Attribution Nidhal Abdulaziz

8EB

ac Power

ac Power

(a) We have used: $10 \sin(\omega t + 40^{\circ})$ volts $\rightarrow 10\underline{/40^{\circ}}$ volts

To be absolutely correct: $10 \sin(\omega t + 40^{\circ})$ volts $\rightarrow 10\underline{/40^{\circ}}$ volts Peak.

Up to now this has not mattered, we have simply been using calculations based on the (phasor) peak value.

Power is different: here we must use volts and amps RMS.

RMS: Root Mean Square, we have already discussed when dealing with power in the resistor and noted the $\sqrt{2}$ relationship.

(b) We are now going to expand in this work by asking the question:

What effect has the phase angle between current and voltage have on our concept of power?

ac Power

Some ***Trivia*** on ac power generation in UK

ac Power

Some ***Trivia*** on ac power generation in UK

Q1. What is the UK’s average yearly electricity power consumption?

Q2. What is the UK’s typical generating capacity?

Q3. How do we generate our electricity (by fuel types)?

ac Power

Some ***Trivia*** on ac power generation in UK

Q1. What is the UK's average yearly electricity power consumption?

Answer: ~ 380 T Watt Hours

(380, 000, 000, 000, 000 Watt Hours)

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Some ***Trivia*** on ac power generation in UK

Q2. What is the UK’s typical generating capacity?

Answer: ~ 76,970 MW …. From some 250 power plants of all types
(plus inter-connectors)

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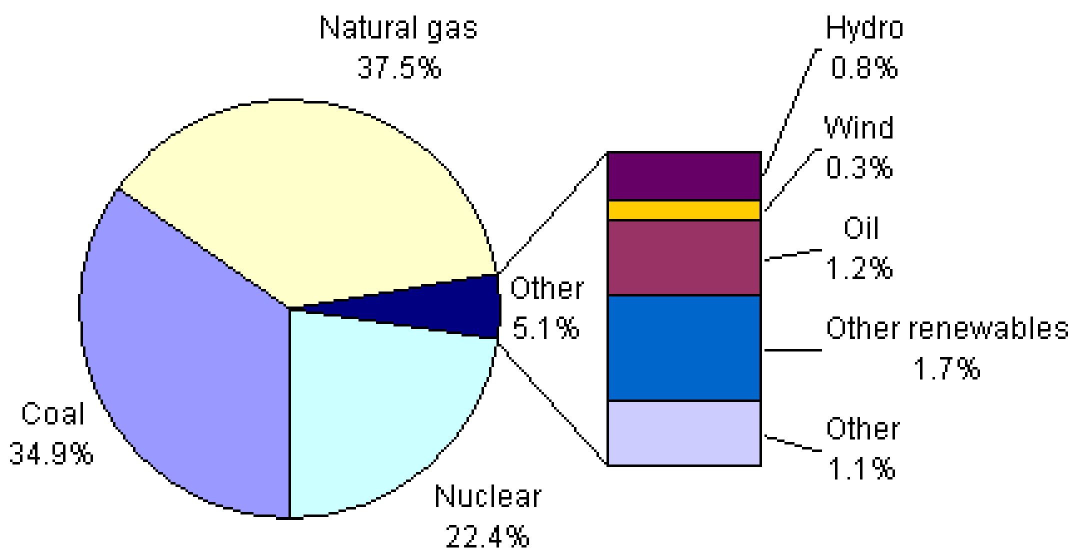
Some ***Trivia*** on ac power generation in UK

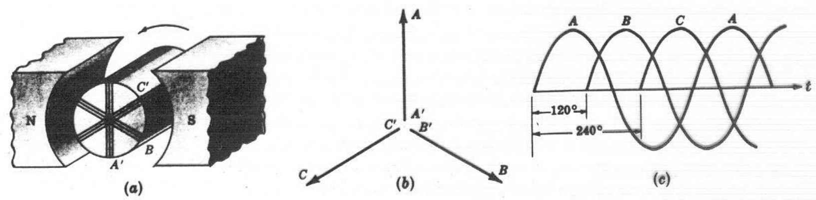
Q3. How do we generate our electricity (by fuel types)?



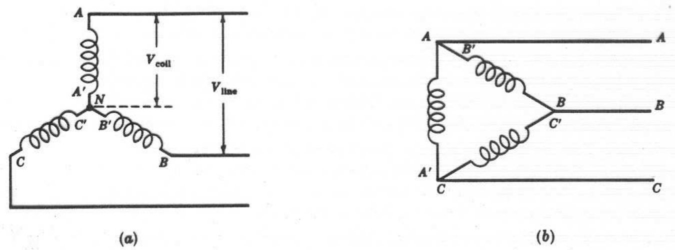
Data is some 3 years old / from the web!

ac Power

Some ***Trivia*** on ac power generation in UK



Three-phase System

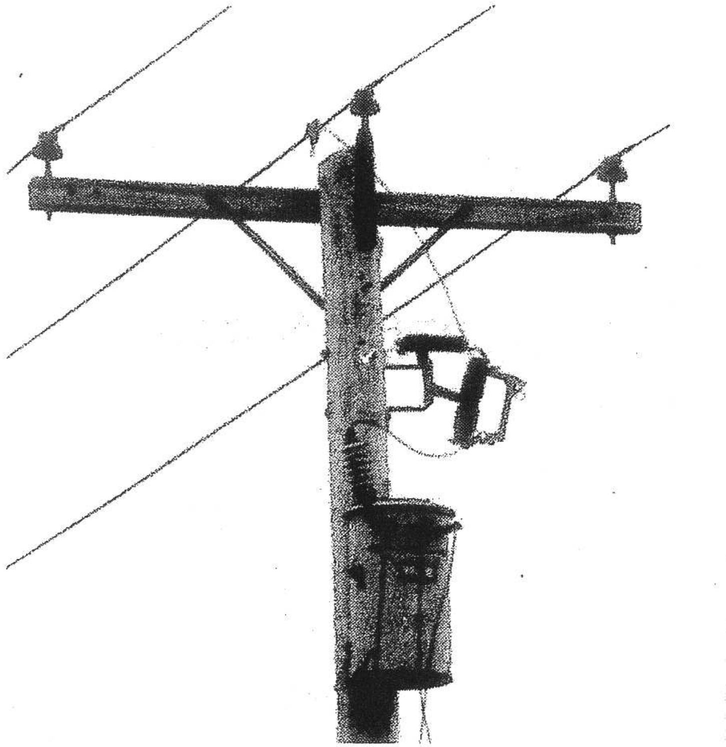


3-Phase Generation

ac Power

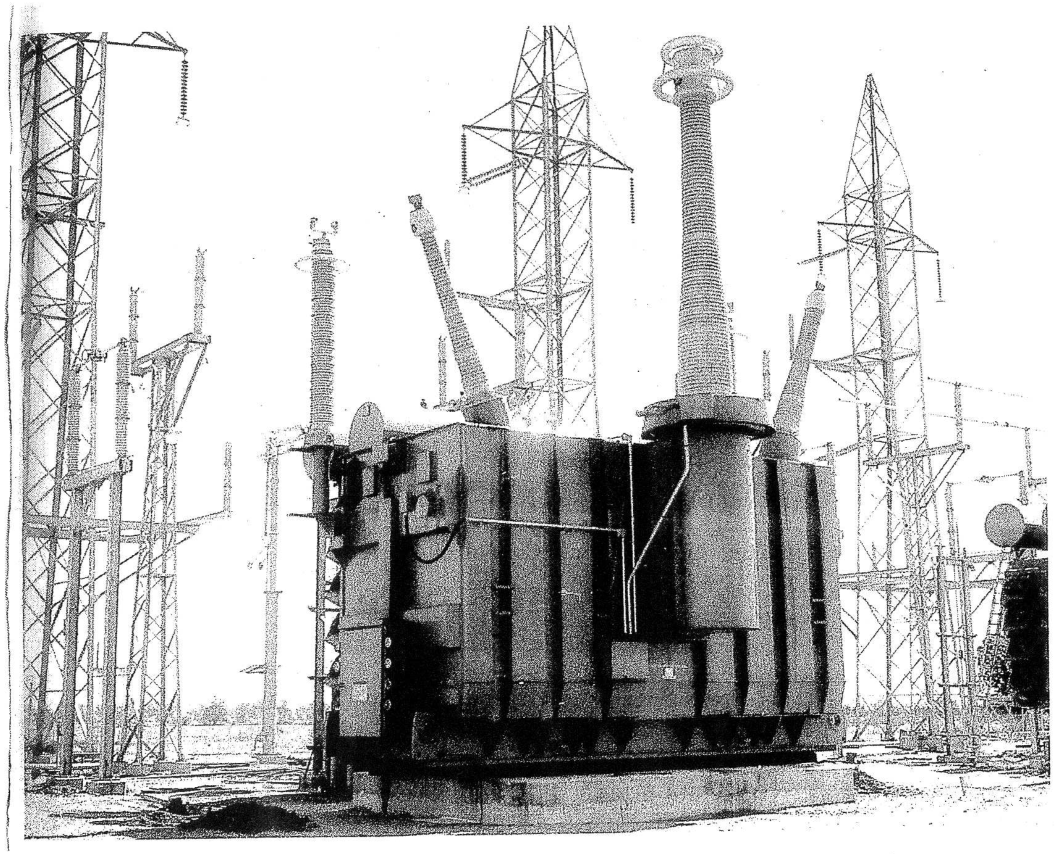
When we are dealing with quantities like **100 kVA** and **1000 MVA** the units we use often become almost insignificant. It is only when one places these numbers in the real world that there significance becomes apparent. As a guide to reality, the following pictures "set the scene" to the *dimensions* indicative of the power industry.

ac Power



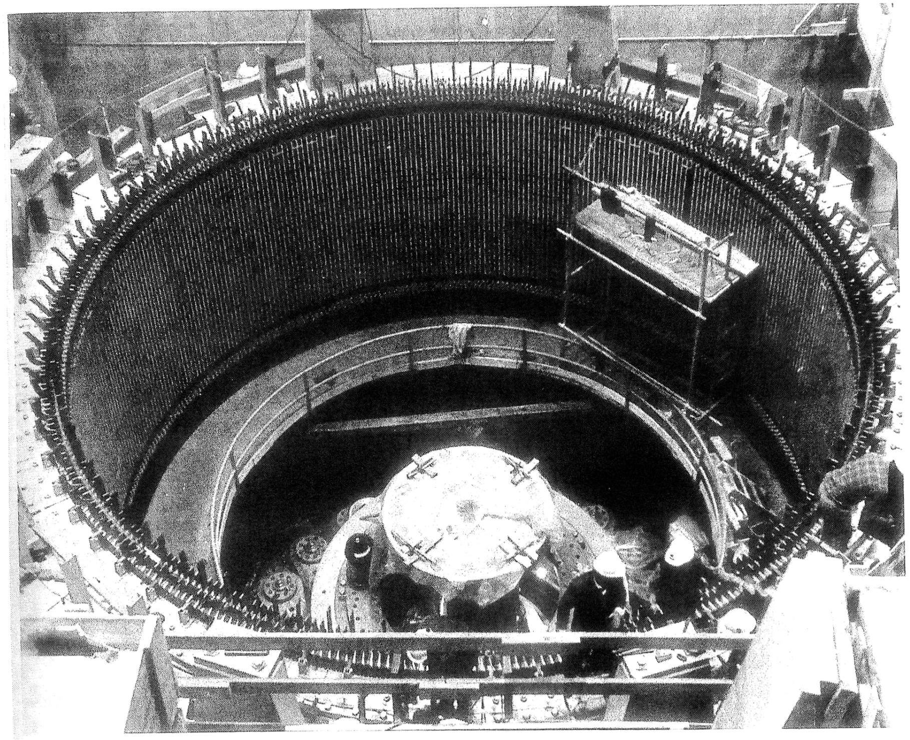
Single phase, pole mounted distribution transformer,
100 KVA, 13.3 kV/240 V

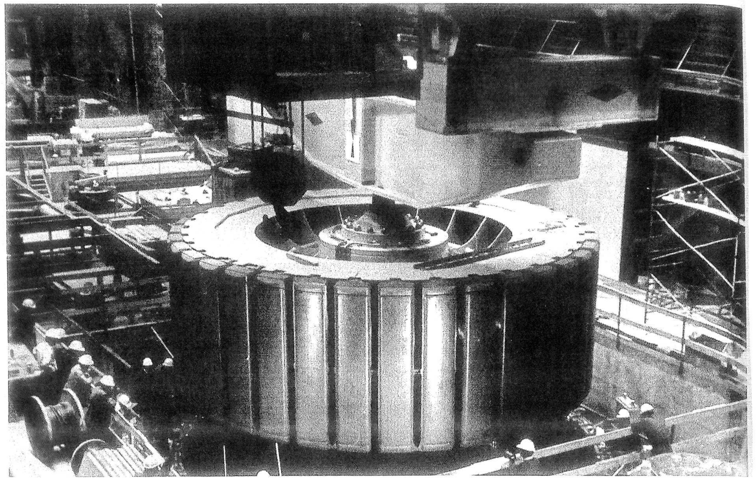
ac Power



Single phase, 1000 MVA, 765/25 kVA transformer (used at a generating station to step down transmission voltages).

ac Power





Stator of a 3-phase, 500 MVA, 0.95 pf, 15 kV, 60 Hz, 200 rpm, generator. Diameter 9.25m, length 2.35 m.).

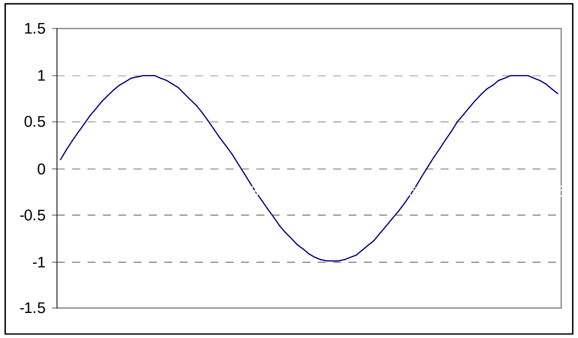
Rotor being lowered into stator. Current in each pole face is 2400 A dc. Total weight of rotor is 600 tonnes. Air gap between rotor and stator is 33mm.

ac Power

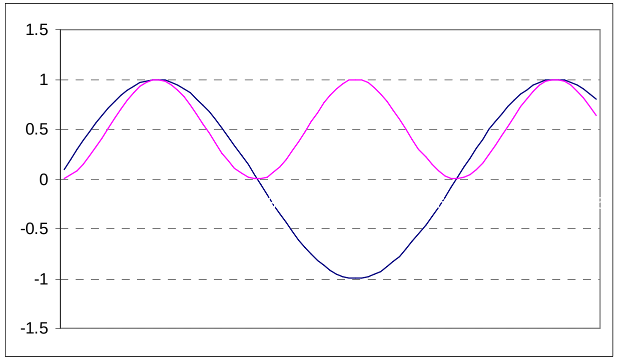
Theory: ac power and the Resistor

ac Power

Introduction: ac power and the Resistor



voltage v(t) = V sin(ωt)



sin (ωt) and sin² (ωt)

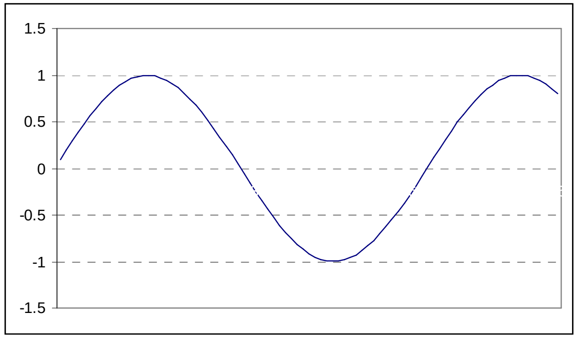
So instantaneous power =

$$\frac{V\sin(\omega t)\times V\sin(\omega t)}{R}$$

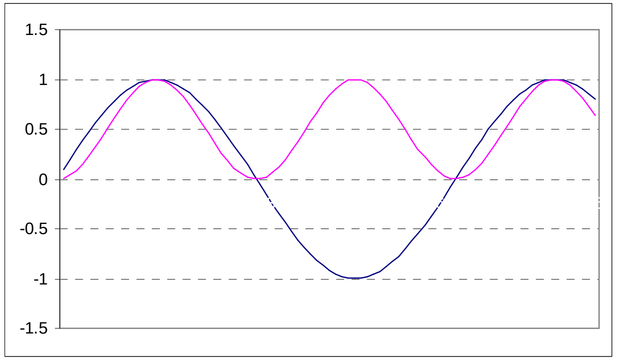
The Average Power is obtained by integrating over time.

ac Power

Introduction: ac power and the Resistor (Revision)



voltage v(t) = V sin(ωt)



sin (ωt) and sin² (ωt)

Now from trigonometry: $\sin^2(\omega t) = 0.5\,(1 - \cos 2\omega t)$ (ONLY for sine waves)

Thus the average value of the $\sin^2$ function is 0.5.

