Chapter 38

Diffraction Patterns and Polarization

Diffraction and Polarization

When plane light waves pass through a small aperture in an opaque barrier, the aperture acts as if it were a point source of light, with waves entering the shadow region behind the barrier.

- This phenomenon, known as diffraction.

Diffraction can be described only with a wave model for light.

A diffraction pattern occurs when the light from an aperture is allowed to fall on a screen .

- The features of this diffraction pattern can be investigated.

Electromagnetic waves are transverse. That is, the electric and magnetic field vectors associated with electromagnetic waves are perpendicular to the direction of wave propagation.

Under certain conditions transverse waves with electric field vectors in all directions can be polarized in various ways .

- Only certain directions of the electric field vectors are present in the polarized wave.

Diffraction Pattern

Light of wavelength comparable to or larger than the width of a slit spreads out in all forward directions upon passing through the slit.

This phenomena is called *diffraction.*

When light passes through a narrow slit, it spreads beyond the narrow path defined by the slit into regions that would be in shadow if light travelled in straight lines.

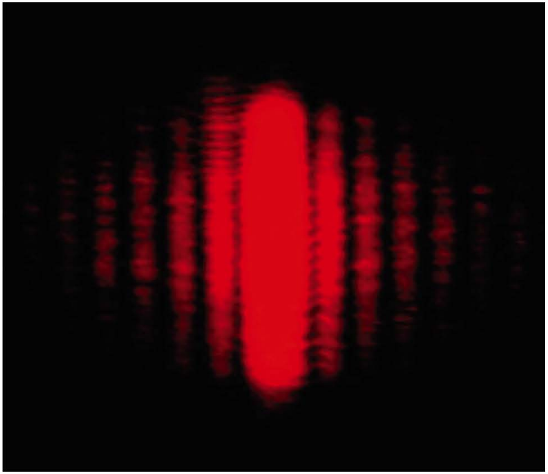
A single slit placed between a distant light source and a screen produces a **diffraction pattern.**

- It will have a broad, intense central band
  - Called the **central maximum**
- The central band will be flanked by a series of narrower, less intense secondary bands.
  - Called **side maxima** or **secondary maxima**
- The central band will also be flanked by a series of dark bands.
  - Called **minima**

Diffraction Pattern, Single Slit

The diffraction pattern consists of the central maximum and a series of secondary maxima and minima.

The pattern is similar to an interference pattern.



The diffraction pattern that appears on a screen when light passes through a narrow vertical slit. The pattern consists of a broad central fringe and a series of less intense and narrower side fringes.

Fraunhofer Diffraction Pattern

A **Fraunhofer diffraction pattern** occurs when the rays leave the diffracting object in parallel directions.

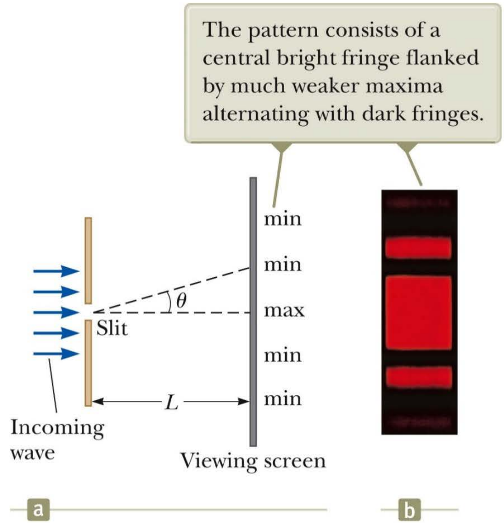
- Screen very far from the slit and the rays reaching the screen are approximately parallel.

In this model, the pattern on the screen is called a **Fraunhofer diffraction pattern.**

A broad central bright fringe is seen along the axis (θ = 0) snd flanked by much weaker bright fringes alternating with dark fringes.

Alternating bright and dark fringes are seen on each side.

The central bright maximum is twice as wide as the secondary maxima.



Single-Slit Diffraction

The finite width of slits is the basis for understanding Fraunhofer diffraction.

- We can explain some important features of its phenomenon by examining waves coming from various portions of the slit.

