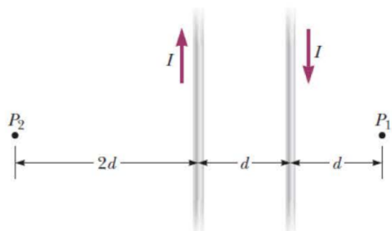
PHYS143
Physics for Engineers
Tutorial - Chapter 30 - Solutions

Question 1

The two wires shown in Figure are separated by $d = 10.0$ cm and carry currents of $I = 5.00$ A in opposite directions. Find the magnitude and direction of the net magnetic field (a) at a point midway between the wires; (b) at point P_1, 10.0 cm to the right of the wire on the right; and (c) at point P_2, 2d = 20.0 cm to the left of the wire on the left.



Assume that the wire on the right is wire 1 and that on the left is wire 2. Also, choose the positive direction for the magnetic field to be out of the page and negative into the page.

(a) At the point half way between the two wires,

$$B_{\text{net}} = -B_1 - B_2 = -\left[\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}\right] = -\frac{\mu_0}{2\pi r}(I_1 + I_2)$$

$$= -\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(5.00 \times 10^{-2}\ \text{m})}(10.0\ \text{A}) = -4.00 \times 10^{-5}\ \text{T}$$

or $\quad B_{\text{net}} = \boxed{40.0\ \mu\text{T into the page}}$

(b) At point P_1, $\quad B_{\text{net}} = +B_1 - B_2 = \dfrac{\mu_0}{2\pi}\left[+\dfrac{I_1}{r_1} - \dfrac{I_2}{r_2}\right]$

$$B_{\text{net}} = \frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\left[\frac{5.00\ \text{A}}{0.100\ \text{m}} - \frac{5.00\ \text{A}}{0.200\ \text{m}}\right]$$

$$= \boxed{5.00\ \mu\text{T out of page}}$$

(c) At point P_2, $\quad B_{\text{net}} = -B_1 + B_2 = \dfrac{\mu_0}{2\pi}\left[-\dfrac{I_1}{r_1} + \dfrac{I_2}{r_2}\right]$

$$B_{\text{net}} = \frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\left[-\frac{5.00\ \text{A}}{0.300\ \text{m}} + \frac{5.00\ \text{A}}{0.200\ \text{m}}\right]$$

$$= \boxed{1.67\ \mu\text{T out of page}}$$

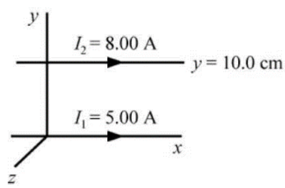
Question 2

Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. The first wire carries a current I_1 = 5.00 A, and the second carries I_2 = 8.00 A. (a) What is the magnitude of the magnetic field created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field created by I_2 at the location of I_1? (d) What is the force per length exerted by I_2 on I_1?

Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

(a) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \boxed{1.00\times10^{-5}\ \text{T out of the page}}$$



(b) $$\vec{\mathbf{F}}_B = I_2\vec{\ell}\times\vec{\mathbf{B}} = (8.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}}\times(1.00\times10^{-5}\ \mathrm{T})\hat{\mathbf{k}}\right]$$
$$= (8.00\times10^{-5}\ \mathrm{N})(-\hat{\mathbf{j}})$$

$$\vec{\mathbf{F}}_B = \boxed{8.00\times10^{-5}\ \text{N toward the first wire}}$$

(c) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}(-\hat{\mathbf{k}}) = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(8.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}(-\hat{\mathbf{k}})$$
$$= (1.60\times10^{-5}\ \mathrm{T})(-\hat{\mathbf{k}})$$

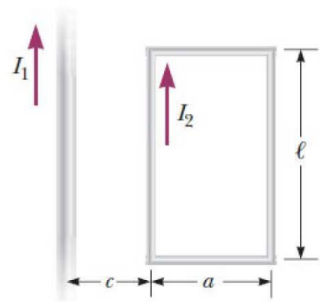
$$\vec{\mathbf{B}} = \boxed{1.60\times10^{-5}\ \text{T into the page}}$$

(d) $$\vec{\mathbf{F}}_B = I_1\vec{\ell}\times\vec{\mathbf{B}} = (5.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}}\times(1.60\times10^{-5}\ \mathrm{T})(-\hat{\mathbf{k}})\right]$$
$$= (8.00\times10^{-5}\ \mathrm{N})(+\hat{\mathbf{j}})$$

$$\vec{\mathbf{F}}_B = \boxed{8.00\times10^{-5}\ \text{N towards the second wire}}$$

Question 3

In Figure, the current in the long, straight wire is I_1 = 5.00 A and the wire lies in the plane of the rectangular loop, which carries a current I_2 = 10.0 A. The dimensions in the figure are c = 0.100 m, a = 0.150 m, and l = 0.450 m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

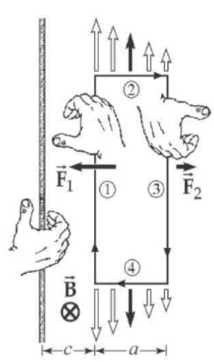


To the right of the long, straight wire, current I_1 creates a magnetic field into the page. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{-a}{c(c+a)}\right]\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \frac{(4\pi\times10^{-7}\ \mathrm{N/A^2})(5.00\ \mathrm{A})(10.0\ \mathrm{A})(0.450\ \mathrm{m})}{2\pi} \times\left(\frac{-0.150\ \mathrm{m}}{(0.100\ \mathrm{m})(0.250\ \mathrm{m})}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \left(-2.70\times10^{-5}\hat{\mathbf{i}}\right)\ \mathrm{N} = \left(-27.0\times10^{-6}\hat{\mathbf{i}}\right)\ \mathrm{N} = \boxed{-27.0\hat{\mathbf{i}}\ \mu\mathrm{N}}$$



Question 4

A solenoid 10.0 cm in diameter and 75.0 cm long is made from copper wire of diameter 0.100 cm, with very thin insulation. The wire is wound onto a cardboard tube in a single layer, with adjacent turns touching each other. What power must be delivered to the solenoid if it is to produce a field of 8.00 mT at its center? ($\rho = 1.7 \times 10^{-8}$)

The number of turns is $N = \frac{75.0 \text{ cm}}{0.100 \text{ cm}} = 750$. We assume that the solenoid is long enough to qualify as a long solenoid. Then the field within it (not close to the ends) is $B = \frac{N\mu_0 I}{\ell}$, so

$$I = \frac{B\ell}{N\mu_0} = \frac{(8.00 \times 10^{-3} \text{ T})(0.750 \text{ m})}{750(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})} = 6.37 \text{ A}$$

The resistance of the wire is

$$R = \frac{\rho\ell_{\text{wire}}}{A} = \frac{(1.7 \times 10^{-8} \ \Omega\cdot\text{m})2\pi(0.050\ 0 \text{ m})750}{\pi(0.050\ 0 \times 10^{-2} \text{ m})^2} = 5.10 \ \Omega$$

The power delivered is

$$P = I\Delta V = I^2 R = (6.37 \text{ A})^2(5.10 \ \Omega) = \boxed{207 \text{ W}}$$

The power required would be smaller if wire were wrapped in several layers.