Chapter 40

Introduction to Quantum Physics

Need for Quantum Physics

Problems remained from classical mechanics that the special theory of relativity didn't explain.

Attempts to apply the laws of classical physics to explain the behavior of matter on the atomic scale were consistently unsuccessful.

Problems included:

- Blackbody radiation
  - The electromagnetic radiation emitted by a heated object
- Photoelectric effect
  - Emission of electrons by an illuminated metal

Quantum Mechanics Revolution

Between 1900 and 1930, another revolution took place in physics.

A new theory called *quantum mechanics* was successful in explaining the behavior of particles of microscopic size.

The first explanation using quantum theory was introduced by Max Planck.

- Many other physicists were involved in other subsequent developments

Blackbody Radiation

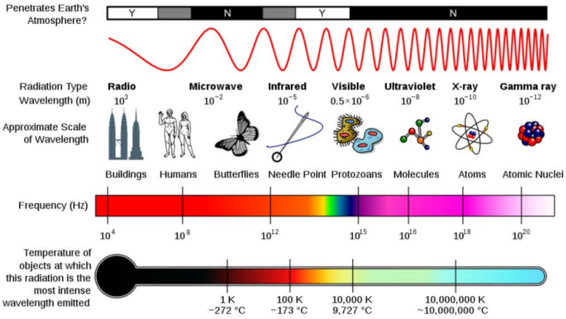
An object at any temperature is known to emit thermal radiation.

- Characteristics depend on the temperature and surface properties.
- The thermal radiation consists of a continuous distribution of wavelengths from all portions of the em spectrum.

At room temperature, the wavelengths of the thermal radiation are mainly in the infrared region.

As the surface temperature increases, the wavelength changes.

- It will glow red and eventually white.



Blackbody Radiation

The basic problem was in understanding the observed distribution in the radiation emitted by a black body.

- Classical physics didn't adequately describe the observed distribution.

A **black body** is an ideal system that absorbs all radiation incident on it.

The electromagnetic radiation emitted by a black body is called **blackbody radiation.**

Blackbody Experiment Results

The following two consistent experimental findings were seen as especially significant:

The total power of the emitted radiation increases with temperature.

- Stefan's law (from Chapter 20):

$$P = \sigma A e T^4$$

- P is the power in Watts radiated at all wavelengths from the surface of an object, $\sigma = 5.670 \times 10^{-8}$ W/m^2.K^4 is the Stefan-Boltzmann constant, A is the surface area of the object in square meters, e is the emissivity of the surface, and T is the surface temperature in kelvins. For blackbody, e =1 exactly.

The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases.

- Wien's displacement law: $\lambda_{max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$

where λ_{max} is the wavelength at which the curve peaks and *T* is the absolute temperature of the surface of the object emitting the radiation.

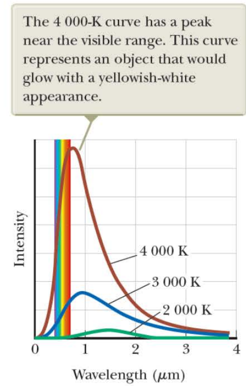
Intensity of Blackbody Radiation, Summary

The intensity increases with increasing temperature.

The amount of radiation emitted increases with increasing temperature.

- The area under the curve

The peak wavelength decreases with increasing temperature.

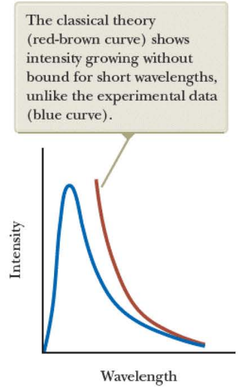


Rayleigh-Jeans Law

An early classical attempt to explain blackbody radiation was the **Rayleigh-Jeans law.**

$$I(\lambda,T) = \frac{2\pi c k_B T}{\lambda^4}$$

where $I(\lambda,T)$ to be the intensity, or power per unit area, k_B is Boltzmann's constant (1.380649×10^{-23} J.K^{-1}).



Example

Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35° C.

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{T}$$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{308\ \mathrm{K}} = 9.41\ \mu\mathrm{m}$$

This radiation is in the infrared region of the spectrum and is invisible to the human eye. Some animals (pit vipers, for instance) are able to detect radiation of this wavelength and therefore can locate warm-blooded prey even in the dark.