The average power is $\dfrac{V^2}{2R}$ Watts and the effective voltage is $\dfrac{V}{\sqrt{2}}$ volts $_{rms}$

($\dfrac{V}{\sqrt{2}}$ is known as the r.m.s. voltage (Root Mean Square))

ac Power

A general passive network

For a sinusoidal voltage $v(t) = V \sin \omega t$ volts applied to a general passive network, let the resulting current be $i(t) = I \sin (\omega t + \theta)$ amps.

The phase angle will be positive or negative depending on the net capacitive or inductive nature of the general passive circuit.

Power is given by $p(t) = v(t).i(t)$.

$$p(t) = v(t).i(t) = (V \sin \omega t).(I \sin (\omega t + \theta))$$

$$= VI \sin \omega t. \sin (\omega t - \theta) \text{ Watts}$$

When we introduce resistor (R = Watts), capacitor (?) and inductor (?) impedance combinations we get a net phase shift ϴ.
We will now examine the significance of the phase shift ϴ on ac power.

ac Power

A general passive network

$$p(t) = v(t).i(t) = (V \sin \omega t).(I \sin (\omega t + \theta)$$

$$= VI \sin \omega t. \sin (\omega t - \theta) \text{ Watts}$$

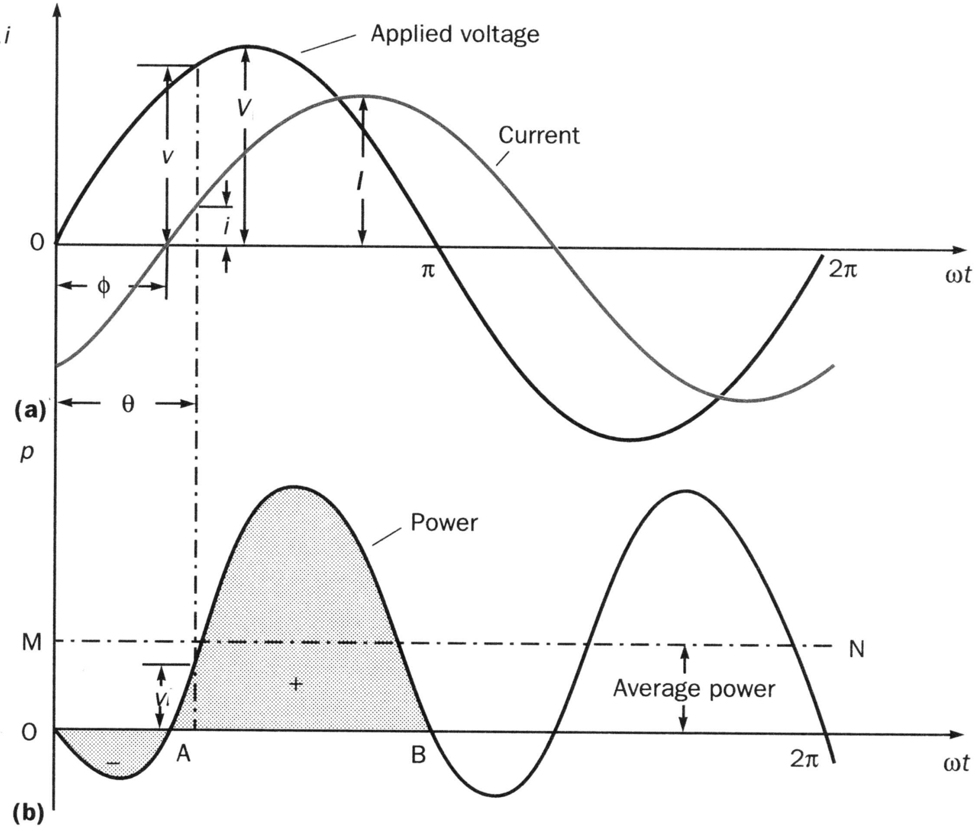
Using the trigonometric expansion:

Sin A sin B = 0.5{cos(A - B) - cos (A + B)} & note that $\cos(-\theta) = \cos(\theta)$.

$$\text{Average power } p(t) = 0.5\, VI \{\cos \theta - \cos (2\omega t - \theta)\} \text{ Watts}$$

ac Power

Average power $p(t) = 0.5\ VI\ \{\cos\theta - \cos(2\omega t - \theta)\}$ Watts



Volts, Current and Power

ac Power

Average power $p(t) = 0.5\ VI\ \{\cos\theta - \cos(2\omega t - \theta)\}$ Watts

(a) The instantaneous power p(t) consists of a cosine term $\{0.5\ VI \cos(2\omega t - \theta)\}$

(b) Also, 0.5 VI term

ac Power

Average power   p(t) = 0.5 VI {cos θ - cos (2ωt - θ)} Watts

(a) The instantaneous power p(t) consists of a sinusoidal term {0.5 VI cos (2ωt - θ)} which has an average value of zero.

→ Average value of cos (2ωt - θ) term → $\frac{1}{\pi}\int_0^{\pi} VI\cos(2\omega t + \theta)\,d\omega t$ → Zero

ac Power

Average power $p(t) = 0.5\ VI\ \{\cos\theta - \cos(2\omega t - \theta)\}$ Watts

(a) The instantaneous power p(t) consists of a sinusoidal term $\{0.5\ VI \cos(2\omega t - \theta)\}$ which has an average value of zero.

$\rightarrow$ Average value of $\cos(2\omega t - \theta)$ term $\rightarrow$ $\dfrac{1}{\pi}\int_0^{\pi} VI\cos(2\omega t + \theta)\,d\omega t$ $\rightarrow$ Zero

(b) Also, 0.5 VI is equivalent to $V_{rms}.I_{rms}$

Thus the *average* value of ac power is $V_{rms}.I_{rms}\cos\theta$ Watts

ac Power: Average Power (P) Watts

Thus the *average* value of ac power is $V_{rms}.I_{rms} \cos\theta$ Watts

This is the ***average power*** and is indicated by the symbol P **Watts**

The term $\cos\theta$ is called the ***power factor***, often abbreviated pf.

The angle between V and I is always between $\pm 90°$. Hence $\cos\theta$ and consequently power are always positive values.

ac Power: power factor

The term cos θ is called the ***power factor***, often abbreviated pf.

The angle between V and I is always between $\pm$ 90°. Hence cos θ and consequently power are always positive values.

Important convention:

To indicate the sign of θ, an ***inductive circuit***, where the current lags the voltage, has a ***lagging power factor***.

In a capacitive circuit the current leads the voltage, and the circuit has a *leading power factor*.

Note the reference!

ac Power: Apparent Power (S)

The product VI is called the ***apparent power*** and is indicated by the symbol **S**.

The units of S are volt-ampere (VA) and kilovolt-ampere (kVA).

All electrical equipment are rated by their VA rating.

ac Power: Reactive Power (Q)

The product VI sin θ is called the ***reactive power*** and is indicated by the symbol **Q**.

The units of Q are volt-amperes-reactive or VAr's (also kVAr).

An alternative name is ***Quadrative Power*** (hence the Q symbol) and this is the energy given and taken back by the ac supply in setting up the

magnetic field: $\frac{1}{2}Li^2$ or the electric field $\frac{1}{2}CV^2$

ac Power: Reactive Power (Q)

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magnetic field: $\frac{1}{2}Li^2$ or the electric field $\frac{1}{2}CV^2$

Likewise as the power dissipated is the *real power* - hence the cosine term; and the power in the reactive terms must naturally be the imaginary or sine term.

As V and I are phasors, and impedance is complex, so not unsurprisingly we also have a **complex power**.

ac Power: Complex Power

S, P and **Q**

Since apparent power (**S**) is calculated without using a multiplying factor of sine or cosine of angle, it is always greater than (or at least equal to) the real power and reactive power.

In fact, apparent power is the hypotenuse of a right triangle whose other two sides are P and Q.

This is called the **power triangle**.

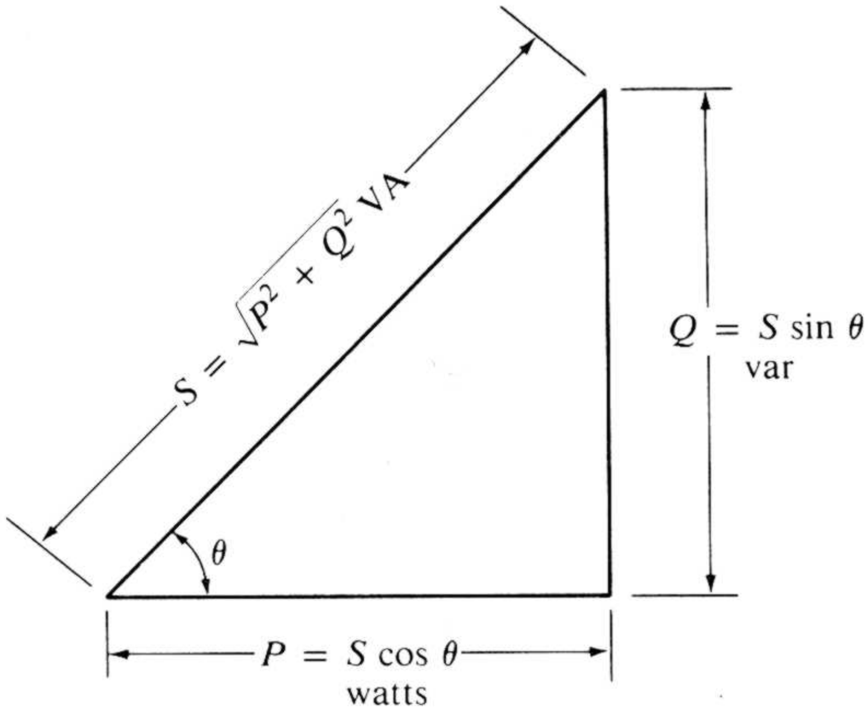
Now:

P = **S** cos θ Watts, **Q** = **S** sin θ VAr, **S** = $\sqrt{P^2 + Q^2}$ VA

ac Power: The Power Triangle

The Power Triangle

The equations associated with the average, apparent and reactive powers can be developed geometrically on a right triangle called a ***power triangle***.



ac Power: The Power Triangle

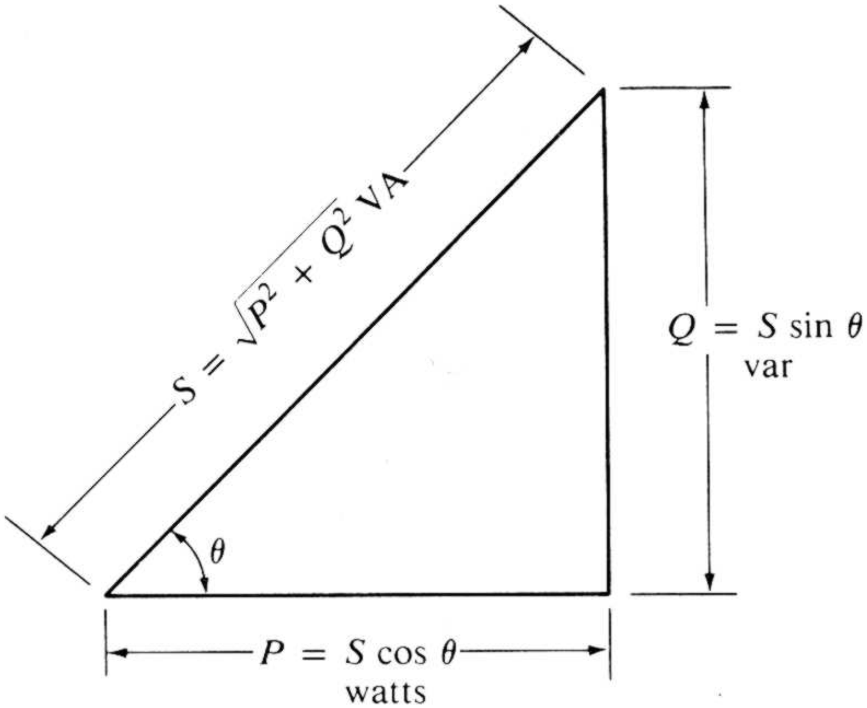
The Power Triangle

The way in which the triangle is drawn differs when drawing Inductive and Capacitive circuits. In power calculations we always *refer to the phase of the Current* ***with respect to the Voltage***.

Thus, we talk about an **inductive** circuit as having a *lagging* pf.

A Capacitive circuit is designated as having a leading pf (and a negative Q).

The significance of positive and negative Q is explained when we consider the alternative version of S, the complex power.



Power Triangle for a capacitive circuit

ac Power: Complex Power

Complex Power

The three sides **S**, **P** and **Q** of the power triangle can be obtained from the product **VI*.**

Now **I*** is the complex conjugate of current

i.e. if I = (A + jB) then I* = (A - jB)

The result of this product is a complex number called the *complex power* S.

Its *real part* equals the average power **P** and its imaginary part is equal to the reactive power **Q.**

ac Power: Complex Power

Complex Power

The three sides **S**, **P** and **Q** of the power triangle can be obtained from the product **VI*.**

Consider an *Inductor*: $I = I\,\underline{/-\theta°}$ Amps and $V = V\underline{/0°}$ volts

Remember we talk about an Inductor as having a lagging power factor

$VI^* = V\underline{/0°} \times I\,\underline{/+\theta°} = VI\underline{/+\theta°}$ $\Rightarrow$ Complex equation of form A **+** jB.

Now (+jB) equates to a *positive* **Q**.

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current

(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

(c) $S = I^2 Z$, $P = I^2 R$ and $Q = I^2 X$

(d) VI^*

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current

•$v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts can be written as $100\underline{/30°}$ volts rms.

In power calculations we should always use rms values of voltage and current.

