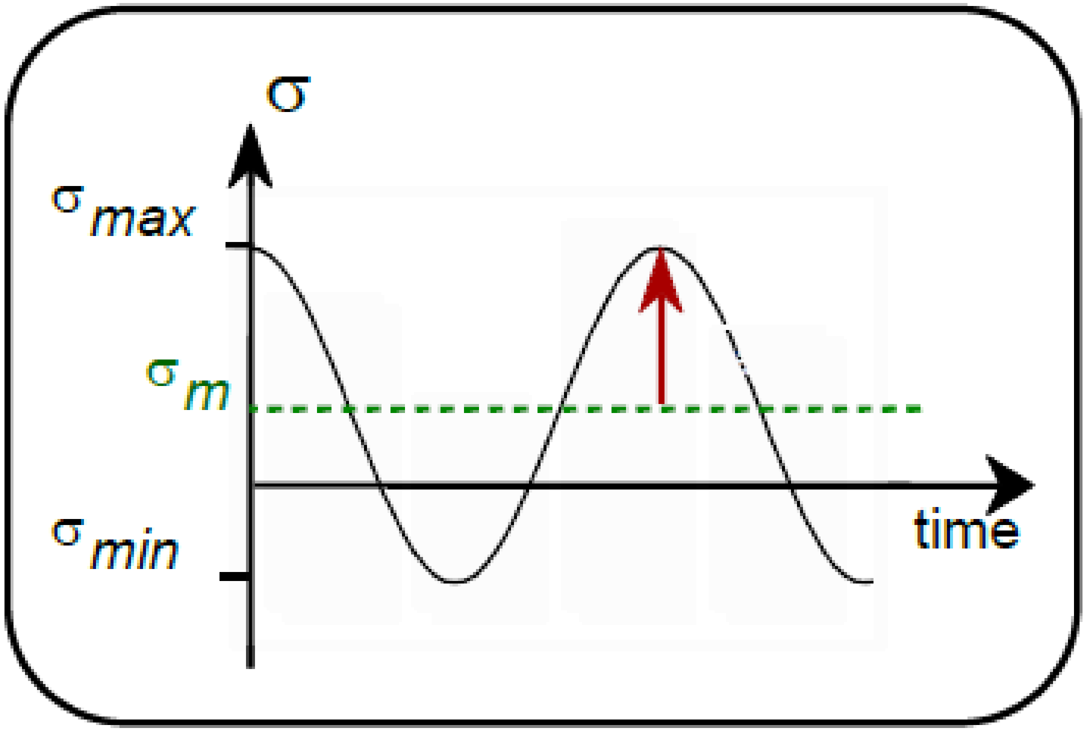
$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

- Mean stress:

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

- Stress ratio:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

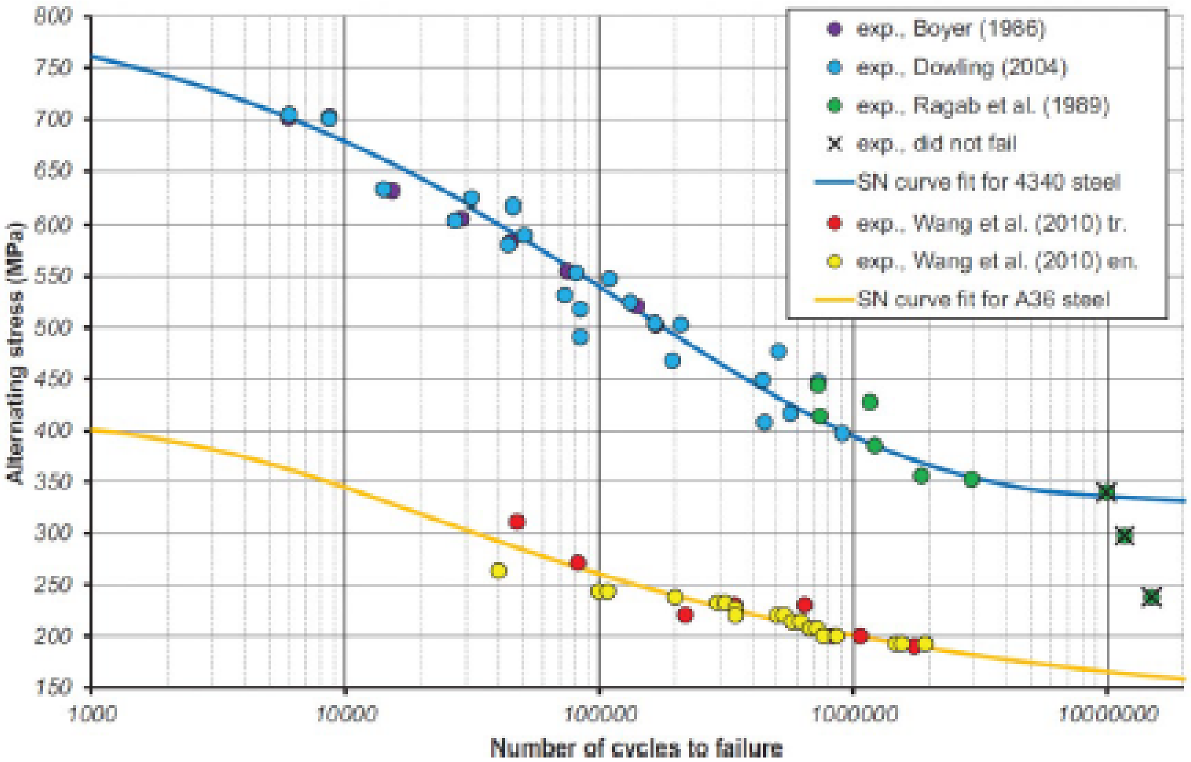


$\boxed{\text{If } R = -1 \text{ then } \sigma_m = 0}$

FATIGUE S-N CURVES

- The sample is subjected to a stress cycle with a given stress amplitude σ_a or stress range $\Delta\sigma$ and the number of cycles to failure N is recorded.

