PHYS143

Physics for Engineers

Tutorial - Chapter 24 - Solutions

Question 1

A flat surface of area 3.20 m^2 is rotated in a uniform electric field of magnitude $E = 6.20 \times 10^5$ N/C. Determine the electric flux through this area (a) when the electric field is perpendicular to the surface and (b) when the electric field is parallel to the surface.

For a uniform electric field passing through a plane surface, $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$, where θ is the angle between the electric field and the normal to the surface.

(a) The electric field is perpendicular to the surface, so $\theta = 0°$:

$$\Phi_E = (6.20 \times 10^5 \text{ N/C})(3.20 \text{ m}^2)\cos 0°$$

$$\Phi_E = \boxed{1.98 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}}$$

(b) The electric field is parallel to the surface: $\theta = 90°$, so $\cos\theta = 0$, and the flux is $\boxed{\text{zero.}}$

Question 2

Consider a thin, spherical shell of radius 14.0 cm with a total charge of 32.0 μC distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

(a) Consider the spherical symmetry of the situation. A gaussian sphere concentric wth the shell, with radius 10.0 cm, encloses 0 charge. Then at the surface of this sphere, inside the charged shell, we have $\vec{\mathbf{E}} = \boxed{0}$.

(b) For a gaussian sphere of radius 20.0 cm, we apply $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}$. The field is radially outward, and $4\pi r^2 E = q/\epsilon_0$:

$$E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(32.0 \times 10^{-6} \text{ C})}{(0.200 \text{ m})^2}$$

$$= 7.19 \times 10^6 \text{ N/C}$$

so $\vec{\mathbf{E}} = \boxed{7.19 \text{ MN/C radially outward}}$

Question 3

A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

(a) At the center of the sphere, the total charge is zero, so

$$E = \frac{k_e Q r}{a^3} = \boxed{0}$$

(b) At a distance of 10.0 cm = 0.100 m from the center,

$$E = \frac{k_e Q r}{a^3} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C})(26.0\times10^{-6}\ \mathrm{C})(0.100\ \mathrm{m})}{(0.400\ \mathrm{m})^3}$$
$$= \boxed{365\ \mathrm{kN/C}}$$

(c) At a distance of 40.0 cm = 0.400 m from the center, all of the charge is enclosed, so

$$E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C})(26.0\times10^{-6}\ \mathrm{C})}{(0.400\ \mathrm{m})^2}$$
$$= \boxed{1.46\ \mathrm{MN/C}}$$

(d) At a distance of 60.0 cm = 0.600 m from the center,

$$E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C})(26.0\times10^{-6}\ \mathrm{C})}{(0.600\ \mathrm{m})^2}$$
$$= \boxed{649\ \mathrm{kN/C}}$$

The direction for each electric field is $\boxed{\text{radially outward}}$.

Question 4

A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from the axis of the rod, where distances are measured perpendicular to the rod's axis.

$\oint E\,dA = E(2\pi r l) = \frac{q_{\text{in}}}{\epsilon_0}$ $\quad E = \frac{q_{\text{in}}/l}{2\pi \epsilon_0 r} = \frac{\lambda}{2\pi \epsilon_0 r}$ for the field outside the metal rod.

(a) At $r = 3.00$ cm, $\quad \vec{\mathbf{E}} = \boxed{0}$

(b) At $r = 10.0$ cm,

$$\vec{\mathbf{E}} = \frac{30.0\times10^{-9}\ \mathrm{C}}{2\pi(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.100\ \mathrm{m})}$$
$$= \boxed{5\,400\ \mathrm{N/C,\ outward}}$$

(c) At $r = 100$ cm,

$$\vec{\mathbf{E}} = \frac{30.0 \times 10^{-9}\ \text{C}}{2\pi\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(1.00\ \text{m})}$$

$$= \boxed{540\ \text{N/C, outward}}$$

Note: we can also use:

$$E = \frac{2k_e\lambda}{r}$$

Question 5

A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicularly to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.

(a) $E = \dfrac{\sigma}{\epsilon_0}$, so

$$\sigma = \left(8.00 \times 10^4\ \text{N/C}\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)$$

$$= 7.08 \times 10^{-7}\ \text{C/m}^2$$

$\sigma = \boxed{708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \dfrac{Q}{A}$, so

$$Q = \sigma A = \left(7.08 \times 10^{-7}\ \text{C/m}^2\right)(0.500\ \text{m})^2$$

$$= 1.77 \times 10^{-7}\ \text{C} = \boxed{177\ \text{nC}}$$

positive on one face and negative on the other.

Question 6

A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm, which is concentric with the solid sphere with a charge of -4.00 μC. Assume both conductors are in an electrostatic equilibrium. Find the electric field at (a) r = 1.00 cm, (b) r = 3.00 cm, (c) r = 4.50 cm, and (d) r = 7.00 cm from the center of this charge configuration.

(a) $\vec{\mathbf{E}} = \boxed{0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00 \times 10^{-6}\ \text{C}\right)}{(0.030\,0\ \text{m})^2} = 7.99 \times 10^7\ \text{N/C}$$

$\vec{\mathbf{E}} = \boxed{79.9\ \text{MN/C radially outward}}$

(c) $\vec{\mathbf{E}} = \boxed{0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00 \times 10^{-6}\ \text{C}\right)}{(0.070\,0\ \text{m})^2} = 7.34 \times 10^6\ \text{N/C}$$

$\vec{\mathbf{E}} = \boxed{7.34\ \text{MN/C radially outward}}$