• $(3 + j4)\ \Omega$ should be expressed in polar form, $z = 5\underline{/53°}\ \Omega$

•$I = V/Z$, $I = 20\underline{/-23°}$ Amps

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

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ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

$S = 100 \times 20 = 2000$ VA

ac Power: Example 1

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(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

$S = 100 \times 20 = 2000$ VA
$P = 2000\cos(53°) = 1200$ Watts

ac Power: Example 1

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(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

$S = 100 \times 20 = 2000$ VA
$P = 2000\cos(53°) = 1200$ Watts
$Q = 2000\sin(53°) = 1600$ VAr

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

$S = 100 \times 20 = 2000$ VA
$P = 2000\cos(53°) = 1200$ Watts
$Q = 2000\sin(53°) = 1600$ VAr
$pf = \cos(53°) = 0.6$

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(b) Determine $S = VI$, $P = VI\cos\theta$ and $Q = VI\sin\theta$

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(d) VI^*

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(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(c) $S = I^2 Z$, $P = I^2 R$ and $Q = I^2 X$

$(3 + j4)\ \Omega = 5\underline{/53°}\ \Omega$

$S = I^2 Z = (20)^2 \times 5 = 2000$ VA

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(c) $S = I^2 Z$, $P = I^2 R$ and $Q = I^2 X$

$S = I^2 Z = (20)^2 \times 5 = 2000$ VA

$P = I^2 R = (20)^2 \times 3 = 1200$ Watts

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(c) $S = I^2 Z$, $P = I^2 R$ and $Q = I^2 X$

$S = I^2 Z = (20)^2 \times 5 = 2000$ VA

$P = I^2 R = (20)^2 \times 3 = 1200$ Watts

$Q = I^2 X = (20)^2 \times 4 = 1600$ Var

ac Power: Example 1

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(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(c) $S = I^2 Z$, $P = I^2 R$ and $Q = I^2 X$

$S = I^2 Z = (20)^2 \times 5 = 2000$ VA

$P = I^2 R = (20)^2 \times 3 = 1200$ Watts

$Q = I^2 X = (20)^2 \times 4 = 1600$ VAr

$pf = \cos(53°) = 0.6$

ac Power: Example 1

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(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(d) VI *

$VI^* = 100\underline{/30°} \times 20\underline{/23°} = 2000\underline{/53°}$ VA = (1200 + j 1600)

ac Power: Example 1

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(d) VI *

$VI^* = 100\underline{/30°} \times 20\underline{/23°} = 2000\underline{/53°}$ VA $= (1200 + j\,1600)$

$(1200 + j\,1600) \Rightarrow$ 1200 Watts and 1600 VAr

pf = cos (53°) = 0.6

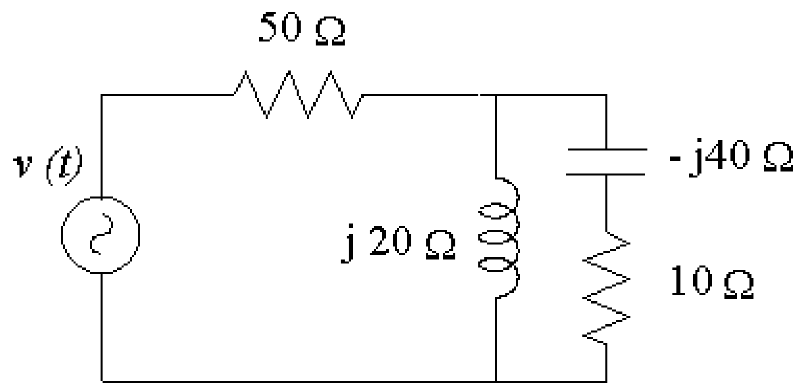
ac Power: Example 1

Class Example 1

ac Power: Example 1

Class Example 1

Determine the power triangle (S, P and Q) for the following circuit.
The supply voltage *v(t)* is given by 100 sin ($2\pi \times 10^3$ t + 45°) volts.



ac Power: Example 1

Class Example 1

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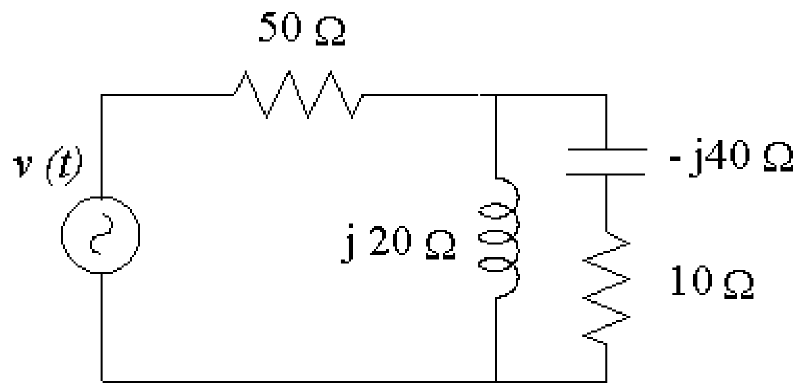


Note: with ALL power questions we must use rms values of current and voltage!

So 100 sin ($2\pi \times 10^3$ t + 45°) volts

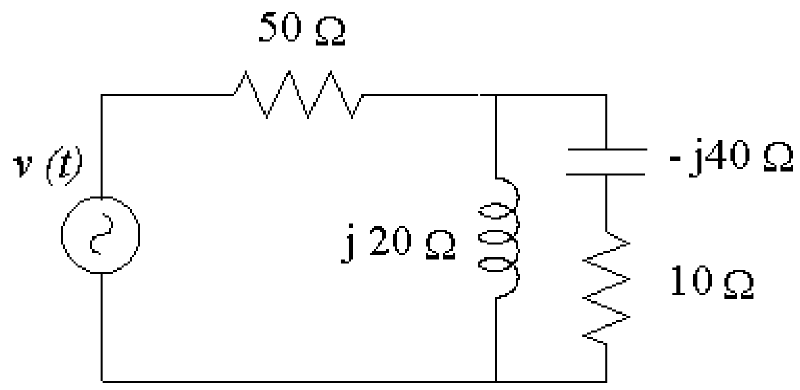
should be taken as

($100/\sqrt{2}$) /$\underline{45^{o}}$ volts rms

ac Power: Example 1

Class Example 1

Determine the power triangle (S, P and Q) for the following circuit.
The supply voltage *v(t)* is given by 100 sin ($2\pi \times 10^3$ t + 45°) volts.



ac Power: Example 1

Class Example 1

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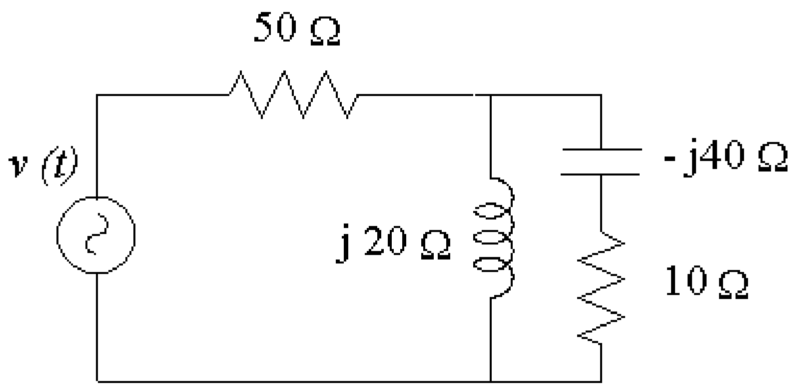


$$\textbf{Z (parallel)} = \frac{(j\,20)\,(10 - j\,40)}{(j\,20) + (10 - j\,40)}$$

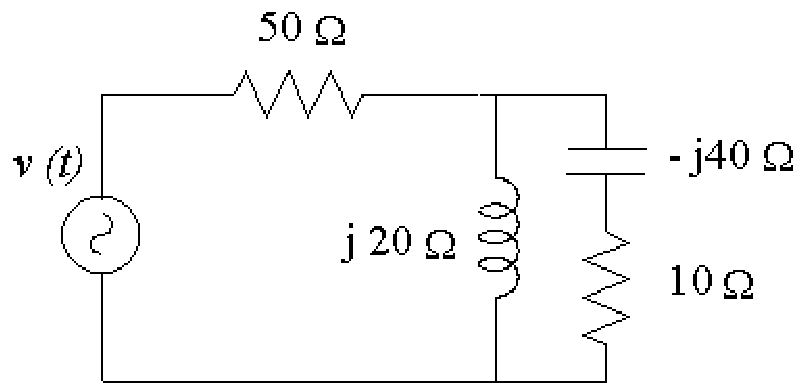
$$\frac{(20\,/90^\circ)\,(41.23\,/-76^\circ)}{(22.36\,/-63^\circ)}$$

$$= 36.88\,/77^\circ = \mathbf{(8.3 + j\,36)\ \Omega}$$

ac Power: Example 1

Class Example 1

Determine the power triangle (S, P and Q) for the following circuit.
The supply voltage *v(t)* is given by 100 sin ($2\pi \times 10^3$ t + 45°) volts.



Z (total) = (50) + (8.3 + j 36)

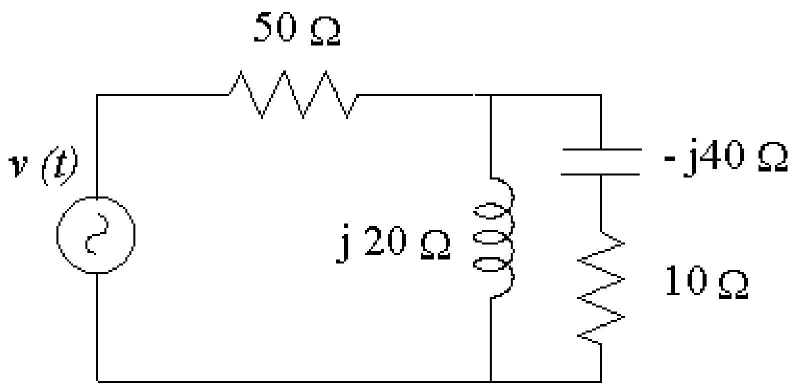
$$= (58.3 + j\,36)\ \Omega$$

$$= 68.51\ \underline{/31.7^{o}}\ \Omega$$

ac Power: Example 1

Class Example 1

Determine the power triangle (S, P and Q) for the following circuit.
The supply voltage *v(t)* is given by $100 \sin(2\pi \times 10^3\, t + 45^\circ)$ volts.



$Z\text{ (total)} = (50) + (8.3 + j\,36)$

$= (58.3 + j\,36)\ \Omega$

$= 68.51\ \underline{/31.7^o}\ \Omega$

$I = V / Z =$

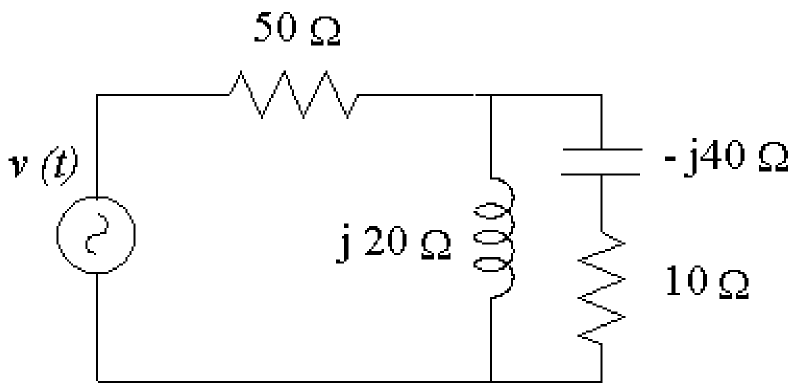
$\mathbf{(100/\sqrt{2})\ \underline{/45^o}} \ / \ 68.51\ \underline{/31.7^o}$

$= 1.03\ \underline{/13.3^o}$ Amps

ac Power: Example 1

Class Example 1

Determine the power triangle (S, P and Q) for the following circuit.
The supply voltage *v(t)* is given by 100 sin ($2\pi \times 10^3$ t + 45°) volts.



I = 1.03 $\angle 13.3^\circ$ Amps

V = $(100/\sqrt{2}) \angle 45^\circ$ volts

S = VI* = $((100/\sqrt{2}) \angle 45^\circ) \times 1.03 \angle -13.3^\circ$

S = 72.84 $\angle 31.7^\circ$ VA

ac Power: Example 1

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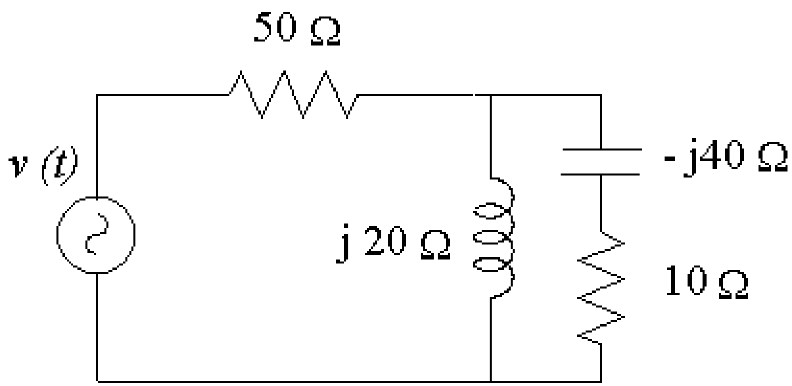


I = **1.03 $\underline{/13.3^{\circ}}$ Amps**

V = $(100/\sqrt{2})\ \underline{/45^{\circ}}$ volts

S = $VI^* = ((100/\sqrt{2})\ \underline{/45^{\circ}}) \times 1.03\ \underline{/-13.3^{\circ}}$

S = 72.84 $\underline{/31.7^{\circ}}$ VA

S = (61.97 + j 38.27)

or 61.97 Watts and 38.27 VAr

ac Power: Example 2

Class Example 2

ac Power: Example 2

Class Example 2

A voltage $20\angle 0^\circ$ volts (rms) is applied to a two branch parallel circuit where:
$Z1 = 4\angle 30^\circ\ \Omega$ and $Z2 = 5\angle -60^\circ\ \Omega$.
Obtain the power triangle for this parallel circuit.

ac Power: Example 2

Class Example 2

A voltage $20/\underline{0^\circ}$ volts is applied to a two branch parallel circuit where:
$Z1 = 4/\underline{30^\circ}\ \Omega$ and $Z2 = 5/\underline{-60^\circ}\ \Omega$.
Obtain the power triangle for this parallel circuit.

Current through Z1

$I_1 = 20/\underline{0^\circ}\ \ /\ 4/\underline{30^\circ}\ \ = 5\ \underline{/-30^\circ}$ A.

$S_1 = VI^*_1 = 20/\underline{0^\circ} \times 5\ \underline{/+30^\circ}$ VA

$= 100\ \underline{/+30^\circ}\ = (86.6 + j\ 50)$ VA

or 86.6 Watts and 50 VAr

ac Power: Example 2

Class Example 2

A voltage $20\underline{/0^\circ}$ volts is applied to a two branch parallel circuit where:
$Z1 = 4\underline{/30^\circ}\ \Omega$ and $Z2 = 5\underline{/-60^\circ}\ \Omega$.
Obtain the power triangle for this parallel circuit.

Current through Z1

$I_1 = 20\underline{/0^\circ}\ /\ 4\underline{/30^\circ}\ = 5\ \underline{/-30^\circ}$ A.

$S_1 = VI^*_1 = 20\underline{/0^\circ} \times 5\ \underline{/+30^\circ}$ VA

$= 100\ \underline{/+30^\circ}\ = (86.6 + j\,50)$ VA

or 86.6 Watts and 50 VAr

Current through Z2

$I_2 = 20\underline{/0^\circ}\ /\ 5\underline{/-60^\circ}\ = 4\ \underline{/+60^\circ}$ A.

$S_2 = VI^*_2 = 20\underline{/0^\circ} \times 4\ \underline{/-60^\circ}$ VA

$= 80\ \underline{/-60^\circ}\ = (40 - j\,69)$ VA

or 40 Watts and 69 VAr

ac Power: Example 2

Class Example 2

A voltage $20\underline{/0^\circ}$ volts is applied to a two branch parallel circuit where:
$Z1 = 4\underline{/30^\circ}\ \Omega$ and $Z2 = 5\underline{/-60^\circ}\ \Omega$.
Obtain the power triangle for this parallel circuit.

$S_1 = 100\ \underline{/+30^\circ} = (86.6 + j\,50)$ VA

or 86.6 Watts and 50 VAr

$S_2 = 80\ \underline{/-60^\circ} = (40 - j\,69)$ VA

or 40 Watts and 69 VAr

$\sum$ Watts = 86.6 + 40 = **126.6 Watts**.

$\sum$ VAr = (50 VAr LAGGING) + (69 VAr LEADING) = + j 50 – j 69 = **- j 19 VAr**

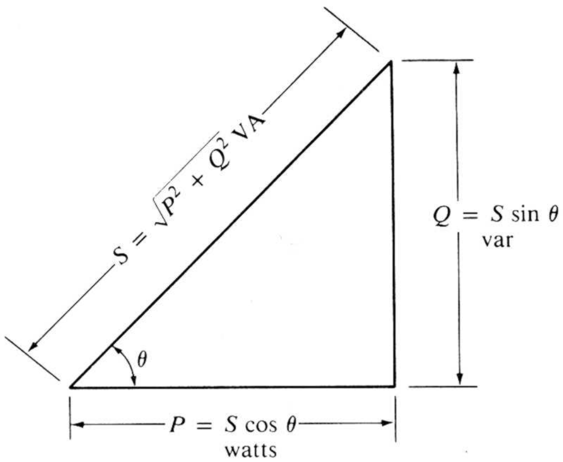
S = (126.6 – j 19) = 128 $\underline{/-8.53^\circ}$ VA

ac Power: Example 3

Example 3: Multiple loads connected to a common voltage supply.

Load 1: 150 kVA at 0.7 power factor lagging
Load 2: 100 kW at unity power factor
Load 3: 75 kVA at 0.8 power factor lagging
Load 4: 50 kW at 0.6 power factor leading

Determine the apparent power (VA) and power factor for the power source.



pf = cos θ (leading)

ac Power: Power Factor Correction

In ordinary residential and industrial applications the load appears ***inductive*** and the current lags the applied voltage. The average power, P, delivered to a load is a measure of the useful work per unit time that the load can perform.

This power is usually transmitted through distribution lines and transformers. Since a transformer, rated in kVA is often used at a fixed voltage, the kVA rating is merely an indication of the maximum current permitted. Theoretically, if a pure inductive or capacitive load were connected, the transformer could be fully loaded while the average load delivered would be zero.