- Plot stress (σ_a or $\Delta\sigma$) against cycles to failure N (log scale).
  - **S-N curve.**



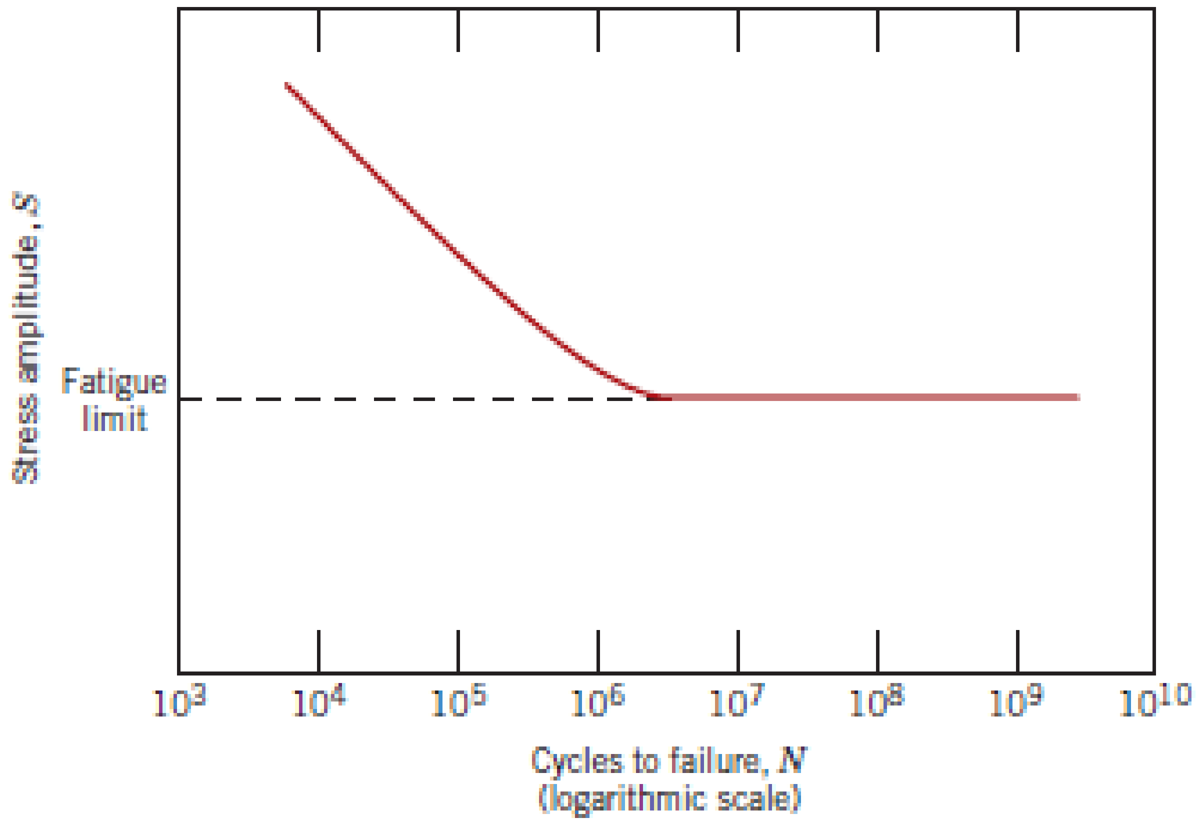
S-N curves of ASTM A36 and AISI 4340 steel

Fatigue Stress Amplitude – Cycles to failure (S-N) curves

<u>Two</u> types of fatigue behaviour observed:

A. Fatigue failure does not occur below a certain stress level.

- –Endurance limit σ_e (also called fatigue limit)
- –Ferrous alloys (carbon steels, stainless steels, alloy steels)
- –Titanium alloys
- –Some polymers

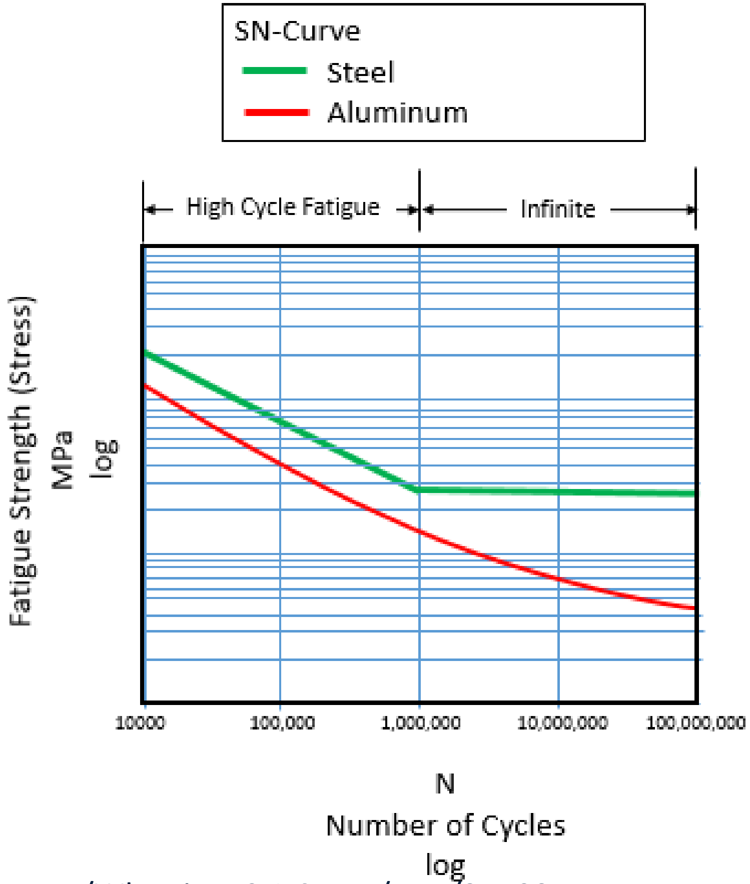


Fatigue S-N curves

Two types of fatigue behaviour observed:

