Chapter 23

Electric Fields

Electricity and Magnetism

The laws of electricity and magnetism play a central role in the operation of many modern devices.

The interatomic and intermolecular forces responsible for the formation of solids and liquids are electric in nature.

Electricity and Magnetism, Some History

Chinese

- Documents suggest that magnetism was observed as early as 2000 BC

Greeks

- Electrical and magnetic phenomena as early as 700 BC
- Experiments with amber and magnetite

1600

- William Gilbert showed electrification effects were not confined to just amber.
- The electrification effects were a general phenomena.

1785

- Charles Coulomb confirmed inverse square law form for electric forces

Electricity and Magnetism, More History

1819

- Hans Oersted found a compass needle deflected when near a wire carrying an electric current.

1831

- Michael Faraday and Joseph Henry showed that when a wire is moved near a magnet, an electric current is produced in the wire.

1873

- James Clerk Maxwell used observations and other experimental facts as a basis for formulating the laws of electromagnetism.
  - Unified electricity and magnetism

Electricity and Magnetism – Forces

The concept of force links the study of electromagnetism to previous study.

The electromagnetic force between charged particles is one of the fundamental forces of nature.

Electric Charges

There are two kinds of electric charges

- Called positive and negative
  - Negative charges are the type possessed by electrons.
  - Positive charges are the type possessed by protons.

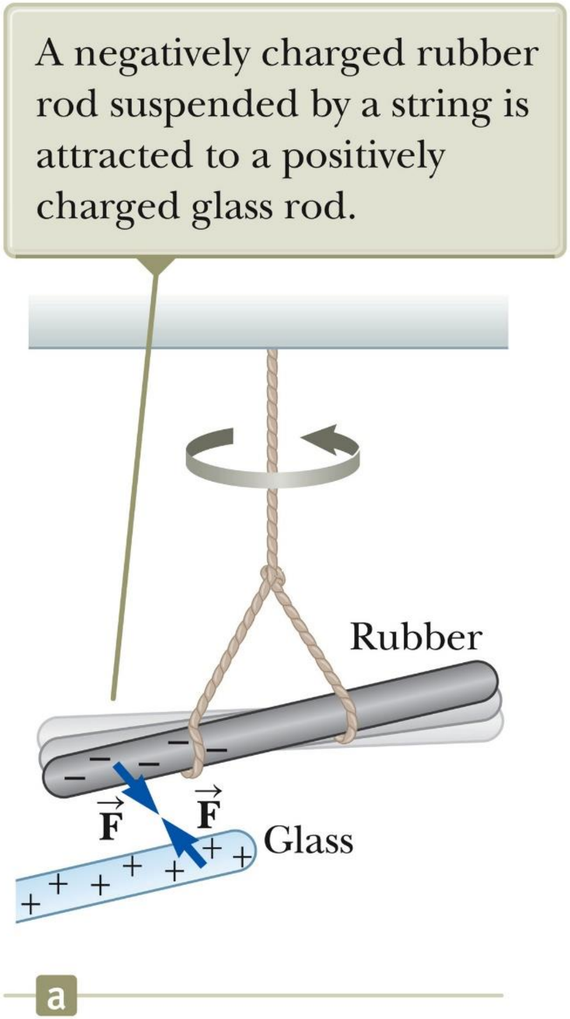
Charges of the same sign repel one another and charges with opposite signs attract one another.

Electric Charges, 2

The rubber rod is negatively charged.

The glass rod is positively charged.

The two rods will attract.

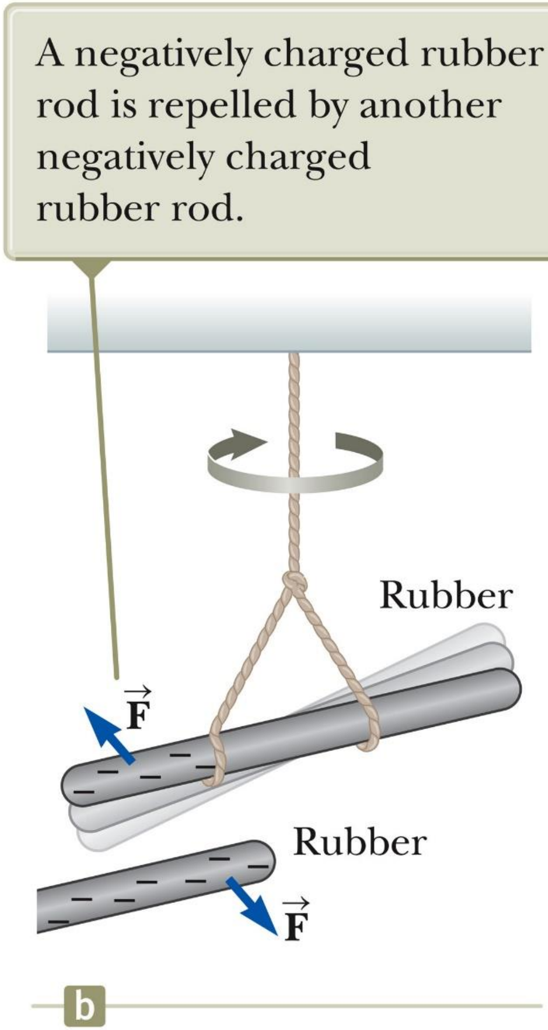


Electric Charges, 3

The rubber rod is negatively charged.

The second rubber rod is also negatively charged.

The two rods will repel.



More About Electric Charges

Electric charge is always conserved in an isolated system.

- For example, charge is not created in the process of rubbing two objects together.
- The electrification is due to a transfer of charge from one object to another.

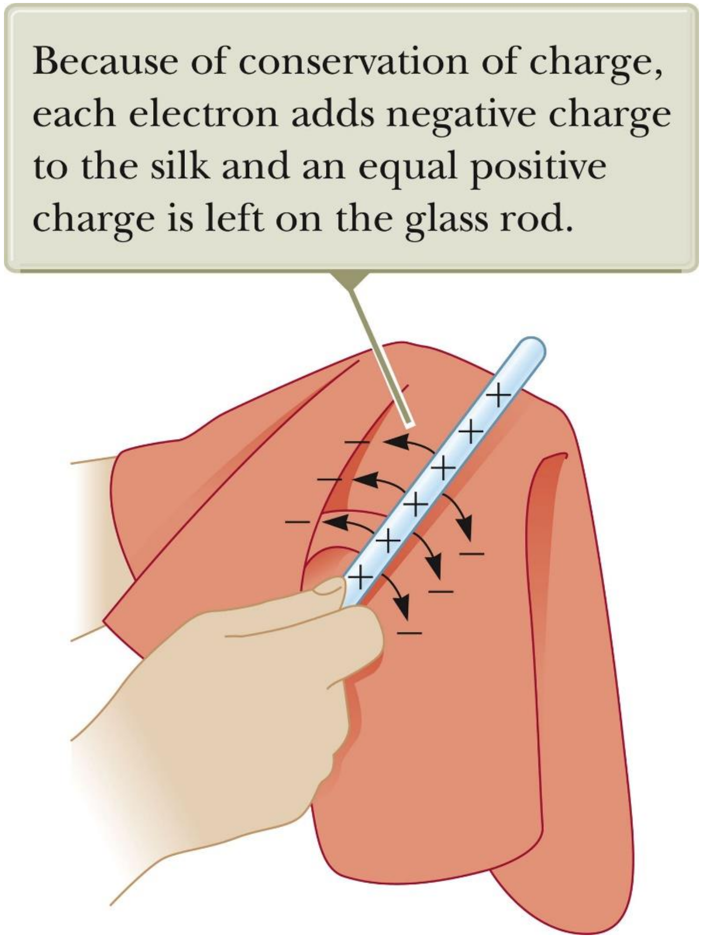
Conservation of Electric Charges

A glass rod is rubbed with silk.

Electrons are transferred from the glass to the silk.

Each electron adds a negative charge to the silk.

An equal positive charge is left on the rod.



Quantization of Electric Charges

The electric charge, q, is said to be quantized.

- q is the standard symbol used for charge as a variable.
- Electric charge exists as discrete packets.
- $q = \pm Ne$
  - N is an integer
  - e is the fundamental unit of charge
  - $|e| = 1.6 \times 10^{-19}$ C
  - Electron: $q = -e$
  - Proton: $q = +e$

Conductors

Electrical conductors are materials in which some of the electrons are free electrons.

- Free electrons are not bound to the atoms.
- These electrons can move relatively freely through the material.
- Examples of good conductors include copper, aluminum and silver.
- When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material.

Insulators

Electrical insulators are materials in which all of the electrons are bound to atoms.

- These electrons can not move relatively freely through the material.
- Examples of good insulators include glass, rubber and wood.
- When a good insulator is charged in a small region, the charge is unable to move to other regions of the material.

Semiconductors

The electrical properties of semiconductors are somewhere between those of insulators and conductors.

Examples of semiconductor materials include silicon and germanium.

- Semiconductors made from these materials are commonly used in making electronic chips.

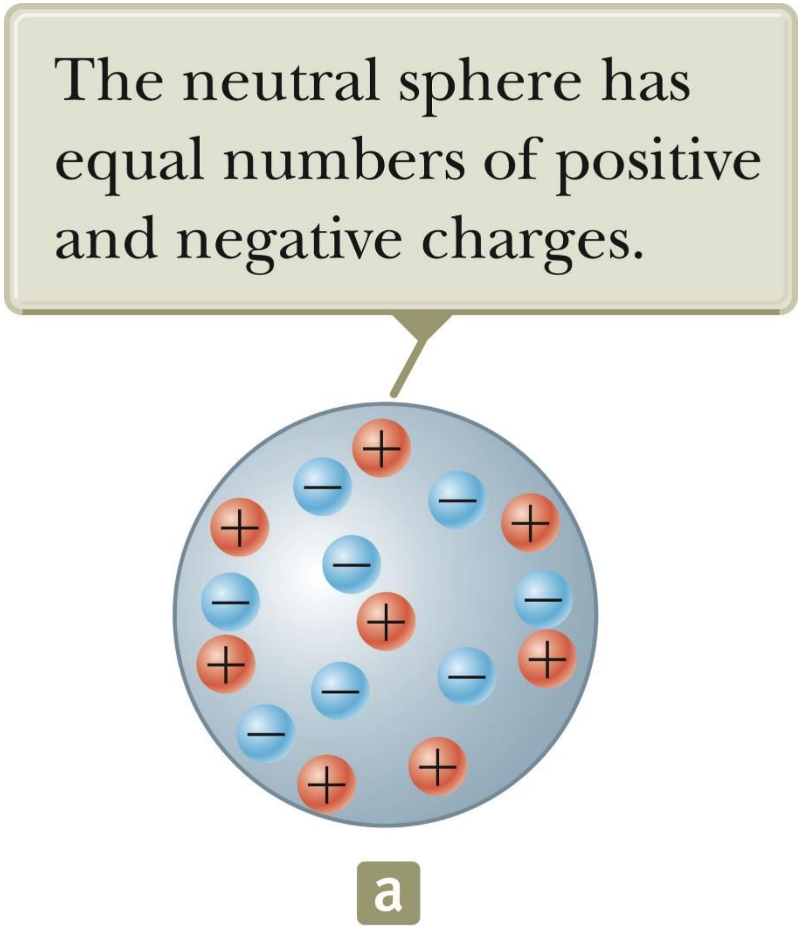
The electrical properties of semiconductors can be changed by the addition of controlled amounts of certain atoms to the material.