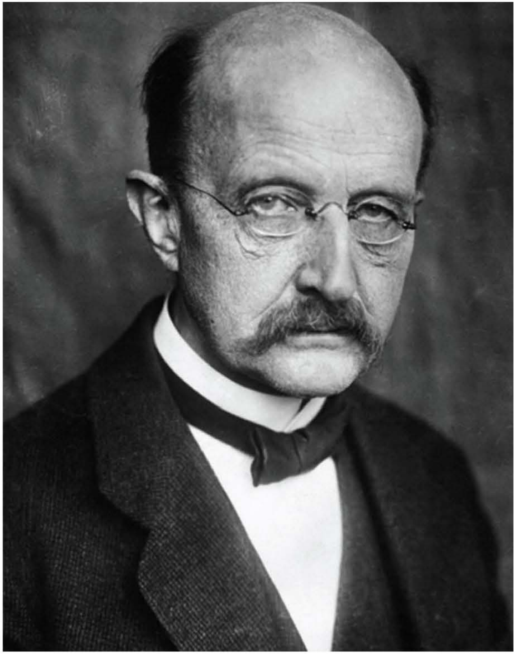
Max Planck

1858 – 1847

German physicist

Introduced the concept of “quantum of action”

In 1918 he was awarded the Nobel Prize for the discovery of the quantized nature of energy.



Planck's Theory of Blackbody Radiation

In 1900 Planck developed a theory of blackbody radiation that leads to an equation for the intensity of the radiation, $I(\lambda, T)$.

Planck's Assumption, 1

The energy of an oscillator can have only certain discrete values E_n.

- $E_n = n\ h\ f$
  - n is a positive integer called the quantum number
  - f is the frequency of oscillation
  - h is Planck's constant
- This says the energy is quantized.
- Each discrete energy value corresponds to a different quantum state.
  - Each quantum state is represented by the quantum number, *n*.

Planck's Assumption, 2

The oscillators emit or absorb energy when making a transition from one quantum state to another.

- The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation.
- An oscillator emits or absorbs energy only when it changes quantum states.
- The energy carried by the quantum of radiation is $E = h f$.

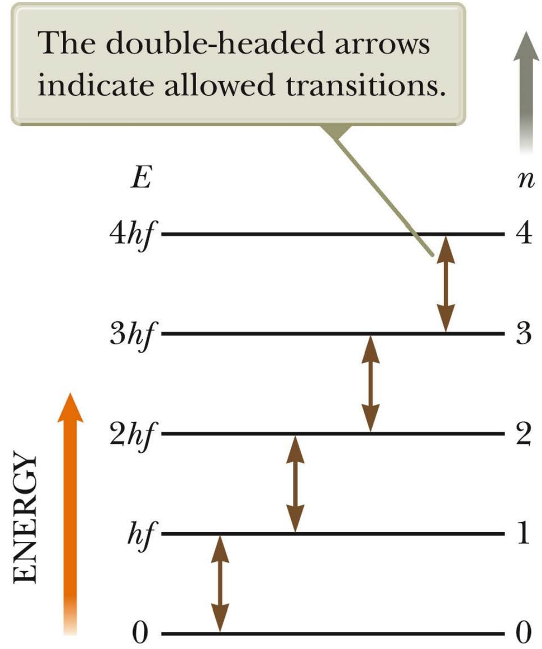
Energy-Level Diagram

An **energy-level diagram** shows the quantized energy levels and allowed transitions.

Energy is on the vertical axis.

Horizontal lines represent the allowed energy levels.

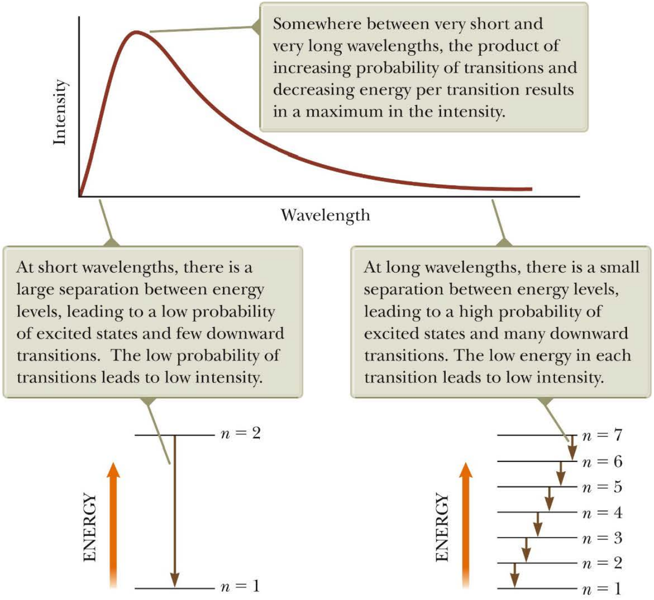
The double-headed arrows indicate allowed transitions.