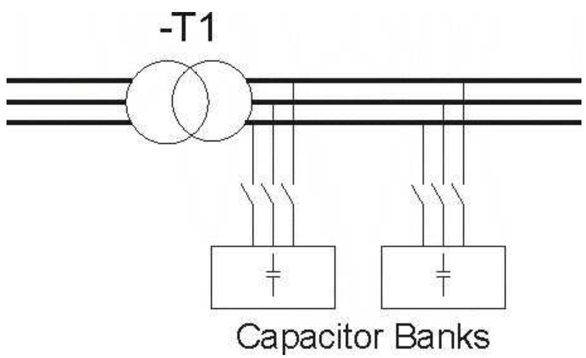
Referring to the power triangle, the hypotenuse S is a measure of the loading of the distribution system, and the side P is a measure of the useful power delivered.

It is therefore desirable to have S as close as P as possible, that is, *to make the angle θ approach zero.* Since pf = cos θ, the pf should approach *unity*.

ac Power: Power Factor Correction

For the usual case of an inductive load, it is often possible *to improve the power factor by placing capacitors in parallel with the load*.

Note that the voltage across the load remains the same, the useful power P also does not change. Since the power factor is increased, the current and apparent power decrease, and a more efficient utilisation of the power distribution system is obtained.



ac Power: Example 4: pf correction on Example 1

A source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30^\circ)$ volts applied across a complex impedance of $(3 + j4)\ \Omega$.

By adding a bank of parallel capacitors, correct the power factor to 0.9.

ac Power: Example 1 - Revisited

Given a source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30°)$ volts is input across a load of complex impedance $(3 + j4)\ \Omega$, determine the power triangle.

(a) Determine the complex current **$I = 20\underline{/-23°}$ Amps**

(d) VI *

$VI^* = 100\underline{/30°} \times 20\underline{/23°} = 2000\underline{/53°}$ VA = $(1200 + j\,1600)$

$(1200 + j\,1600) \Rightarrow$ 1200 Watts and 1600 VAr

$pf = \cos(53°) = 0.6$

ac Power: Example 4: pf correction on Example 1

A source voltage of $v(t) = 100\sqrt{2}\sin(\omega t + 30^\circ)$ volts applied across a complex impedance of $(3 + j4)\ \Omega$.

By adding a bank of parallel capacitors, correct the power factor to 0.9.

We are required to determine:

The new value for the apparent power, S' after the correction is introduced and the VAr of the capacitors.

Now a pf = 0.9 infers that $\cos\theta = 0.9$, and thus $\theta = 26^\circ$.

If we redraw the power triangle such that instead of a 56° phase angle we have a phase angle of 26°.

ac Power: Example 4: pf correction on Example 1

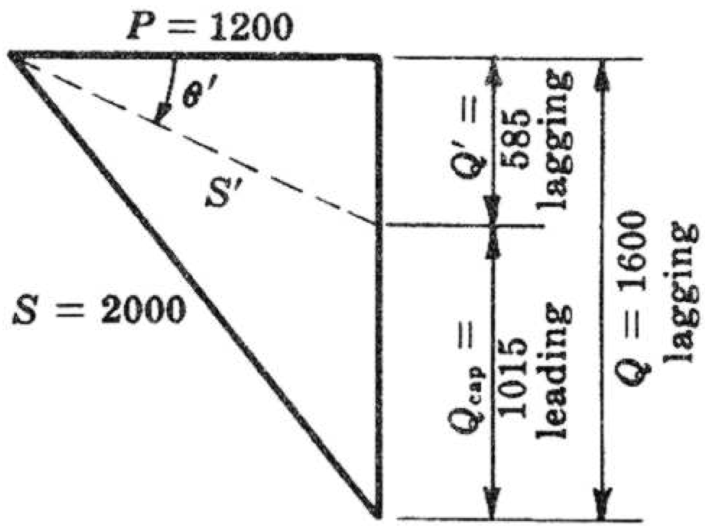
If we redraw the power triangle such that instead of a 56° phase angle we have a phase angle of 26°.

$$S' = \frac{P}{\cos\theta'} = \frac{1200}{\cos 26^\circ} = 1333 \text{ VA}$$

Q' = S' sin 26° = 585 VAr Lagging

Capacitor VAr's = (1600 - 585)
= 1015 VAr Leading

Since P remains unchanged, the work also remains unchanged after the correction of pf. The value of S has been reduced from 2000 to **1333 VA**.



Resonance

Resonance

In circuits that contain resistance, inductance and capacitance a resonance condition can occur.

Resonance can occur for both series connected and parallel connected components.

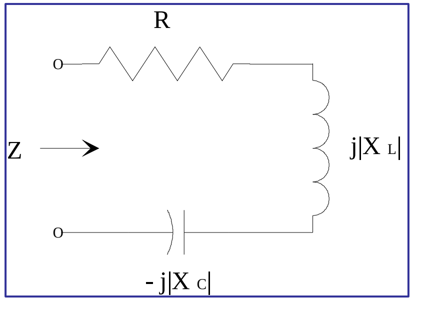
A circuit is said to be in resonance when the applied voltage ***E*** and the resulting current ***I*** are in phase. Thus at resonance the equivalent complex impedance of the circuit consists of only resistance ***R***.

Resonance

Series RLC Circuit

We know from theory that inductive and capacitive reactance behave differently as a function of frequency.

(a) Inductor $X_L = j\omega L$
(b) Capacitor $X_c = - j\,(1/\omega\,L)$



Series RLC circuit

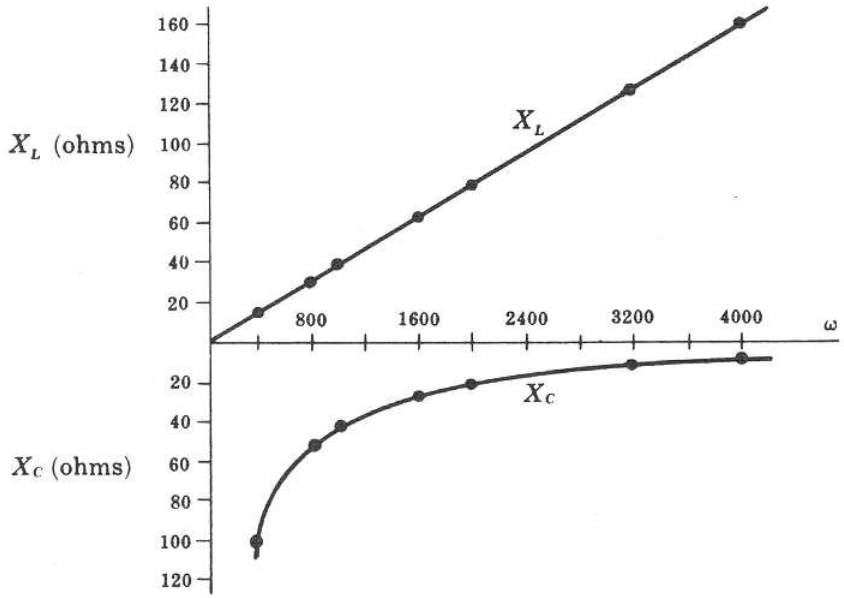
Resonance

Series RLC Circuit

Example: Consider the plot of inductive & capacitive reactance versus frequency for L = 40mH and C = 25μF.

ω rad/sec	X_L ohms	X_C ohms
400	16	100
800	32	50
1000	40	40
1600	64	25
2000	80	20
3200	128	12.5
4000	160	10

(a)



(b)

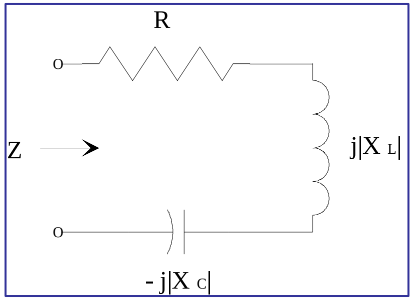
For ω between 400 and 4000 rad/s.

Resonance

Series RLC Circuit

We know from theory that inductive and capacitive reactance behave differently as a function of frequency.

(a) Inductor $X_L = j\omega L$
(b) Capacitor X_c = - j (1/ω L)



Series RLC circuit

Resonance

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We know from theory that inductive and capacitive reactance behave differently as a function of frequency.

(a) Inductor $X_L = j\omega L$
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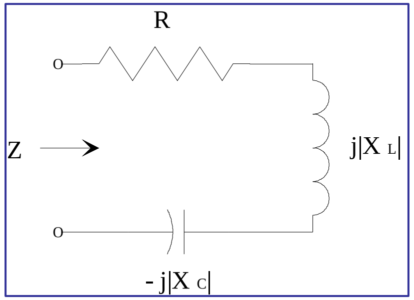
For a series RLC circuit the supply voltage is equal to the sum of the voltages V_R, V_L and V_C and as the current I is common:

$$\therefore |Z| = \sqrt{\left\{ R^2 + \left(2\pi\ f\ L - \frac{1}{2\pi\ f\ C} \right)^2 \right\}}$$



Series RLC circuit

$$Z = \frac{V}{I} = R + j\,(2\pi\ f\ L) + \frac{1}{j(2\pi\ f\ C)}$$

Resonance

Series RLC Circuit

We know from theory that inductive and capacitive reactance behave differently as a function of frequency.

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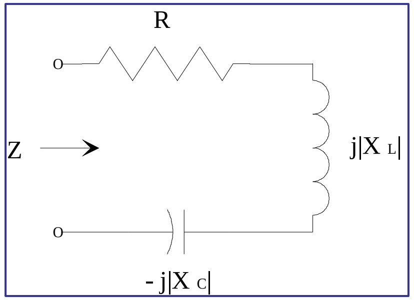

Series RLC circuit

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$$\therefore |Z| = \sqrt{\left\{ R^2 + \left(2\pi\, f\, L - \frac{1}{2\pi\, f\, C} \right)^2 \right\}}$$

$$Z = \frac{V}{I} = R + j\,(2\pi\, f\, L) + \frac{1}{j\,(2\pi\, f\, C)}$$

Resonance is defined as that condition when the inductive and the capacitive impedances cancel. The circuit appears to be only resistive. This results in the maximum current flowing into the circuit for a given voltage.

Resonance

Series RLC Circuit: Resonant frequency

When the inductive reactance equals the capacitive reactance:

$$\boxed{2\pi f L = \frac{1}{2\pi f C}} \quad \text{or} \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{(LC)}}}$$

Resonance

Series RLC Circuit: Resonant frequency

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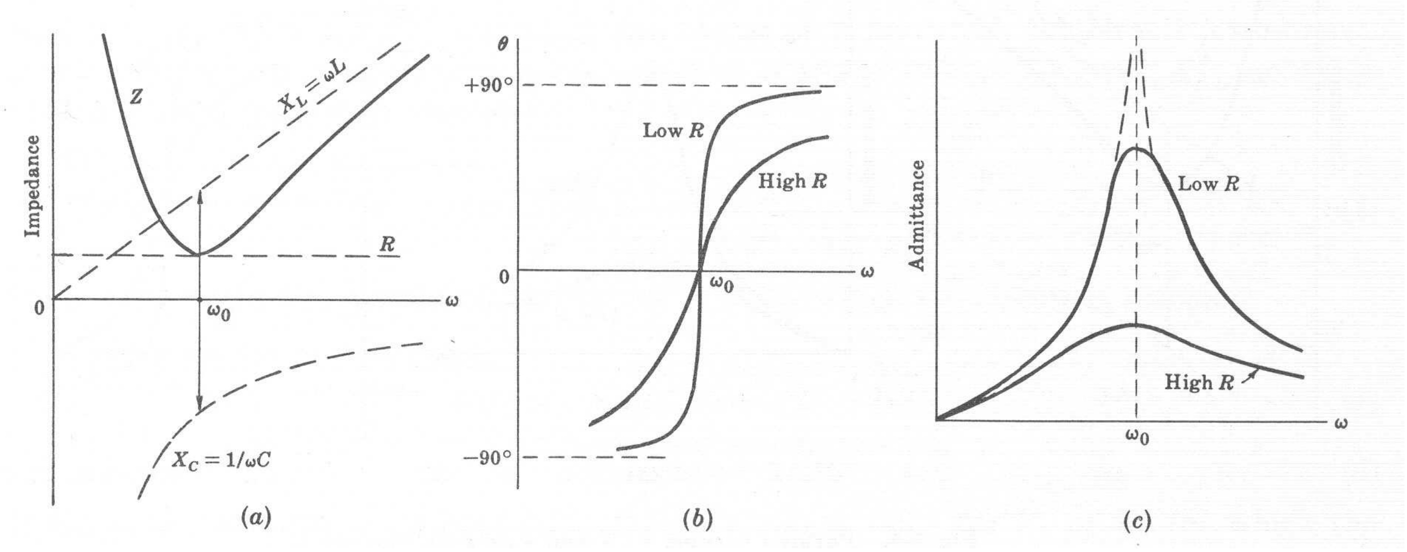


Series circuit: (a) Impedance (b) phase angle (c) Admittance as a function of ω

Resonance

Series RLC Circuit: Oscillation of energy at resonance

When the inductive reactance equals the capacitive reactance:

Energy is stored in the inductive and capacitive reactance of L and C and oscillates between them at resonance.

The maximum magnetic energy stored in the inductor is $\frac{1}{2} LI^2{}_{MAX}$ Joules and the maximum electrostatic energy stored in the capacitor is $\frac{1}{2} CV^2{}_{MAX}$ Joules.

Losses are experienced as the current passes through the resistor when passing energy between L and C.

Resonance

Series RLC Circuit: Oscillation of energy at resonance

The Quality Factor (Q) of coils, capacitors and circuits is defined by:

$$Q = 2\pi \frac{\max imum\ stored\ energy}{energy\ dissipated\ per\ cycle}$$

Resonance

Series RLC Circuit: Oscillation of energy at resonance

The Quality Factor (Q) of coils, capacitors and circuits is defined by:

$$Q = 2\pi \frac{\max imum\ stored\ energy}{energy\ dissipated\ per\ cycle}$$

Now the energy dissipated per cycle in a RL or RC circuit is given by the product of the average power in the resistor (I^2 rms.R or $(I_{MAX}/\sqrt{2})^2$.R) and the period T (or 1/f). For an RL and an RC circuit:

$$\boxed{Q_{RL} = 2\pi \frac{\frac{1}{2} L I_{MAX}^2}{(I_{MAX}^2/2)R(1/f)} = \frac{2\pi f L}{R} = \frac{\omega L}{R}}$$

$$\boxed{Q_{RC} = 2\pi \frac{\frac{1}{2} C V_{MAX}^2}{(I_{MAX}^2/2)R(1/f)} = 2\pi \frac{\frac{1}{2} I_{MAX}^2/\omega^2 C}{(I_{MAX}^2/2)(R/f)} = \frac{1}{\omega C R}}$$

Resonance

Series RLC Circuit: Oscillation of energy at resonance

Q-factor can also be more generally described as the ratio of reactive power (in L or C) to the average power in the resistor at resonance.

$$\boxed{Q_0 = \frac{reactive\ power}{average\ power} = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R}}$$

$$\boxed{Q_0 = \frac{reactive\ power}{average\ power} = \frac{I^2 X_C}{I^2 R} = \frac{X_C}{R} = \frac{1}{\omega_0 CR}}$$

Inductive reactance at resonance

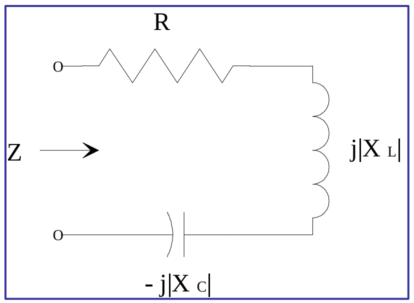
Capacitive reactance at resonance

Resonance

Series RLC Circuit: Q and voltage magnification

At resonance the voltage across the inductor (L) and the capacitor (C) can be much greater than the applied voltage, E.

