PHYS143
Physics for Engineers
Tutorial - Chapter 44 – Solutions

Question 1

Calculate the difference in binding energy per nucleon for the nuclei ${}^{23}_{11}\text{Na}$ and ${}^{23}_{12}\text{Mg}$ (Mass of Neutral atom: Na: 22.989769, Mg: 22.994124, H: 1.007 825, and 1.008 665 for neutron).

We use

$$E_b(\text{MeV}) = \left[ZM(\text{H}) + Nm_n - M\left({}^{A}_{Z}\text{X}\right)\right](931.494\ \text{MeV/u})$$

Then, for ${}^{23}_{11}\text{Na}$,

$$E_b\left({}^{23}_{11}\text{Na}\right) = \left[11M(\text{H}) + 12m_n - M\left({}^{23}_{11}\text{Na}\right)\right](931.494\ \text{MeV/u})$$

$$= \left[11(1.007\,825\ \text{u}) + 12(1.008\,665\ \text{u}) - 22.989\,769\ \text{u}\right] \times (931.494\ \text{MeV/u})$$

$$= 186.565\ \text{MeV}$$

and $$\frac{E_b}{A} = \frac{186.565\ \text{MeV}}{23} = 8.11\ \text{MeV}$$

For ${}^{23}_{12}\text{Mg}$,

$$E_b = E_b\left({}^{23}_{12}\text{Mg}\right)$$

$$= \left[12M(\text{H}) + 11m_n - M\left({}^{23}_{12}\text{Mg}\right)\right](931.494\ \text{MeV/u})$$

$$= \left[12(1.007\,825\ \text{u}) + 11(1.008\,665\ \text{u}) - 22.994\,124\ \text{u}\right] \times (931.494\ \text{MeV/u})$$

$$= 181.726\ \text{MeV}$$

and $$\frac{E_b}{A} = \frac{181.726\ \text{MeV}}{23} = 7.90\ \text{MeV}$$

The difference is

$$\frac{\Delta E_b}{A} = \frac{E_b\left({}^{23}_{11}\text{Na}\right) - E_b\left({}^{23}_{12}\text{Mg}\right)}{A}$$

$$= 8.11\ \text{MeV} - 7.90\ \text{MeV} = 0.210\ \text{MeV}$$

Question 2

Calculate the minimum energy required to remove a neutron from the ${}^{43}_{20}\text{Ca}$ nucleus. (Mass of Neutral atom: ${}^{43}_{20}\text{Ca}: 42.958767$, ${}^{42}_{20}\text{Ca}: 41.958618$, and 1.008 665 for neutron)

Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is ΔE_n, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + \Delta E_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass}(\text{n})$$

$$\Delta E_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass}(\text{n}) - \text{mass}\left({}^{43}_{20}\text{Ca}\right)$$

$$\Delta E_n = \left(41.958\,618\text{ u} + 1.008\,665\text{ u} - 42.958\,767\text{ u}\right) \times \left(931.5\text{ MeV/u}\right)$$

$$= \boxed{7.93\text{ MeV}}$$

Question 3

A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. Find (a) the decay constant and (b) the half-life. (c) How many atoms of the isotope were contained in the freshly prepared sample? (d) What is the sample's activity 30.0 h after it is prepared?

(a) From $R = R_0 e^{-\lambda t}$, the decay constant is

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0\text{ mCi}}{8.00\text{ mCi}}\right) = 5.58\times10^{-2}\text{ h}^{-1}$$

$$= \boxed{1.55\times10^{-5}\text{ s}^{-1}}$$

(b) The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$$

(c) The number of original atoms can be found if we convert the initial activity from curies into becquerels (decays per second):

1 Ci $\equiv 3.70 \times 10^{10}$ Bq.

$$R_0 = 10.0\text{ mCi} = \left(10.0\times10^{-3}\text{ Ci}\right)\left(3.70\times10^{10}\text{ Bq/Ci}\right)$$

$$= 3.70\times10^{8}\text{ Bq}$$

Since $R_0 = \lambda N_0$, the original number of nuclei is

$$N_0 = \frac{R_0}{\lambda} = \frac{3.70\times10^{8}\text{ decays/s}}{1.55\times10^{-5}\text{ s}} = \boxed{2.39\times10^{13}\text{ atoms}}$$

(d) The decay rate after thirty hours is

$$R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp\left[\left(-5.58\times10^{-2}\text{ h}^{-1}\right)(30.0\text{ h})\right]$$
$$= \boxed{1.88\text{ mCi}}$$

Question 4

The radioactive isotope ^{198}Au has a half-life of 64.8 h. A sample containing this isotope has an initial activity (t = 0) of 40.0 μCi. Calculate the number of nuclei that decay in the time interval between t_1 = 10.0 h and t_2 = 12.0 h.

The number of nuclei that decay during the interval will be

$$\Delta N = N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$$

First we find the decay constant λ:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{64.8\text{ h}} = 0.010\,7\text{ h}^{-1} = 2.97\times10^{-6}\text{ s}^{-1}$$

Now we find N_0:

$$N_0 = \frac{R_0}{\lambda} = \frac{(40.0\,\mu\text{Ci})}{2.97\times10^{-6}\text{ s}^{-1}}\left(\frac{3.70\times10^4\text{ s}^{-1}}{\mu\text{Ci}}\right)$$
$$= 4.98\times10^{11}\text{ nuclei}$$

Substituting in these values,

$$N_1 - N_2 = \left(4.98\times10^{11}\right)\left[e^{-(\ln 2/64.8\text{ h})(10.0\text{ h})} - e^{-(\ln 2/64.8\text{ h})(12.0\text{ h})}\right]$$

$$N_1 - N_2 = \boxed{9.47\times10^9\text{ nuclei}}$$

Question 5

Determine which decays can occur spontaneously.

