Chapter 30

Sources of the Magnetic Field

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Magnetic Fields

The origin of the magnetic field is moving charges.

The magnetic field due to various current distributions can be calculated.

Ampère's law is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

Magnetic effects in matter can be explained on the basis of atomic magnetic moments.

Biot-Savart Law – Introduction

Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet.

They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current.

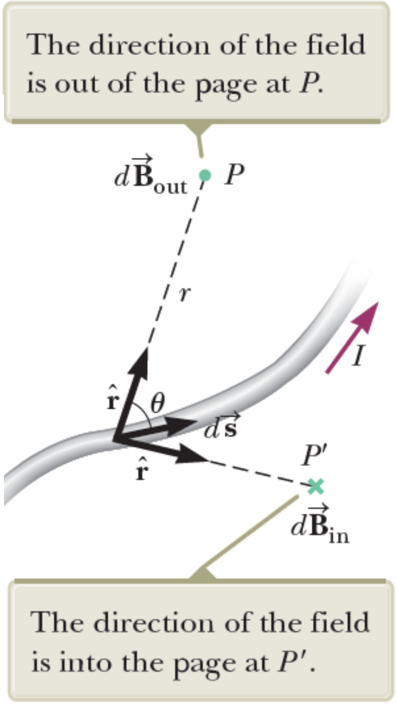
The magnetic field described by the Biot-Savart Law is the field *due to* a given current carrying conductor.

- Do not confuse this field with any external field applied to the conductor from some other source.

Biot-Savart Law – Observations

That expression is based on the following experimental observations for the magnetic field $d\vec{\mathbf{B}}$ at a point P associated with a length element $d\vec{\mathbf{s}}$ of a wire carrying a steady current I

- The vector $d\vec{\mathbf{B}}$ is perpendicular both to $d\vec{\mathbf{s}}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\vec{\mathbf{s}}$ toward P.
- The magnitude of $d\vec{\mathbf{B}}$ is inversely proportional to r^2, where r is the distance from $d\vec{\mathbf{s}}$ to P.
- The magnitude of $d\vec{\mathbf{B}}$ is proportional to the current I and to the magnitude ds of the length element $d\vec{\mathbf{s}}$.
- The magnitude of $d\vec{\mathbf{B}}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$.



Biot-Savart Law – Equation

The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

The constant μ_o is called the **permeability of free space.**

$\mu_o = 4\pi \times 10^{-7}$ T· m / A

Total Magnetic Field

$d\vec{\mathbf{B}}$ is the field created by the current in the length segment ds.

To find the total field, sum up the contributions from all the current elements I $d\vec{\mathbf{s}}$

$$\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

- The integral is over the entire current distribution.

The law is also valid for a current consisting of charges flowing through space.

- For example, this could apply to the beam in an accelerator.

Magnetic Field Compared to Electric Field

Distance

- The magnitude of the magnetic field varies as the inverse square of the distance from the source.
- The electric field due to a point charge also varies as the inverse square of the distance from the charge.

Direction

- The electric field created by a point charge is radial in direction.
- The magnetic field created by a current element is perpendicular to both the length element $d\vec{\mathbf{s}}$ and the unit vector. $\hat{\mathbf{r}}$

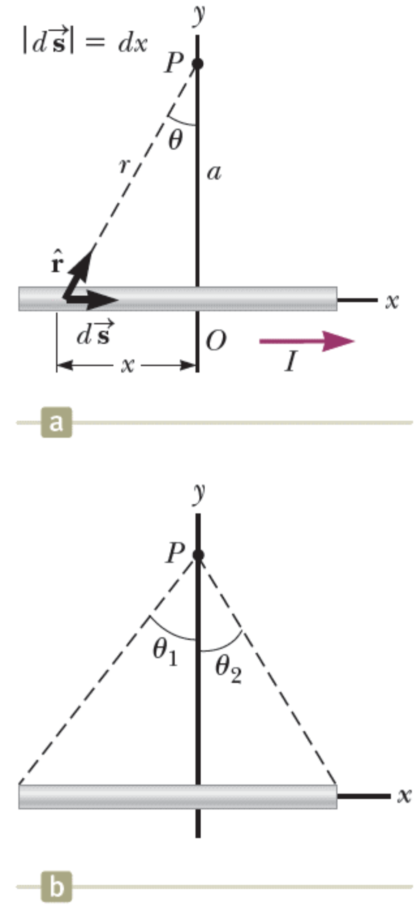
Magnetic Field Compared to Electric Field, cont.

Source

- An electric field is established by an isolated electric charge.
- The current element that produces a magnetic field must be part of an extended current distribution.
  - Therefore you must integrate over the entire current distribution.

Magnetic Field Surrounding a Thin, Straight Conductor, Example

Consider a thin, straight wire of finite length carrying a constant current I and placed along the x axis as shown in Figure 30.3. Determine the magnitude and direction of the magnetic field at point P due to this current.



Analyze Let's start by considering a length element $d\vec{\mathbf{s}}$ located a distance r from P. The direction of the magnetic field at point P due to the current in this element is out of the page because $d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$ is out of the page. In fact, because *all* the current elements $I\,d\vec{\mathbf{s}}$ lie in the plane of the page, they all produce a magnetic field directed out of the page at point P. Therefore, the direction of the magnetic field at point P is out of the page and we need only find the magnitude of the field.

Magnetic Field Surrounding a Thin, Straight Conductor, Example

Evaluate the cross product in the Biot–Savart law:

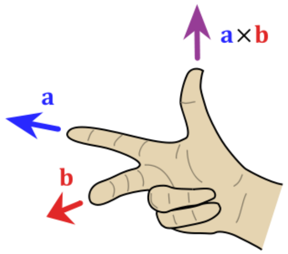
$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx \cos\theta)\hat{\mathbf{k}}$$

Substitute into Equation 30.1:

$$(1) \quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

From the geometry in Figure 30.3a, express r in terms of θ:

$$(2) \quad r = \frac{a}{\cos\theta}$$



Notice that $\tan\theta = -x/a$ from the right triangle in Figure 30.3a (the negative sign is necessary because $d\vec{\mathbf{s}}$ is located at a negative value of x) and solve for x:

$$x = -a\tan\theta$$

Find the differential dx:

$$(3) \quad dx = -a\sec^2\theta\, d\theta = -\frac{a\, d\theta}{\cos^2\theta}$$

Substitute Equations (2) and (3) into the expression for the z component of the field from Equation (1):

$$(4) \quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\, d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$$

Integrate Equation (4) over all length elements on the wire, where the subtending angles range from θ_1 to θ_2 as defined in Figure

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \boxed{\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)}$$

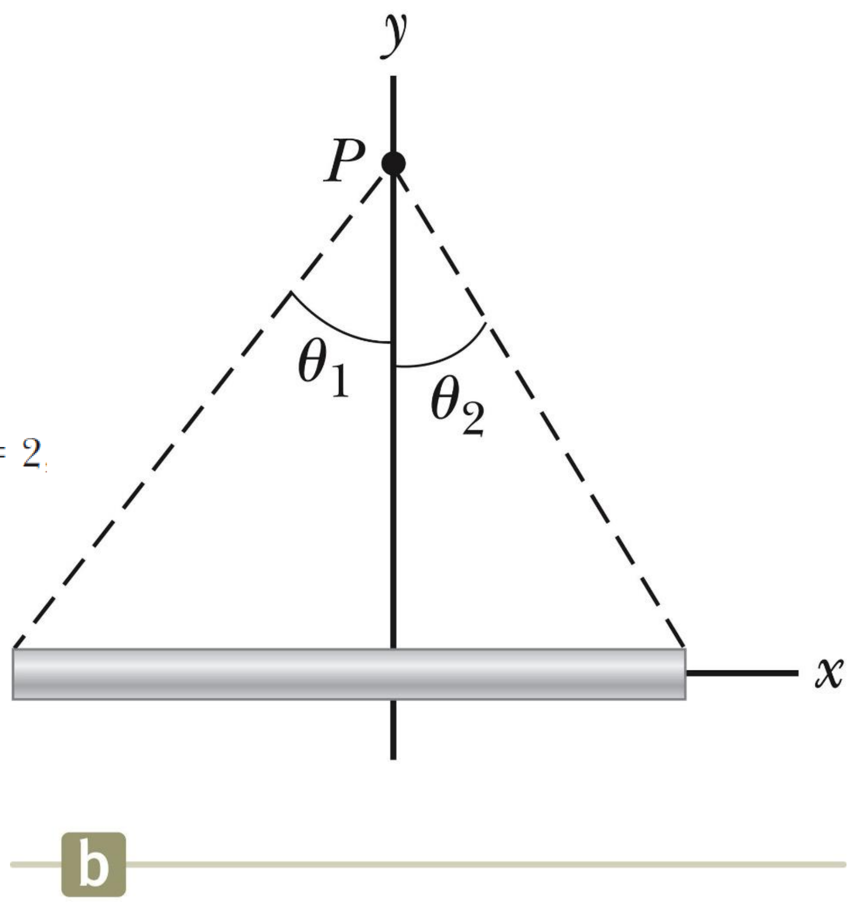
Magnetic Field for a Long, Straight Conductor, Special Case

If the conductor is an infinitely long, straight wire, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

