Alternating Current and Voltage

Alternating Current and Voltage

Introduction

The simple world of dc circuits leads to very narrow-minded understanding of circuit theory and the power of electrical engineering.

In reality the word is an ***ac world*** and only when we consider alternating waveforms will electrical engineering come alive!

Alternating Current and Voltage

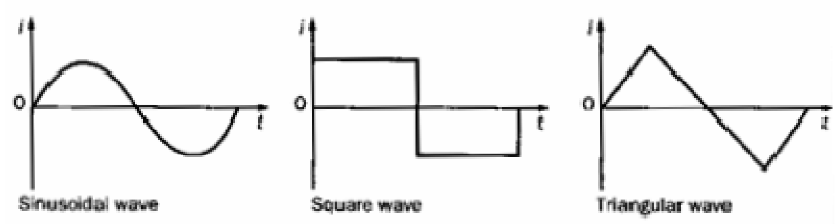
Waveforms

If a current is varied in a repetitive manner then it is known as an alternating current, commonly abbreviated to ac. Current flows first in one direction and then in the other (electrons moving in one direction and then in the other), and the cycle of variation is exactly the same for each direction.

The curves relating current to time are known as ***waveforms***.

These can be of a variety of shapes as illustrated or even more complicated.

To avoid dc the shape above the zero line should be identical to that below the zero line.



Alternating Current and Voltage

The sinusoid

The most common and most useful form is the sinusoid as:

- This can be easily generated
- It can form other interesting waveforms
- It is a very naturally occurring shape
- It is mathematically simple

As ac waveforms are so universal we must learn how to define the terms before we can begin to analyse circuits operating with ac rather than dc.

Alternating Current and Voltage

Generating ac

Before looking at the generation of ac waveforms we should remind ourselves of a basic principle.

This was the relationship: $E = Blu$

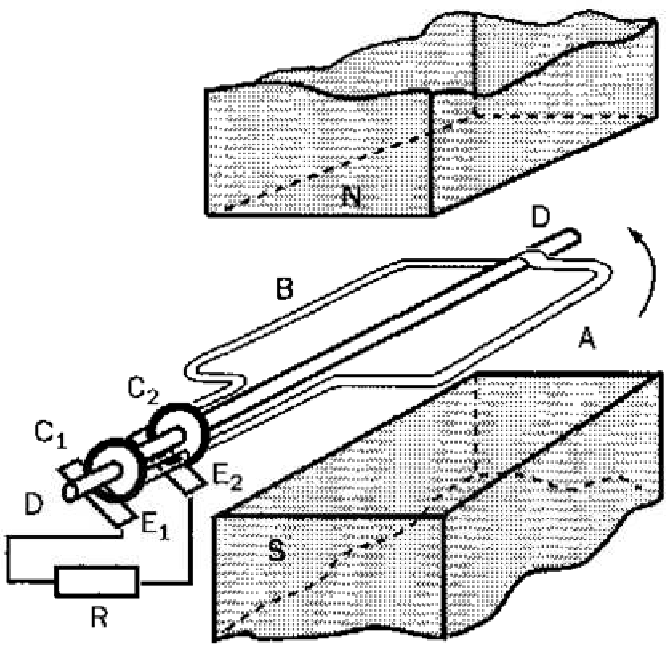
where E is the voltage generated by a wire, length l (m) passing through a magnetic field of strength B (teslas) at a velocity u (m/sec)

Alternating Current and Voltage

Generating ac

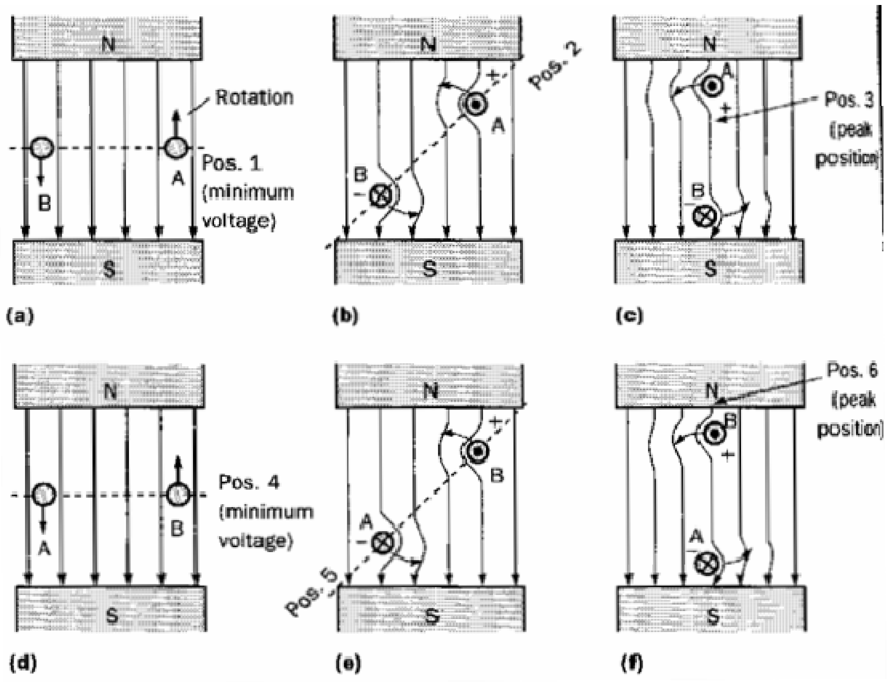
If we now consider a loop of conductor, AB, carried on a spindle, DD, rotated at a constant speed in an anticlockwise direction in a uniform magnetic field created by the poles of a magnet as shown:

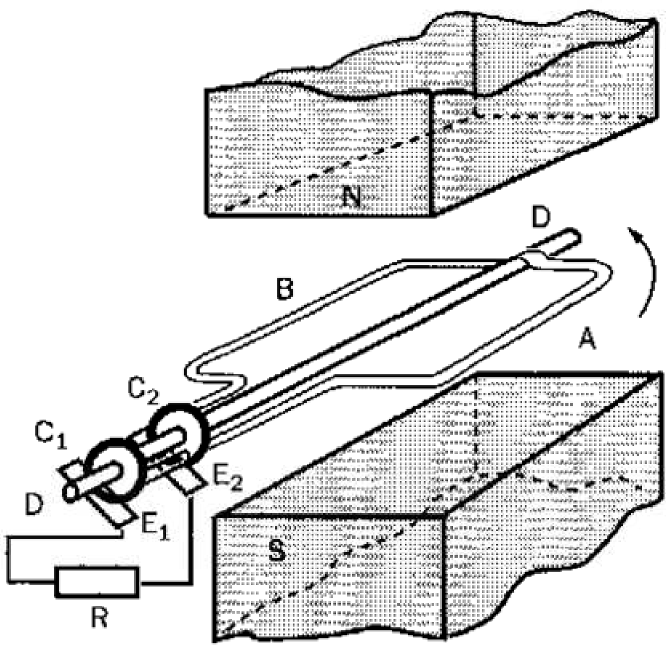
We may observe that when the loop is horizontal, no flux is being cut and no emf can be generated. Whereas, if the loop is vertical, then flux is being cut at a maximum rate. At any angle in between these limits, the flux is being cut at some rate less than the maximum but more than zero. It is not difficult to imagine that the signal that will emerge will be sinusoidal.



Alternating Current and Voltage

Generating ac



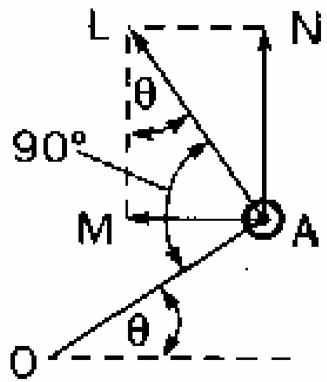


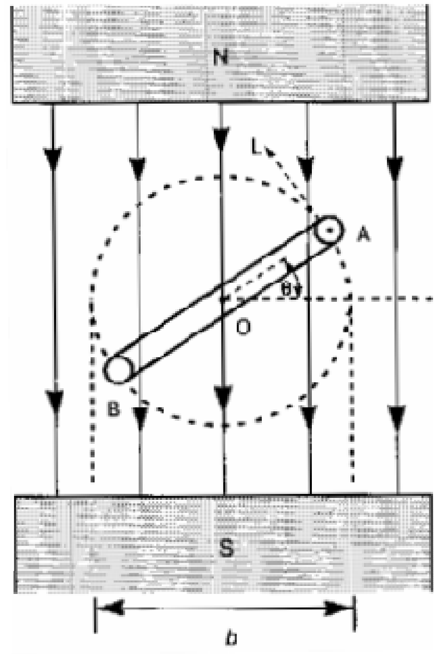
Emf in a rotating coil

Alternating Current and Voltage

Generating ac

Taking the A part of the loop at an angle θ to the horizontal, we can see that if AL represents velocity u, then the horizontal component of this at angle θ is AM = AL sin θ.





Instantaneous value

of generated emf

Alternating Current and Voltage

Generating ac

Going back to the equation, $E = Blu$, we now have the emf generated by one side of the loop as: $Blu \sin \theta$ volts

For both halves of the loop this doubles to: $e = 2Blu \sin \theta$ volts

The maximum value of voltage generated is when the sine function is unity and gives: $E_m = 2Blu$ volts

If the loop has a breadth of b meters and has N turns and the rotational rate is n revs/s, then the circumference of the circle the loop makes is πb meters and this gives a speed of (πbn) meters/sec so,

$$e = 2Bl(\pi bn) \sin \theta \text{ volts and } E_{max} = 2Bl\pi bn \text{ volts}$$

Noting the area of the loop to be $A = bl$ gives for an N turn coil:

$e = 2\pi BAnN \sin \theta$ volts and $E_m = 2\pi BAnN$ volts

Alternating Current and Voltage

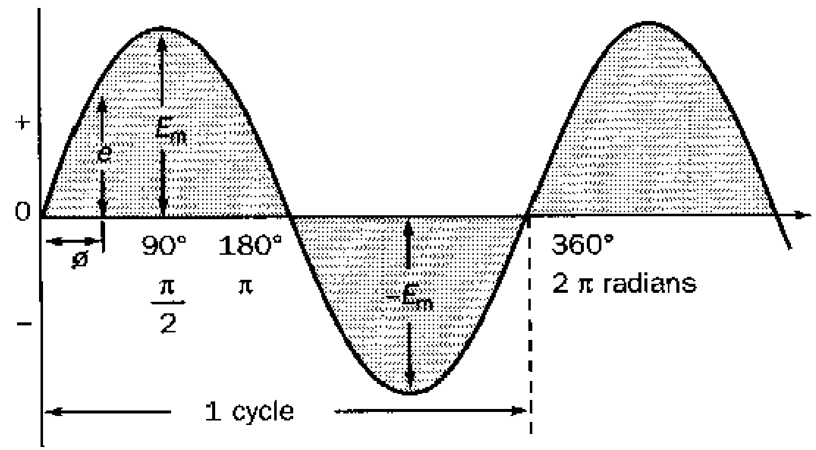
Waveform Terms and Definitions

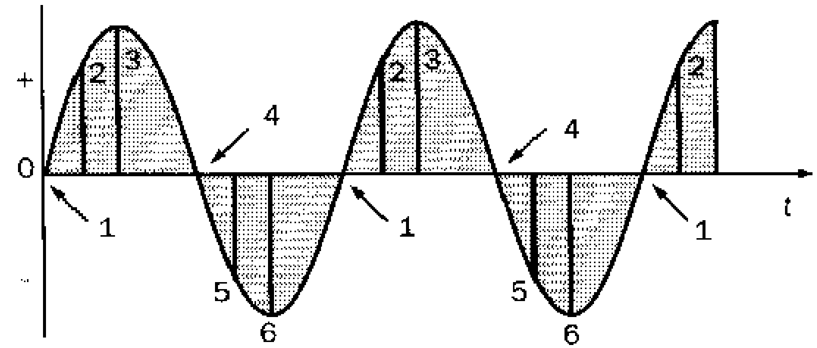
- *Waveform*: The variation of voltage or current against time as a graph
- *Cycle*: A complete repetition of the waveform assuming periodicity
- *Period*: The duration of one cycle of the waveform
- *Instantaneous value*: The magnitude at any given instant. Positive or negative.
- *Peak Value*: The maximum value the function can reach
- *Peak-to-peak*: The measure of the range between the maximum and minimum values the function reaches
- *Peak Amplitude*: The maximum instantaneous value measured from the mean value of the waveform
- *Frequency* : (Hertz (Hz)) This is the inverse of the period. If a waveform has a frequency of 50Hz the period is 1/50 = 20ms.
  In other words the waveform repeats every 20ms

Alternating Current and Voltage

Waveform Terms and Definitions

Sine wave





Alternating Current and Voltage

Example1

(a) A coil is wound with 200 turns on a square former with sides 60mm in length. Calculate the maximum value of the emf generated in the coil when it is rotated at 2500r/min in a uniform magnetic field of density 0.9 T.
(b) What is the frequency of this emf?

Solution in Hughes.

2EO1 Circuits and Modelling 2

AC THEORY

2EO1 Circuits and Modelling 2

So far you have studied only the "dc world" where:

•Series and parallel resistive (R) circuits.

•Network theorems: Thevenin and Norton

•Introduced to the Inductor (L) and the Capacitor (C)

•Transients…. (an introduction to temporal variability)

All of these subjects constitute the "fundamentals" on which we can develop the skills required to understand electrical engineering.

BUT batteries (dc) and series/parallel resistive circuits are only a lead in to the "real" world of the electrical engineering. It is not until we introduce time-varying waveforms that electrical engineering will come alive!

2EO1 Circuits and Modelling 2

“Circuits and Modelling” is about how the three passive components:

Resistor (R) the Capacitor (L) and the Inductor (C)

take on their true significance when they are excited by time varying waveforms.

To (ac) the Capacitor and the Inductor become “frequency dependant”.

When circuits can obtain a frequency dependence then we have the basis for radio, communications, mobile phones, computers – indeed every but of electrical engineering you can think of – full stop!

But all of (ac) is based on some very basic (but essential) concepts.

2EO1 Circuits and Modelling 2

The maths you already have → complex numbers.

$$(e^{j\Theta} = \cos(\Theta) + j \sin(\Theta))$$

The waveforms: well the building blocks are just the sine and the cosine.

$$(\sin(\omega t) \text{ and } \cos(\omega t))$$

This bit of the course is about taking you through the essential steps to set you up to "understand" electrical engineering.

And it has to be understood. You can struggle through this course just doing memory work. But if you try to follow what we say to you over the next weeks we hope you will have a depth of knowledge that will be able to take with you on your future careers.

For the EE among you this is what (I hope) you have come to university to study!

Sinusoidal waveforms and the phasor

The Sine and cosine waveforms are universally accepted as the fundamental alternating waveforms associated with all aspects of electrical engineering theory.

• sinusoids are readily handled mathematically (trigonometric formulae)

• through the application of Fourier techniques all waveform shapes can be considered as composed of summations of sinusoids of varying frequency, amplitude and phase

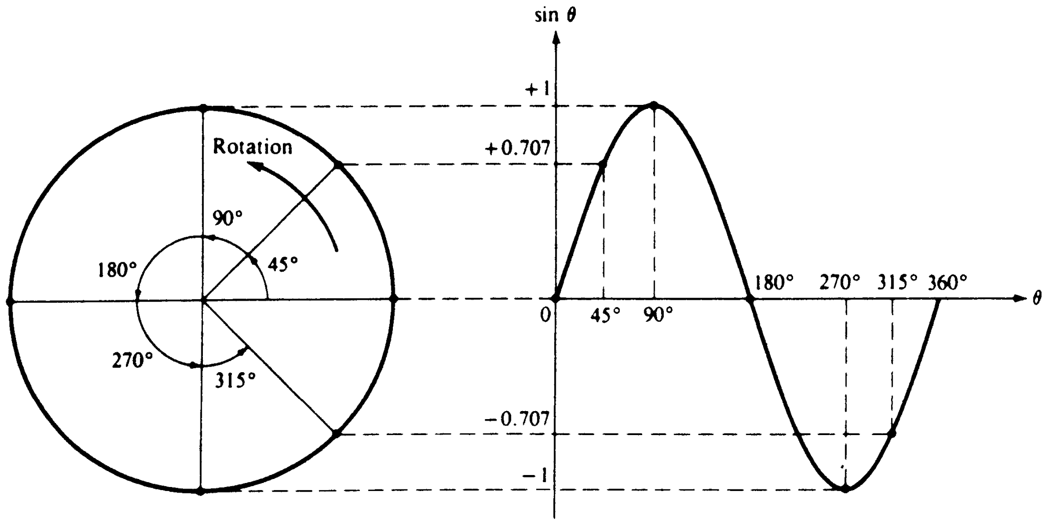
• the natural response of many electrical and non-electrical circuits is to generate a sinusoidal responses

Sinusoidal waveforms and the phasor

• The sinusoidal and co sinusoidal waveforms can be best understood by considering them as being represented by the horizontal and vertical projection of a rotating phasor.

• Imagine a line rotating in an anticlockwise direction, so that its tip traces out a circle.

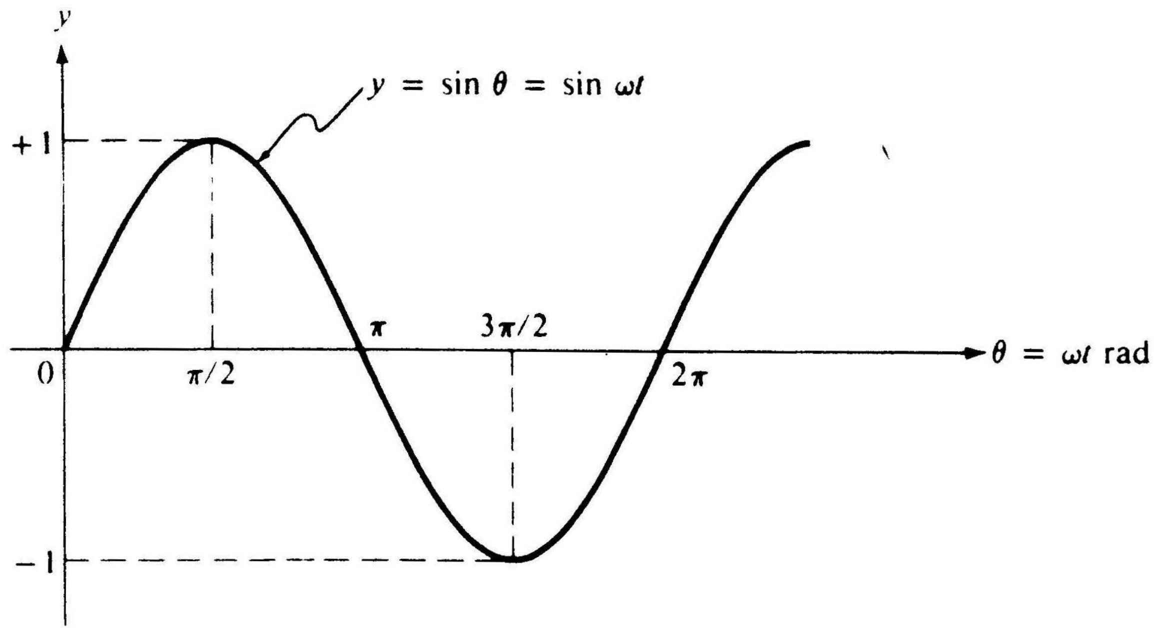
• The resultant waveform is the plot sin (ϴ) versus ϴ.



Sinusoidal waveforms and the phasor

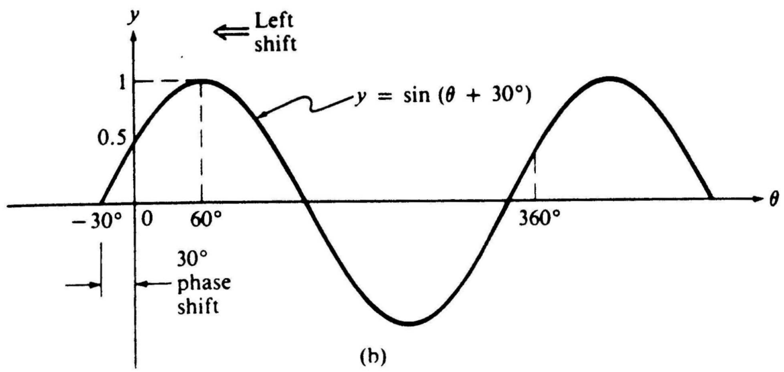
A sine wave can be expressed as a function of time by writing

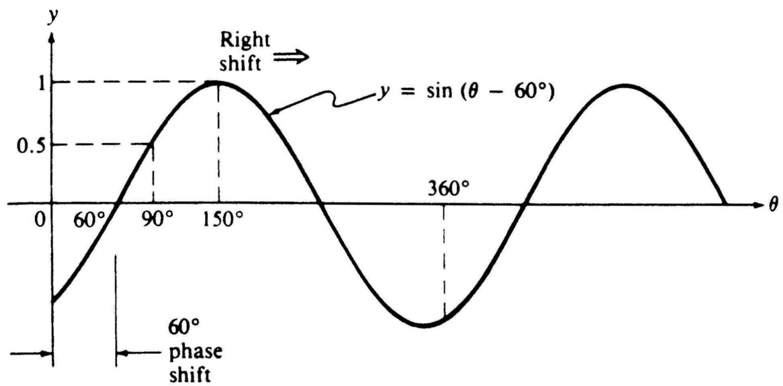
$$\sin \Theta = \sin \omega t$$



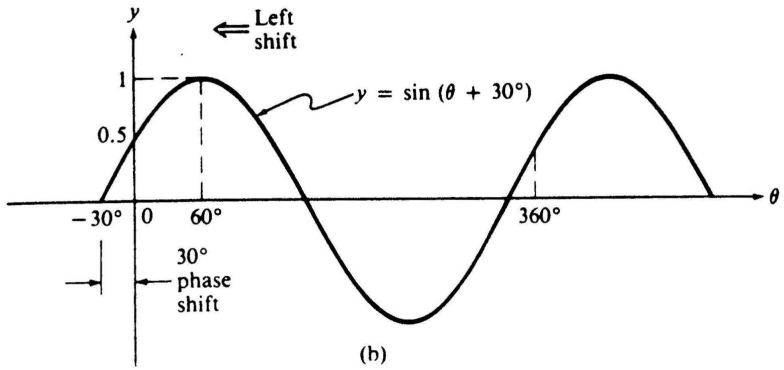
Sinusoidal waveforms and the phasor

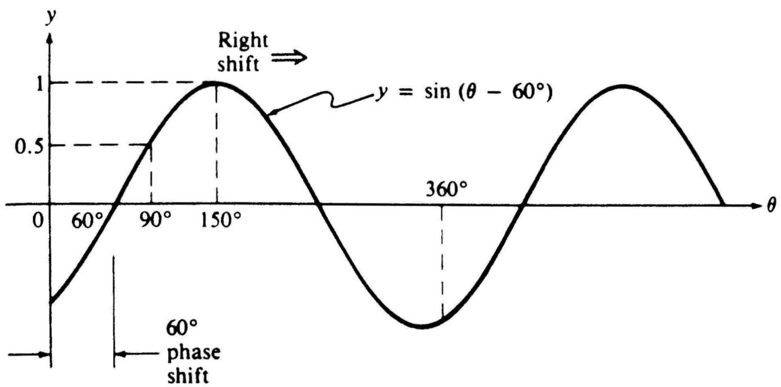
A phase angle Φ can be added to the variable ϴ or ωt to cause the sine wave to shift to the left along the horizontal axis for positive phase angles. A negative phase angle causes the sine wave to shift to the right.





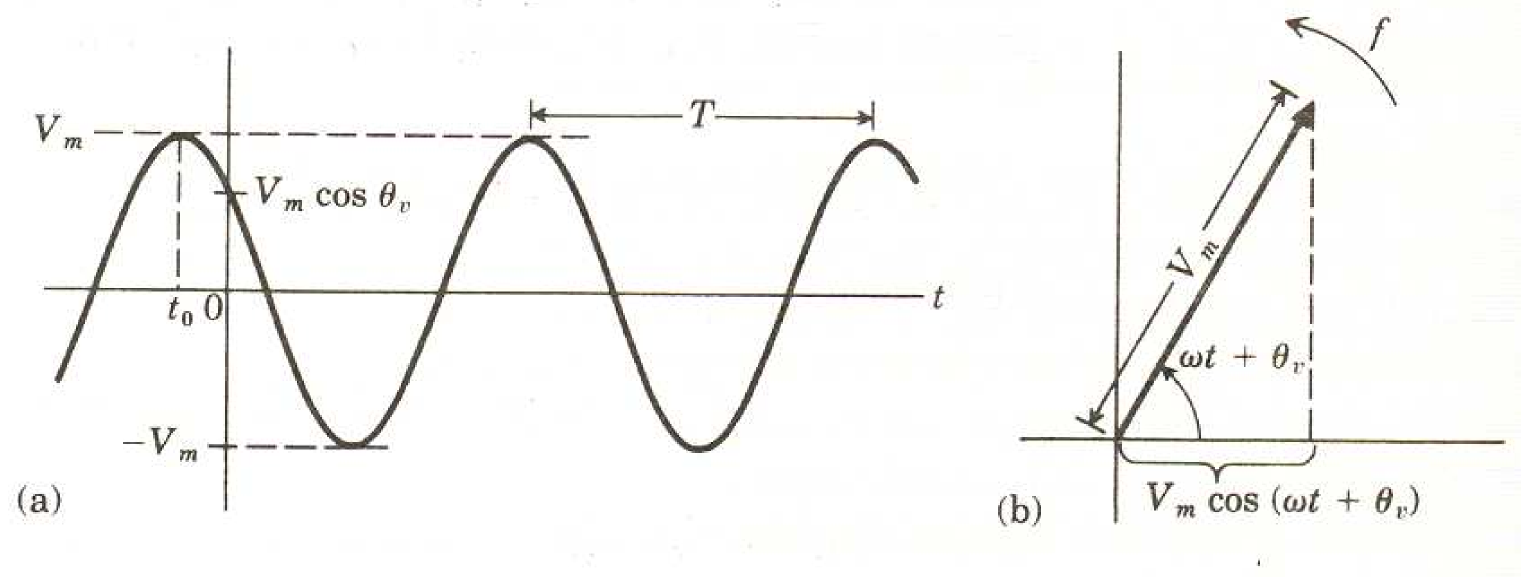
Sinusoidal waveforms and the phasor





This PHASE effect – the phase difference between two sinusoidal (co sinusoidal) waveforms is fundamental to all we will do in Electrical Engineering.

Sinusoidal waveforms and the phasor



Sinusoidal waveforms and the phasor

The sine wave is an example of a periodic waveform.

If the value of a periodic sine wave is f (t_1) at time t_1 and is similar at times

$(t_1 + nT)$, where n is an integer, then T is known as the period or periodic time of the function.

The frequency, f, of an alternating waveform is the number of cycles that occur in 1 s. Frequency is inversely proportional to period. Thus $f = 1/T$. The units of frequency are cycles per second or in SI units Hz.

One cycle is the same as 2π radians. The number of radians produced in 1 s is (2π) times (f), or $\omega = 2\pi f$ radians per second

ω is the angular frequency.

Now sin ωt is sine (angle) and angle has units of radians. Remember to set your calculator to radian measurement!

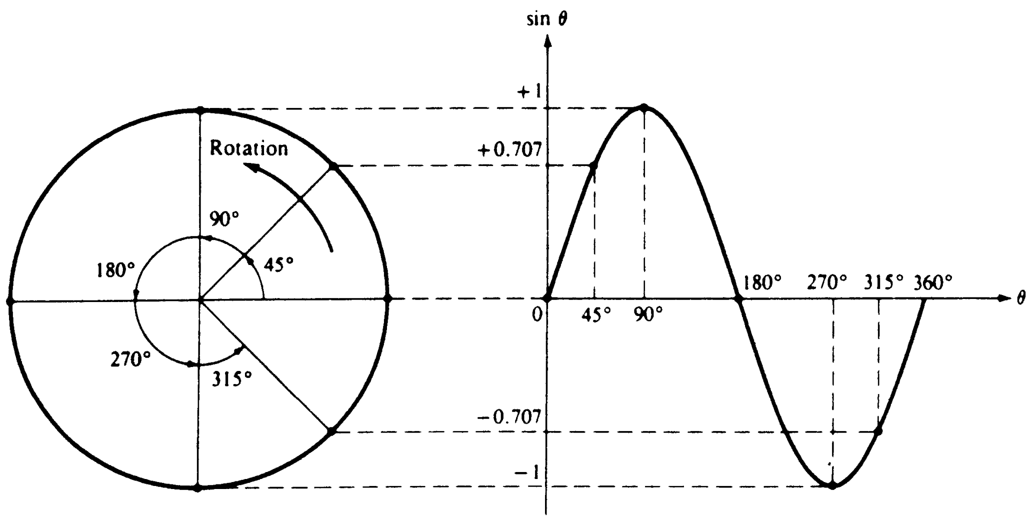
Sinusoidal waveforms and the phasor

The sinusoidal wave, V sin ($\omega t + \Phi$) volts is described by the three variables:

(a) Amplitude: V is Volts peak

(b) Frequency: ω is in rads^{-1}

(c) Phase: Φ is in units of radians



V sin ($\omega t + \Phi$) volts

Sinusoidal waveforms and the phasor

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Thus $10 \sin(100\pi t + 45^{\circ})$ volts is a 50 Hz signal with a phase of 45° at time $t = 0$

However please note that the equation is dimensionally **incorrect**. X

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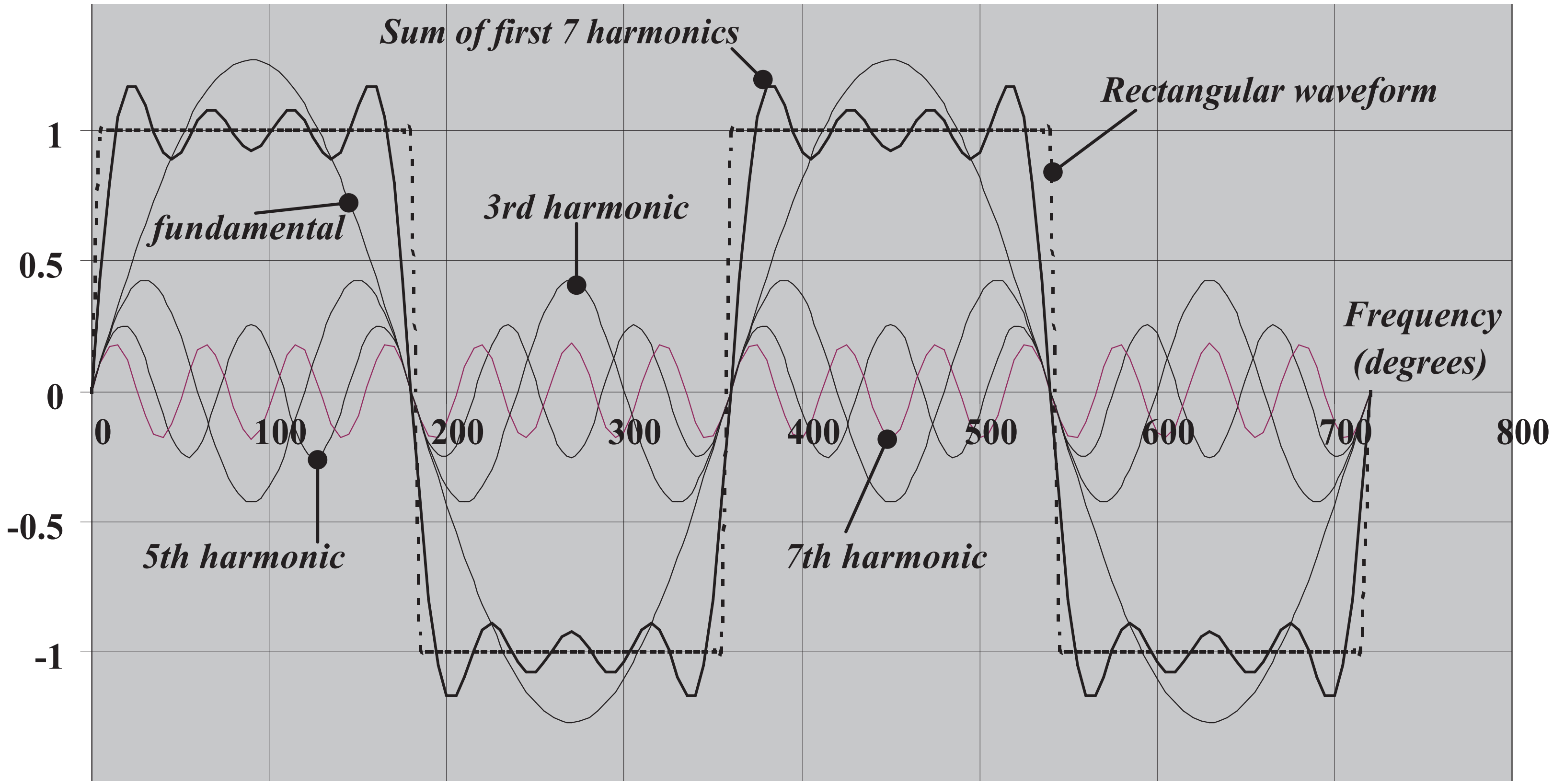
However please note that the equation is dimensionally **incorrect.** X

Phase must be in radians.

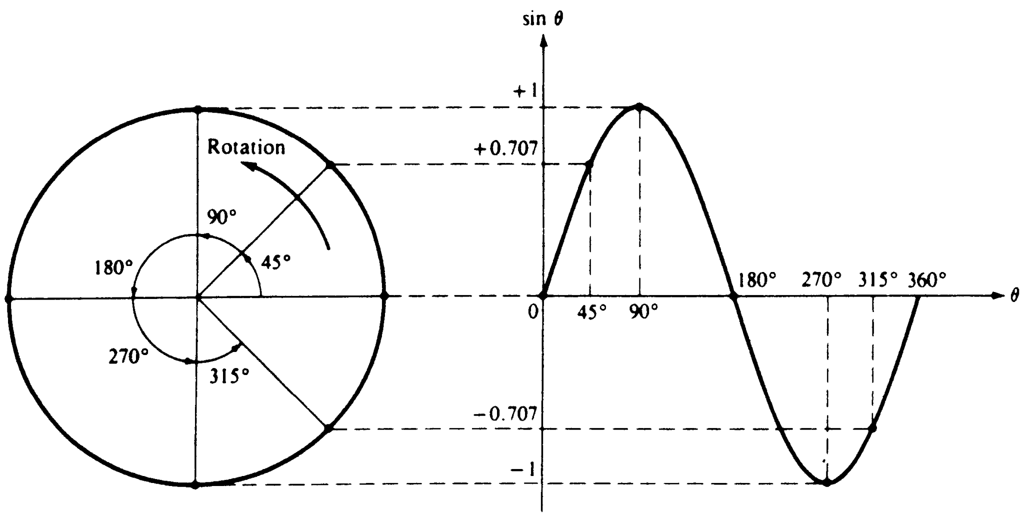
So take care that you use your calculator set to the correct units!

