Chapter 34

Electromagnetic Waves

Electromagnetic Waves

Mechanical waves require the presence of a medium.

Electromagnetic waves can propagate through empty space.

Maxwell's equations form the theoretical basis of all electromagnetic waves that propagate through space at the speed of light.

Hertz confirmed Maxwell's prediction when he generated and detected electromagnetic waves in 1887.

Electromagnetic waves are generated by oscillating electric charges.

- The waves radiated from the oscillating charges can be detected at great distances.

Electromagnetic waves carry energy and momentum.

Electromagnetic waves cover many frequencies.

James Clerk Maxwell

1831 – 1879

Scottish theoretical physicist

Developed the electromagnetic theory of light

His successful interpretation of the electromagnetic field resulted in the field equations that bear his name.

Also developed and explained

- Kinetic theory of gases
- Nature of Saturn’s rings
- Color vision

Modifications to Ampère's Law

Ampère's Law is used to analyze magnetic fields created by currents:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

In this equation, the line integral is over any closed path through which conduction current passes.

But, this form is valid only if any electric fields present are constant in time.

Maxwell modified the equation to include time-varying electric fields.

Maxwell's modification was to add a term.

Modifications to Ampère's Law, cont

The additional term included a factor called the **displacement current**, I_d.

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \qquad \Phi_E \equiv \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

This term was then added to Ampère's Law.

This showed that magnetic fields are produced both by conduction currents and by time-varying electric fields.

The general form of Ampère's Law is

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(I + I_d) = \mu_0 I + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$$

- Sometimes called Ampère-Maxwell Law

Maxwell's Equations

In his unified theory of electromagnetism, Maxwell showed that electromagnetic waves are a natural consequence of the fundamental laws expressed in these four equations:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

◀ Gauss's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

◀ Gauss's law in magnetism

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

◀ Faraday's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

◀ Ampère–Maxwell law

Maxwell's Equation 1 – Gauss' Law

The total electric flux through any closed surface equals the net charge inside that surface divided by ε_o

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

This relates an electric field to the charge distribution that creates it.

Maxwell's Equation 2 – Gauss' Law in Magnetism

The net magnetic flux through a closed surface is zero.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

The number of magnetic field lines that enter a closed volume must equal the number that leave that volume.

If this weren't true, there would be magnetic monopoles found in nature.

- There haven't been any found

Maxwell's Equation 3 – Faraday's Law of Induction

Describes the creation of an electric field by a time-varying magnetic field.

The emf, which is the line integral of the electric field around any closed path, equals the rate of change of the magnetic flux through any surface bounded by that path.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

One consequence is the current induced in a conducting loop placed in a time-varying magnetic field.

Maxwell's Equation 4 – Ampère-Maxwell Law

Describes the creation of a magnetic field by a changing electric field and by electric current.

The line integral of the magnetic field around any closed path is the sum of μ_o times the net current through that path and $\varepsilon_o\mu_o$ times the rate of change of electric flux through any surface bounded by that path.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

Lorentz Force Law

Once the electric and magnetic fields are known at some point in space, the force acting on a particle of charge *q* can be found.

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Maxwell’s equations with the Lorentz Force Law completely describe all classical electromagnetic interactions.

Properties of em Waves

The solutions of Maxwell's third and fourth equations are wave-like, with both *E* and *B* satisfying a wave equation.

Electromagnetic waves travel at the speed of light:

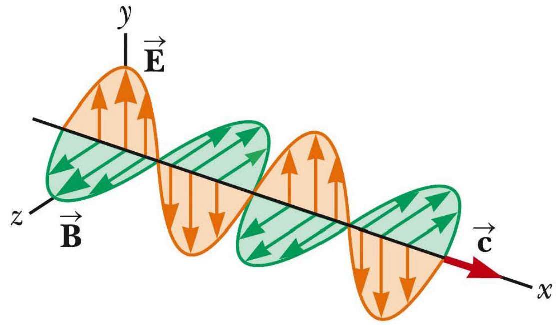
$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

- This comes from the solution of Maxwell's equations.

Properties of em Waves, 2

The components of the electric and magnetic fields of plane electromagnetic waves are perpendicular to each other and perpendicular to the direction of propagation.

- This can be summarized by saying that electromagnetic waves are transverse waves.