The field becomes

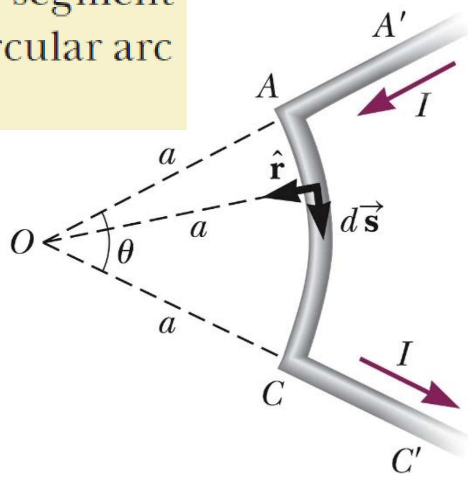
$$B = \frac{\mu_o I}{2\pi a}$$

Because $(\sin\theta_1 - \sin\theta_2) = [\sin\pi/2 - \sin(-\pi/2)] = 2$



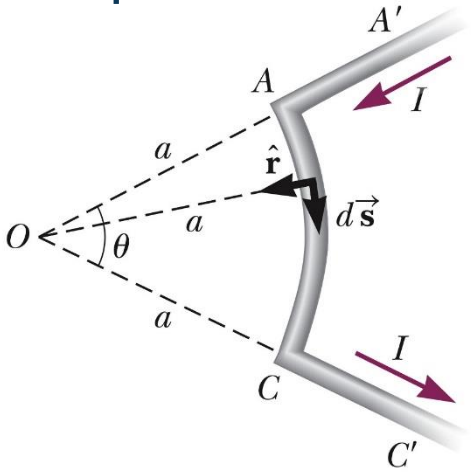
Magnetic Field Due to a Curved Wire Segment, Example

Calculate the magnetic field at point O for the current-carrying wire segment shown in Figure 30.4. The wire consists of two straight portions and a circular arc of radius a, which subtends an angle θ.



The magnetic field at O due to the current in the straight segments AA' and CC' is zero because $d\vec{\mathbf{s}}$ is parallel to $\hat{\mathbf{r}}$ along these paths, which means that $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = 0$ for these paths. Therefore, we expect the magnetic field at O to be due only to the current in the curved portion of the wire.

Magnetic Field Due to a Curved Wire Segment, Example



Analyze Each length element $d\vec{\mathbf{s}}$ along path AC is at the same distance a from O, and the current in each contributes a field element $d\vec{\mathbf{B}}$ directed into the page at O. Furthermore, at every point on AC, $d\vec{\mathbf{s}}$ is perpendicular to $\hat{\mathbf{r}}$; hence, $|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$.

From Equation 30.1, find the magnitude of the field at O due to the current in an element of length ds:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds}{a^2}$$

Integrate this expression over the curved path AC, noting that I and a are constants:

$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2}s$$

From the geometry, note that $s = a\theta$ and substitute:

$$B = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \boxed{\frac{\mu_0 I}{4\pi a}\theta}$$

Section 30.1

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Magnetic Field for a Circular Loop of Wire

Consider the previous result, with a full circle

- $\theta = 2\pi$

$$B = \frac{\mu_o I}{4\pi a}\theta = \frac{\mu_o I}{4\pi a}2\pi = \frac{\mu_o I}{2a}$$

This is the field at the center of the loop.

Magnetic Field on the Axis of Circular Current Loop, Example

Consider a circular wire loop of radius a located in the yz plane and carrying a steady current I as in Figure 30.5. Calculate the magnetic field at an axial point P a distance x from the center of the loop.

Analyze In this situation, every length element $d\vec{\mathbf{s}}$ is perpendicular to the vector $\hat{\mathbf{r}}$ at the location of the element. Therefore, for any element, $|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = (ds)(1)\sin 90° = ds$. Furthermore, all length elements around the loop are at the same distance r from P, where $r^2 = a^2 + x^2$.

Use Equation 30.1 to find the magnitude of $d\vec{\mathbf{B}}$ due to the current in any length element $d\vec{\mathbf{s}}$:

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}$$

Find the x component of the field element:

$$dB_x = \frac{\mu_0 I}{4\pi}\frac{ds}{(a^2 + x^2)}\cos\theta$$

Integrate over the entire loop:

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{a^2 + x^2}$$

From the geometry, evaluate $\cos\theta$:

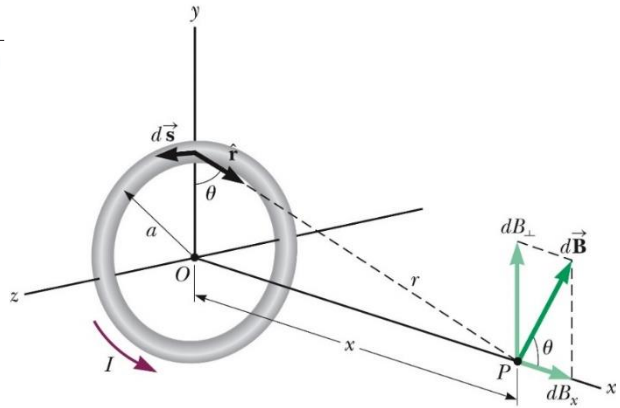
$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

Substitute this expression for $\cos\theta$ into the integral and note that x and a are both constant:

$$B_x = \frac{\mu_0 I}{4\pi}\oint \frac{ds}{a^2 + x^2}\left[\frac{a}{(a^2 + x^2)^{1/2}}\right] = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}\oint ds$$

Integrate around the loop:

$$B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}(2\pi a) = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}}$$



Comparison of Loops

Consider the field at the center of the current loop.

At this special point, $x = 0$

Then,

$$B_x = \frac{\mu_o I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_o I}{2a}$$

- This is exactly the same result as from the curved wire.

Magnetic Field Lines for a Loop

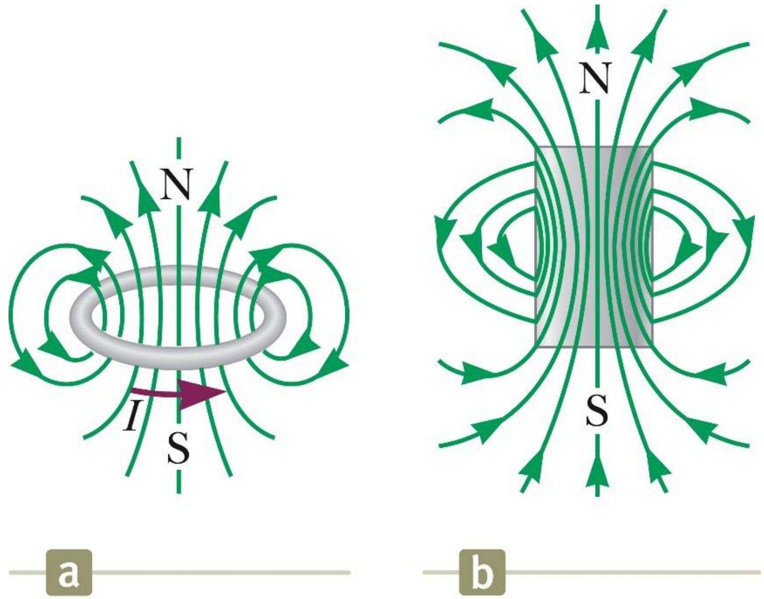


Figure (a) shows the magnetic field lines surrounding a current loop.

Figure (b) compares the field lines to that of a bar magnet.

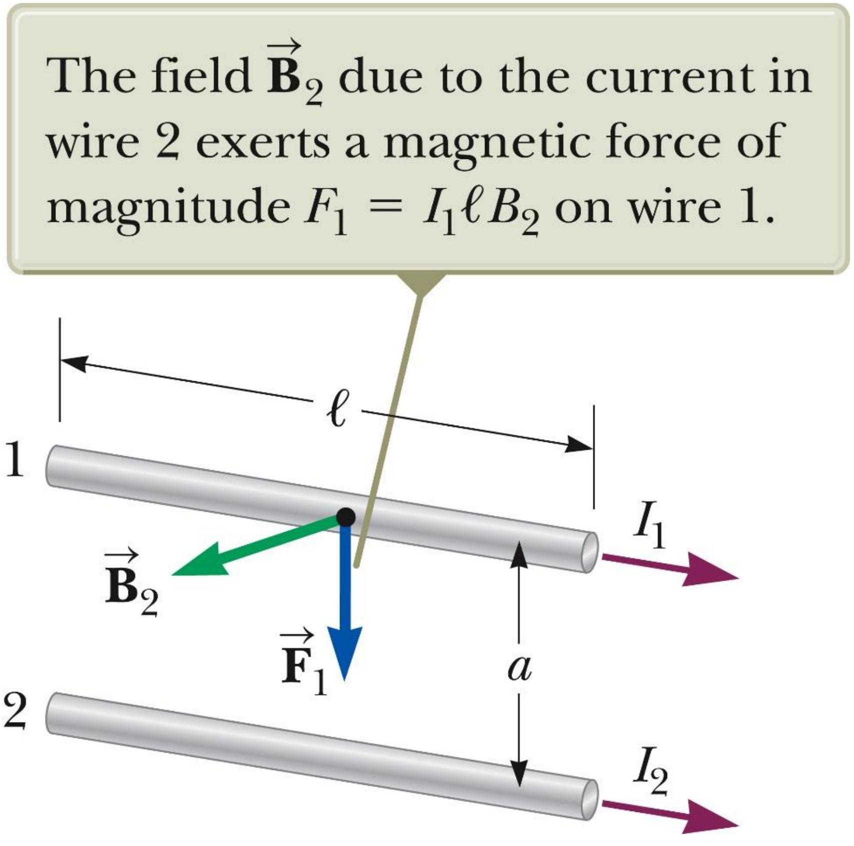
Notice the similarities in the patterns.