Sinusoidal waveforms and the phasor

"All waveform shapes can be considered as composed of summations of sinusoids of varying frequency, amplitude and phase."



Sinusoidal waveforms and the phasor: Summary



The horizontal projection of a rotating line or *phasor*

The phasor is a convenient mathematical model for sinusoidal alternating waveforms and can be represented as

$$v\ (t) = V \text{ exp. } j\ (\omega t + \Phi)$$

where V is the peak voltage, ω is the angular frequency (rads^{-1}) and Φ is some reference phase angle.

Sinusoidal waveforms and the phasor: Summary

$$v\,(t) = V \exp.\, j\,(\omega t + \Phi)$$

where V is the peak voltage, ω is the angular frequency (rad s^{-1}) and Φ is some reference phase angle.

As (ωt) has units of radians, so the phase angle Φ, must also have units of radians.

For convenience we refer to the horizontal (right-hand) x-axis as the reference, i.e. a phase of zero degrees.

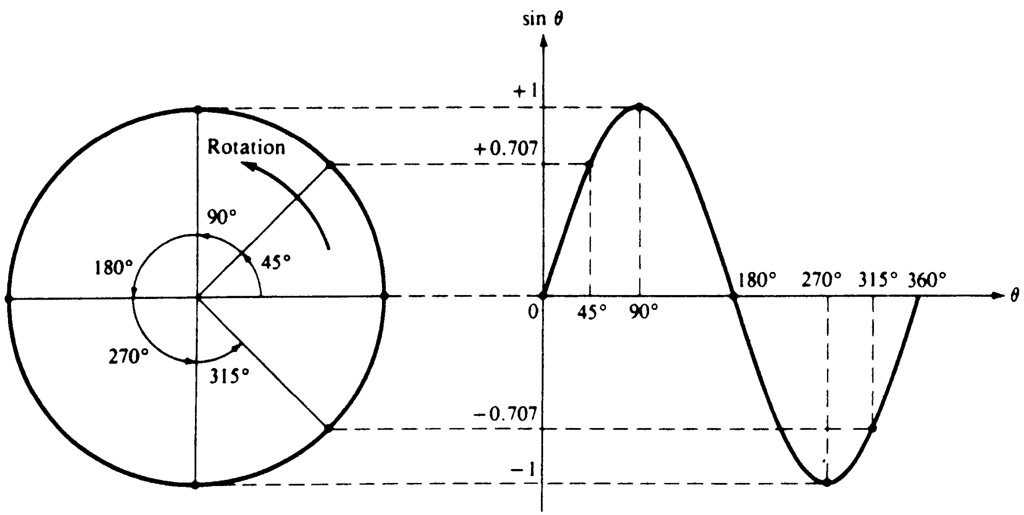
$$v(t) = V \exp.\, j(\omega t + \Phi) = V\,\{\cos\,(\omega t + \Phi) + j \sin\,(\omega t + \Phi)\}$$

The cosine wave can be considered as the *real* part: Re[v(t)]

The sine wave the *imaginary* part: Im[v(t)]

The designations real (Re) and imaginary (Im) are simply there to differentiate between the two directional components.

Sinusoidal waveforms and the phasor: Summary



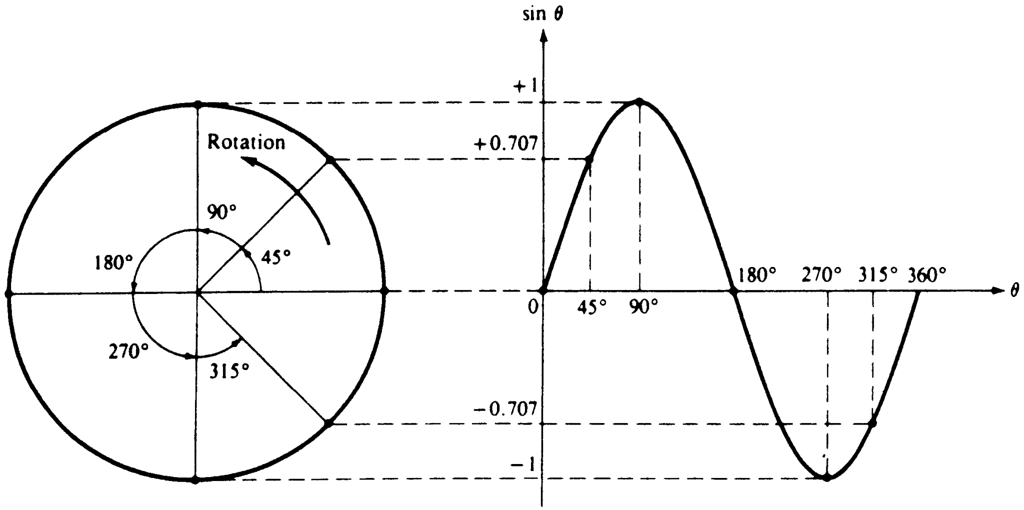
The horizontal projection of a rotating line or *phasor*

As alternating circuit theory is founded on phasors,

i.e. a two dimensional co-ordinate axis system

mathematical manipulations must inevitably use complex number theory.

Sinusoidal waveforms and the phasor



The horizontal projection of a rotating line or *phasor*

Phasor Demonstration

Sinusoidal waveforms and the phasor

Phase Angles and the phasor representation

In using the phasor as a mathematical tool to represent the sine and cosine functions we will restrict ourselves (for the moment) to:

a. linear circuits that have multiple “input” and “output” phasors all of which rotate at the *same angular frequency*

b. Multiple phasors can be “frozen in time”; it is only the relative phase between one phasor and another that is of importance in ac theory.

Sinusoidal waveforms and the phasor

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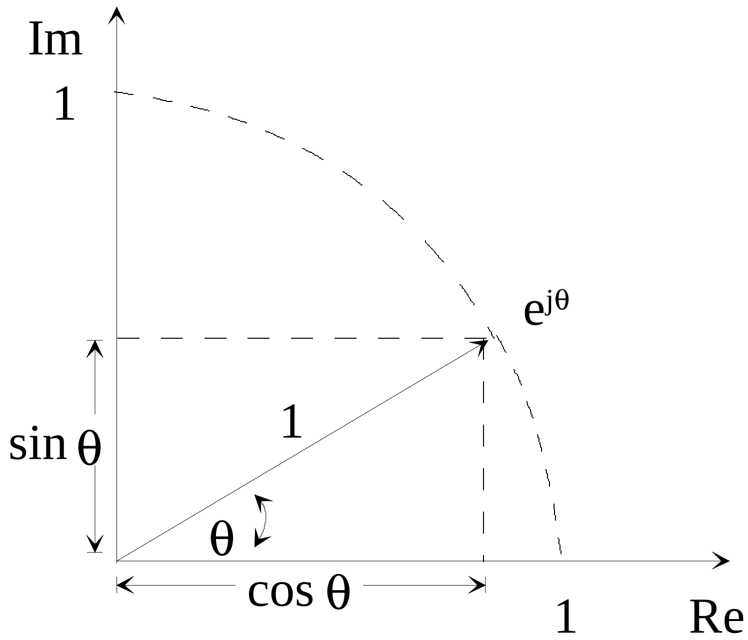
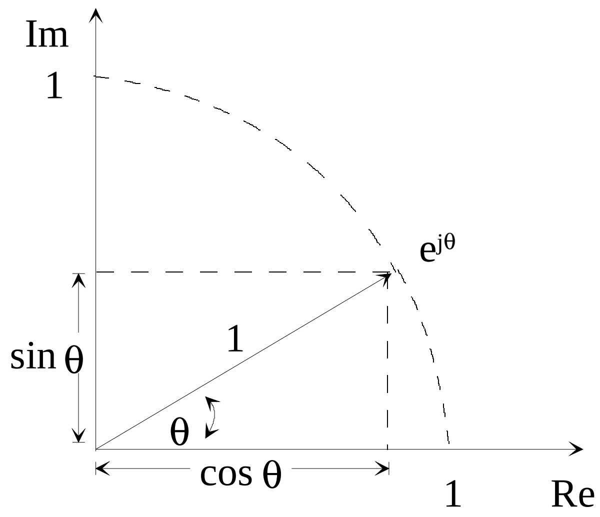


$$\left| e^{j\theta} \right| = 1$$

$$\angle e^{j\theta} = \theta$$

Sinusoidal waveforms and the phasor



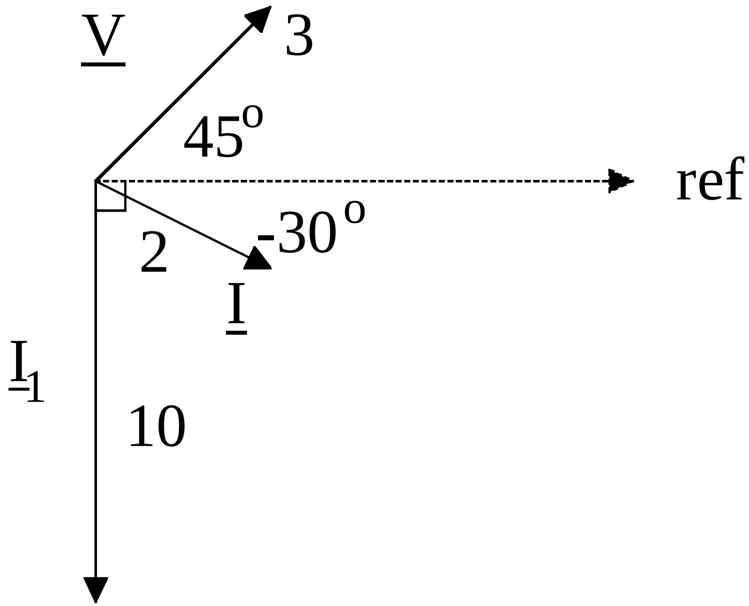
$$\left| e^{j\theta} \right| = 1$$

$$\angle e^{j\theta} = \theta$$

polar form	exponential form	rectangular form
vector	*exponential functions*	*complex numbers*
$\underline{V} \equiv V \underline{/\Theta}$	$V = e^{j\Theta}$	$V\{\cos\Theta + j\sin\Theta\}$ $\rightarrow$ $Re[e^{j\Theta}] + j\,Im[e^{j\Theta}]$

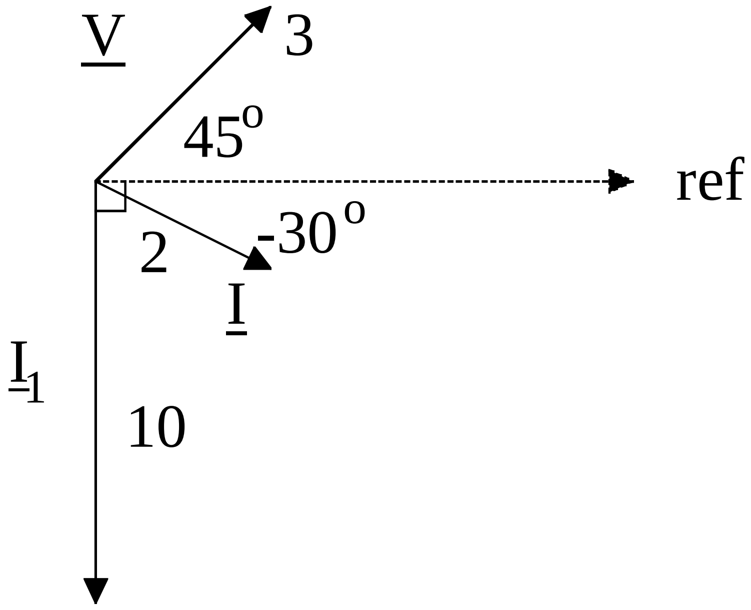
Sinusoidal waveforms and the phasor

Example 1



Sinusoidal waveforms and the phasor

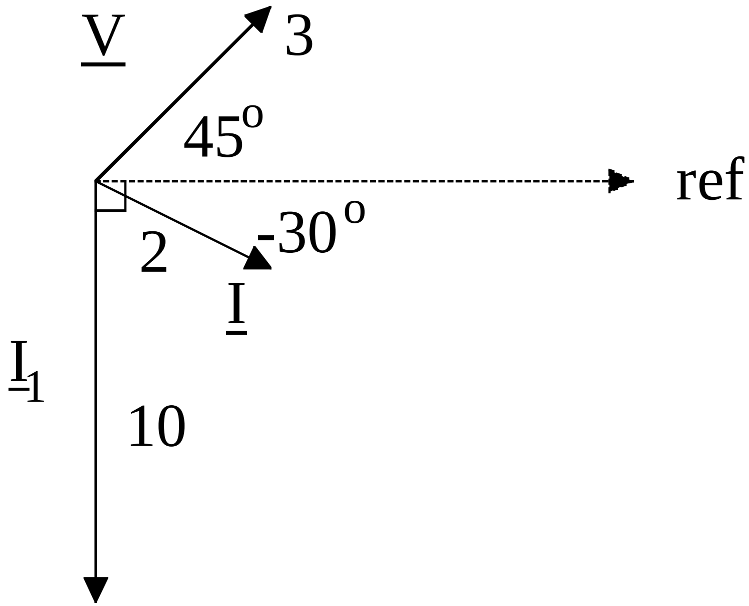
Example 1



$v(t) = 3\cos(\omega t + 45^o) \rightarrow \underline{V} = 3\,/\underline{45^o}$ volts

Sinusoidal waveforms and the phasor

Example 1

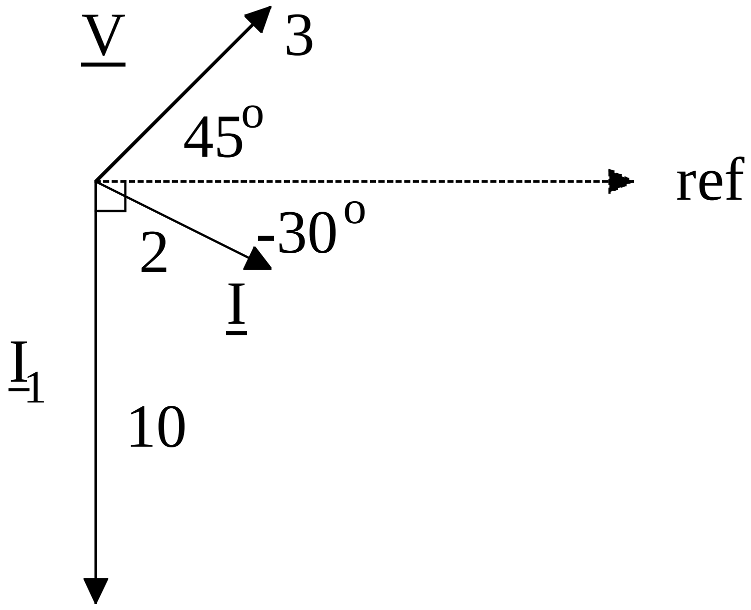


$v(t) = 3 \cos(\omega t + 45^{o}) \rightarrow \underline{V} = 3\ /\underline{45^{o}}$ volts

$i(t) = 2 \cos(\omega t - 30^{o}) \rightarrow \underline{I} = 2\ /\underline{-30^{o}}$ amps

Sinusoidal waveforms and the phasor

Example 1



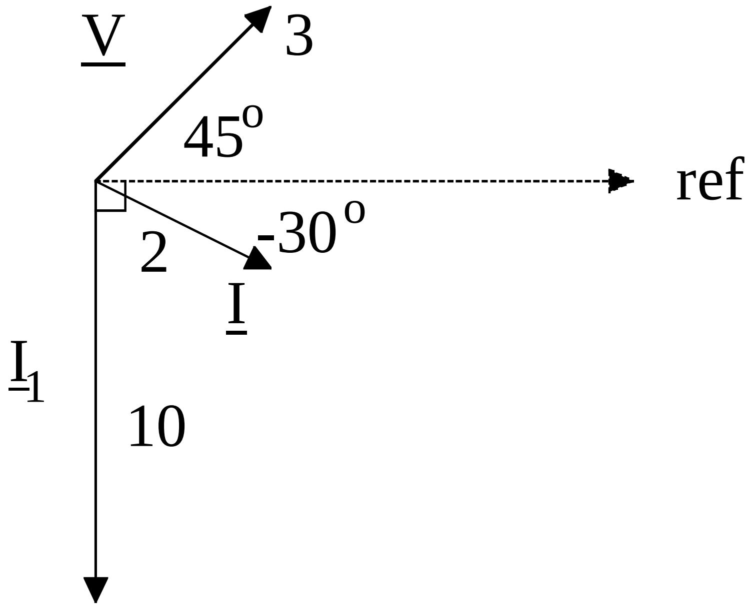
$v(t) = 3 \cos(\omega t + 45^o) \rightarrow \underline{V} = 3\ /\underline{45^o}$ volts

$i(t) = 2 \cos(\omega t - 30^o) \rightarrow \underline{I} = 2\ /\underline{-\ 30^o}$ amps

$I_1(t) = 10 \sin(\omega t) = 10 \cos(\omega t - 90^o) \rightarrow \underline{I_1} = 10/\underline{-90^o}$ amps

Sinusoidal waveforms and the phasor

Example 1



$\underline{V}$ LEADS $\underline{I}$

and

$\underline{I}$ LEADS $\underline{I}_1$

Sinusoidal waveforms and the phasor

Example 2

Given that $y(t) = 1 \sin (3141.6)t$

Determine:

1. The angular frequency (ω)

2. The frequency (f in Hz)

3. The period of the waveform (T)

Sinusoidal waveforms and the phasor

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Sinusoidal waveforms and the phasor

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2. The frequency (f in Hz) $f = \omega/2\pi = 314.6/2\pi = 500$ Hz

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Sinusoidal waveforms and the phasor

Example 2

Given that $y(t) = 1 \sin (3141.6)t$

Determine:

1. The angular frequency (ω) — $\omega = 314.6$ rad s^{-1}

2. The frequency (f in Hz) — $f = \omega/2\pi = 314.6/2\pi = 500$ Hz

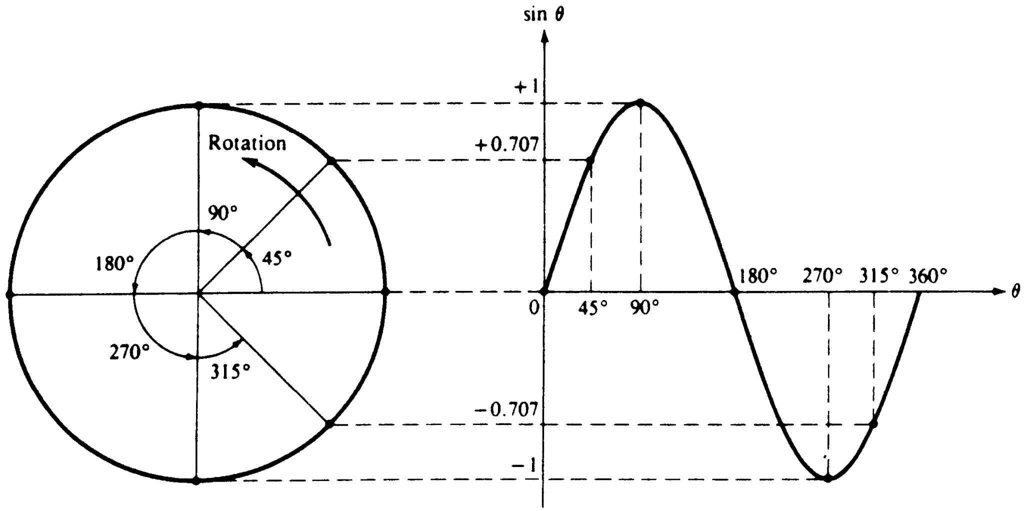
3. The period of the waveform (T) — $T = 1/f = 1/500 = 2$ ms

Sinusoidal waveforms and the phasor

Phase relationships between sinusoids of the same frequency

When we write V sin ($\omega t + \Phi$) we understand from the mathematics of this equation

- waveform shape is sinusoidal
- it can be pictured as a phasor of magnitude V volts rotating anticlockwise at an angular rotation of ω rads^{-1}

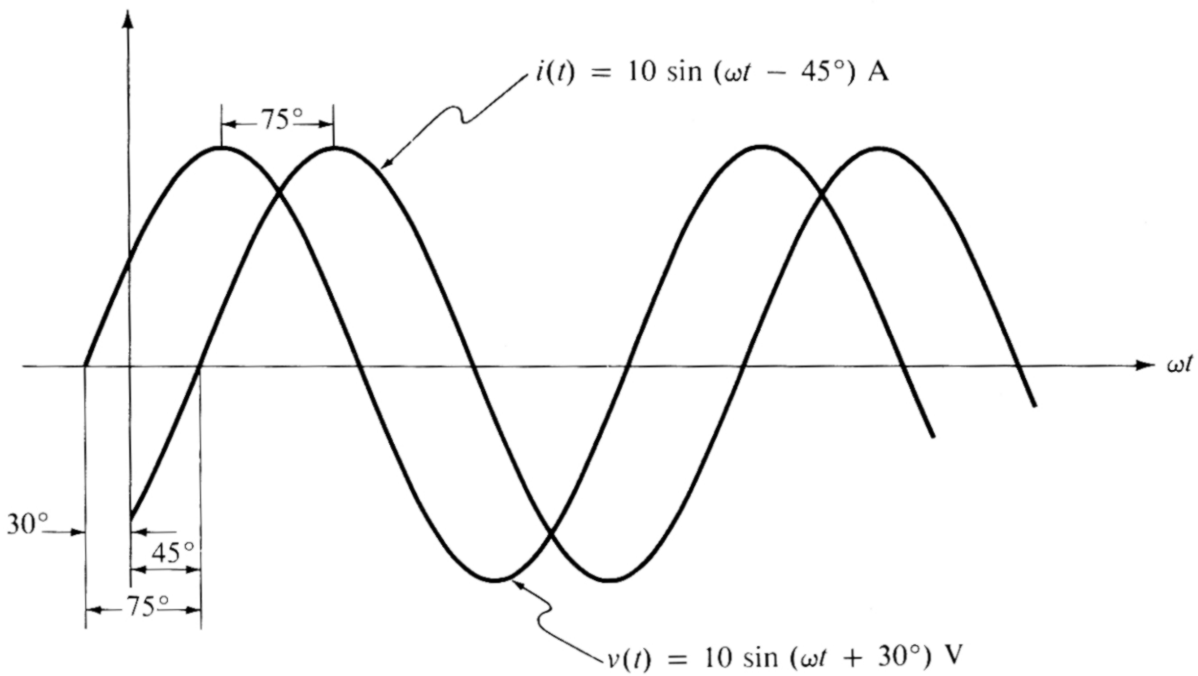


But what about the phase term (Φ)?

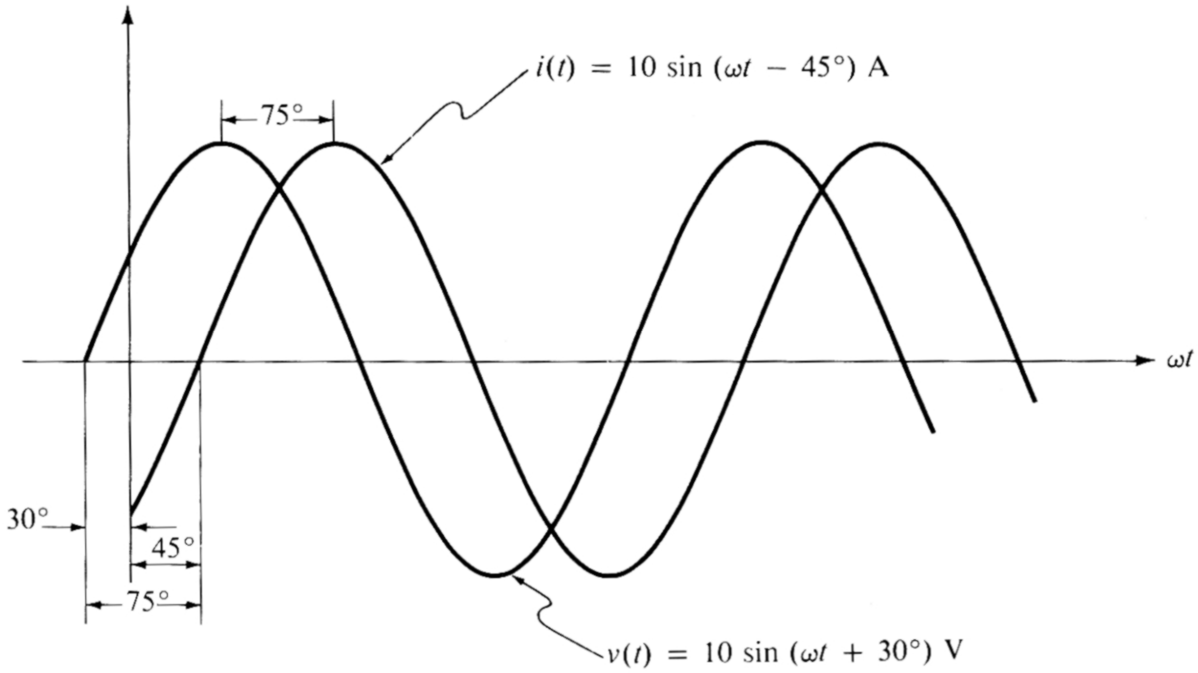
Sinusoidal waveforms and the phasor

But what about the phase term (Φ)?

- One interpretation is that the temporal waveform at t = 0 (?) has a value of sin (Φ).
- But it is usually phase difference that matters in electrical engineering; i.e. our interest lies in the phase of one phasor relative to another.



Sinusoidal waveforms and the phasor



A voltage 10 sin (ωt + 30º) volts, and a current 10 sin (ωt - 45º) amps.

v (t) leads i(t).

Sinusoidal waveforms and the phasor

Example 1: Leading and Lagging

What is the phase relationship between:

$v(t) = 10 \sin(\omega t + 30^{o})$ volts and $i(t) = 5 \sin(\omega t + 70^{o})$ amps ?

Sinusoidal waveforms and the phasor

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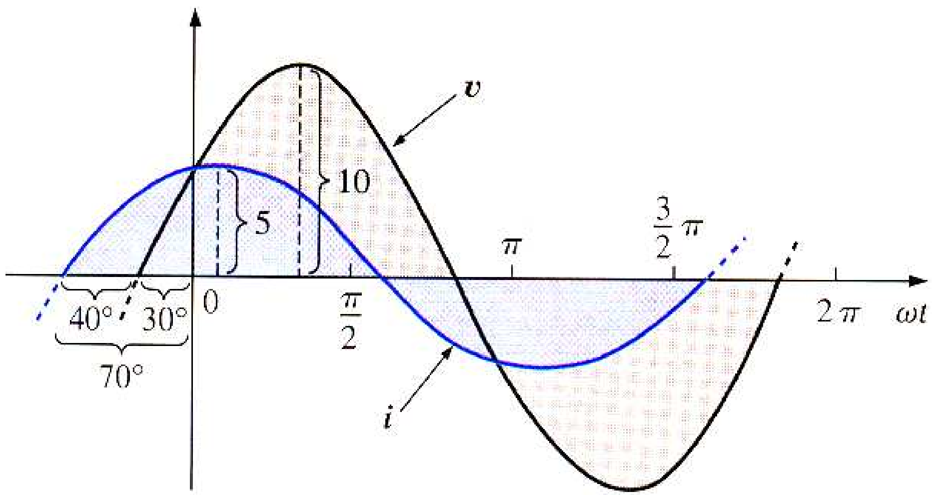

i(t) LEADS v(t) by 40º

Sinusoidal waveforms and the phasor

Example 2: Leading and Lagging

What is the phase relationship between:

$v(t) = 10 \sin(\omega t - 20^{\circ})$ volts and $i(t) = 15 \sin(\omega t + 60^{\circ})$ amps ?

Sinusoidal waveforms and the phasor

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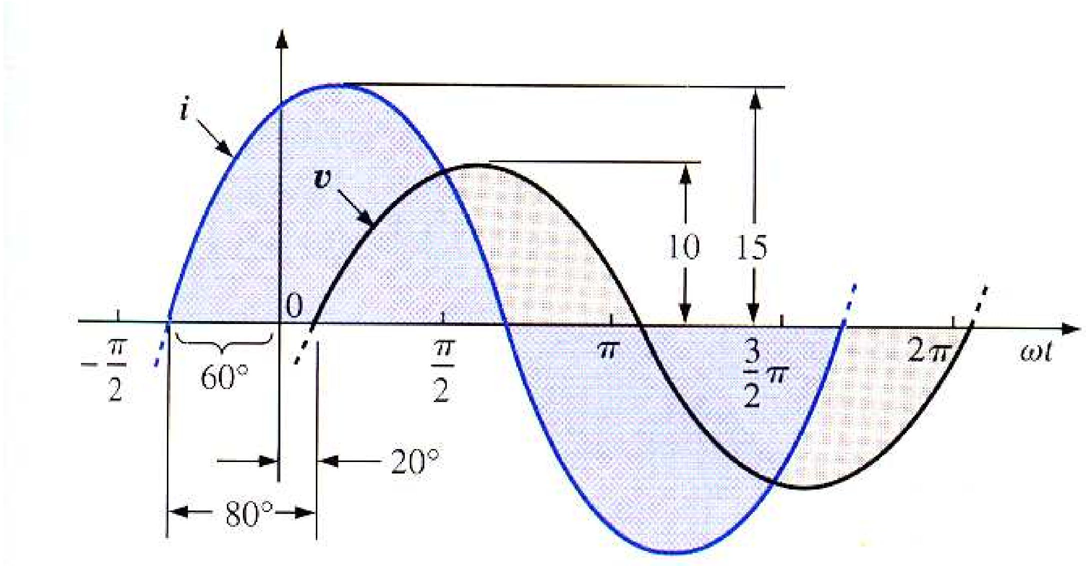


i(t) LEADS v(t) by 80º

Sinusoidal waveforms and the phasor

Example 3: Leading and Lagging

What is the phase relationship between:

$v(t) = 3 \sin(\omega t - 10^{o})$ volts and $i(t) = 2 \cos(\omega t + 10^{o})$ amps ?

Note: you cannot mix sine and cosine terms!

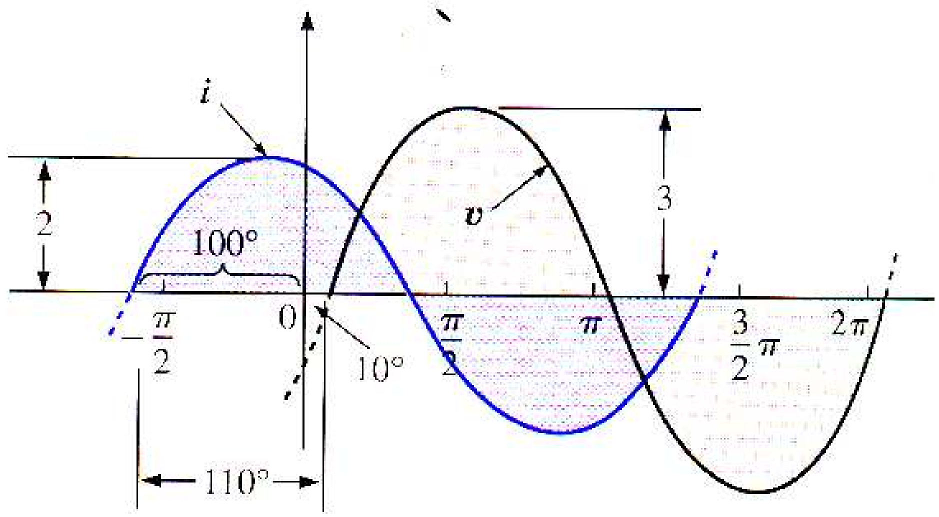
Sinusoidal waveforms and the phasor

Example 3: Leading and Lagging

What is the phase relationship between:

$v(t) = 3 \sin(\omega t - 10^o)$ volts and $i(t) = 2 \cos(\omega t + 10^o)$ amps ?

Note: $\cos(\omega t + 10^o) = \sin(\omega t + 10^o + 90^o) = \sin(\omega t + 100^o)$



i(t) LEADS v(t) by 110º

Sinusoidal waveforms and the phasor

Example 4: Leading and Lagging

What is the phase relationship between:

$v(t) = 2 \sin(\omega t + 10^\circ)$ volts and $i(t) = -1 \sin(\omega t + 30^\circ)$ amps ?

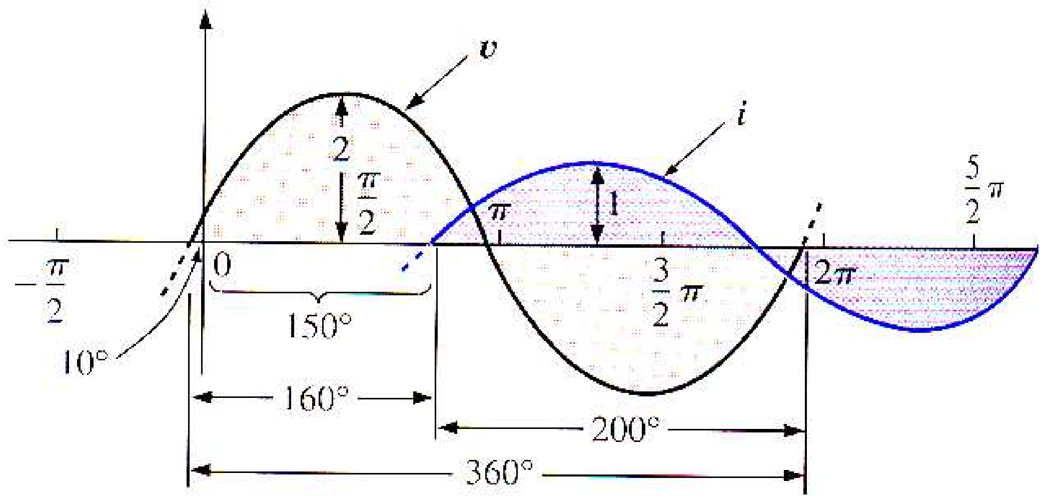
Sinusoidal waveforms and the phasor

Example 4: Leading and Lagging

What is the phase relationship between:

$v(t) = 2 \sin(\omega t + 10^o)$ volts and $i(t) = -1 \sin(\omega t + 30^o)$ amps ?