B. Fatigue failure occurs at all levels of applied stress.

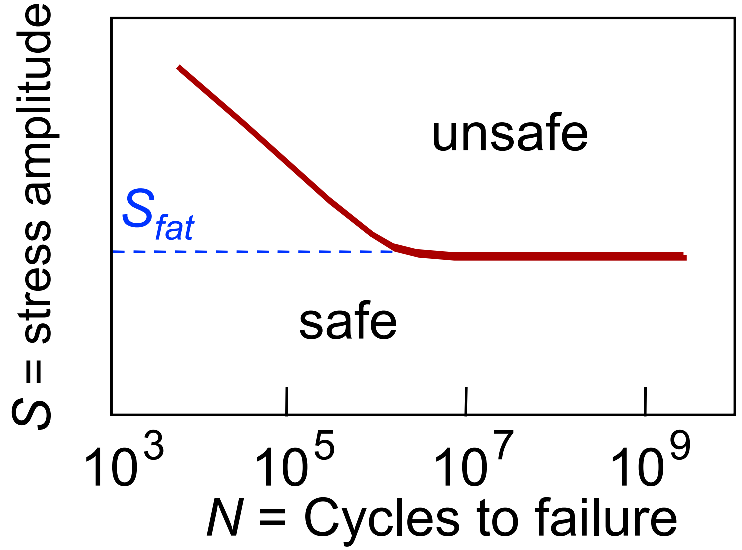
- No endurance limit
- Define a *fatigue strength* σ_f at a certain number of cycles
- Aluminium alloys
- Copper alloys
- Magnesium alloys
- Nickel alloys



https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/What-is-a-SN-Curve/ta-p/355935

Types of Fatigue Behavior

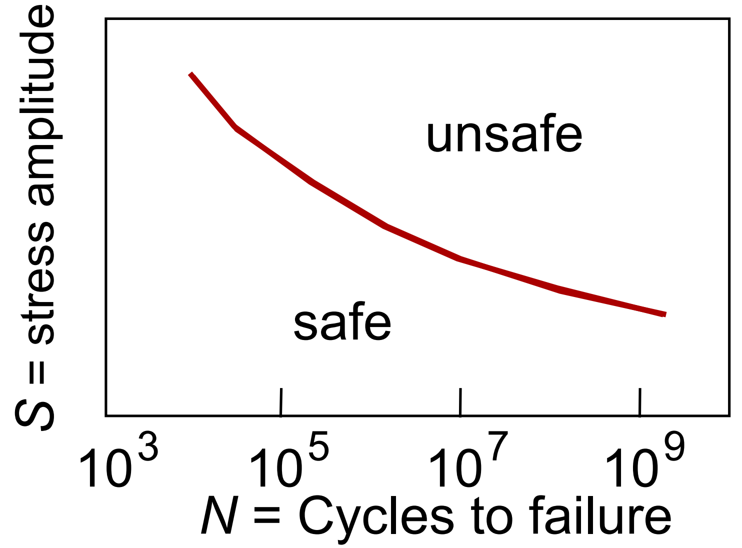
- Fatigue limit, S_{fat}:
  --no fatigue if $S < S_{fat}$



case for **steel** (typ.)

Adapted from Fig. 8.19(a), *Callister & Rethwisch 8e.*

- For some materials, there is no fatigue limit!