Magnetic Force Between Two Parallel Conductors

Two parallel wires each carry a steady current.

The field $\vec{\mathbf{B}}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$.

Because $\vec{\ell}$ is perpendicular to $\vec{\mathbf{B}}_2$ in this situation



Magnetic Force Between Two Parallel Conductors, cont.

Substituting the equation for the magnetic field (B_2), $B = \frac{\mu_o I}{2\pi a}$, gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a} \ell$$

- Parallel conductors carrying currents in the same direction attract each other.
- Parallel conductors carrying current in opposite directions repel each other.

Magnetic Force Between Two Parallel Conductors, final

The result is often expressed as the magnetic force between the two wires, $F_{B.}$

This can also be given as the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

The derivation assumes both wires are long compared with their separation distance.

- Only one wire needs to be long.
- The equations accurately describe the forces exerted on each other by a long wire and a straight, parallel wire of limited length, ℓ.

Definition of the Ampere

The force between two parallel wires can be used to define the ampere.

When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2 x 10^{-7} N/m, the current in each wire is defined to be 1 A.

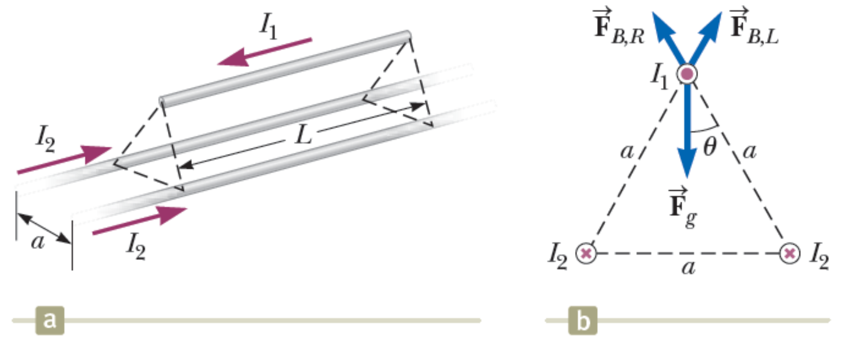
Definition of the Coulomb

The SI unit of charge, the coulomb, is defined in terms of the ampere.

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

Suspending a Wire, Example

Two infinitely long, parallel wires are lying on the ground a distance $a = 1.00$ cm apart as shown in Figure 30.8a. A third wire, of length $L = 10.0$ m and mass 400 g, carries a current of $I_1 = 100$ A and is levitated above the first two wires, at a horizontal position midway between them. The infinitely long wires carry equal currents I_2 in the same direction, but in the direction opposite that in the levitated wire. What current must the infinitely long wires carry so that the three wires form an equilateral triangle?



Analyze The horizontal components of the magnetic forces on the levitated wire cancel. The vertical components are both positive and add together. Choose the z axis to be upward through the top wire in Figure 30.8b and in the plane of the page.

Find the total magnetic force in the upward direction on the levitated wire:

$$\vec{\mathbf{F}}_B = 2\left(\frac{\mu_0 I_1 I_2}{2\pi a}\ell\right)\cos\theta\,\hat{\mathbf{k}} = \frac{\mu_0 I_1 I_2}{\pi a}\ell\cos\theta\,\hat{\mathbf{k}}$$

Find the gravitational force on the levitated wire:

$$\vec{\mathbf{F}}_g = -mg\hat{\mathbf{k}}$$

Apply the particle in equilibrium model by adding the forces and setting the net force equal to zero:

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_B + \vec{\mathbf{F}}_g = \frac{\mu_0 I_1 I_2}{\pi a}\ell\cos\theta\,\hat{\mathbf{k}} - mg\hat{\mathbf{k}} = 0$$

Solve for the current in the wires on the ground:

$$I_2 = \frac{mg\pi a}{\mu_0 I_1 \ell\cos\theta}$$

Substitute numerical values:

$$I_2 = \frac{(0.400\text{ kg})(9.80\text{ m/s}^2)\pi(0.010\,0\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A})(10.0\text{ m})\cos 30.0^\circ}$$

$$= \boxed{113\text{ A}}$$

Andre-Marie Ampère

1775 – 1836

French physicist

Credited with the discovery of electromagnetism

- The relationship between electric current and magnetic fields

Also worked in mathematics

Magnetic Field for a Long, Straight Conductor: Direction

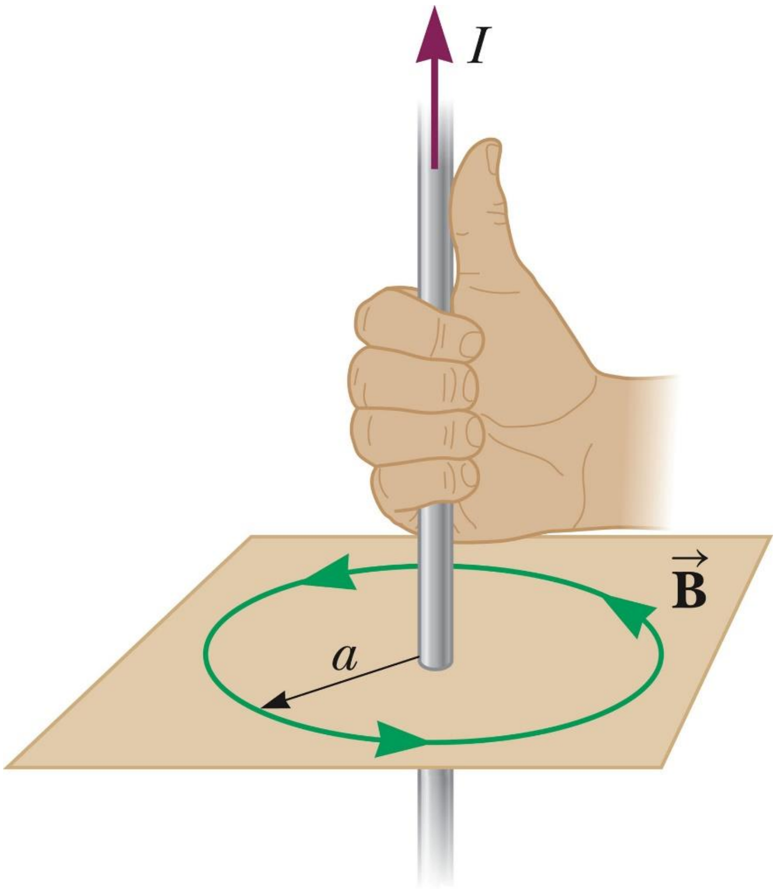
The magnetic field lines are circles concentric with the wire.

The field lines lie in planes perpendicular to the wire.

The magnitude of the field is constant on any circle of radius *a.*

The right-hand rule for determining the direction of the field is shown.

Put the thumb of your right hand in the direction of the current through the amperian loop and your fingers curl in the direction you should integrate around the loop.



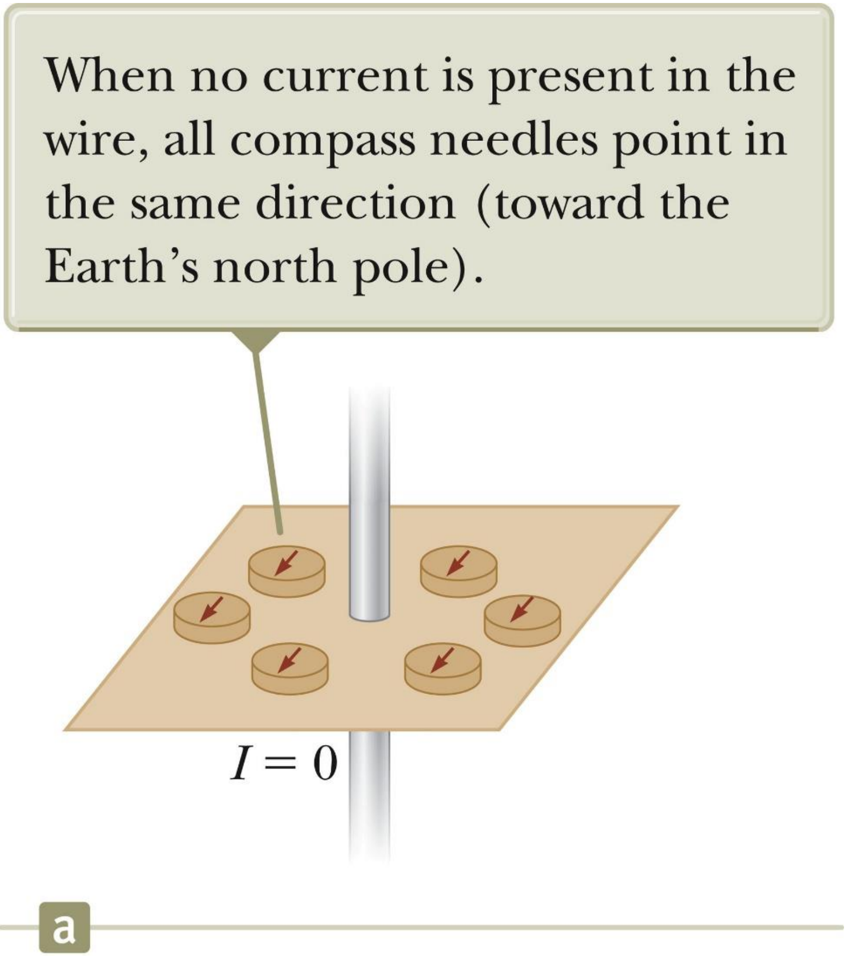
Magnetic Field of a Wire

A compass can be used to detect the magnetic field.

