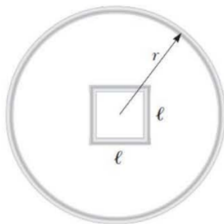
PHYS143
Physics for Engineers
Tutorial - Chapter 31 - Solutions

Question 1

A square, single-turn wire loop l = 1.00 cm on a side is placed inside a solenoid that has a circular cross section of radius r = 3.00 cm as shown in the end view of Figure. The solenoid is 20.0 cm long and wound with 100 turns of wire. (a) If the current in the solenoid is 3.00 A, what is the magnetic flux through the square loop? (b) If the current in the solenoid is reduced to zero in 3.00 s, what is the magnitude of the average induced emf in the square loop?



The initial magnetic field inside the solenoid is

$$B = \mu_0 nI = \left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{100}{0.200\ \mathrm{m}}\right)(3.00\ \mathrm{A})$$

$$= 1.88 \times 10^{-3}\ \mathrm{T}$$

(a) $\Phi_B = BA\cos\theta = \left(1.88 \times 10^{-3}\ \mathrm{T}\right)\left(1.00 \times 10^{-2}\ \mathrm{m}\right)^2 \cos 0°$

$$= \boxed{1.88 \times 10^{-7}\ \mathrm{T\cdot m^2}}$$

(b) When the current is zero, the flux through the loop is $\Phi_B = 0$ and the average induced emf has been

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{\left|0 - 1.88 \times 10^{-7}\ \mathrm{T\cdot m^2}\right|}{3.00\ \mathrm{s}} = \boxed{6.28 \times 10^{-8}\ \mathrm{V}}$$

Question 2

A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30.0° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.400 s, an emf of magnitude 80.0 mV is induced in the coil. What is the total length of the wire in the coil?

Faraday's law, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$, becomes here

$$\mathcal{E} = -N\frac{d}{dt}\left(BA\cos\theta\right) = -NA\cos\theta\frac{dB}{dt}$$

The magnitude of the emf is

$$|\mathcal{E}| = NA\cos\theta\left(\frac{\Delta B}{\Delta t}\right)$$

The area is

$$A = \frac{|\mathcal{E}|}{N\cos\theta\left(\frac{\Delta B}{\Delta t}\right)}$$

$$A = \frac{80.0\times10^{-3}\ \text{V}}{50\left(\cos 30.0^\circ\right)\left(\frac{600\times10^{-6}\ \text{T} - 200\times10^{-6}\ \text{T}}{0.400\ \text{s}}\right)} = 1.85\ \text{m}^2$$

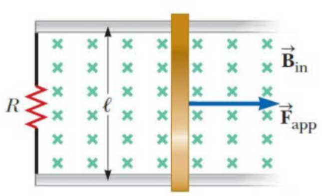
Each side of the coil has length $d = \sqrt{A}$, so the total length of the wire is

$$L = N(4d) = 4N\sqrt{A} = (4)(50)\sqrt{1.85\ \text{m}^2} = \boxed{272\ \text{m}}$$

Question 3

(a) Consider the arrangement shown in Figure. Assume that R = 6.00 Ω, l = 1.20 m, and a uniform 2.50-T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

(b) The resistor is R = 6.00 Ω, and a 2.50-T magnetic field is directed perpendicularly downward, into the paper. Let l = 1.20 m. (i) Calculate the applied force required to move the bar to the right at a constant speed of 2.00 m/s. (ii) At what rate is energy delivered to the resistor?

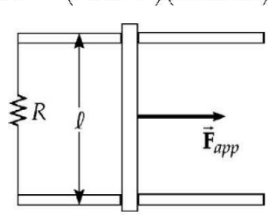


(a) See ANS. FIG. P31.26. The current is given by

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$$

Solving for the velocity gives

$$v = \frac{IR}{B\ell} = \frac{(0.500\ \text{A})(6.00\ \Omega)}{(2.50\ \text{T})(1.20\ \text{m})} = \boxed{1.00\ \text{m/s}}$$



(b) (i) Refer to ANS. FIG. P31.26 above. At constant speed, the net force on the moving bar equals zero, or

$$\left|\vec{\mathbf{F}}_{\text{app}}\right| = I\left|\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right|$$

where the current in the bar is $I = \mathcal{E}/R$ and the motional emf is $\mathcal{E} = B\ell v$. Therefore,

$$F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50\text{ T})^2(1.20\text{ m})^2(2.00\text{ m/s})}{6.00\ \Omega}$$
$$= 3.00\text{ N}$$

The applied force is $\boxed{3.00\text{ N to the right}}$.

(ii) $P = I^2R = \dfrac{B^2\ell^2v^2}{R} = 6.00\text{ W}$ or $P = Fv = \boxed{6.00\text{ W}}$

Question 4

A conducting rod of length *l* moves on two horizontal, frictionless rails as shown in Figure of Question 6. If a constant force of 1.00 N moves the bar at 2.00 m/s through a magnetic field $\vec{\mathbf{B}}$ that is directed into the page, (a) what is the current through the 8.00-Ω resistor R? (b) What is the rate at which energy is delivered to the resistor?

The magnetic force on the rod is given by

$$F_B = I\ell B$$

and the motional emf by

$$\mathcal{E} = B\ell v$$

The current is given by $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$, so $B = \dfrac{IR}{\ell v}$.

(a) $F_B = \dfrac{I^2\ell R}{\ell v}$ and $I = \sqrt{\dfrac{F_B v}{R}} = \sqrt{\dfrac{(1.00\text{ N})(2.00\text{ m/s})}{8.00\ \Omega}} = \boxed{0.500\text{ A}}$

(b) The rate at which energy is delivered to the resistor is the power delivered, given by

$$P = I^2R = (0.500\text{ A})^2(8.00\ \Omega) = \boxed{2.00\text{ W}}$$