Charging by Induction

Charging by induction requires no contact with the object inducing the charge.

Assume we start with a neutral metallic sphere.

- The sphere has the same number of positive and negative charges.



Charging by Induction, 2

B:

A charged rubber rod is placed near the sphere.

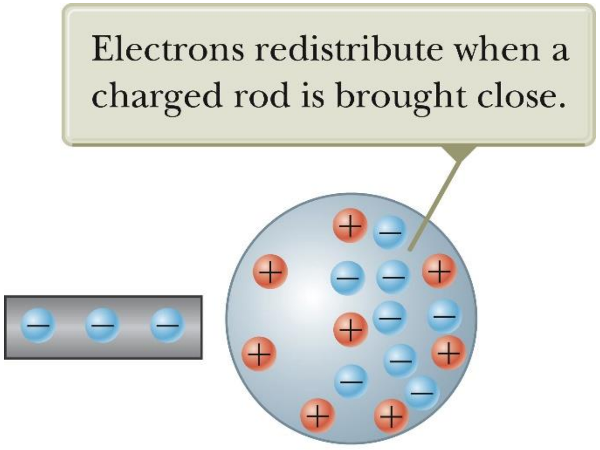
- It does **not** touch the sphere.

The electrons in the neutral sphere are redistributed.

C:

The sphere is grounded.

Some electrons can leave the sphere through the ground wire.



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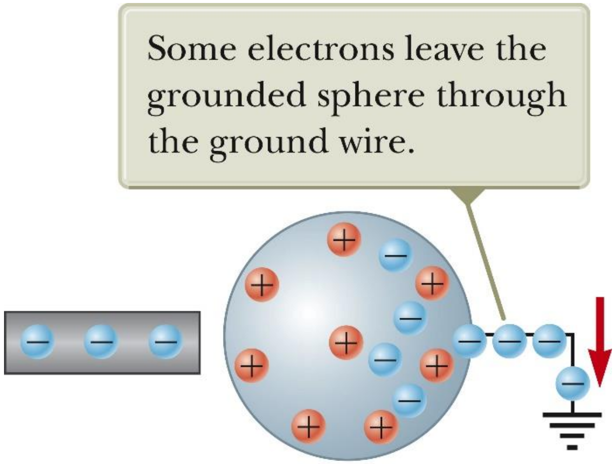


c

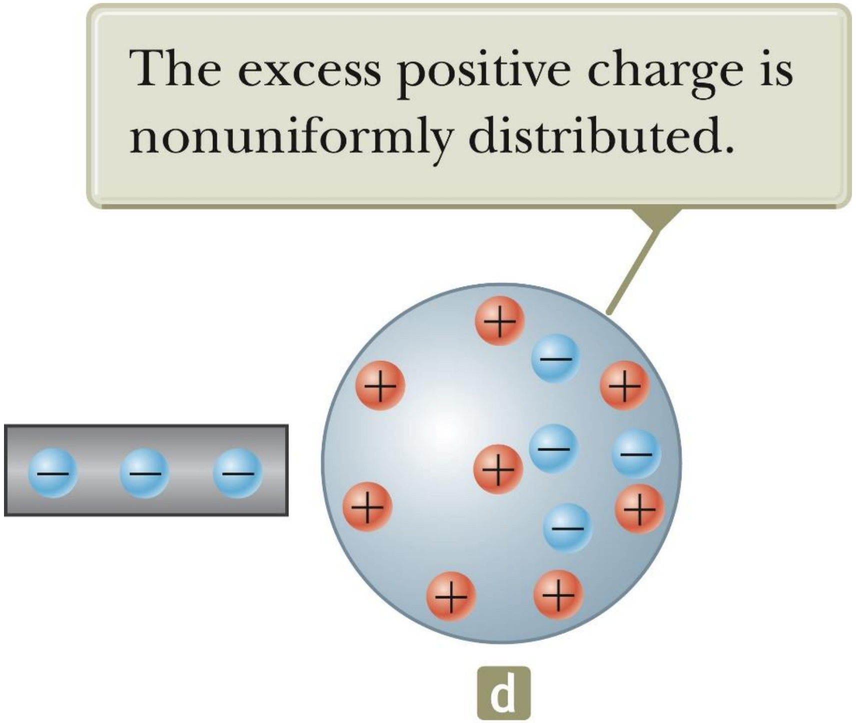
Charging by Induction, 3

The ground wire is removed.

There will now be more positive charges.

The charges are not uniformly distributed.

The positive charge has been ***induced*** in the sphere.



Charging by Induction, 4

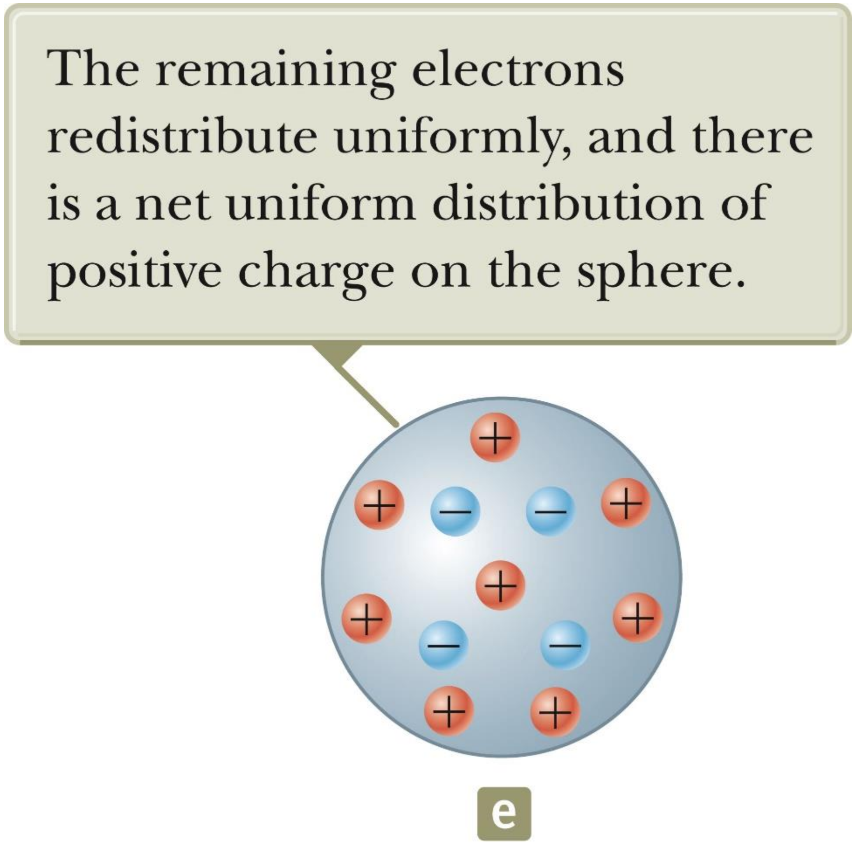
The rod is removed.

The electrons remaining on the sphere redistribute themselves.

There is still a net positive charge on the sphere.

The charge is now uniformly distributed.

Note the rod lost none of its negative charge during this process.

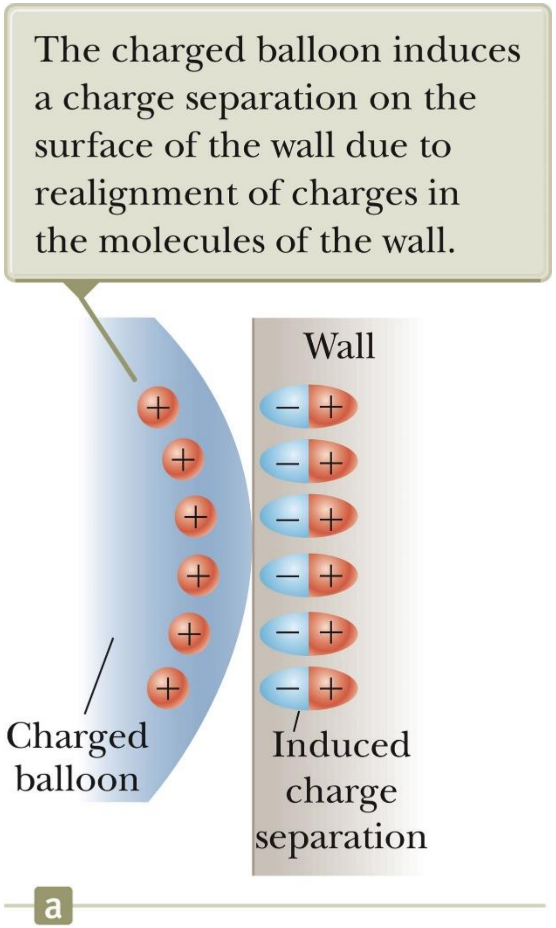


Charge Rearrangement in Insulators

A process similar to induction can take place in insulators.

The charges within the molecules of the material are rearranged.

The proximity of the positive charges on the surface of the object and the negative charges on the surface of the insulator results in an attractive force between the object and the insulator.