When there is no current in the wire, there is no field due to the current.

The compass needles all point toward the Earth's north pole.

- Due to the Earth's magnetic field



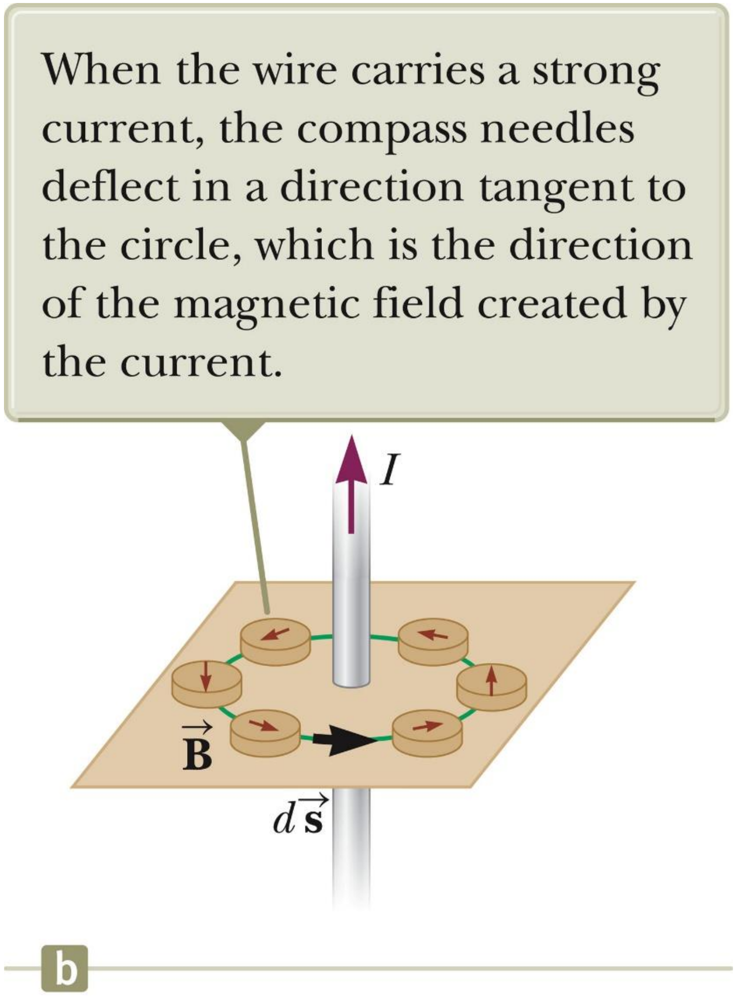
Magnetic Field of a Wire, cont.

Here the wire carries a strong current.

The compass needles deflect in a direction tangent to the circle.

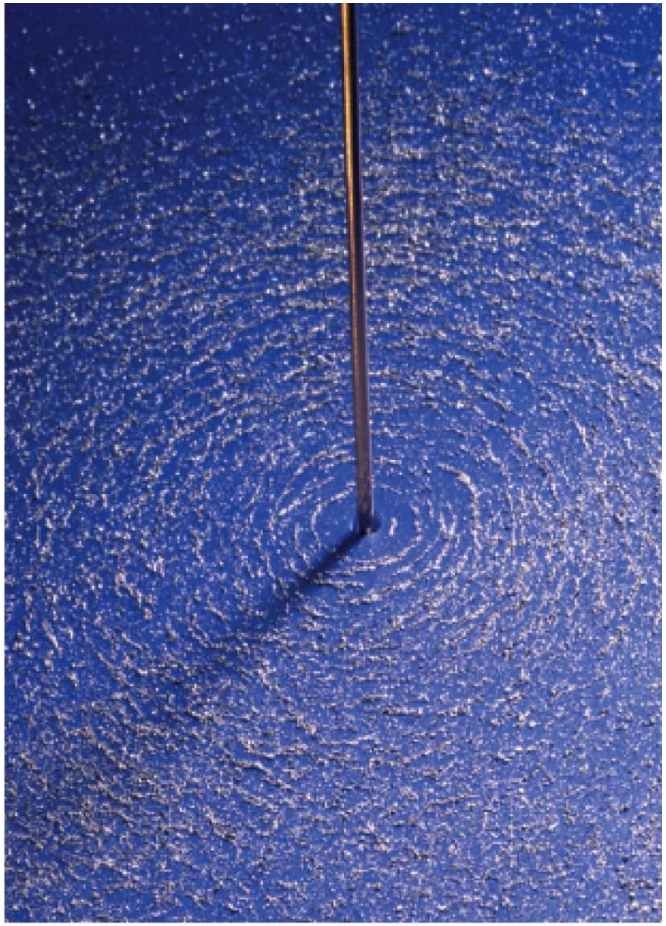
This shows the direction of the magnetic field produced by the wire.

If the current is reversed, the direction of the needles also reverse.



Magnetic Field of a Wire, final

The circular magnetic field around the wire is shown by the iron filings.



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Ampere's Law

The product of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ can be evaluated for small length elements $d\vec{\mathbf{s}}$ on the circular path defined by the compass needles and sum the products for all elements over the closed circular path.

Along this path, the vectors $\vec{\mathbf{B}}$ and $d\vec{\mathbf{s}}$ are parallel at each point, so $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\, ds$

Furthermore, the magnitude of $\vec{\mathbf{B}}$ is constant on this circle and is given by

$$B = \frac{\mu_0 I}{2\pi a}$$

Therefore, the sum of the products *B ds* over the closed path, which is equivalent to the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Ampere's Law

The line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

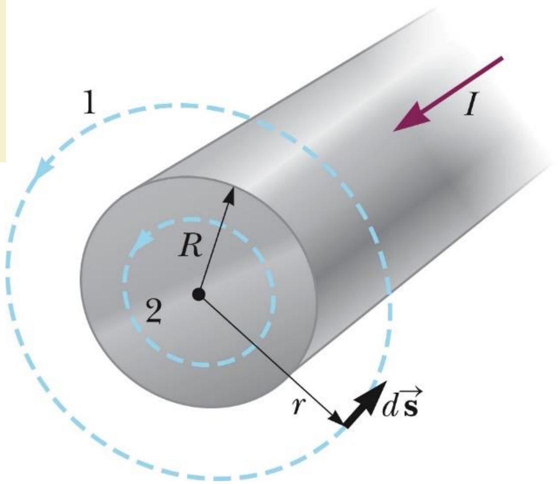
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

Ampere's law describes the creation of magnetic fields by all continuous current configurations.

- Most useful for this course if the current configuration has a high degree of symmetry.

The Magnetic Field Created by a Long Current-Carrying Wire, Example

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. 30.13). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.



From symmetry, $\vec{\mathbf{B}}$ must be constant in magnitude and parallel to $d\vec{\mathbf{s}}$ at every point on this circle.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \boxed{\frac{\mu_0 I}{2\pi r}} \quad (\text{for } r \geq R)$$

Now consider the interior of the wire, where $r < R$. Here the current I' passing through the plane of circle 2 is less than the total current I.

Set the ratio of the current I' enclosed by circle 2 to the entire current I equal to the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:

$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

$$I' = \frac{r^2}{R^2} I$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I\right)$$

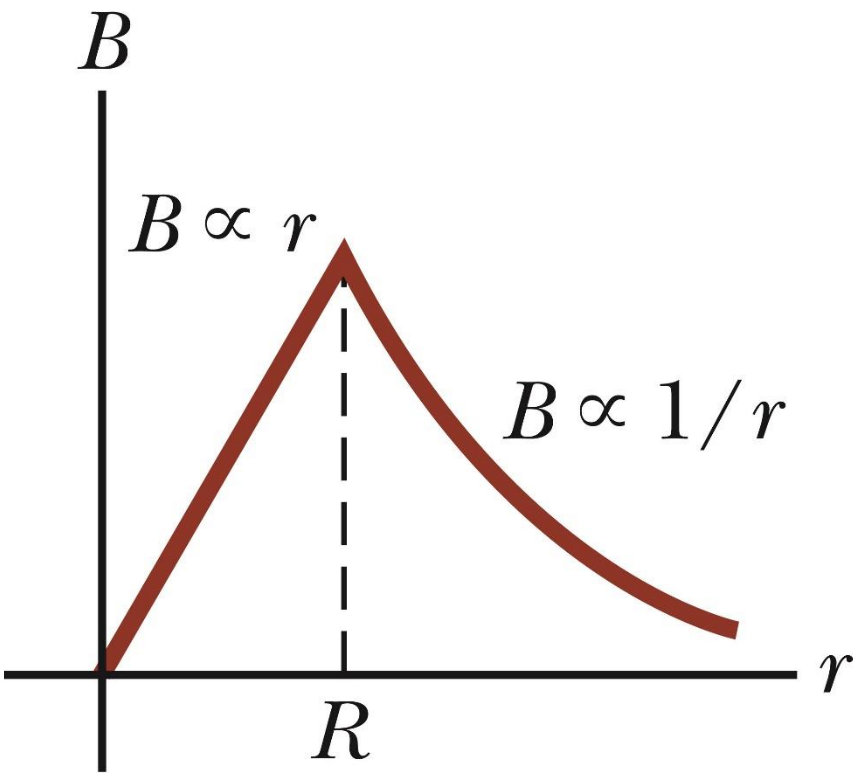
$$B = \boxed{\left(\frac{\mu_0 I}{2\pi R^2}\right) r} \quad (\text{for } r < R)$$

Field Due to a Long Straight Wire – Results Summary

The field is proportional to r inside the wire.