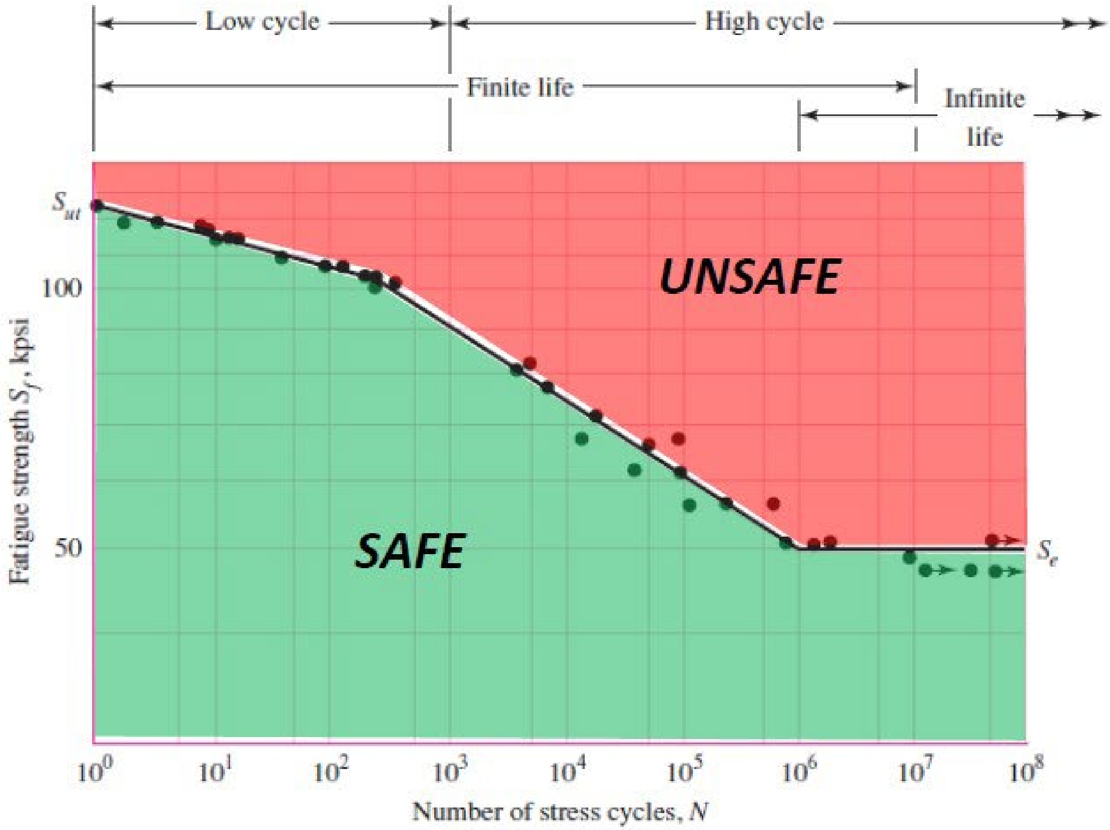


case for **Al** (typ.)

Adapted from Fig. 8.19(b), *Callister & Rethwisch 8e.*

Using S-N Curves

- Loading conditions ***below*** the curve = ***safe.***
- Loading conditions ***above*** the curve = ***unsafe.***



Using S-N Curves

- Data in ***S-N*** curves is for a specific loading type under certain atmospheric conditions.
  - Should be given on the curve.
- Usually data is for:
  - Fully reversed loading.
  - Mean stress ($\sigma_m = 0$).
  - Stress ratio (R = -1).

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

$$R = -1$$

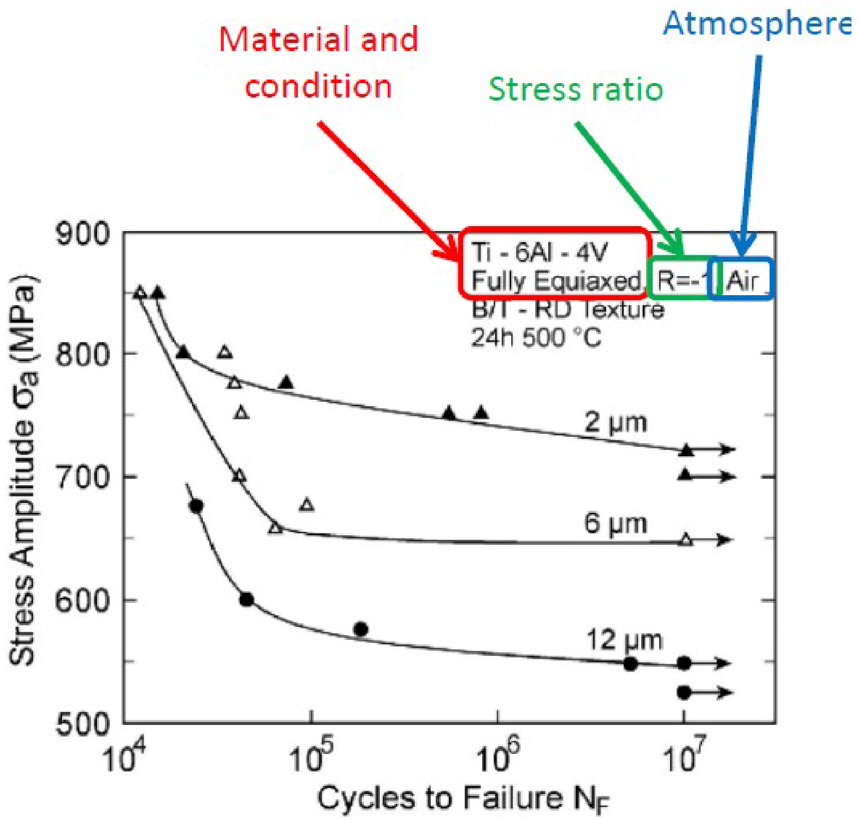


Image from: Lutjering, G. and J.C. Williams, 5. Alpha+Beta Alloys, in Titanium. 2007, Springer: Berlin. p. 203-258.

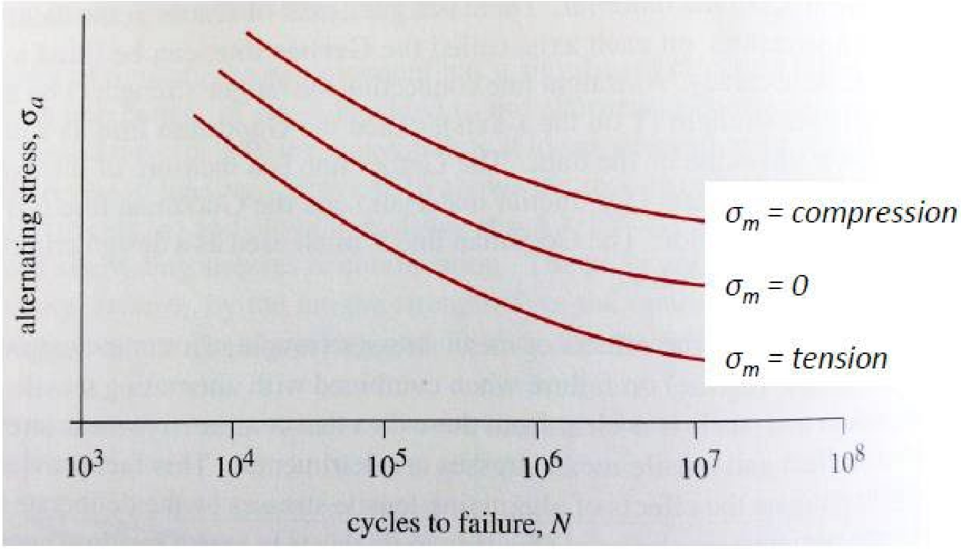
Using S-N Curves

- Often fatigue tests are conducted with rotating samples
  - Fully reversed loading
  - Mean stress $\sigma_m = 0$
  - Stress ratio $R = -1$