Charles Coulomb

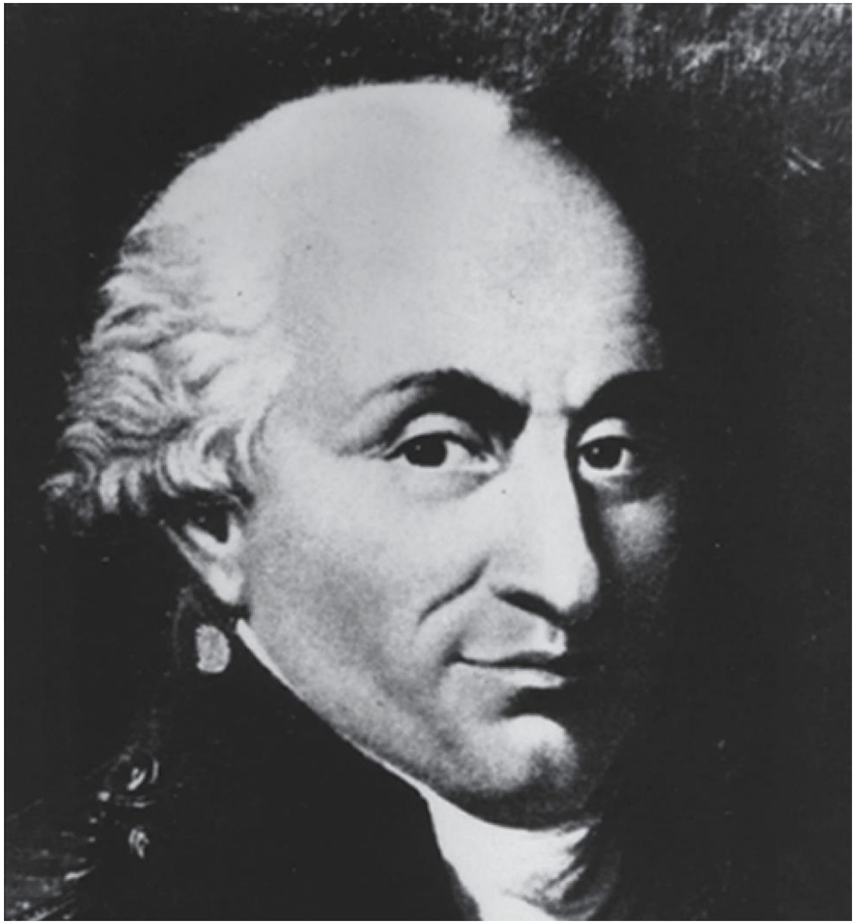
1736 – 1806

French physicist

Major contributions were in areas of electrostatics and magnetism

Also investigated in areas of

- Strengths of materials
- Structural mechanics
- Ergonomics



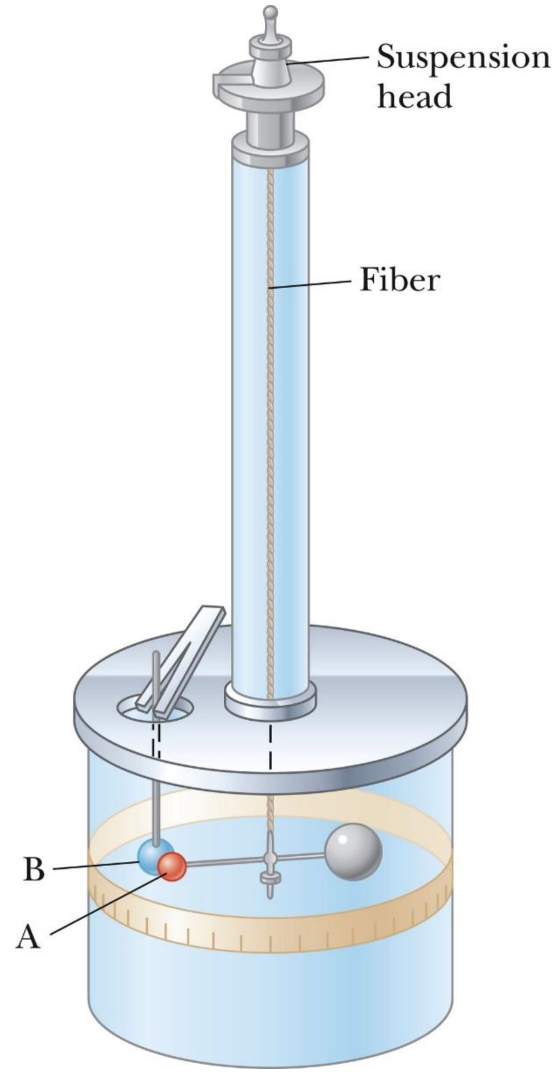
Coulomb's Law

Charles Coulomb measured the magnitudes of electric forces between two small charged spheres.

The force is inversely proportional to the square of the separation r between the charges and directed along the line joining them.

The force is proportional to the product of the charges, q_1 and q_2, on the two particles.

The electrical force between two stationary point charges is given by Coulomb's Law.



Point Charge

The term **point charge** refers to a particle of zero size that carries an electric charge.

- The electrical behavior of electrons and protons is well described by modeling them as point charges.

Coulomb’s Law, cont.

The force is attractive if the charges are of opposite sign.

The force is repulsive if the charges are of like sign.

The force is a conservative force.

Coulomb's Law, Equation

Mathematically,

$$F_e = k_e \frac{|q_1|\,|q_2|}{r^2}$$

The SI unit of charge is the **coulomb** ©.

k_e is called the **Coulomb constant.**

- k_e = 8.9876 x 10^9 N·m^2/C^2 = 1/(4$\pi\varepsilon_o$)
- ε_o is the **permittivity of free space.**
- ε_o = 8.8542 x 10^{-12} C^2 / N·m^2

Coulomb's Law, Example

Example 23.1 **The Hydrogen Atom**

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitude of the electric force between the two particles.

SOLUTION

Use Coulomb's law to find the magnitude of the electric force:

$$F_e = k_e \frac{|e||-e|}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(5.3 \times 10^{-11} \text{ m})^2}$$

$$= \boxed{8.2 \times 10^{-8} \text{ N}}$$

Coulomb's Law, Notes

Remember the charges need to be in coulombs.

- *e* is the smallest unit of charge.
  - except quarks
- $e = 1.6 \times 10^{-19}$ C
- So 1 C needs 6.24×10^{18} electrons or protons

Typical charges can be in the μC range.

Remember that force is a *vector* quantity.

Particle Summary

TABLE 23.1 *Charge and Mass of the Electron, Proton, and Neutron*

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,176\,5 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,5 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

The electron and proton are identical in the magnitude of their charge, but very different in mass.

The proton and the neutron are similar in mass, but very different in charge.

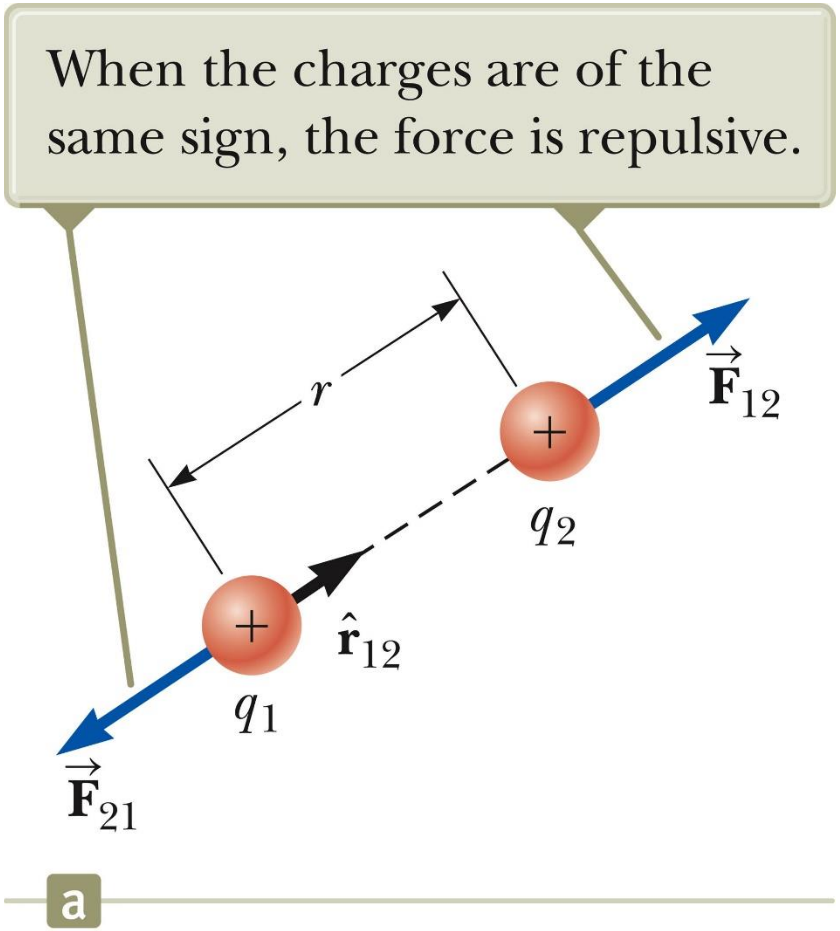
Vector Nature of Electric Forces

In vector form,

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}$$

$\hat{\mathbf{r}}_{12}$ is a unit vector directed from q_1 to q_2.

The like charges produce a repulsive force between them.



Vector Nature of Electrical Forces, cont.

Electrical forces obey Newton's Third Law.

The force on q_1 is equal in magnitude and opposite in direction to the force on q_2

- $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$

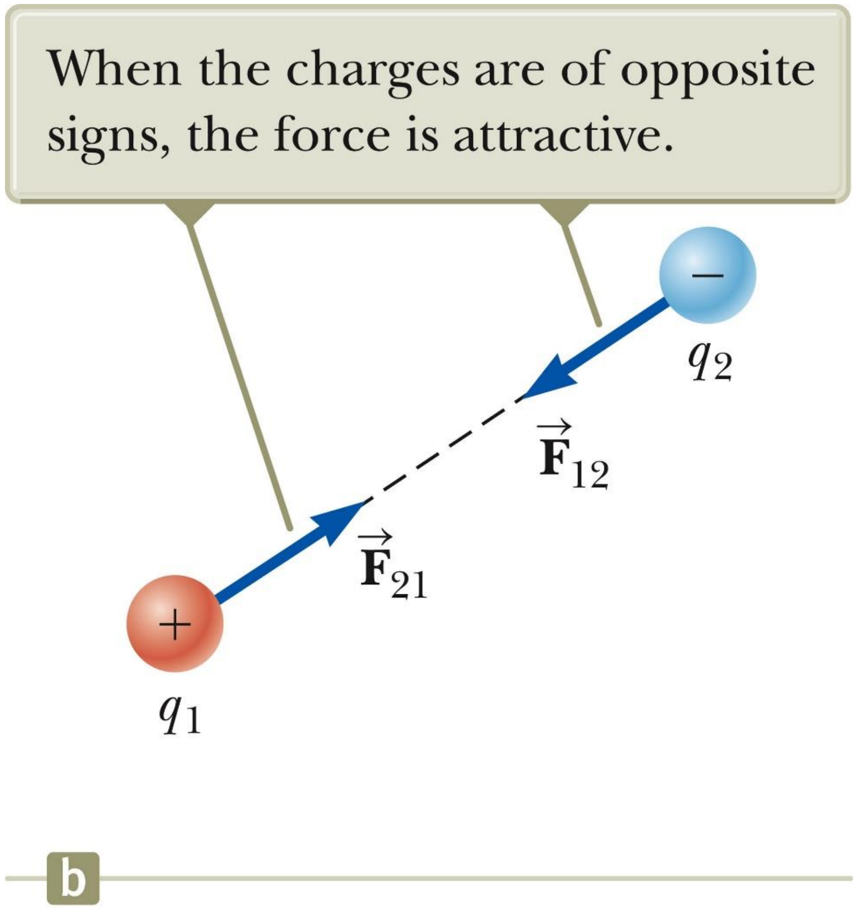
With like signs for the charges, the product $q_1 q_2$ is positive and the force is repulsive.