Note: $-1 \sin(\omega t + 30^o) = 1 \sin(\omega t + 30^o - 180^o) = 1 \sin(\omega t - 150^o)$



v(t) LEADS i(t) by 160º

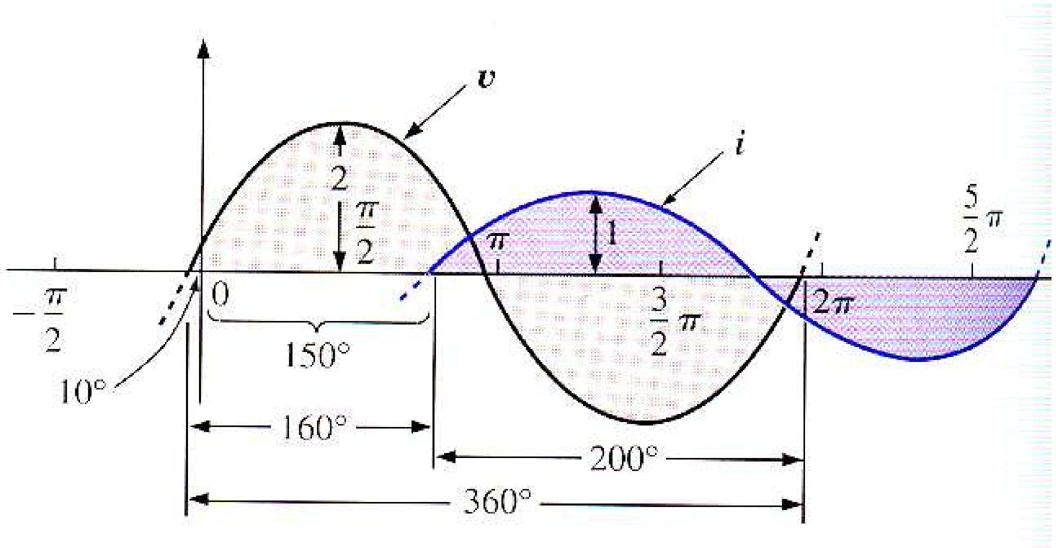
Sinusoidal waveforms and the phasor

Example 4a: Leading and Lagging

What is the phase relationship between:

$v(t) = 2 \sin(\omega t + 10^{o})$ volts and $i(t) = -1 \sin(\omega t + 30^{o})$ amps ?

Note: $-1 \sin(\omega t + 30^{o}) = 1 \sin(\omega t + 30^{o} + 180^{o}) = 1 \sin(\omega t + 210^{o})$

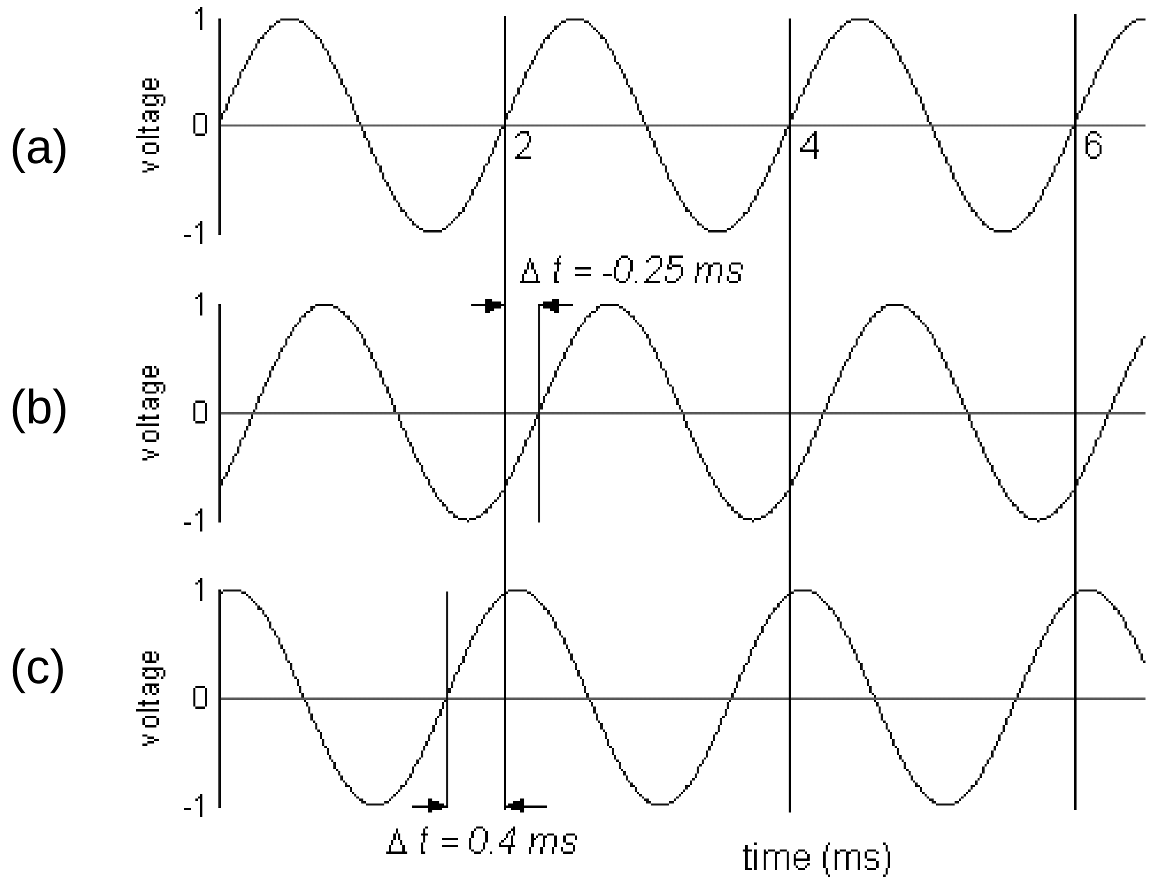


i(t) LEADS v(t) by 200º

Sinusoidal waveforms and the phasor

Class example

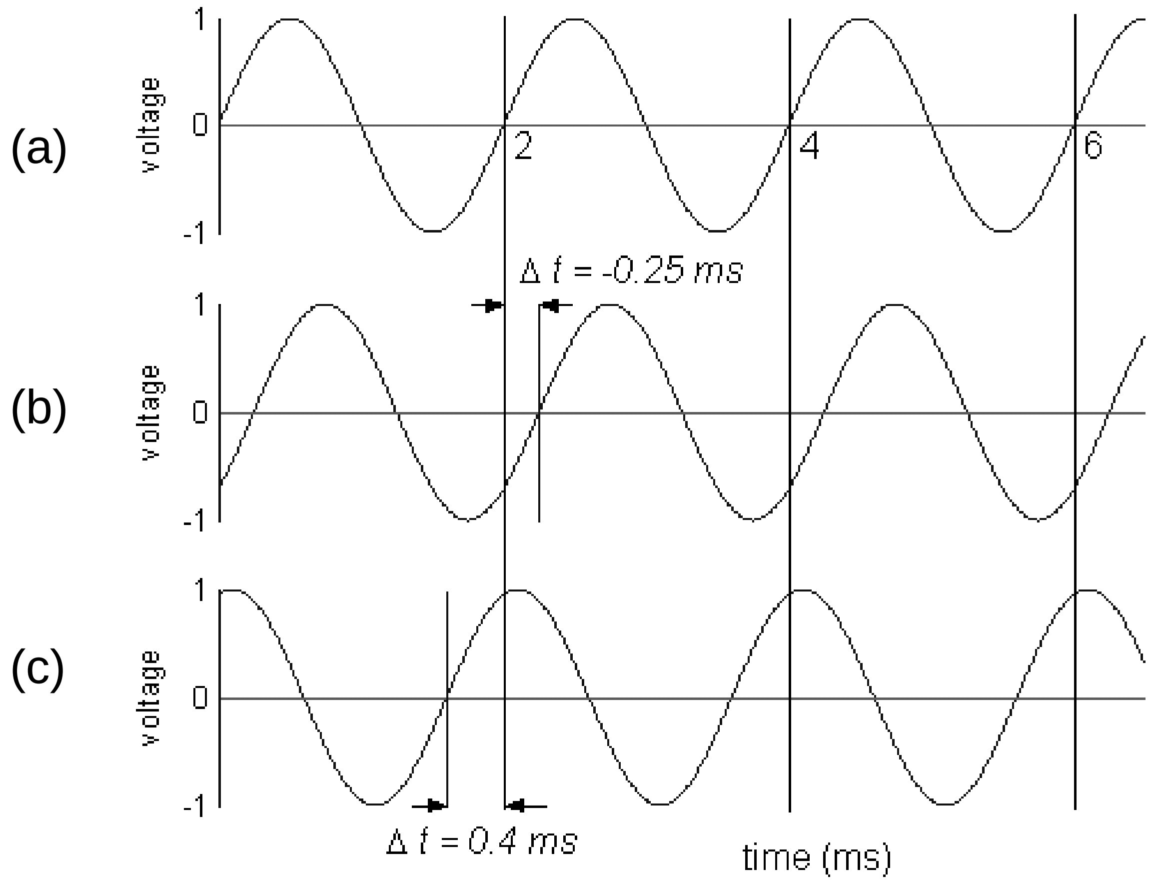
What is the phase of waveform (c) relative to that of waveform (a)?



Sinusoidal waveforms and the phasor

Class example

What is the phase of waveform (c) relative to that of waveform (a)?



Answer:

(c) LEADS (a) by

0.4 ms.

As the period is 2 ms:

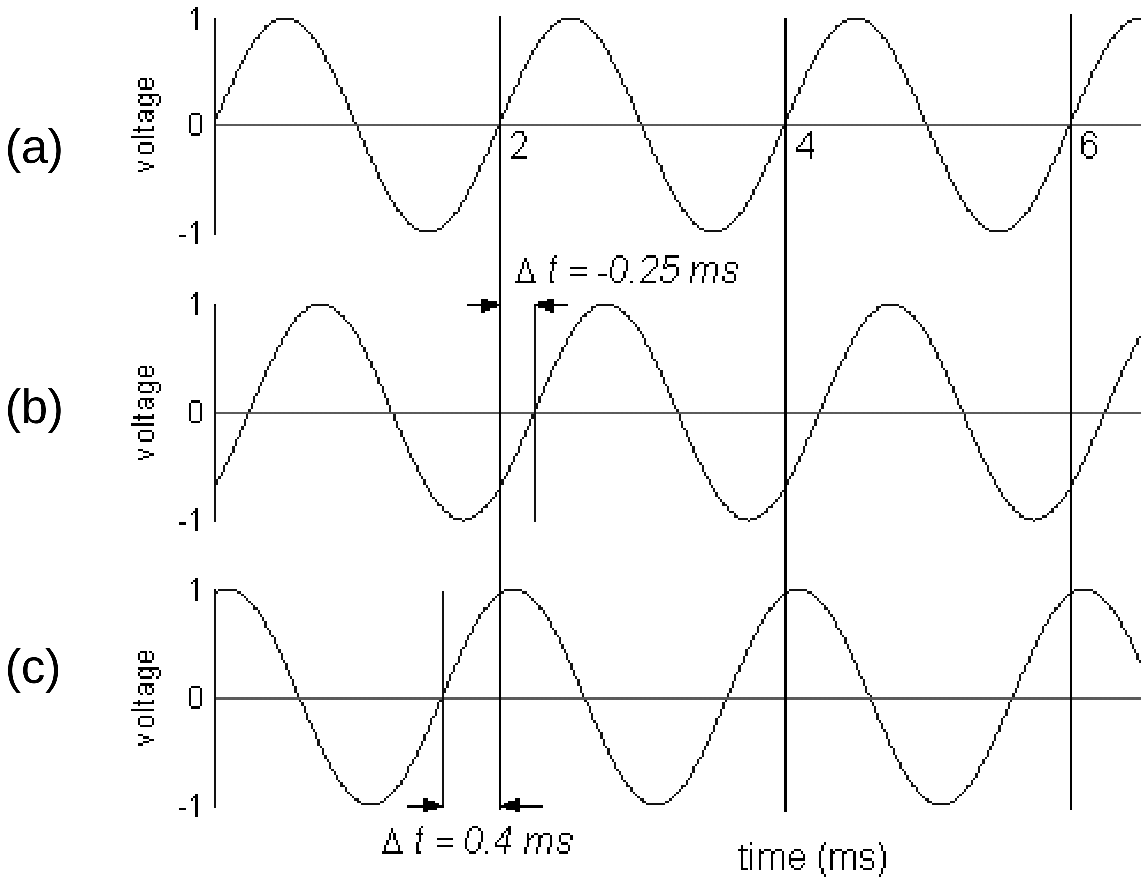
2 ms → 360º (or 2π rad)

So 0.4 ms → 72º

Sinusoidal waveforms and the phasor

Class example

What is the phase difference between waveforms (b) and (c)?



Sinusoidal waveforms and the phasor

Class example

What is the phase difference between waveforms (b) and (c)?

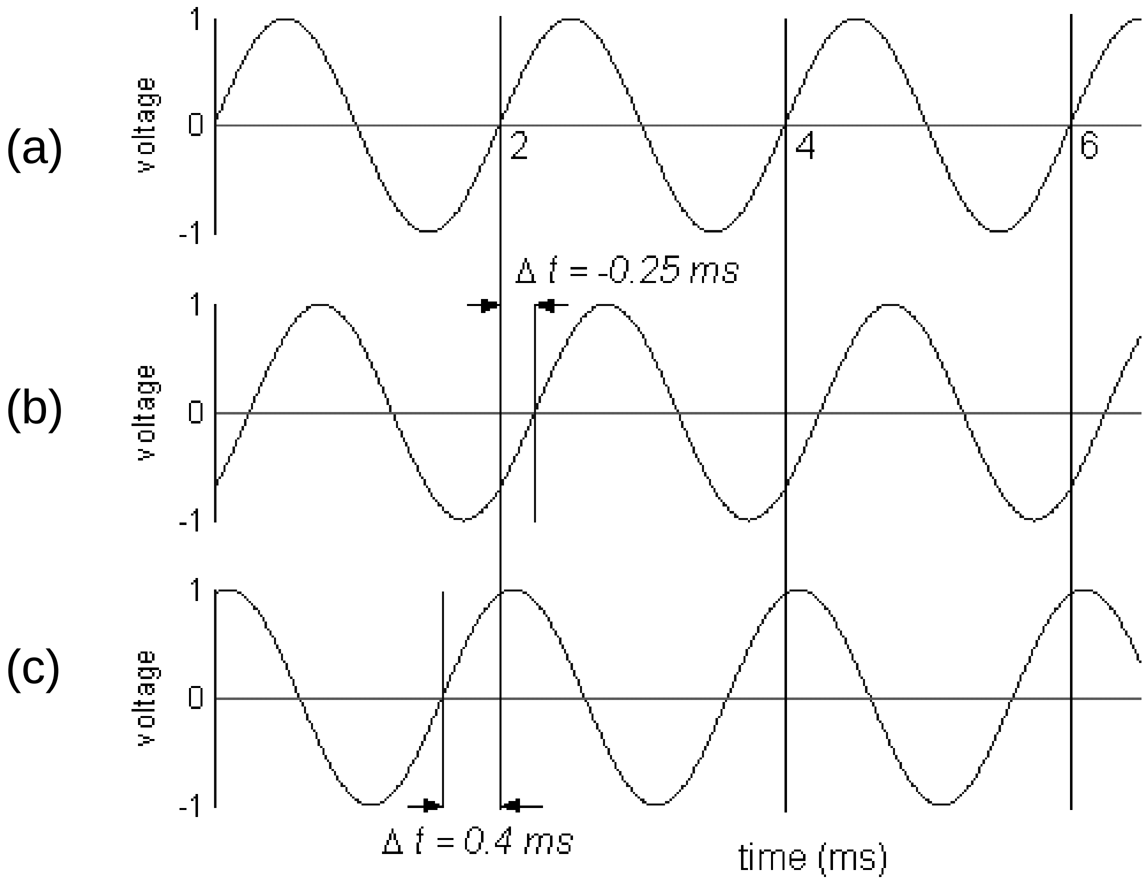
Answer:

(c) LEADS (a) by 72º

But (b) LAGS (a) by:

(0.25/2) X 2π = π/4 (45º)

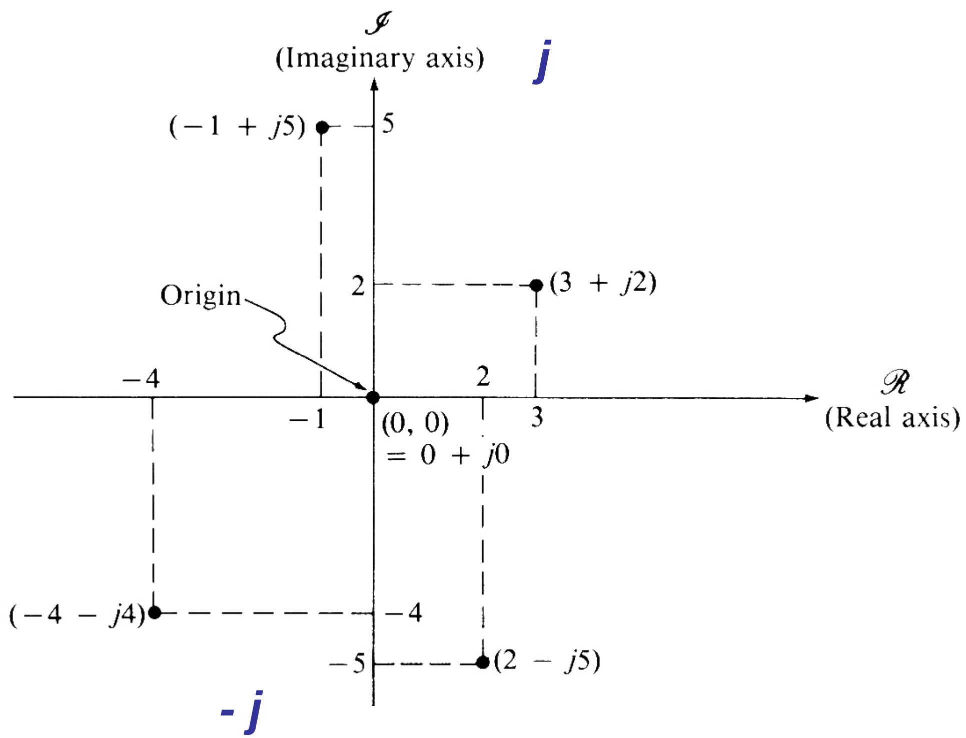
Hence phase difference between (b) and (c) is 72º + 45º = **117º**



Sinusoidal waveforms and the phasor

REVISION: Complex Numbers (The *j* 90º operator)

The complex plane is a rectangular co-ordinate system in which real numbers are plotted along the horizontal (real) axis and imaginary numbers along the vertical axis.



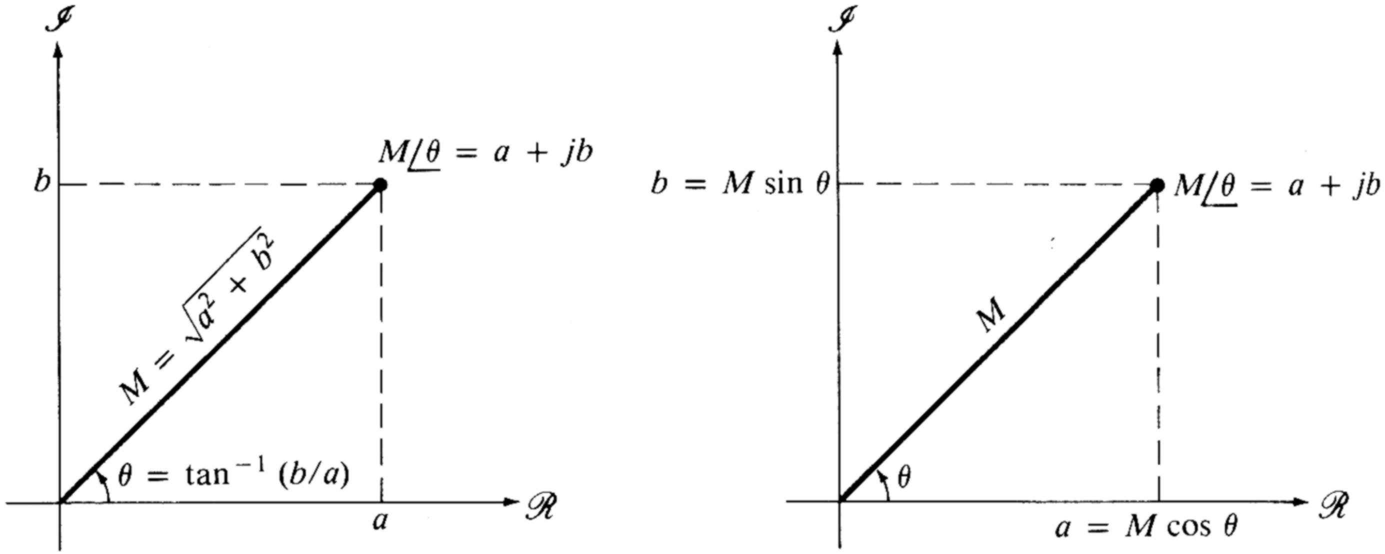
$j = \sqrt{(-1)}$

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

The representation (a + j b) is called the rectangular form of a complex number. Every complex number can also be represented in polar form:

y = M /<u>Ɵº</u> where M is the magnitude of y and Ɵº is its *angle*.



Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Addition and Subtraction can only be performed in **rectangular** co-ordinates.

Example:

Determine the sum of the two phasor voltages:

$V_1(t) = 12\underline{/\ -30^o}$ volts and $v_2\ (t) = 20\ /\ \underline{45^o}$ volts

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Addition and Subtraction can only be performed in **rectangular** co-ordinates.

Example:

Determine the sum of the two phasor voltages:

$V_1(t) = 12\underline{/ -30^o}$ volts and $v_2\ (t) = 20\ /\ \underline{45^o}$ volts

This requires the conversion of the voltages from polar to rectangular format:

$\therefore 12\underline{/ -30^\circ}$ volts + $20\ \underline{/ 45^\circ}$ volts => (10.4 - j 6) + (14.1 + j 14.1)

= **(24.5 + j 8.1)** volts

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Addition and Subtraction can only be performed in **rectangular** co-ordinates.

Example:

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= **(24.5 + j 8.1)** volts

To get the answer back in polar notation, we do the reverse

(24.5 + j 8.1) => $\mathbf{25.8\ \underline{/\ 18.3}^\circ}$ volts.

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Multiplication and Division can only be performed in **polar** co-ordinates.

Example:

180 $\underline{/\ 27°}$ amps divided by 1.5 $\underline{/\ 85°}$ amps

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Multiplication and Division can only be performed in **polar** co-ordinates.

Example:

180 $\underline{/\ 27°}$ amps divided by 1.5 $\underline{/\ 85°}$ amps

This is straight forward as both currents are in polar co-ordinates.

$\Rightarrow$ 180/1.5 /$\underline{(27° - 85°)}$ = 120 $\underline{/\ -58°}$ amps.

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Multiplication and Division can only be performed in **polar** co-ordinates.

Example:

If (1 + j 2) was multiplied by (2 + j 6) then we have to do the rectangular to polar conversion and then the multiplication.

Sinusoidal waveforms and the phasor

REVISION: Complex Numbers

Everything we do in ac theory will rely on complex manipulation and

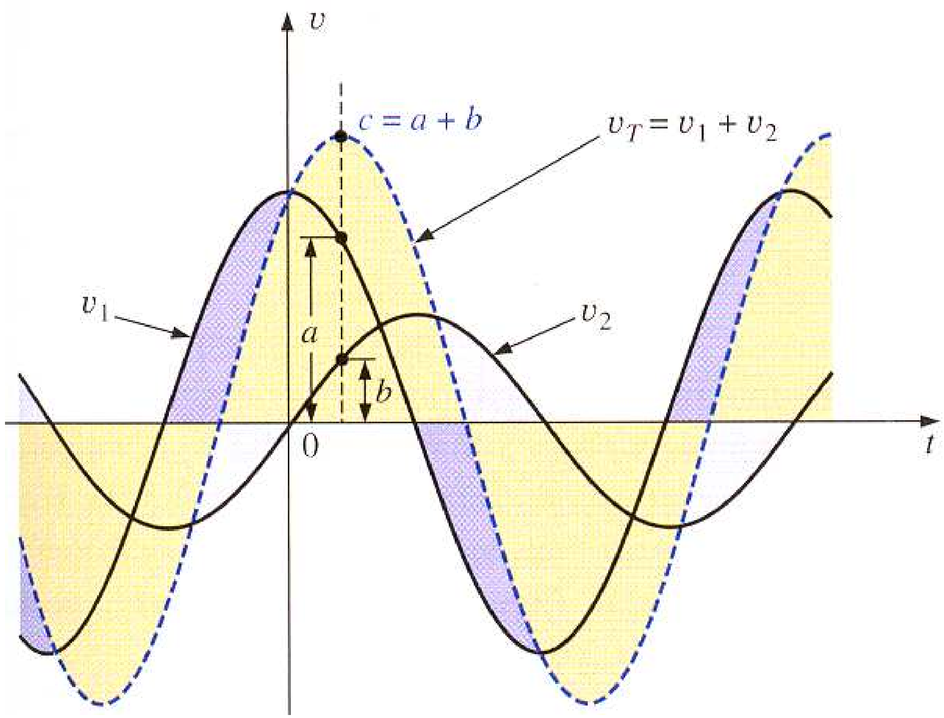
rectangular ⇔ polar conversions.

Learn to use your calculator to do this manipulation

Sinusoidal waveforms and the phasor

Example: Phasors and sinusoidal waveforms

The addition of sinusoidal voltages and currents is ***constantly*** required in ac circuit analysis. One perfectly valid method of adding such waveforms is to place both sinusoidal waveforms on the same set of axis and add algebraically the magnitude of each at every point along the abscissa.



Sinusoidal waveforms and the phasor

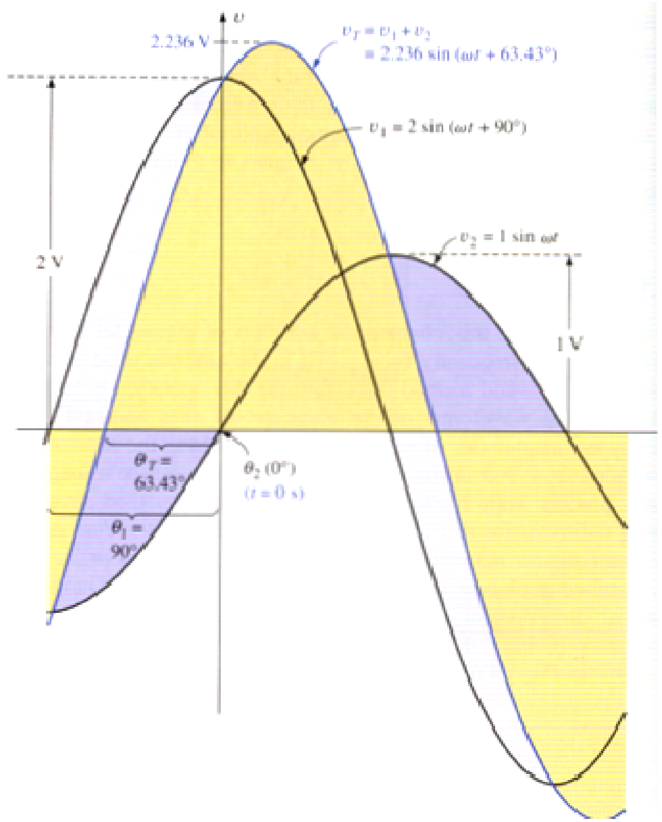
Example 1: Phasors and sinusoidal waveforms

Consider this example of a

1 volt peak sine wave and a

2 volt peak cosine wave

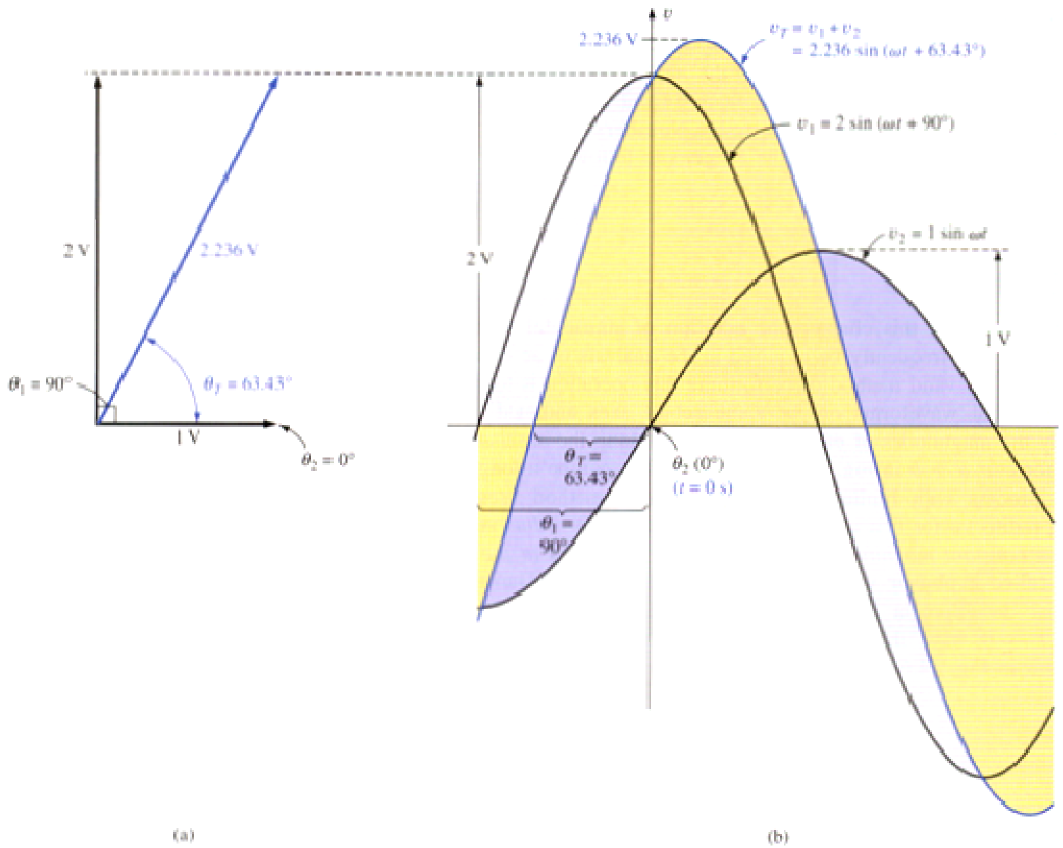
added together.



Sinusoidal waveforms and the phasor

Example 1: Phasors and sinusoidal waveforms

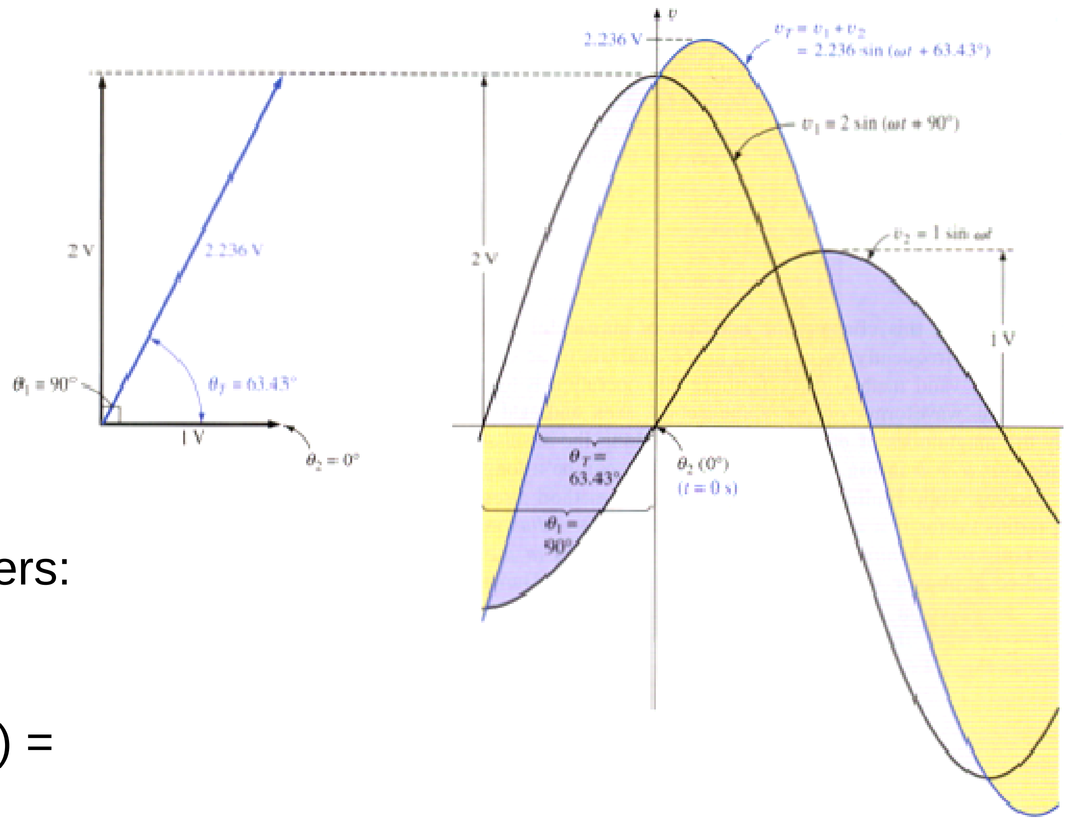
We see this as the addition of two phasors in quadrature with each other.



Sinusoidal waveforms and the phasor

Example 1: Phasors and sinusoidal waveforms

We see this as a phasor addition of two phasors in quadrature with each other.



