PHYS143
Physics for Engineers
Tutorial - Chapter 39 – Solutions

Question 1

A star is 5.00 ly from the Earth. At what speed must a spacecraft travel on its journey to the star such that the Earth–star distance measured in the frame of the spacecraft is 2.00 ly?

In the rest frame of the spacecraft, the Earth-star gap travels past it at speed u. The distance from Earth to the star is a proper length in the Earth's frame:

$$L = \frac{L_P}{\gamma} = L_P\sqrt{1-\left(\frac{u}{c}\right)^2}$$

Solving for the speed of the spacecraft gives,

$$u = c\sqrt{1-\left(\frac{L}{L_P}\right)^2} = c\sqrt{1-\left(\frac{2.00\text{ ly}}{5.00\text{ ly}}\right)^2} = \boxed{0.917c}$$

Question 2

At what speed does a clock move if it is measured to run at a rate one-half the rate of a clock at rest with respect to an observer?

A clock running at one-half the rate of a clock at rest takes twice the time to register the same time interval: $\Delta t = 2\Delta t_p$.

$$\Delta t = \frac{\Delta t_p}{\left[1-(v/c)^2\right]^{1/2}} \qquad \text{so} \qquad v = c\left[1-\left(\frac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$$

For $\Delta t = 2\Delta t_p$,

$$v = c\left[1-\left(\frac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1-\frac{1}{4}\right]^{1/2} = \boxed{0.866c}$$

Question 3

A physicist drives through a stop light. When he is pulled over, he tells the police officer that the Doppler shift made the red light of wavelength 650 nm appear green to him, with a wavelength of 520 nm. The police officer writes out a traffic citation for speeding. How fast was the physicist traveling, according to his own testimony?

For the light as observed, $\lambda = 650$ nm and $\lambda' = 520$ nm. From

$$f' = \frac{c}{\lambda'} = \sqrt{\frac{1+v/c}{1-v/c}}f = \sqrt{\frac{1+v/c}{1-v/c}}\frac{c}{\lambda}$$

Solving for the velocity,

$$\sqrt{\frac{1+v/c}{1-v/c}}=\frac{\lambda}{\lambda'} \quad\rightarrow\quad 1+\frac{v}{c}=\left(\frac{\lambda}{\lambda'}\right)^2\left(1-\frac{v}{c}\right)$$

Then,

$$\frac{v}{c}\left[1+\left(\frac{\lambda}{\lambda'}\right)^2\right]=\left(\frac{\lambda}{\lambda'}\right)^2-1$$

$$\frac{v}{c}=\frac{\left(\frac{\lambda}{\lambda'}\right)^2-1}{1+\left(\frac{\lambda}{\lambda'}\right)^2}=\frac{\left(\frac{650\text{ nm}}{520\text{ nm}}\right)^2-1}{1+\left(\frac{650\text{ nm}}{520\text{ nm}}\right)^2}=0.220$$

or $$v=\boxed{0.220c}=6.59\times10^7\text{ m/s}$$

Question 4

The identical twins Speedo and Goslo join a migration from the Earth to Planet X, 20.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same moment on different spacecraft. Speedo's spacecraft travels steadily at $0.950c$ and Goslo's at $0.750c$. (a) Calculate the age difference between the twins after Goslo's spacecraft lands on Planet X. (b) Which twin is older?

(a) In the Earth frame, Speedo's trip lasts for a time

$$\Delta t=\frac{\Delta x}{v}=\frac{20.0\text{ ly}}{0.950\text{c}}=21.05\text{ yr}$$

Speedo's age advances only by the proper time interval

$$\Delta t_p=\frac{\Delta t}{\gamma}=21.05\text{ yr}\sqrt{1-(0.950)^2}=6.574\text{ yr}$$

during his trip. Similarly for Goslo,

$$\Delta t_p=\frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}}=\frac{20.0\text{ ly}}{0.750\text{ ly/yr}}\sqrt{1-(0.750)^2}=17.64\text{ yr}$$

While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\text{ ly/yr}}-\frac{20.0\text{ ly}}{0.950\text{ ly/yr}}=5.614\text{ yr}$$

From their departure to when the twins meet, Speedo has aged (6.574 yr + 5.614 yr) = 12.19 yr, and Goslo has aged 17.64 years, for an age difference of

$$17.64\text{ yr}-(6.574\text{ yr}+5.614\text{ yr})=\boxed{5.45\text{ yr}}$$

(b) $\boxed{\text{Goslo}}$ is older.