The voltage across the capacitor: $V_C = I \times \frac{1}{j\omega C}$ and $I = \frac{E}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$



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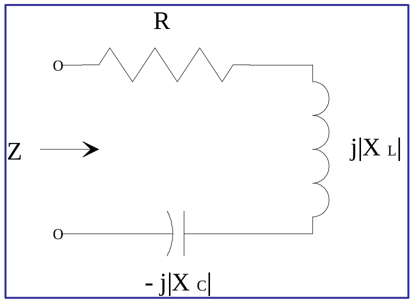
$$V_C = \frac{E}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \times \frac{1}{j\omega C} = \frac{E}{\left(1 - \omega^2 LC\right) + j\omega CR}$$

$$\therefore \quad |V_C| = \frac{E}{\sqrt{\left(1 - \omega^2 LC\right)^2 + (\omega CR)^2}}$$



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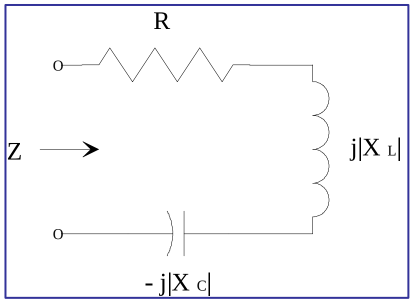
$$V_C = \frac{E}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \times \frac{1}{j\omega C} = \frac{E}{\left(1 - \omega^2 LC\right) + j\omega CR}$$

$$\therefore \quad |V_C| = \frac{E}{\sqrt{\left(1 - \omega^2 LC\right)^2 + (\omega CR)^2}}$$

Now at resonance: $\omega_0 = \frac{1}{\sqrt{(LC)}}$



Resonance

Series RLC Circuit: Q and voltage magnification

At resonance the voltage across the inductor (L) and the capacitor (C) can be much greater than the applied voltage, E.

The voltage across the capacitor: $|V_C| = \dfrac{E}{\sqrt{\left(1-\omega^2 LC\right)^2 + (\omega CR)^2}}$

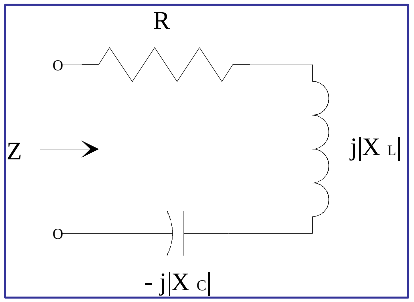
$$\boxed{\omega_0 = \frac{1}{\sqrt{(LC)}}}$$

$$\therefore |V_C| = \frac{E}{\sqrt{(\omega_0 CR)^2}} = \frac{E}{\omega_0 CR} = QE \quad \text{where } Q = \frac{1}{\omega_0 CR}$$

Now Q is the Q-factor or voltage

magnification: $Q = \dfrac{1}{\omega_0 CR} = \dfrac{\omega_0 L}{R} = \dfrac{X}{R}$ also $\omega_0 = \dfrac{1}{\sqrt{(LC)}}$

$$\boxed{\therefore Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$



Resonance

Series RLC Circuit: Q and voltage magnification

At resonance the voltage across the inductor (L) and the capacitor (C) can be much greater than the applied voltage, E.

The voltage across the capacitor: $|V_C| = \dfrac{E}{\sqrt{(1-\omega^2 LC)^2 + (\omega CR)^2}}$

$$\boxed{\omega_0 = \frac{1}{\sqrt{(LC)}}}$$

$$\therefore |V_C| = \frac{E}{\sqrt{(\omega_0 CR)^2}} = \frac{E}{\omega_0 CR} = QE \quad where \;\; Q = \frac{1}{\omega_0 CR}$$

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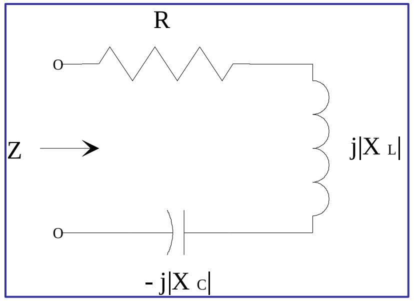

$$\boxed{\therefore Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

In a series circuit this Q can be very large!

Resonance

Series RLC Circuit: Q and voltage magnification

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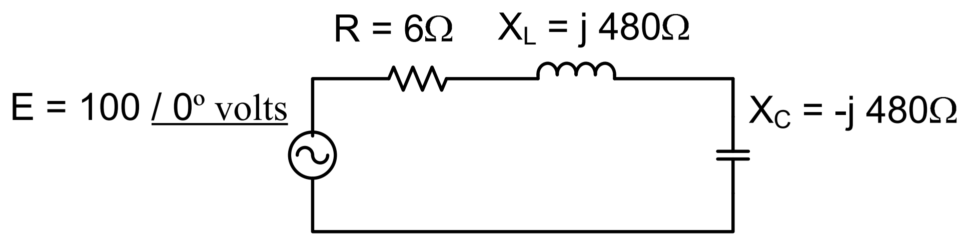
In a series circuit this Q can be very large!

Example:

Consider the RLC circuit:



Resonance

Series RLC Circuit: Q and voltage magnification

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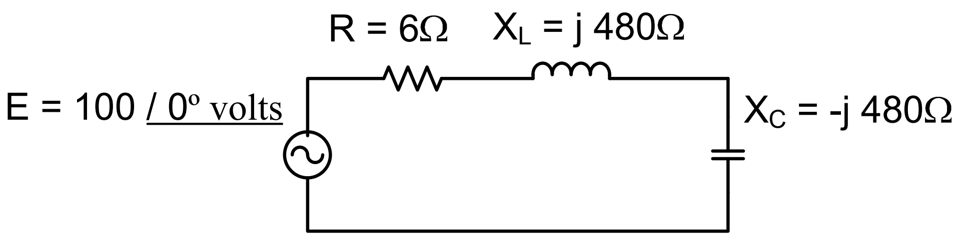


$$Q = \frac{X}{R} = \frac{480}{6} = 80$$

and $V_L = V_C = QE = 8kV$

Current is given by: 100/6 = 16.7A at resonance.

Resonance

Series RLC Circuit: Bandwidth

The bandwidth of a circuit is defined as the frequency range between half-power points. As power is I^2R then half-power will occur when the current falls to the square-root of the maximum current.

The maximum current will occur at resonance and be given by $I_{max} = E/R$.

Therefore at the half-power point: $I' = \dfrac{I_{max}}{\sqrt{2}}$

Resonance

Series RLC Circuit: Bandwidth

The bandwidth of a circuit is defined as the frequency range between half-power points. As power is I^2R then half-power will occur when the current falls to the square-root of the maximum current.

The maximum current will occur at resonance and be given by $I_{max} = E/R$.

Therefore at the half-power point: $I' = \frac{I_{\max}}{\sqrt{2}}$

At half-power the reactive and resistive terms are equal i.e. $Z = R\,(1 \pm j1)$

Thus at half-power the phase angles of the current are 45º with respect to E

Resonance

Series RLC Circuit: Bandwidth

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R\left(1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)\right) \; and \;\; at\, half - power\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right) = \pm 1$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Therefore: $Q\left[\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right] = \pm 1$

Resonance

Series RLC Circuit: Bandwidth

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R\left(1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)\right) \; and \;\; at\, half - power \left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right) = \pm 1$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Therefore: $Q\left[\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right] = \pm 1$

Turning this into a quadratic gives: $\omega^2 \pm \omega\frac{\omega_0}{Q} - \omega_0{}^2 = 0 \; and\ solving\ gives \;\; \omega = \pm\frac{\omega_0}{2Q} + \omega_0\sqrt{1 \mp \frac{1}{4Q^2}}$

$$\boxed{The\, two\, solutions\, are: \omega_1 = -\frac{\omega_0}{2Q} + \omega_0\sqrt{1 + \frac{1}{4Q^2}} \;\; and \;\; \omega_2 = \frac{\omega_0}{2Q} + \omega_0\sqrt{1 - \frac{1}{4Q^2}}}$$

Resonance

Series RLC Circuit: Bandwidth

$$The\,two\,solutions\,are: \omega_1 = -\frac{\omega_0}{2Q} + \omega_0\sqrt{1+\frac{1}{4Q^2}} \;\; and \;\; \omega_2 = \frac{\omega_0}{2Q} + \omega_0\sqrt{1-\frac{1}{4Q^2}}$$

Hence the bandwidth is: $\boxed{\omega_2 - \omega_1 = \frac{\omega_0}{Q}}$

So the quality factor can be given as the ratio of the resonant frequency to bandwidth:

$$\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{bandwidth}}$$

Resonance

Series RLC Circuit: Bandwidth

$$The\,two\,solutions\,are: \omega_1 = -\frac{\omega_0}{2Q} + \omega_0\sqrt{1+\frac{1}{4Q^2}} \quad and \quad \omega_2 = \frac{\omega_0}{2Q} + \omega_0\sqrt{1-\frac{1}{4Q^2}}$$

Hence the bandwidth is: $\boxed{\omega_2 - \omega_1 = \frac{\omega_0}{Q}}$

So the quality factor can be given as the ratio of the resonant frequency to bandwidth:

$$\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{bandwidth}}$$

The resonant frequency is given as the geometric mean of ω_2 -and ω_1:

$$\omega_0 = \sqrt{\omega_1\,\omega_2} \qquad\qquad f_0 = \sqrt{f_1\,f_2}$$

The higher the Q the higher the lower the bandwidth!

Resonance

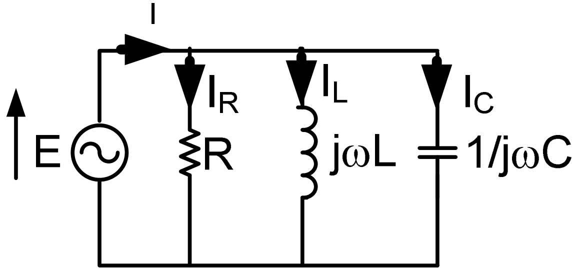
Parallel RLC Circuit

The admittance of the three elements is
$Y = G + j\,(\omega C - 1/\omega L) = G + j\,B$.

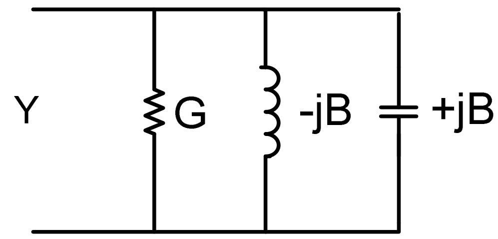
$B_L = 1/\omega L$ and $B_C = \omega C$. $G = 1/R$.

The circuit is in resonance when $B = 0$.
Thus as in the series circuit the resonant frequency is given by:

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{(LC)}}}$$



Ideal parallel RLC



Admittance circuit

Resonance

Parallel RLC Circuit

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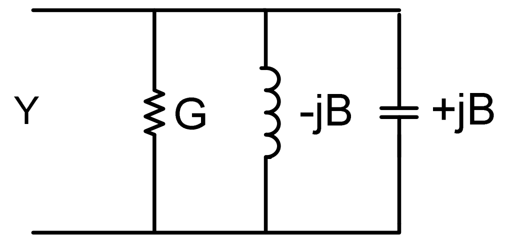
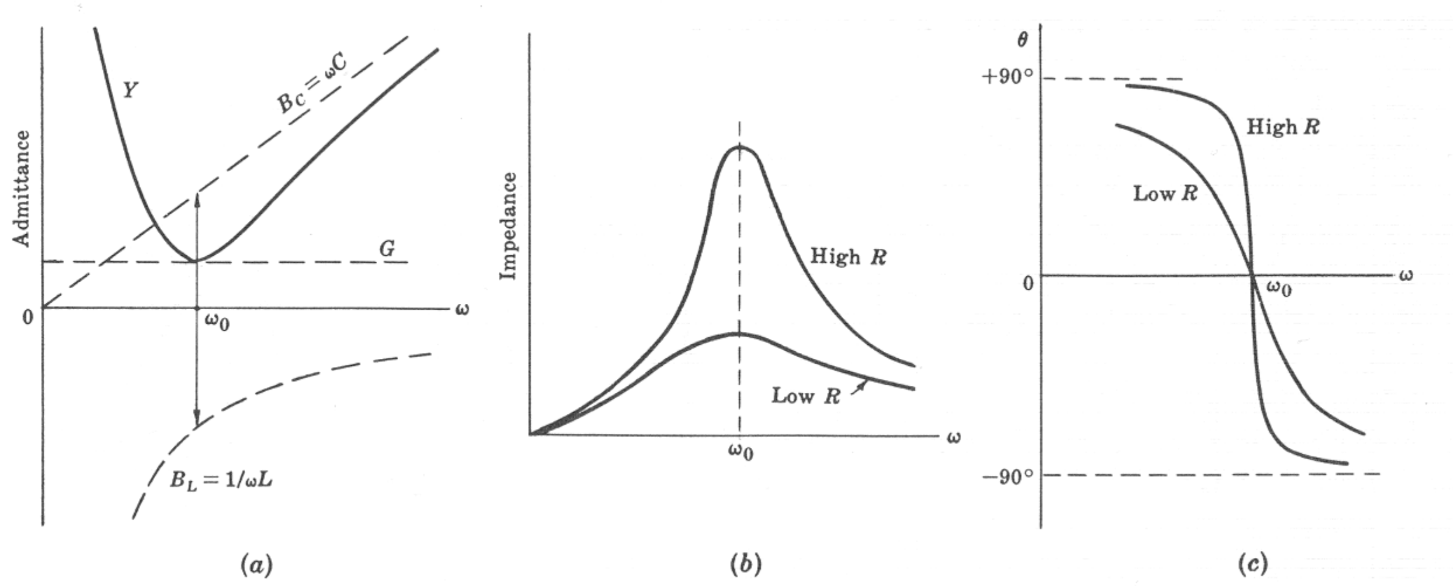
$Y = G + j\,(\omega C - 1/\omega L) = G + j\,B.$



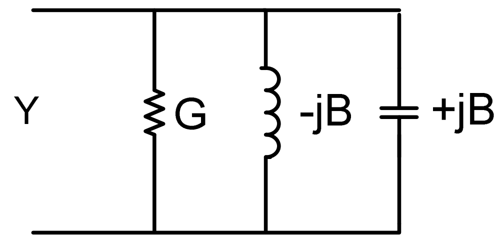
Admittance circuit



Parallel circuit (a) Admittance (b) Impedance (c) Phase angle as a function of ω

Resonance

Parallel RLC Circuit: Q and current magnification



Admittance circuit

At resonance the magnitudes of the inductive and capacitive reactance are equal i.e. $|X_L| = |X_C|$

The phases will be of opposite sense.
The magnitude of the current in the inductor and capacitor will be equal.

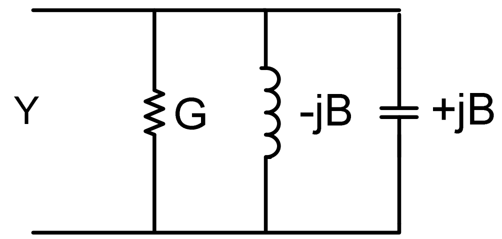
At resonance: $E = \dfrac{I}{G}$

Thus the current in the inductor will be:

$$I_L = \frac{E}{j\omega_0 L} \quad \therefore |I_L| = \frac{I}{\omega_0 LG} = Q\,I \quad where\, Q\, is\, the\, current\,\, magnification$$

Resonance

Parallel RLC Circuit: Q and current magnification



Admittance circuit

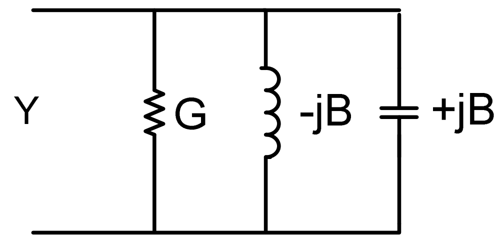
$$I_L = \frac{E}{j\omega_0 L} \quad \therefore |I_L| = \frac{I}{\omega_0 LG} = Q\,I \quad \textit{where } Q \textit{ is the current magnification}$$

$$\therefore Q = \frac{1}{\omega_0 LG} = \frac{\omega_0 C}{G} = \frac{B}{G} = \frac{R}{X}$$

where B is inductive or capacitive susceptance and X is the inductive or capacitive reactance.

Resonance

Parallel RLC Circuit: Q and current magnification



Admittance circuit

$$I_L = \frac{E}{j\omega_0 L} \quad \therefore |I_L| = \frac{I}{\omega_0 LG} = Q\,I \quad where\, Q\, is\, the\ current\ magnification$$

$$\therefore Q = \frac{1}{\omega_0 LG} = \frac{\omega_0 C}{G} = \frac{B}{G} = \frac{R}{X}$$

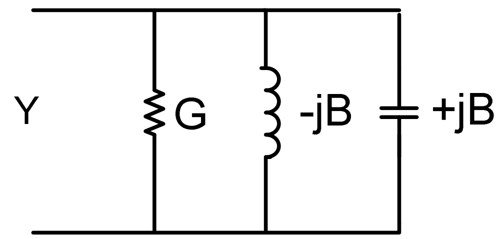
where B is inductive or capacitive susceptance and X is the inductive or capacitive reactance.