Vector Nature of Electrical Forces, 3

Two point charges are separated by a distance *r.*

The unlike charges produce an attractive force between them.

With unlike signs for the charges, the product q_1q_2 is negative and the force is attractive.



A Final Note about Directions

The sign of the product of q_1q_2 gives the *relative* direction of the force between q_1 and $q_{2.}$

The *absolute* direction is determined by the actual location of the charges.

Multiple Charges

The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present.

- Remember to add the forces *as vectors.*

The resultant force on q_1 is the vector sum of all the forces exerted on it by other charges.

For example, if four charges are present, the resultant force on one of these equals the vector sum of the forces exerted on it by each of the other charges.

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

Multiple Charges, Example

Example 23.2 **Find the Resultant Force**

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.00\ \mu C$, $q_2 = -2.00\ \mu C$, and $a = 0.100$ m. Find the resultant force exerted on q_3.

SOLUTION

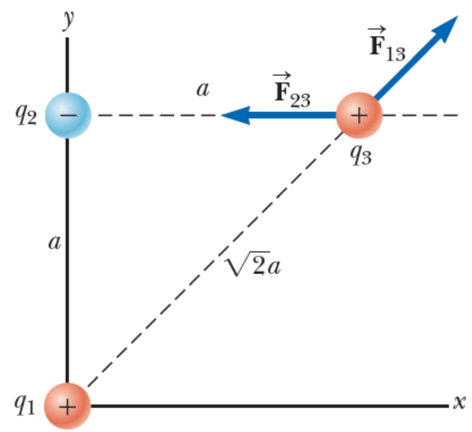


Figure 23.7 (Example 23.2) The force exerted by q_1 on q_3 is $\vec{F}_{13}$. The force exerted by q_2 on q_3 is $\vec{F}_{23}$. The resultant force $\vec{F}_3$ exerted on q_3 is the vector sum $\vec{F}_{13} + \vec{F}_{23}$.

Multiple Charges, Example (cont'd)

Use Equation 23.1 to find the magnitude of $\vec{\mathbf{F}}_{23}$:

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(2.00 \times 10^{-6}\ \mathrm{C})(5.00 \times 10^{-6}\ \mathrm{C})}{(0.100\ \mathrm{m})^2} = 8.99\ \mathrm{N}$$

Find the magnitude of the force $\vec{\mathbf{F}}_{13}$:

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}\,a)^2}$$

$$= (8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.00 \times 10^{-6}\ \mathrm{C})(5.00 \times 10^{-6}\ \mathrm{C})}{2(0.100\ \mathrm{m})^2} = 11.2\ \mathrm{N}$$

Find the x and y components of the force $\vec{\mathbf{F}}_{13}$:

$$F_{13x} = (11.2\ \mathrm{N}) \cos 45.0^\circ = 7.94\ \mathrm{N}$$

$$F_{13y} = (11.2\ \mathrm{N}) \sin 45.0^\circ = 7.94\ \mathrm{N}$$

Find the components of the resultant force acting on q_3:

$$F_{3x} = F_{13x} + F_{23x} = 7.94\ \mathrm{N} + (-8.99\ \mathrm{N}) = -1.04\ \mathrm{N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.94\ \mathrm{N} + 0 = 7.94\ \mathrm{N}$$

Express the resultant force acting on q_3 in unit-vector form:

$$\vec{\mathbf{F}}_3 = \boxed{(-1.04\,\hat{\mathbf{i}} + 7.94\,\hat{\mathbf{j}})\ \mathrm{N}}$$

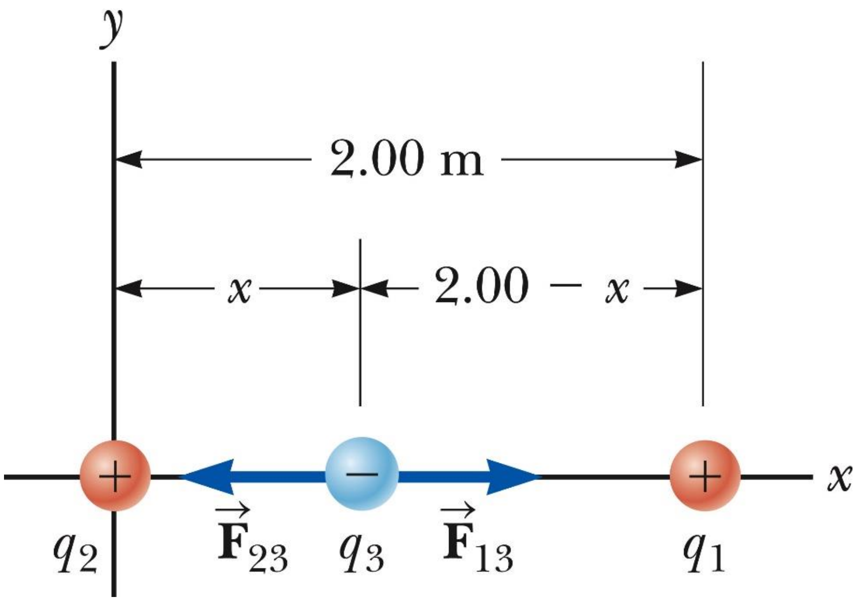
Zero Resultant Force, Example

Where is the resultant force equal to zero?

- The magnitudes of the individual forces will be equal.
- Directions will be opposite.

Will result in a quadratic

Choose the root that gives the forces in opposite directions.



Analyze Write an expression for the net force on charge q_3 when it is in equilibrium:

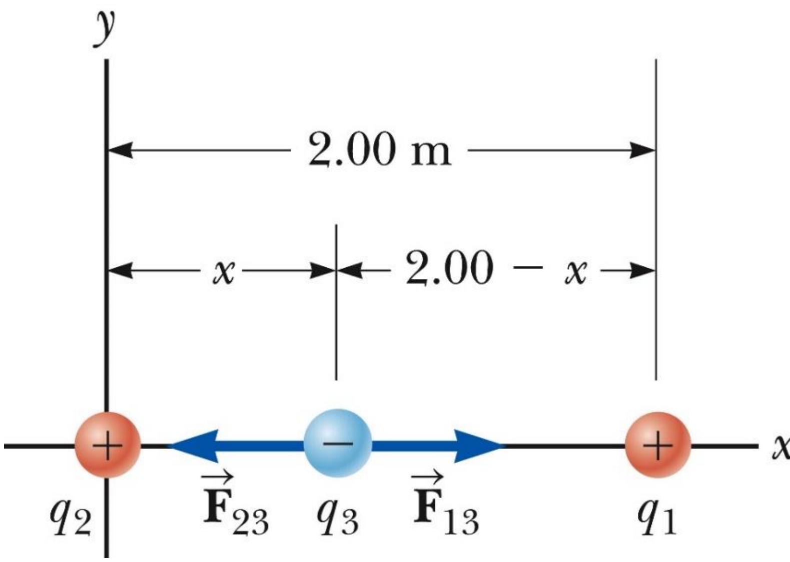
$$\vec{F}_3 = \vec{F}_{23} + \vec{F}_{13} = -k_e \frac{|q_2||q_3|}{x^2}\hat{\mathbf{i}} + k_e \frac{|q_1||q_3|}{(2.00 - x)^2}\hat{\mathbf{i}} = 0$$

Move the second term to the right side of the equation and set the coefficients of the unit vector $\hat{\mathbf{i}}$ equal:

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

continued

Zero Resultant Force, Example (cont'd)



Eliminate k_e and $|q_3|$ and rearrange the equation: $(2.00 - x)^2|q_2| = x^2|q_1|$

Take the square root of both sides of the equation: $(2.00 - x)\sqrt{|q_2|} = \pm x\sqrt{|q_1|}$

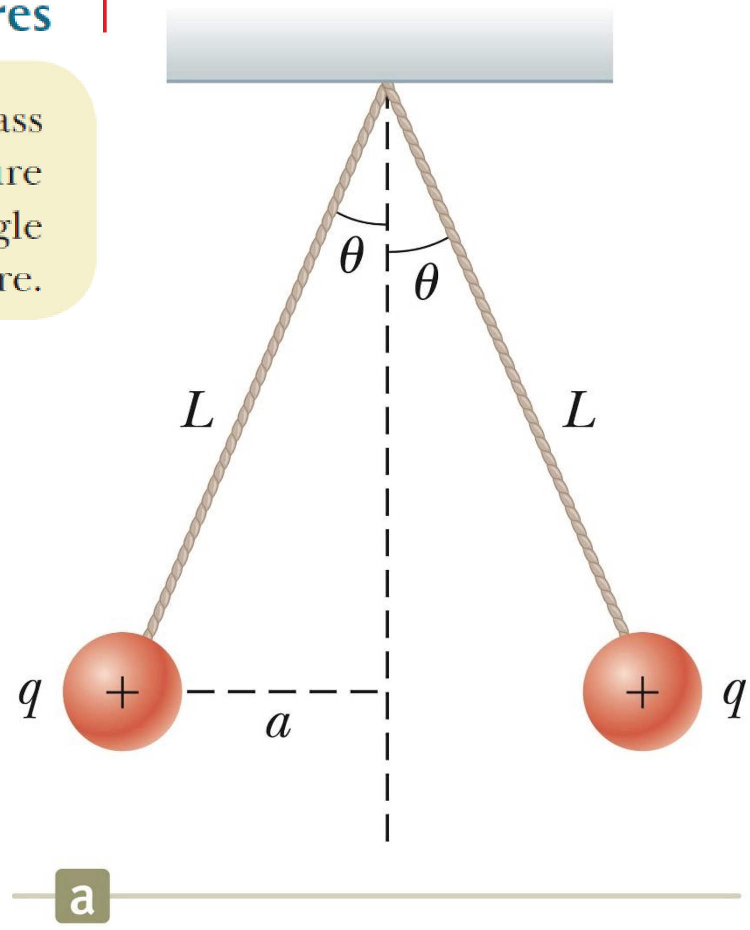
Solve for x: $$x = \frac{2.00\sqrt{|q_2|}}{\sqrt{|q_2|} \pm \sqrt{|q_1|}}$$

Substitute numerical values, choosing the plus sign: $$x = \frac{2.00\sqrt{6.00 \times 10^{-6}\ \mathrm{C}}}{\sqrt{6.00 \times 10^{-6}\ \mathrm{C}} + \sqrt{15.0 \times 10^{-6}\ \mathrm{C}}} = 0.775\ \mathrm{m}$$

Electrical Force with Other Forces, Example

Example 23.4 **Find the Charge on the Spheres**

Two identical small charged spheres, each having a mass of 3.00×10^{-2} kg, hang in equilibrium as shown in Figure 23.9a. The length L of each string is 0.150 m, and the angle θ is 5.00°. Find the magnitude of the charge on each sphere.