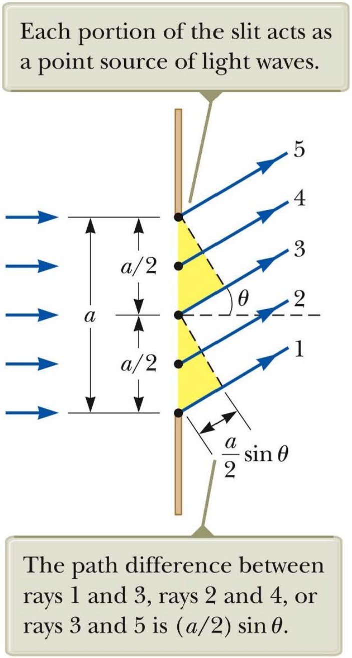
Each portion of the slit acts as a source of light waves.

Therefore, light from one portion of the slit can interfere with light from another portion.

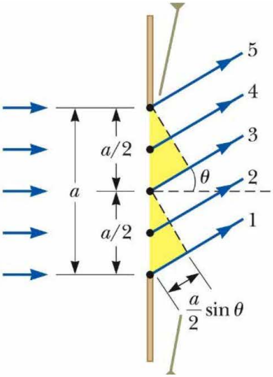
The resultant light intensity on a viewing screen depends on the direction θ.

The diffraction pattern is actually an interference pattern.

- The different sources of light are different portions of the single slit.



Single-Slit Diffraction, Analysis



All the waves are in phase as they leave the slit.

Wave 1 travels farther than wave 3 by an amount equal to the path difference.

- $(a/2)\sin\theta$, a is the width of the slit

Similarly, the path difference between rays 2 and 4 is also $(a/2)\sin\theta$, as is that between rays 3 and 5.

If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results.

In general, destructive interference occurs for a single slit of width a when

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

There is no central dark fringe. Corresponds to no m = 0 in the equation.

Single-Slit Diffraction, Intensity

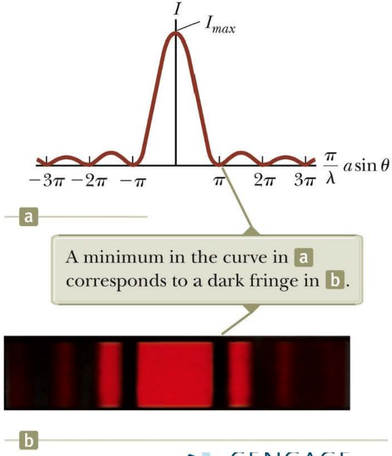
The intensity can be expressed as $$I = I_{\text{max}}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$$

where I_{max} is the intensity at θ = 0 (the central maximum) and λ is the wavelength of light used to illuminate the slit.

Minima occur at $$\frac{\pi a \sin\theta_{\text{dark}}}{\lambda} = m\pi \quad \text{or} \quad \sin\theta_{\text{dark}} = m\frac{\lambda}{a}$$

Most of the light intensity is concentrated in the central maximum.

The graph shows a plot of light intensity vs. (π /λ) a sin θ.



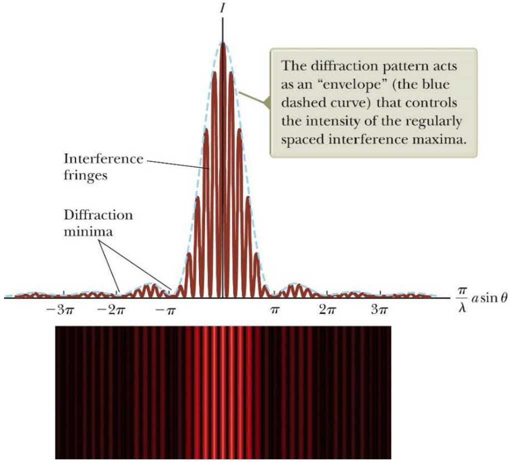
Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, consideration must be made of

- The diffraction patterns due to individual slits.
- The interference due to the wave coming from different slits.

To determine the maximum intensity:

$$I = I_{\max} \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a \sin\theta / \lambda)}{\pi a \sin\theta / \lambda}\right]^2$$



The factor in the square brackets represents the single-slit diffraction pattern.

- This acts as the envelope for the two-slit interference pattern (the $\cos^2$ term).

Two-Slit Diffraction Patterns, Maxima and Minima

To find which interference maximum coincides with the first diffraction minimum.

$$\frac{d \sin \theta}{a \sin \theta} = \frac{m\lambda}{\lambda} \rightarrow \frac{d}{a} = m$$

a is the slit width
d is the distance between the two slits

- The conditions for the interference maximum
  - $d \sin \theta = m \lambda$
- The conditions for the first diffraction minimum
  - $a \sin \theta = \lambda$

For example, if d/a = 18 μm / 3.0 μm = 6. Therefore, the sixth interference maximum (if we count the central maximum as *m* = 0) is aligned with the first diffraction minimum and is dark.