The figure represents a sinusoidal em wave moving in the *x* direction with a speed *c*.

Properties of em Waves, 3

The magnitudes of the electric and magnetic fields in empty space are related by the expression:

$$c = E/B$$

- This comes from the solution of the partial differentials obtained from Maxwell's equations.

Electromagnetic waves obey the superposition principle.

The wavelength and frequency of an electromagnetic wave are related as

$$\lambda = \frac{c}{f}$$

Example

Suppose you are located 180 m from a radio transmitter. (a) How many wavelengths are you from the transmitter if the station calls itself 1150 AM? (The AM band frequencies are in kilohertz.) (b) What if this station is 98.1 FM? (The FM band frequencies are in megahertz.)

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{1\ 150\times 10^{3}\ \text{s}^{-1}} = 261\ \text{m} \quad \Rightarrow \quad \frac{180\ \text{m}}{261\ \text{m}} = \boxed{0.690\ \text{wavelengths}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{98.1\times 10^{6}\ \text{s}^{-1}} = 3.06\ \text{m} \quad \Rightarrow \quad \frac{180\ \text{m}}{3.06\ \text{m}} = \boxed{58.9\ \text{wavelengths}}$$

Example

An electromagnetic wave in vacuum has an electric field amplitude of 220 V/m. Calculate the amplitude of the corresponding magnetic field.

$$\frac{E}{B} = c \quad \text{or} \quad \frac{220 \text{ V/m}}{B} = 3.00 \times 10^8 \text{ m/s},$$

$$B = 7.33 \times 10^{-7} \text{ T} = \boxed{733 \text{ nT}}$$

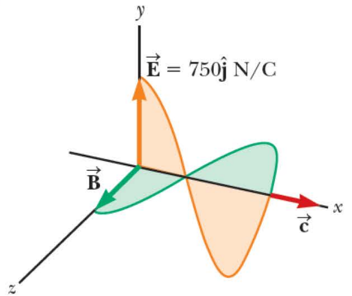
An Electromagnetic Wave, Example

A sinusoidal electromagnetic wave of frequency 40.0 MHz travels in free space in the *x* direction as in Figure 34.9.

(A) Determine the wavelength and period of the wave.

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{40.0 \times 10^6 \text{ Hz}} = \boxed{7.50 \text{ m}}$$

$$T = \frac{1}{f} = \frac{1}{40.0 \times 10^6 \text{ Hz}} = \boxed{2.50 \times 10^{-8} \text{ s}}$$



(B) At some point and at some instant, the electric field has its maximum value of 750 N/C and is directed along the *y* axis. Calculate the magnitude and direction of the magnetic field at this position and time.

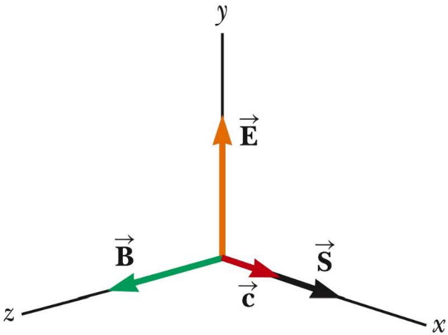
$$B_{\max} = \frac{E_{\max}}{c} = \frac{750 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.50 \times 10^{-6} \text{ T}}$$

Poynting Vector, 1

Electromagnetic waves carry energy.

As they propagate through space, they can transfer that energy to objects in their path.

The rate of transfer of energy by an em wave is described by a vector, $\vec{\mathbf{S}}$, called the **Poynting vector.**



The Poynting vector is defined as

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_o} \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

Its direction is the direction of propagation.

This is time dependent.

- Its magnitude varies in time.
- Its magnitude reaches a maximum at the same instant as $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$.

.

Poynting Vector, 2

The magnitude of the vector represents the rate at which energy passes through a unit surface area perpendicular to the direction of the wave propagation.

- Therefore, the magnitude represents the *power per unit area.*

The SI units of the Poynting vector are $J/(s \cdot m^2) = W/m^2$.

Intensity

The wave *intensity, I*, is the time average of S (the Poynting vector) over one or more cycles.

$$I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

Energy Density

The energy density, *u*, is the energy per unit volume.