Question 5
A red light flashes at position x_R = 3.00 m and time t_R = 1.00 × 10^{-9} s, and a blue light flashes at x_B = 5.00 m and t_B = 9.00 × 10^{-9} s, all measured in the S reference frame. Reference frame S' moves uniformly to the right and has its origin at the same point as S at $t = t' = 0$. Both flashes are observed to occur at the same place in S'. (a) Find the relative speed between S and S'. (b) Find the location of the two flashes in frame S'. (c) At what time does the red flash occur in the S' frame?

(a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$$\Delta x' = \gamma(\Delta x - v\Delta t):\ 0 = \gamma\left[2.00\text{ m} - v\left(8.00\times10^{-9}\text{ s}\right)\right]$$

$$v = \frac{2.00\text{ m}}{8.00\times10^{-9}\text{ s}} = \boxed{2.50\times10^{8}\text{ m/s}}$$

so

$$\gamma = \frac{1}{\sqrt{1-\left(2.50\times10^{8}\text{ m/s}\right)^2/\left(3.00\times10^{8}\text{ m/s}\right)^2}} = 1.81$$

(b) Again from the Lorentz transformation, $x' = \gamma(x - vt)$:

$$x' = 1.81\left[3.00\text{ m} - \left(2.50\times10^{8}\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right] = \boxed{4.98\,\text{m}}$$

(c) $t' = \gamma\left(t - \dfrac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^{8}\text{ m/s}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\,\text{s}}$$

Question 6
Keilah, in reference frame S, measures two events to be simultaneous. Event A occurs at the point (150.0 m, 0, 0) at the instant 9:00:00 Universal time on January 15, 2013. Event B occurs at the point (50 m, 0, 0) at the same moment. Torrey, moving past with a velocity of 0.800c $\hat{\mathbf{i}}$, also observes the two events. In her reference frame S', which event occurred first and what time interval elapsed between the events?

We use the Lorentz transformation. In frame S, we may take $t = 0$ for both events, so the coordinates of event A are (x = 150.0 m, y = 0, z = 0, t = 0), and the coordinates of event B are (x = 50 m, y = 0, z = 0, t = 0). The time coordinates of event A in frame S′ are

$$t'_A = \gamma\left(t_A - \frac{v}{c^2}x_A\right)$$
$$= \frac{1}{\sqrt{1-(0.800)^2}}\left(0 - \frac{0.800c}{c^2}(150 \text{ m})\right)$$
$$= 1.667\left(-\frac{120 \text{ m}}{3.00\times10^8 \text{ m/s}}\right)$$
$$= -6.67\times10^{-7} \text{ s}$$

The time coordinates of event B in frame S′ are

$$t'_B = \gamma\left(t_B - \frac{v}{c^2}x_B\right)$$
$$= \frac{1}{\sqrt{1-(0.800)^2}}\left(0 - \frac{0.800c}{c^2}(50.0 \text{ m})\right)$$
$$= 1.667\left(-\frac{40.0 \text{ m}}{3.00\times10^8 \text{ m/s}}\right)$$
$$= -2.22\times10^{-7} \text{ s}$$

We see that event B occurred earlier. The time elapsed between the events was

$$\Delta t' = t'_A - t'_B = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = -\gamma\frac{v}{c^2}\Delta x$$

$$= -1.667\left(\frac{80.0 \text{ m}}{3.00\times10^8 \text{ m/s}}\right) = -4.44\times10^{-7} \text{ s} = \boxed{444 \text{ ns}}$$

Question 7

A spacecraft is launched from the surface of the Earth with a velocity of $0.600c$ at an angle of 50.0° above the horizontal positive x axis. Another spacecraft is moving past with a velocity of $0.700c$ in the negative x direction. Determine the magnitude and direction of the velocity of the first spacecraft as measured by the pilot of the second spacecraft.

Let frame S be the Earth frame of reference. Then $v = -0.700c$.

The components of the velocity of the first spacecraft are

$$u_x = (0.600c)\cos 50.0° = 0.386c$$

and

$$u_y = (0.600c)\sin 50.0° = 0.460c.$$

As measured from the S' frame of the second spacecraft,

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.386c - (-0.700c)}{1 - [(0.386c)(-0.700c)/c^2]}$$
$$= \frac{1.086c}{1.27} = 0.855c$$

and

$$u'_y = \frac{u_y}{\gamma\left(1 - u_x v/c^2\right)} = \frac{0.460c\sqrt{1-(0.700)^2}}{1-(0.386)(-0.700)}$$

$$= \frac{0.460c(0.714)}{1.27} = 0.258c$$

The magnitude of $\vec{\mathbf{u}}'$ is $\sqrt{(0.855c)^2 + (0.285c)^2} = \boxed{0.893c}$

and its direction is at $\tan^{-1}\left(\frac{0.258c}{0.855c}\right) = \boxed{16.8^\circ \text{ above the } x' \text{ axis}}$.