The spheres are in equilibrium.

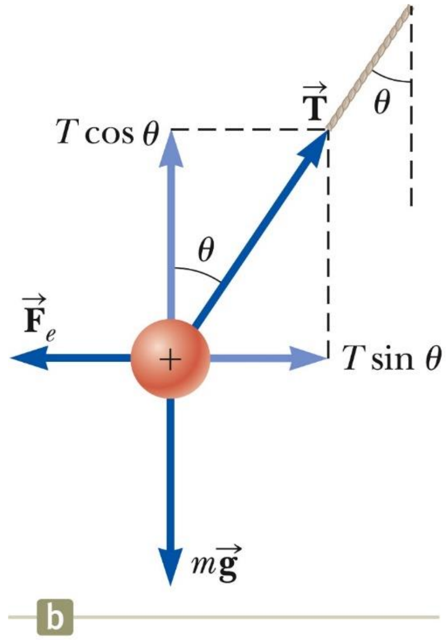
Since they are separated, they exert a repulsive force on each other.

- Charges are like charges

Model each sphere as a particle in equilibrium.

Proceed as usual with equilibrium problems, noting one force is an electrical force.

Electrical Force with Other Forces, Example cont.



The force diagram includes the components of the tension, the electrical force, and the weight.

Solve for $|q|$

If the charge of the spheres is not given, you cannot determine the sign of q, only that they both have same sign.

From the particle in equilibrium model, set the net force on the left-hand sphere equal to zero for each component:

(1) $\sum F_x = T\sin\theta - F_e = 0 \rightarrow T\sin\theta = F_e$

(2) $\sum F_y = T\cos\theta - mg = 0 \rightarrow T\cos\theta = mg$

Divide Equation (1) by Equation (2) to find F_e:

(3) $\tan\theta = \dfrac{F_e}{mg} \rightarrow F_e = mg\tan\theta$

$$F_e = mg\tan\theta = (3.0 \times 10^{-2}\text{ kg})(9.80\text{ m/s}^2)\tan(5.0°) = 2.6 \times 10^{-2}\text{ N}$$

$$a = L\sin\theta = (0.15\text{ m})\sin(5.0°) = 0.013\text{ m}$$

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6 \times 10^{-2}\text{ N})(0.026\text{ m})^2}{8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 1.96 \times 10^{-15}\text{ C}^2$$

$$|q| = \boxed{4.4 \times 10^{-8}\text{ C}}$$

Electric Field – Introduction

The electric force is a field force.

Field forces can act through space.

- The effect is produced even with no physical contact between objects.

Faraday developed the concept of a field in terms of electric fields.

Electric Field – Definition

An **electric field** is said to exist in the region of space around a charged object.

- This charged object is the **source charge.**

When another charged object, the **test charge**, enters this electric field, an electric force acts on it.

Electric Field – Definition, cont

The electric field is defined as the electric force on the test charge per unit charge.

The electric field vector, $\vec{\mathbf{E}}$, at a point in space is defined as the electric force acting on a positive test charge, q_o, placed at that point divided by the test charge:

$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_o}$$

Electric Field, Notes

$\vec{\mathbf{E}}$ is the field produced by some charge or charge distribution, separate from the test charge.

The existence of an electric field is a property of the source charge.

- The presence of the test charge is not necessary for the field to exist.

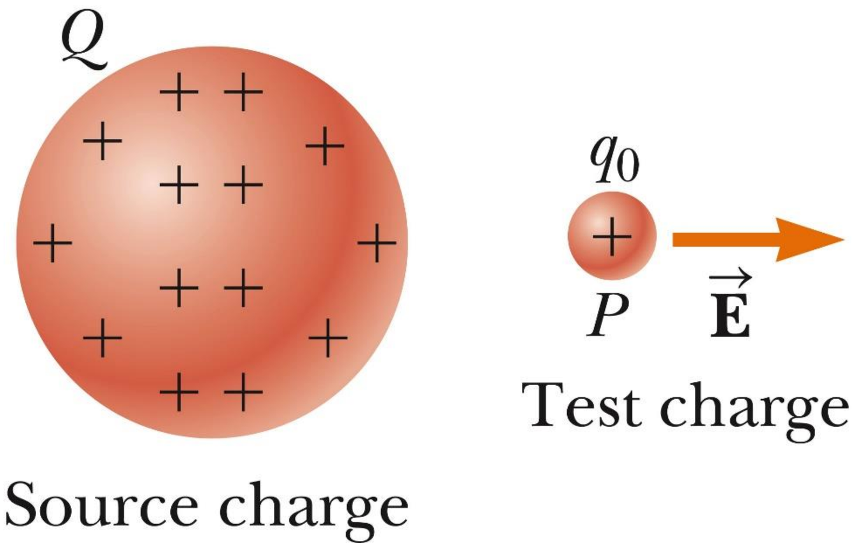
The test charge serves as a detector of the field.

Electric Field Notes, Final

The direction of $\vec{\mathbf{E}}$ is that of the force on a positive test charge.

The SI units of $\vec{\mathbf{E}}$ are N/C.

We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.



Relationship Between F and E

If an arbitrary charge *q* is placed in an electric field $\vec{\mathbf{E}}$, it experiences an electric force given by

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$$

- This is valid for a point charge only.
- One of zero size
- For larger objects, the field may vary over the size of the object.

If *q* is positive, the force and the field are in the same direction.

If *q* is negative, the force and the field are in opposite directions.

Electric Field, Vector Form

Remember Coulomb's law, between the source and test charges, can be expressed as

$$\vec{\mathbf{F}}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_o} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

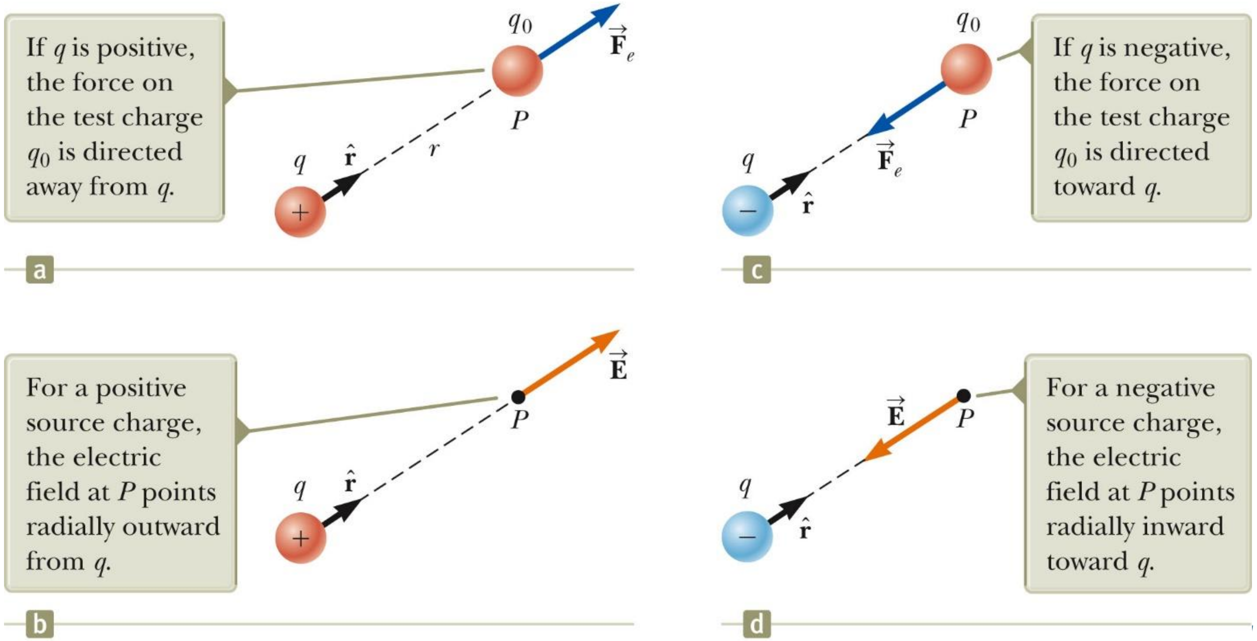
More About Electric Field Direction

a) q is positive, the force is directed away from q.

b) The direction of the field is also away from the positive source charge.

c) q is negative, the force is directed toward q.

d) The field is also toward the negative source charge.



Electric Fields from Multiple Charges

At any point *P*, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

where r_i is the distance from the ith source charge q_i to the point P and $\hat{\mathbf{r}}_i$ is a unit vector directed from q_i toward P.

Electric Fields from Multiple Charges, Example

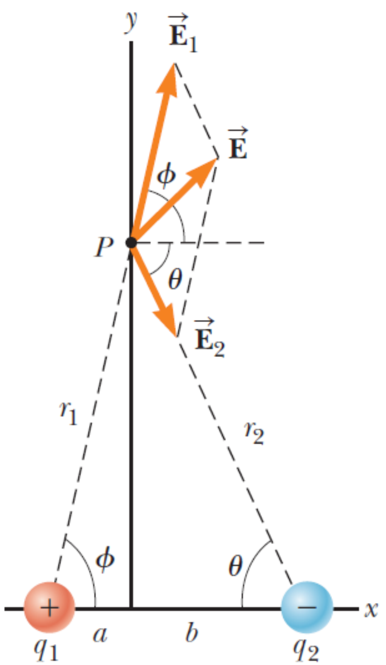
Example 23.6 Electric Field Due to Two Charges

Charges q_1 and q_2 are located on the x axis, at distances a and b, respectively, from the origin as shown in Figure 23.12.