For the electric field, $u_E = \frac{1}{2} \varepsilon_o E^2$

For the magnetic field, $u_B = B^2 / 2\mu_o$

Since $B = E/c$ and $c = 1/\sqrt{\mu_o \varepsilon_o}$

$$u_B = u_E = \tfrac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

The instantaneous energy density associated with the magnetic field of an em wave equals the instantaneous energy density associated with the electric field.

- In a given volume, the energy is shared equally by the two fields.

Energy Density, cont.

The **total instantaneous energy density** is the sum of the energy densities associated with each field.

- $u = u_E + u_B = \varepsilon_o E^2 = B^2 / \mu_o$

When this is averaged over one or more cycles, the total average per unit volume becomes

$$u_{\text{avg}} = \epsilon_0 (E^2)_{\text{avg}} = \frac{1}{2}\epsilon_0 E_{\text{max}}^2 = \frac{B_{\text{max}}^2}{2\mu_0}$$

In terms of I, $I = S_{\text{avg}} = c u_{\text{avg}}$

The intensity of an em wave equals the average energy density multiplied by the speed of light.

Momentum

Electromagnetic waves transport momentum as well as energy.

As this momentum is absorbed by some surface, pressure is exerted on the surface.

Assuming the wave transports a total energy T_{ER} to the surface in a time interval Δt, the total momentum is $p = T_{ER} / c$ for complete absorption.

Pressure and Momentum

Pressure, P, is defined as the force per unit area

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{c}\frac{\left(dT_{ER}/dt\right)}{A}$$

But the magnitude of the Poynting vector is $(dT_{ER}/dt)/A$ and so $P = S / c$.

- For a perfectly absorbing surface

For a perfectly reflecting surface, $p = 2T_{ER} /c$ and $P = 2S/c$

For a surface with a reflectivity somewhere between a perfect reflector and a perfect absorber, the pressure delivered to the surface will be somewhere in between S/c and 2S/c.

For direct sunlight, the radiation pressure is about 5×10^{-6} N/m^2 .

Example

When a high-power laser is used in the Earth's atmo-sphere, the electric field associated with the laser beam can ionize the air, turning it into a conducting plasma that reflects the laser light. In dry air at 0° C and 1 atm, electric breakdown occurs for fields with amplitudes above about 3.00 MV/m. (a) What laser beam intensity will produce such a field? (b) At this maximum intensity, what power can be delivered in a cylindrical beam of diameter 5.00 mm? $\mu_0 = 4\pi \times 10^{-7}$ T.m/A.

$$I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{\left(3.00 \times 10^6 \text{ V/m}\right)^2}{2\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)\left(3.00 \times 10^8 \text{ m/s}\right)} \Rightarrow I = \boxed{1.19 \times 10^{10} \text{ W/m}^2}$$

$$P = IA = I\pi r^2 = \left(1.19 \times 10^{10} \text{ W/m}^2\right)\pi\left(\frac{5.00 \times 10^{-3} \text{ m}}{2}\right)^2$$

$$= \boxed{2.34 \times 10^5 \text{ W}}$$

Example

At what distance from a 100-W spherical electromagnetic wave point source does E_{max} = 15.0 V/m?

$$\text{Power} = IA = \frac{E_{\max}^2}{2\mu_0 c}(4\pi r^2)$$

$$r = \sqrt{\frac{P\mu_0 c}{2\pi E_{\max}^2}} = \sqrt{\frac{(100\text{ W})(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(3.00\times 10^{8}\text{ m/s})}{2\pi(15.0\text{ V/m})^2}}$$

$$= \boxed{5.16\text{ m}}$$

Example

Assuming the antenna of a 10.0-kW radio station radiates spherical electromagnetic waves, (a) compute the maximum value of the magnetic field 5.00 km from the antenna.

(a) The intensity of the broadcast waves is $I = \dfrac{B_{\max}^2 c}{2\mu_0} = \dfrac{P}{4\pi r^2}$

$$B_{\max} = \sqrt{\left(\frac{P}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\left(\frac{P}{2\pi r^2}\right)\left(\frac{\mu_0}{c}\right)}$$

$$= \sqrt{\frac{(10.0\times 10^3\ \text{W})(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(5.00\times 10^3\ \text{m})^2(3.00\times 10^8\ \text{m/s})}} = \boxed{5.16\times 10^{-10}\ \text{T}}$$