(a) (b)

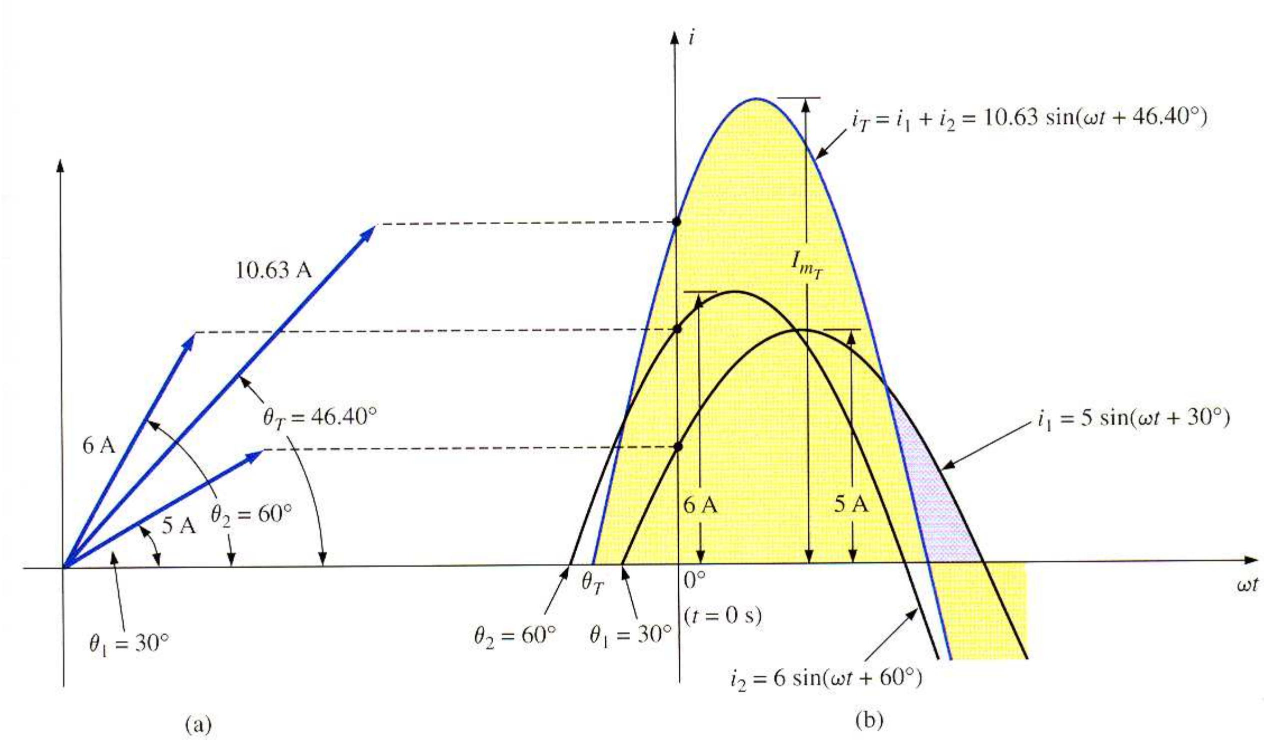
Using complex numbers:

$1\underline{/0^\circ} + 2\underline{/90^\circ} = (1 + j\,2) =$

$\rightarrow$ **$2.236\underline{/63.43^\circ}$ volts**.

Sinusoidal waveforms and the phasor

Example 2: Phasors and sinusoidal waveforms



Adding two sinusoidal currents: phase angle other than 90º

Sinusoidal waveforms and the phasor

Example 2: Phasors and sinusoidal waveforms

i_1 (t) = 5/<u>30º</u> A

I_2 (t) = 6/<u>60º</u> A

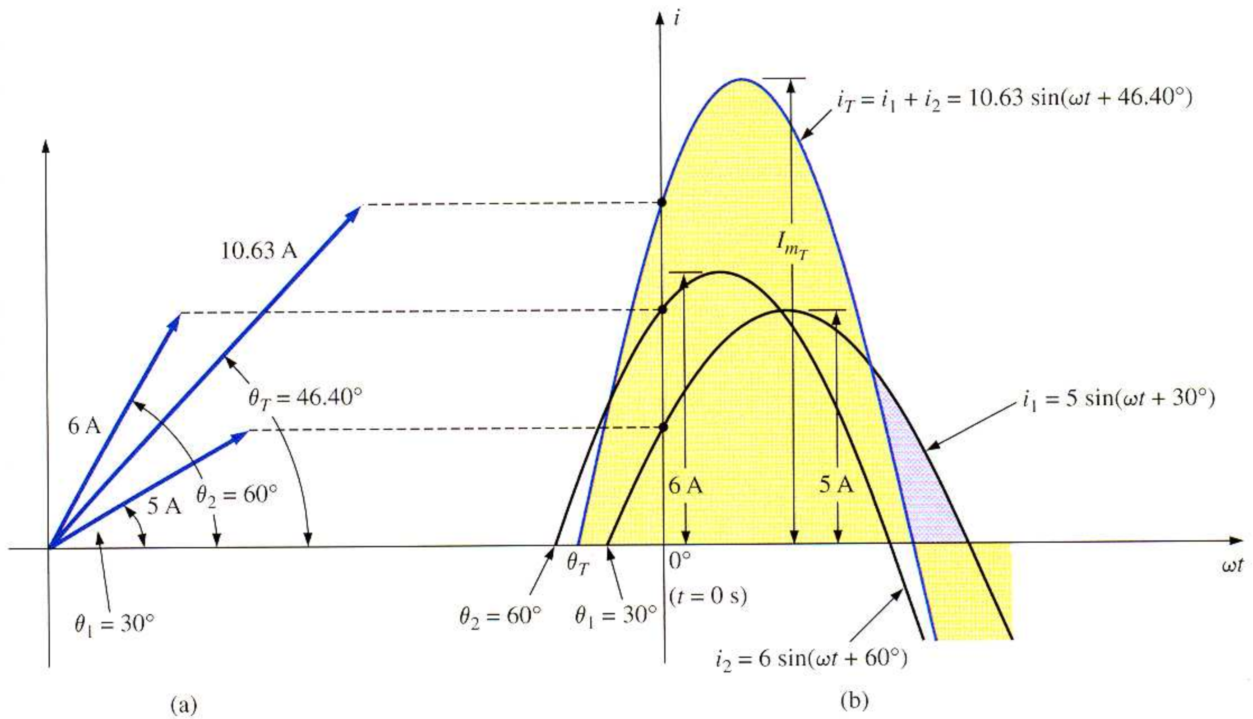
i_1 (t) + i_2 (t) =

(4.33 + j 2.5) +

(3.000 + j 5.196)

= (7.33 + j 7.696)

→ **10.628/<u>46.39º</u> A**



Adding two sinusoidal currents: phase angle other than 90º

Sinusoidal waveforms and the phasor

Example 3: Kirchhoff's Law

A circuit consists of two series components. A supply voltage of 340 cos ωt volts is applied to this series circuit. If the voltage developed across one of the components is 250 cos (ωt + 30º) volts, what is the voltage across the other component?

Sinusoidal waveforms and the phasor

Example 3: Kirchhoff's Law

A circuit consists of two series components. A supply voltage of 340 cos ωt volts is applied to this series circuit. If the voltage developed across one of the components is 250 cos (ωt + 30º) volts, what is the voltage across the other component?

In other words: $340 \cos \omega t$ volts = $250 \cos (\omega t + 30^\circ) + v(t)$ volts

So $v(t) = 340 \cos \omega t - 250 \cos (\omega t + 30^\circ)$

Sinusoidal waveforms and the phasor

Example 3: Kirchhoff’s Law

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So $v(t) = 340 \cos \omega t - 250 \cos (\omega t + 30^\circ)$

Convert to phasor format: $v(t) = 340\underline{/0^\circ} - 250\underline{/30^\circ}$

Sinusoidal waveforms and the phasor

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So $v(t) = 340 \cos \omega t - 250 \cos(\omega t + 30^\circ)$ volts

Convert to phasor format: $v(t) = 340\underline{/0^\circ} - 250\underline{/30^\circ}$

But we can only add in rectangular components, so:

$v(t) = (340 + j\,0) – (216.5 + j\,125) = (123.5 – j\,125) \rightarrow 175.7\underline{/-45.3^\circ}$ volts

Sinusoidal waveforms and the phasor

Example 3: Kirchhoff's Law

A circuit consists of two series components. A supply voltage of 340 cos ωt volts is applied to this series circuit. If the voltage developed across one of the components is 250 cos (ωt + 30º) volts, what is the voltage across the other component?

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Convert to phasor format: v(t) = $340\underline{/0^{\circ}} - 250\underline{/30^{\circ}}$

But we can only add in rectangular components, so:

v(t) = (340 + j 0) – (216.5 + j 125) = (123.5 – j 125) → $175.7\underline{/-45.3^{\circ}}$ volts

Returning to cosine waves……

v(t) = 175.7 cos (ωt – 45.3º) volts

The Passive components

There are only three passive components in all of electrical engineering.

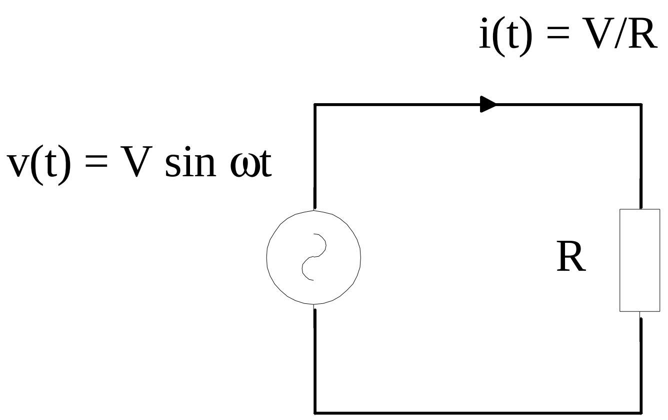
Clearly we have to understand in depth how all three behave, particularly as we change the frequency of the excitation waveform.

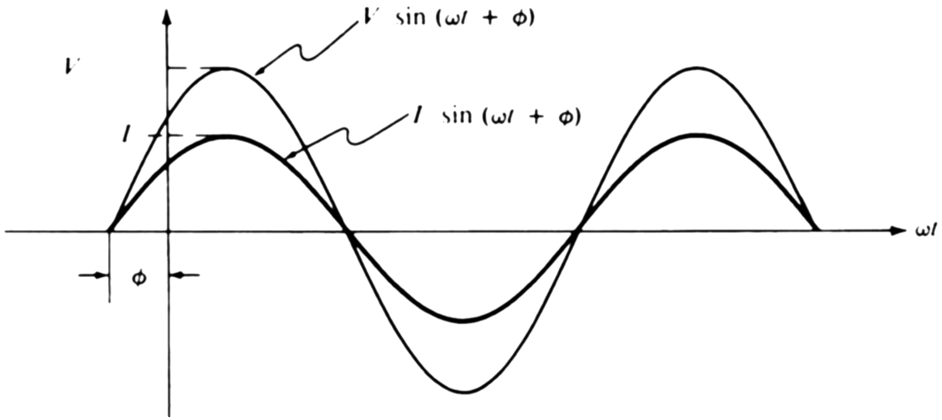
We will only consider sine and cosine waveform excitation this year.

The Passive components: Resistor

ac voltage and current in the Resistor

Ohm's Law can be applied to an ac circuit containing a resistance to determine the ac current in the resistance when an ac voltage is connected.





$i(t) = V/R \sin \omega t$

In a resistor voltage and current are in phase

The Passive components: Resistor

ac voltage and current in the Resistor

You will recall that in a dc circuit power can be calculated using any of the three relationships:

VI **I^2R** **V^2/R** Watts.

In ac circuits both voltage and current are time-varying quantities, and so therefore is power.

The power at any instant, *the instantaneous power*, can be computed using instantaneous values of voltage and/or current.

$$\text{ac Power, } p(t) = v(t).i(t) = (V \sin \omega t)(I \sin \omega t) = VI \sin^2 \omega t \quad \text{Watts}$$

The Passive components: Resistor

ac voltage and current in the Resistor

ac Power, $p(t) = v(t).i(t) = (V \sin \omega t)(I \sin \omega t) = VI \sin^2 \omega t$ Watts

Now the $\sin^2 \omega t$ term can be viewed in two ways:

Firstly, although $\sin \omega t$ goes both positive and negative in the function $\sin^2 \omega t$ all such values are *squared,* so the function is always positive.

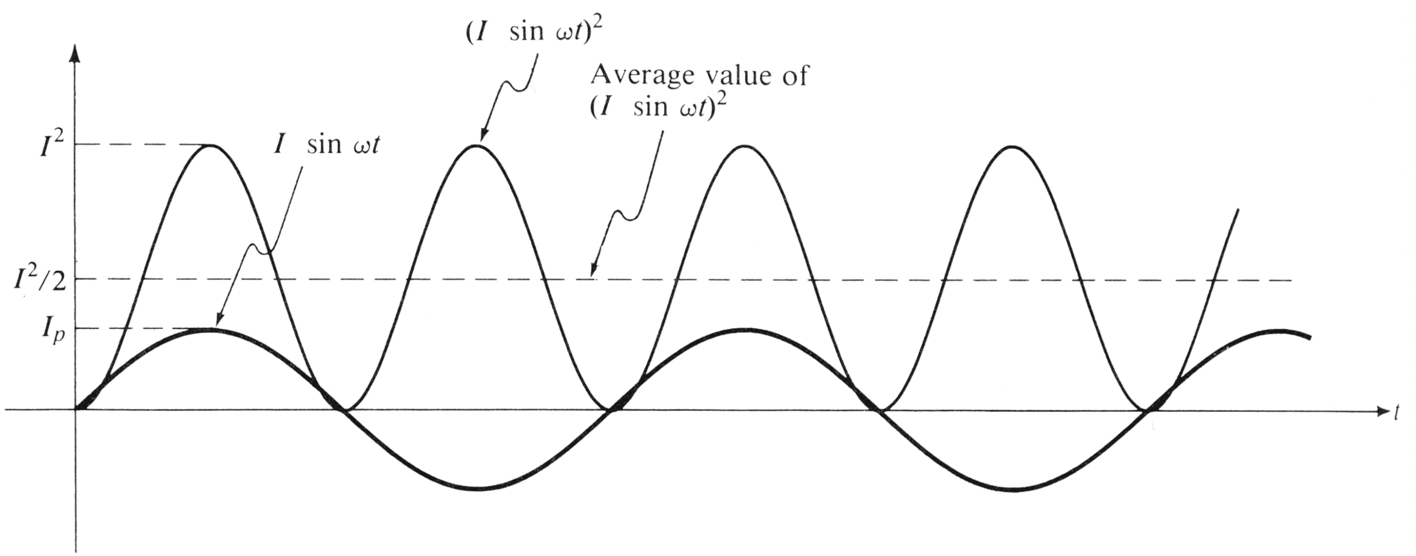
The Passive components: Resistor

ac voltage and current in the Resistor

ac Power, p(t) = v(t).i(t) = (V sin ωt)(I sin ωt) = VI $\sin^2$ ωt   Watts

Now the $\sin^2$ ωt term can be viewed in two ways:

Firstly, although sin ωt goes both positive and negative, in the function $\sin^2$ ωt all such values are *squared,* so the function is always positive.



The Passive components: Resistor

ac voltage and current in the Resistor

$$\text{ac Power, } p(t) = v(t).i(t) = (V \sin \omega t)(I \sin \omega t) = VI \sin^2 \omega t \quad \text{Watts}$$

Now the $\sin^2 \omega t$ term can be viewed in two ways:

Secondly, we can use standard trigonometric formulae to expand $\sin^2 \omega t$:

$$\sin^2 \omega t = 0.5\,(1 - \cos 2\omega t)$$

Thus ac power $= VI \sin^2 \omega t = 0.5VI\,(1 - \cos 2\omega t)$ Watts.

Remember V and I are peak values!

The Passive components: Resistor

ac voltage and current in the Resistor

ac Power, $p(t) = v(t).i(t) = (V \sin \omega t)(I \sin \omega t) = VI \sin^2 \omega t$ Watts

Now the $\sin^2 \omega t$ term can be viewed in two ways:

Secondly, we can use standard trigonometric formulae to expand $\sin^2 \omega t$:

$$\sin^2 \omega t = 0.5\,(1 - \cos 2\omega t)$$

Thus ac power $= VI \sin^2 \omega t = 0.5VI\,(1 - \cos 2\omega t)$ Watts.

Now, consider the $\cos 2\omega t$ term.

The average value of $\cos 2\omega t$ over some period of time (t>> a period) is zero (equal positive and negative areas).

Thus average ac power = 0.5VI.

The Passive components: Resistor

ac voltage and current in the Resistor

The average ac power = 0.5VI.

This relates to rms (root mean square values) of voltage and current:

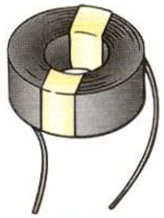
So rms value of voltage will be: $V_{rms} = V/\sqrt{2}$

rms value of current will be $I_{rms} = I/\sqrt{2}$

Thus ac power is given by p = 0.5 VI (Watts) or $V_{rms}\ I_{rms}$

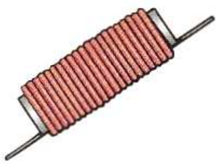
The Passive components: Inductor

Type: Open Core Coil
Typical Values: 3 mH to 40 mH
Applications: Used in low-pass filter circuits. Found in speaker crossover networks.

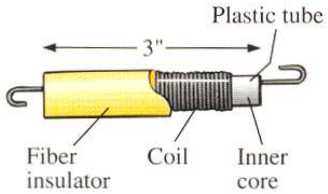


Type: Toroid Coil
Typical Values: 1 mH to 30 mH
Applications: Used as a choke in AC power lines circuits to filter transient and reduce EMI interference. This coil is found in many electronic appliances.

Type: Hash Choke Coil
Typical Values: 3 μH to 1 mH
Applications: Used in AC supply lines that deliver high currents.



Type: Delay Line Coil
Typical Values: 10 μH to 50 μH
Applications: Used in color televisions to correct for timing differences between the color signal and black and white signal.

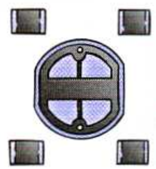


Type: Common Mode Choke Coil
Typical Values: 0.6 mH to 50 mH
Applications: Used in AC line filters, switching power supplies, battery charges and other electronic equipment.

Type: RF Chokes
Typical Values: 10 μH to 50 μH
Applications: Used in radio, television, and communication circuits. Found in AM, FM, and UHF circuits.

Type: Moiled Coils
Typical Values: 0.1 μH to 100 μH
Applications: Used in a wide variety of circuit such as oscillators, filters, pass-band filters, and others.

Type: Surface Mounted Inductors
Typical Values: 0.01 μH to 100 μH
Applications: Found in many electronic circuits that require miniature components on multilayered PCB.



Type: Adjustable RF Coil
Typical Values: 1 μH to 100 μH
Applications: Variable inductor used in oscillators and various RF circuits such as CB transceivers, televisions, and radios.

The Passive components: Inductor

ac voltage and current in the Inductor

Whereas in the resistor the Voltage and Current are related by the linear relationship we call Resistance (Ω), for the Inductor (and the Capacitor) the situation is not so straightforward.

For an Inductor the voltage to current relationship is:

$$V = L\frac{di}{dt}$$

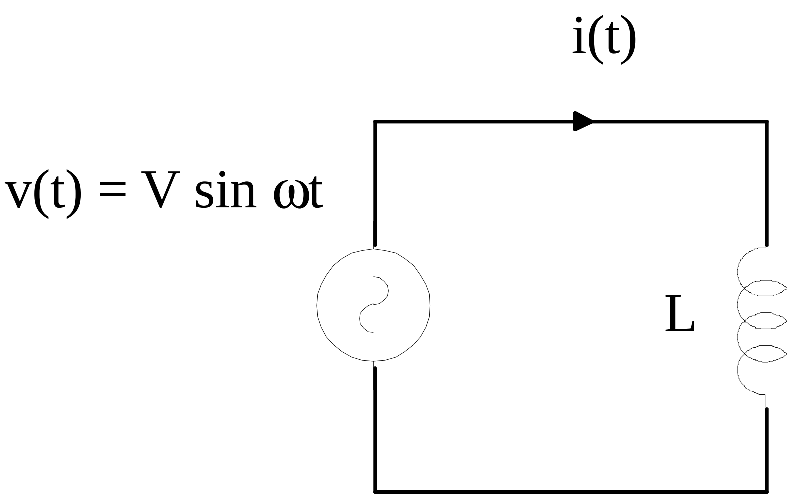
where *V* is the voltage across the inductor and *i* is the current through the inductor

The Passive components: Inductor

ac voltage and current in the Inductor

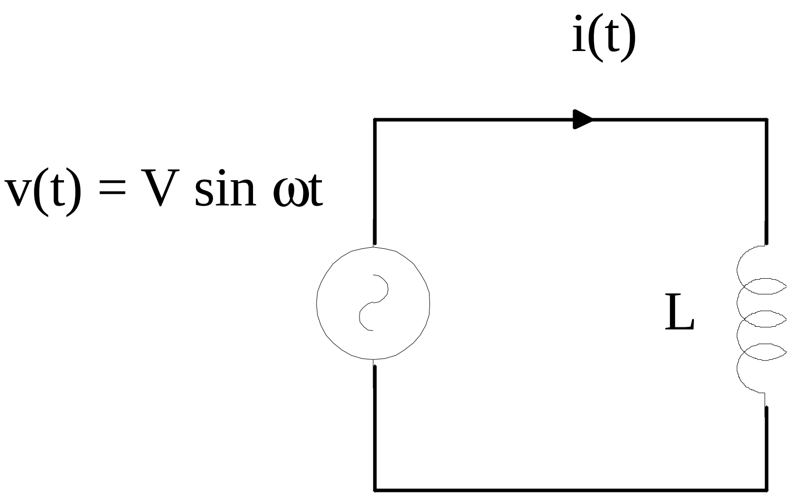
$$V = L \frac{di}{dt}$$

So let us return to our basic circuit with a voltage v(t) = V sin ωt volts applied across an inductor (L).



The Passive components: Inductor

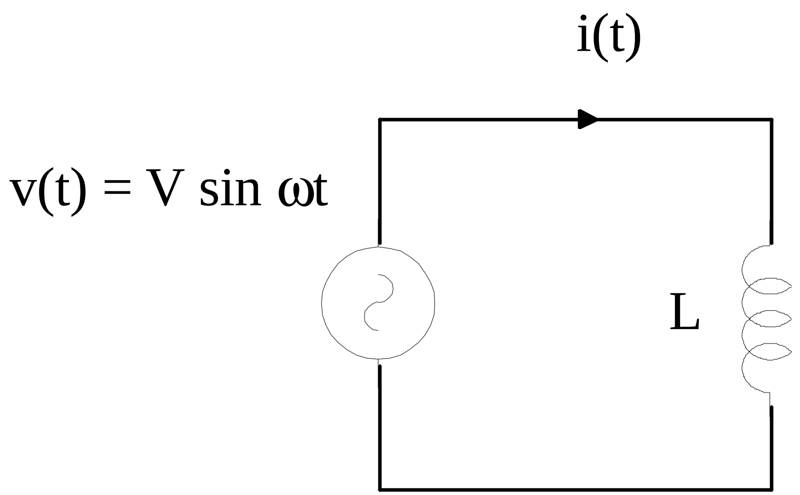
ac voltage and current in the Inductor



Both sides of the equation, $V = L \dfrac{di}{dt}$ must agree.

The Passive components: Inductor

ac voltage and current in the Inductor



<u>Both sides of the equation</u>, $V = L \dfrac{di}{dt}$ <u>must agree</u>.

If the voltage is sinusoidal then the current must be I sin (ωt - $\pi/2$),

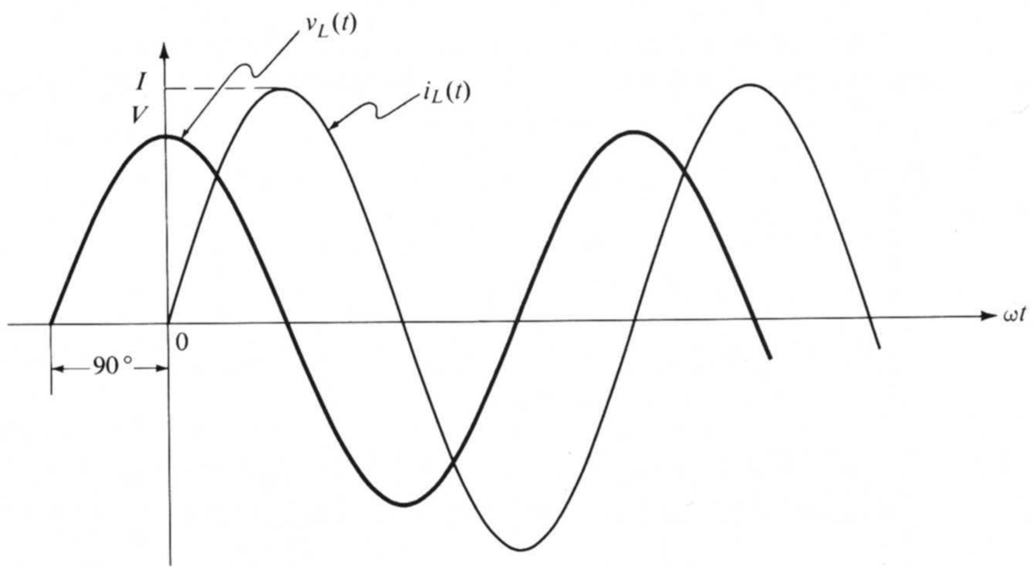
$$V \sin \omega t = L \frac{d}{dt}\{-I \cos \omega t\}$$

The Passive components: Inductor

ac voltage and current in the Inductor

If the voltage is sinusoidal then the current must be I sin (ωt - π/2),

$$V \sin \omega t = L \frac{d}{dt} \{-I \cos \omega t\}$$



In an Inductor the voltage LEADS the current by 90°

The Passive components: Inductor

ac voltage and current in the Inductor (Let us get the MATHS Right!)

If the voltage is sinusoidal then the current must be I sin (ωt - π/2),

$$V \sin \omega t = L \frac{d}{dt} \{-I \cos \omega t\}$$

Now $\frac{d}{dt} \{-I \cos \omega t\} = \omega I \sin \omega t$

so substituting in the above equation gives:

$$V \sin \omega t = L \frac{d}{dt} \{-I \cos \omega t\} = L\,\omega\,(I \sin \omega t)$$

So the fundamental relationship between Voltage and Current for an inductor is ωL (which naturally has units of Ω.)

ωL is known as the Inductive Reactance and is denoted X_L.

The Passive components: Inductor

ac voltage and current in the Inductor

So the *key features for the Inductor* are:

In an Inductor the Voltage LEADS the Current by 90°

(It is perhaps worthy of note that by convention for Inductive circuits we usually refer to the inductor as a LAGGING circuit element i.e. we say ***Current LAGS Voltage***.)

The relationship between Voltage and Current is the

Inductive Reactance: $X_L = (\omega L)\ \Omega$

As $X_L = (\omega L)\ \Omega$ the value of Inductive Reactance

is directly proportional to frequency

The Passive components: Inductor

Power in an Inductor

The instantaneous power delivered from the supply, v(t), is:

$$\text{ac power } p(t) = v(t).i(t) = (V \sin \omega t).\ (-I \cos \omega t) \text{ Watts}$$

Refer to the trigonometric identity:

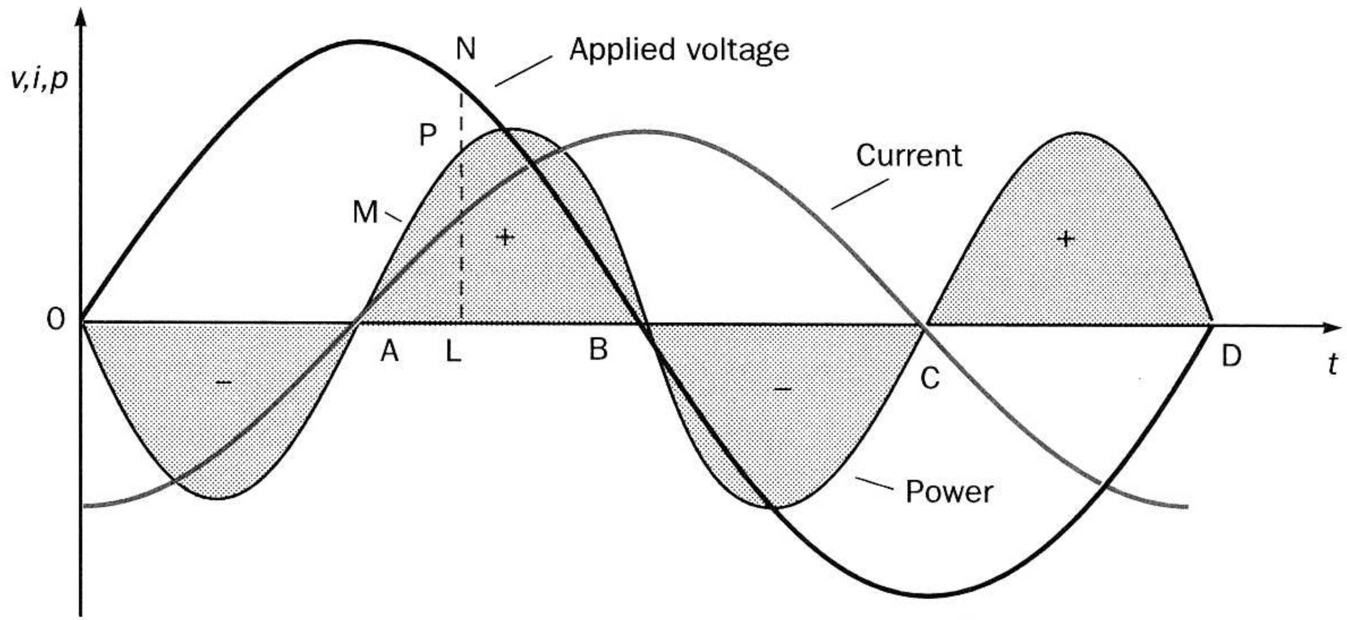
$$\sin A \cos B = 0.5\ [\sin(A - B) + \sin(A + B)]$$

$$\text{Then } p(t) = (V \sin \omega t).\ (-I \cos \omega t) = -0.5VI\ [\sin 2\omega t] \text{ Watts}$$

The Passive components: Inductor

Power in an Inductor

$$p(t) = (V \sin \omega t).(-I \cos \omega t) = -0.5VI [\sin 2\omega t] \text{ Watts}$$



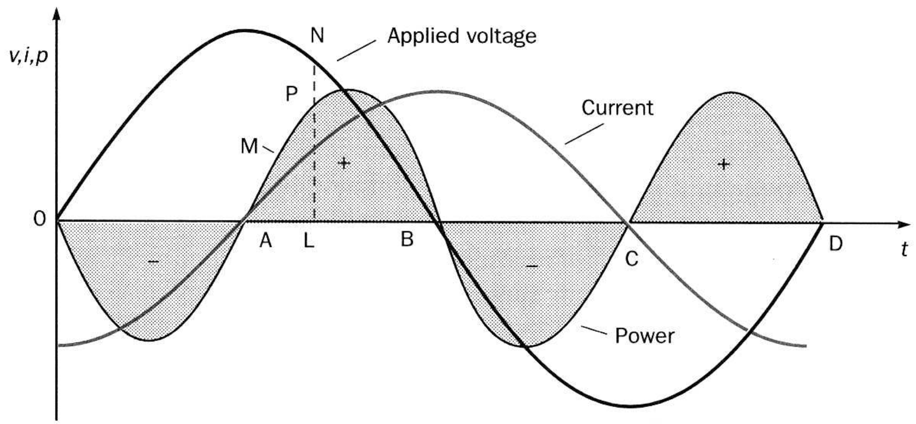
Voltage, current and power in an Inductor

The Passive components: Inductor

Power in an Inductor

We have already proved that a sine or cosine term has an average value of zero.

Thus the perfect Inductor dissipates no power.
This is entirely due to the 90° relationship between Voltage and Current. This is not surprising as 90° infers that the quantities are orthogonal.



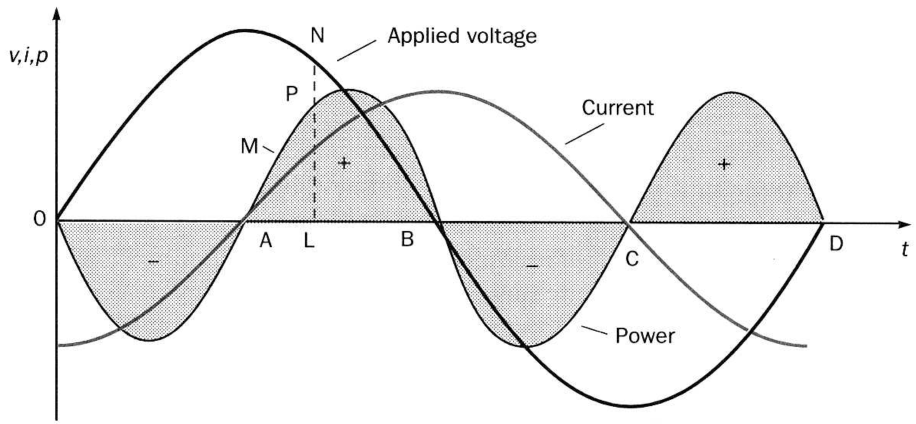
Voltage, current and power in an Inductor

The Passive components: Inductor

Power in an Inductor

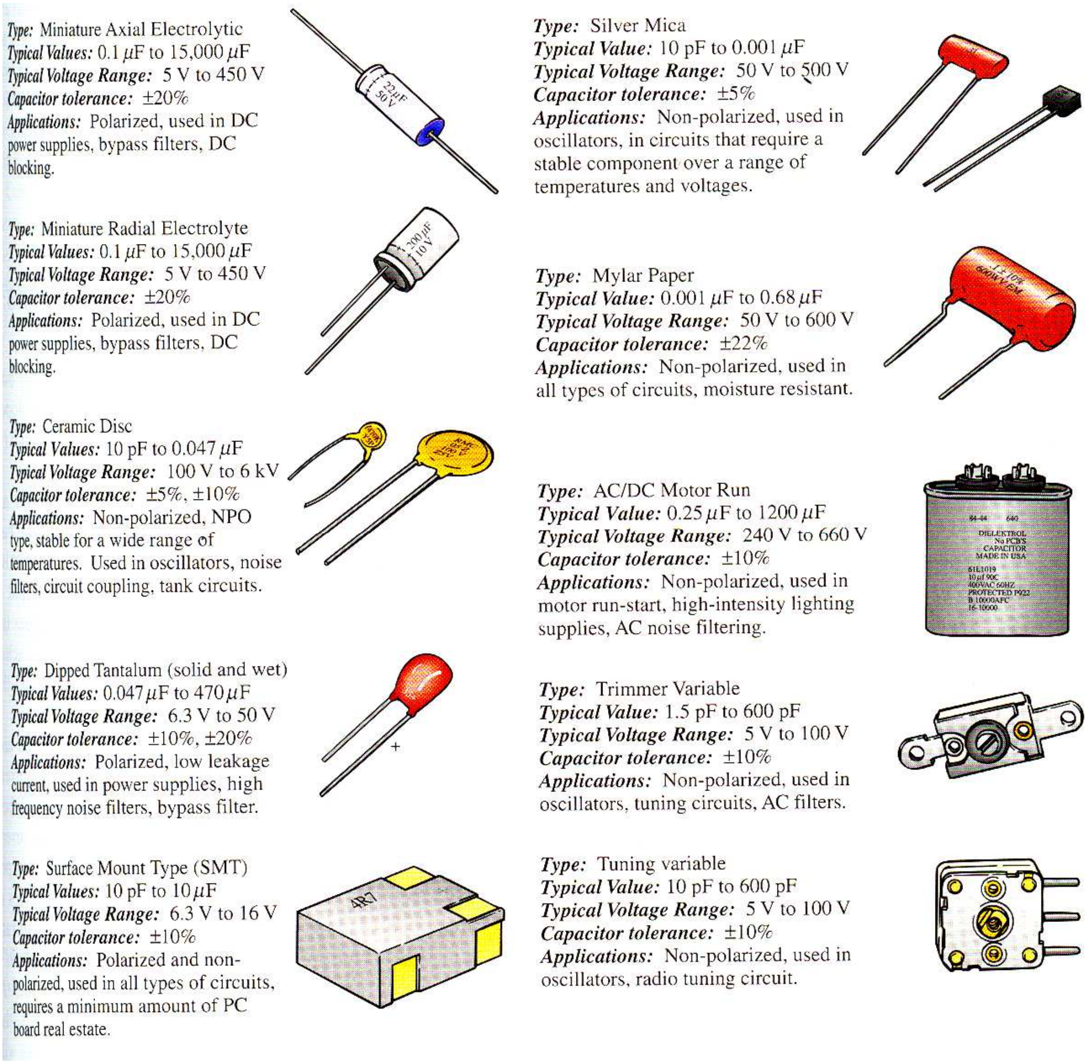
During the time that the voltage and current are both *positive* the power p(t) is positive and power and energy is delivered from the source to the Inductor and stored in the magnetic field.