Example

Coherent microwaves of wavelength 5.00 cm enter a tall, narrow window in a building otherwise essentially opaque to the microwaves. If the window is 36.0 cm wide, what is the distance from the central maximum to the first-order minimum along a wall 6.50 m from the window?

For destructive interference,

$$\sin\theta = m\frac{\lambda}{a} = \frac{\lambda}{a} = \frac{5.00 \text{ cm}}{36.0 \text{ cm}} = 0.139$$

and $\theta = 7.98^\circ$. Then, $\frac{y}{L} = \tan\theta$

gives
$$y = L\tan\theta = (6.50 \text{ m})\tan 7.98^\circ = 0.912 \text{ m}$$
$$= \boxed{91.2 \text{ cm}}$$

Example

Helium–neon laser light (λ = 632.8 nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit?

With $m = 1$,

$$\sin\theta = \frac{\lambda}{a} = \frac{6.328\times10^{-7}\text{ m}}{3.00\times10^{-4}\text{ m}} = 2.11\times10^{-3}$$

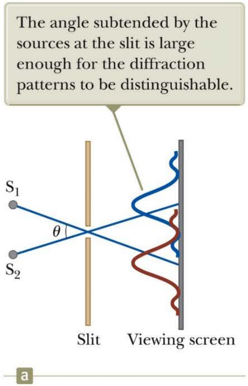
Then,

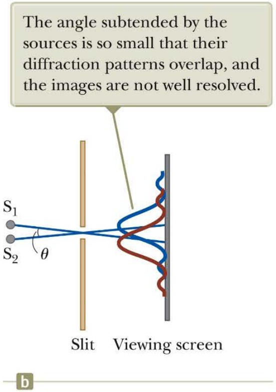
$$\frac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta \approx \theta \text{ (for small } \theta) \quad \rightarrow \quad y = 2.11\text{ mm}$$

$$2y = \boxed{4.22\text{ mm}}$$

Resolution

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.





The images are far enough apart to keep their central maxima from overlapping.

The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable.

The images are resolved.

The sources are so close together that their central maxima do overlap.

The angle subtended by the sources is so small that their diffraction patterns overlap.

The images are not resolved.

Resolution

If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished.

- The images are said to be resolved.

If the two sources are close together, the two central maxima overlap and the images are not resolved.

Resolution, Rayleigh's Criterion

To determine whether two images are resolved, the following condition is often used:

> When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**

The angle of separation, θ_{min}, is the angle subtended by the sources for which the images are just resolved.

Since $\lambda << a$ in most situations, $\sin\theta$ is very small and $\sin\theta \approx \theta$.

Therefore, the limiting angle (in rad) of resolution for a slit of width a is

$$\theta_{min} = \lambda / a$$

To be resolved, the angle subtended by the two sources must be greater than $\theta_{min.}$

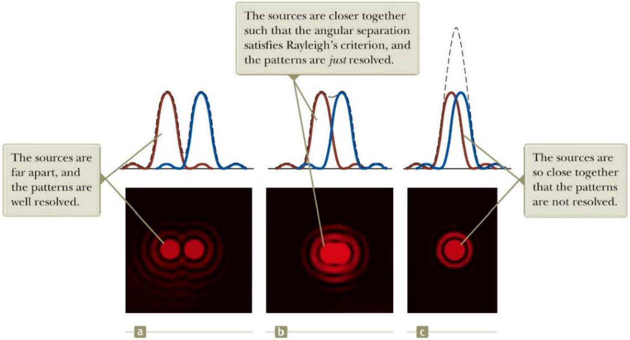
Circular Apertures

The diffraction pattern of a circular aperture consists of a central bright disk surrounded by progressively fainter bright and dark rings.

The limiting angle of resolution of the circular aperture is

$$\theta_{\min} = 1.22\,\frac{\lambda}{D}$$

- D is the diameter of the aperture.



Example

The objective lens of a certain refracting telescope has a diameter of 58.0 cm. The telescope is mounted in a satellite that orbits the Earth at an altitude of 270 km to view objects on the Earth's surface. Assuming an average wavelength of 500 nm, find the minimum distance between two objects on the ground if their images are to be resolved by this lens.