(A) Find the components of the net electric field at the point P, which is at position $(0, y)$.

SOLUTION

Figure 23.12 (Example 23.6) The total electric field $\vec{\mathbf{E}}$ at P equals the vector sum $\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$, where $\vec{\mathbf{E}}_1$ is the field due to the positive charge q_1 and $\vec{\mathbf{E}}_2$ is the field due to the negative charge q_2.



Electric Fields from Multiple Charges, Example (cont'd)

Analyze Find the magnitude of the electric field at P due to charge q_1:

$$E_1 = k_e \frac{|q_1|}{r_1^{\,2}} = k_e \frac{|q_1|}{a^2 + y^2}$$

Find the magnitude of the electric field at P due to charge q_2:

$$E_2 = k_e \frac{|q_2|}{r_2^{\,2}} = k_e \frac{|q_2|}{b^2 + y^2}$$

Write the electric field vectors for each charge in unit-vector form:

$$\vec{\mathbf{E}}_1 = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi\, \hat{\mathbf{i}} + k_e \frac{|q_1|}{a^2 + y^2} \sin\phi\, \hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2 = k_e \frac{|q_2|}{b^2 + y^2} \cos\theta\, \hat{\mathbf{i}} - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta\, \hat{\mathbf{j}}$$

Write the components of the net electric field vector:

$$(1)\quad E_x = E_{1x} + E_{2x} = \boxed{k_e \frac{|q_1|}{a^2 + y^2} \cos\phi + k_e \frac{|q_2|}{b^2 + y^2} \cos\theta}$$

$$(2)\quad E_y = E_{1y} + E_{2y} = \boxed{k_e \frac{|q_1|}{a^2 + y^2} \sin\phi - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta}$$

Electric Field – Continuous Charge Distribution

The distances between charges in a group of charges may be much smaller than the distance between the group and a point of interest.

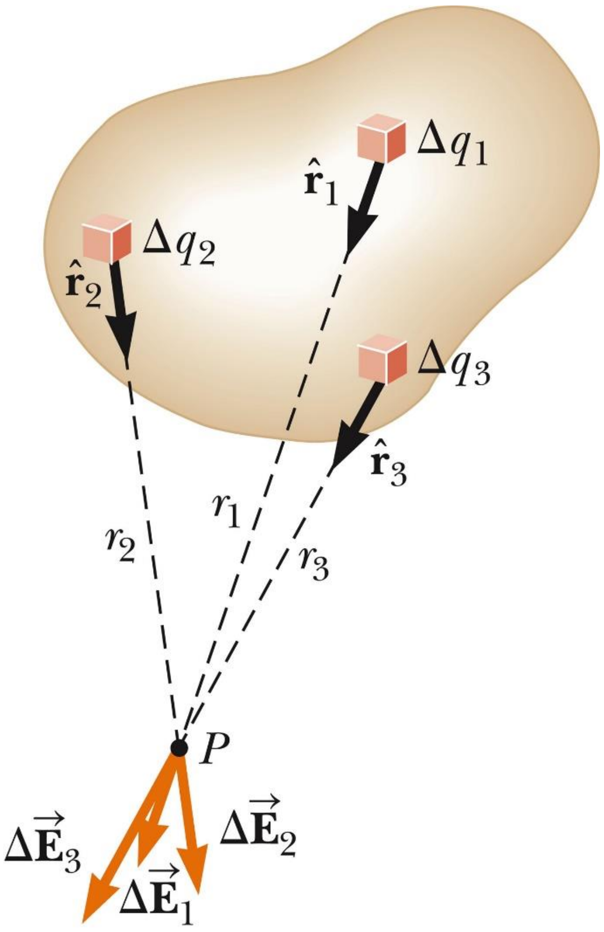
In this situation, the system of charges can be modeled as continuous.

The system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

Electric Field – Continuous Charge Distribution, cont

Procedure:

- Divide the charge distribution into small elements, each of which contains Δq.
- Calculate the electric field due to one of these elements at point *P.*
- Evaluate the total field by summing the contributions of all the charge elements.



Electric Field – Continuous Charge Distribution, equations

For the individual charge elements

$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

Because the number of elements is very large and the charge distribution is continuous

$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Charge Densities

Volume charge density: when a charge is distributed evenly throughout a volume

- $\rho \equiv Q / V$ with units C/m^3

Surface charge density: when a charge is distributed evenly over a surface area

- $\sigma \equiv Q / A$ with units C/m^2

Linear charge density: when a charge is distributed along a line

- $\lambda \equiv Q / \ell$ with units C/m

Amount of Charge in a Small Volume

If the charge is nonuniformly distributed over a volume, surface, or line, the amount of charge, dq, is given by

- For the volume: $dq = \rho \, dV$
- For the surface: $dq = \sigma \, dA$
- For the length element: $dq = \lambda \, d\ell$

Problem-Solving Strategy

Conceptualize

- Establish a mental representation of the problem.
- Image the electric field produced by the charges or charge distribution.

Categorize

- Individual charge?
- Group of individual charges?
- Continuous distribution of charges?

Problem-Solving Strategy, cont

Analyze

- **Analyzing a group of individual charges**:
  - Use the superposition principle, find the fields due to the individual charges at the point of interest and then add them as vectors to find the resultant field.
  - Be careful with the manipulation of vector quantities.
- **Analyzing a continuous charge distribution:**
  - The vector sums for evaluating the total electric field at some point must be replaced with vector integrals.
  - Divide the charge distribution into infinitesimal pieces, calculate the vector sum by integrating over the entire charge distribution.
- **Symmetry:**
  - Take advantage of any symmetry to simplify calculations.

Problem Solving Hints, final

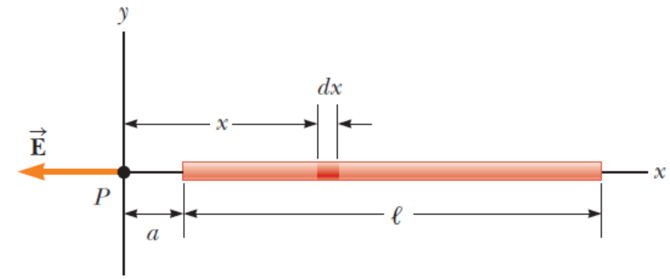
Finalize

- Check to see if the electric field expression is consistent with your mental representation.
- Check to see if the solution reflects any symmetry present.
- Image varying parameters to see if the mathematical result changes in a reasonable way.

Example – Charged Rod

Example 23.7 The Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end (Fig. 23.15).



Analyze Let's assume the rod is lying along the x axis, dx is the length of one small segment, and dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$.

Find the magnitude of the electric field at P due to one segment of the rod having a charge dq:

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

Find the total field at P using[4] Equation 23.11:

$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

Noting that k_e and $\lambda = Q/\ell$ are constants and can be removed from the integral, evaluate the integral:

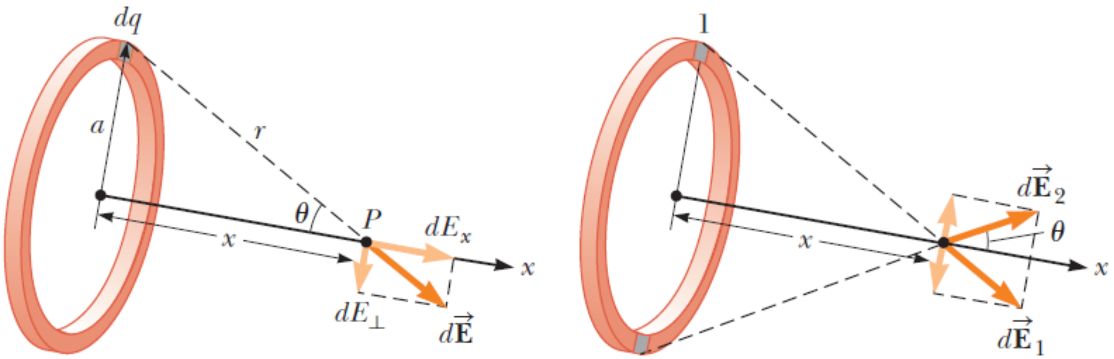
$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$(1) \quad E = k_e \frac{Q}{\ell} \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

Example – Charged Ring

Example 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.16a).



Analyze Evaluate the parallel component of an electric field contribution from a segment of charge dq on the ring:

$$(1)\quad dE_x = k_e \frac{dq}{r^2}\cos\theta = k_e \frac{dq}{a^2 + x^2}\cos\theta$$

From the geometry in Figure 23.16a, evaluate $\cos\theta$:

$$(2)\quad \cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$$

Substitute Equation (2) into Equation (1):

$$dE_x = k_e \frac{dq}{a^2 + x^2}\left[\frac{x}{(a^2 + x^2)^{1/2}}\right] = \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Integrate over the circumference of the ring to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

$$(3)\quad E = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}}\, Q}$$

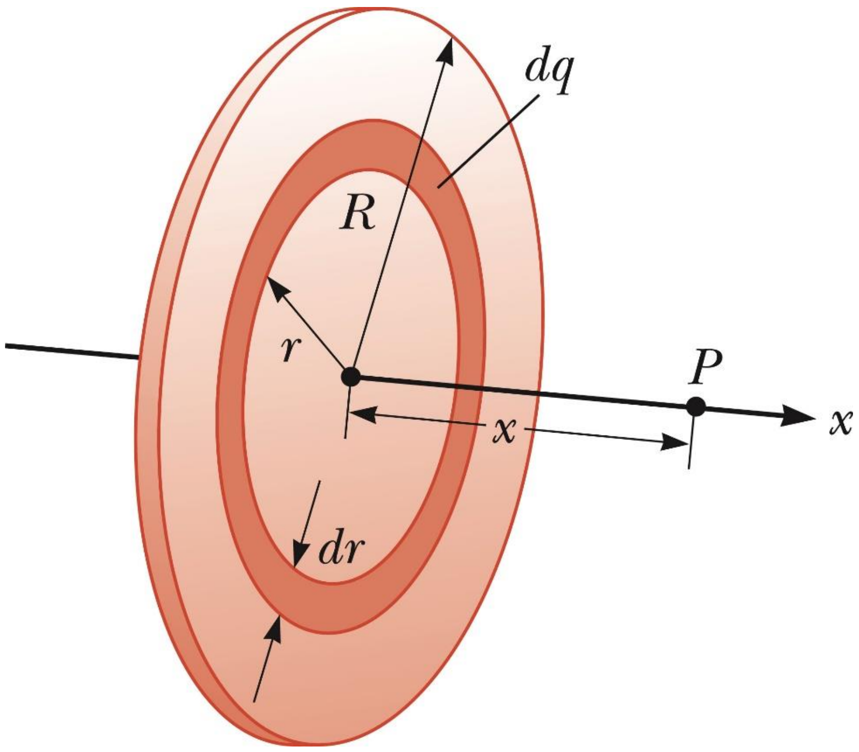
Example – Charged Disk

The disk has a radius *R* and a uniform charge density *σ*.