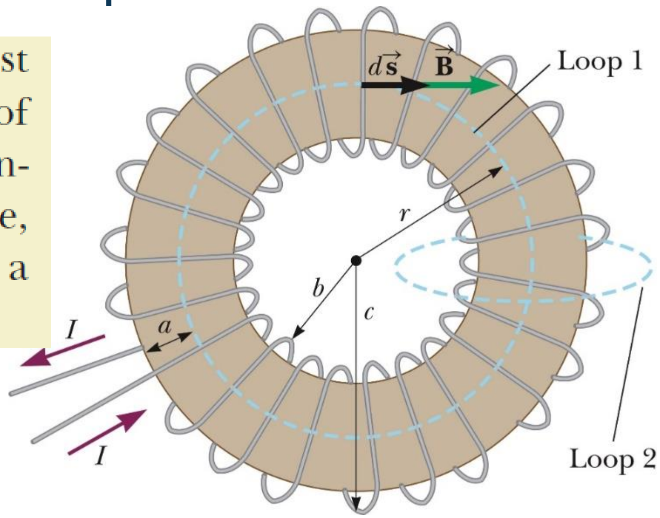
The field varies as 1/r outside the wire.

Both equations are equal at r = R.



The Magnetic Field Created by a Toroid, Example

A device called a *toroid* (Fig. 30.15) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material. For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance r from the center.



Analyze Consider the circular amperian loop (loop 1) of radius r in the plane of Figure 30.15. By symmetry, the magnitude of the field is constant on this circle and tangent to it, so $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\,ds$. Furthermore, the wire passes through the loop N times, so the total current through the loop is NI.

Apply Ampère's law to loop 1:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r) = \mu_0 NI$$

Solve for B:

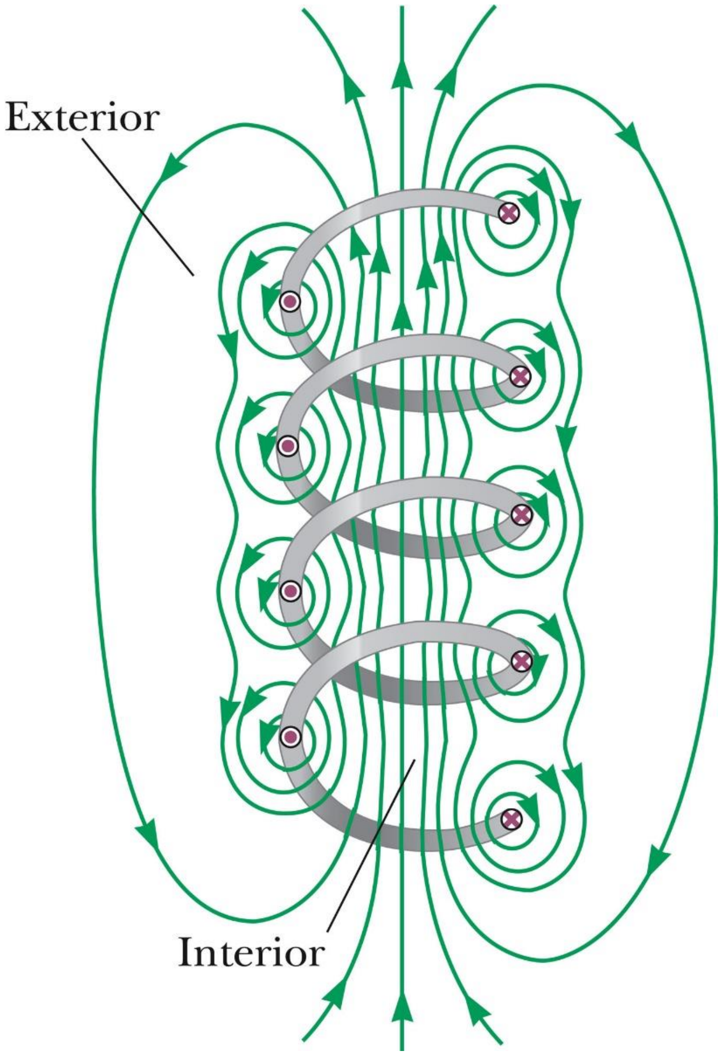
$$B = \boxed{\frac{\mu_0 NI}{2\pi r}}$$

Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix.

A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire.

- The interior of the solenoid



Magnetic Field of a Solenoid, Description

The field lines in the interior are

- Nearly parallel to each other
- Uniformly distributed
- Close together

This indicates the field is strong and almost uniform.

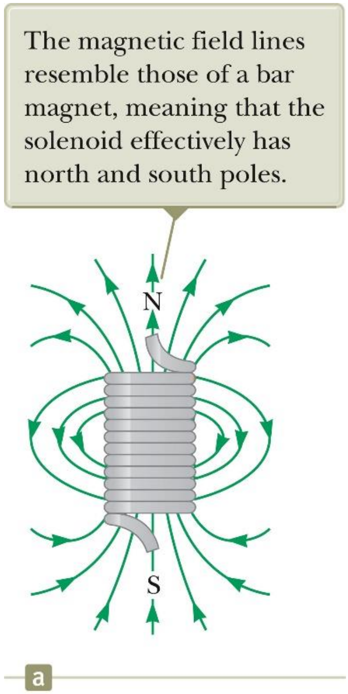
Magnetic Field of a Tightly Wound Solenoid

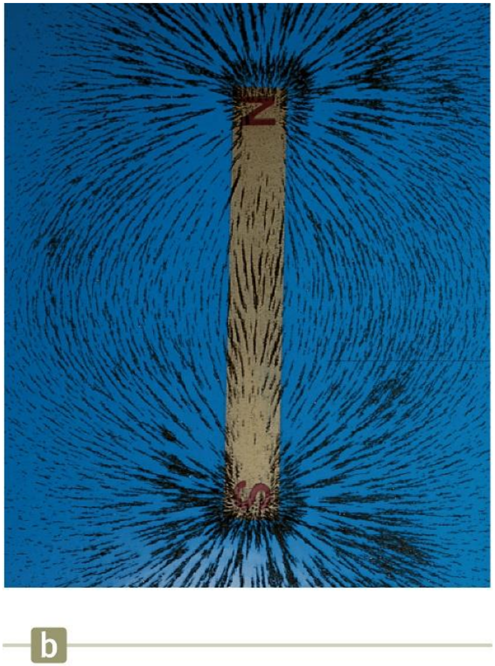
The field distribution is similar to that of a bar magnet.

As the length of the solenoid increases,

- The interior field becomes more uniform.
- The exterior field becomes weaker.

The magnetic field lines in the exterior space form circles around the solenoid. The planes of these circles are perpendicular to the page.

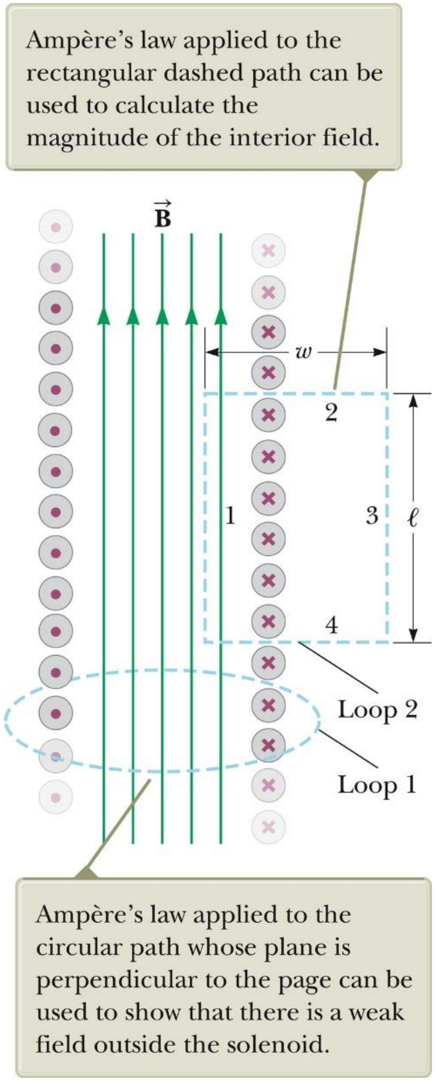




Ideal Solenoid – Characteristics

An ideal solenoid is approached when:

- The turns are closely spaced.
- The length is much greater than the radius of the turns.



Ampere’s Law Applied to a Solenoid

Consider an amperian loop (loop 1 in the diagram) surrounding the ideal solenoid.

- The loop encloses a small current as the charges in the wire move coil by coil along the length of the solenoid.
- There is a weak field external to the solenoid.
- A second layer of turns of wire could be used to eliminate the field.

Ampere’s law can also be used to find the interior magnetic field of the solenoid.

