PHYS143

Physics for Engineers

Tutorial - Chapter 32 - Solutions

Question 1

A solenoid of radius 2.50 cm has 400 turns and a length of 20.0 cm. Find (a) its inductance and (b) the rate at which current must change through it to produce an emf of 75.0 μV.

(a) The inductance of the solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(400)^2\left[\pi\left(2.50\times10^{-2}\ \mathrm{m}\right)^2\right]}{0.200\ \mathrm{m}}$$

$$= 1.97\times10^{-3}\ \mathrm{H} = \boxed{1.97\ \mathrm{mH}}$$

(b) From $|\mathcal{E}| = L\left(\Delta i/\Delta t\right)$,

$$\frac{\Delta I}{\Delta t} = \frac{|\mathcal{E}|}{L} = \frac{75.0\times10^{-6}\ \mathrm{V}}{1.97\times10^{-3}\ \mathrm{H}} = 38.0\times10^{-3}\ \mathrm{A/s} = \boxed{38.0\ \mathrm{mA/s}}$$

Question 2

A 40.0-mA current is carried by a uniformly wound air-core solenoid with 450 turns, a 15.0-mm diameter, and 12.0-cm length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn, and (c) the inductance of the solenoid. (d) What If? If the current were different, which of these quantities would change?

(a) $B = \mu_0 n i = \left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\dfrac{450}{0.120\ \mathrm{m}}\right)\left(0.040\,0\ \mathrm{A}\right) = \boxed{188\ \mu\mathrm{T}}$

(b) $\Phi_B = BA = B\pi\left(\dfrac{15.0\times10^{-3}\mathrm{m}}{2}\right)^2 = \boxed{3.33\times10^{-8}\ \mathrm{T\cdot m^2}}$

(c) $L = \dfrac{N\Phi_B}{i} = \dfrac{450\Phi_B}{0.040\,0\ \mathrm{A}} = \boxed{0.375\ \mathrm{mH}}$

(d) B and Φ_B are proportional to current; L is independent of current.

Question 3

A 12.0-V battery is connected into a series circuit containing a 10.0-Ω resistor and a 2.00-H inductor. In what time interval will the current reach (a) 50.0% and (b) 90.0% of its final value?

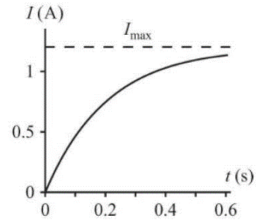
(a) At time t,

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

Where $\tau = \dfrac{L}{R} = \dfrac{2.00\ \mathrm{H}}{10.0\ \Omega} = 0.200\ \mathrm{s}$

After a long time,

$$I_i = \frac{\mathcal{E}}{R}\left(1 - e^{-\infty}\right) = \frac{\mathcal{E}}{R}$$



At $i(t) = 0.500 I_i$

$$(0.500)\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

so $\quad 0.500 = 1 - e^{-t/\tau}$

Isolating the constants on the right,

$$e^{-t/\tau} = 0.500$$

$$\ln\left(e^{-t/\tau}\right) = \ln(0.500)$$

$$t = \tau\left[-\ln(0.500)\right] = (0.200 \text{ s})\left[-\ln(0.500)\right] = \boxed{0.139 \text{ s}}$$

(b) Similarly, to reach 90% of I_i, $\quad 0.900 = 1 - e^{-t/\tau} \quad \rightarrow \quad e^{-t/\tau} = 0.100$

and $\quad t = -\tau \ln(0.100)$

Thus,

$$t = -(0.200 \text{ s})\ln(0.100) = \boxed{0.461 \text{ s}}$$

Question 4

On a clear day at a certain location, a 100-V/m vertical electric field exists near the Earth's surface. At the same place, the Earth's magnetic field has a magnitude of 0.500×10^{-4} T. Compute the energy densities of (a) the electric field and (b) the magnetic field.

