PHYS143
Physics for Engineers
Tutorial - Chapter 33 - Solutions

Question 1

The output voltage of an AC source is given by $\Delta v = 120 \sin 30.0\pi t$, where Δv is in volts and t is in seconds. The source is connected across a 0.500-H inductor. Find (a) the frequency of the source, (b) the rms voltage across the inductor, (c) the inductive reactance of the circuit, (d) the rms current in the inductor, and (e) the maximum current in the inductor.

We are given: $\Delta v = 120 \sin 30.0\pi t$ where Δv is in volts and t in seconds, and $L = 0.500$ H.

(a) By inspection, $\omega = 30\pi$ rad/s, so

$$f = \frac{\omega}{2\pi} = \frac{30\pi \text{ rad/s}}{2\pi} = \boxed{15.0 \text{ Hz}}.$$

(b) Also by inspection, $\Delta V_{L,\max} = 120$ V, so that

$$\Delta V_{L,\text{rms}} = \frac{\Delta V_{L,\max}}{\sqrt{2}} = \frac{120 \text{ V}}{\sqrt{2}} = \boxed{84.9 \text{ V}}$$

(c) $X_L = 2\pi f L = \omega L = (30\pi \text{ rad/s})(0.500 \text{ H}) = \boxed{47.1\ \Omega}$

(d) $I_{\text{rms}} = \dfrac{\Delta V_{L,\text{rms}}}{X_L} = \dfrac{84.9 \text{ V}}{47.1\ \Omega} = \boxed{1.80 \text{ A}}$

(e) $I_{\max} = \sqrt{2}\, I_{\text{rms}} = \sqrt{2}(1.80 \text{ A}) = \boxed{2.55 \text{ A}}$

Question 2

A source delivers an AC voltage of the form $\Delta v = 98.0 \sin 80\pi t$, where Δv is in volts and t is in seconds, to a capacitor. The maximum current in the circuit is 0.500 A. Find (a) the rms voltage of the source, (b) the frequency of the source, and (c) the value of the capacitance.

(a) By inspection, $\Delta V_{C,\max} = 98.0$ V, so

$$\Delta V_{C,\text{rms}} = \frac{\Delta V_{C,\max}}{\sqrt{2}} = \frac{98.0 \text{ V}}{\sqrt{2}} = \boxed{69.3 \text{ V}}.$$

(b) Also by inspection, $\omega = 80\pi$ rad/s, so

$$f = \frac{\omega}{2\pi} = \frac{80\pi \text{ rad/s}}{2\pi \text{ rad}} = \boxed{40.0 \text{ Hz}}$$

(c) We can find the capacitive reactance from

$$X_C = \frac{\Delta V_{C,\max}}{I_{\max}} = \frac{98.0 \text{ V}}{0.500 \text{ A}} = 196\ \Omega$$

and since

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

solving for the capacitance gives

$$C = \frac{1}{\omega X_C} = \frac{1}{(80\pi \text{ rad/s})(196\ \Omega)} = 2.03 \times 10^{-5} \text{ F} = \boxed{20.3\ \mu\text{F}}$$

Question 3

An AC voltage of the form Δv = 90.0 sin 350t, where Δv is in volts and t is in seconds, is applied to a series RLC circuit. If R = 50.0 Ω, C = 25.0 µF, and L = 0.200 H, find (a) the impedance of the circuit, (b) the rms current in the circuit, and (c) the average power delivered to the circuit.

