PHYS143
Physics for Engineers
Tutorial - Chapter 42 – Solutions

Question 1
An isolated atom of a certain element emits light of wavelength 520 nm when the atom falls from its fifth excited state into its second excited state. The atom emits a photon of wavelength 410 nm when it drops from its sixth excited state into its second excited state. Find the wavelength of the light radiated when the atom makes a transition from its sixth to its fifth excited state.

(a) The fifth excited state must lie above the second excited state by the photon energy

$$E_{52} = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{520\times10^{-9}\text{ m}}$$
$$= 3.82\times10^{-19}\text{ J}$$

The sixth excited state exceeds the second in energy by

$$E_{62} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{410\times10^{-9}\text{ m}} = 4.85\times10^{-19}\text{ J}$$

Then the sixth excited state is above the fifth by

$$(4.85-3.82)\times10^{-19}\text{ J} = 1.03\times10^{-19}\text{ J}$$

In the 6 to 5 transition the atom emits a photon with the infrared wavelength

$$\lambda = \frac{hc}{E_{65}} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.03\times10^{-19}\text{ J}}$$
$$= 1.94\times10^{-6}\text{ m} = \boxed{1.94\ \mu\text{m}}$$

Question 2
A photon is emitted when a hydrogen atom undergoes a transition from the n = 5 state to the n = 3 state. Calculate (a) the energy (in electron volts), (b) the wavelength, and (c) the frequency of the emitted photon.

The allowed energy levels of the hydrogen atom are given by

$$E_n = -\frac{13.6\text{ eV}}{n^2} \quad \text{where } n = 1, 2, 3\ldots$$

In a transition for higher state n_i to lower state n_f, a photon of energy ΔE is emitted, and energy is conserved:

$$E_i + \Delta E = E_f$$

or

$$\Delta E = E_f - E_i = -\frac{13.6\text{ eV}}{n_f^2} - \left(-\frac{13.6\text{ eV}}{n_i^2}\right) = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

(a) For the transition $n_i = 5$ to $n_f = 3$,

$$\Delta E = 13.6\text{ eV}\left(\frac{1}{3^2} - \frac{1}{5^2}\right) = \boxed{0.967\text{ eV}}$$

(b) To find the wavelength of the emitted photon, we use:

$$\Delta E = 0.967\text{ eV} = hf = \frac{hc}{\lambda}$$

Solving,

$$\lambda = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{0.967\text{ eV}} = 1\,282\text{ nm} = \boxed{1.28\ \mu\text{m}}$$

(c) The frequency of the emitted photon is

$$f = \frac{c}{\lambda} = \frac{3.00\times 10^8\text{ m/s}}{1\,282\times 10^{-9}\text{ m}} = \boxed{2.34\times 10^{14}\text{ Hz}}$$

Question 3

For a hydrogen atom in its ground state, compute (a) the orbital speed of the electron, (b) the kinetic energy of the electron, and (c) the electric potential energy of the atom. ($a_0 = 0.0529$ nm)

(a) From $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$ where

$$r_1 = (1)^2 a_0 = 0.005\,29\text{ nm} = 5.29\times 10^{-11}\text{ m}$$

Substituting numerical values,

$$v_1 = \sqrt{\frac{(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\text{ C})^2}{(9.11\times 10^{-31}\text{ kg})(5.29\times 10^{-11}\text{ m})}}$$
$$= \boxed{2.19\times 10^6\text{ m/s}}$$

(b) The kinetic energy of the electron is

$$K_1 = \frac{1}{2}m_e v_1^2 = \frac{1}{2}(9.11\times 10^{-31}\text{ kg})(2.19\times 10^6\text{ m/s})^2$$
$$= 2.18\times 10^{-18}\text{ J} = \boxed{13.6\text{ eV}}$$

(c) The electric potential energy of the atom is

$$U_1 = -\frac{k_e e^2}{r_1} = -\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{5.29\times10^{-11}\ \mathrm{m}}$$
$$= -4.35\times10^{-18}\ \mathrm{J} = \boxed{-27.2\ \mathrm{eV}}$$

Question 4

A hydrogen atom is in its first excited state (n = 2). Calculate (a) the radius of the orbit, (b) the linear momentum of the electron, (c) the angular momentum of the electron, (d) the kinetic energy of the electron, (e) the potential energy of the system, and (f) the total energy of the system. ($a_0 = 0.0529$ nm)

We note, during our calculations, that the nominal velocity of the electron is less than 1% of the speed of light; therefore, we do not need to use relativistic equations.