(a) The energy density stored by the electric field is

$$u_E = \epsilon_0 \frac{E^2}{2} = \left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{(100 \text{ V/m})^2}{2}\left(\frac{\text{J/C}}{\text{V}}\right)^2\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$= 4.43 \times 10^{-8} \frac{\text{J}}{\text{m}^3} = \boxed{44.3 \text{ nJ/m}^3}$$

(b) The energy density stored by the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(0.500 \times 10^{-4} \text{ T}\right)^2}{2\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)}\left(\frac{\text{N/A}\cdot\text{m}}{\text{T}}\right)$$

$$= 9.95 \times 10^{-4} \frac{\text{N}}{\text{m}^2}\left(\frac{\text{m}}{\text{m}}\right) = 9.95 \times 10^{-4} \frac{\text{N}\cdot\text{m}}{\text{m}^3} = \boxed{995 \ \mu\text{J/m}^3}$$

Question 5

Two solenoids A and B, spaced close to each other and sharing the same cylindrical axis, have 400 and 700 turns, respectively. A current of 3.50 A in solenoid A produces an average flux of 300 μWb through each turn of A and a flux of 90.0 μWb through each turn of B. (a) Calculate the mutual inductance of the two solenoids. (b) What is the inductance of A? (c) What emf is induced in B when the current in A changes at the rate of 0.500 A/s?

(a) The mutual inductance of the coils is

$$M = \frac{N_B \Phi_{BA}}{i_A} = \frac{700\left(90.0\times10^{-6}\,\text{Wb}\right)}{3.50\text{ A}} = \boxed{18.0\text{ mH}}$$

(b) The inductance of coil A is

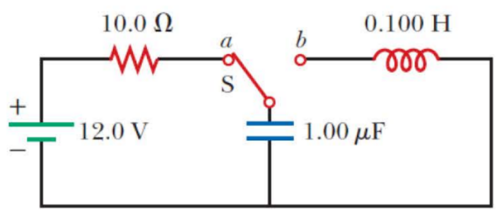
$$L_A = \frac{N_A \Phi_A}{i_A} = \frac{400(300\times10^{-6})}{3.5} = 34.3\,mH$$

(c) The emf induced in the other coil is

$$|\varepsilon_B| = \left|-M\frac{di_A}{dt}\right| = (18.0\text{ mH})(0.500\text{ A/s}) = \boxed{9.00\text{ mV}}$$

Question 6

The switch in Figure is connected to position a for a long time interval. At t = 0, the switch is thrown to position b. After this time, what are (a) the frequency of oscillation of the LC circuit, (b) the maximum charge that appears on the capacitor, (c) the maximum current in the inductor, and (d) the total energy the circuit possesses at t = 3.00 s?



(a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.100\text{ H})(1.00\times10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$$

(b) The maximum charge on the capacitor is

$$Q = C\mathcal{E} = \left(1.00\times10^{-6}\text{ F}\right)(12.0\text{ V}) = \boxed{12.0\ \mu\text{C}}$$

(c) To find the maximum current I_i, we equate

$$\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_i^2$$

Then solve for I_i to obtain

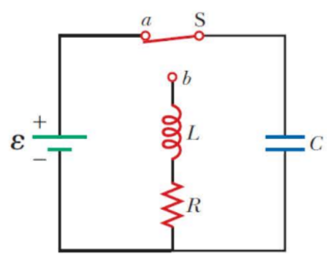
$$I_i = \mathcal{E}\sqrt{\frac{C}{L}} = 12.0\text{ V}\sqrt{\frac{1.00\times10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$$

(d) The total energy the circuit possesses at $t = 3.00$ s and at all times is

$$U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}(1.00\times10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{72.0\ \mu\text{J}}$$

Question 7

In Figure, let R = 7.60 Ω, L = 2.20 mH, and C = 1.80 µF. (a) Calculate the frequency of the damped oscillation of the circuit when the switch is thrown to position b. (b) What is the critical resistance for damped oscillations?



(a) The frequency of damped oscillations is given by Equation 32.32:

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$= \sqrt{\frac{1}{(2.20\times10^{-3}\text{H})(1.80\times10^{-6}\text{F})} - \left(\frac{7.60}{2(2.20\times10^{-3}\text{H})}\right)^2}$$

$$= 1.58\times10^4\ \text{rad/s}$$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \frac{1.58\times10^4\ \text{rad/s}}{2\pi} = \boxed{2.51\text{ kHz}}$.

(b) Critical damping occurs when $\omega_d = 0$, or when

$$R_c = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4(2.20\times10^{-3}\text{H})}{1.80\times10^{-6}\text{F}}} = \boxed{69.9\ \Omega}$$