By substituting $\omega_0 = \dfrac{1}{\sqrt{(LC)}}$

$$\boxed{\therefore\ Q = \frac{1}{G}\sqrt{\frac{C}{L}} = R\sqrt{\frac{C}{L}}}$$

Resonance

Parallel RLC Circuit: Q and current magnification



Admittance circuit

$$\boxed{\therefore\ Q = \frac{1}{G}\sqrt{\frac{C}{L}} = R\sqrt{\frac{C}{L}}}$$

In general for a parallel circuit Q = R/X and for a high Q-factor R is large. Hence the circuit must be driven by a *current source* in order to exhibit a resonant peak and be selective.

Resonance

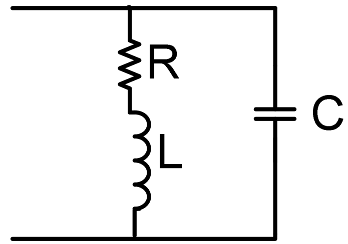
Parallel RLC Circuit: The two branch parallel circuit

This is the real-world circuit!

We are going to analyse it in two ways:

(a) Parallel to series conversion

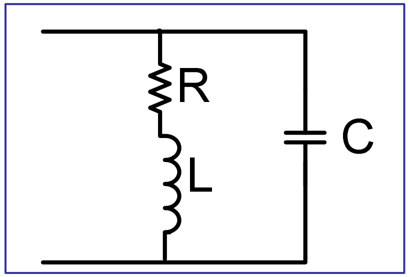
(b) Direct calculation of impedance Z



Two branch parallel circuit

Resonance

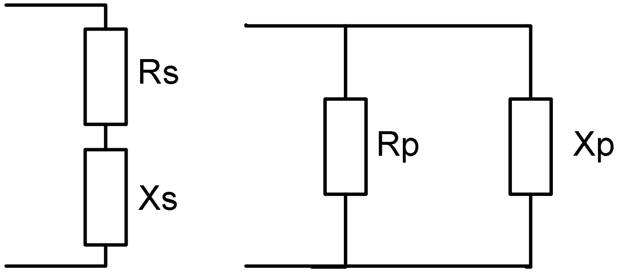
Parallel RLC Circuit: The two branch parallel circuit



Two branch parallel circuit

(a) Parallel to series conversion

For equivalence: $R_S + jX_S$ must be equal to the parallel combination of R_P and jX_P.



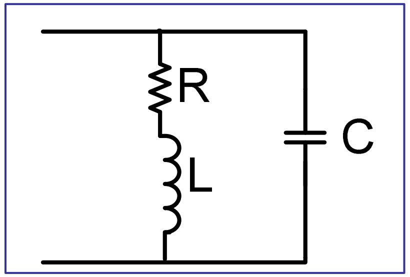
Series and Parallel circuits

$$R_S + jX_S = \frac{R_P(jX_P)}{R_P + jX_P} = \frac{(R_P - jX_P)R_P(jX_P)}{R_P^{\,2} + X_P^{\,2}}$$

$$= \frac{R_P X_P^{\,2}}{R_P^{\,2} + X_P^{\,2}} + j\frac{R_P^{\,2} X_P}{R_P^{\,2} + X_P^{\,2}} = \frac{R_P}{1 + \frac{R_P^{\,2}}{X_P^{\,2}}} + j\frac{X_P}{1 + \frac{X_P^{\,2}}{R_P^{\,2}}}$$

Resonance

Parallel RLC Circuit: The two branch parallel circuit

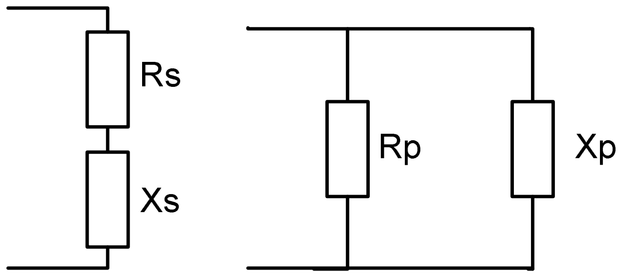


Two branch parallel circuit

(a) Parallel to series conversion

$$R_S + jX_S \Rightarrow$$

$$= \frac{R_P X_P^{\,2}}{R_P^{\,2} + X_P^{\,2}} + j\frac{R_P^{\,2} X_P}{R_P^{\,2} + X_P^{\,2}} = \frac{R_P}{1 + \frac{R_P^{\,2}}{X_P^{\,2}}} + j\frac{X_P}{1 + \frac{X_P^{\,2}}{R_P^{\,2}}}$$



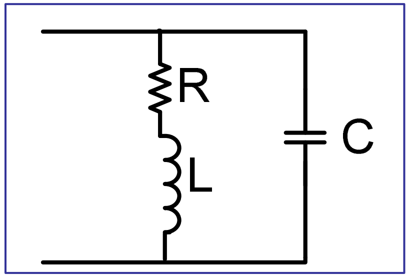
Series and Parallel circuits

In a parallel circuit: $Q = \frac{R_P}{X_P}$

$$\boxed{R_S + jX_S = \frac{R_P}{1+Q^2} + j\frac{X_P}{1+\frac{1}{Q^2}}}$$

Resonance

Parallel RLC Circuit: The two branch parallel circuit



Two branch parallel circuit

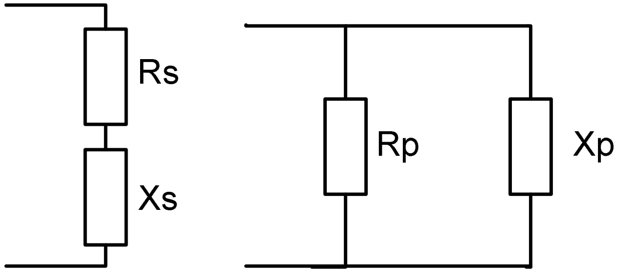
(a) Parallel to series conversion

$$R_S + jX_S = \frac{R_P}{1+Q^2} + j\frac{X_P}{1+\frac{1}{Q^2}}$$

$$\therefore R_S = \frac{R_P}{1+Q^2} \cong \frac{R_P}{Q^2} \quad for\ high\ Q$$

$$\therefore X_S = \frac{X_P}{1+\frac{1}{Q^2}} \cong X_P \quad for\ high\ Q$$

or $R_P = Q^2 R_S$



Series and Parallel circuits

Resonance

Parallel RLC Circuit: The two branch parallel circuit

(a) Parallel to series conversion

$$R_S + jX_S = \frac{R_P}{1+Q^2} + j\frac{X_P}{1+\frac{1}{Q^2}}$$

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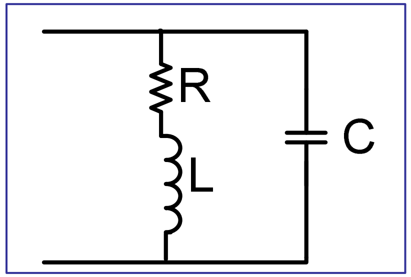
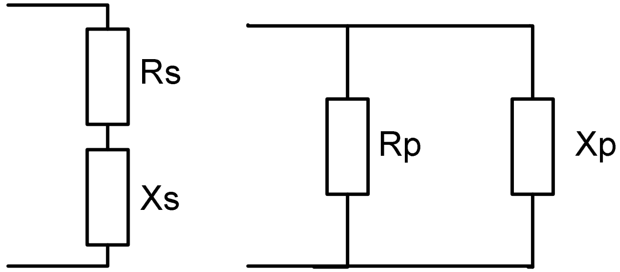


Two branch parallel circuit

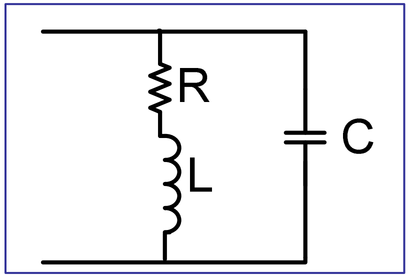


Series and Parallel circuits

Note: the Q factor is the same whether the circuit is considered series or parallel. For the high Q condition: $R_P = Q^2 R_S$ the term R_P is often called the *dynamic resistance* of the circuit.

Resonance

Parallel RLC Circuit: The two branch parallel circuit



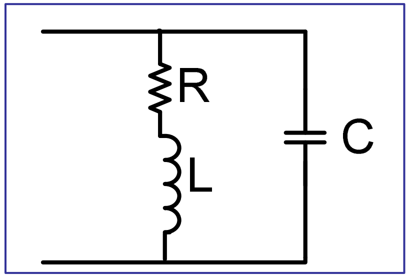
Two branch parallel circuit

(b) Direct calculation of impedance Z

$$Z = \frac{(R + j\omega L)\left(\frac{1}{j\omega C}\right)}{(R + j\omega L)\left(\frac{1}{j\omega C}\right)} = \frac{\left(\frac{L}{C} - j\frac{R}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Resonance

Parallel RLC Circuit: The two branch parallel circuit



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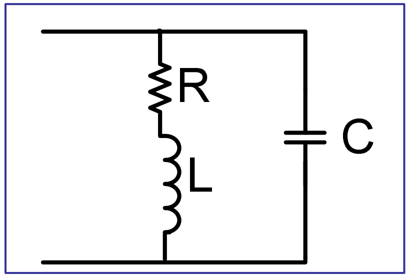
If we assume that at resonance all the j terms are zero, we get: $\boxed{Z_{\text{Re}\,sonance} = \frac{L}{CR}}$

Resonance

Parallel RLC Circuit: The two branch parallel circuit



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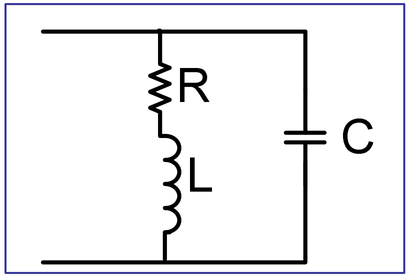
If we assume that at resonance all the j terms are zero, we get: $\boxed{Z_{\mathrm{Re}\,sonance} = \frac{L}{CR}}$

Hence at resonance the impedance of the parallel network is equivalent to a resistor of value $Z_{\text{Resonance}}$.

Resonance

Parallel RLC Circuit: The two branch parallel circuit



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If we assume that at resonance all the j terms are zero, we get: $Z_{\mathrm{Re}\,sonance} = \frac{L}{CR}$

Hence at resonance the impedance of the parallel network is equivalent to a resistor of value $Z_{Resonance}$.

The lower the value of the resistance of the coil, the higher is the dynamic resistance of the parallel circuit. Thus for high Q coils we require the inductors series resistance to be as small as possible.

Signals and Waveforms

Signals and Waveforms

Signals

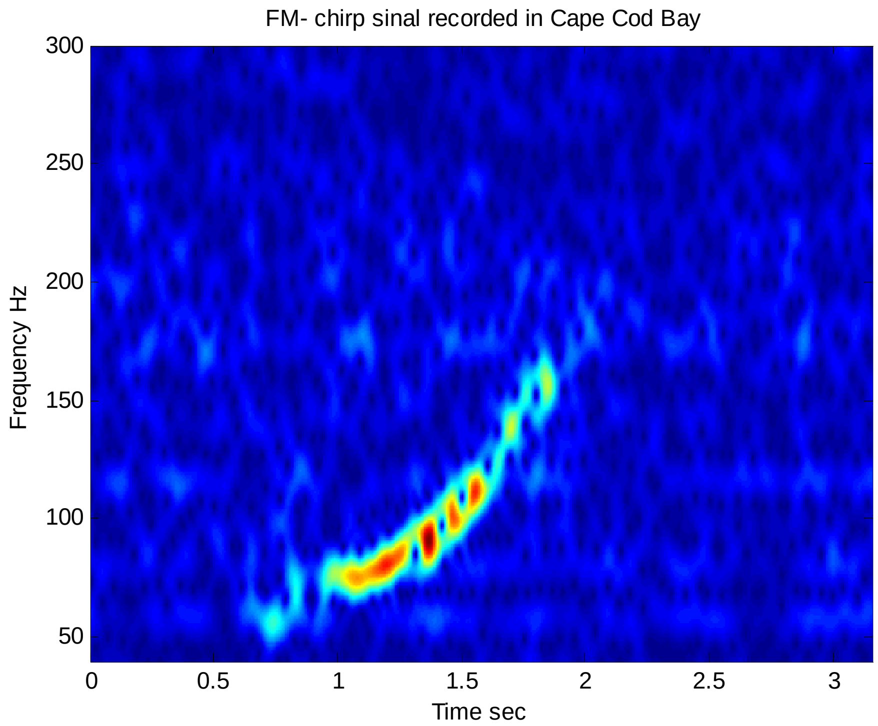
A *signal* can be regarded as the variation of any measurable quantity that conveys *information* concerning the behaviour of a related system.

Whether we are dealing with engineering *systems*, biological systems, transport, communication or economic systems, we rely on the interpretation of signal records, charts, graphs and displays to increase understanding, to make decisions and, when appropriate, to verify that a particular system is performing to *specification*.

Signal analysis is a big field in (electrical) engineering!