(a) $^{40}_{20}\text{Ca} \rightarrow \text{e}^+ + {}^{40}_{19}\text{K}$ (b) $^{98}_{44}\text{Ru} \rightarrow {}^4_2\text{He} + {}^{94}_{42}\text{Mo}$

(c) $^{144}_{60}\text{Nd} \rightarrow {}^4_2\text{He} + {}^{140}_{58}\text{Ce}$

Atomic Masses: ^{40}Ca (39.962 591), ^{40}K (39.963 998), ^{98}Ru (97.905 287), ^{94}Mo (93.905 088), ^{144}Nd (143.910 087), ^{140}Ce (139.905 439), m_e (0.000549), M_α (4.002603).

(a) For this e$^+$ decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2$$
$$= \left[39.962\,591\text{ u} - 39.963\,999\text{ u} - 2\left(0.000\,549\text{ u}\right)\right]$$
$$\times\left(931.5\text{ MeV/u}\right)$$

$$Q = -2.33\text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For this alpha decay,

$$Q = (M_X - M_Y - M_\alpha)c^2$$
$$= [97.905\,287 \text{ u} - 4.002\,603 \text{ u} - 93.905\,088 \text{ u}] \times (931.5 \text{ MeV/u})$$
$$Q = -2.24 \text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For this alpha decay,

$$Q = (M_X - M_Y - M_\alpha)c^2$$
$$= [143.910\,083 \text{ u} - 4.002\,603 \text{ u} - 139.905\,434 \text{ u}] \times (931.5 \text{ MeV/u})$$
$$Q = 1.91 \text{ MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

Question 6

A ${}^3_1$H nucleus beta decays into ${}^3_2$He by creating an electron and an antineutrino according to the reaction

$$ {}^3_1\text{H} \rightarrow {}^3_2\text{He} + e^- + \bar{\nu}$$

Determine the total energy released in this decay. Atomic Masses: ^{3}H (3.016 049), ^{3}He (3.016 029).

(a) The reaction is ${}^3_1\text{H} \rightarrow {}^3_2\text{He} + e^- + \bar{\nu}$.

Adding one electron, the reaction becomes

$${}^3_1\text{H nucleus} + e^- \rightarrow {}^3_2\text{He nucleus} + 2e^- + \bar{\nu}$$

Ignoring the slight difference in ionization energies, we have

$${}^3_1\text{H atom} \rightarrow {}^3_2\text{He atom} + \bar{\nu}$$

(b) The total energy released is the Q value:

$$Q = (M_{\text{H-3}} - M_{\text{He-3}})c^2$$
$$Q = (3.016\,049 \text{ u} - 3.016\,029 \text{ u})(931.5 \text{ MeV/u})$$
$$= 0.018\,6 \text{ MeV} = \boxed{18.6 \text{ keV}}$$

Question 7

Identify the unknown nuclide or particle (X).

(a) $X \rightarrow {}^{65}_{28}\text{Ni} + \gamma$ (b) ${}^{215}_{84}\text{Po} \rightarrow X + \alpha$

(c) $X \rightarrow {}^{55}_{26}\text{Fe} + e^+ + \nu$

Total Z and A are conserved.

(a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z = 0$ and $A = 0$. Keeping the total values of Z and A for the system conserved requires $Z = 28$ and $A = 65$ for X. With this atomic number it must be nickel, and the nucleus must be in an excited state, so X is $\boxed{{}^{65}_{28}\mathrm{Ni}^*}$.

(b) An alpha particle, $\alpha = {}^{4}_{2}\mathrm{He}$, has $Z = 2$ and $A = 4$. Total initial Z is 84, and total initial A is 215, so for X we require

$$Z = 84 = Z_X + 2 \quad \rightarrow \quad Z_X = 82 \quad \rightarrow \quad \text{Pb, and}$$

$$A = 215 = A_X + 4 \quad \rightarrow \quad A_X = 211, \quad \rightarrow \quad \text{X is } \boxed{{}^{211}_{82}\mathrm{Pb}}.$$

(c) A positron, $e^+ = {}^{0}_{1}e$, has charge the same as a nucleus with $Z = 1$. A neutrino, ${}^{0}_{0}\nu$, has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z = 26 + 1 + 0 = 27$; so, X is Co. And $A = 55 + 0 + 0 = 55$: X is $\boxed{{}^{55}_{27}\mathrm{Co}}$.

Question 8

The nucleus ${}^{15}_{8}\mathrm{O}$ decays by electron capture. The nuclear reaction is written

$${}^{15}_{8}\mathrm{O} + e^- \rightarrow {}^{15}_{7}\mathrm{N} + \nu$$

(a) Write the process going on for a single particle within the nucleus. (b) Disregarding the daughter's recoil, determine the energy of the neutrino. Atomic masses: ${}^{15}\mathrm{O}$ (15.003 066), ${}^{15}\mathrm{N}$ (15.000 109)

(a) The reaction for one particle is $\boxed{e^- + p \rightarrow n + \nu}$.

(b) For nuclei, ${}^{15}_{8}\mathrm{O} + e^- \rightarrow {}^{15}_{7}\mathrm{N} + \nu$.

Add seven electrons to both sides to obtain

$${}^{15}_{8}\mathrm{O}\text{ atom} \rightarrow {}^{15}_{7}\mathrm{N}\text{ atom} + \nu$$

$$Q = (15.003\,065\text{ u} - 15.000\,109\text{ u})(931.5\text{ MeV/u})$$
$$= \boxed{2.75\text{ MeV}}$$