Planck's Model, Graph

At low frequencies (long wavelengths), according to assumption 2, the energy levels are close together as on the right in Figure.

Now, consider high-frequency radiation, that is, radiation with short wavelength. To obtain this radiation, the allowed energies are very far apart as on the left in Figure



Planck's Wavelength Distribution Function

Planck generated a theoretical expression for the wavelength distribution.

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 \left(e^{hc/\lambda k_B T} - 1\right)}$$

- h = 6.626 x 10^{-34} J·s
- h is a fundamental constant of nature.

At long wavelengths, Planck's equation reduces to the Rayleigh-Jeans expression.

At short wavelengths, it predicts an exponential decrease in intensity with decreasing wavelength.

- This is in agreement with experimental results.

Photoelectric Effect

The **photoelectric effect** occurs when light incident on certain metallic surfaces causes electrons to be emitted from those surfaces.

- The emitted electrons are called **photoelectrons.**
  - They are no different than other electrons.
  - The name is given because of their ejection from a metal by light in the photoelectric effect.

Photoelectric Effect Feature 1

Dependence of photoelectron kinetic energy on light intensity

- *Classical Prediction*
  - Electrons should absorb energy continually from the electromagnetic waves.
  - As the light intensity incident on the metal is increased, the electrons should be ejected with more kinetic energy.
- *Experimental Result*
  - The maximum kinetic energy is independent of light intensity.
  - The maximum kinetic energy is proportional to the stopping potential (ΔV_s).

$$K_{\max} = e\,\Delta V_s$$

  - The **stopping potential** is defined as the **potential** necessary to **stop** any electron (or, in other words, to **stop** even the electron with the most kinetic energy) from 'reaching the other side'.

Photoelectric Effect Feature 2

Time interval between incidence of light and ejection of photoelectrons

- *Classical Prediction*
  - At low light intensities, a measurable time interval should pass between the instant the light is turned on and the time an electron is ejected from the metal.
  - This time interval is required for the electron to absorb the incident radiation before it acquires enough energy to escape from the metal.
- *Experimental Result*
  - Electrons are emitted almost instantaneously, even at very low light intensities.

Photoelectric Effect Feature 3

Dependence of ejection of electrons on light frequency

- *Classical Prediction*
  - Electrons should be ejected at any frequency as long as the light intensity is high enough because energy is transferred to the metal regardless of the incident light frequency.
- *Experimental Result*
  - No electrons are emitted if the incident light falls below some **cutoff frequency, f_c.**
  - The cutoff frequency is characteristic of the material being illuminated.
  - No electrons are ejected below the cutoff frequency regardless of intensity.

Photoelectric Effect Feature 4

Dependence of photoelectron kinetic energy on light frequency

- *Classical Prediction*
  - There should be no relationship between the frequency of the light and the electric kinetic energy.
  - The kinetic energy should be related to the intensity of the light.
- *Experimental Result*
  - The maximum kinetic energy of the photoelectrons increases with increasing light frequency.

Photoelectric Effect Features, Summary

The experimental results contradict all four classical predictions.

Einstein extended Planck's concept of quantization to electromagnetic waves.

All electromagnetic radiation of frequency f from any source can be considered a stream of quanta, now called *photons.*

Each photon has an energy E and moves at the speed of light in a vacuum.

- $E = hf$

A photon of incident light gives all its energy to a single electron in the metal.

Photoelectric Effect, Work Function

Electrons ejected from the surface of the metal and not making collisions with other metal atoms before escaping possess the maximum kinetic energy K_{max}.

$K_{max} = hf - \varphi$

- φ is called the work function of the metal.
- The work function represents the minimum energy with which an electron is bound in the metal.

TABLE 40.1

Work Functions of Selected Metals

Metal	ϕ (eV)
Na	2.46
Al	4.08
Fe	4.50
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

Note: Values are typical for metals listed. Actual values may vary depending on whether the metal is a single crystal or polycrystalline. Values may also depend on the face from which electrons are ejected from crystalline metals. Furthermore, different experimental procedures may produce differing values.