Signals and Waveforms

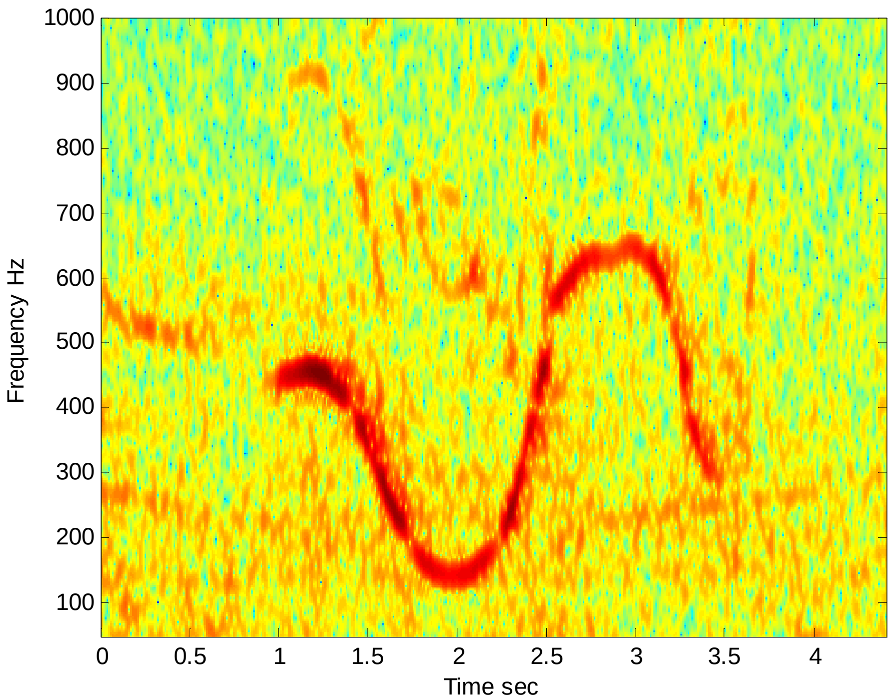
Examples of Biological signals



Right whale vocalisation

Signals and Waveforms

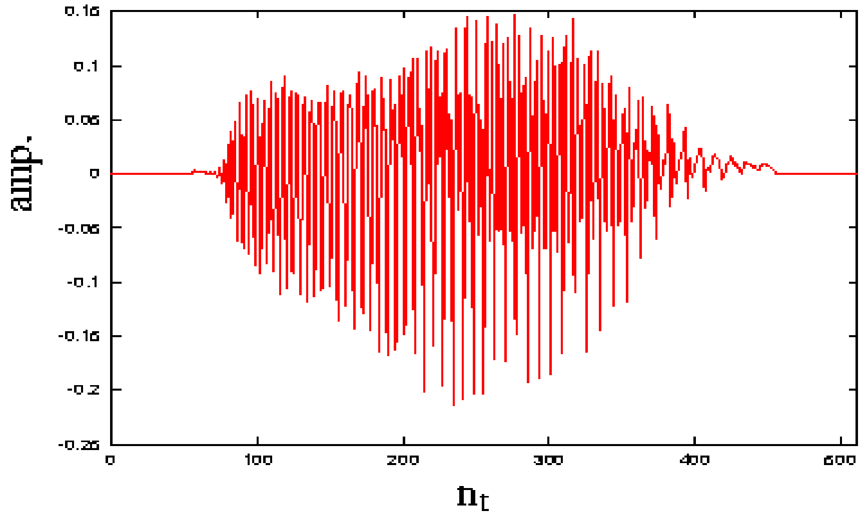
Examples of Biological signals



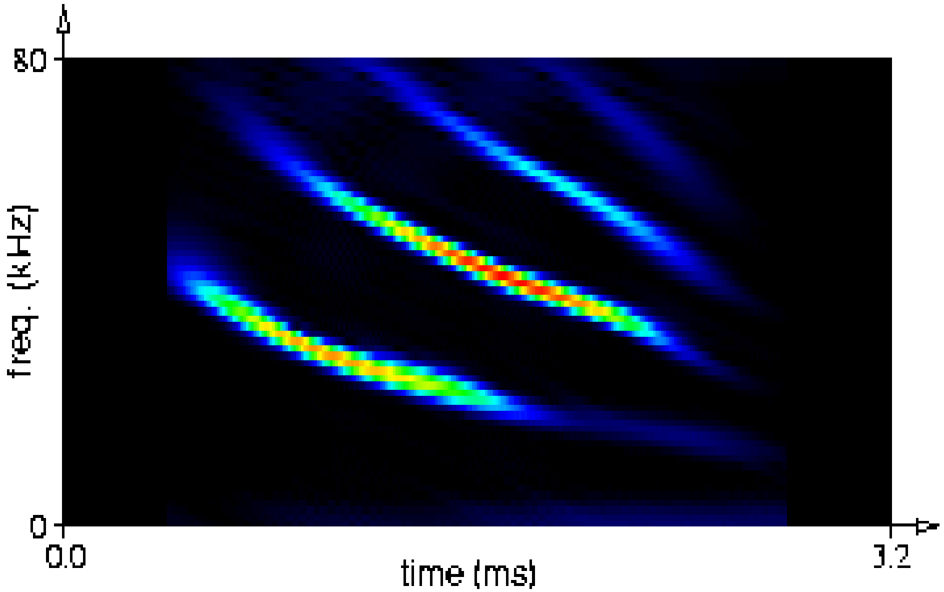
Bowhead whale vocalisation

Signals and Waveforms

Examples of Biological signals



2.5 ms acoustic burst



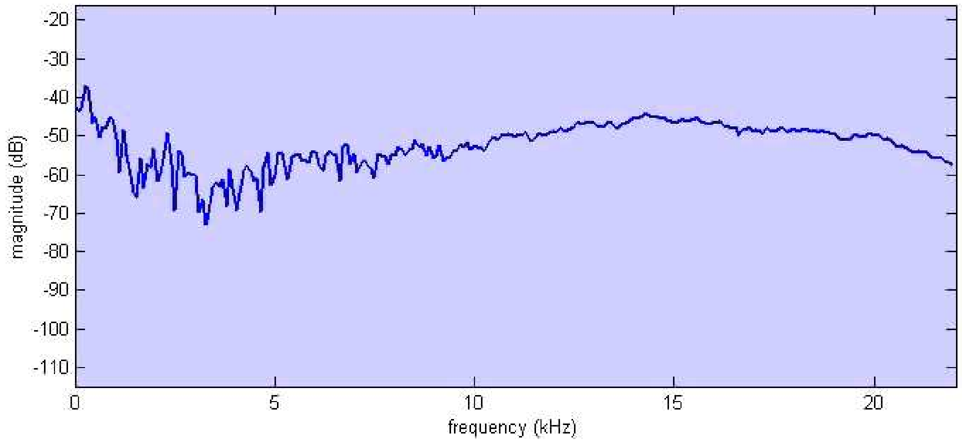
Bat chirp: frequency versus time

Big brown bat (Eptesicus fuscus)

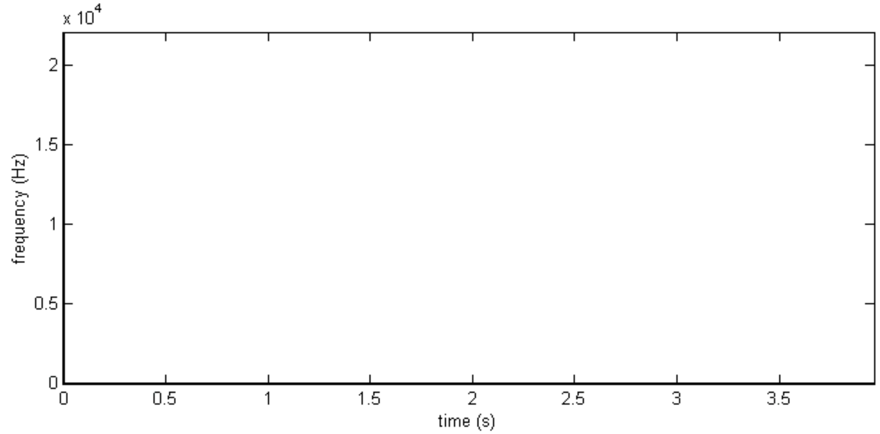
Signals and Waveforms

Time and Frequency

Amplitude vs time



Amplitude vs frequency as a function of time



Frequency versus time

Signals and Waveforms

Signals

Effective interpretation of a signal requires careful judgement and a knowledge of the various signal-processing techniques that might be applied to reveal its information content.

It may be decided for example to rid a signal of short-term fluctuations which obscure its underlying long-term behaviour.

A more complex series of operations could be defined to separate a wanted signal from an instructive interfering signal.

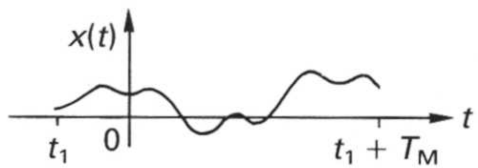
Further processing might then be applied to determine the peak value, average value, average power, rate of change or any other aspect of a signal that may be considered significant and yet might not be apparent on first inspection.

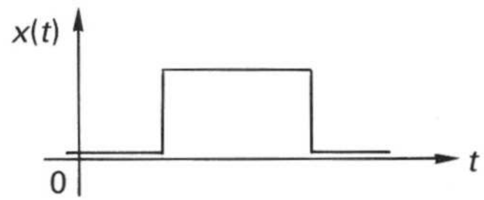
Signals and Waveforms

Signals: continuous-time

A continuous-time signal can be represented mathematically as a function of a continuous time variable. The graph of a continuous-time signal x (t) is thus defined at each and every instant over a measurement interval extending from $t = t_1$ to $t = t_1 + T_M$

Most naturally occurring signals, such as speech and music signals, fall into this category and are known as continuous-time analogue signals.





continuous-time analogue signals

Signals and Waveforms

Periodic and aperiodic signals

The values of a time-dependent ***periodic*** signal repeat every T_0 seconds where T_0 is known as the period of the signal. Many signals of natural origin can be modelled as having periodic behaviour and it is also common practice to generate periodic signals in the laboratory for the purpose of system testing and evaluation.

A mathematical model describing the waveform of a continuous-time periodic signal has the property:

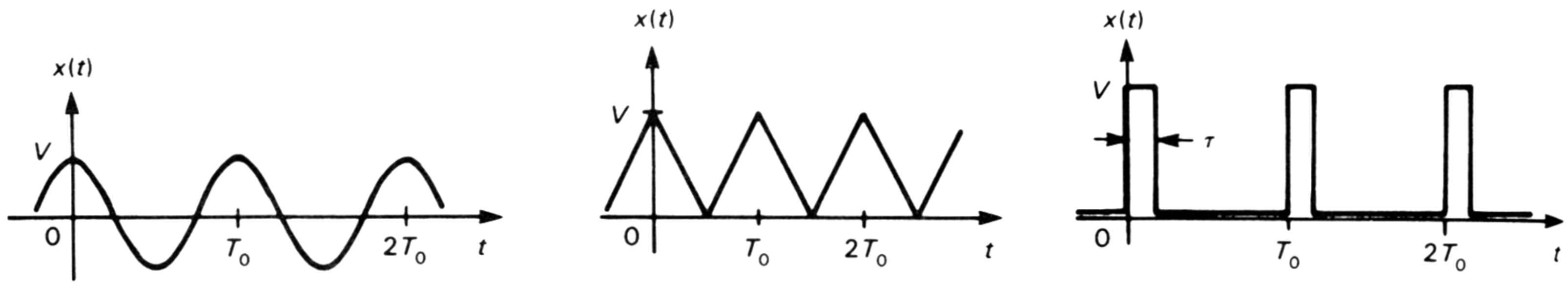
$$x(t) = x(t + T_0)$$

for all time (t).

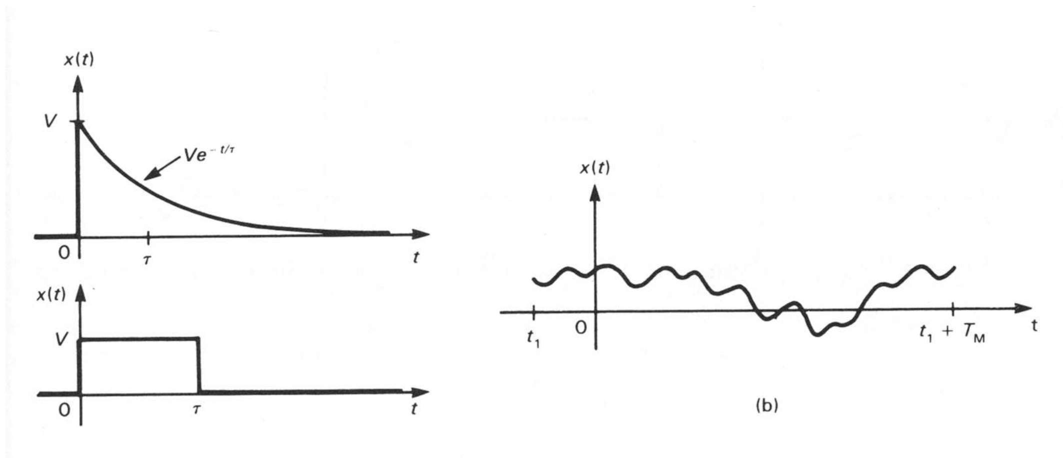
If a signal is ***aperiodic*** however, its waveform will not repeat within the measurement interval and fails to do so even if the interval is made arbitrarily long.

Signals and Waveforms

Periodic and aperiodic signals



(a) Periodic signal



(b) Aperiodic signal

A study of the properties of fluctuating signals and, indeed, all signals with a random content brings us into the area of probability and statistics.

Signals and Waveforms

Average value

The graph of a time-dependent signal varies above and below a well-defined level known as the average or mean value x_0. For a continuous-time signal x (t) recorded over an interval $t = t_1$ to $t = t_1 + T_M$ the average value is given by the signal processing operation:

$$\boxed{x_0 = \frac{1}{T_M}\int_{t_1}^{t_1+T_M} x(t)\,dt.}$$

Example 1

Calculate the average value of the sinusoidal voltage over all time.

A sinusoidal function has a period given by $T = \dfrac{2\pi}{\omega}$

Signals and Waveforms

$$x_0 = \frac{1}{T_M}\int_{t_1}^{t_1+T_M} x(t)dt.$$

Average value

Example 1

Calculate the average value of the sinusoidal voltage over all time.

A sinusoidal function has a period given by T = $\dfrac{2\pi}{\omega}$

The sinusoidal waveform is periodic so we only need to calculate the average over one period.

$$V_{av} = \frac{1}{T}\int_0^T V\sin\omega.t.dt = \left[\frac{-V}{T}.\frac{1}{\omega}.\cos\omega t\right]_0^T = \left[\frac{-V}{T}.\frac{T}{2\pi}.\cos\frac{2\pi}{T}t\right]_0^T = zero$$

This is as expected as for a sine function, during one period, the positive area above the axis is equal to the negative area below the axis. What it does show is that average may not necessarily be a very useful quantity measurement.

Signals and Waveforms

Average value

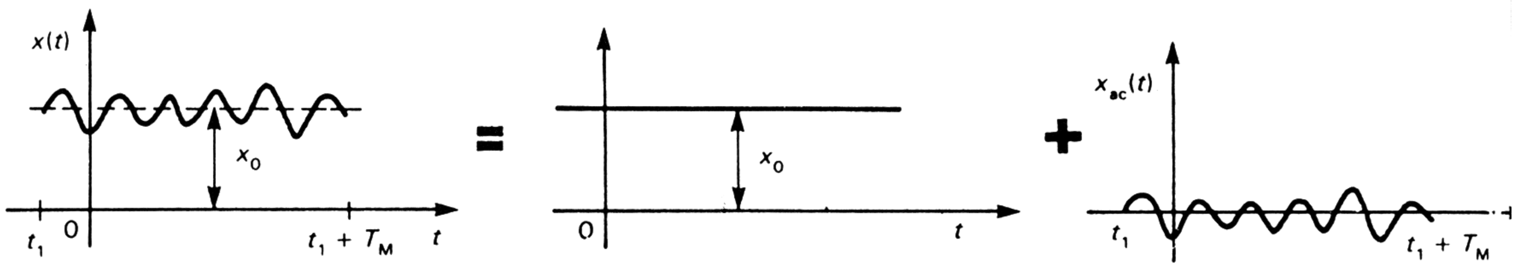
$$x_0 = \frac{1}{T_M} \int_{t_1}^{t_1+T_M} x(t)dt.$$

If we extract the average value from a signal, then we are left with a time-varying signal

x_{ac} (t) defined by:

$$x_{ac}(t) = x(t) - x_0.$$

In this context, we call x_0 the direct component and x_{ac} (t) the alternating component of the signal. In principal, any signal record of finite length can be represented as the sum of direct and alternating components.



Representation of a signal x (t) as a sum of direct and alternating components.

Signals and Waveforms

Energy signals and power signals

We define an energy signal to be one for which the total energy is finite. This is given the value given by the integral:

$$\boxed{E_{tot} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

The energy in a signal is thus defined to be the area contained by the graph of the ***squared magnitude*** of the signal and the time axis and is a wholly positive quantity. Pulse-like signals that have a finite duration or decay exponentially to zero have finite energy.

Signals and Waveforms

Energy signals and power signals

We define an energy signal to be one for which the total energy is finite. This is given the value given by the integral:

$$\boxed{E_{tot} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

Example 2

What is the total energy associated with the *exponential decaying pulse*:

$$v(t) = Ve^{\frac{-t}{\tau}} \; for \; t \geq 0$$

$$v(t) = 0 \quad for \; t < 0$$

Energy signals and power signals

Example 2

What is the total energy associated with the *exponential decaying pulse*:

$$E_{tot} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$v(t) = Ve^{\frac{-t}{\tau}} \; for \; t \geq 0$$

$$v(t) = 0 \quad for \; t < 0$$

$$= \int_{0}^{\infty} e^{-2t/\tau} dt = \frac{V^2 \tau}{2} ...... finite \;\; energy$$

Signals and Waveforms

Energy signals and power signals

If we attempt to evaluate: $\boxed{E_{tot} = \int_{-\infty}^{\infty} |x(t)|^2 dt}$ for a periodic signal or for any

other signal that does not die away with time, the energy accumulates as longer and longer stretches of signal are taken into account and, in the limit, the *total energy is infinite*.

