PHYS143
Physics for Engineers
Tutorial - Chapter 34 - Solutions

Question 1

High-power lasers in factories are used to cut through cloth and metal. One such laser has a beam diameter of 1.00 mm and generates an electric field having an amplitude of 0.700 MV/m at the target. Find (a) the amplitude of the magnetic field produced, (b) the intensity of the laser, and (c) the power delivered by the laser.

(a) $B_{\max} = \dfrac{E_{\max}}{c}$: $B_{\max} = \dfrac{7.00\times 10^{5}\ \text{N/C}}{3.00\times 10^{8}\ \text{m/s}} = \boxed{2.33\ \text{mT}}$

(b) $I = \dfrac{E_{\max}^{2}}{2\mu_0 c}$:

$$I = \frac{\left(7.00\times 10^{5}\ \text{V/m}\right)^{2}}{2\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times 10^{8}\ \text{m/s}\right)} = 6.50\times 10^{8}\ \text{W/m}^{2}$$
$$= \boxed{650\ \text{MW/m}^{2}}$$

(c) $I = \dfrac{P}{A}$:

$$P = IA = \left(6.50\times 10^{8}\ \text{W/m}^{2}\right)\left[\frac{\pi}{4}\left(1.00\times 10^{-3}\ \text{m}\right)^{2}\right] = \boxed{511\ \text{W}}$$

Question 2

At one location on the Earth, the rms value of the magnetic field caused by solar radiation is 1.80 μT. From this value, calculate (a) the rms electric field due to solar radiation, (b) the average energy density of the solar component of electromagnetic radiation at this location, and (c) the average magnitude of the Poynting vector for the Sun's radiation.

(a) $E_{\text{rms}} = cB_{\text{rms}} = \left(3.00\times 10^{8}\ \text{m/s}\right)\left(1.80\times 10^{-6}\ \text{T}\right) = \boxed{540\ \text{V/m}}$

(b) From,

$$u_{\text{avg}} = \frac{\left(B_{\max}\right)^{2}}{2\mu_0} = \frac{\left(B_{\text{rms}}\right)^{2}}{\mu_0} = \frac{\left(1.80\times 10^{-6}\ \text{T}\right)^{2}}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = \boxed{2.58\ \mu\text{J/m}^{3}}$$

(c) $S_{\text{avg}} = cu_{\text{avg}} = \left(3.00\times 10^{8}\ \text{m/s}\right)\left(2.58\times 10^{-6}\ \text{J/m}^{3}\right) = \boxed{773\ \text{W/m}^{2}}$

Question 3

(a) A 25.0-mW laser beam of diameter 2.00 mm is reflected at normal incidence by a perfectly reflecting mirror. Calculate the radiation pressure on the mirror.
(b) A radio wave transmits 25.0 W/m^2 of power per unit area. A flat surface of area A is perpendicular to the direction of propagation of the wave. Assuming the surface is a perfect absorber, calculate the radiation pressure on it.

(a) The intensity of the beam is $I = \frac{P_{\text{power}}}{\pi r^2}$, where $r = 1.00 \times 10^{-3}$ m. The radiation pressure on the mirror is

$$P = \frac{2S}{c} = \frac{2I}{c} = \frac{2P_{\text{power}}}{\pi r^2 c}$$

$$= \frac{2(25.0 \times 10^{-3}\ \text{W})}{\pi(1.00 \times 10^{-3}\ \text{m})^2(3.00 \times 10^8\ \text{m/s})} = \boxed{5.31 \times 10^{-5}\ \text{N/m}^2}$$

(b) For complete absorption,

$$P = \frac{S}{c} = \frac{25.0\ \text{W/m}^2}{3.00 \times 10^8\ \text{m/s}} = 8.33 \times 10^{-8}\ \text{N/m}^2 = \boxed{83.3\ \text{nPa}}$$

Question 4

A 15.0-mW helium–neon laser emits a beam of circular cross section with a diameter of 2.00 mm. (a) Find the maximum electric field in the beam. (b) What total energy is contained in a 1.00-m length of the beam? (c) Find the momentum carried by a 1.00-m length of the beam.

(a) $I = \frac{P}{\pi r^2} = \frac{E_{\text{max}}^2}{2\mu_0 c}$, and $r = 1.00 \times 10^{-3}$ m:

$$E_{\text{max}} = \sqrt{\frac{2\mu_0 c P}{\pi r^2}}$$

$$= \sqrt{\frac{2\left[4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right](3.00 \times 10^8\ \text{m/s})(15.0 \times 10^{-3}\ \text{W})}{\pi(1.00 \times 10^{-3}\ \text{m})^2}}$$

$$= 1.90 \times 10^8\ \text{J} = \boxed{1.90\ \text{kN/C}}$$

(b) The beam carries power P. The amount of energy ΔE in the length of a beam of length ℓ is the amount of power that passes a point in time interval $\Delta t = \ell/c$:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\ell/c}$$

or $$\Delta E = \frac{P\ell}{c} = \frac{15.0 \times 10^{-3}\ \text{W}}{3.00 \times 10^8\ \text{m/s}}(1.00\ \text{m}) = \boxed{50.0\ \text{pJ}}.$$

(c) From our result in part (b), the momentum and energy carried a light beam are related by

$$p = \frac{T_{\text{ER}}}{c} = \frac{\Delta E}{c} = \frac{50.0 \times 10^{-12}\ \text{J}}{3.00 \times 10^8\ \text{m/s}} = \boxed{1.67 \times 10^{-19}\ \text{kg}\cdot\text{m/s}}$$

Question 5

A plane electromagnetic wave of intensity 6.00 W/m^2, moving in the x direction, strikes a small perfectly reflecting pocket mirror, of area 40.0 cm^2, held in the yz plane. (a) What momentum does the wave transfer to the mirror each second? (b) Find the force the wave exerts on the mirror.

(a) The magnitude of the momentum transferred to the assumed totally reflecting surface in time interval Δt is

$$\Delta p = \frac{2T_{ER}}{c} = \frac{2SA\Delta t}{c}$$

Then the momentum transfer is

$$\Delta \vec{\mathbf{p}} = \frac{2\vec{\mathbf{S}}A\Delta t}{c} = \frac{2(6.00\ \hat{\mathbf{i}}\ \mathrm{W/m^2})(40.0\times 10^{-4}\ \mathrm{m^2})(1.00\ \mathrm{s})}{3.00\times 10^{8}\ \mathrm{m/s}}$$

$$\Delta \vec{\mathbf{p}} = \boxed{1.60\times 10^{-10}\ \hat{\mathbf{i}}\ \mathrm{kg\cdot m/s\ each\ second}}$$

(b) The force is

$$\vec{\mathbf{F}} = PA\hat{\mathbf{i}} = \frac{2SA}{c}\hat{\mathbf{i}} = \frac{2\left(6.00\ \mathrm{W/m^2}\right)\left(40.0\times 10^{-4}\ \mathrm{m^2}\right)(1.00\ \mathrm{s})}{3.00\times 10^{8}\ \mathrm{m/s}}$$

$$= \boxed{1.60\times 10^{-10}\ \hat{\mathbf{i}}\ \mathrm{N}}$$