PHYS143
Physics for Engineers
Tutorial - Chapter 26 - Solutions

Question 1

When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

We have $Q = C\Delta V$ and $C = \epsilon_0 A / d$. Thus, $Q = \epsilon_0 A\Delta V/d$

The surface charge density on each plate has the same magnitude, given by

$$\sigma = \frac{Q}{A} = \frac{\epsilon_0 \Delta V}{d}$$

Thus,

$$d = \frac{\epsilon_0 \Delta V}{Q/A} = \frac{(8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(150\ \mathrm{V})}{(30.0\times 10^{-9}\ \mathrm{C/cm^2})}$$

$$d = \left(4.43\times 10^{-2}\ \frac{\mathrm{V\cdot C\cdot cm^2}}{\mathrm{N\cdot m^2}}\right)\frac{(1\ \mathrm{m^2})}{(10^4\ \mathrm{cm^2})}\frac{\mathrm{J}}{\mathrm{V\cdot C}}\frac{\mathrm{N\cdot m}}{\mathrm{J}} = \boxed{4.43\ \mu\mathrm{m}}$$

Question 2

An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

(a) The potential difference between two points in a uniform electric field is $\Delta V = Ed$, so

$$E = \frac{\Delta V}{d} = \frac{20.0\ \mathrm{V}}{1.80\times 10^{-3}\ \mathrm{m}} = \boxed{1.11\times 10^4\ \mathrm{V/m}}$$

(b) The electric field between capacitor plates is $E = \frac{\sigma}{\epsilon_0}$, so $\sigma = \epsilon_0 E$:

$$\sigma = (8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(1.11\times 10^4\ \mathrm{V/m}) = 9.83\times 10^{-8}\ \mathrm{C/m^2}$$
$$= \boxed{98.3\ \mathrm{nC/m^2}}$$

(c) For a parallel-plate capacitor, $C = \frac{\epsilon_0 A}{d}$:

$$C = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(7.60\times10^{-4}\ \mathrm{m^2}\right)}{1.80\times10^{-3}\ \mathrm{m}}$$
$$= 3.74\times10^{-12}\ \mathrm{F} = \boxed{3.74\ \mathrm{pF}}$$

(d) The charge on each plate is $Q = C\Delta V$:

$$Q = \left(3.74\times10^{-12}\ \mathrm{F}\right)(20.0\ \mathrm{V}) = \boxed{74.7\ \mathrm{pC}}$$

Question 3

An isolated, charged conducting sphere of radius 12.0 cm creates an electric field of 4.90×10^4 N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?

(a) The electric field outside a spherical charge distribution of radius R is $E = k_e q/r^2$. Therefore,

$$q = \frac{Er^2}{k_e} = \frac{\left(4.90\times10^{4}\ \mathrm{N/C}\right)(0.210\ \mathrm{m})^2}{8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}} = 0.240\ \mu\mathrm{C}$$

Then

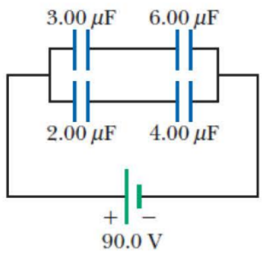
$$\sigma = \frac{q}{A} = \frac{0.240\times10^{-6}\ \mathrm{C}}{4\pi(0.120\ \mathrm{m})^2} = \boxed{1.33\ \mu\mathrm{C/m^2}}$$

(b) For an isolated charged sphere of radius R,

$$C = 4\pi\epsilon_0 r = 4\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.120\ \mathrm{m}) = \boxed{13.3\ \mathrm{pF}}$$

Question 4

For the system of four capacitors shown in Figure, find (a) the equivalent capacitance of the system, (b) the charge on each capacitor, and (c) the potential difference across each capacitor.



(a) The equivalent capacitance of the series combination in the upper branch is

$$\frac{1}{C_{\text{upper}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.00\ \mu\mathrm{F}} + \frac{1}{6.00\ \mu\mathrm{F}} \rightarrow C_{\text{upper}} = 2.00\ \mu\mathrm{F}$$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} \rightarrow C_{\text{lower}} = 1.33\ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00\ \mu\text{F} + 1.33\ \mu\text{F} = \boxed{3.33\ \mu\text{F}}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. Each of the capacitors in series combination holds the same charge as that on the equivalent capacitor. For the upper branch:

$$Q_3 = Q_6 = Q_{\text{upper}} = C_{\text{upper}}(\Delta V) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C s}$$

so, $\boxed{180\ \mu\text{C on the 3.00-}\mu\text{F and the 6.00-}\mu\text{F capacitors}}$

For the lower branch:

$$Q_2 = Q_4 = Q_{\text{lower}} = C_{\text{lower}}(\Delta V) = (1.33\ \mu\text{F})(90.0\ \text{V}) = 120\ \mu\text{C}$$

so, $\boxed{120\ \mu\text{C on the 2.00-}\mu\text{F and 4.00-}\mu\text{F capacitors}}$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$\boxed{60.0\ \text{V across the 3.00-}\mu\text{F and the 2.00-}\mu\text{F capacitors}}$

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$\boxed{30.0\ \text{V across the 6.00-}\mu\text{F and the 4.00-}\mu\text{F capacitors}}$

Question 5

Two capacitors, C_1 = 18.0 μF and C_2 = 36.0 μF, are connected in series, and a 12.0-V battery is connected across the two capacitors. Find (a) the equivalent capacitance and (b) the energy stored in this equivalent capacitance. (c) Find the energy stored in each individual capacitor.

