Chapter 33

Alternating Current Circuits

Alternating Current Circuits

Electrical appliances in the house use alternating current (AC) circuits.

If an AC source applies an alternating voltage to a series circuit containing resistor, inductor, and capacitor, what are the amplitude and time characteristics of the alternating current?

Other devices will be discussed

- Transformers
- Power transmission
- Electrical filters

AC Circuits

An AC circuit consists of a combination of circuit elements and a power source.

The power source provides an alternating voltage, Δv.

Notation note:

- Lower case symbols will indicate instantaneous values.
- Capital letters will indicate fixed values.

AC Voltage

The output of an AC power source is sinusoidal and varies with time according to the following equation:

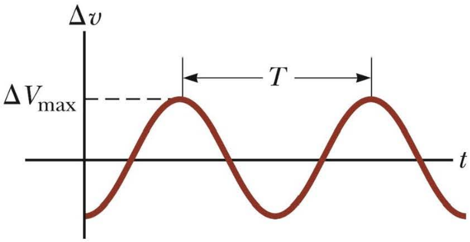
- $\Delta v = \Delta V_{max} \sin \omega t$
  - Δv is the instantaneous voltage.
  - ΔV_{max} is the maximum output voltage of the source.
    - Also called the **voltage amplitude**
  - ω is the angular frequency of the AC voltage.

The angular frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- f is the frequency of the source.
- T is the period of the source.

The voltage is positive during one half of the cycle and negative during the other half.



Resistors in an AC Circuit

Consider a circuit consisting of an AC source and a resistor.

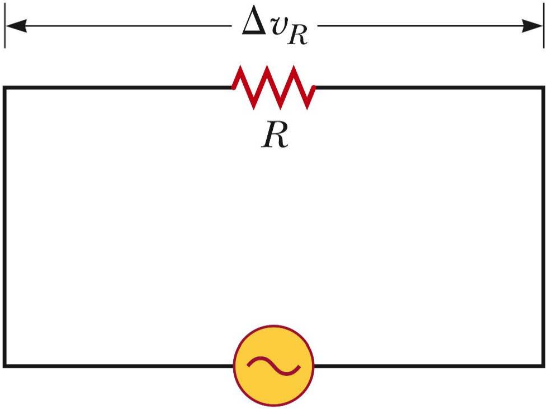
The AC source is symbolized by 

The instantaneous current in the resistor is

$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{\max}}{R} \sin \omega t = I_{\max} \sin \omega t$$

The instantaneous voltage across the resistor is also given as

$$\Delta v_R = i_R R = I_{\max} R \sin \omega t$$



Resistors in an AC Circuit, final

The graph shows the current through and the voltage across the resistor.

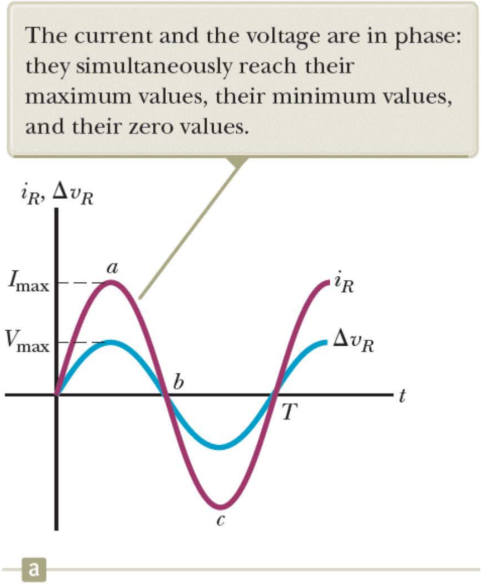
The current and the voltage reach their maximum values at the same time.

The current and the voltage are said to be *in phase.*

For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.

The direction of the current has no effect on the behavior of the resistor.

Resistors behave essentially the same way in both DC and AC circuits.