- How does data compare to other loading conditions
  - Other loading cycles:

    Mean stress $\sigma_m \neq 0$

    Stress ratio $R \neq -1$



- Many engineering applications involve HCF where $\boldsymbol{\sigma_m \neq 0}$
  - Is the data useless?

Image from: Norton, R.L., *Machine design : an integrated approach* . *3rd* ed., [International ed.] ed. 2006, Boston, Mass. ; Pearson.

Using S-N Curves

- Where $\boldsymbol{\sigma_m \neq 0}$ we use Goodman's rule:

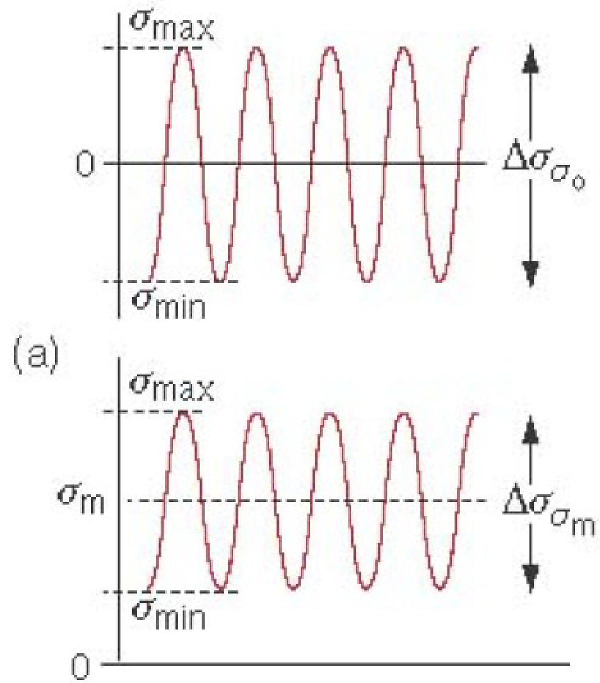
Stress range at a mean stress of $\boldsymbol{\sigma_m}$

Mean stress $\boldsymbol{\sigma_m}$

$$\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)$$

Stress range at a mean stress of ***zero***

Tensile strength of material



- While the above equation is in terms of stress range it clearly holds for stress amplitude (as $\Delta\sigma = 2\sigma_a$ for all cases).

Image from: Ashby, M., H. Shercliff, and D. Cebon, *Materials: Engineering, Science, Processing and Design*. 2nd ed. 2010, Oxford: Butterworth-Heinemann.

S-N Curve: Exam Example

- Stress Range:

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

- Stress Amplitude:

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max}-\sigma_{min}}{2}$$

- Mean Stress:

$$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2}$$

$$\boxed{\Delta\sigma_{\sigma_m} = \Delta\sigma_{\sigma_0}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)}$$

- High strength low alloy steel.

$$\sigma_{ts} = 855\ MPa$$

- Subjected to cyclic tensile stress.

$$\sigma_{min} = 120\ MPa$$
$$\sigma_{max} = 780\ MPa$$

$$\sigma_m = 450\ MPa \qquad \sigma_a = 330\ MPa$$

- Stress amplitude (***= Δσ/2***) with zero mean stress that gives equivalent fatigue behaviour:

$$\left(\frac{\Delta\sigma_{\sigma_0}}{2}\right) = \frac{\left(\frac{\Delta\sigma_{\sigma_m}}{2}\right)}{\left(1-\frac{\sigma_m}{\sigma_{ts}}\right)} = \frac{330MPa}{\left(1-\frac{450MPa}{855MPa}\right)} = 697\ \text{MPa}$$

Fatigue of cracked components

- Engineering structures and components are rarely defect-free.
  - Usually contain internal flaws and cracks.
  - These defects may be so small they go undetected.
- Cyclic loading will cause the cracks to grow.
  - At some point the crack size may reach the critical crack length for fast fracture.
  - Need to know the number of cycles (or time) until these cracks reach the critical length.
- Consider the stress intensity factor:

Chapter 13

$$K = Y\sigma\sqrt{\pi a}$$

- For fatigue loadings we have the fatigue stress intensity factor:

