PHYS143
Physics for Engineers
Tutorial - Chapter 29 - Solutions

Question 1

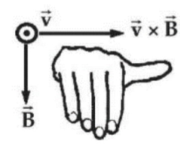
A proton moves perpendicular to a uniform magnetic field $\vec{\mathbf{B}}$ at a speed of 1.00×10^{7} m/s and experiences an acceleration of 2.00×10^{13} m/s^2 in the positive x direction when its velocity is in the positive z direction. Determine the magnitude and direction of the field. $(m_p = 1.67 \times 10^{-27})$.

$$F = ma = \left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{13}\ \text{m/s}^2\right) = 3.34\times10^{-14}\ \text{N} = qvB\sin 90°$$

$$B = \frac{F}{qv} = \frac{3.34\times10^{-14}\ \text{N}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{7}\ \text{m/s}\right)} = 2.09\times10^{-2}\ \text{T} = 20.9\times10^{-3}\ \text{T}$$
$$= 20.9\ \text{mT}$$

From ANS. FIG. P29.11, the right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction. Therefore,

$$\boxed{\vec{\mathbf{B}} = -20.9\ \hat{\mathbf{j}}\ \text{mT}}$$



ANS. FIG. P29.11

Question 2

A cyclotron designed to accelerate protons has an outer radius of 0.350 m. The protons are emitted nearly at rest from a source at the center and are accelerated through 600 V each time they cross the gap between the dees. The dees are between the poles of an electromagnet where the field is 0.800 T. (a) Find the cyclotron frequency for the protons in this cyclotron. Find (b) the speed at which protons exit the cyclotron and (c) their maximum kinetic energy. (d) How many revolutions does a proton make in the cyclotron? (e) For what time interval does the proton accelerate?

Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by

$$\sum F = ma:$$

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$|q|B = m\frac{v}{r} = m\omega$$

(a) $\omega = \dfrac{|q|B}{m} = \dfrac{(1.60\times10^{-19}\ \text{C})(0.800\ \text{N}\cdot\text{s/C}\cdot\text{m})}{(1.67\times10^{-27}\ \text{kg})}\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)$

$= \boxed{7.66\times10^{7}\ \text{rad/s}}$

(b) $v = \omega r = (7.66\times10^{7}\ \text{rad/s})(0.350\ \text{m})\left(\dfrac{1}{1\ \text{rad}}\right) = \boxed{2.68\times10^{7}\ \text{m/s}}$

(c) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(1.67\times10^{-27}\ \text{kg})(2.68\times10^{7}\ \text{m/s})^2\left(\dfrac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right)$

$= \boxed{3.76\times10^{6}\ \text{eV}}$

(d) The kinetic energy of the proton changes by $\Delta K = e\Delta V = e(600\ \text{V}) = 600$ eV twice during each revolution, so the number of revolutions is

$$\frac{3.76\times10^{6}\ \text{eV}}{2(600\ \text{eV})} = \boxed{3.13\times10^{3}\ \text{revolutions}}$$

(e) From $\theta = \omega\,\Delta t$,

$$\Delta t = \frac{\theta}{\omega} = \frac{3.13\times10^{3}\ \text{rev}}{7.66\times10^{7}\ \text{rad/s}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = \boxed{2.57\times10^{-4}\ \text{s}}$$

Question 3

A velocity selector consists of electric and magnetic fields described by the expressions $\vec{\mathbf{E}} = E\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = B\hat{\mathbf{j}}$ with $B = 15.0$ mT. Find the value of E such that a 750-eV electron moving in the negative x direction is undeflected. $(m_e = 9.11\times10^{-31})$

For the electron to travel undeflected, we require $F_B = F_e$, so

$$qvB = qE$$

where $v = \sqrt{\dfrac{2K}{m}}$ and K is kinetic energy of the electron. Then,

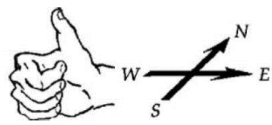
$$E = vB = \sqrt{\frac{2K}{m}}B = \sqrt{\frac{2(750\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}}(0.015\,0\ \text{T})$$

$= \boxed{244\ \text{kV/m}}$

Question 4

A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are (a) the direction and (b) the magnitude of the minimum magnetic field needed to lift this wire vertically upward?

(a) The magnetic force must be upward to lift the wire. For current in the south direction, the magnetic field must be $\boxed{\text{east}}$ to produce an upward force, as shown by the right-hand rule in the figure.



(b) $F_B = ILB\sin\theta$ with $F_B = F_g = mg$

$$mg = ILB\sin\theta \quad \text{so} \quad \frac{m}{L}g = IB\sin\theta \quad \rightarrow \quad B = \frac{m}{L}\frac{g}{I\sin\theta}$$

$$B = \frac{m}{L}\frac{g}{I\sin\theta} = \left(\frac{0.500\times10^{-3}\text{ kg}}{1.00\times10^{-2}\text{ m}}\right)\left(\frac{9.80\text{ m/s}^2}{(2.00\text{ A})\sin 90.0^\circ}\right) = \boxed{0.245\text{ T}}$$

Question 5

A wire is formed into a circle having a diameter of 10.0 cm and is placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find (a) the maximum torque on the wire and (b) the range of potential energies of the wire–field system for different orientations of the circle.

(a) From $\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}}$, so the maximum magnitude of the torque on the loop is

$$\tau = \left|\vec{\mu}\times\vec{\mathbf{B}}\right| = \mu B\sin\theta = NIAB\sin\theta$$

$$\begin{aligned}\tau_{\max} &= NIAB\sin 90.0^\circ \\ &= 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) \\ &= \boxed{118\ \mu\text{N}\cdot\text{m}}\end{aligned}$$

(b) The potential energy is given by

$$U = -\vec{\mu}\cdot\vec{\mathbf{B}}$$

so $-\mu B \le U \le +\mu B$

Now, since

$$\begin{aligned}\mu B &= (NIA)B \\ &= 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) \\ &= 118\ \mu\text{J}\end{aligned}$$

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$.