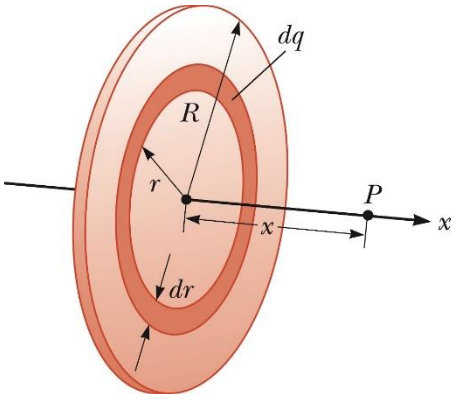
Choose *dq* as a ring of radius *r.*

The ring has a surface area $2\pi r\, dr$.

Integrate to find the total field.



Example – Charged Disk



Analyze Find the amount of charge dq on the surface area of a ring of radius r and width dr as shown in Figure 23.17:

$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

Use this result in the equation given for E_x in Example 23.8 (with a replaced by r and Q replaced by dq) to find the field due to the ring:

$$dE_x = \frac{k_e x}{(r^2 + x^2)^{3/2}}(2\pi\sigma r\, dr)$$

To obtain the total field at P, integrate this expression over the limits $r = 0$ to $r = R$, noting that x is a constant in this situation:

$$E_x = k_e x \pi\sigma \int_0^R \frac{2r\, dr}{(r^2 + x^2)^{3/2}}$$

$$= k_e x \pi\sigma \int_0^R (r^2 + x^2)^{-3/2} d(r^2)$$

$$= k_e x \pi\sigma \left[\frac{(r^2 + x^2)^{-1/2}}{-1/2}\right]_0^R = \boxed{2\pi k_e \sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$

Electric Field Lines

Field lines give us a means of representing the electric field pictorially.

The electric field vector is tangent to the electric field line at each point.

- The line has a direction that is the same as that of the electric field vector.

The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

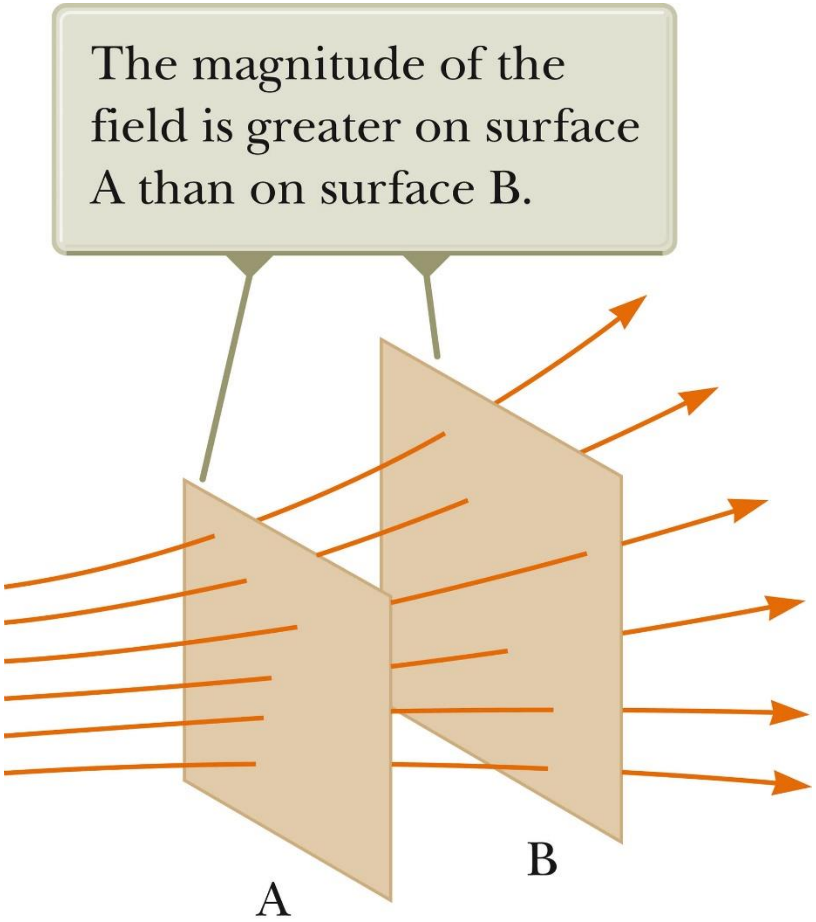
Electric Field Lines, General

The density of lines through surface A is greater than through surface B.

The magnitude of the electric field is greater on surface A than B.

The lines at different locations point in different directions.

- This indicates the field is nonuniform.



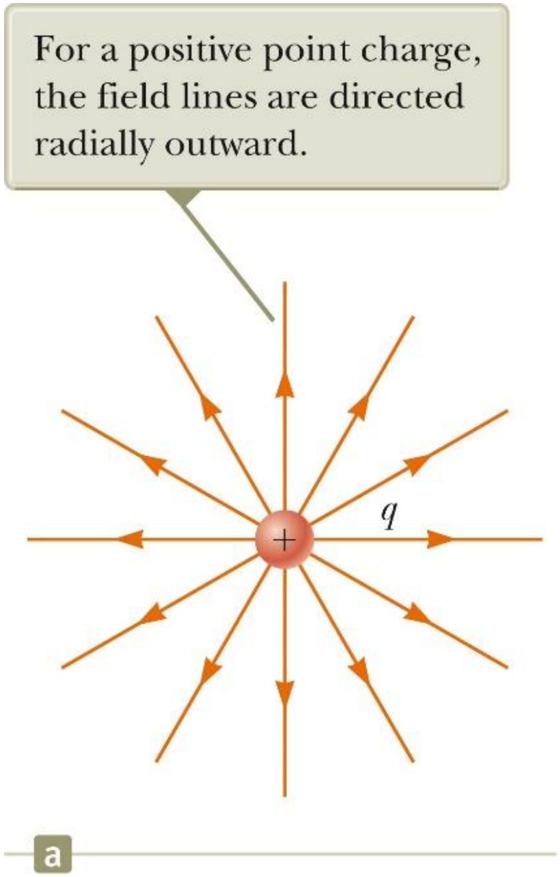
Electric Field Lines, Positive Point Charge

The field lines radiate outward in all directions.

- In three dimensions, the distribution is spherical.

The lines are directed away from the source charge.

- A positive test charge would be repelled away from the positive source charge.

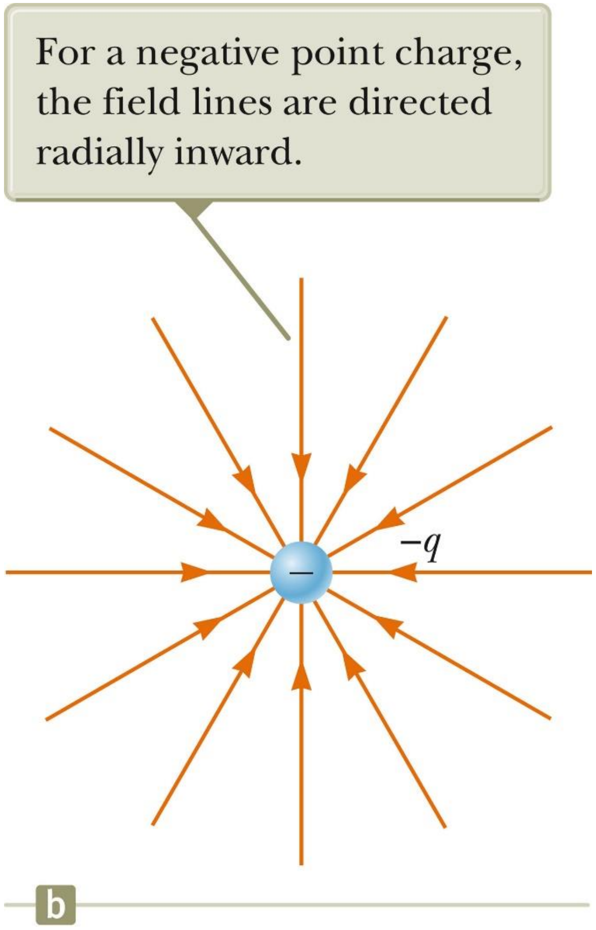


Electric Field Lines, Negative Point Charge

The field lines radiate inward in all directions.

The lines are directed toward the source charge.

- A positive test charge would be attracted toward the negative source charge.



Electric Field Lines – Rules for Drawing

The lines must begin on a positive charge and terminate on a negative charge.

- In the case of an excess of one type of charge, some lines will begin or end infinitely far away.

The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

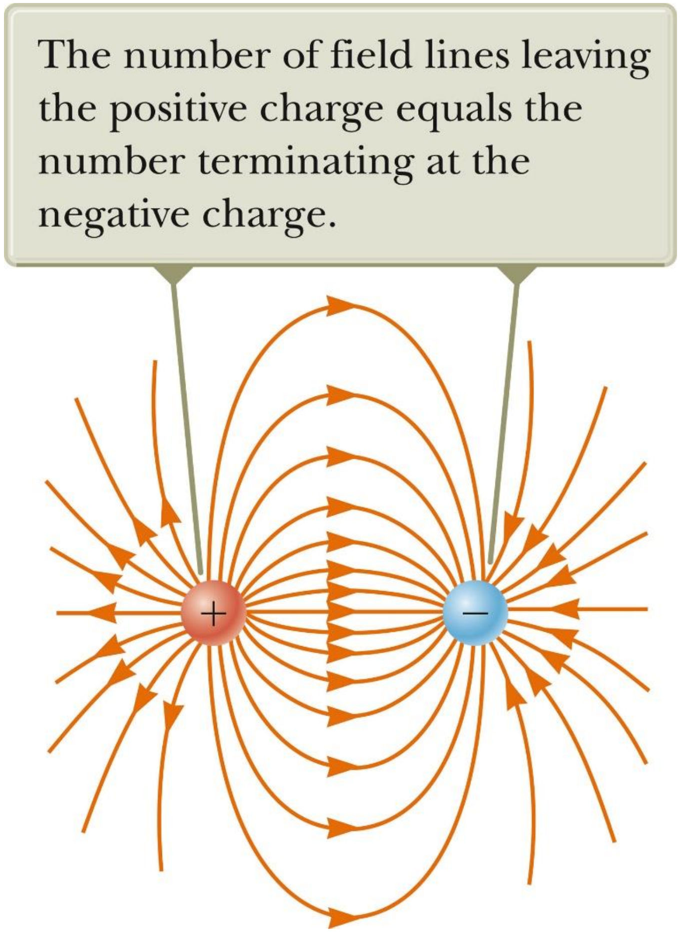
No two field lines can cross.