Using Rayleigh's criterion,

$$\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{y}{L}.$$

Therefore,

$$y = 1.22\left(\frac{\lambda}{D}\right)L = 1.22\left(\frac{500\times10^{-9}\ \text{m}}{58.0\times10^{-2}\ \text{m}}\right)\left(270\times10^{3}\ \text{m}\right)$$

$$= \boxed{0.284\ \text{m}}$$

Example

A pinhole camera has a small circular aperture of diameter D. Light from distant objects passes through the aperture into an otherwise dark box, falling on a screen at the other end of the box. The aperture in a pinhole camera has diameter D = 0.600 mm. Two point sources of light of wavelength 550 nm are at a distance L from the hole. The separation between the sources is 2.80 cm, and they are just resolved by the camera. What is L?

Using Rayleigh's criterion, $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{y}{L}$. Therefore,

$$L = \frac{yD}{1.22\lambda} = \frac{(2.80\times10^{-2}\ \text{m})(0.600\times10^{-3}\ \text{m})}{1.22(550\times10^{-9}\ \text{m})} = \boxed{25.0\ \text{m}}$$

Diffraction Grating

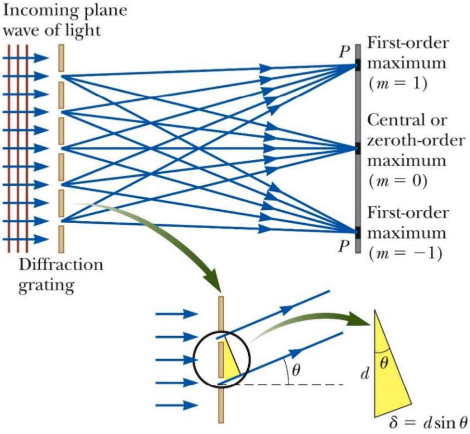
The **diffraction grating,** a useful device for analysing light sources, consists of a large number of equally spaced parallel slits.

A plane wave is incident from the left, normal to the plane of the grating.

The pattern observed on the screen far to the right of the grating is the result of the combined effects of interference and diffraction.

Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.

The waves from all slits are in phase as they leave the slits.



The condition for *maxima* is

- $d \sin \theta_{bright} = m\lambda$,   m = 0, $\pm$1, $\pm$2, …

The integer *m* is the *order number* of the diffraction pattern.

Diffraction Grating, Intensity

We can use the previous expression to calculate the wavelength if we know the grating spacing *d* and the angle θ_{bright}.

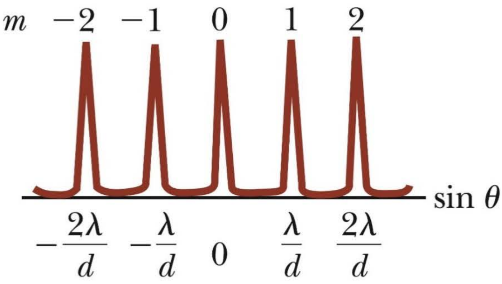
All the wavelengths are seen at *m* = 0.

- This is called the zeroth-order maximum.

The first-order maximum corresponds to *m* = 1.

Characteristics of the intensity pattern:

- The sharp peaks are in contrast to the broad, bright fringes characteristic of the two-slit interference pattern.
- Because the principle maxima are so sharp, they are much brighter than two-slit interference patterns.
- The width of the intensity maxima decreases as the number of slits increases.



Intensity versus sin θ for a diffraction grating. The zeroth-, first-, and second-order maxima are shown.

The Orders of a Diffraction Grating, Example

Monochromatic light from a helium–neon laser ($\lambda = 632.8$ nm) is incident normally on a diffraction grating containing 6 000 grooves per centimeter. Find the angles at which the first- and second-order maxima are observed.

Calculate the slit separation as the inverse of the number of grooves per centimeter:

$$d = \frac{1}{6\,000}\text{ cm} = 1.667 \times 10^{-4}\text{ cm} = 1\,667\text{ nm}$$

Solve for $\sin\theta$ and substitute numerical values for the first-order maximum ($m = 1$) to find θ_1:

$$\sin\theta_1 = \frac{(1)\lambda}{d} = \frac{632.8\text{ nm}}{1\,667\text{ nm}} = 0.379\,7$$

$$\theta_1 = \boxed{22.31°}$$

Repeat for the second-order maximum ($m = 2$):

$$\sin\theta_2 = \frac{(2)\lambda}{d} = \frac{2(632.8\text{ nm})}{1\,667\text{ nm}} = 0.759\,4$$

$$\theta_2 = \boxed{49.41°}$$

Example

A helium–neon laser (λ = 632.8 nm) is used to calibrate a diffraction grating. If the first-order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

The first order maximum occurs at 20.5°, so $\sin\theta = \sin 20.5° = 0.350,$

and

$$d = \frac{\lambda}{\sin\theta} = \frac{632.8 \text{ nm}}{0.350} = 1.81 \times 10^3 \text{ nm}$$

Therefore, the line spacing

$= \boxed{1.81\ \mu\text{m}}$

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Example

White light is spread out into its spectral components by a diffraction grating. If the grating has 2 000 grooves per centimeter, at what angle does red light of wavelength 640 nm appear in first order?

