PHYS143
Physics for Engineers
Tutorial - Chapter 37 – Solutions

Question 1

Two slits are separated by 0.320 mm. A beam of 500-nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range -30.0° < θ < 30.0°.

The angular locations of the bright fringes (or maxima) is given by:

$$d\sin\theta = m\lambda$$

Solving for m and substituting 30.0° gives

$$m = \frac{d\sin\theta}{\lambda} = \frac{(3.20\times10^{-4}\ \text{m})\sin 30.0^\circ}{500\times10^{-9}\ \text{m}} = 320$$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead at $\theta = 0$. There are therefore $\boxed{641\ \text{maxima}}$.

Question 2

Light of wavelength 530 nm illuminates a pair of slits separated by 0.300 mm. If a screen is placed 2.00 m from the slits, determine the distance between the first and second dark fringes.

The location of the dark fringe of order m (measured from the position of the central maximum) is given by

$$(y_{\text{dark}})_m = \left(m + \frac{1}{2}\right)\left(\frac{L\lambda}{d}\right)$$

where $m = 0, \pm1, \pm2, \ldots$ Thus, the spacing between the first and second dark fringes will be

$$\Delta y = (y_{\text{dark}})_{m=1} - (y_{\text{dark}})_{m=0}$$
$$= \left(1 + \frac{1}{2}\right)\left(\frac{L\lambda}{d}\right) - \left(0 + \frac{1}{2}\right)\left(\frac{L\lambda}{d}\right) = \frac{L\lambda}{d}$$

or $$\Delta y = \frac{(5.30\times10^{-7}\ \text{m})(2.00\ \text{m})}{0.300\times10^{-3}\ \text{m}} = 3.53\times10^{-3}\ \text{m} = \boxed{3.53\ \text{mm}}$$

Question 3

(a) A Young's interference experiment is performed with blue-green argon laser light. The separation between the slits is 0.500 mm, and the screen is located 3.30 m from the slits. The first bright fringe is located 3.40 mm from the center of the interference pattern. What is the wavelength of the argon laser light?

(b) Light of wavelength 620 nm falls on a double slit, and the first bright fringe of the interference pattern is seen at an angle of 15.0° with the horizontal. Find the separation between the slits.

(a) The location of the bright fringes for small angles is given by:

$$y_{\text{bright}} = \frac{\lambda L}{d} m$$

For $m = 1$,

$$\lambda = \frac{y_{\text{bright}}}{L} d = \frac{(3.40 \times 10^{-3}\ \text{m})(0.500 \times 10^{-3}\ \text{m})}{3.30\ \text{m}} = \boxed{515\ \text{nm}}$$

(b) We do not use the small-angle approximation $\sin\theta \approx \tan\theta$ here because the angle is greater than 10°. For the first bright fringe, $m = 1$, and we have

$$d\sin\theta = m\lambda = \lambda$$

and $$d = \frac{\lambda}{\sin\theta} = \frac{620 \times 10^{-9}\ \text{m}}{\sin 15.0^\circ} = 2.40 \times 10^{-6}\ \text{m}$$

Question 4

Young's double-slit experiment is performed with 589-nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

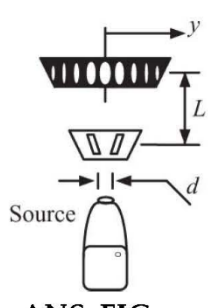
In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$, the first minimum is described by $m = 0$ and the tenth by $m = 9$:

$$\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right) = 9.5\frac{\lambda}{d}$$

Also, $\tan\theta = \frac{y}{L}$. But, for small θ, $\sin\theta \approx \tan\theta$.

Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$:

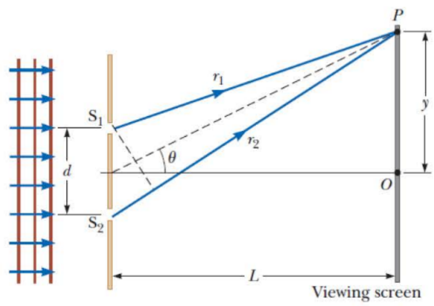
$$d = \frac{9.5(5\ 890 \times 10^{-10}\ \text{m})(2.00\ \text{m})}{7.26 \times 10^{-3}\ \text{m}} = 1.54 \times 10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}}$$



ANS. FIG.

Question 5

In Figure (not to scale), let L = 1.20 m and d = 0.120 mm and assume the slit system is illuminated with monochromatic 500-nm light. Calculate the phase difference between the two wave fronts arriving at P when (a) θ = 0.500° and (b) y = 5.00 mm. (c) What is the value of θ for which the phase difference is 0.333 rad? (d) What is the value of θ for which the path difference is λ/4?