Phasor Diagram

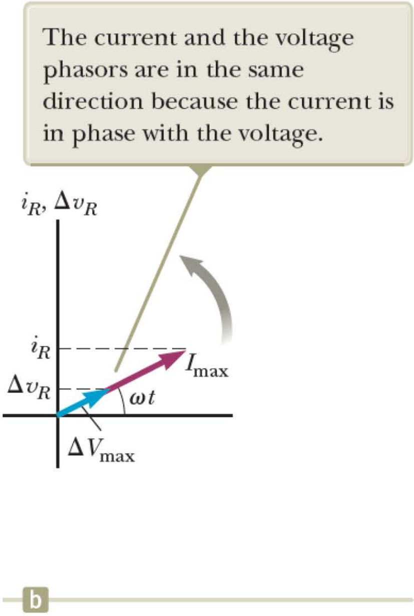
To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used.

A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents.

The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.

The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

The horizontal coordinate does not represent anything.



rms Current and Voltage

The average current in one cycle is zero.

Resistors experience a temperature increase which depends on the magnitude of the current, but not the direction of the current.

The power is related to the square of the current.

The rms current is the average of importance in an AC circuit.

- *rms* stands for root mean square

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

Alternating voltages can also be discussed in terms of rms values.

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

Power

The rate at which electrical energy is delivered to a resistor in the circuit is given by

- $P = i^2 R$
  - i is the *instantaneous current.*
  - The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value.
  - The maximum current occurs for a small amount of time.
- The average power delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$

This equation states that an alternating current whose maximum value is 2.00 A delivers to a resistor the same power as a direct current that has a value of (0.707)(2.00 A) = 1.41 A.

Notes About rms Values

rms values are used when discussing alternating currents and voltages because

- AC ammeters and voltmeters are designed to read rms values.
- Many of the equations that will be used have the same form as their DC counterparts.

Example

When an AC source is connected across a 12.0-Ω resistor, the rms current in the resistor is 8.00 A. Find (a) the rms voltage across the resistor, (b) the peak voltage of the source, (c) the maximum current in the resistor, and (d) the average power delivered to the resistor.

(a) The rms voltage across the resistor is given by

$$\Delta V_{R,\mathrm{rms}} = I_{\mathrm{rms}} R = (8.00\ \mathrm{A})(12.0\ \Omega) = \boxed{96.0\ \mathrm{V}}$$

(b) $\Delta V_{R,\max} = \sqrt{2}\left(\Delta V_{R,\mathrm{rms}}\right) = \sqrt{2}\,(96.0\ \mathrm{V}) = \boxed{136\ \mathrm{V}}$

(c) $I_{\max} = \sqrt{2} I_{\mathrm{rms}} = \sqrt{2}\,(8.00\ \mathrm{A}) = \boxed{11.3\ \mathrm{A}}$

(d) $P_{\mathrm{avg}} = I_{\mathrm{rms}}^2 R = (8.00\ \mathrm{A})^2 (12.0\ \Omega) = \boxed{768\ \mathrm{W}}$

Example

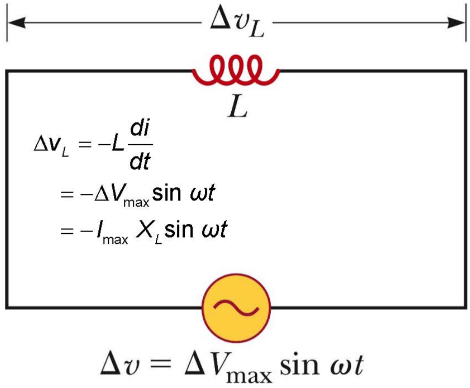
(a) What is the resistance of a lightbulb that uses an average power of 75.0 W when connected to a 60.0-Hz power source having a maximum voltage of 170 V? (b) What If? What is the resistance of a 100-W lightbulb?

