Module 2 Exam: Magnetism
Formulae Sheet

Constants:

$k_e = 8.9876 \times 10^{9}\ N.m^2/C^2$

$\epsilon_0 = 8.8542 \times 10^{-12}\ C^2/N.m^2$

$e = 1.60218 \times 10^{-19}\ C$

$m_e = 9.1094 \times 10^{-31}\ Kg$

$m_p = 1.67262 \times 10^{-27}\ Kg$

$1\ ev = 1.60 \times 10^{-19}\ J$

$\mu_0 = 4\pi \times 10^{-7}\ T.m\ /\ A$

Avogadro's number of atoms ($N_A = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$)

Formulas:

$$\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

$$\vec{\boldsymbol{\mu}}_{\text{coil}} = NI\vec{\mathbf{A}}$$

$$U_B = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

$$\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$\frac{m}{q} = \frac{rB_o}{v} = \frac{rB_o B}{E}$$

$$\omega = \frac{qB}{m}$$

$$\theta = \omega \Delta t,$$

$$K = \frac{1}{2} mv^2$$

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$$

$$I = nqv_d A$$

$$R = \frac{\rho \ell_{wire}}{A}$$

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

$$L = \frac{N\Phi_B}{i}$$

$$L = -\frac{\mathcal{E}_L}{di/dt}$$

$$\tau = \frac{L}{R}$$

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt}$$

Magnetic Field for a Long, Straight Conductor:

$$B = \frac{\mu_o I}{2\pi a}$$

Magnetic Field for a Circular Loop of Wire:

$$B = \frac{\mu_o I}{4\pi a}\theta = \frac{\mu_o I}{4\pi a}2\pi = \frac{\mu_o I}{2a}$$

Magnetic Field on the Axis of Circular Current Loop:

$$B_x = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2 + x^2)^{3/2}}(2\pi a) = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}}$$

Magnetic Force Between Two Parallel Conductors:

$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

Ampere's Law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

The Magnetic Field Created by a Toroid:

$$B = \boxed{\frac{\mu_0 N I}{2\pi r}}$$

Ampere's Law Applied to a Solenoid:

$$\boxed{B = \mu_0\frac{N}{\ell}I = \mu_0 n I}$$

Magnetic Flux Through a Rectangular Loop

$$\Phi_B = \frac{\mu_0 I b}{2\pi}\int_c^{a+c}\frac{dr}{r} = \frac{\mu_0 I b}{2\pi}\ln r\Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi}\ln\left(\frac{a+c}{c}\right) = \boxed{\frac{\mu_0 I b}{2\pi}\ln\left(1 + \frac{a}{c}\right)}$$

Motional emf Induced in a Rotating Bar:

$$\mathcal{E} = B\int v\,dr = B\omega\int_0^\ell r\,dr = \frac{1}{2}B\omega\ell^2$$

Faraday's Law

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

Motional emf Induced in a Rotating Bar

$$\mathcal{E} = B\int v\,dr = B\omega\int_0^\ell r\,dr = \frac{1}{2}B\omega\ell^2$$

Electric Field Induced by a Changing Magnetic Field in a Solenoid

$$E = \frac{\mu_0 n I_{\max}\omega R^2}{2r}\sin\omega t \quad (\text{for } r > R)$$

$$E = \frac{\mu_0 n I_{\max}\omega}{2} r\sin\omega t \quad (\text{for } r < R)$$

For a Solenoid:

$$L = \mu_0\frac{(n\ell)^2}{\ell}A = \mu_0 n^2 A\ell = \mu_0 n^2 V$$

Induced emf in a Rotating Loop:

$$\varepsilon = NBA\omega sin(\omega t)$$

Charging Inductor:

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$$

Discharging Inductor:

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau} = I_i e^{-t/\tau}$$

Magnetic Field Energy:

$$U = \frac{1}{2}\mu_o n^2 V\left(\frac{B}{\mu_o n}\right)^2 = \frac{B^2}{2\mu_o}V$$

Magnetic Field Energy Density:

$$u_B = \frac{U}{V} = \frac{B^2}{2\mu_o}$$

Angular frequency of oscillation for the RLC circuit

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

Critical resistance of oscillation for the RLC circuit

$$R_C = \sqrt{4L/C}$$

$$P_{av} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2\left(\omega^2 - \omega_o^2\right)^2}$$

The Q-factor:

$$Q = \omega_o / \Delta\omega = (\omega_o L) / R$$

For Transformer:

$$R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

Maxwell's Equations:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0\mu_0 \frac{d\Phi_E}{dt}$$

$$c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_o} \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

$$I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0}$$

$$I = S_{\text{avg}} = cu_{\text{avg}}$$

For a perfectly reflecting surface: ***$p = 2T_{ER}/c$ and $P = 2S/c$***

For a perfectly absorbing surface: ***$p = T_{ER}/c$ and $P = S/c$***

Motional emf:

$$\mathcal{E} = -B\ell v$$

Oscillation in LC Circuit:

$$U = U_E + U_B = \frac{q^2}{2C} + \tfrac{1}{2}Li^2 = \frac{Q_{\max}^2}{2C} \qquad \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_i^2 \qquad Q = C\mathcal{E}$$

$$\omega = 1/\sqrt{LC} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max} \qquad \Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

$X_L = \omega L$

$X_C = 1/\omega C$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2} \qquad \varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$P_{avg} = \frac{1}{2} I_{max}\, \Delta V_{max} \cos\varphi = I_{rms} \Delta V_{rms} \cos\varphi$

$$\Delta v_2 = \frac{N_2}{N_1}\Delta v_1 \qquad c = E/B \qquad \lambda = \frac{c}{f}$$