Part A MCQ (30%)

- **Only the students who take the final exam online** are required to justify their choices in the MCQ part (by showing their work).

(6*pts*) **Problem 1**
If

$$a_n = 2n\left(\frac{1}{n} + \sin\frac{1}{n}\right), \quad \text{evaluate } \lim_{n\to\infty} a_n.$$

(a) a_n converges to 2

(b) a_n converges to 4

(c) a_n converges to 0

(d) a_n converges to $1+\pi$

(e) a_n diverges

(6*pts*) **Problem 2**
The sum of the geometric series

$$4 + 3 + \frac{9}{4} + \frac{27}{16} + ...$$

is

(a) $\frac{175}{16}$

(b) 19

(c) $\frac{143}{16}$

(d) 13

(e) 16

(6*pts*) **Problem 3**

The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{7^n (x+3)^n}{\sqrt{n}}$ is

(a) $\frac{1}{3}$

(b) $\frac{7}{3}$

(c) $\frac{1}{7}$

(d) 3

(e) 21

(6*pts*) **Problem 4**

The power series representation of the function $\frac{x^3}{3+x}$ is equal to

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{3^n}$

(b) $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{3^{n+1}}$

(c) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+3}}{3^{n+1}}$

(d) $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^{n+1}$

(e) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+3}}{3^n}$

(6pts) **Problem 5**
The coefficient of x^4 in Maclaurin series of the function $f(x) = \cos(5x^2)$ equal to

(a) $\dfrac{-25}{2}$

(b) $\dfrac{-4}{5}$

(c) $\dfrac{24}{5}$

(d) $\dfrac{-1}{24}$

(e) $\dfrac{1}{12}$

Part B Written Questions (70%)

(15pts)**Problem 1**
Find the interval of convergence of the following power series

1. $\sum_{n=0}^{\infty} \frac{x^n}{n2^n}$

2. $\sum_{n=0}^{\infty} \frac{(x+2)^n}{n!}$

(15pts)**Problem 2**
Solve the initial value problem for the separable equation below

$$\frac{dy}{dx} = 3x^2y^2, \qquad y(0) = \frac{1}{2}.$$

(20*pts*)**Problem 3**
Show that the differential equation is exact and solve the equation.

$$(\cos y + y \cos x)\, dx + (\sin x - x \sin y)\, dy = 0$$

(20*pts*)**Problem 4**
Solve the initial value problem for the Bernoulli equation below

$$x\frac{dy}{dx} - 2y = 4x^3y^{1/2}, \qquad y(1) = 0.$$