$$\Delta V_{\text{rms}} = \frac{170\ \text{V}}{\sqrt{2}} = 120\ \text{V}$$

$$P = \frac{(\Delta V_{\text{rms}})^2}{R} \quad \rightarrow \quad R = \frac{(120\ \text{V})^2}{75.0\ \text{W}} = \boxed{193\ \Omega}$$

$$R = \frac{(120\ \text{V})^2}{100\ \text{W}} = \boxed{144\ \Omega}$$

Inductors in an AC Circuit



Kirchhoff's loop rule can be applied and gives:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max} \sin \omega t$$

The equation obtained from Kirchhoff's loop rule can be solved for the current

$$i_L = \frac{\Delta V_{\max}}{L}\int \sin \omega t \; dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_{max} = \frac{\Delta V_{max}}{\omega L}$$

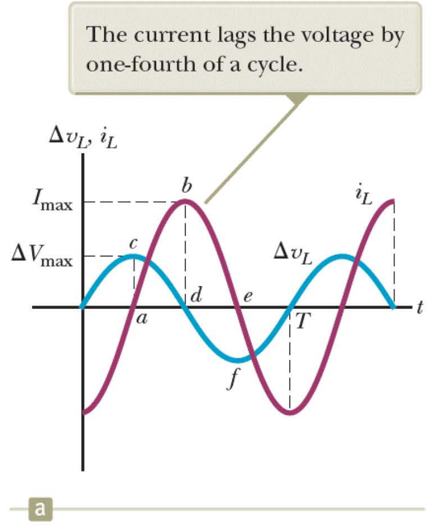
This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $(\pi/2)$ rad = 90°.

Phase Relationship of Inductors in an AC Circuit

The current is a maximum when the voltage across the inductor is zero.

- The current is momentarily not changing

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$).

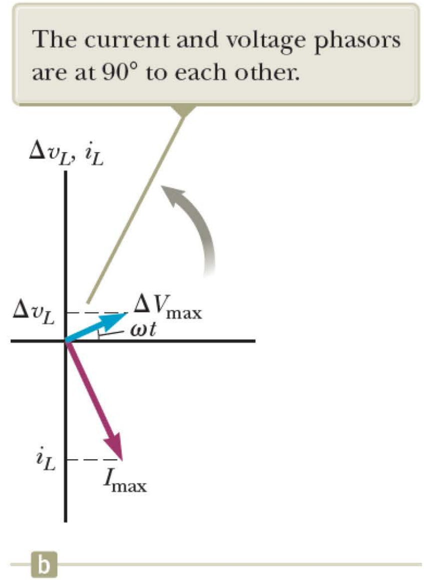


Phasor Diagram for an Inductor

The phasors are at 90° with respect to each other.

This represents the phase difference between the current and voltage.

Specifically, the current lags behind the voltage by 90°.



Inductive Reactance

The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.

Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.

The factor is the **inductive reactance** and is given by:

$$X_L = \omega L$$

Current can be expressed in terms of the inductive reactance:

$$I_{max} = \frac{\Delta V_{max}}{X_L} \text{ or } I_{rms} = \frac{\Delta V_{rms}}{X_L}$$

As the frequency increases, the inductive reactance increases

- This is consistent with Faraday's Law:
  - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

Example

An AC source has an output rms voltage of 78.0 V at a frequency of 80.0 Hz. If the source is connected across a 25.0-mH inductor, what are (a) the inductive reactance of the circuit, (b) the rms current in the circuit, and (c) the maximum current in the circuit?

$$X_L = 2\pi f L = 2\pi(80.0\ \text{Hz})(25.0 \times 10^{-3}\ \text{H}) = \boxed{12.6\ \Omega}$$

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{78.0\ \text{V}}{X_L} = \boxed{6.21\ \text{A}}$$

$$I_{\max} = \sqrt{2}\, I_{\text{rms}} = \sqrt{2}(6.21\ \text{A}) = \boxed{8.78\ \text{A}}$$