During the time that the voltage and current have opposite signs the power p(t) is *negative* the stored energy is returned from the Inductance back to the source.



Voltage, current and power in an Inductor

The Passive components: Capacitor



Type: Miniature Axial Electrolytic
Typical Values: 0.1 μF to 15,000 μF
Typical Voltage Range: 5 V to 450 V
Capacitor tolerance: ±20%
Applications: Polarized, used in DC power supplies, bypass filters, DC blocking.

Type: Miniature Radial Electrolyte
Typical Values: 0.1 μF to 15,000 μF
Typical Voltage Range: 5 V to 450 V
Capacitor tolerance: ±20%
Applications: Polarized, used in DC power supplies, bypass filters, DC blocking.

Type: Ceramic Disc
Typical Values: 10 pF to 0.047 μF
Typical Voltage Range: 100 V to 6 kV
Capacitor tolerance: ±5%, ±10%
Applications: Non-polarized, NPO type, stable for a wide range of temperatures. Used in oscillators, noise filters, circuit coupling, tank circuits.

Type: Dipped Tantalum (solid and wet)
Typical Values: 0.047 μF to 470 μF
Typical Voltage Range: 6.3 V to 50 V
Capacitor tolerance: ±10%, ±20%
Applications: Polarized, low leakage current, used in power supplies, high frequency noise filters, bypass filter.

Type: Surface Mount Type (SMT)
Typical Values: 10 pF to 10 μF
Typical Voltage Range: 6.3 V to 16 V
Capacitor tolerance: ±10%
Applications: Polarized and non-polarized, used in all types of circuits, requires a minimum amount of PC board real estate.

Type: Silver Mica
Typical Value: 10 pF to 0.001 μF
Typical Voltage Range: 50 V to 500 V
Capacitor tolerance: ±5%
Applications: Non-polarized, used in oscillators, in circuits that require a stable component over a range of temperatures and voltages.

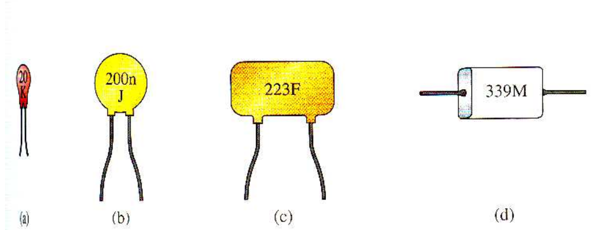
Type: Mylar Paper
Typical Value: 0.001 μF to 0.68 μF
Typical Voltage Range: 50 V to 600 V
Capacitor tolerance: ±22%
Applications: Non-polarized, used in all types of circuits, moisture resistant.

Type: AC/DC Motor Run
Typical Value: 0.25 μF to 1200 μF
Typical Voltage Range: 240 V to 660 V
Capacitor tolerance: ±10%
Applications: Non-polarized, used in motor run-start, high-intensity lighting supplies, AC noise filtering.

Type: Trimmer Variable
Typical Value: 1.5 pF to 600 pF
Typical Voltage Range: 5 V to 100 V
Capacitor tolerance: ±10%
Applications: Non-polarized, used in oscillators, tuning circuits, AC filters.

Type: Tuning variable
Typical Value: 10 pF to 600 pF
Typical Voltage Range: 5 V to 100 V
Capacitor tolerance: ±10%
Applications: Non-polarized, used in oscillators, radio tuning circuit.

Different types of Capacitor (note tolerances)



Different ways of indicating capacitor value

The Passive components: Capacitor

ac voltage and current in the Capacitor

For the Capacitor the voltage to current relationship is:

$$V = \frac{1}{C}\int i\,dt$$

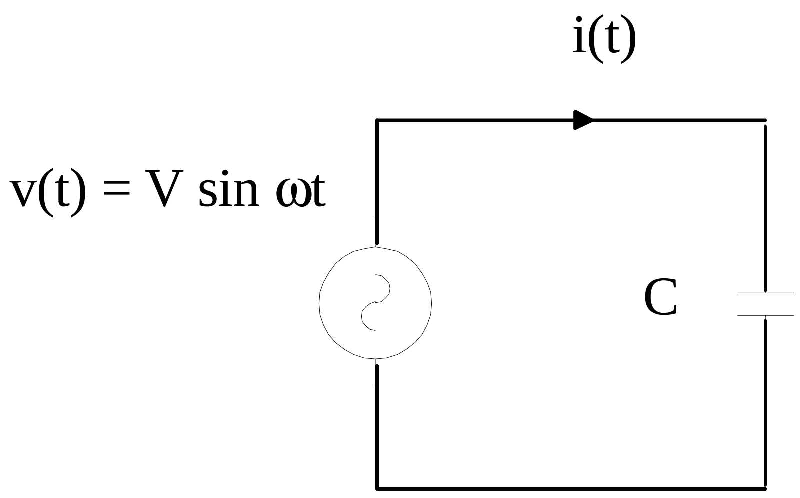
where *V* is the voltage across the Capacitor and *i* is the Current through the Capacitor.

The Passive components: Capacitor

ac voltage and current in the Capacitor

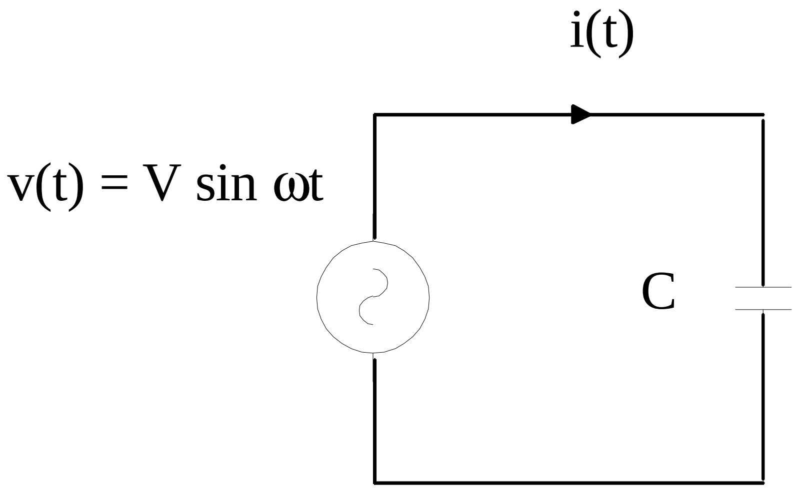
$$V = \frac{1}{C}\int i\,dt$$

If we return to our basic circuit with a voltage v(t) = V sin ωt volts applied across a Capacitor (C).



The Passive components: Capacitor

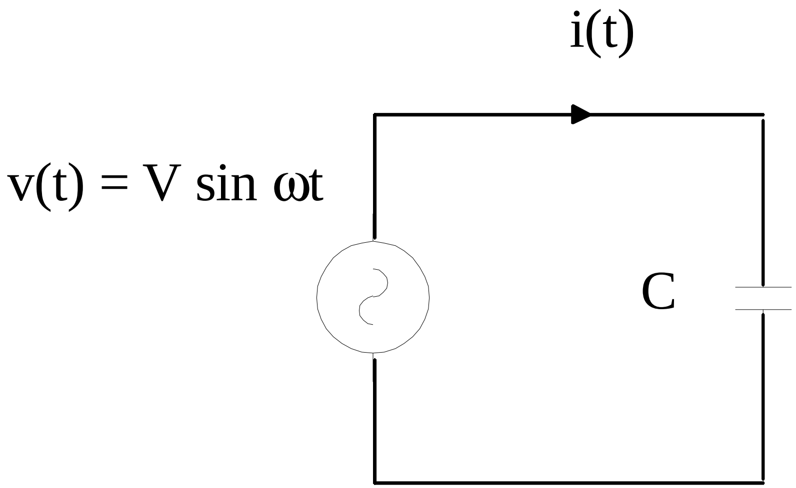
ac voltage and current in the Capacitor



<u>Both sides of the equation</u>, v = $\frac{1}{C}\int i dt$ <u>must agree</u>

The Passive components: Capacitor

ac voltage and current in the Capacitor



<u>Both sides of the equation</u>, v = $\frac{1}{C}\int idt$ <u>must agree</u>

If the voltage is sinusoidal the current must be co sinusoidal

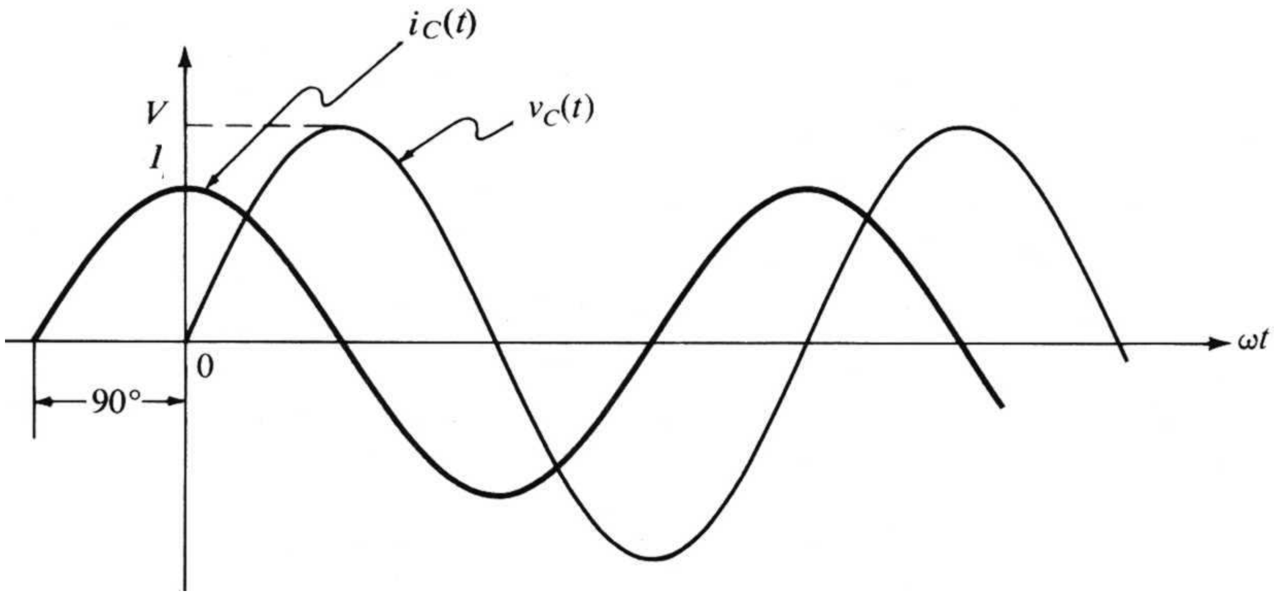
$$V \sin \omega t = \frac{1}{C}\int idt = \frac{1}{C}\int \{I \cos \omega t . dt\}$$

The Passive components: Capacitor

ac voltage and current in the Capacitor

If the voltage is sinusoidal the current must be co sinusoidal

$$V \sin \omega t = \frac{1}{C}\int i dt \quad = \quad \frac{1}{C}\int \{I \cos \omega t . dt\}$$



In a Capacitor the voltage LAGS the current by 90°

The Passive components: Capacitor

ac voltage and current in the Capacitor (Let us get the MATHS Right!)

If the voltage is sinusoidal the current must be co sinusoidal

$$V \sin \omega t = \frac{1}{C}\int i dt \quad = \quad \frac{1}{C}\int \{\mathrm{I} \cos \omega t . dt\}$$

Now $\int \{\mathrm{I} \cos \omega t . dt\}$ = $\frac{1}{\omega}$ I sin ωt, so substituting in the above equation

gives: $V \sin \omega t = \frac{1}{C}\int i dt \quad = \quad \frac{1}{C}\int \{\mathrm{I} \cos \omega t . dt\} \quad = \quad \frac{I}{\omega C} \sin \omega t$

So the fundamental relationship between Voltage and Current for a Capacitor

is $\frac{1}{\omega C}$ which is known as the Capacitive reactance, X_C.

The Passive components: Capacitor

ac voltage and current in the Capacitor (Let us get the MATHS Right!)

So the *key features for the Capacitor* are:

In a Capacitor the Voltage LAGS the Current by 90°

The relationship between Voltage and Current is the Capacitive Reactance,

$$X_C = \frac{1}{\omega C} \quad \Omega$$

As $\frac{1}{\omega C}$ Ω the value of Capacitive Reactance is *inversely* proportional

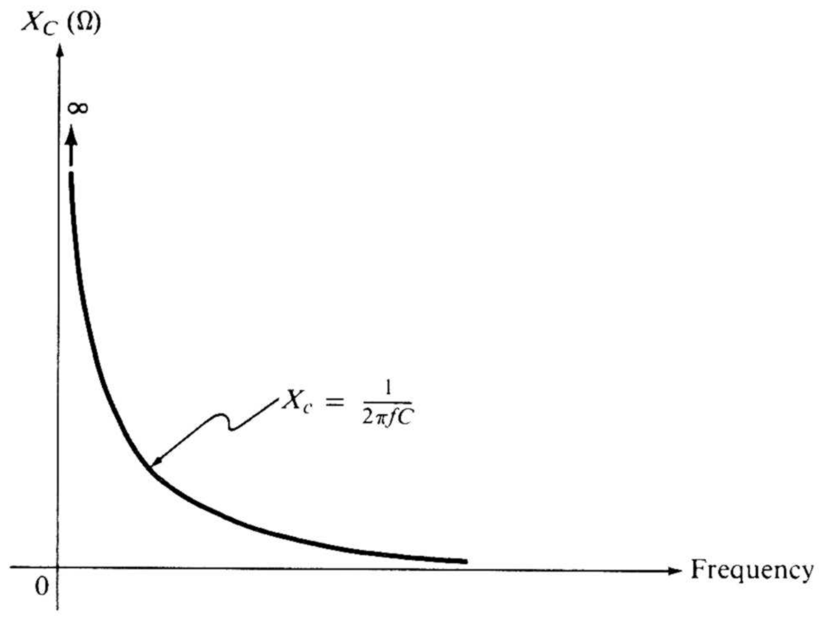
to frequency

The Passive components: Capacitor

ac voltage and current in the Capacitor

As $\frac{1}{\omega C}$ Ω the value of Capacitive Reactance is *inversely* proportional

to frequency.



Plot of Capacitive Reactance, X_C, versus frequency

The Passive components: Capacitor

Power in a Capacitor

The instantaneous power delivered from the supply, v(t), is given by:

$$\text{ac power } p(t) = v(t).i(t) = (V \sin \omega t).\ (I \cos \omega t\ ) \text{ Watts}$$

Refer to the trigonometric identity:

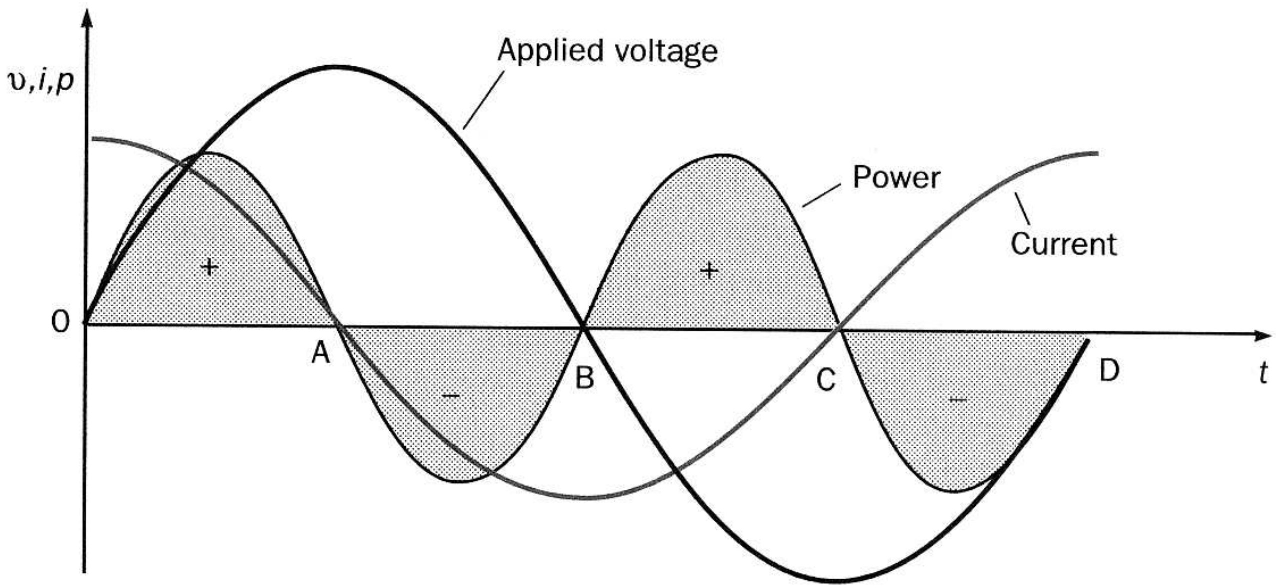
$$\sin A \cos B = 0.5 \sin (A - B) + \sin (A + B)]$$

$$\text{Thus, } p(t) = (V \sin \omega t).\ (I \cos \omega t\ ) = 0.5VI\ [\sin 2\omega t\ ] \text{ Watts}$$

The Passive components: Capacitor

Power in a Capacitor

Thus, $p(t) = (V \sin \omega t).(I \cos \omega t) = 0.5VI [\sin 2\omega t]$ Watts



Voltage, current and power in a Capacitor

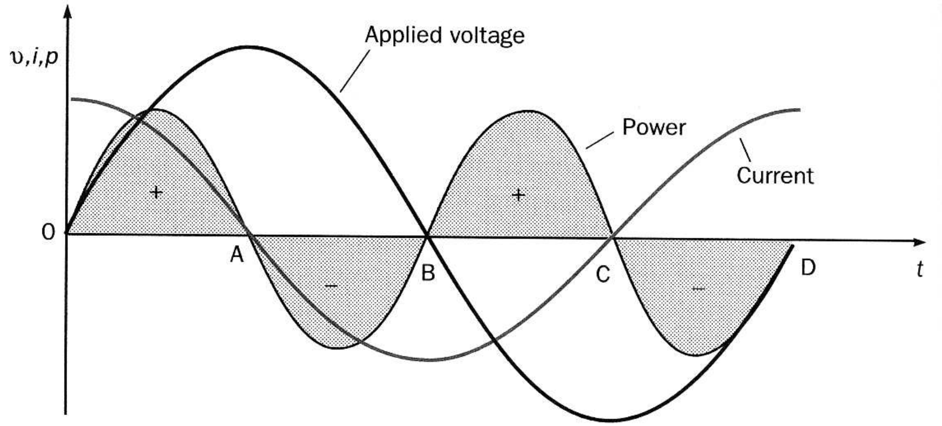
The Passive components: Capacitor

Power in a Capacitor

Now this is the same result (except for the change in sign) as for the Inductor.

Again this is not surprising as 90° infers the two quantities are orthogonal.

Thus the perfect Capacitor dissipates no power, taking energy from the power supply during part of the ac cycle and returning it back to the source during another part of the cycle.



Voltage, current and power in a Capacitor

The Passive components

What about this (j) operator thing?

ac theory has revealed the distinct differences between the three passive components. The key facts are summarised in the following table.

Property	**Resistor**	**Inductor**	**Capacitor**
v versus *i* *relationship*	V = I r	$V = L\frac{di}{dt}$	$v = \frac{1}{C}\int idt$
Average Power dissipated	$v_{rms}i_{rms}$	Zero	Zero
v versus *i* *<u>phase</u> relationship*	*v* in phase with *i*	*v* leads *i* by 90 ° (π/2)	*v* lags *i* by 90 ° (π/2)
Reactance (Ω)	-	$X_L = (\omega L)$	$X_C = \frac{1}{\omega C}$
Reactance versus frequency (ω)	-	proportional	Inversely proportional
This (j) thing?	?	?	?

The Passive components

What about this (j) operator thing?

Clearly in ac theory we need to take into account the phase angles of 90° between the voltage and currents in the reactive devices and the zero phase shift in the resistor.

The obvious way to present this information is by using the *complex plane*. All we need is a reference datum!

The universally accepted approach is to use the Real axis of the complex plane to represent ***the current axis.*** This is an important fact to remember as in order to establish any rule-base you must *always* remember to *what* you are referring.

By making the real axis a current reference we see that for the resistor (voltage in phase with current) we can represent the *phasors* for voltage and current as both on the real axis.

The Passive components

What about this (j) operator thing?

Now for the **Inductor**, the voltage leads the current by 90°.

The (j) operator is of course just a mathematical representation of a 90° anticlockwise rotation.

Reference to current, the inductor voltage is 90° ahead or + j. Thus we display Inductor voltage on the + j axis of the complex plane.

Inductive reactance becomes: $X_L = +j\,\omega L$

The Passive components

What about this (j) operator thing?

For the **Capacitor**, the voltage lags the current by 90°. We thus place the capacitor voltage on the - j plane.

Thus Capacitive Reactance becomes:

$$X_C = -j \frac{1}{\omega C}$$

Note: Frequency Dependence of L & C

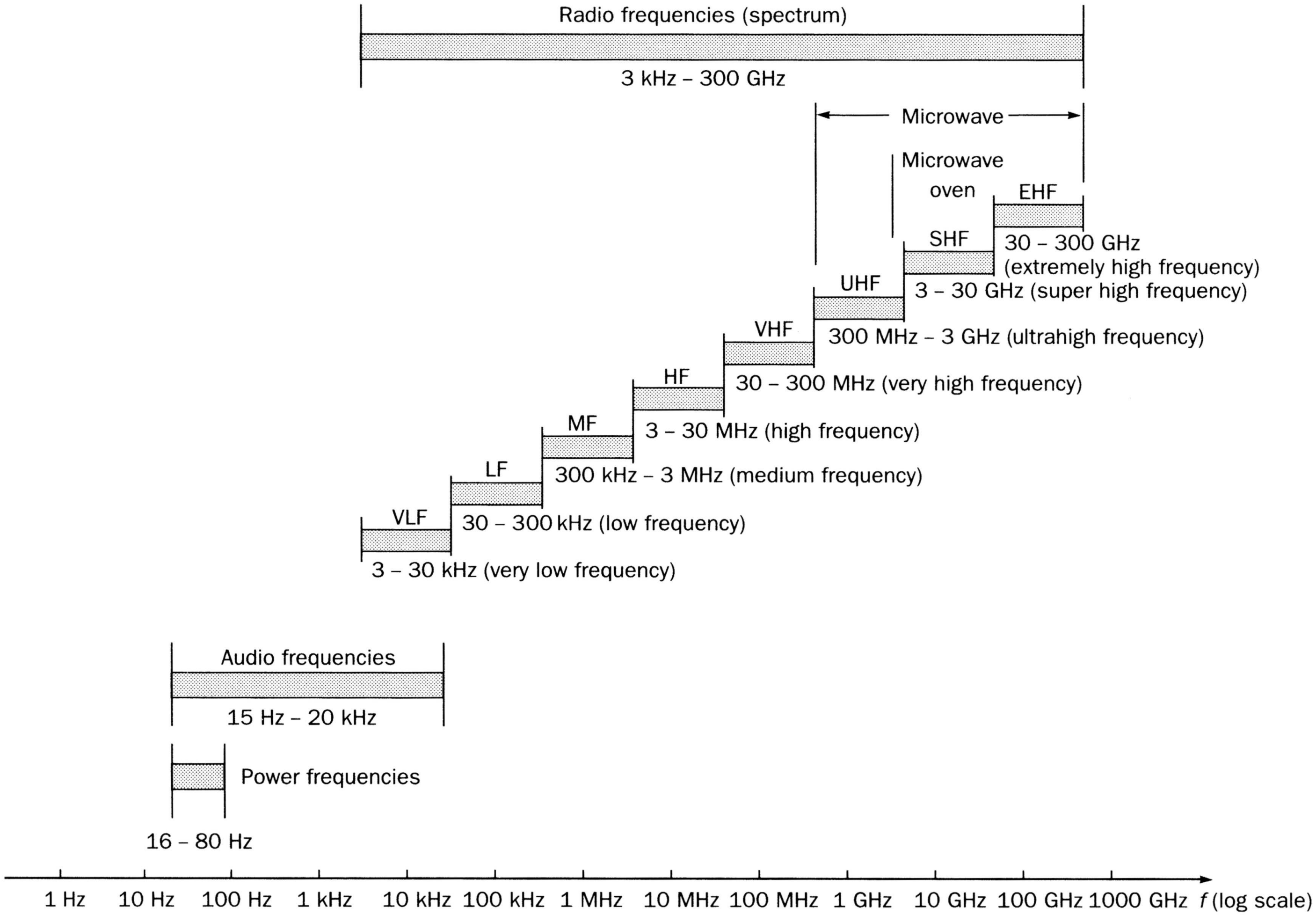
From the introduction to this subject we noted that:

"When circuits can obtain a frequency dependence then we have the basis for radio, communications, mobile phones, computers – indeed every but of electrical engineering you can think of – full stop!"

The range of frequencies used in electrical engineering literally covers the "frequency spectrum".

Note: Frequency Dependence of L & C

The range of frequencies used in electrical engineering literally covers the "frequency spectrum".

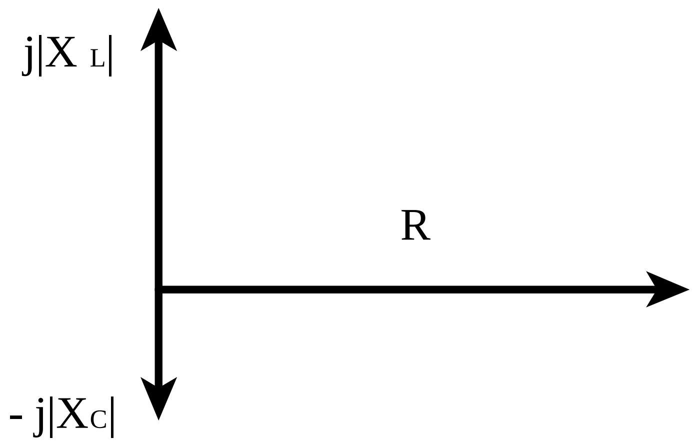


The Passive components

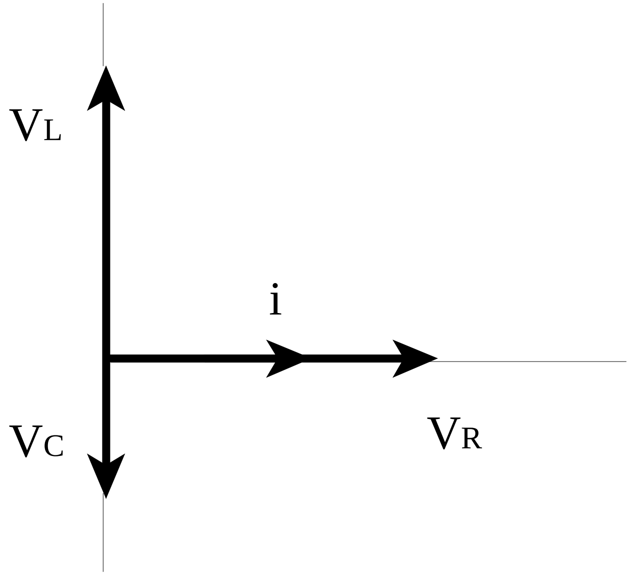
What about this (j) operator thing?

The complex plane will from now on always be our means of handling currents and voltages and resistance's and reactance's.

The ac world is a world of complex mathematics (though not necessarily computationally complexity!).

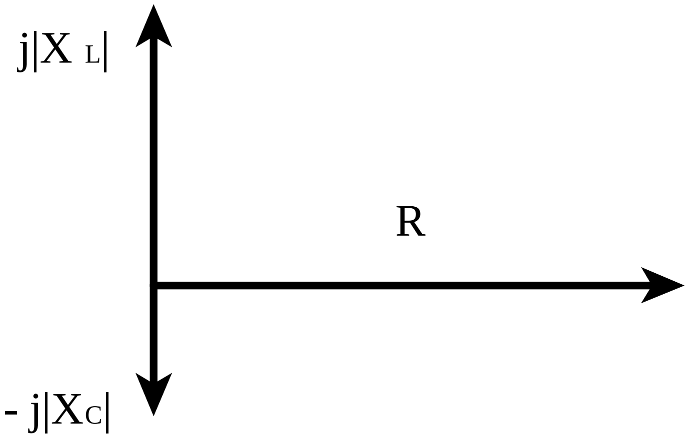


The Impedance Plane

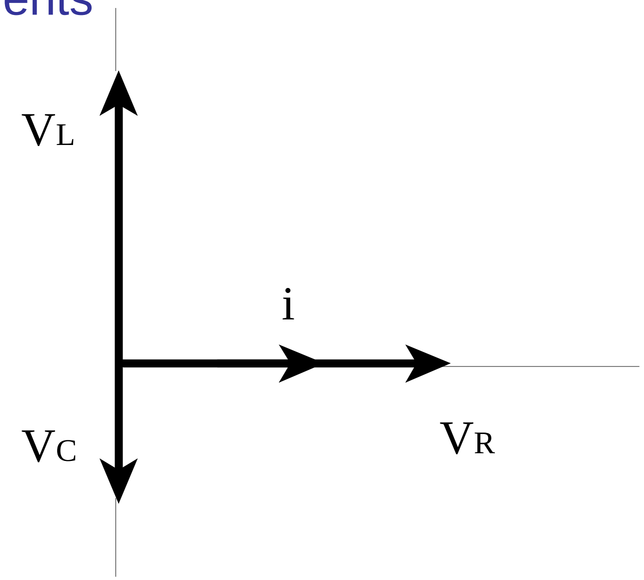


The Voltage Plane *(series circuits)*

The Passive components



The Impedance Plane



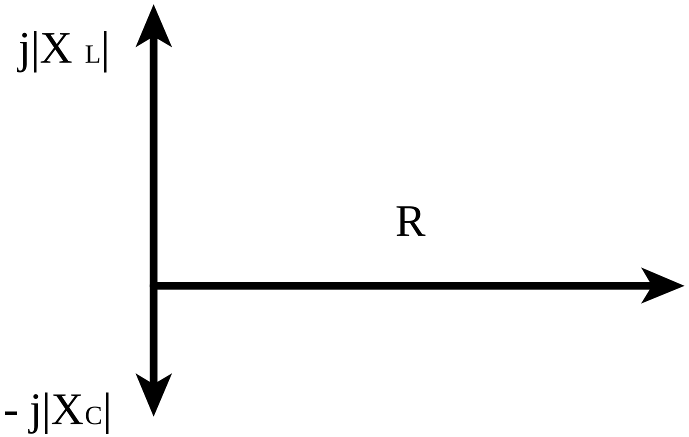
The Voltage Plane (*series circuits*)

Aide-memoire to voltage/currents/lead/lag/Inductor/Capacitor.

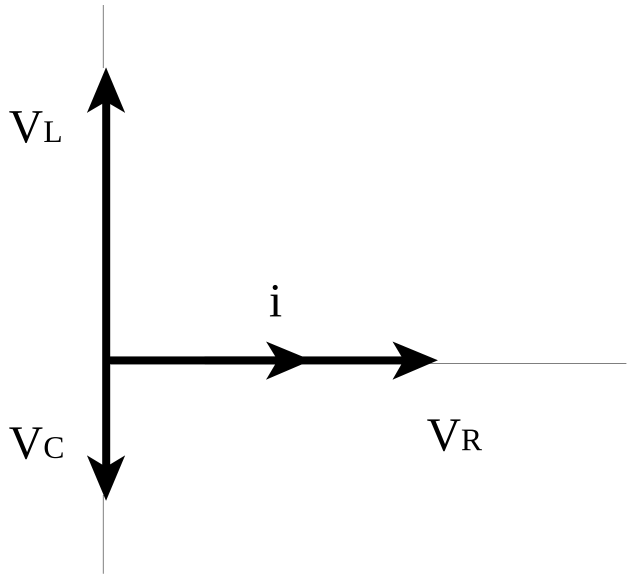
One memory trick is to remember "CIVIL". Reading from left to right:

"In a **C**apacitor **I** (current) leads **V**oltage → (**V**)oltage leads **I** in an **L** (Inductor)"

Impedance



The Impedance Plane



The Voltage Plane (*series circuits*)

The Inductor and Capacitor have Inductive and Capacitive Reactance of

$X_L = j\,\omega L\ \Omega$ and $X_C = -j\,\dfrac{1}{\omega C}\ \Omega$

This led us to the concept of complex impedances and the use of the complex plane, above. Note: for series circuits!

Impedance

The Inductor and Capacitor have Inductive and Capacitive Reactance of

$X_L = j\,\omega L\ \Omega$ and $X_C = -j\,\dfrac{1}{\omega C}\ \Omega$

Very few circuits are pure R, L or C.

Most circuits are combinations of these R, L and C and either:

Series or *Parallel* or combinations of Series/Parallel circuitry.

The rules for series and parallel circuits are identical to those established in the dc circuits part of your course.

However the *difference* in the ac circuits is that all voltages and currents *must be treated as phasors*.