Photon Model Explanation of the Photoelectric Effect

Dependence of photoelectron kinetic energy on light intensity

- K_{max} is independent of light intensity.
- K depends on the light frequency and the work function.

Time interval between incidence of light and ejection of the photoelectron

- Each photon can have enough energy to eject an electron immediately.

Dependence of ejection of electrons on light frequency

- There is a failure to observe photoelectric effect below a certain cutoff frequency, which indicates the photon must have more energy than the work function in order to eject an electron.
- Without enough energy, an electron cannot be ejected, regardless of the fact that many photons per unit time are incident on the metal in a very intense light beam.

Photon Model Explanation of the Photoelectric Effect, cont.

Dependence of photoelectron kinetic energy on light frequency

- Since $K_{max} = hf - \varphi$
- A photon of higher frequency carries more energy.
  - A photoelectron is ejected with higher kinetic energy.
  - Once the energy of the work function is exceeded.
- There is a linear relationship between the maximum electron kinetic energy and the frequency.

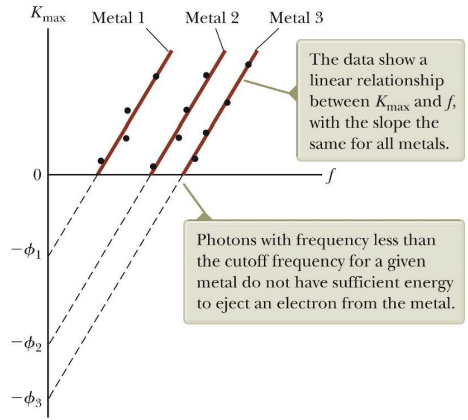
Cutoff Frequency

The lines show the linear relationship between K and f.

The slope of each line is h.

The x-intercept is the **cutoff frequency.**

- This is the frequency below which no photoelectrons are emitted.



Cutoff Frequency and Wavelength

The cutoff frequency is related to the work function through $f_c = \varphi / h$.

The cutoff frequency corresponds to a **cutoff wavelength.**

$$\lambda_c = \frac{c}{f_c} = \frac{hc}{\varphi}$$

Wavelengths greater than λ_c incident on a material having a work function φ do not result in the emission of photoelectrons.

Example

A sodium surface is illuminated with light having a wavelength of 300 nm. As indicated in Table 40.1, the work function for sodium metal is 2.46 eV. ($h = 6.62607004 \times 10^{-34}$ m^2 kg / s)

(A) Find the maximum kinetic energy of the ejected photoelectrons.

$$E = hf = \frac{hc}{\lambda}$$

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi = \frac{1\,240 \text{ eV} \cdot \text{nm}}{300 \text{ nm}} - 2.46 \text{ eV} = \boxed{1.67 \text{ eV}}$$

(B) Find the cutoff wavelength λ_c for sodium.

$$\lambda_c = \frac{hc}{\phi} = \frac{1\,240 \text{ eV} \cdot \text{nm}}{2.46 \text{ eV}} = \boxed{504 \text{ nm}}$$

Arthur Holly Compton

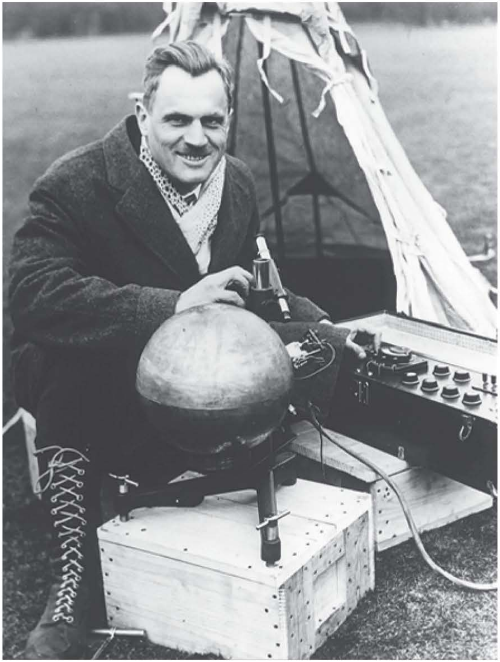
1892 – 1962

American physicist

Director of the lab at the University of Chicago

Discovered the Compton Effect

Shared the Nobel Prize in 1927



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The Compton Effect, Introduction

Compton and Debye extended Einstein's idea of photon momentum, equal to $E/c = hf / c$.