Capacitors in an AC Circuit

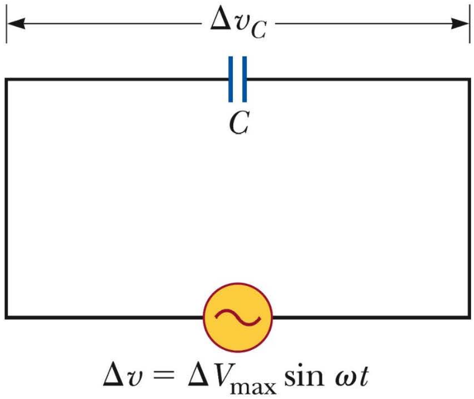
The circuit contains a capacitor and an AC source.

Kirchhoff's loop rule gives:

$\Delta v + \Delta v_c = 0$ and so

$\Delta v = \Delta v_C = \Delta V_{\max} \sin \omega t$

- Δv_c is the instantaneous voltage across the capacitor.



The charge is $q = C\Delta V_{max} \sin \omega t$

The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

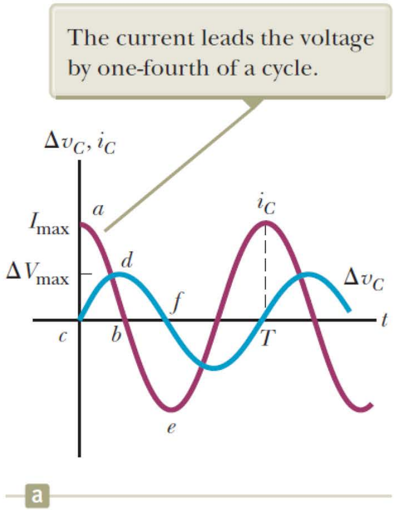
$$\text{or } i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The current is $\pi/2$ rad = 90° out of phase with the voltage

More About Capacitors in an AC Circuit

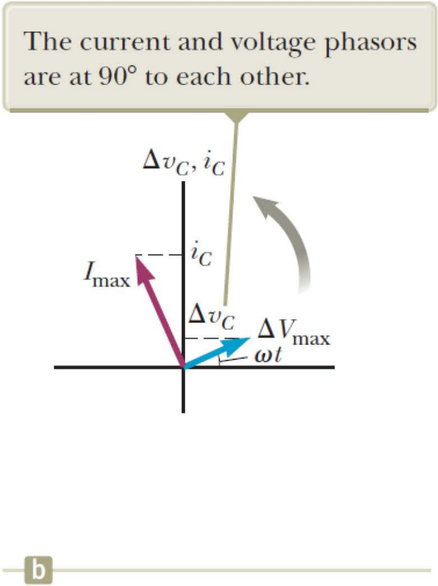
The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

The current leads the voltage by 90º.



Phasor Diagram for Capacitor

The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°.



Capacitive Reactance

The maximum current in the circuit occurs at cos $\omega t = 1$ which gives

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

Voltage Across a Capacitor

The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$.

$$X_C = \frac{1}{\omega C}$$

As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.

As the frequency approaches zero, X_C approaches infinity and the current approaches zero.

- This would act like a DC voltage and the capacitor would act as an open circuit.

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Example

What maximum current is delivered by an AC source with ΔV_{max} = 48.0 V and f = 90.0 Hz when connected across a 3.70-μF capacitor?

We combine the steps in the equation

$$I_{max} = \frac{\Delta V_{max}}{X_C} = \Delta V_{max}\omega C = \Delta V_{max}(2\pi f C)$$

$$I_{max} = (48.0\text{ V})(2\pi)(90.0\text{ Hz})\left(3.70\times10^{-6}\text{ F}\right) = \boxed{100\text{ mA}}$$

Example

An AC source with an output rms voltage of 36.0 V at a frequency of 60.0 Hz is connected across a 12.0-μF capacitor. Find (a) the capacitive reactance, (b) the rms current, and (c) the maximum current in the circuit, (d) Does the capacitor have its maximum charge when the current has its maximum value. Explain.