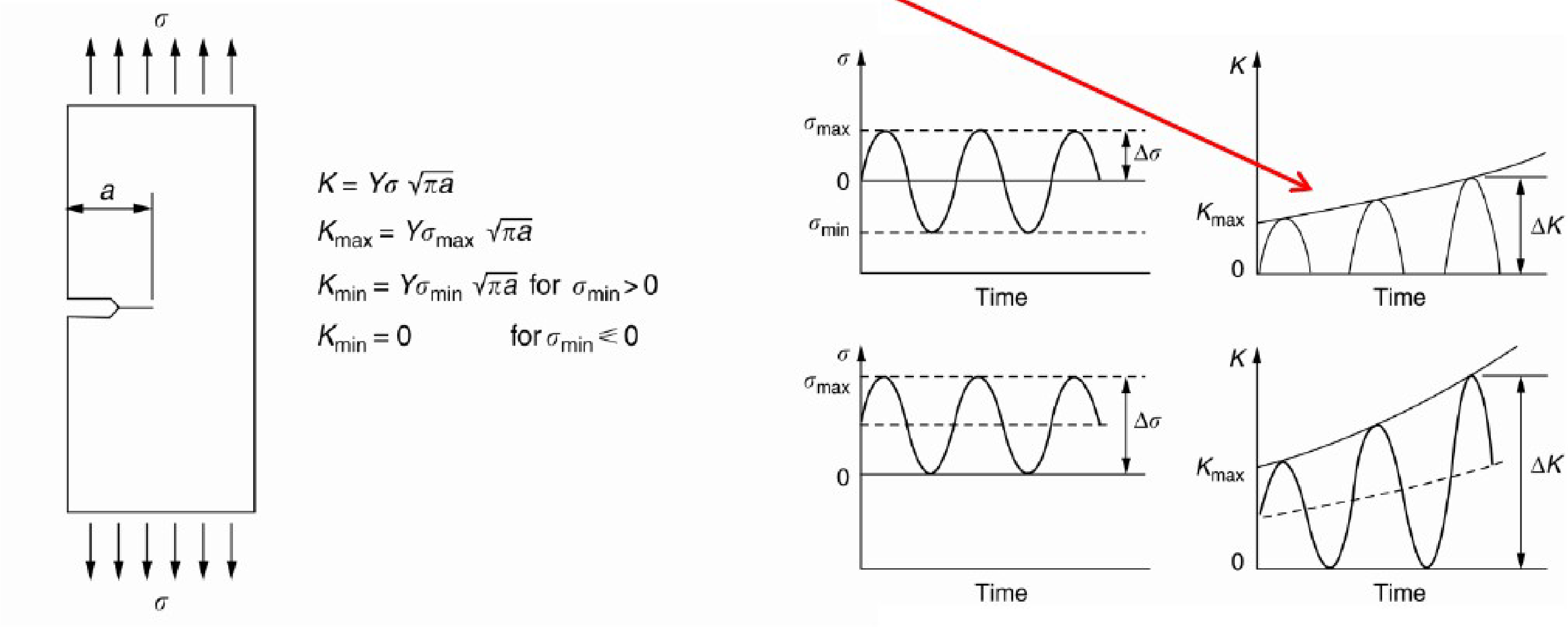
$$\Delta K = K_{max} - K_{min} = Y\Delta\sigma\sqrt{\pi a}$$

Fatigue of cracked components

- Fatigue stress intensity factor:

$$\Delta K = K_{max} - K_{min} = Y\Delta\sigma\sqrt{\pi a}$$

  - Crack grows under $\boxed{\text{tension}}$ → ***a*** increases
  - With repeated loading ***ΔK*** increases (same loading each cycle)

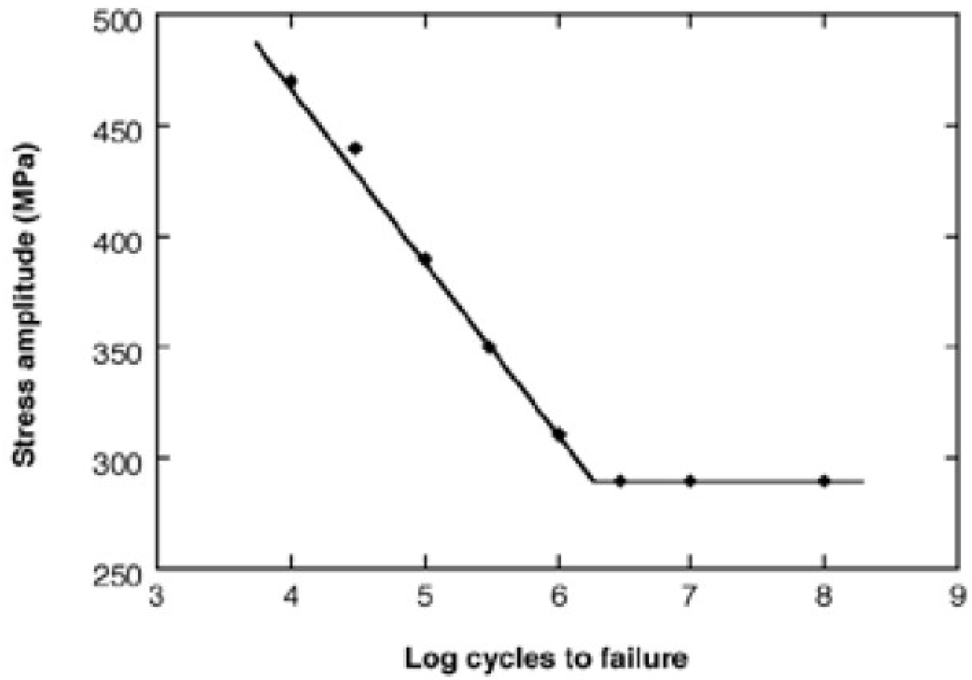


S-N curve: Example 4.3

- A steel shaft operates at continuously with rotational speed of 600 rpm.