Impedance: Series ac circuits

The total impedance Z of a series circuit containing the three passive components:

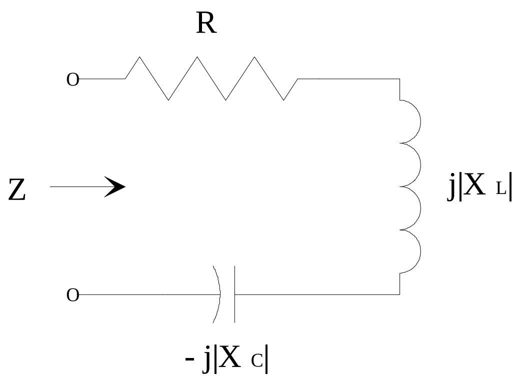
resistance R Ω

inductive reactance $X_L = j\omega L\ \Omega$, and

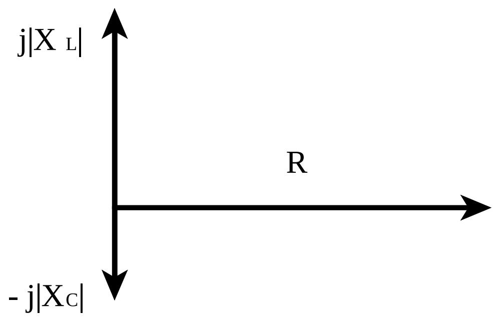
capacitive reactance $X_C = -j\ (1/\ \omega C)\ \Omega$

can be found by combining the phasor forms of each of these components on an *impedance diagram.*

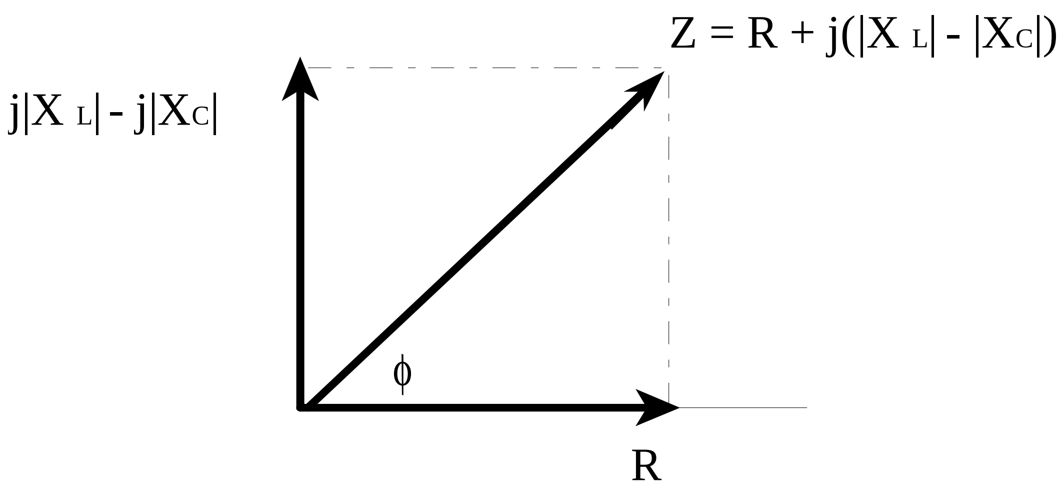
Impedance: Series ac circuits



Series R, L and C

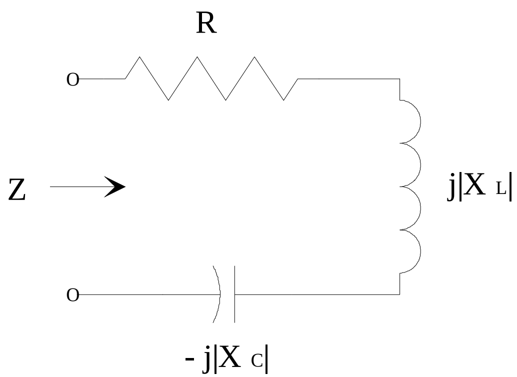


The Impedance Plane

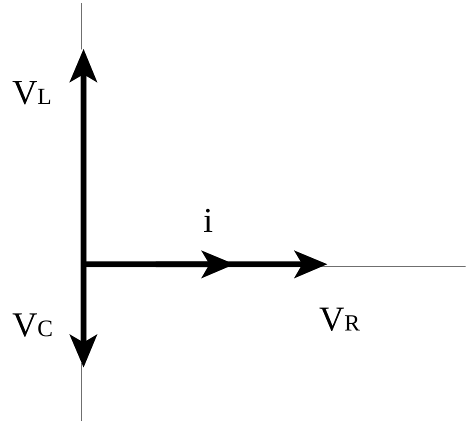


Series RLC circuit impedance diagram

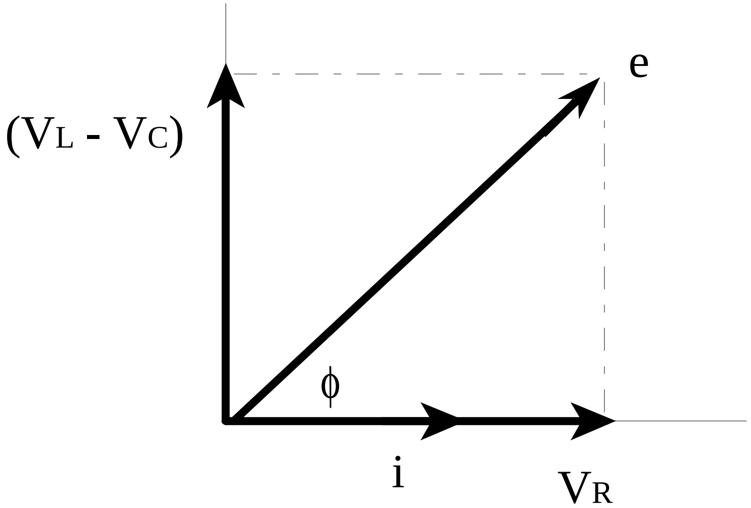
Impedance: Series ac circuits



Series R, L and C

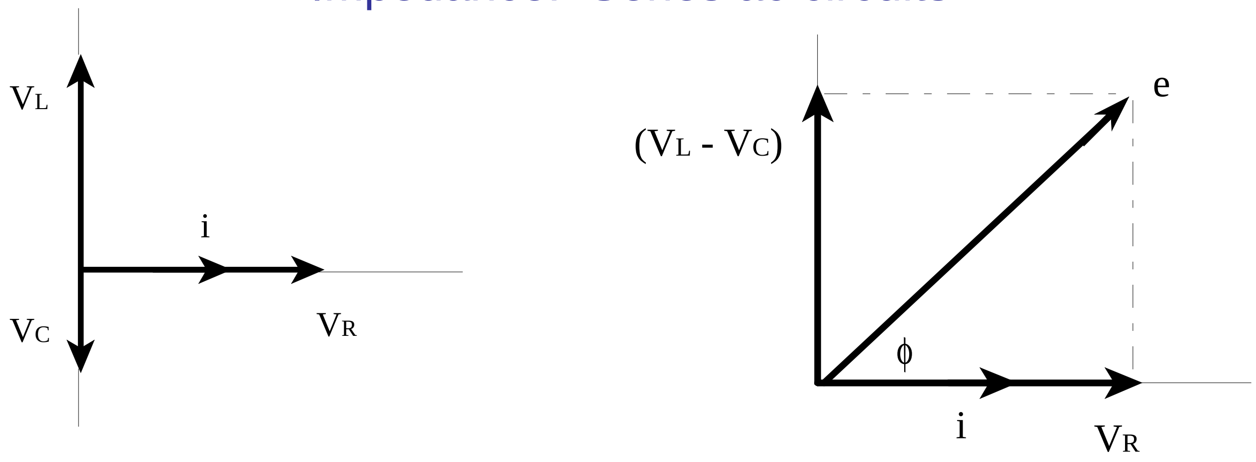


The Voltage Plane (*series circuits*)



Voltage relationships in series RLC circuit

Impedance: Series ac circuits



The Voltage Plane *(series circuits)*

- The voltage across the inductor V_L leads the current *i* through it by 90 degrees
- The current *i* in the capacitor leads the voltage V_C across it by 90 degrees
- The current *i* in a series circuit is in phase with the voltage across the resistor V_R
- The applied voltage *e* is the phasor sum of all the voltage drops in the circuit
- The angle θ between the applied voltage *e* and the current *i* is the same as the angle of the total impedance of the circuit

Impedance: Series ac circuits

Example 1

Impedance: Series ac circuits

Example 1

A coil having a resistance of 12 Ω and an inductance of 0.1 H, is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The reactance and Impedance of the coil

(b) The current flowing in the circuit

(c) The phase difference between the current and the applied voltage

Impedance: Series ac circuits

Example 1

A coil having a resistance of 12 Ω and an inductance of 0.1 H, is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The reactance and Impedance of the coil

Inductive reactance $X_L = \omega L = 2\pi f\, L = 2\pi\,(50)\,(0.1) = 10\pi =$ **+j 31.42Ω**

The coil consists of a series circuit of resistance 12 Ω and an inductive reactance of +j 31.42Ω.

Thus the Impedance **Z = (12 + j 31.42) Ω**.

Impedance: Series ac circuits

Example 1

A coil having a resistance of 12 Ω and an inductance of 0.1 H, is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The reactance and Impedance of the coil **{Z = (12 + j 31.42) Ω}**

(b) The current flowing in the circuit

V = I Z

$Z = (12 + j\,31.42) = 33.6\underline{/69^o}\ \Omega$

$I = V/Z = 100\underline{/0^o} \div 33.6\underline{/69^o}$

$I = 2.97\underline{/-69^o}$ A.

Impedance: Series ac circuits

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$I = 2.97\underline{/-69^{o}}$ A.

(c) The phase difference between the current and the applied voltage

VOLTAGE LEADS THE CURRENT BY 69º

Impedance: Series ac circuits

Example 2

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H and a capacitor of 100μF is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The circuit Impedance

(b) The current flowing in the circuit

(c) The voltage across the R, L and C

(d) the phase difference between the current and the supply voltage

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H and a capacitor of 100μF is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The circuit Impedance

$R = 12\ \Omega.$

$X_L = \omega L = 2\pi f\ L = 2\pi\ (50)\ (0.15) = 15\pi = \mathbf{+j\ 47.12\Omega}$

$X_C = 1/(2\pi\ (50)\ 100\ X\ 10^{-6}) = 1/(\pi\ x\ 10^{-2}) = 100/\pi = \mathbf{-\ j\ 31.83\ \Omega}$

$\mathbf{Z = 12 + j\ 47.12 - j\ 31.83} = \mathbf{(12 + j\ 15.29)\ \Omega}$

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H (+j 47.12) and a capacitor of 100μF (- j 31.83) is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The circuit Impedance **Z = (12 + j 15.29) Ω**

(b) The current flowing in the circuit

V = I Z

$Z = (12 + j\ 15.29) = 19.436\underline{/51.87^{\circ}}\ \Omega$

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H (+j 47.12) and a capacitor of 100μF (- j 31.83) is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The circuit Impedance **$Z = (12 + j\,15.29)\ \Omega$**

(b) The current flowing in the circuit

$V = I\,Z$ $Z = (12 + j\,15.29) = 19.436\underline{/51.87^{\circ}}\ \Omega$

$I = V/Z$ $= (100\underline{/0^{\circ}}) \div (19.436\underline{/51.87^{\circ}}) =$ **$5.145\underline{/-51.87^{\circ}}$ A.**

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H (+j 47.12) and a capacitor of 100μF (- j 31.83) is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The circuit Impedance **Z = (12 + j 15.29) Ω**

(b) The current flowing in the circuit **I = 5.145$\underline{/- 51.87^{\circ}}$ A.**

(c) The voltage across the R, L and C

Voltage across Resistor:

Voltage across Inductor:

Voltage across Capacitor:

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H (+j 47.12) and a capacitor of 100μF (- j 31.83) is connected across a 100 volt peak 50 Hz supply.

Calculate:

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(b) The current flowing in the circuit **I = 5.145$\underline{/-51.87^{o}}$ A.**

(c) The voltage across the R, L and C

Voltage across Resistor: IR = (5.145$\underline{/-51.87^{o}}$) x 12 = 61.8 $\underline{/-51.87^{o}}$ volts

Voltage across Inductor:

Voltage across Capacitor:

Impedance: Series ac circuits

Example 2

A circuit having a resistance of 12 Ω, an inductance of 0.15 H (+j 47.12) and a capacitor of 100μF (- j 31.83) is connected across a 100 volt peak 50 Hz supply.

Calculate:

(a) The circuit Impedance **$Z = (12 + j\,15.29)\ \Omega$**

(b) The current flowing in the circuit **$I = 5.145\underline{/-51.87^\circ}$ A**.

(c) The voltage across the R, L and C

Voltage across Resistor: $IR = (5.145\underline{/-51.87^\circ}) \times 12 = 61.8\ \underline{/-51.87^\circ}$ volts

Voltage across Inductor: $IX_L = (5.145\underline{/-51.87^\circ}) \times \mathbf{47.12\underline{/90^\circ}} = 242.5\underline{/38.13^\circ}$ volts

Voltage across Capacitor:

Impedance: Series ac circuits

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(c) The voltage across the R, L and C

Voltage across Resistor: IR = (5.145/- 51.87º) x 12 = 61.8 /- 51.87º volts

Voltage across Inductor: IX_L= (5.145/- 51.87º) x **47.12/90º** = 242.5/38.13º volts

Voltage across Capacitor: IX_C= (5.145/- 51.87º) x **31.83/-90º** = 163.76/-141.87º volts

Impedance: Series ac circuits

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(c) The voltage across the R (61.8 /- 51.87º), L (242.5/38.13º) and C (163.76/-141.87º)

(d) the phase difference between the current and the supply voltage

(i) v = 100/0º volts and i = 5.145/- 51.87º A ….so Φ = 51.87º

Impedance: Series ac circuits

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(ii) R (61.8 volts) and "X" = (242.5 – 163.76) volts. ("X" = 78.74 volts)

Impedance: Series ac circuits

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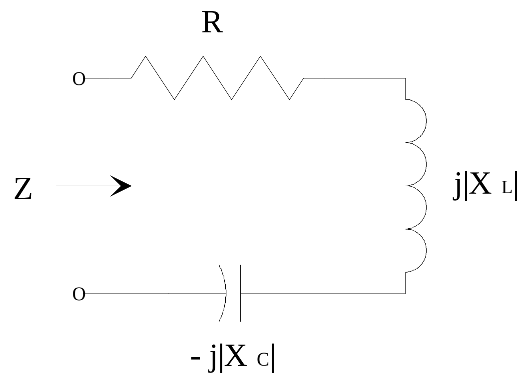
(d) the phase difference between the current and the supply voltage

(i) $v = 100\underline{/0^{o}}$ volts and $i = 5.145\underline{/-51.87^{o}}$ A ….so $\Phi = 51.87^{o}$

(ii) R (61.8 volts) and "X" = (242.5 – 163.76) volts. ("X" = 78.74 volts)

So $\tan \Phi = 78.76/61.8 = 1.274$……thus $\Phi = 51.8^{o}$✓

Impedance: Series ac circuits



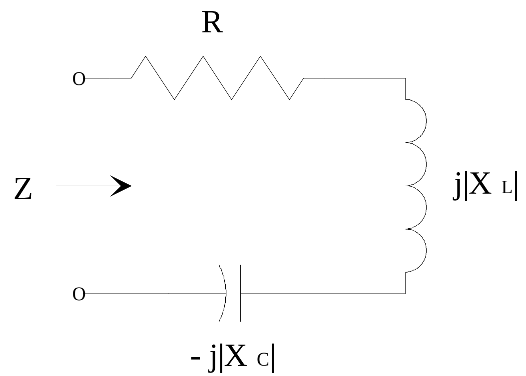
Series Resonance

Consider the RLC series circuit connected to a sinusoidal signal generator whose frequency can be varied over a wide range of frequencies.

As the frequency is increased, the magnitude of the inductive reactance increases according to $|X_L| = \omega L$, whereas the magnitude of the capacitive reactance term decreases according to $|X_C| = 1/\ \omega C$.

At *some* frequency, which we shall call f_S, $|X_L|$ will have exactly the same magnitude as $|X_C|$. The frequency at which $|X_L| = |X_C|$ is called the *resonant frequency* and at this frequency (only) the circuit will have no reactance term and will thus be purely resistive.

Impedance: Series ac circuits



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This will be a situation where the circuit will have a minimum impedance (resistance) and thus a maximum current. Since the voltage across the resistor is $V_R = iR$, it also follows that the voltage across the resistor will have a maximum value at resonance.

Impedance: Series ac circuits

Class Example 1

Impedance: Series ac circuits

Class Example 1

The series circuit of $R = 10\Omega$ and $C = 40\mu F$ has an applied voltage:

$$v(t) = 500 \cos(2500t - 20^\circ) \text{ volts.}$$

Determine:

(i) the capacitive reactance

(ii) the circuit impedance

(iii) the current through the series circuit.

Draw the phasor diagram of i and v.

Impedance: Series ac circuits

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Impedance: Series ac circuits

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$X_C = 1/(2500 \times 40 \times 10^{-6}) =$ **$-j\,10\ \Omega$**

Impedance: Series ac circuits

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Impedance: Series ac circuits

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Determine:

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(ii) the circuit impedance

$\mathbf{Z =} (10 - j10) = 10\sqrt{2}\ \underline{/-45^o}\ \Omega$

Impedance: Series ac circuits

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Impedance: Series ac circuits

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(iii) the current through the series circuit.

V = I Z

$\mathbf{I = V/Z} = (500\underline{/-20^\circ})\ /\ (10\sqrt{2}\ \underline{/-45^\circ}) = \mathbf{25\sqrt{2}\underline{/25^\circ}\ A.}$

Impedance: Series ac circuits

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The series circuit of $R = 10\Omega$ and $C = 40\mu F$ has an applied voltage:

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$V = I\ Z$

$I = V/Z = (\mathbf{500\underline{/-20°}}) / (10\sqrt{2}\ \underline{//-45°}) =$ **$25\sqrt{2}\ /\underline{25°}$ A.**

Draw the <u>phasor diagram</u> of i and v.

Impedance: Series ac circuits

Class Example 2

Impedance: Series ac circuits

Class Example 2

A U.K. mains connected series circuit consists of a resistor, $R = 100\Omega$ and an inductor, $L = 0.1$ H. What is the circuit impedance?

Impedance: Series ac circuits

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$X_L = +j\,\omega L = +j\,(2\pi f)\,L = 2\pi\,(50)\,(0.1) = 10\pi =$ **+j 31.42Ω**

Impedance: Series ac circuits

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Circuit impedance (Z) = (100 + j 31.42) Ω

Polar:

Impedance: Series ac circuits

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Circuit impedance (Z) = (100 + j 31.42) Ω

Polar: 104.82 $\underline{/17.4^{o}}$ Ω

Impedance: Series ac circuits

Class Example 3

Impedance: Series ac circuits

Class Example 3

Sometimes rather than quoting the inductor (H) and capacitor (F) values we simply quote their reactive impedance's at the operating frequency of the circuit.

For example: If a 240 $\underline{/\ 0^\circ}$ volt rms ac supply is connected across a series circuit consisting of a 15 Ω resistor and a 15 Ω inductive reactance.
Determine:

(i) The supply current
(ii) Resistor and inductor voltages
(iii) the voltage phasor diagram

Impedance: Series ac circuits

Class Example 3

If a 240 $\underline{/\ 0^\circ}$ volt rms ac supply is connected across a series circuit consisting of a 15 Ω resistor and a 15 Ω inductive reactance.

Determine:

(i) The supply current

$Z = (15 + j15) = 21.1\ \underline{/45^\circ}\ \Omega$

$I = 240\ \underline{/\ 0^\circ}\ /\ 21.1\ \underline{/45^\circ} = 11.3\ /\underline{-\ 45^\circ}$ A rms

Impedance: Series ac circuits

Class Example 3

If a 240 $\underline{/ 0^\circ}$ volt rms ac supply is connected across a series circuit consisting of a 15 Ω resistor and a 15 Ω inductive reactance.

Determine:

(i) The supply current $Z = (15 + j15) = 21.1 \underline{/45^o}\ \Omega$ & $I = 11.3 \underline{/- 45^o}$ A rms

(ii) Resistor and inductor voltages

Voltage across Resistor: $IR = (11.3 \underline{/- 45^o}) \times 15 = 169.5 \underline{/- 45^o}$ volts

Impedance: Series ac circuits

Class Example 3

If a 240 $\angle 0^\circ$ volt rms ac supply is connected across a series circuit consisting of a 15 Ω resistor and a 15 Ω inductive reactance.

Determine:

(i) The supply current $Z = (15 + j15) = 21.1 \angle 45^\circ\ \Omega$ & $I = 11.3 \angle -45^\circ$ A rms

(ii) Resistor and inductor voltages

Voltage across Resistor: $IR = (11.3 \angle -45^\circ) \times 15 = 169.5 \angle -45^\circ$ volts

Voltage across Inductor: $IX_L = (11.3 \angle -45^\circ) \times 15 \angle 90^\circ = 169.5 \angle 45^\circ$ volts

Impedance: Series ac circuits

Class Example 3

If a **240 /0°** volt rms ac supply is connected across a series circuit consisting of a 15 Ω resistor and a 15 Ω inductive reactance.

Determine:

(i) The supply current Z = (15 + j15) = 21.1 /45º Ω & I = 11.3 /- 45º A rms

(ii) Resistor and inductor voltages

Voltage across Resistor: IR = (11.3 /- 45º) x 15 = **169.5 /- 45º** volts

Voltage across Inductor: IX_L= (11.3 /- 45º) x 15/90º = **169.5 /45º** volts

(iii) the voltage phasor diagram

Impedance: Series ac circuits

Class Example 4

Impedance: Series ac circuits

Class Example 4

A series circuit has a voltage of 120 $\underline{/30°}$ volts rms. applied across it and a current of 3 $\underline{/ -15°}$ Amps rms. flowing.
Determine the circuit impedance in terms of resistance, and inductive or capacitance reactance.

Impedance: Series ac circuits

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A series circuit has a voltage of 120 $\underline{/30^\circ}$ volts rms. applied across it and a current of 3 $\underline{/ -15^\circ}$ Amps rms. flowing.
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V = I Z

Thus Z = V/I

Impedance: Series ac circuits

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A series circuit has a voltage of 120 $\underline{/30^\circ}$ volts rms. applied across it and a current of 3 $\underline{/-15^\circ}$ Amps rms. flowing.
Determine the circuit impedance in terms of resistance, and inductive or capacitance reactance.

V = I Z

Thus Z = V/I

Z = (120 $\underline{/30^\circ}$) / (3 $\underline{/-15^\circ}$) = 40$\underline{/45^\circ}$ Ω

Impedance: Series ac circuits

Class Example 4

A series circuit has a voltage of 120 $\underline{/30^\circ}$ volts rms. applied across it and a current of 3 $\underline{/ -15^\circ}$ Amps rms. flowing.
Determine the circuit impedance in terms of resistance, and inductive or capacitance reactance.

V = I Z

Thus Z = V/I

$Z = (120\ \underline{/30^\circ}) / (3\ \underline{/ -15^\circ}) = 40\underline{/45^o}\ \Omega$

Only meaningful if we express impedance in rectangular format:

Hence, $40\underline{/45^o} = (28.3 + j\ 28.3)$

i.e. 28.3 Ω resistor in series with an inductive reactance of + j 28.3 Ω

Impedance: Series ac circuits

Class Example 5

Impedance: Series ac circuits

Class Example 5

Construct the voltage/current phasor diagram and the impedance diagram for a series circuit that has a voltage of $v(t) = 311 \sin(2500t + 170^\circ)$ volts across it and a current of $i(t) = 15.5 \sin(2500t - 145^\circ)$ Amps flowing.

Impedance: Series ac circuits

Class Example 5

Construct the voltage/current phasor diagram and the impedance diagram for a series circuit that has a voltage of v(t) = 311 sin (2500t + 170°) volts across it and a current of i(t) = 15.5 sin (2500t - 145°) Amps flowing.

v = 311 $\underline{/+ 170^{o}}$ volts

i = 15.5 $\underline{/- 145^{o}}$ Amps

Impedance: Series ac circuits

Class Example 5

Construct the voltage/current phasor diagram and the impedance diagram for a series circuit that has a voltage of $v(t) = 311 \sin(2500t + 170^\circ)$ volts across it and a current of $i(t) = 15.5 \sin(2500t - 145^\circ)$ Amps flowing.

$v = 311 \underline{/+170^\circ}$ volts

$i = 15.5 \underline{/-145^\circ}$ Amps

Hence $Z = 20 \underline{/-45^\circ}\ \Omega$

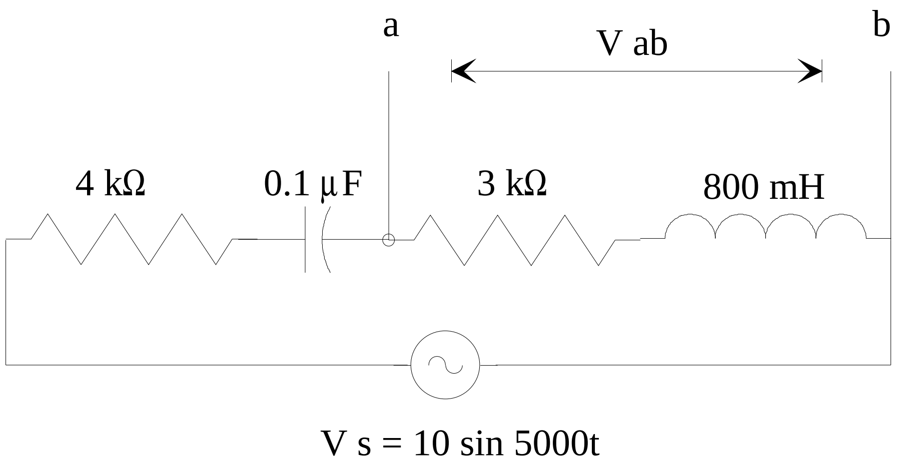
Impedance: Series ac circuits

Class Example 6

Impedance: Series ac circuits

Class Example 6

In the circuit determine the voltage V_{ab} in terms of the applied voltage V_S.



Impedance: Series ac circuits

Class Example 6

In the circuit determine the voltage V_{ab} in terms of the applied voltage V_S.

(i) Calculate Reactance's/Impedances

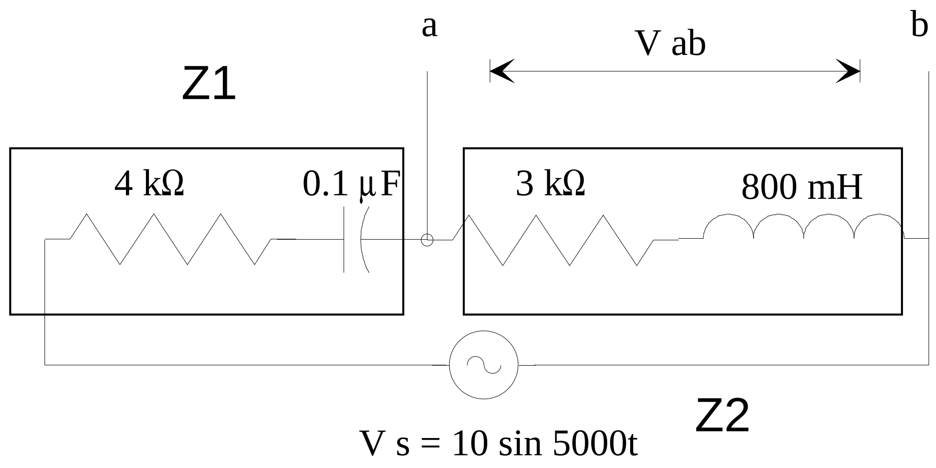
(ii) Consider "potential divider"

Where

Z1 =

Z2 =

Hence V_{ab}



Impedance: Series ac circuits

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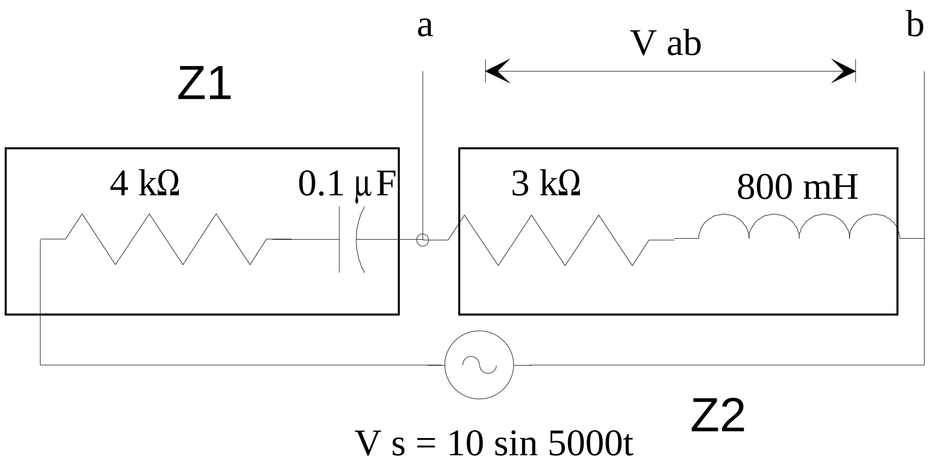

X_C = -j (1/ωC) =

1/ (5000 x 0.1 x 10^{-6}) = **- j 2000Ω**

X_L = ωL = (5000 x 800 x 10^{-3}) =

+ j 4000 Ω

Z1 = (4000 – j 2000) Ω

Z2 = (3000 + j 4000) Ω

Impedance: Series ac circuits

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In the circuit determine the voltage V_{ab} in terms of the applied voltage V_S.

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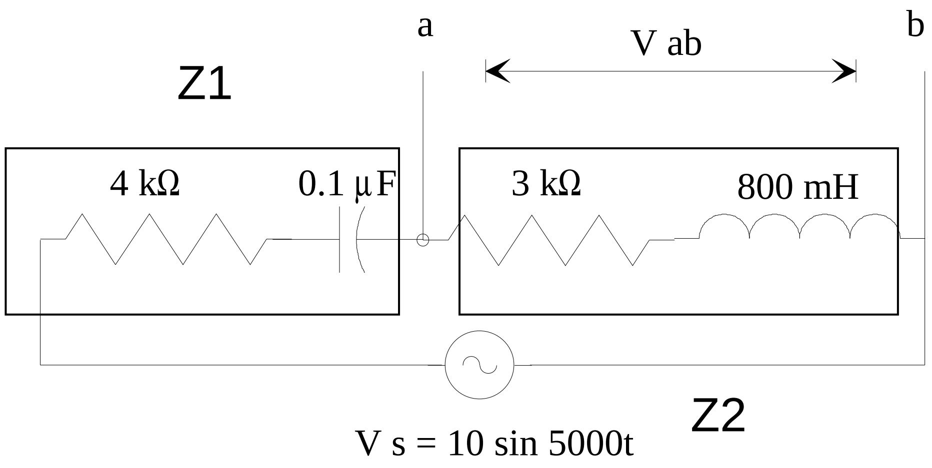

Z1 = (4000 – j 2000) Ω

Z2 = (3000 + j 4000) Ω

(ii) Consider "potential divider"

$V_{ab} = 10\ /\underline{0^o} \times \{(Z2)/(Z1 + Z2)\}$

$= 10\ /\underline{0^o} \times [\{\mathbf{(3000 + j\,4000)\ /\ [(4000 - j\,2000) + (3000 + j\,4000)]}$

Impedance: Series ac circuits

Class Example 6

In the circuit determine the voltage V_{ab} in terms of the applied voltage V_S.