- Consider a rectangle with side ℓ parallel to the interior field and side w perpendicular to the field.
  - This is loop 2 in the diagram.
- The side of length ℓ inside the solenoid contributes to the field.
  - This is side 1 in the diagram.
  - Sides 2, 3, and 4 give contributions of zero to the field.

Ampere's Law Applied to a Solenoid, cont.

Applying Ampere's Law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path 1}} ds = B\ell$$

The total current through the rectangular path equals the current through each turn multiplied by the number of turns.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

Solving Ampere's law for the magnetic field is

$$\boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 n I}$$

- $n = N / \ell$ is the number of turns per unit length.

This is valid only at points near the center of a very long solenoid.

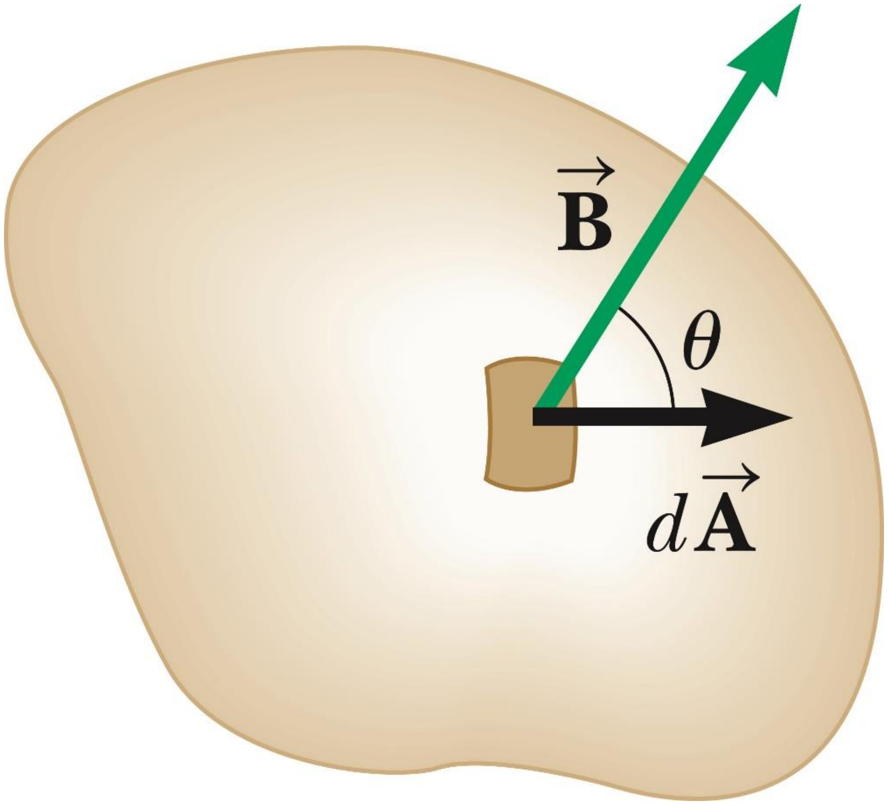
Magnetic Flux

The magnetic flux associated with a magnetic field is defined in a way similar to electric flux.

Consider an area element dA on an arbitrarily shaped surface.

The magnetic field in this element is $\vec{\mathbf{B}}$.

$d\vec{\mathbf{A}}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area *dA.*



Magnetic Flux, cont.

The magnetic flux Φ_B is

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

The unit of magnetic flux is $T \cdot m^2$ = Wb

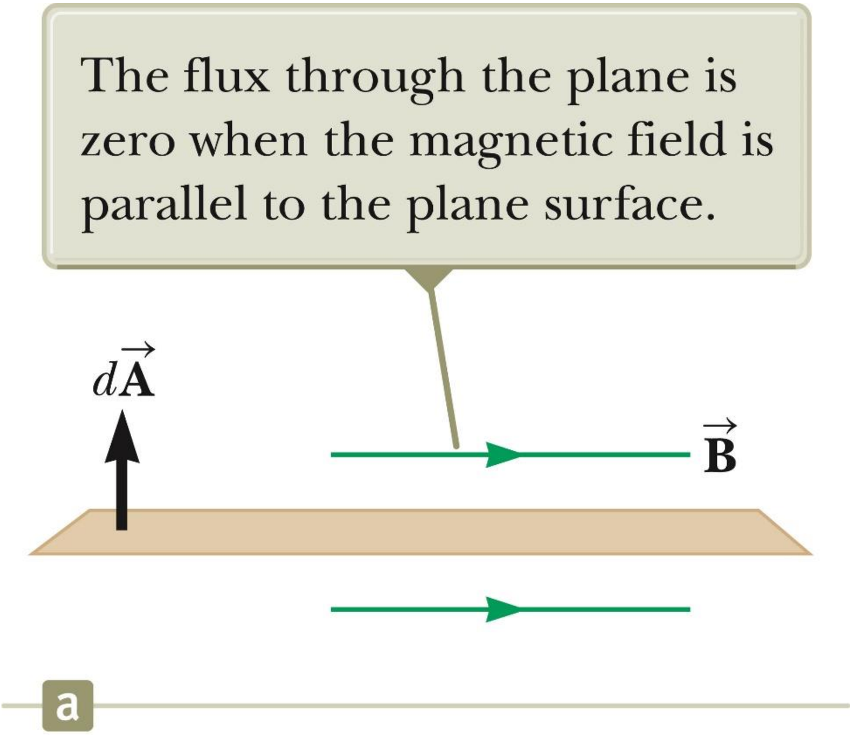
- Wb is a weber

Magnetic Flux Through a Plane, 1

A special case is when a plane of area *A* makes an angle θ with $d\vec{\mathbf{A}}$.

The magnetic flux is $\Phi_B = BA \cos \theta$.

In this case, the field is parallel to the plane and $\Phi_B = 0$.

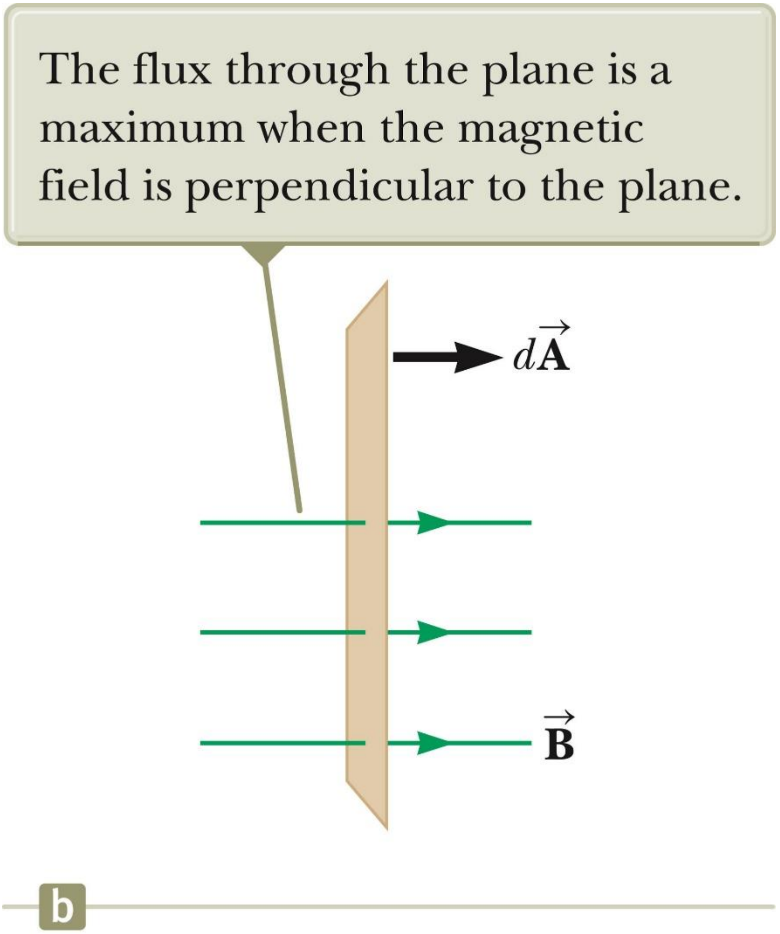


Magnetic Flux Through A Plane, 2

The magnetic flux is $\Phi_B = BA \cos \theta$.

In this case, the field is perpendicular to the plane and $\Phi = BA$.

- This is the maximum value of the flux.



Gauss' Law in Magnetism

Magnetic fields do not begin or end at any point.

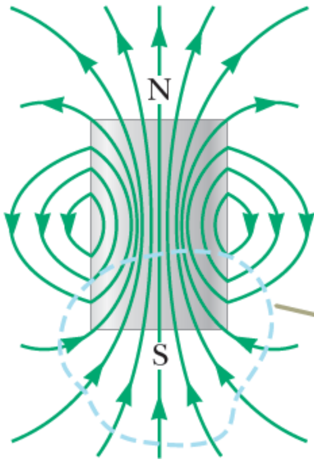
- Magnetic field lines are continuous and form closed loops.
- The number of lines entering a surface equals the number of lines leaving the surface.

Gauss' law in magnetism says the magnetic flux through any closed surface is always zero:

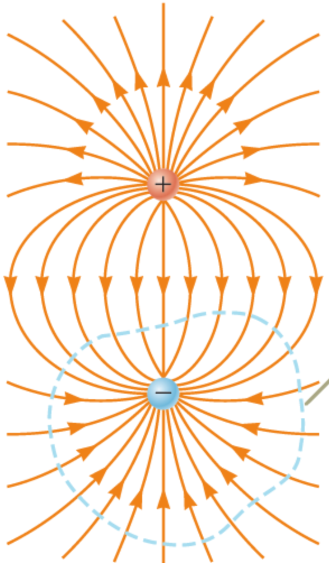
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

This indicates that isolated magnetic poles (monopoles) have never been detected.