Using the *S-N* curve provided determine the maximum continuous life for a stress amplitude of:

a) 450 MPa
b) 380 MPa
c) 310 MPa



Example: Solution

a) For $\sigma_a = \mathbf{450\ MPa} \rightarrow N_f \approx 10^{4.2}\ cycles$

$$N_f \approx 16000\ cycles$$

$$t_f = \frac{16000\ cycles}{600\ cycles/min} = 26\ min 40 sec$$

b) For $\sigma_a = \mathbf{380 MPa} \rightarrow N_f \approx 10^{5.1}$

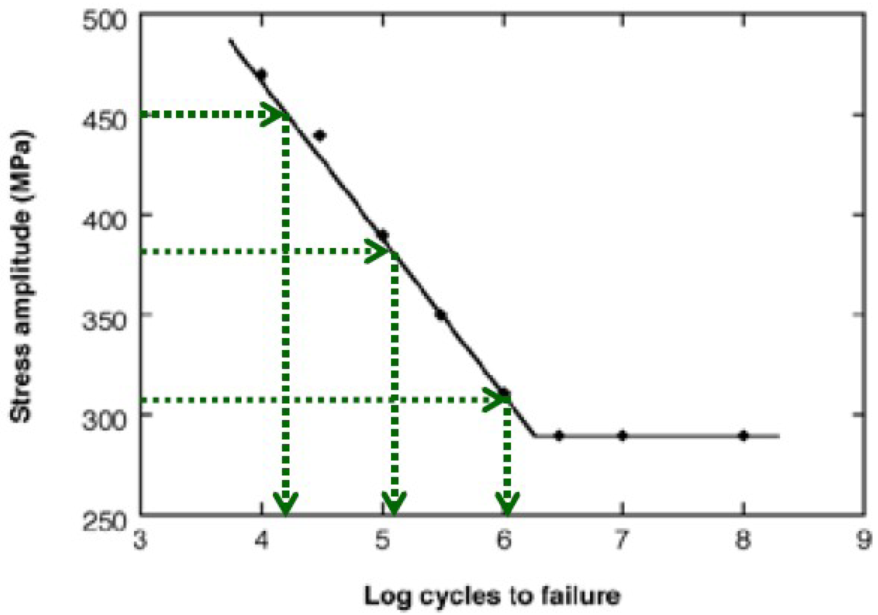
$$N_f \approx 1.26 \times 10^5\ cycles$$