Given $v = \Delta V_{\max} \sin(\omega t) = (90.0 \text{ V})\sin(350t)$, observe that $\Delta V_{\max} = 90.0$ V and $\omega = 350$ rad/s. Also, the net reactance is

$$X_L - X_C = 2\pi f L - \frac{1}{2\pi f C} = \omega L - \frac{1}{\omega C}$$

(a) To find the impedance, we first compute

$$\begin{aligned} X_L - X_C &= \omega L - \frac{1}{\omega C} \\ &= (350 \text{ rad/s})(0.200 \text{ H}) - \frac{1}{(350 \text{ rad/s})(25.0 \times 10^{-6} \text{ F})} \\ &= -44.3\ \Omega \end{aligned}$$

so the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (-44.3\ \Omega)^2} = \boxed{66.8\ \Omega}$$

(b) The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\max}/\sqrt{2}}{Z} = \frac{90.0 \text{ V}}{\sqrt{2}(66.8\ \Omega)} = \boxed{0.953 \text{ A}}$$

(c) The phase difference between the applied voltage and the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{-44.3\ \Omega}{50.0\ \Omega}\right) = -41.5^\circ$$

so the average power delivered to the circuit is

$$\begin{aligned} P_{\text{avg}} &= I_{\text{rms}} \Delta V_{\text{rms}} \cos\phi = I_{\text{rms}} \left(\frac{\Delta V_{\max}}{\sqrt{2}}\right) \cos\phi \\ &= (0.953 \text{ A})\left(\frac{90.0 \text{ V}}{\sqrt{2}}\right)\cos(-41.5^\circ) = \boxed{45.4 \text{ W}} \end{aligned}$$

Question 4

A series RLC circuit has components with the following values: L = 20.0 mH, C = 100 nF, R = 20.0 Ω, and ΔV_{max} = 100 V, with $\Delta v = \Delta V_{max} \sin \omega t$. Find (a) the resonant frequency of the circuit, (b) the amplitude of the current at the resonant frequency, (c) the Q of the circuit, and (d) the amplitude of the voltage across the inductor at resonance.

We are given $L = 0.020\,0$ H, $C = 100 \times 10^{-9}$ F, $R = 20.0\ \Omega$, and $\Delta V_{max} = 100$ V.

(a) The resonant frequency for a series *RLC* circuit is

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \boxed{3.56\text{ kHz}}$$

(b) At resonance,

$$I_{max} = \frac{\Delta V_{max}}{R} = \boxed{5.00\text{ A}}$$

(c) From,

$$Q = \frac{\omega_0 L}{R} = \boxed{22.4}$$

(d) At resonance, the amplitude of the voltage across the inductor is

$$\Delta V_{L,\,max} = X_L I_{max} = \omega_0 L I_{max} = \boxed{2.24\text{ kV}}$$

Question 5

A step-down transformer is used for recharging the batteries of portable electronic devices. The turns ratio N_2/N_1 for a particular transformer used in a DVD player is 1:13. When used with 120-V (rms) household service, the transformer draws an rms current of 20.0 mA from the house outlet. Find (a) the rms output voltage of the transformer and (b) the power delivered to the DVD player.

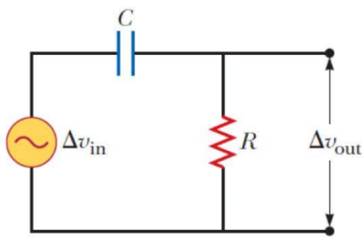
(a) The output voltage is found from $\Delta v_2 = \frac{N_2}{N_1}\Delta v_1$. Therefore,

$$\Delta v_2 = \frac{1}{13}(120\text{ V}) = \boxed{9.23\text{ V}}$$

(b) Assuming an ideal transformer, $P_2 = P_1$. Therefore,

$$\Delta V_2 I_2 = \Delta V_1 I_1 = (120\text{ V})(0.0200\text{ A}) = \boxed{2.40\text{ W}}$$

Question 6
The RC high-pass filter shown in Figure has a resistance R = 0.500 Ω and a capacitance C = 613 μF. What is the ratio of the amplitude of the output voltage to that of the input voltage for this filter for a source frequency of 600 Hz?



For this *RC* high-pass filter, the voltage gain ratio is

$$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{I_{max}R}{I_{max}Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$$

With a capacitance of 613 μF and a frequency of 600 Hz, the capacitive reactance is

$$X_C = \frac{1}{2\pi(600\text{ Hz})(6.13\times 10^{-4}\text{ F})} = 0.433\ \Omega$$

and

$$\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}$$