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Z2 = (3000 + j 4000) Ω

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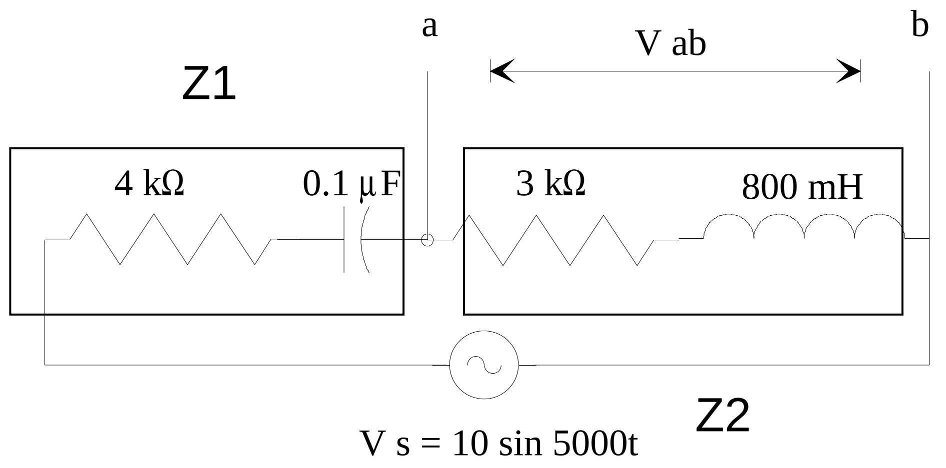


$V_{ab} = 10\ /\underline{0^{\circ}} \times \{(Z2)/(Z1 + Z2)\}$

$= 10\ /\underline{0^{\circ}} \times [\{(3000 + j\,4000) / [(4000 – j\,2000) + (3000 + j\,4000)]$

$\rightarrow$ 6.87 $/\underline{37^{\circ}}$ volts or **6.87 sin (5000t + 37º) volts**

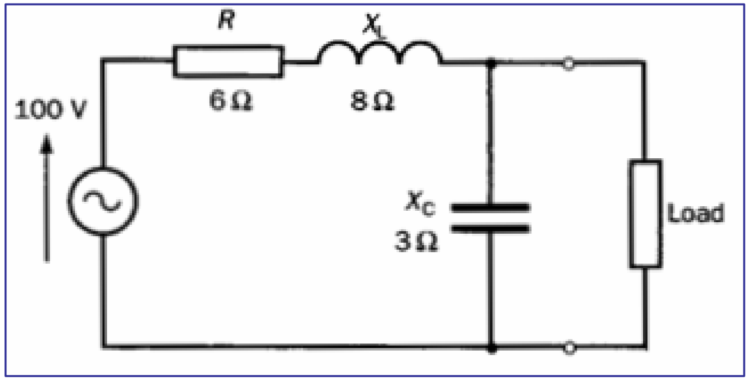
Thevenin

Thevenin

Determine the Thevenin equivalent for the network below.
Hughes: Example 15.8

L = j8Ω

C = -j3Ω



(a) ac network with load resistor

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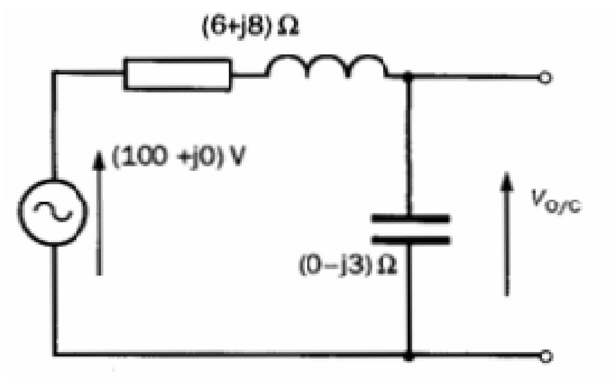
Hughes: Example 15.8

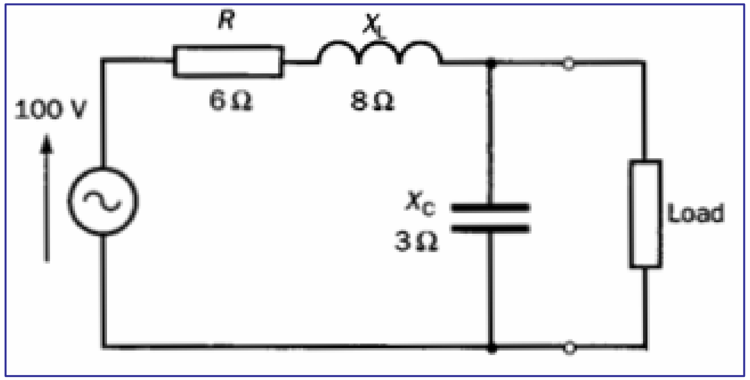
Step 1



(b) Thevenin Voltage: remove the load resistor & determine the open circuit voltage

L = j8Ω

C = -j3Ω



(a) ac network with load resistor

Thevenin

Determine the Thevenin equivalent for the network below.
Hughes: Example 15.8

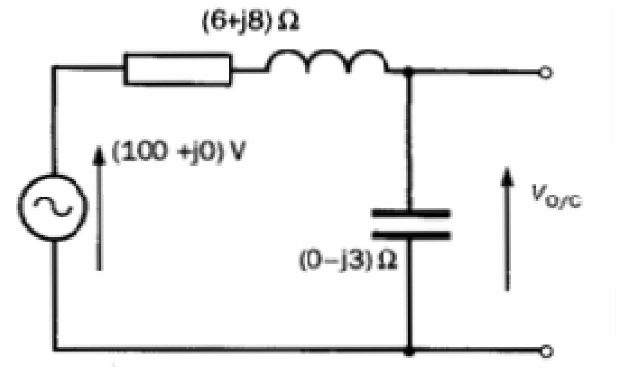
Step 1

(b) Potential division of 100/0º volts by (6 +j8) and (-j3)

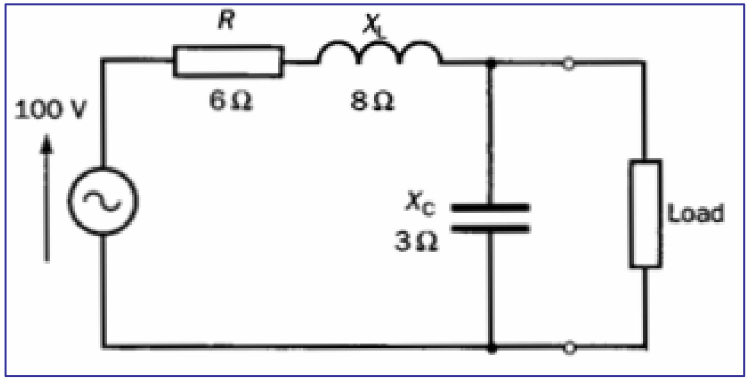
$$V_T = \left(\frac{(100\angle 0^\circ)(-j3)}{(0-j3)+(6+j8)}\right) = \left(\frac{(100\angle 0^\circ)(3\angle -90^\circ)}{(6+j5)}\right) = \left(\frac{300\angle -90^\circ)}{(7.81\angle 39.8^\circ)}\right) = (38.4\angle -129.8^\circ)volts$$

L = j8Ω

C = -j3Ω



(b) Thevenin Voltage: remove the load resistor & determine the open circuit voltage



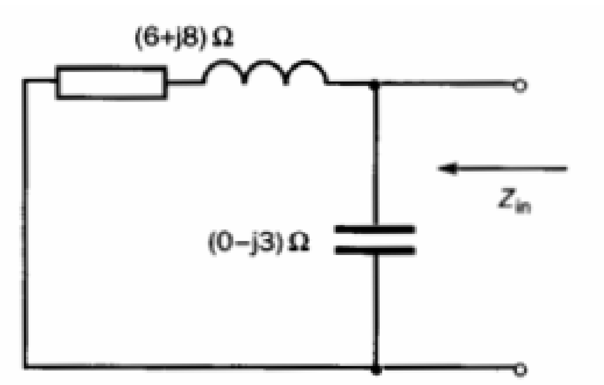
(a) ac network with load resistor

Thevenin

Determine the Thevenin equivalent for the network below.
Hughes: Example 15.8

Step 2

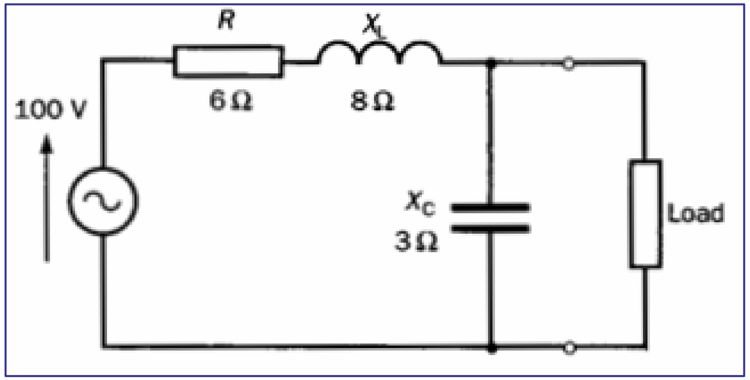
(c) Z_T= (6 +j8) in parallel with (-j3)



(c) Thevenin Impedance: voltage source short circuited

L = j8Ω

C = -j3Ω



(a) ac network with load resistor

Thevenin

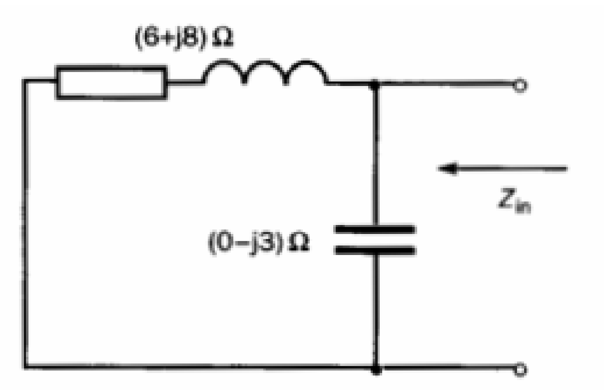
Determine the Thevenin equivalent for the network below.
Hughes: Example 15.8

Step 2

(c) Z_T= (6 +j8) in parallel with (-j3)

$$\boxed{\left(\frac{(6+j8)(-j3)}{(6+j8)+(-j3)}\right)=\left(\frac{(10\angle 53°)(3\angle -90°)}{(7.81\angle 39.8°)}\right)=\left(\frac{(30\angle -37°)}{(7.81\angle 39.8°)}\right)=3.84\angle -76.8°\Omega}$$

or (0.876 –j3.73)Ω



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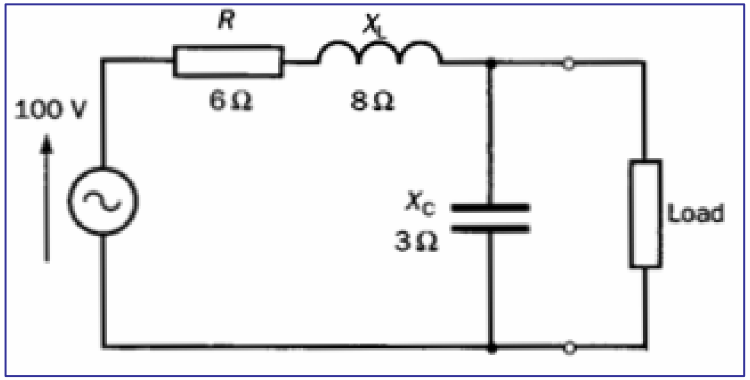

(a) ac network with load resistor

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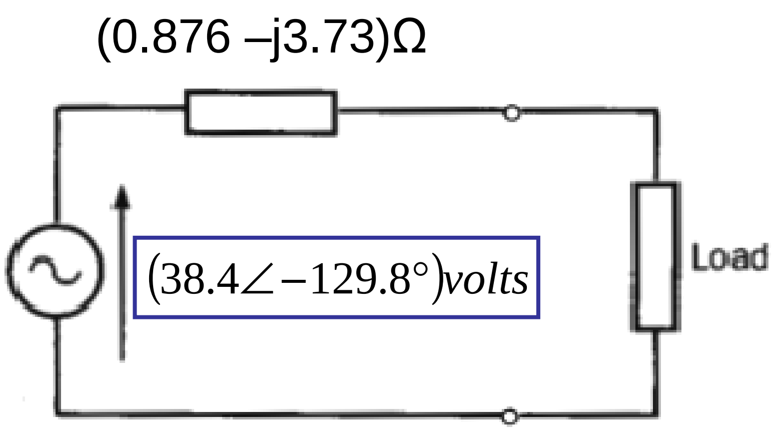
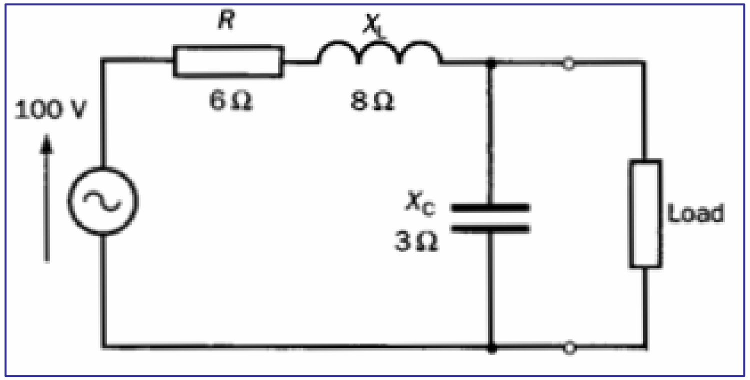


(d) Thevenin Equivalent Circuit

L = j8Ω

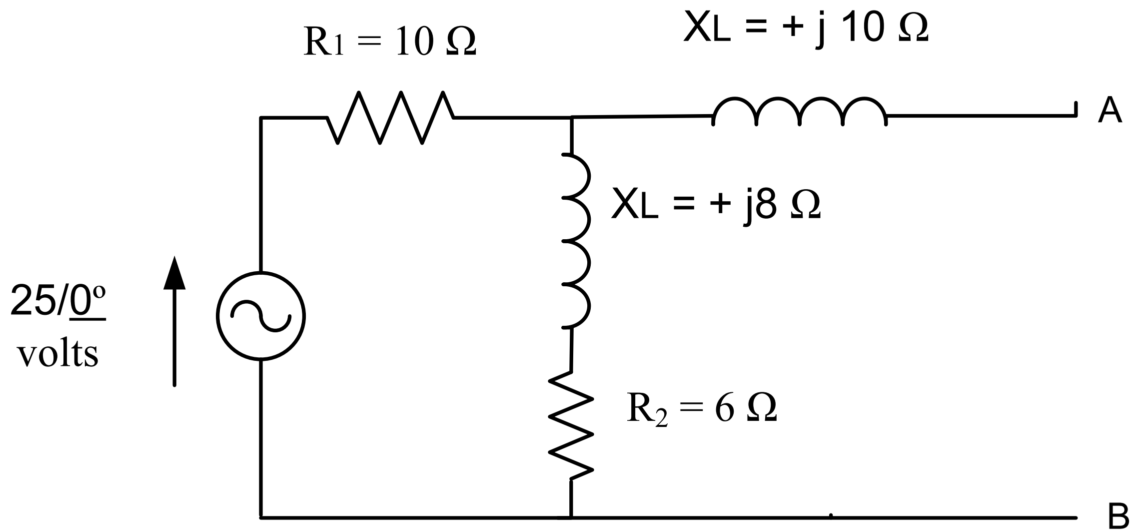
C = -j3Ω



(a) ac network with load resistor

Thevenin

Determine the Thevenin equivalent for the network below.



(a) Thevenin Voltage: determine the open circuit voltage
(b) Thevenin Impedance: with the voltage source short circuited

Impedance: Parallel ac circuits

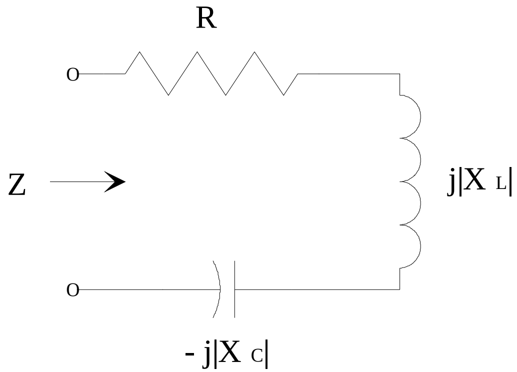
Impedance: Parallel ac circuits (a)

To date we have concentrated on SERIES circuits, where the current flows through each of the elements: for this reason we adopted a CURRENT reference in our impedance diagrams.

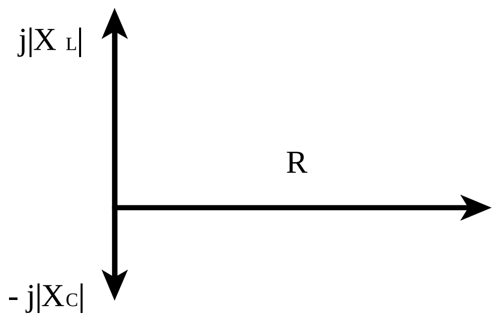
We have to review this reference choice when we deal with parallel circuits!

But, first a reminder of SERIES circuit nomenclature:

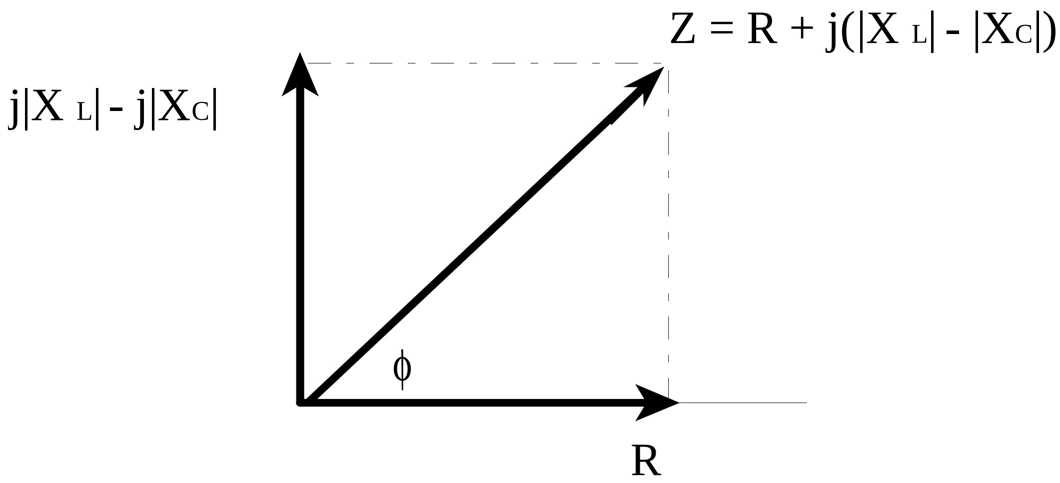
Impedance: Series ac circuits



Series R, L and C

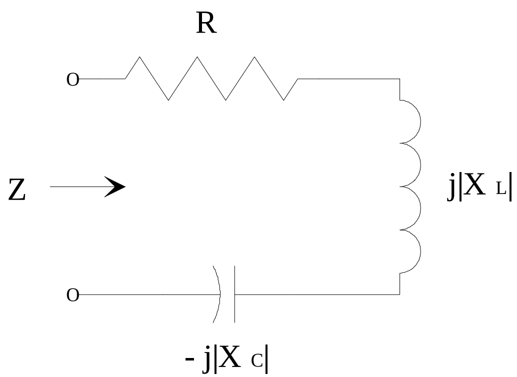


The Impedance Plane

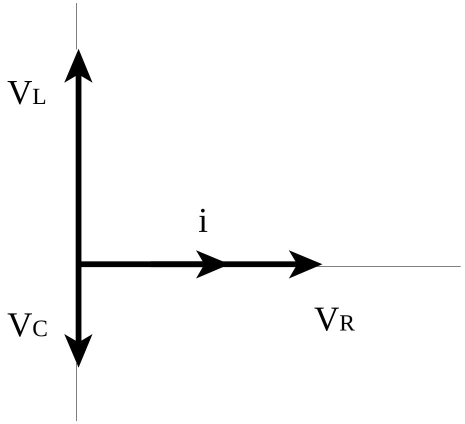


Series RLC circuit impedance diagram

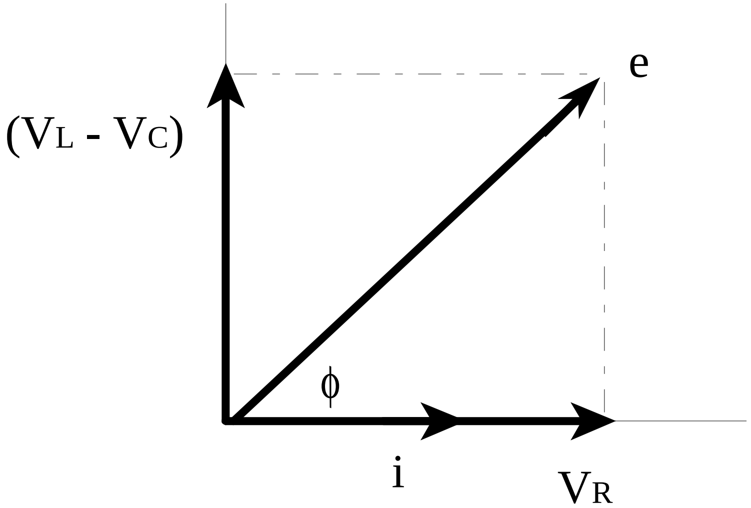
Impedance: Series ac circuits



Series R, L and C



The Voltage Plane (*series circuits*)

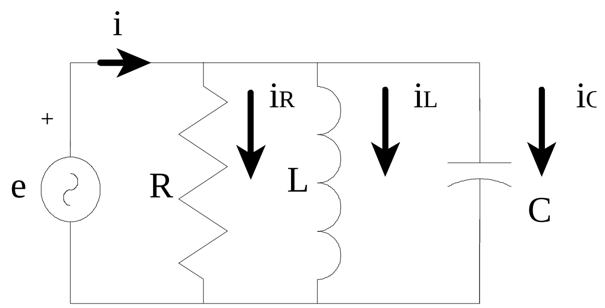


Voltage relationships in series RLC circuit

Impedance: Parallel ac circuits

The fundamental difference between parallel ac circuits and series ac circuit theory is that for the parallel circuit the voltage across each branch of the parallel circuit is identical.

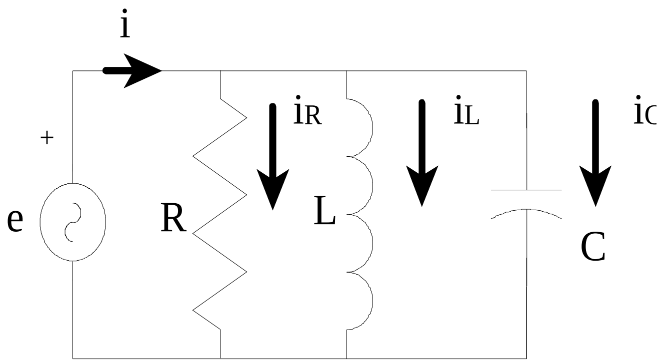
In parallel ac circuit theory we have to obey Kirchhoff's current law, but once again, *in phasor format*.

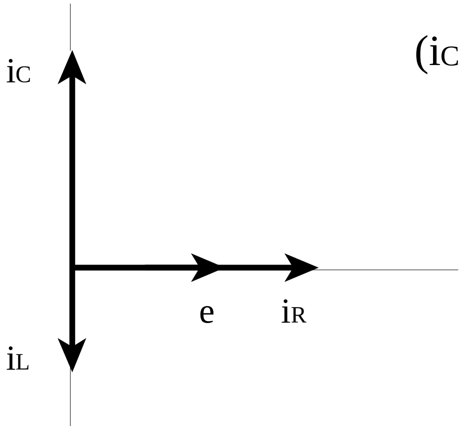


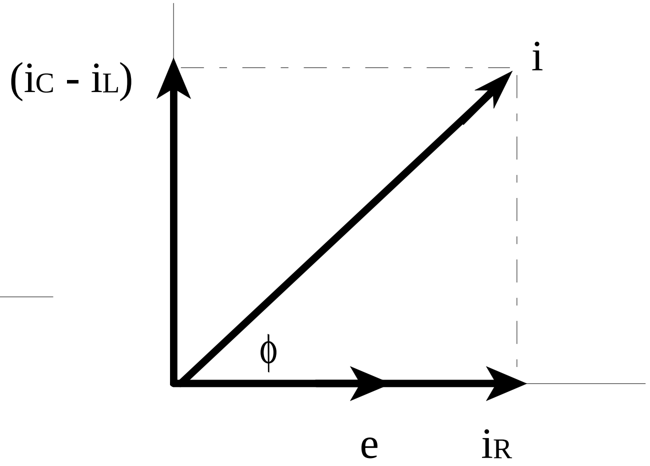
In the parallel circuit, it is the ***voltage*** that is the common term.

We thus take the ***voltage* across the Resistor as the reference.**

Impedance: Parallel ac circuits







In the parallel circuit, it is the ***voltage*** that is the common term.

In the capacitor the current is *leading* the voltage by 90° and thus must appear as a (+ j) term relative to the real axis. Likewise the Inductor has a current that *lags* voltage, hence the (- j) term.

This nomenclature is contrary to the previously adopted system used with series circuits. For the parallel circuit configurations however it is sensible to adopt the above phasor diagram when describing the current flow.

Impedance: Parallel ac circuits

Impedance Z_1 and Z_2 in parallel

When we have two parallel connected components having impedances Z_1 and Z_2, the total equivalent impedance of such a circuit is given by Z total, where

$$Z\ \text{total} = \frac{Z_1 . Z_2}{(Z_1 + Z_2)} \quad \Omega$$

All the arithmetic operations in the above equation must be performed in *phasor* format.

Parallel ac circuits: (b) Impedance / Admittance

Impedance and Admittance

An alternative approach often used in parallel circuits is to use not impedance terms but *admittance*.

The reciprocal of resistance R is called *conductance* **G**, and it has the units of Siemens (S).

Parallel ac circuits: (b) Impedance / Admittance

Impedance and Admittance

An alternative approach often used in parallel circuits is to use not impedance terms but *admittance*.

The reciprocal of resistance R is called *conductance* **G**, and it has the units of Siemens (S).

Similarly, the reciprocal of reactance is called *susceptance*, **B** = 1/X.

There is naturally *inductive* susceptance and *capacitive* susceptance.

The general term for conductance, susceptance and combinations of these is admittance, which has the symbol **Y**.

Parallel ac circuits

Impedance and Admittance

Since $i = \dfrac{e}{Z}$ Ω

and Z = $\dfrac{1}{Y}$ Ω

The relationship between current, voltage and admittance is $i = e\,Y$

Admittance (**Y**) = Conductance (**G**) + Susceptance (**B**)

Parallel ac circuits

Impedance and Admittance

Admittance (**Y**) = Conductance (**G**) + Susceptance (**B**)

Now:

$$R + jX = \frac{1}{G + jB}$$

Remember that we have a complex number to invert!

$$R + jX = \frac{1}{G + jB} = \frac{G - jB}{G^2 + B^2}$$

Alternatively you can use ***polar manipulation*** to do the inversion

$$Z = \frac{1}{Y}$$

Parallel ac circuits

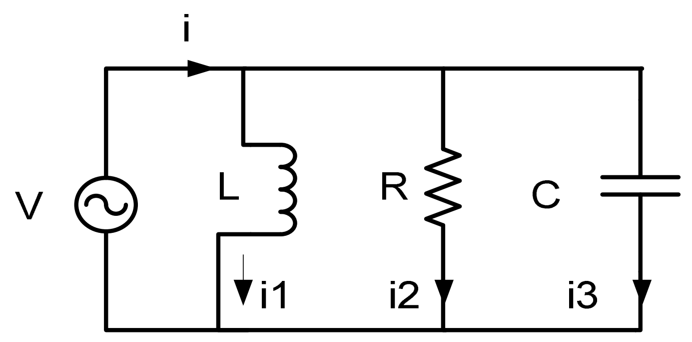
Admittance (R, L and C)

For a parallel circuit the voltage is the common element and the currents add:

$$I = I_1 + I_2 + I_3$$

It is much simpler to work in admittance terms:

For the resistor: $Y_R = 1/R$
For the inductor $Y_L = 1/j\,\omega\,L$
For the capacitor $Y_C = j\,\omega\,C$



Parallel ac circuits

Admittance (R, L and C)

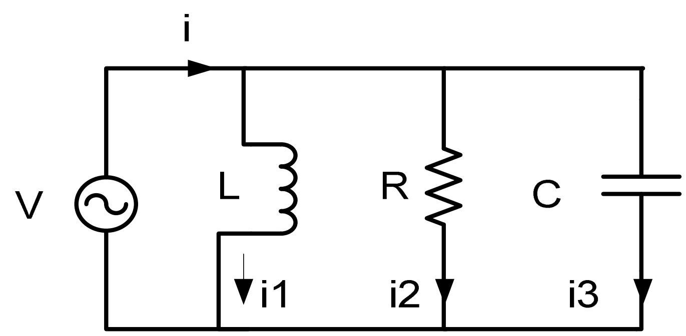
The total admittance is $Y = (1/R + 1/j\omega L + j\,\omega\,C)$ which can be written as a complex number $G + jB$ where B in this case is $(\omega\,C – 1/\,\omega\,L)$.

Note the negative sign.

$I = V/Z$ or VY so $I = V\,(1/R + j\,(\omega\,C – 1/\,\omega\,L))$

$I_1 = V\,/\,(j\,\omega\,L)$ $I_2 = V/R$ $I_3 = V\,(j\,\omega\,C)$

The relationships between current and voltage in the components are exactly as before.



Parallel ac circuits

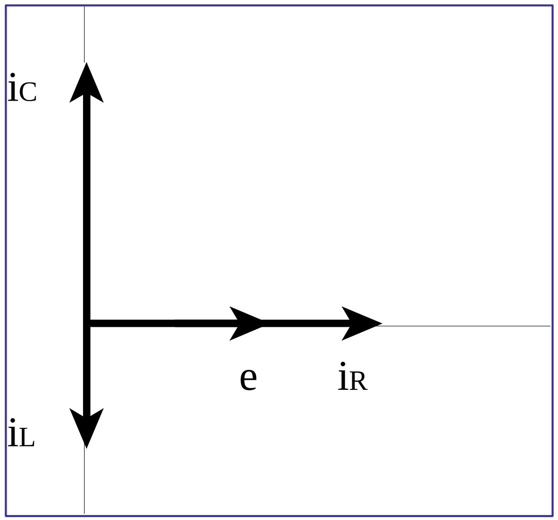
Admittance (R, L and C)

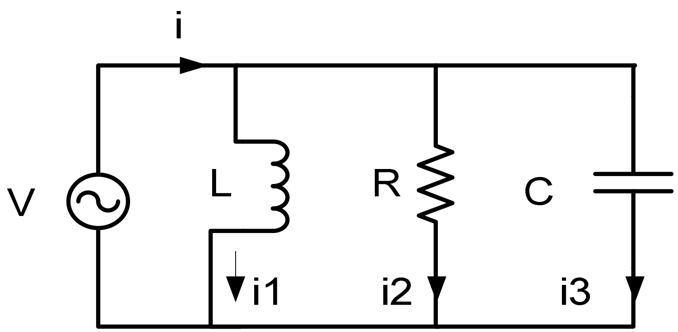
The total admittance is $Y = (1/R + 1/j\omega L + j\,\omega\,C)$ which can be written as a complex number $G + jB$ where B in this case is $(\omega\,C - 1/\,\omega\,L)$.

Note the negative sign.

$I = V/Z$ or VY so $I = V\,(1/R + j\,(\omega\,C - 1/\,\omega\,L))$

$I_1 = V\,/\,(j\,\omega\,L)$ $\quad I_2 = V/R$ $\quad I_3 = V\,(j\,\omega\,C)$

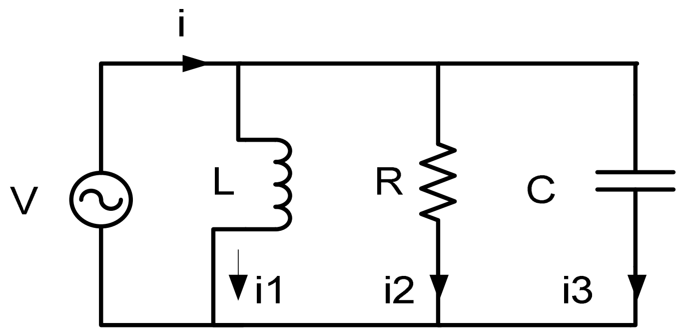




Parallel ac circuits

Admittance (R, L and C) Example

Determine the supply current for an input voltage of 1000 $\underline{/30^o}$ volts.



ω = 1000 rad/s; R = 100Ω; L = 100mH; C = 20μF

Parallel ac circuits

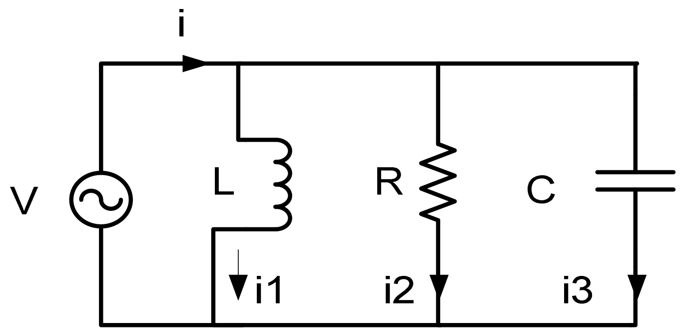
Admittance (R, L and C) Example

Determine the supply current for an input voltage of 1000 $\underline{/30^{o}}$ volts.

$R = 100\Omega$

$X_L = j\ \omega L = +\ j\ 100\ \Omega$

$X_C = -\ j\ (1/\omega C) = -\ j\ 50\ \Omega$



$\boxed{\omega = 1000 \text{ rad/s};\ R = 100\Omega;\ L = 100\text{mH};\ C = 20\mu F}$

Parallel ac circuits

Admittance (R, L and C) Example

Determine the supply current for an input voltage of 1000 $\underline{/30^o}$ volts.

$R = 100\Omega$

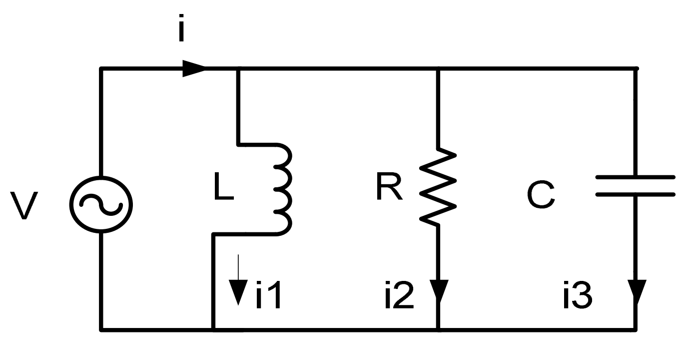
$X_L = j\ \omega L = +\ j\ 100\ \Omega$

$X_C = -\ j\ (1/\omega C) = -\ j\ 50\ \Omega$

$Y_R = 0.01\ S$

$Y_L = -\ j\ 0.01\ S$

$Y_C = +\ j\ 0.02\ S$



$\omega = 1000$ rad/s; $R = 100\Omega$; $L = 100mH$; $C = 20\mu F$

Parallel ac circuits

Admittance (R, L and C) Example

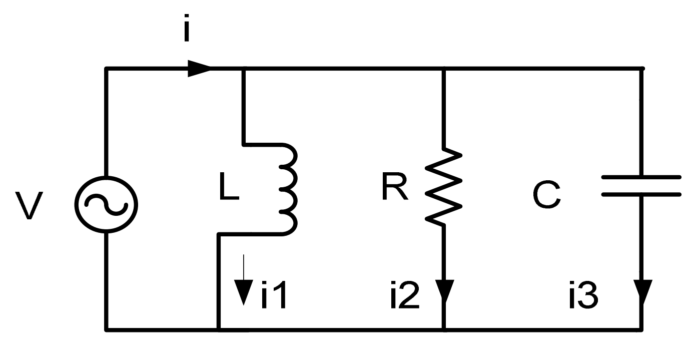
Determine the supply current for an input voltage of 1000 $\underline{/30^o}$ volts.

$R = 100\Omega$ $\qquad$ $Y_R = 0.01$ S

$X_L = j\,\omega L = +\,j\,100\ \Omega$ $\qquad$ $Y_L = -\,j\,0.01$ S

$X_C = -\,j\,(1/\omega C) = -\,j\,50\ \Omega$ $\qquad$ $Y_C = +\,j\,0.02$ S

$$Y = (1/R + 1/j\omega L + j\,\omega\,C) = (0.01 - j\,0.01 + j\,0.02) = (0.01 + j0.01) = 0.0141\ \underline{/45^o}\ S$$



$\omega = 1000$ rad/s; $R = 100\Omega$; $L = 100$mH; $C = 20\mu F$

Parallel ac circuits

Admittance (R, L and C) Example

Determine the supply current for an input voltage of 1000 $\underline{/30^o}$ volts.

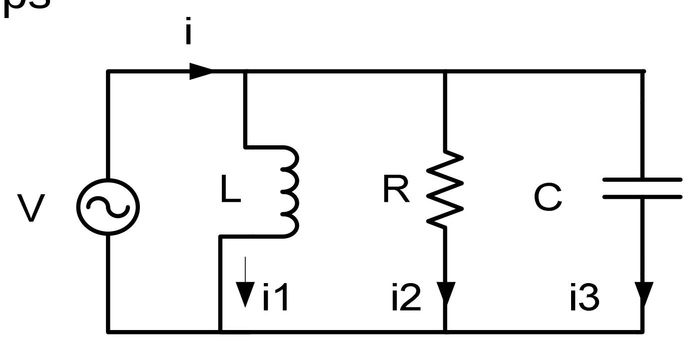
$R = 100\Omega$ | $Y_R = 0.01$ S

$X_L = j\,\omega L = +\,j\,100\ \Omega$ | $Y_L = -\,j\,0.01$ S

$X_C = -\,j\,(1/\omega C) = -\,j\,50\ \Omega$ | $Y_C = +\,j\,0.02$ S

$Y = (1/R + 1/j\omega L + j\,\omega\,C) = (0.01 - j\,0.01 + j\,0.02) = (0.01 + j0.01) = 0.0141\ /\underline{45^o}$ S

$I = VY = 1000\ /\underline{30^o}\ \times 0.0141\ /\underline{45^o} = 14.1\ \underline{/75^o}$ Amps

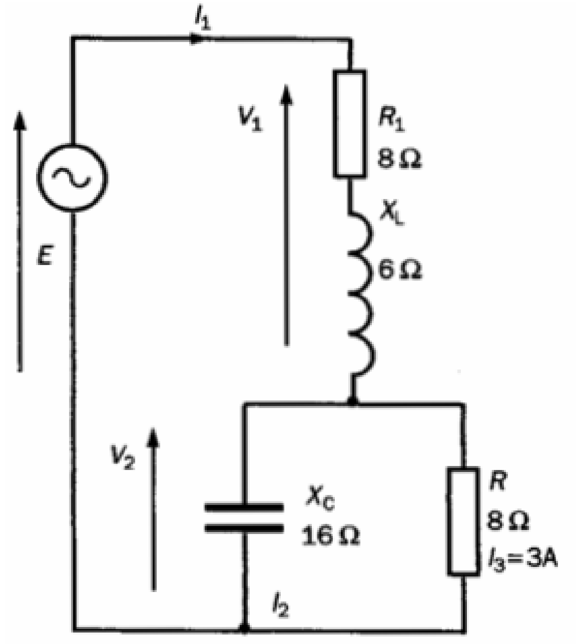


$\omega = 1000$ rad/s; $R = 100\Omega$; $L = 100$mH; $C = 20\mu$F ac Circuit Examples / Hughes

ac Circuit Examples / Hughes

Example 15.1

Determine the supply current and source emf.