$$t_f = \frac{1.26 \times 10^5\ cycles}{600\ cycles/min} = 3hrs29min$$

c) For $\sigma_a = \mathbf{310 MPa} \rightarrow N_f \approx 10^6$

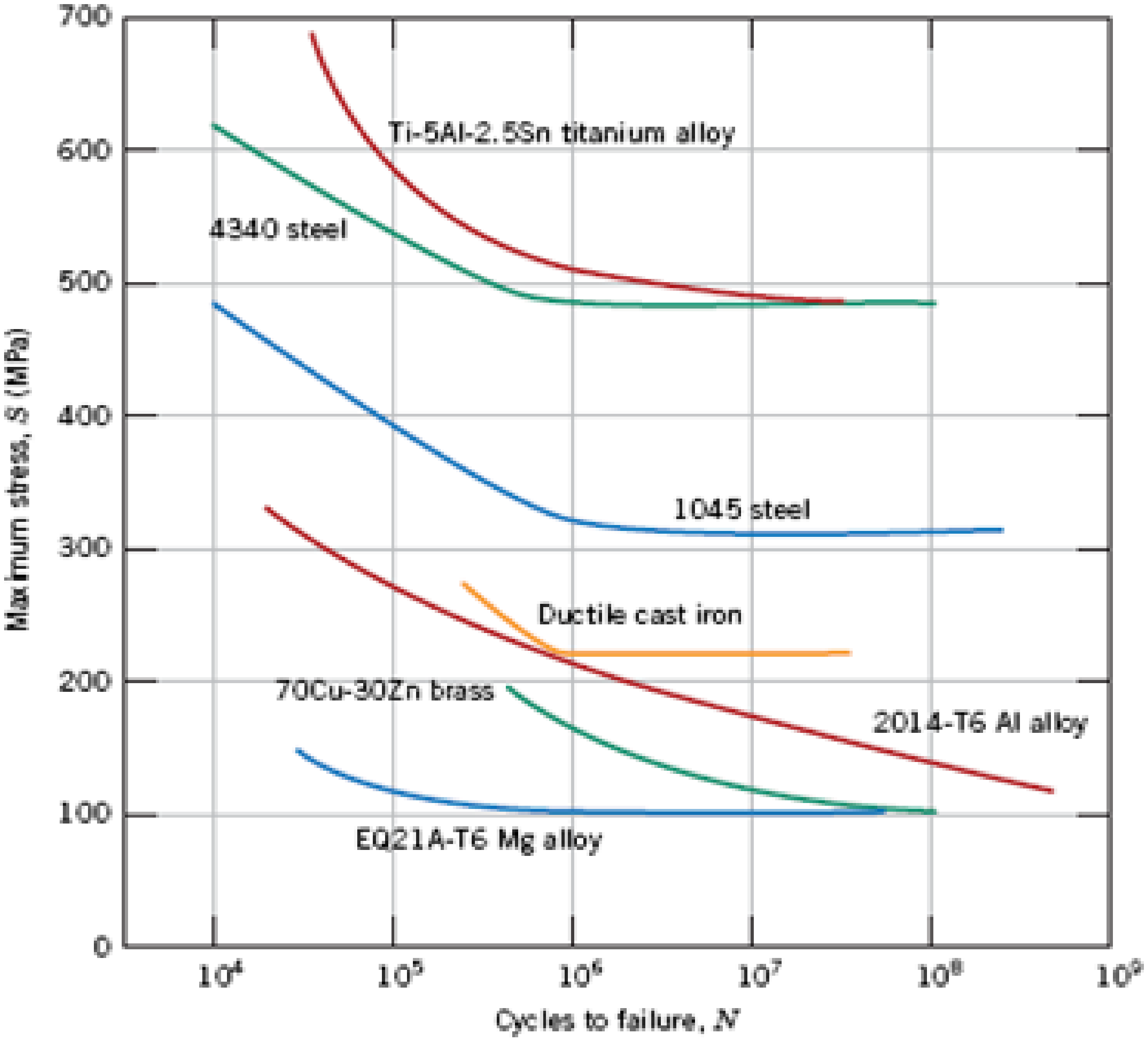
$$N_f \approx 1.0 \times 10^6\ cycles$$

$$t_f = \frac{1.0 \times 10^6\ cycles}{600\ cycles/min} = 27hrs46min$$



Fracture Mechanics

Exam Example – Tutorial 4



A cylindrical tie rod is made from AISI 4340 quenched and tempered steel and will be subjected to a fully reversed cyclic loading of $F = \pm 200\ kN$.

What is the minimum required diameter of the tie rod if it must survive 10^5 loading cycles?

Stop and check video on moodle

Lecture 4: Stop and Check

+ Distance Learning Resources folder- Understanding Fatigue Failure and S-N Curves

SUMMARY

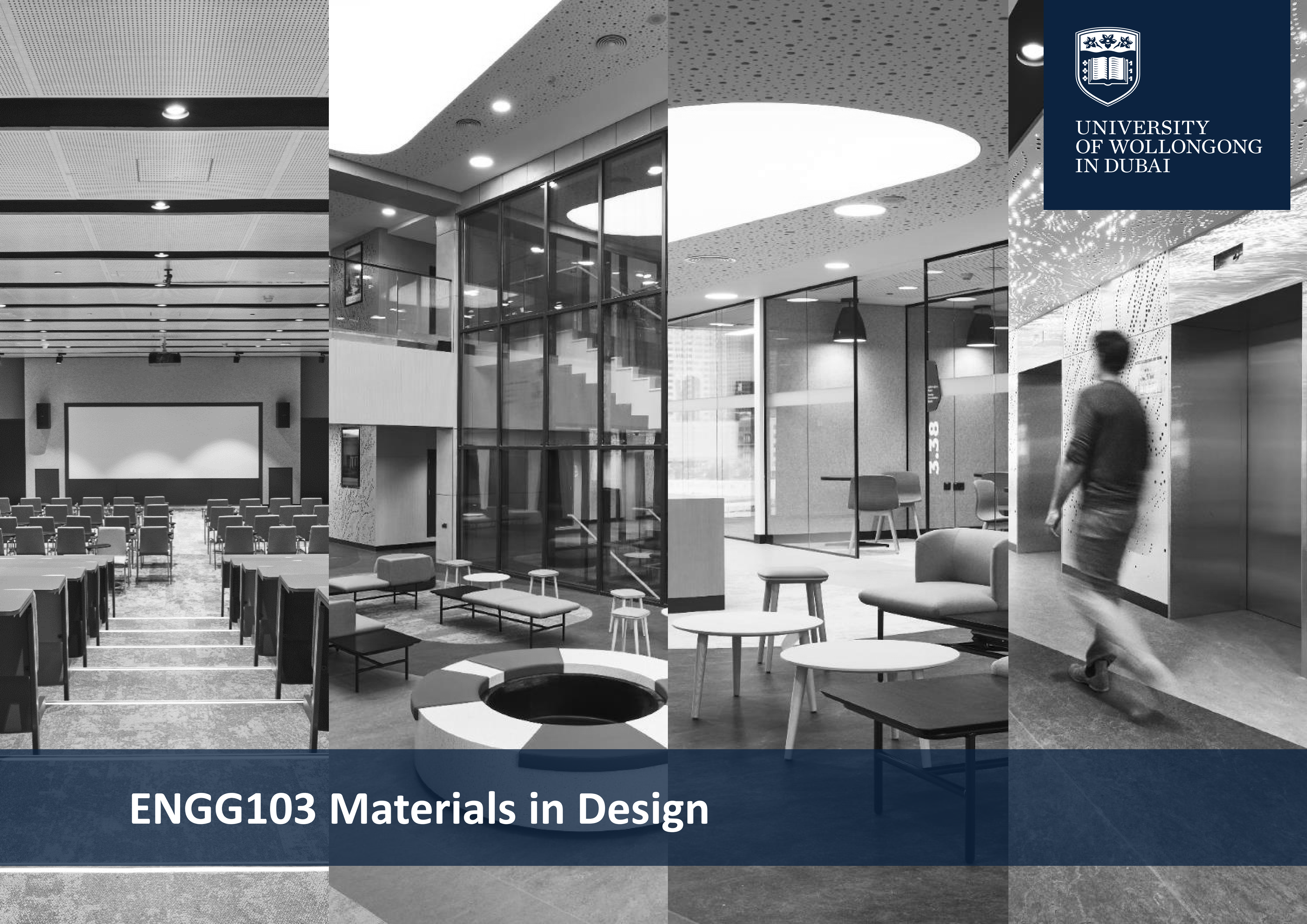
- Engineering materials not as strong as predicted by theory
- Flaws act as stress concentrators that cause failure at stresses lower than theoretical values.
- Sharp corners produce large stress concentrations and premature failure.
- Failure type depends on T and σ:

  -For simple fracture (noncyclic σ and $T < 0.4T_m$), failure stress decreases with:

  - increased maximum flaw size,
  - decreased T,
  - increased rate of loading.

  - For fatigue (cyclic σ):

    - cycles to fail decreases as $\Delta\sigma$ increases.



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