The ruling engine that cut the diffraction grating (or the aluminum plate from which the gelatin or plastic was cast) sliced each centimeter into two thousand divisions. So the grating spacing is

$$d = \frac{1.00 \times 10^{-2}\ \text{m}}{2\ 000} = 5.00 \times 10^{-6}\ \text{m}$$

The light is deflected according to $d \sin\theta = m\lambda$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{1\left(640 \times 10^{-9}\ \text{m}\right)}{5.00 \times 10^{-6}\ \text{m}}\right] = \boxed{7.35^\circ}$$

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Diffraction of X-Rays by Crystals

X-rays are electromagnetic waves of very short wavelength.

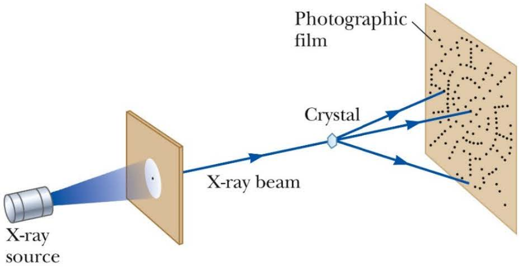
Max von Laue suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays.

Subsequent experiments confirmed this prediction.

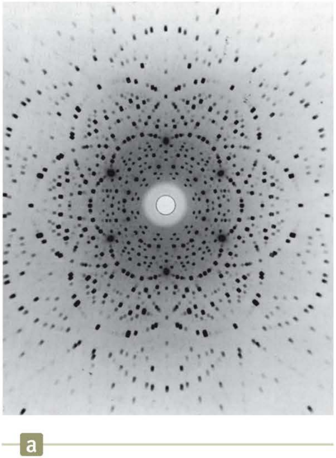
The diffraction patterns from crystals are complex because of the three-dimensional nature of the crystal structure.

Experimental arrangement for observing x-ray diffraction from a crystal.

One can deduce the crystalline structure by analyzing the positions and intensities of the various spots in the pattern.

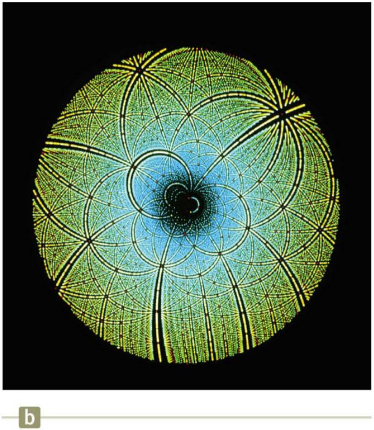


Laue Pattern for Beryl



A Laue pattern of a single crystal of the mineral beryl (beryllium aluminium silicate). Each dot represents a point of constructive interference.

Laue Pattern for Rubisco



A Laue pattern of the enzyme Rubisco, produced with a wideband x-ray spectrum. This enzyme is present in plants and takes part in the process of photosynthesis. The Laue pattern is used to determine the crystal structure of Rubisco.

X-Ray Diffraction, Equations

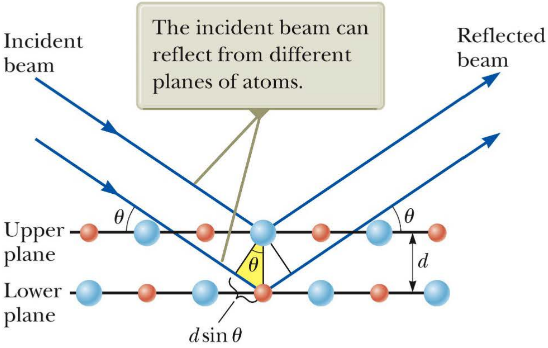
This is a two-dimensional description of the reflection of the x-ray beams.

Suppose an incident x-ray beam makes an angle θ with one of the planes.

The beam can be reflected from both the upper plane and the lower one, but the beam reflected from the lower plane travels farther than the beam reflected from the upper plane.