L = j 6 Ω and C = -j 16 Ω

Example 15.1

Determine the supply current and source emf.

VOLTAGE ACROSS $R = IR$

$I_3 = 3A = 3\angle 0°$

$\therefore V_2 = 3\angle 0° \times 8 = 24\angle 0°$ V

$I_2 = V_2 / X_C = \dfrac{24\angle 0°}{-j16}$

$= \dfrac{24\angle 0°}{16\angle -90°} = 1.5\angle 90°$ A

$\underline{\underline{I_1 = I_2 + I_3 = (3 + j1.5)\text{ A}}}$

$V_1 = I_1 (R + jX_L) = I_1 (8 + j6)$

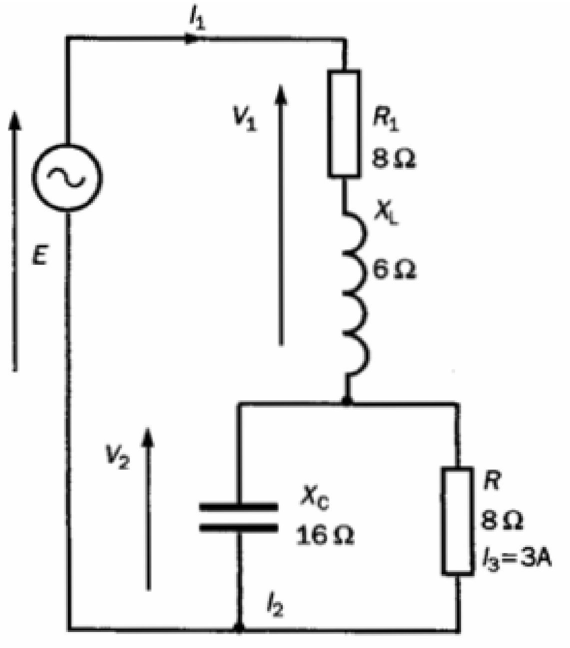
$V_1 = (3 + j1.5)(8 + j6)$

$= 3.35\angle 26.6° \times 10\angle 36.9°$

$= 33.5\angle 63.5° = (15 + j30)$ V

$E = V_1 + V_2 = 24 + (15 + j30)$

$\underline{\underline{= 39 + j30 = 49.2\angle 37.6° \text{ V}}}$

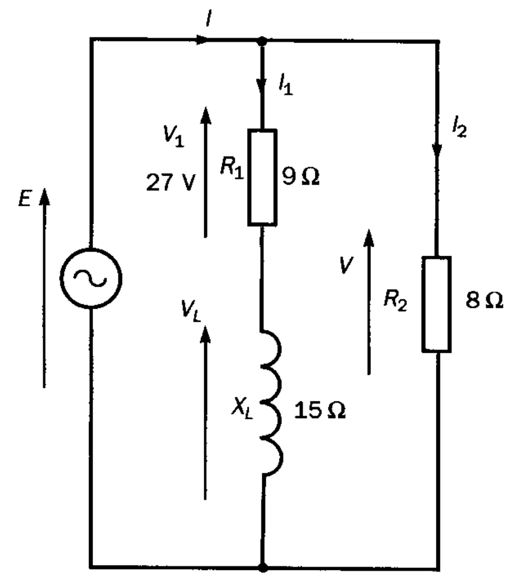


L = j 6 Ω and C = -j 16 Ω

ac Circuit Examples / Hughes

Example 15.2

Determine I_1, E, I_2 and I



L = j 15 Ω

ac Circuit Examples / Hughes

Example 15.2

Determine I_1, E, I_2 and I

$$I_1 = V_1/R_1 = \frac{27\angle 0^\circ}{9\angle 0^\circ} = \underline{\underline{3\angle 0^\circ \text{ A}}}$$

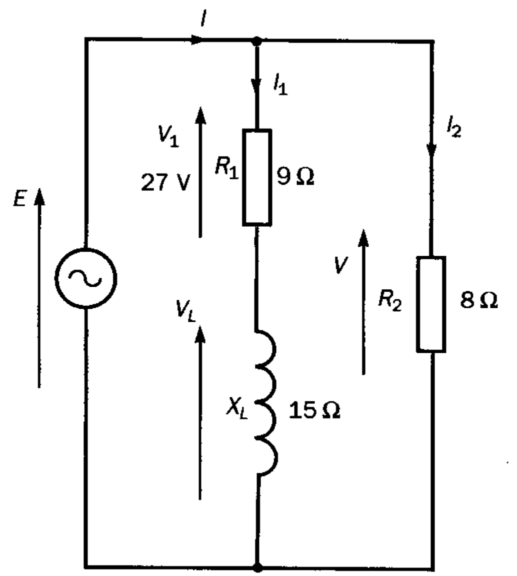
$$V_L = I_1 X_L = 3\angle 0^\circ \times j15$$
$$= 3\angle 0^\circ \times 15\angle 90^\circ = 45\angle 90^\circ \text{ V}$$

$$V_1 + V_L = (27 + j0) + (0 + j45)$$
$$= (27 + j45)\text{ V} = \underline{\underline{52.5\angle 59^\circ \text{ V}}}$$

$$V = E \quad \text{So } I_2 = V/R_2 = \frac{52.5\angle 59^\circ}{8}$$

$$= 6.56\angle 59^\circ \text{ A}$$

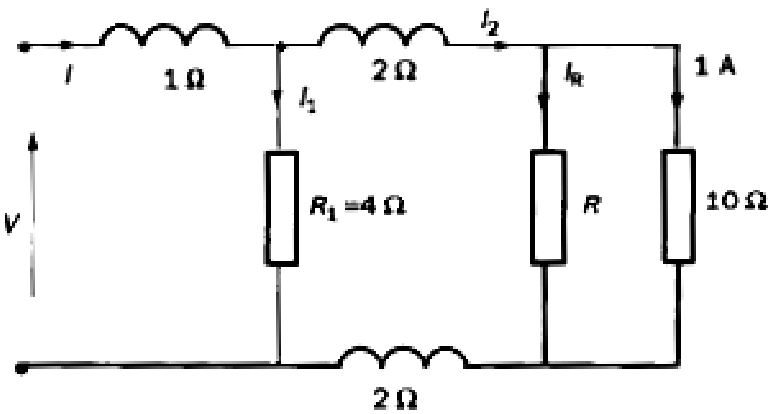
$$I = I_1 + I_2 = 3 + 6.56\angle 59^\circ$$
$$= 3 + (3.38 + j5.62)$$
$$= \underline{\underline{6.38 + j5.62 = 8.5\angle 41.4^\circ \text{A}}}$$



L = j 15 Ω

Example 15.3

If the power dissipated in R is 20W, determine I and V



L_1 = j 1 Ω; L_2 = j 2Ω; L_3 = j 2Ω

Example 15.3

If the power dissipated in R is 20W, determine I and V

$V_R = 10 \times 1 = 10V$

So Voltage Across $R = 10V$

As Power $= V^2/R = 20W$

$R = V^2/20 = 100/20 = 5\Omega$

Current $= 2A$

$\therefore I_2 = 1 + 2 = 3A$

Voltage Across $R_1 = I_2(j2) + I_2(j2) + V_R$

$= (10 + 12j)\ V = 15.62\angle 50.2^\circ$

$$I_1 = V_1/R_1 = \frac{15.62\angle 50.2}{4} = 3.9\angle 50.2^\circ$$

$I = I_1 + I_2 = 3.9\angle 50.2^\circ + 3 = (2.5 + j3) + 3$

$\therefore \underline{\underline{I = (5.5 + j3)\,A = 6.3\angle 28.6^\circ A}}$

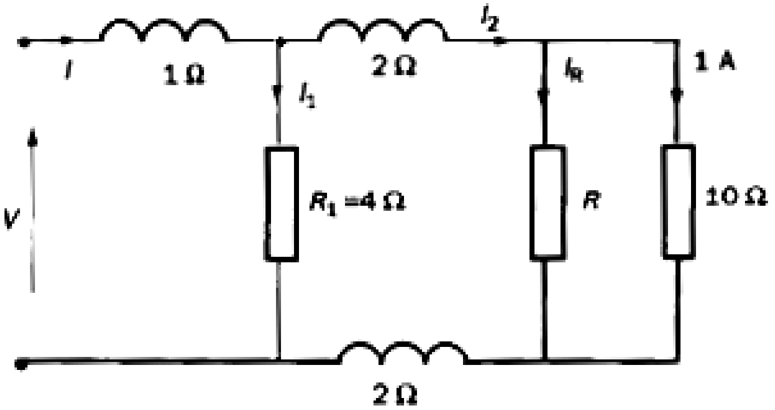
Voltage Across j1 Inductor $= 6.3\angle 28.6 \times 1\angle 90^\circ$

$= 6.3\angle 118.6^\circ$

$V = 6.3\angle 118.6^\circ + (10 + j12)$

$= (-3 + j5.53) + (10 + j12) = 7 + j17.53$

$\underline{\underline{= 18.9\angle 68.2^\circ}}\ V$

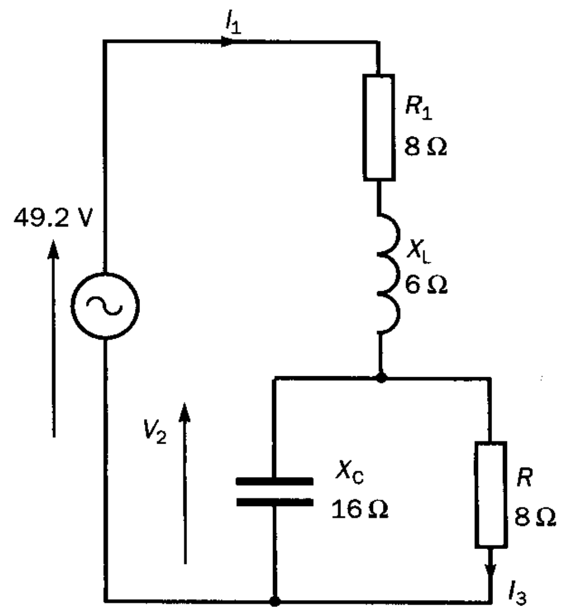


$L_1 = j\,1\ \Omega;\ L_2 = j\,2\Omega;\ L_3 = j\,2\Omega$

ac Circuit Examples / Hughes

Example 15.4

Determine the supply current I_1 and the branch current I_3



L = j 6 Ω and C = -j 16 Ω

ac Circuit Examples / Hughes

Example 15.4

Determine the supply current I_1 and the branch current I_3

USING ADMITTANCE

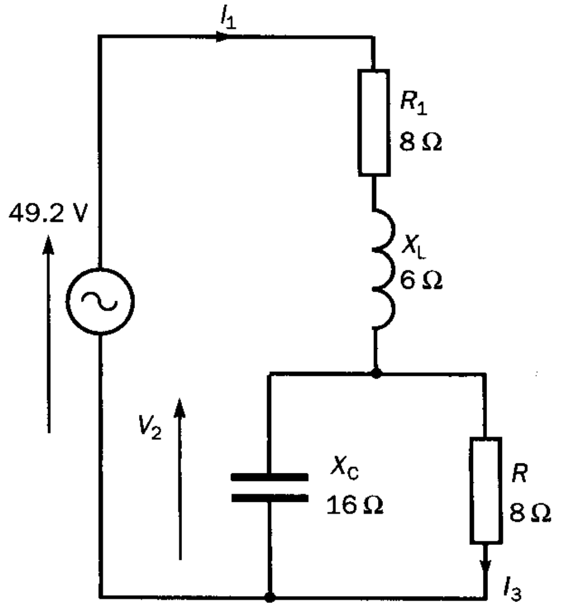
$$Y = \frac{1}{R} + \frac{1}{X_c} \quad \text{FOR PARALLEL}$$

$$Y = \frac{1}{8} + \frac{1}{-j16} = \frac{1}{8} + j\frac{1}{16}$$

$$= \frac{1}{16}(2+j) = \frac{2.24}{16} \angle 26.57^\circ$$

$$X = \frac{1}{Y} = \frac{16}{2.24} \angle -26.57^\circ$$

$$= 7.14 \angle -26.57^\circ = (6.4 - j3.2)$$



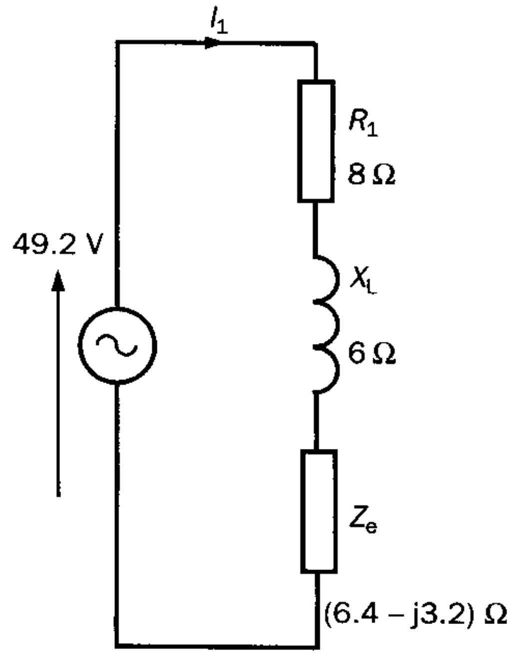
L = j 6 Ω and C = -j 16 Ω

Example 15.4

Determine the supply current I_1 and the branch current I_3

$$I_1 = \frac{49.2}{Z} = \frac{49.2}{(14.4 + j2.8)} = \frac{49.2}{14.7\angle 11^\circ}$$

$$= \underline{\underline{3.35\angle -11^\circ \text{ A} = (3.3 - j0.64)}}$$



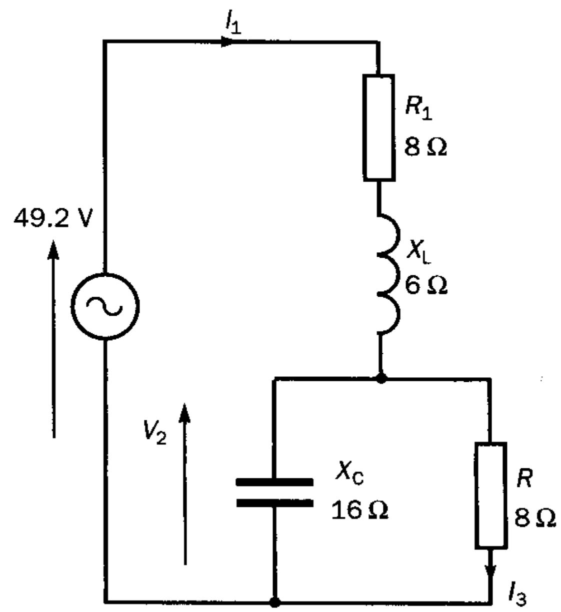
Total series impedance = (14.4 + j 2.8) Ω

Example 15.4

Determine the supply current I_1 and the branch current I_3

$$I_1 = \frac{49.2}{Z} = \frac{49.2}{(14.4 + j2.8)} = \frac{49.2}{14.7\angle 11^\circ}$$

$$= \underline{\underline{3.35\angle -11^\circ \text{ A} = (3.3 - j0.64)}}$$



Using current division the above current can be split into the 2 component branches

Total series impedance = (14.4 + j 2.8) Ω

Example 15.4

Determine the supply current I_1 and the branch current I_3

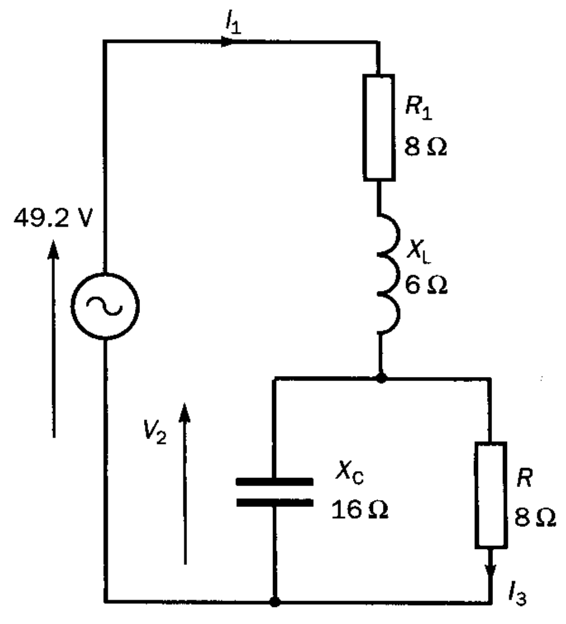
$$I_1 = \frac{49.2}{Z} = \frac{49.2}{(14.4 + j2.8)} = \frac{49.2}{14.7\angle 11^\circ}$$

$$= \underline{\underline{3.35\angle -11^\circ \text{ A} = (3.3 - j0.64)}}$$

$$I_3 = \frac{X_c}{R + X_c} I_1 = \frac{-j16}{(8 - j16)} 3.35\angle -11^\circ$$

$$= \frac{16\angle -90^\circ}{17.9\angle -63.4^\circ} \times 3.35\angle -11^\circ$$

$$= \underline{\underline{3\angle -57.6^\circ}}$$



Total series impedance = (14.4 + j 2.8) Ω

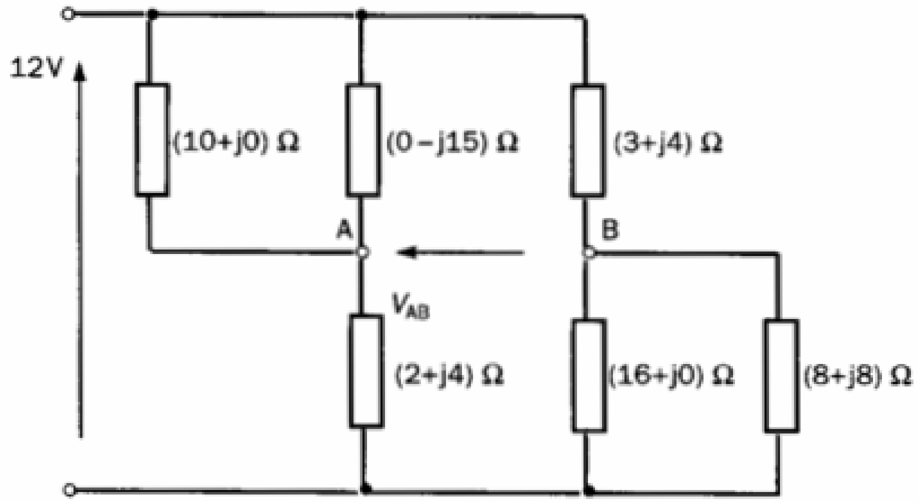
Example 15.5

Determine V_{AB}

Notice the parallel combinations.

You should practice this using both the admittance method and the parallel method

$Z = Z_1 \times Z_2/(Z_1 + Z_2)$



Example 15.5

Determine V_{AB}

$$V_{AC} = \frac{(2+j4)}{(2+j4)+(6.92-j4.62)} \times 12$$

$$= \frac{(24+j48)}{(8.92-j0.62)}$$

$$= \frac{53.67\angle 63.43^\circ}{8.94\angle -4^\circ}$$

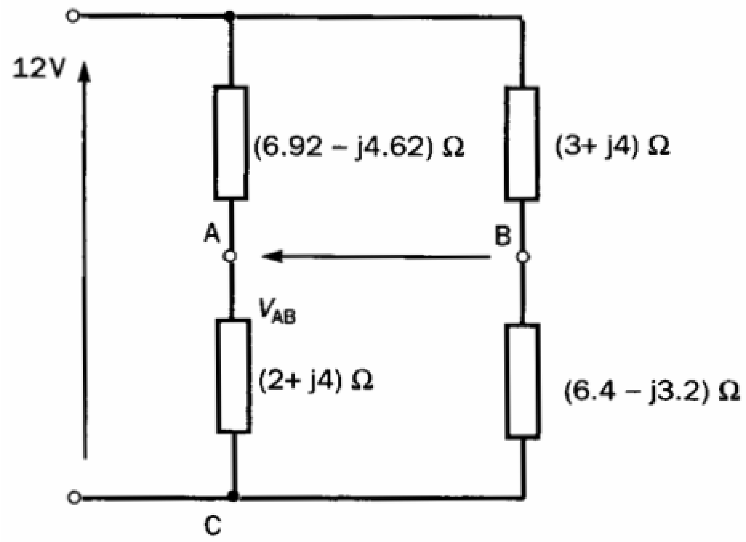
$$= 6\angle 67.43^\circ$$

SIMILARLY $V_{BC} = 2.4\angle 0^\circ$

$$V_{AB} = V_{AC} - V_{BC} = (2.62 + j5.54) - 2.4$$

$$= 0.22 + j5.54$$

$$= 5.55\angle 87.7^\circ \text{ V}$$



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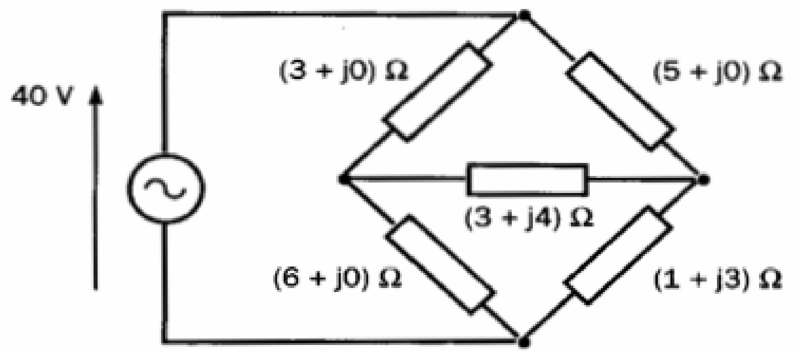
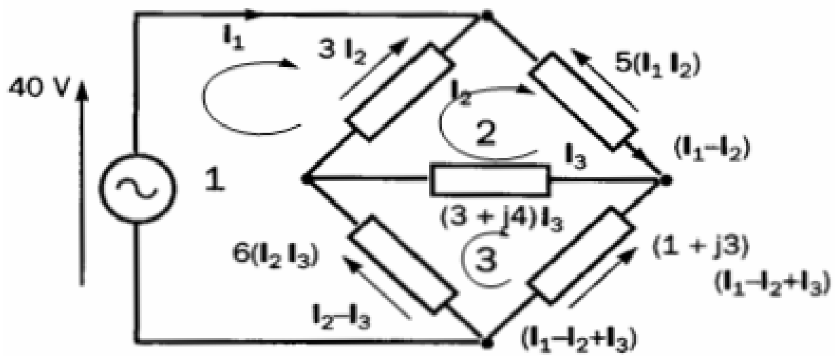
Example 15.6

Calculate the current in the (3 + j4)Ω impedance





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Example 15.6

Calculate the current in the (3 + j4)Ω impedance

Loop 1

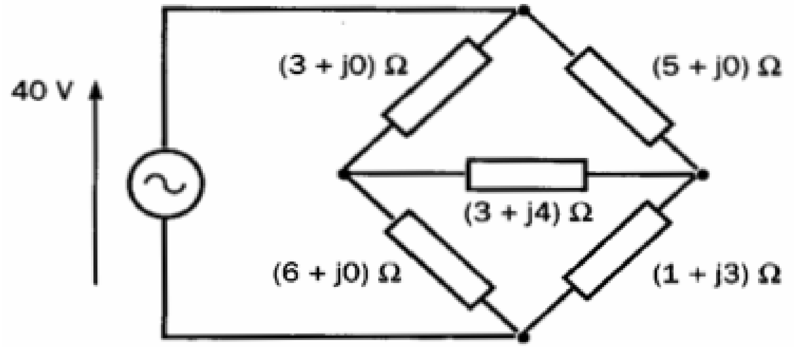
$$40 = 9I_1 - 3I_2 - 6I_3$$

Loop 2

$$0 = (11+j4)I_2 - 3I_1 - (3+j4)I_3$$

Loop 3

$$0 = (10+j7)I_3 - (3+j4)I_2 - 6I_1$$

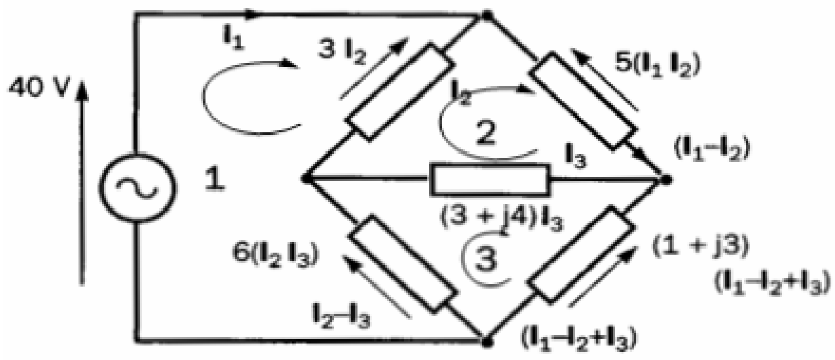


Hence

① $40 = 9I_1 - 3I_2 - 6I_3$

② $0 = -3I_1 + (11+j4)I_2 - (3+j4)I_3$

③ $0 = -6I_1 - (3+j4)I_2 + (10+j7)I_3$



Example 15.6

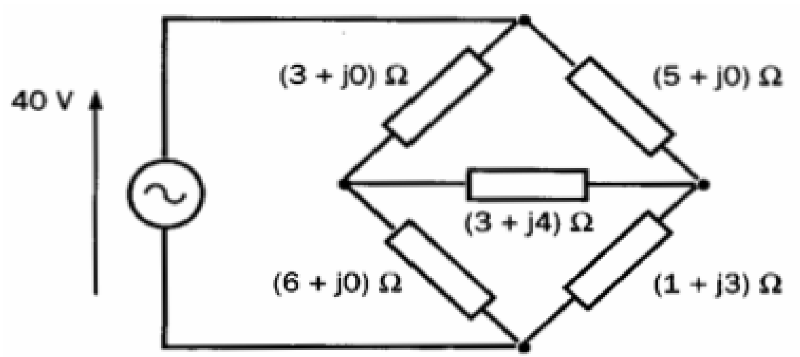
Calculate the current in the (3 + j4)Ω impedance

HENCE

① $40 = 9I_1 - 3I_2 - 6I_3$

② $0 = -3I_1 + (11+j4)I_2 - (3+j4)I_3$

③ $0 = -6I_1 - (3+j4)I_2 + (10+j7)I_3$



① + 3×② → $40 = (30+j12)I_2 - (15+j12)I_3$

③ − 2×② → $0 = -(25+j12)I_2 + (16+j15)I_3$

HENCE $I_2 = \frac{(16+j15)}{(25+j12)} I_3 = \frac{21.93\angle 43.3}{27.73\angle 25.6} I_3$

$I_2 = 0.791\angle 17.56^\circ \, I_3$

AND $40 = [(30+j12)(0.791\angle 17.56) - (15+j12)]I_3$

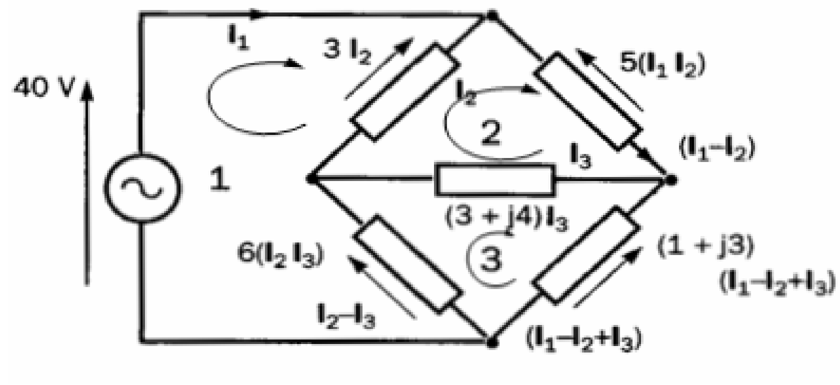
$\therefore \; 40 = [32.31\angle 21.8 \times 0.791\angle 17.56 - (15+j12)]I_3$

$= [25.55\angle 39.36 - (15+j12)]\, I_3$

$= [19.75 + j16.2 - 15 - j12]\, I_3$

$= (4.75 + j4.2)\, I_3$

HENCE $I_3 = \frac{40}{6.34\angle 41.5} = 6.3\angle -41.5$



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Example 15.6

Calculate the current in the (3 + j4)Ω impedance

$$I_2 = 0.791\angle 17.56\ I_3$$

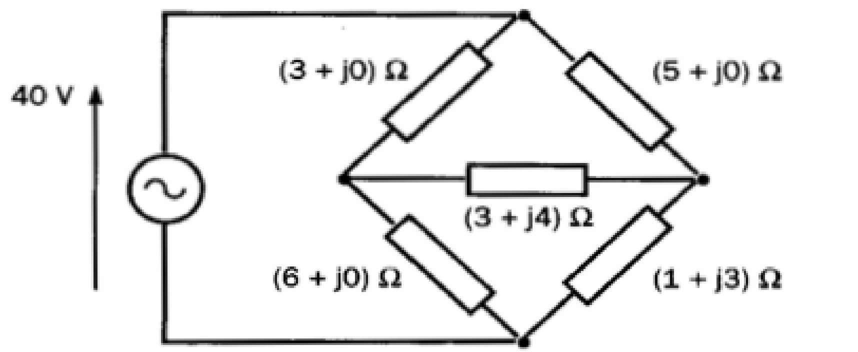
$$= 0.791\angle 17.56 \times 6.3\angle -41$$

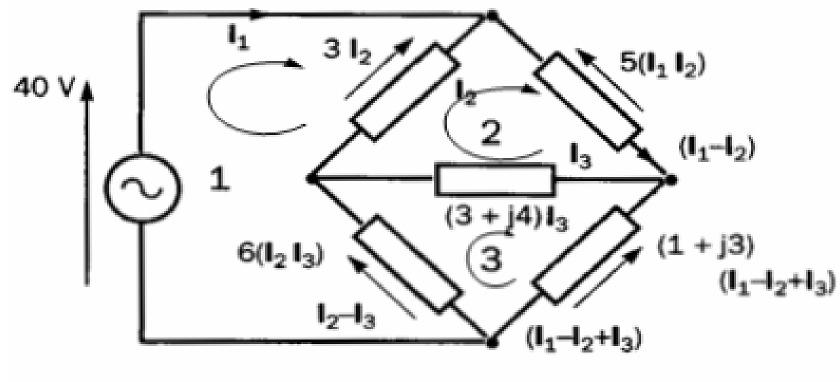
$$= 4.982\angle -23.44$$

$$= 4.57 - j1.98$$

$$I_3 - I_2 = 0.185 - j2.153$$

$$= 2.16\angle -85^\circ \text{ A}$$





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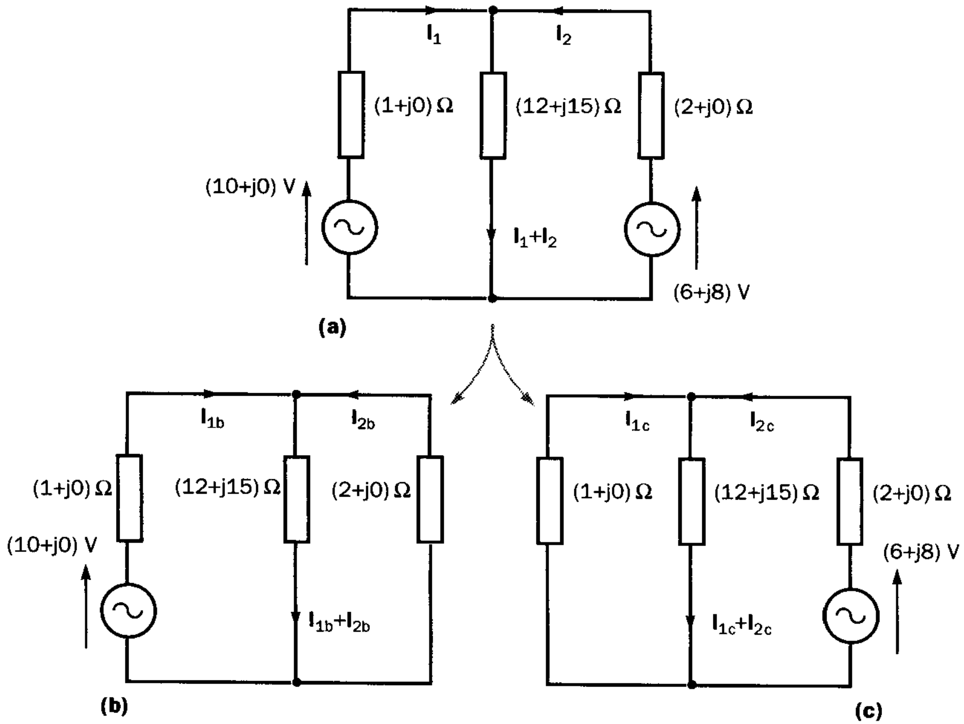
Example 15.7

Determine, by using superposition, the currents in the network



Example 15.7

Determine, by using superposition, the currents in the network

For circuit (b)

b) $Z = (1+j0) + (12+j15) \| (2+j0)$

$$= 1 + \frac{24+j30}{14+j15} = 1 + \frac{38.4\angle 51.3}{20.5\angle 46.97}$$

$$= 1 + 1.87\angle 4.33 = 2.87 + j0.14$$

$$I_{1b} = \frac{10+j0}{2.87+j0.14} = (3.48 - j0.17)\ A$$

$$-I_{2b} = (3.48 - j0.17) \times \frac{12+j15}{(12+j15)+(2+j0)}$$

$$= (3.24 + j0.09)\ A$$

AND $I_{1b} + I_{2b} = (3.48 - j0.17) - (3.24 + j0.09)$

$$= (0.24 - j0.26)\ A$$

ETC.