The two groups of experimenters accumulated evidence of the inadequacy of the classical wave theory.

The classical wave theory of light failed to explain the scattering of x-rays from electrons.

Compton Effect, Classical Predictions

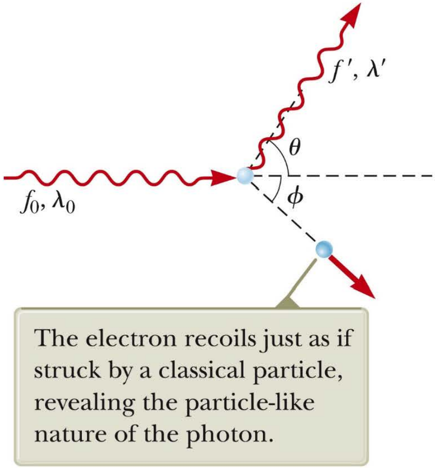
According to the classical theory, em waves of frequency f incident on electrons should have two effects:

- Have radiation pressure that should cause the electrons to accelerate in the direction of propagation of the waves.
- The oscillating electric field of the incident radiation should set the electrons into oscillation at the apparent frequency f', where f' is the frequency in the frame of the moving electrons.

Compton Effect, Observations

Because different electrons move at different speeds after the interaction, depending on the amount of energy absorbed from the electromagnetic waves, the scattered wave frequency at a given angle to the incoming radiation should show a distribution of Doppler-shifted values.

Contrary to this prediction, Compton's experiments showed that, at any given angle, only *one* frequency of radiation is observed.



Compton Effect, Explanation

The results could be explained by treating the photons as point-like particles having energy hf and momentum hf/c.

Assume the energy and momentum of the isolated system of the colliding photon-electron are conserved.

This scattering phenomena is known as the **Compton effect.**

Compton Shift Equation

The graphs show the scattered x-rays for various angles.

The shifted peak, λ', is caused by the scattering of free electrons.

$$\lambda' - \lambda_o = \frac{h}{m_e c}(1 - \cos\theta)$$

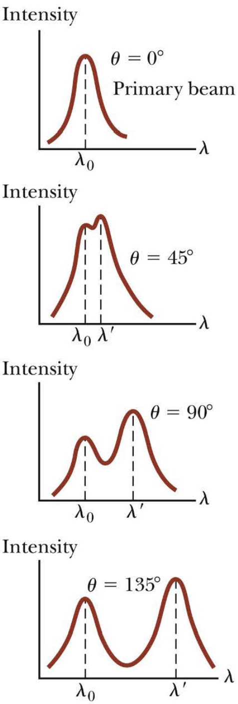
- This is called the **Compton shift equation.**

Compton Wavelength

The factor $h/m_e c$ in the equation is called the **Compton wavelength** of the electron and is

$$\lambda_C = \frac{h}{m_e c} = 0.002\ 43\,\text{nm}$$

The unshifted wavelength, λ_o, is caused by x-rays scattered from the electrons that are tightly bound to the target atoms.



Example

X-rays of wavelength λ_0 = 0.200 000 nm are scattered from a block of material. The scattered x-rays are observed at an angle of 45.0 to the incident beam. Calculate their wavelength.

$$(1) \quad \lambda' = \lambda_0 + \frac{h(1 - \cos\theta)}{m_e c}$$

$$\lambda' = 0.200\,000 \times 10^{-9}\text{ m} + \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(1 - \cos 45.0^\circ)}{(9.11 \times 10^{-31}\text{ kg})(3.00 \times 10^{8}\text{ m/s})}$$

$$= 0.200\,000 \times 10^{-9}\text{ m} + 7.10 \times 10^{-13}\text{ m} = \boxed{0.200\,710\text{ nm}}$$

Louis de Broglie

1892 – 1987

French physicist

Originally studied history

Was awarded the Nobel Prize in 1929 for his prediction of the wave nature of electrons

Wave Properties of Particles

Louis de Broglie postulated that because photons have both wave and particle characteristics, perhaps all forms of matter have both properties.

- According to de Broglie, electrons, just like light, have a dual particle–wave nature.