- Perhaps they do not exist
- Certain theories do suggest the possible existence of magnetic monopoles.



The net magnetic flux through a closed surface surrounding one of the poles or any other closed surface is zero.



The electric flux through a closed surface surrounding one of the charges is not zero.

Magnetic Flux Through a Rectangular Loop, Example

A rectangular loop of width a and length b is located near a long wire carrying a current I (Fig. 30.21). The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

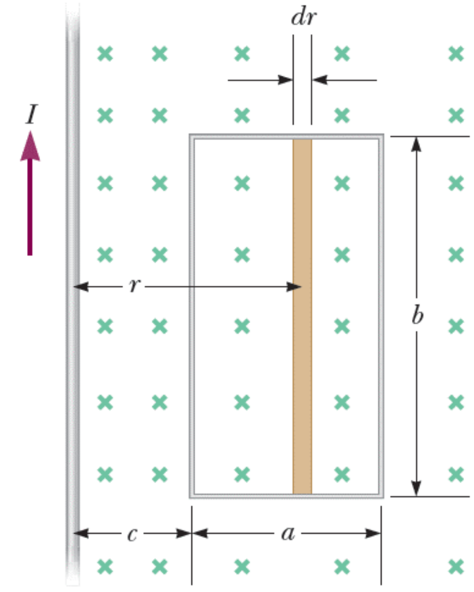
$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

Integrate from $r = c$ to $r = a + c$:

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi}\int \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi}\int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi}\ln r\Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi}\ln\left(\frac{a+c}{c}\right) = \boxed{\frac{\mu_0 I b}{2\pi}\ln\left(1+\frac{a}{c}\right)}$$



$$dA = b\,dr$$

Magnetic Moments

In general, any current loop has a magnetic field and thus has a magnetic dipole moment.

This includes atomic-level current loops described in some models of the atom.

This will help explain why some materials exhibit strong magnetic properties.

Magnetic Moments – Classical Atom

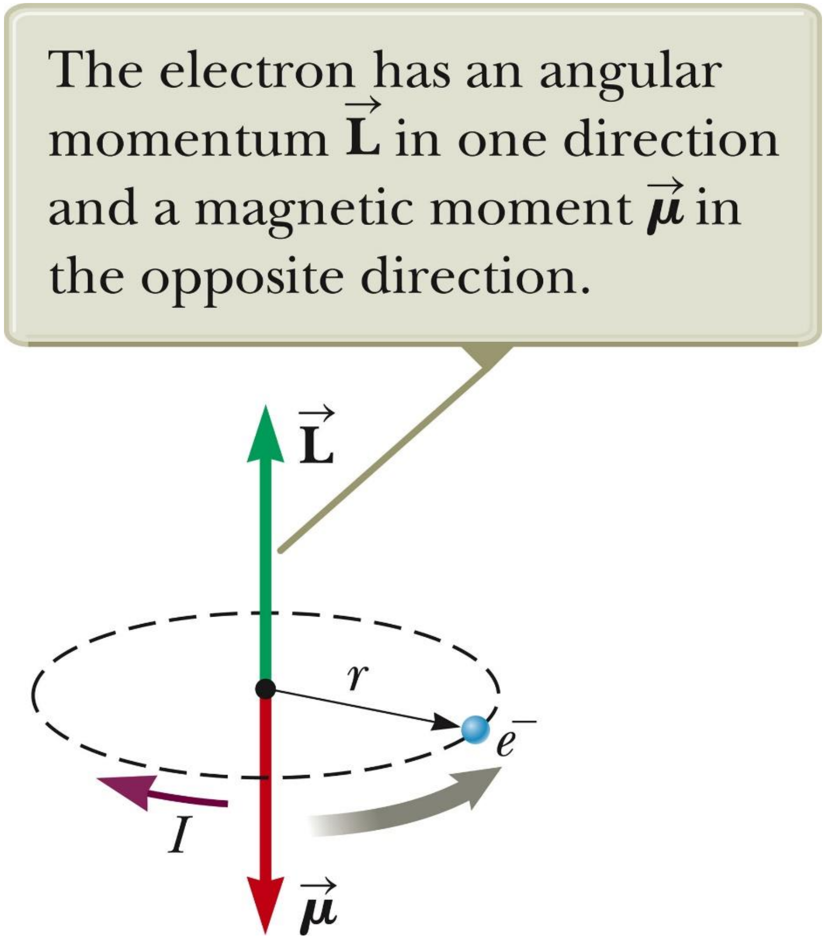
The electrons move in circular orbits.

The orbiting electron constitutes a tiny current loop.

The magnetic moment of the electron is associated with this orbital motion.

$\vec{\mathbf{L}}$ is the angular momentum.

$\vec{\mu}$ is magnetic moment.



Magnetic Moments – Classical Atom, cont.

This model assumes the electron moves:

- with constant speed *v*
- in a circular orbit of radius *r*
- travels a distance $2\pi r$ in a time interval *T*

The current associated with this orbiting electron is

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

The magnetic moment is $\mu = I\,A = \frac{1}{2} evr$, $A = \pi r^2$

The magnetic moment can also be expressed in terms of the angular momentum (*L*).

$$\mu = \left(\frac{e}{2m_e}\right) L \qquad L = m_e v r$$

Magnetic Moments – Classical Atom, final

The magnetic moment of the electron is proportional to its orbital angular momentum.

- The vectors $\vec{\mathbf{L}}$ and $\vec{\mu}$ point in *opposite* directions.
- Because the electron is negatively charged

Quantum physics indicates that angular momentum is quantized and is equal to multiples of

$$\hbar = h/2\pi = 1.05 \times 10^{-34}\,\mathrm{J}$$

where *h* is Planck's constant.

The smallest nonzero value of the electron's magnetic moment resulting from its orbital motion is

$$\mu = \sqrt{2}\,\frac{e}{2m_e}\,\hbar$$

Magnetic Moments of Multiple Electrons

In most substances, the magnetic moment of one electron is canceled by that of another electron orbiting in the opposite direction.

The net result is that the magnetic effect produced by the orbital motion of the electrons is either zero or very small.

Electron Spin

Electrons (and other particles) have an intrinsic property called **spin** that also contributes to their magnetic moment.

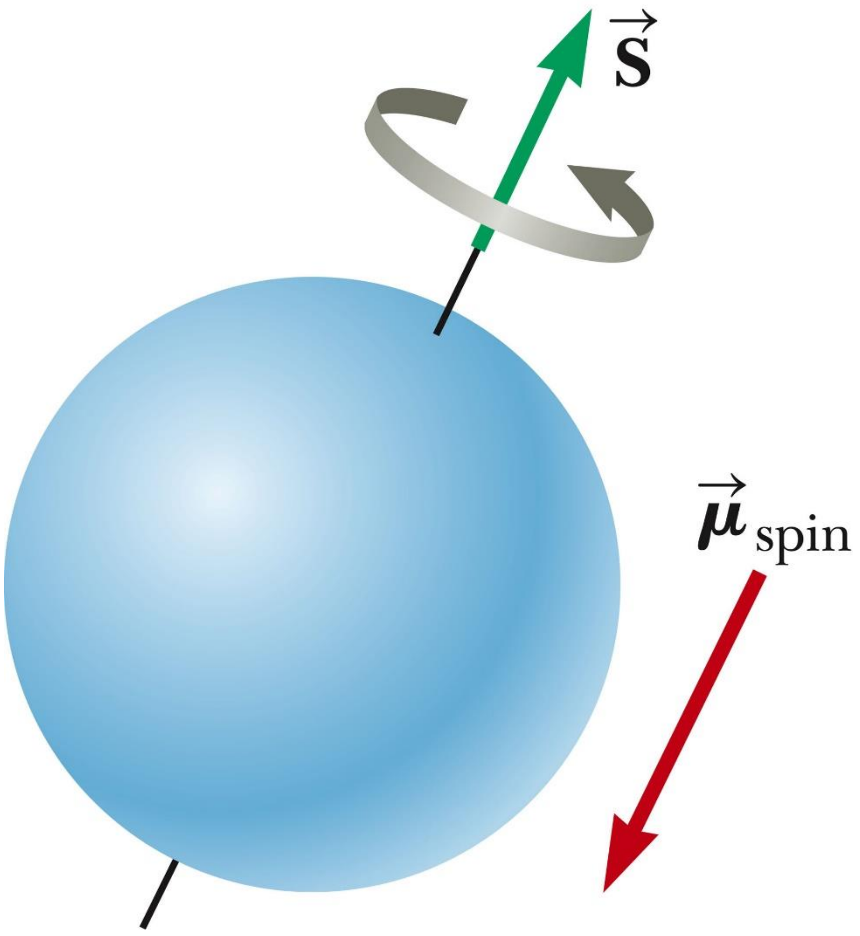
- The electron is not physically spinning.
- It has an intrinsic angular momentum as if it were spinning.
- Spin angular momentum is actually a relativistic effect

Electron Spin, cont.

The classical model of electron spin is the electron spinning on its axis.