(a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{18.0\ \mu\text{F}} + \frac{1}{36.0\ \mu\text{F}} \rightarrow C_{\text{eq}} = \boxed{12.0\ \mu\text{F}}$$

(b) This series combination is connected to a 12.0-V battery, the total stored energy is

$$U_{E,\,eq} = \frac{1}{2}C_{eq}(\Delta V)^2 = \frac{1}{2}\left(12.0\times10^{-6}\ \text{F}\right)(12.0\ \text{V})^2 = \boxed{8.64\times10^{-4}\ \text{J}}$$

(c) Capacitors in series carry the same charge as their equivalent capacitor. The charge stored on each of the two capacitors in the series combination is

$$Q_1 = Q_2 = Q_{total} = C_{eq}(\Delta V) = (12.0\ \mu\text{F})(12.0\ \text{V})$$
$$= 144\ \mu\text{C} = 1.44\times10^{-4}\ \text{C}$$

and the energy stored in each of the individual capacitors is:

18.0 μF capacitor:

$$U_{E1} = \frac{Q_1^2}{2C_1} = \frac{\left(1.44\times10^{-4}\ \text{C}\right)^2}{2\left(18.0\times10^{-6}\ \text{F}\right)} = \boxed{5.76\times10^{-4}\ \text{J}}$$

36.0 μF capacitor:

$$U_{E2} = \frac{Q_2^2}{2C_2} = \frac{\left(1.44\times10^{-4}\ \text{C}\right)^2}{2\left(36.0\times10^{-6}\ \text{F}\right)} = \boxed{2.88\times10^{-4}\ \text{J}}$$

Question 6

(a) How much charge can be placed on a capacitor with air between the plates before it breaks down if the area of each plate is 5.00 cm^2? (for air: κ = 1.00, with E_{max} = 3.00 × 10^6 V/m.)

$Q_{max} = C\Delta V_{max}$, but $\Delta V_{max} = E_{max}d$.

Also, $C = \dfrac{\kappa\epsilon_0 A}{d}$.

Thus, $Q_{max} = \dfrac{\kappa\epsilon_0 A}{d}\left(E_{max}d\right) = \kappa\epsilon_0 AE_{max}$.

With air between the plates, the dielectric constant is κ = 1.00, and the dielectric strength is $E_{max} = 3.00\times10^6$ V/m. Therefore,

$$Q_{max} = \kappa\epsilon_0 AE_{max}$$
$$= \left(8.85\times10^{-12}\ \text{F/m}\right)\left(5.00\times10^{-4}\ \text{m}^2\right)\left(3.00\times10^6\ \text{V/m}\right)$$
$$= \boxed{13.3\ \text{nC}}$$

Question 7

A small, rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge has coordinates (-1.20 mm, 1.10 mm) and the negative charge is at the point (1.40 mm, -1.30 mm). (a) Find the electric dipole moment of the object. The object is placed in an electric field $\vec{\mathbf{E}} = (7.80 \times 10^3\ \hat{\mathbf{i}} - 4.90 \times 10^3\ \hat{\mathbf{j}})$ N/C. (b) Find the torque acting on the object. (c) Find the potential energy of the object–field system when the object is in this orientation. (d) Assuming the orientation of the object can change, find the difference between the maximum and minimum potential energies of the system.

(a) The displacement from negative to positive charge is

$$2\vec{\mathbf{a}} = \left(-1.20\hat{\mathbf{i}} + 1.10\hat{\mathbf{j}}\right)\text{ mm} - \left(1.40\hat{\mathbf{i}} - 1.30\hat{\mathbf{j}}\right)\text{ mm}$$
$$= \left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right) \times 10^{-3}\text{ m}$$

The electric dipole moment is $\vec{\mathbf{p}} = 2\vec{\mathbf{a}}q$

$$\vec{\mathbf{p}} = \left(3.50 \times 10^{-9}\text{ C}\right)\left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right) \times 10^{-3}\text{ m}$$
$$= \boxed{\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right) \times 10^{-12}\text{ C}\cdot\text{m}}$$

(b) The torque exerted by the field on the dipole is

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$
$$= \left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right) \times 10^{-12}\text{ C}\cdot\text{m}\right] \times \left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right) \times 10^{3}\text{ N/C}\right]$$
$$= \left(+44.6\hat{\mathbf{k}} - 65.5\hat{\mathbf{k}}\right) \times 10^{-9}\text{ N}\cdot\text{m} = \boxed{-2.09 \times 10^{-8}\,\hat{\mathbf{k}}\text{ N}\cdot\text{m}}$$

(c) Relative to zero energy when it is perpendicular to the field, the dipole has potential energy

$$U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$$
$$= -\left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right) \times 10^{\ 12}\text{ C}\cdot\text{m}\right]$$
$$\cdot\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right) \times 10^{3}\text{ N/C}\right]$$
$$= (71.0 + 41.2) \times 10^{-9}\text{ J} = \boxed{112\text{ nJ}}$$

(d) For convenience we compute the magnitudes

$$\left|\vec{\mathbf{p}}\right| = \sqrt{(9.10)^2 + (8.40)^2} \times 10^{-12}\text{ C}\cdot\text{m} = 12.4 \times 10^{-12}\text{ C}\cdot\text{m}$$

and $\left|\vec{\mathbf{E}}\right| = \sqrt{(7.80)^2 + (4.90)^2} \times 10^{3}\text{ N/C} = 9.21 \times 10^{3}\text{ N/C}$

The maximum potential energy occurs when the dipole moment is opposite in direction to the field, and is

$$U_{\text{max}} = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}} = -\left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right|(-1) = \left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right| = 114\text{ nJ}$$

The minimum potential energy configuration is the stable equilibrium position with the dipole aligned with the field. The value is $U_{\text{min}} = -114\text{ nJ}$. Then the difference, representing the range of potential energies available to the dipole, is $U_{\text{max}} - U_{\text{min}} = \boxed{228\text{ nJ}}$.