(a) By Bohr's theory,

$$r_n = n^2 a_0$$
$$r_2 = (2)^2(0.052\,9\ \mathrm{nm}) = \boxed{0.212\ \mathrm{nm}}$$

(b) Since $m_e v r = n\hbar$,

$$p = m_e v = \frac{n\hbar}{r} = \frac{2\left(1.054\,6\times10^{-34}\ \mathrm{J\cdot s}\right)}{2.12\times10^{-10}\ \mathrm{m}}$$
$$= \boxed{9.97\times10^{-25}\ \mathrm{kg\cdot m/s}}$$

(c)
$$L_2 = m_e v_2 r_2 = \left(9.97\times10^{-25}\ \mathrm{kg\cdot m/s}\right)\left(0.212\times10^{-9}\ \mathrm{m}\right)$$
$$= \boxed{2.11\times10^{-34}\ \mathrm{kg\cdot m^2/s}}$$

(d) Next, the speed is

$$v = \frac{p}{m_e} = \frac{9.97\times10^{-25}\ \mathrm{kg\cdot m/s}}{9.11\times10^{-31}\ \mathrm{kg}} = 1.09\times10^6\ \mathrm{m/s}$$

So the kinetic energy is $K = \frac{1}{2}m_e v^2$:

$$K = \frac{\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(1.09\times10^6\mathrm{m/s}\right)^2}{2}$$
$$= \frac{5.45\times10^{-19}\ \mathrm{J}}{1.602\times10^{-19}\ \mathrm{J/eV}} = \boxed{3.40\ \mathrm{eV}}$$

(e) The electric potential energy is

$$U = -\frac{k_e e^2}{r} = -\frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.602\times10^{-19}\ \mathrm{C}\right)^2}{2.12\times10^{-10}\ \mathrm{m}}$$
$$= -1.09\times10^{-18}\ \mathrm{J} = \boxed{-6.80\ \mathrm{eV}}$$

(f) Thus the total energy is

$$E = K + U = -5.45\times10^{-19}\ \mathrm{J} = \boxed{-3.40\ \mathrm{J}}$$

Question 5
List the possible sets of quantum numbers for the hydrogen atom associated with (a) the 3d subshell and (b) the 3p subshell.

(a) In the $3d$ subshell, $n = 3$ and $\ell = 2$, we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	–1	–1	–2	–2
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(a total of 10 states.)

(b) In the $3p$ subshell, $n = 3$ and $\ell = 1$, we have

n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	0	0	–1	–1
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(a total of 6 states.)

Question 6
Find all possible values of (a) L, (b) L_z , and (c) θ for a hydrogen atom in a 3d state.

(a) For a $3d$ state, $n = 3$ and $\ell = 2$. Therefore,

$$L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J} \cdot \text{s}$$

(b) m_ℓ can have the values –2, –1, 0, 1, and 2,

so $\boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

(c) Using the relation $\cos\theta = \dfrac{L_z}{L}$, we find the possible values of θ:

$\boxed{145°,\ 114°,\ 90.0°,\ 65.9°,\ \text{and } 35.3°}$

Question 7

A hydrogen atom is in its fifth excited state, with principal quantum number 6. The atom emits a photon with a wavelength of 1 090 nm. Determine the maximum possible magnitude of the orbital angular momentum of the atom after emission.

The 5th excited state has $n = 6$, energy $E_6 = \frac{-13.6 \text{ eV}}{(6)^2} = -0.378 \text{ eV}$.

The atom loses this much energy:

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{(1\,090 \times 10^{-9} \text{ m})(1.60 \times 10^{-19} \text{ J/eV})} = 1.14 \text{ eV}$$

to end up with energy $-0.378 \text{ eV} - 1.14 \text{ eV} = -1.52 \text{ eV}$

which is the energy in state 3: $-\frac{13.6 \text{ eV}}{3^3} = -1.51 \text{ eV}$

While $n = 3$, ℓ can be as large as 2, giving angular momentum

$$L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar = 2.58 \times 10^{-34} \text{ J}\cdot\text{s}}$$