The magnitude of the spin angular momentum is

$$S = \frac{\sqrt{3}}{2}\hbar$$



Electron Spin and Magnetic Moment

The magnetic moment characteristically associated with the spin of an electron has the value

$$\mu_{\text{spin}} = \frac{e\hbar}{2m_e}$$

This combination of constants is called the **Bohr magneton μ_B.**

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\,\mathrm{J/T}$$

Electron Magnetic Moment, final

The total magnetic moment of an atom is the vector sum of the orbital and spin magnetic moments.

Some examples are given in the table at right.

The magnetic moment of a proton or neutron is much smaller than that of an electron and can usually be neglected.

TABLE 30.1

Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

Ferromagnetism

Some substances exhibit strong magnetic effects called ferromagnetism.

Some examples of ferromagnetic materials are:

- iron
- cobalt
- nickel
- gadolinium
- dysprosium

They contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field.

Once the moments are aligned, the substance remains magnetized after the external field is removed.

Domains

All ferromagnetic materials are made up of microscopic regions called **domains.**

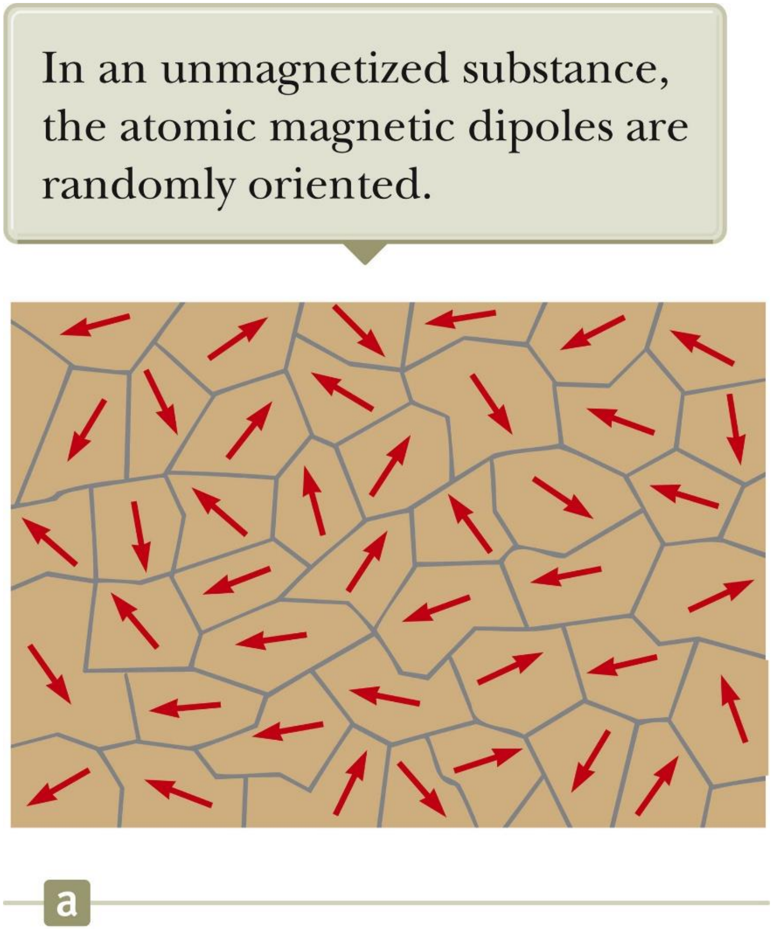
- The domain is an area within which all magnetic moments are aligned.

The boundaries between various domains having different orientations are called **domain walls.**

Domains, Unmagnetized Material

The magnetic moments in the domains are randomly aligned.

The net magnetic moment is zero.

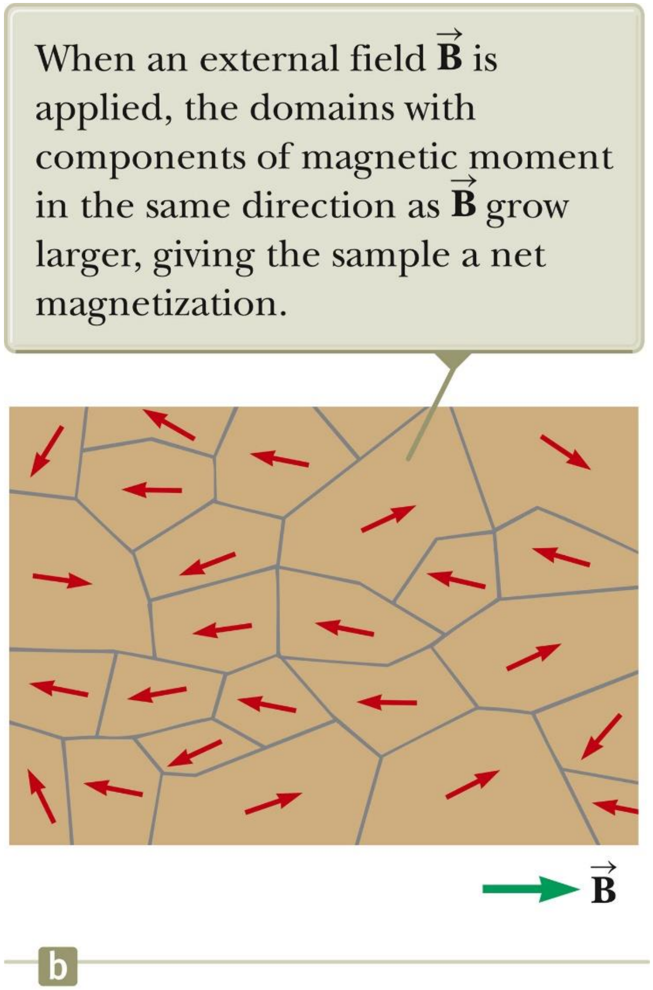


Domains, External Field Applied

A sample is placed in an external magnetic field.

The size of the domains with magnetic moments aligned with the field grows.

The sample is magnetized.

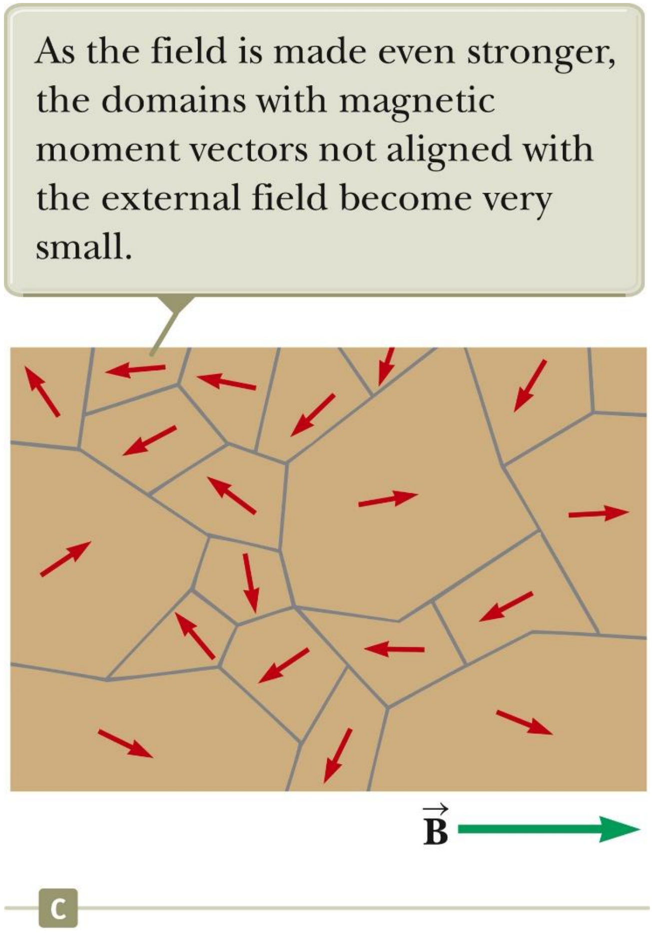


Domains, External Field Applied, cont.

The material is placed in a stronger field.

The domains not aligned with the field become very small.

When the external field is removed, the material may retain a net magnetization in the direction of the original field.



Curie Temperature

The **Curie temperature** is the critical temperature above which a ferromagnetic material loses its residual magnetism.

- The material will become paramagnetic.

Above the Curie temperature, the thermal agitation is great enough to cause a random orientation of the moments and the substance becomes paramagnetic.

Table of Some Curie Temperatures

TABLE 30.2 *Curie Temperatures for Several Ferromagnetic Substances*

Substance	T_{Curie} (K)
Iron	1 043
Cobalt	1 394
Nickel	631
Gadolinium	317
Fe_2O_3	893

Paramagnetism

Paramagnetic substances have a weak magnetism resulting from the presence of atoms (or ions) that have permanent magnetic moments.

These moments interact only weakly with one another and are randomly oriented in the absence of an external magnetic field.

When a paramagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field.

- The alignment process competes with thermal motion which randomizes the moment orientations.

Diamagnetism

When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field.

Diamagnetic substances are weakly repelled by a magnet.

- Weak, so only present when ferromagnetism or paramagnetism do not exist

Meissner Effect

Certain types of superconductors also exhibit perfect diamagnetism in the superconducting state.

- This is called the **Meissner effect.**

If a permanent magnet is brought near a superconductor, the two objects repel each other.

In the Meissner effect, the small magnet at the top induces currents in the superconducting disk below, which is cooled to 321°F (77 K). The currents create a repulsive magnetic force on the magnet causing it to levitate above the superconducting disk.