The condition for *constructive interference* is $2d \sin \theta = m\lambda$ where $m = 1, 2, 3$

This condition is known as **Bragg's law.**



If the wavelength and diffraction angle are measured, this can be used to calculate the spacing between atomic planes.

Example

Potassium iodide (KI) has the same crystalline structure as NaCl, with atomic planes separated by 0.353 nm. A monochromatic x-ray beam shows a first-order diffraction maximum when the grazing angle is 7.60° . Calculate the x-ray wavelength.

The grazing angle is measured from the surface, as shown in Figure 38.23. Then, from

$$2d\sin\theta = m\lambda$$

$$\lambda = \frac{2d\sin\theta}{m}$$

$$= \frac{2\left(0.353\times10^{-9}\ \text{m}\right)\sin 7.60°}{1} = 9.34\times10^{-11}\ \text{m} = \boxed{0.093\,4\ \text{nm}}$$

Example

Monochromatic x-rays (λ = 0.166 nm) from a nickel target are incident on a potassium chloride (KCl) crystal surface. The spacing between planes of atoms in KCl is 0.314 nm. At what angle (relative to the surface) should the beam be directed for a second-order maximum to be observed?

From

$$2d\sin\theta = m\lambda,$$

$$\sin\theta = \frac{m\lambda}{2d} = \frac{2(0.166\text{ nm})}{2(0.314\text{ nm})} = 0.529$$

and

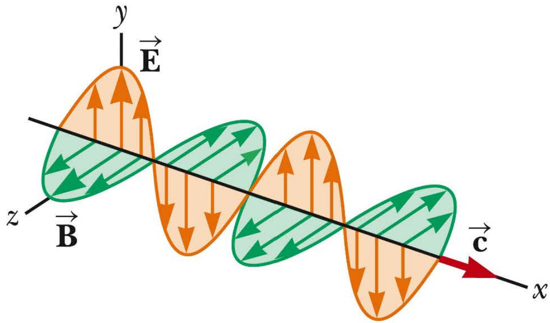
$$\boxed{\theta = 31.9°}$$

Polarization of Light Waves

A wave is said to be *linearly polarized* if the resultant electric field vibrates in the same direction *at all times* at a particular point.

The plane formed by the field and the direction of propagation is called the *plane of polarization* of the wave.

If the wave in Figure represents the resultant of all individual waves, the plane of polarization is the xy plane.



Polarization by Reflection

When an unpolarized light beam is reflected from a surface, the reflected light may be

- Completely polarized
- Partially polarized
- Unpolarized

The polarization depends on the angle of incidence.

- If the angle is 0°, the reflected beam is unpolarized.
- For other angles, there is some degree of polarization.
- For one particular angle, the beam is completely polarized.

Polarization by Reflection, cont.

The angle of incidence for which the reflected beam is completely polarized is called the polarizing angle, $\theta_{p.}$

Brewster's law relates the polarizing angle to the index of refraction for the material.

$$\tan\theta_p = \frac{n_2}{n_1}$$

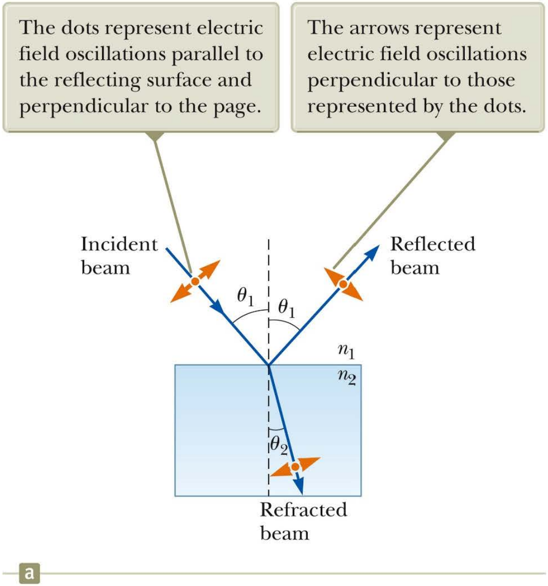
θ_p may also be called Brewster's angle.

Polarization by Reflection, Partially Polarized Example

Unpolarized light is incident on a reflecting surface.

The reflected beam is partially polarized.

The refracted beam is partially polarized



Polarization by Reflection, Completely Polarized Example

Unpolarized light is incident on a reflecting surface.

The reflected beam is completely polarized.

The refracted beam is perpendicular to the reflected beam.

The angle of incidence is Brewster's angle.

https://www.youtube.com/watch?v=DjnDX28l4xA