For a voltage signal v (t), developed across a resistance *R*, the instantaneous power is defined to be the quantity:

$$P(t) = \frac{|v(t)|^2}{R} \quad Watts$$

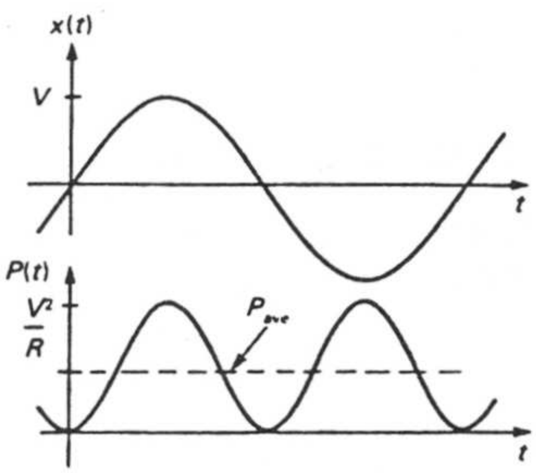
Signals and Waveforms

Energy signals and power signals

For a voltage signal v (t), developed across a resistance *R*, *the instantaneous power* is defined to be the quantity:

$$P(t) = \frac{|v(t)|^2}{R} \quad Watts$$

The graph of instantaneous power is everywhere positive and it can be averaged over any time interval of interest to give the average power delivered to the load resistance:



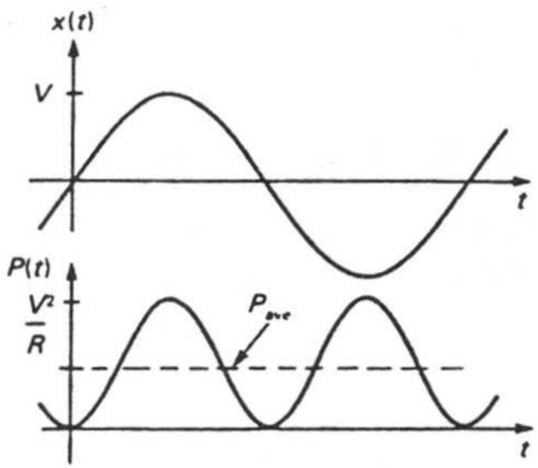
$$P_{av.} = \frac{1}{T_M} \int_{t_1}^{t_1+T_M} P(t)dt = \frac{1}{T_M} \int_{t_1}^{t_1+T_M} \frac{|v(t)|^2}{R} dt.$$

Average power for a sine wave

Signals and Waveforms

Energy signals and power signals

$$P(t) = \frac{|v(t)|^2}{R} \quad Watts$$



Average power for a sine wave

$$\boxed{P_{av.} = \frac{1}{T_M}\int_{t_1}^{t_1+T_M} P(t)dt = \frac{1}{T_M}\int_{t_1}^{t_1+T_M} \frac{|v(t)|^2}{R} dt.}$$

Signals which do not die away with time are known as *power signals because their average power is finite* even for very long measurement intervals. In particular, for periodic signals, the average power may be found by averaging P (t) over a very long interval containing many periods of the signal by taking an average over a single period. For a periodic signal with period To therefore:

$$\boxed{P_{av.} = \frac{1}{T_o}\int_{t_1}^{t_1+T_o} \frac{|v(t)|^2}{R} dt.}$$

Signals and Waveforms

$$P_{av.} = \frac{1}{T_o} \int_{t_1}^{t_1+T_o} \frac{|v(t)|^2}{R} dt.$$

Energy signals and power signals

Example 3

Find the average power associated with the sinusoidal signal v (t) = V cos ωt.

Signals and Waveforms

$$P_{av.} = \frac{1}{T_o} \int_{t_1}^{t_1+T_o} \frac{|v(t)|^2}{R} \, dt.$$

Energy signals and power signals

Example 3

Find the average power associated with the sinusoidal signal v (t) = V cos ωt.

$$P(t) = \frac{|V \cos \omega t|^2}{R}$$

Using the trigonometric identity: cos 2A = (1 + cos 2A)

The instantaneous power is given by: $P(t) = \frac{|V \cos \omega t|^2}{R} = \frac{V^2}{2R} + \frac{V^2}{2R} \cos 2\omega t.$

Signals and Waveforms

Energy signals and power signals

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The instantaneous power is given by:

$$P(t) = \frac{|V \cos \omega t|^2}{R} = \frac{V^2}{2R} + \frac{V^2}{2R} \cos 2\omega t.$$

If we average P (t) over any interval of length $T_0 = 2\pi/\omega$, the time-dependent component averages to *zero* and we obtain the average power $V^2 / 2R$ Watts.

Now, the usual convention in signal analysis is to calculate power on the assumption of a 1Ω resistive load. On this basis, the average power of the sinusoid becomes $V^2 /2$ which is identical to the ***mean-square value*** obtained by averaging the square of the signal.

Signals and Waveforms

The root mean square (rms.) value of a waveform

As we have seen when dealing with the sinusoid taking the average value is of little significance.

We also saw in that the average power of the sinusoid was achieved by considering the mean-square value of the signal.

In ac work it is ***power*** that matters and not just for the sinusoid, but for ***all*** waveform types.

Signals and Waveforms

The root mean square (rms.) value of a waveform

Consider the following definition:

A current *I* (t) in a pure resistor *R* results in a power *p (t)* with an average value *P*. This same *P* could be produced in *R* by a constant current *I*.

Then ***I*** **(t)** is said to have an effective value I_{RMS} equivalent to this constant current ***I***.

The same applies to voltage functions where the effective value is V_{RMS}.

The general function y (t) with period T has an effective value Y_{RMS} given by:

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2 .dt}$$

Signals and Waveforms

The root mean square (rms.) value of a waveform

Example 4

A sinusoid $v(t) = V \cos \omega t$ is applied to a resistor R ohms. Calculate the rms. value of $v(t)$ and the power delivered to the resistance.

The root mean square (rms.) value of a waveform

Example 4

A sinusoid v (t) = V cos ωt is applied to a resistor R ohms. Calculate the rms. value of v (t) and the power delivered to the resistance.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 .dt}$$

Signals and Waveforms

The root mean square (rms.) value of a waveform

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$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 .dt}$$

$$\therefore V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 \cos^2 \omega t .dt} \qquad = \sqrt{\frac{1}{T}\int_0^T \frac{V^2}{2}(1-\cos 2\omega t).dt}$$

The root mean square (rms.) value of a waveform

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Now remember that the average value of cos A over one period is zero.

Thus: $\boxed{v_{rms} = \frac{V}{\sqrt{2}} \approx 0.707\ \ volts}$ $\boxed{\therefore\ Power = \frac{V_{rms}^2}{R} = \frac{V^2}{2R}\ \ Watts}$

Signals and Waveforms

The root mean square (rms.) value of a waveform

$$\boxed{v_{rms} = \frac{V}{\sqrt{2}} \approx 0.707 \;\; volts}$$

$$\boxed{\therefore \; Power = \frac{V_{rms}^2}{R} = \frac{V^2}{2R} \;\; Watts}$$

Now it is useful to know the relationship between V (peak) and V $_{rms}$ for a ***sinusoid*** is a √ 2 relationship, but this is ***only*** true for a perfect sinusoid.

Supposing the waveform is not sinusoidal - then you simply have to calculate:

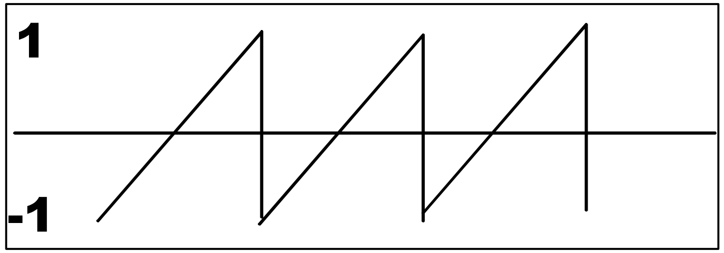
$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2 .dt}$$

Signals and Waveforms

The root mean square (rms.) value of a waveform

Example

A Sawtooth Wave y (t) = $\frac{2}{T}t$



Sawtooth

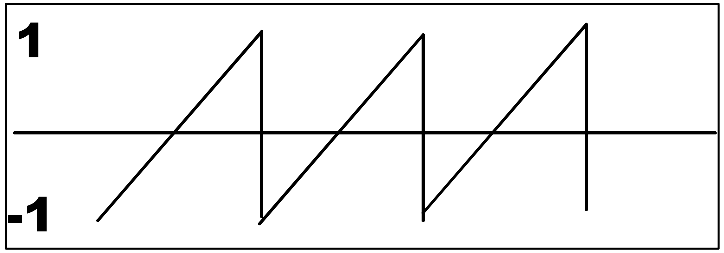
Signals and Waveforms

The root mean square (rms.) value of a waveform

Example

A Sawtooth Wave y (t) = $\dfrac{2}{T}t$

Squaring this gives: $$y^2 = \frac{4}{T^2}t^2$$



Sawtooth

Signals and Waveforms

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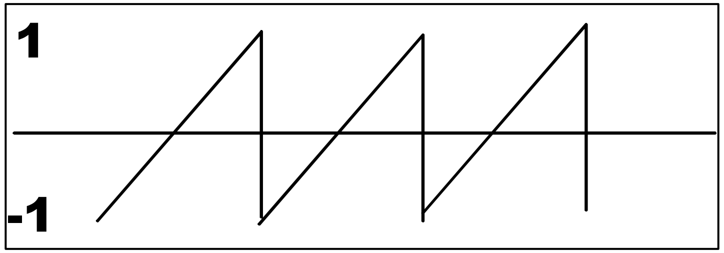


Sawtooth

Squaring this gives: $y^2 = \frac{4}{T^2}t^2$

Integrating this squared function gives:

$$Average = \frac{2}{T}\int_0^{\frac{T}{2}} \frac{4}{T^2}t^2\ dt = \frac{2}{T}\left[\frac{4}{3T^2}\right]_0^{\frac{T}{2}} = \frac{1}{3}$$

rms value is $\frac{1}{\sqrt{3}}$ times the peak value for a sawtooth

The root mean square (rms.) value of a waveform

Example 5

A current having the following steady state values in amperes for equal intervals of time changing instantaneously from one value to the next:

0, 10, 20, 30, 20, 10, 0, -10, -20, -30, -20, -10, 0 etc.

Calculate the average and rms values for this current waveform.

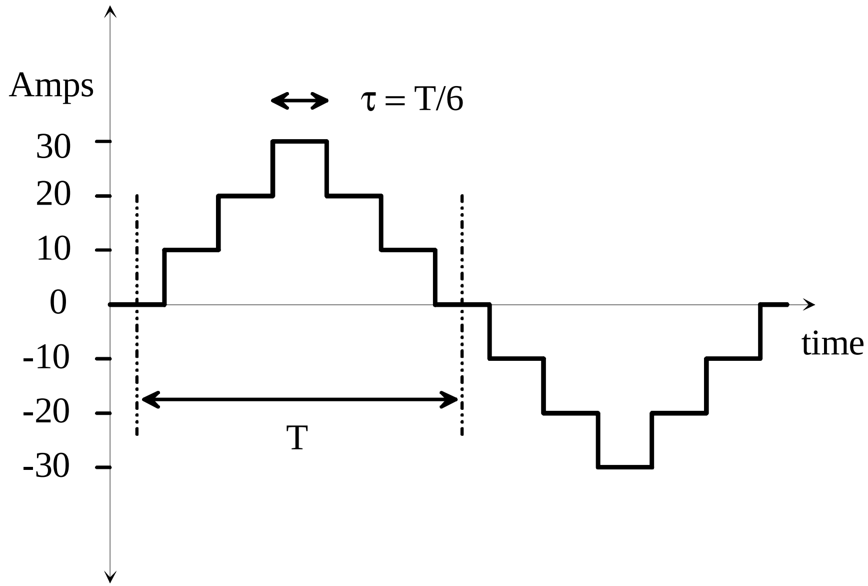
The root mean square (rms.) value of a waveform

Example 5

Calculate the average and rms values for this current waveform.

(a) Average value

This is given by: $$x_0 = \frac{1}{T_M} \int_{t_1}^{t_1+T_M} x(t)dt.$$



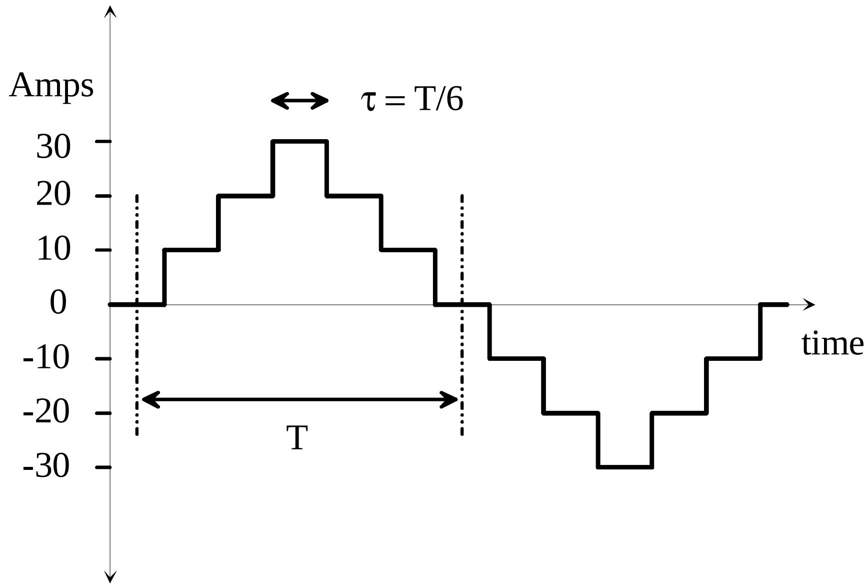
The root mean square (rms.) value of a waveform

Example 5

Calculate the average and rms values for this current waveform.

(a) Average value

This is given by: $$x_0 = \frac{1}{T_M}\int_{t_1}^{t_1+T_M} x(t)dt.$$



For a full cycle the average value is obviously zero.

For a half cycle the average is =

$$\frac{0+10+20+30+20+10}{6} = \frac{90}{6} = 15\ A$$

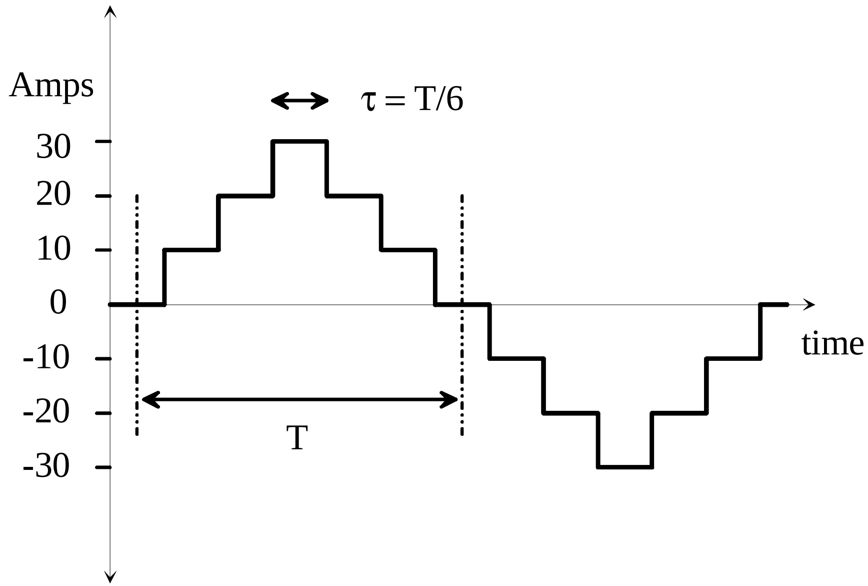
Signals and Waveforms

The root mean square (rms.) value of a waveform

Example 5

Calculate the average and rms values for this current waveform.

(b) rms value $= \sqrt{\frac{1}{T}\int_0^T y(t)^2 .dt}$



Signals and Waveforms

The root mean square (rms.) value of a waveform

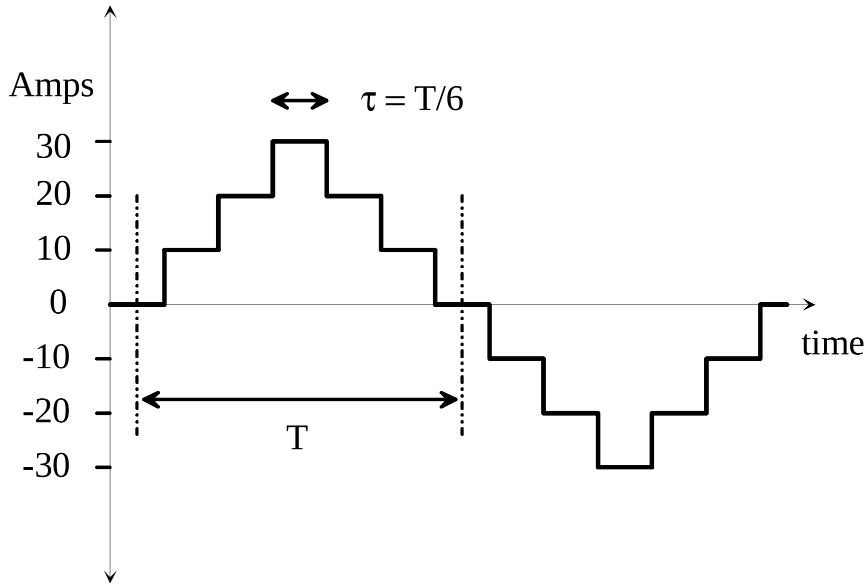
Example 5

Calculate the average and rms values for this current waveform.

(b) rms value $= \sqrt{\frac{1}{T}\int_0^T y(t)^2.dt}$

$$(rms)^2 = \frac{0^2 + 10^2 + 20^2 + 30^2 + 20^2 + 10^2}{6} = 316$$

So rms value = √316 = 17.8 A



Due to symmetry you only need to take one-half of the waveform

END

Signals and Waveforms

The rest is up to you!