Remember field lines are **not** material objects, they are a pictorial representation used to qualitatively describe the electric field.

Electric Field Lines – Dipole

The charges are equal and opposite.

The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.



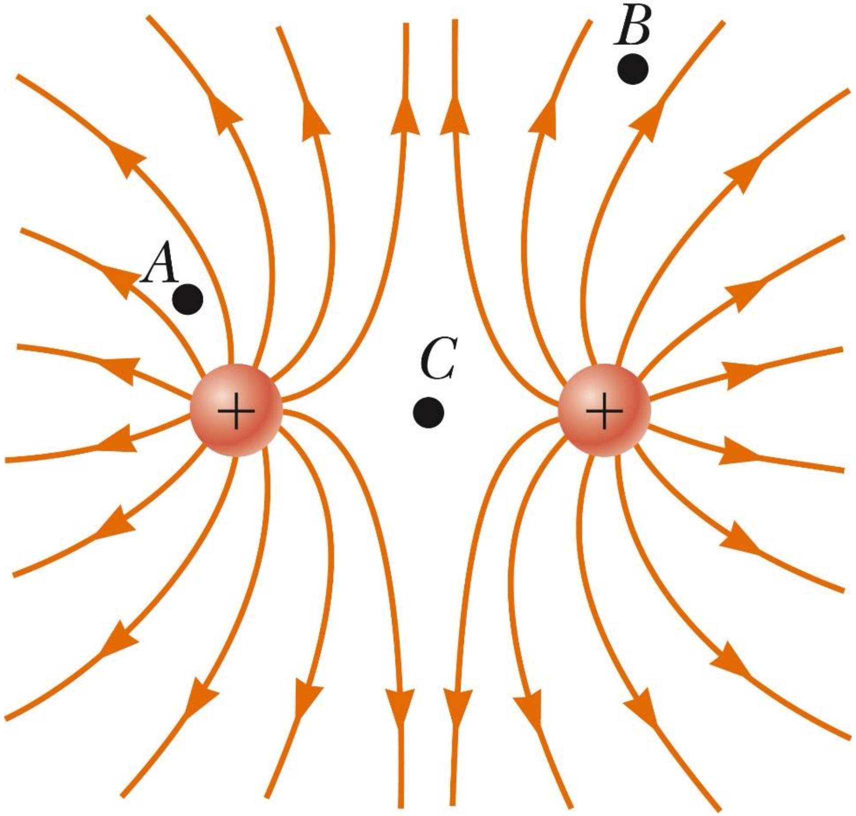
Electric Field Lines – Like Charges

The charges are equal and positive.

The same number of lines leave each charge since they are equal in magnitude.

At a great distance, the field is approximately equal to that of a single charge of 2*q.*

Since there are no negative charges available, the field lines end infinitely far away.

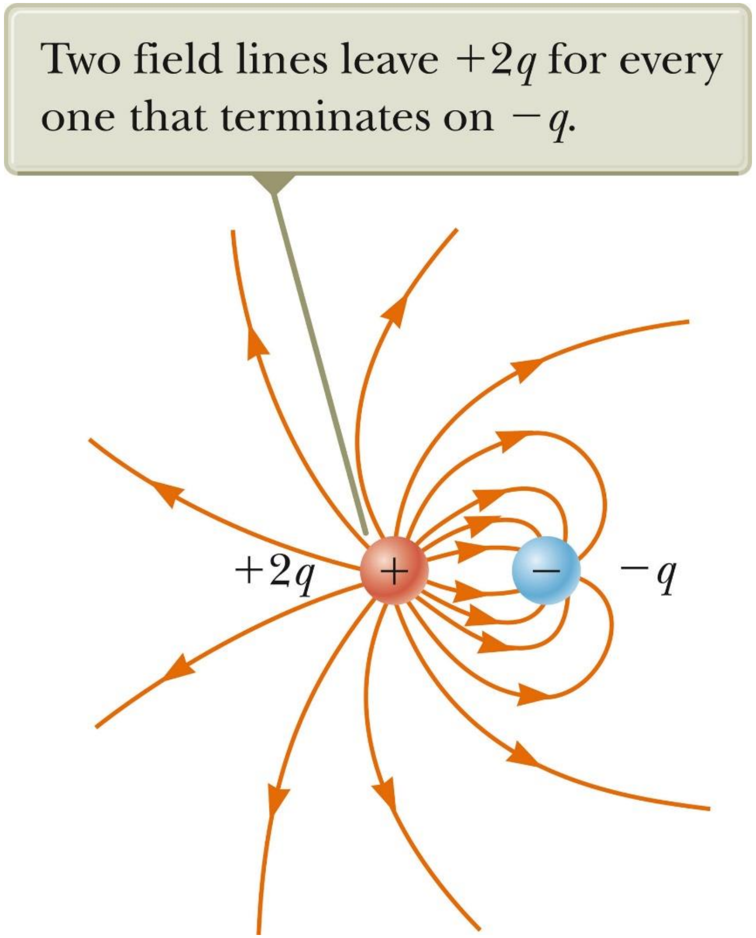


Electric Field Lines, Unequal Charges

The positive charge is twice the magnitude of the negative charge.

Two lines leave the positive charge for each line that terminates on the negative charge.

At a great distance, the field would be approximately the same as that due to a single charge of $+q$.



Motion of Charged Particles

When a charged particle is placed in an electric field, it experiences an electrical force.

If this is the only force on the particle, it must be the net force.

The net force will cause the particle to accelerate according to Newton's second law.

Motion of Particles, cont

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \frac{q\vec{\mathbf{E}}}{m}$$

m is the particle mass.

If the field is uniform (constant in magnitude and direction), then the acceleration is constant.

The particle under constant acceleration model can be applied to the motion of the particle.

- The electric force causes a particle to move according to the models of forces and motion.

If the particle has a positive charge, its acceleration is in the direction of the field.

If the particle has a negative charge, its acceleration is in the direction opposite of the electric field.

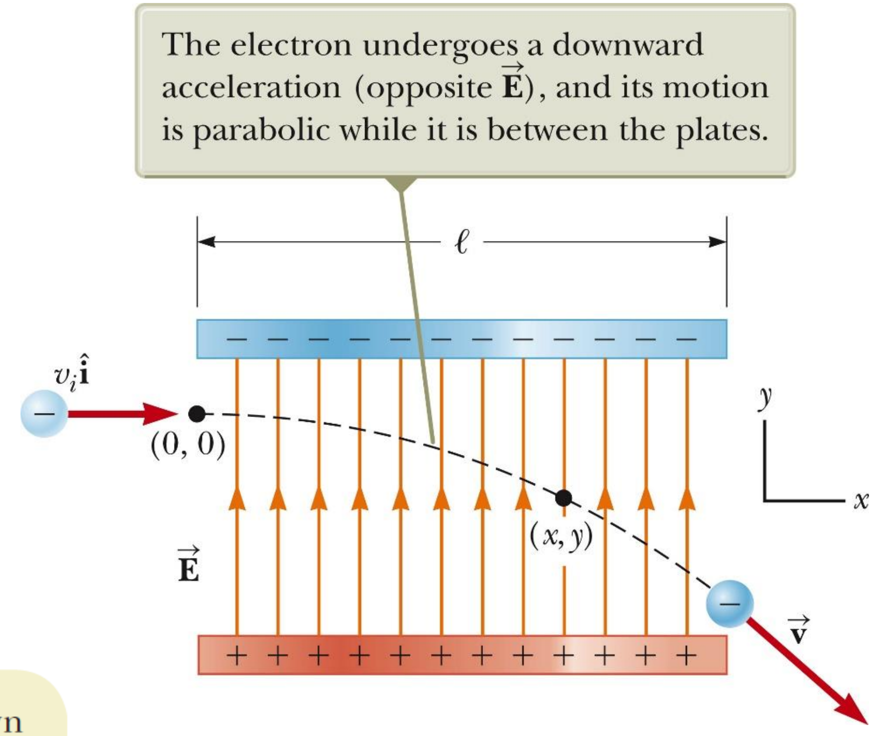
Electron in a Uniform Field, Example

The electron is projected horizontally into a uniform electric field.

The electron undergoes a downward acceleration.

- It is negative, so the acceleration is opposite the direction of the field.

Its motion is parabolic while between the plates.



Example 23.11 **An Accelerated Electron** AM

An electron enters the region of a uniform electric field as shown in Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

Electron in a Uniform Field, Example (cont'd)

Analyze From the particle in a field model, we know that the direction of the electric force on the electron is downward in Figure 23.24, opposite the direction of the electric field lines. From the particle under a net force model, therefore, the acceleration of the electron is downward.

The particle under a net force model was used to develop Equation 23.12 in the case in which the electric force on a particle is the only force. Use this equation to evaluate the y component of the acceleration of the electron:

$$a_y = -\frac{eE}{m_e}$$

Substitute numerical values:

$$a_y = -\frac{(1.60 \times 10^{-19}\ \mathrm{C})(200\ \mathrm{N/C})}{9.11 \times 10^{-31}\ \mathrm{kg}} = \boxed{-3.51 \times 10^{13}\ \mathrm{m/s^2}}$$

(B) Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Analyze Solve Equation 2.7 for the time at which the electron arrives at the right edges of the plates:

$$x_f = x_i + v_x t \quad \rightarrow \quad t = \frac{x_f - x_i}{v_x}$$

Substitute numerical values:

$$t = \frac{\ell - 0}{v_x} = \frac{0.100\ \mathrm{m}}{3.00 \times 10^{6}\ \mathrm{m/s}} = \boxed{3.33 \times 10^{-8}\ \mathrm{s}}$$

Electron in a Uniform Field, Example (cont'd)

(C) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

Analyze Use Equation 2.16 to describe the position of the particle at any time t:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Substitute numerical values:

$$y_f = 0 + 0 + \frac{1}{2}(-3.51 \times 10^{13}\ \text{m/s}^2)(3.33 \times 10^{-8}\ \text{s})^2$$

$$= -0.019\,5\ \text{m} = \boxed{-1.95\ \text{cm}}$$