For a double-slit system, the path difference of the two wave fronts arriving at a screen is $\delta = d\sin\theta$ and the phase difference is

$$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}d\sin\theta \approx \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right)$$

(a) For $\theta = 0.500°$,

$$\phi = \frac{2\pi}{\lambda}d\sin\theta$$

$$\phi = \frac{2\pi}{\left(500\times10^{-9}\text{ m}\right)}\left(0.120\times10^{-3}\text{ m}\right)\sin\left(0.500°\right) = \boxed{13.2\text{ rad}}$$

(b) $$\phi \approx \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right) = \frac{2\pi}{\left(500\times10^{-9}\text{ m}\right)}\left(0.120\times10^{-3}\text{ m}\right)\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right)$$

$$= \boxed{6.28\text{ rad}}$$

(c) If $\phi = 0.333\text{ rad} = \frac{2\pi d\sin\theta}{\lambda}$, then

$$\theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{\left(500\times10^{-9}\text{ m}\right)\left(0.333\text{ rad}\right)}{2\pi\left(0.120\times10^{-3}\text{ m}\right)}\right]$$

$$\theta = \boxed{1.27\times10^{-2\,\circ}}$$

(d) If $d\sin\theta = \frac{\lambda}{4}$, then

$$\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{500\times10^{-9}\text{ m}}{4\left(0.120\times10^{-3}\text{ m}\right)}\right]$$

$$\theta = \boxed{5.97\times10^{-2\,\circ}}$$

Question 6

A soap bubble (n = 1.33) floating in air has the shape of a spherical shell with a wall thickness of 120 nm. (a) What is the wavelength of the visible light that is most strongly reflected? (b) Explain how a bubble of different thickness could also strongly reflect light of this same wavelength. (c) Find the two smallest film thicknesses larger than 120 nm that can produce strongly reflected light of the same wavelength.

(a) With phase reversal in the reflection at the outer surface of the soap film and no reversal on reflection from the inner surface, the condition for constructive interference in the light reflected from the soap bubble is

$$2t = \left(m + \frac{1}{2}\right)\lambda_n = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad \rightarrow \quad 2nt = \left(m + \frac{1}{2}\right)\lambda$$

$$\lambda = \frac{2nt}{\left(m + \frac{1}{2}\right)}$$

where $m = 0, 1, 2, \ldots$. For the lowest order reflection ($m = 0$), and the wavelength is

$$\lambda = \frac{2nt}{(0 + 1/2)} = \frac{2(1.33)(120\text{ nm})}{1/2} = \boxed{638\text{ nm}}$$

(b) $\boxed{\text{A thicker film would require a higher order of reflection, so use a larger value of } m.}$

(c) From (a) above, for a given wavelength, the thickness would be

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(m + \frac{1}{2}\right)\frac{638\text{ nm}}{2(1.33)}$$

The next greater thickness of soap film that can strongly reflect 638 nm light corresponds to $m = 1$, giving

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(1 + \frac{1}{2}\right)\frac{638\text{ nm}}{2(1.33)} = \boxed{360\text{ nm}}$$

and the third such thickness (corresponding to $m = 2$) is

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n} = \left(2 + \frac{1}{2}\right)\frac{638\text{ nm}}{2(1.33)} = \boxed{600\text{ nm}}$$

Question 7

A thin film of oil (n = 1.25) is located on smooth, wet pavement. When viewed perpendicular to the pavement, the film reflects most strongly red light at 640 nm and reflects no green light at 512 nm. How thick is the oil film? (n for water is 1.33).

The layers are air, oil, and water. Because 1 < 1.25 < 1.33, light reflected both from the top and from the bottom surface of the oil suffers phase reversal. For constructive interference we require

$$2t = \frac{m\lambda_{\text{cons}}}{n} \Rightarrow \lambda_{cons} = \frac{2nt}{m}$$

and for destructive interference,

$$2t = \frac{\left[m + (1/2)\right]\lambda_{\text{des}}}{n} \Rightarrow \lambda_{dest} = \frac{2nt}{m+1/2}$$

Then,

$$\frac{\lambda_{cons}}{\lambda_{dest}} = \frac{m + 1/2}{m} = 1 + \frac{1}{2m} = \frac{640\text{ nm}}{512\text{ nm}} \Rightarrow m = 2$$

Therefore, $t = \dfrac{2(640\text{ nm})}{2(1.25)} = \boxed{512\text{ nm}}$.