The **de Broglie wavelength** of a particle is (p: momentum, m: mass of a particle, u: velocity of a particle)

$$\lambda = \frac{h}{p} = \frac{h}{mu}$$

Frequency of a Particle

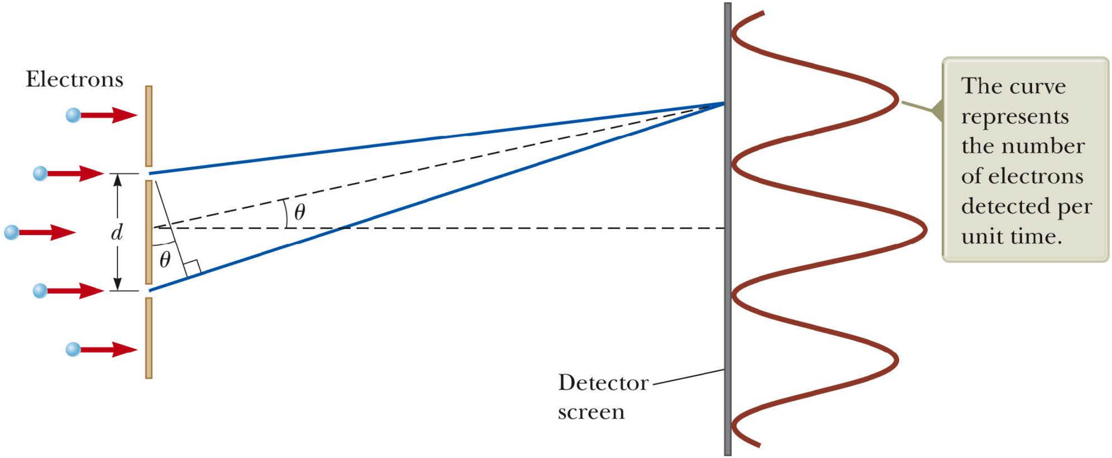
In an analogy with photons, de Broglie postulated that a particle would also have a frequency associated with it

$$f = \frac{E}{h}$$

These equations present the dual nature of matter:

- Particle nature, p and E
- Wave nature, λ and f

Electron Diffraction, Set-Up & Experiment



Parallel beams of mono-energetic electrons that are incident on a double slit.

The slit widths are small compared to the electron wavelength.

An electron detector is positioned far from the slits at a distance much greater than the slit separation.

Electron Diffraction, cont.

If the detector collects electrons for a long enough time, a typical wave interference pattern is produced.

This is distinct evidence that electrons are interfering, a wave-like behavior.

The interference pattern becomes clearer as the number of electrons reaching the screen increases.

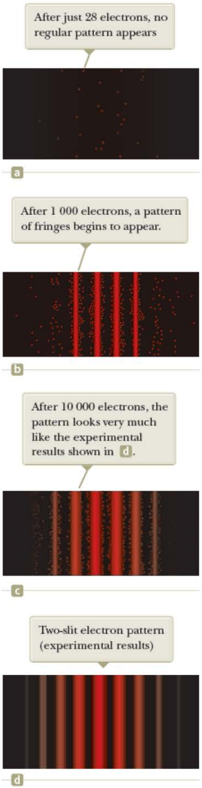
Electron Diffraction, Equations

A maximum occurs when $d \sin \theta = m\lambda$

- This is the same equation that was used for light.

This shows the dual nature of the electron.

- The electrons are detected as particles at a localized spot at some instant of time.
- The probability of arrival at that spot is determined by finding the intensity of two interfering waves.



Example

(A) Calculate the de Broglie wavelength for an electron (m_e = 9.11 × 10^{-31} kg) moving at 1.00 × 10^7 m/s.

$$\lambda = \frac{h}{m_e u} = \frac{6.626 \times 10^{-34}\,\mathrm{J \cdot s}}{(9.11 \times 10^{-31}\,\mathrm{kg})(1.00 \times 10^{7}\,\mathrm{m/s})} = \boxed{7.27 \times 10^{-11}\,\mathrm{m}}$$

(B) A rock of mass 50 g is thrown with a speed of 40 m/s. What is its de Broglie wavelength?

$$\lambda = \frac{h}{mu} = \frac{6.626 \times 10^{-34}\,\mathrm{J \cdot s}}{(50 \times 10^{-3}\,\mathrm{kg})(40\,\mathrm{m/s})} = \boxed{3.3 \times 10^{-34}\,\mathrm{m}}$$