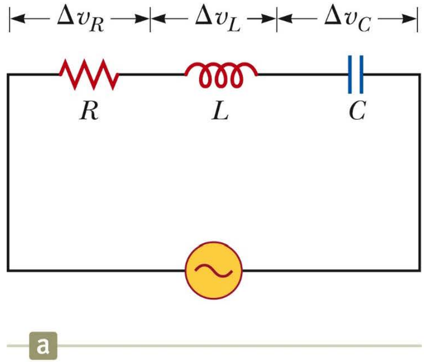
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \text{Hz})(12.0\times10^{-6}\ \text{F})} = \boxed{221\ \Omega}$$

$$I_{\text{rms}} = \frac{\Delta V_{C,\text{rms}}}{X_C} = \frac{36.0\ \text{V}}{221\ \Omega} = \boxed{0.163\ \text{A}}$$

$$I_{\max} = \sqrt{2}\, I_{\text{rms}} = \boxed{0.230\ \text{A}}$$

No. Current leads voltage, and thus charge, by 90° in a capacitor. The current reaches its maximum value one-quarter cycle before the voltage reaches its maximum value. From the definition of capacitance, the capacitor reaches its maximum charge when the voltage across it is also a maximum. Consequently, the maximum charge and the maximum current do not occur at the same time.

The *RLC* Series Circuit



The resistor, inductor, and capacitor can be combined in a circuit.

The current and the voltage in the circuit vary sinusoidally with time.

The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.

The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$.

- φ is the **phase angle** between the current and the applied voltage.

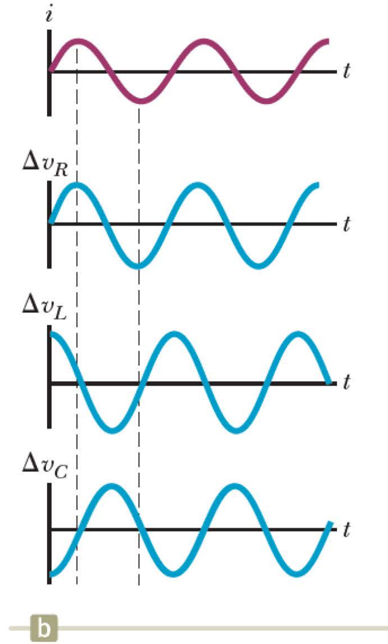
Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.

i and *v* Phase Relationships – Graphical View

The instantaneous voltage across the resistor is in phase with the current.

The instantaneous voltage across the inductor leads the current by 90°.

The instantaneous voltage across the capacitor lags the current by 90°.



i and *v* Phase Relationships – Equations

The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

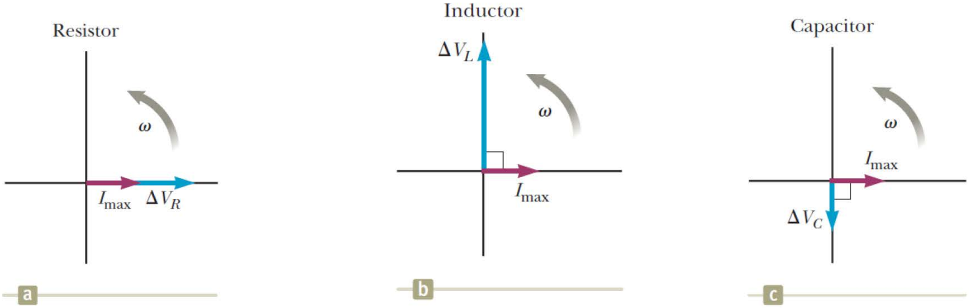
$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

The sum of these voltages must equal the instantaneous voltage from the AC source.

Because of the different phase relationships with the current, they cannot be added directly.

Phasor Diagrams

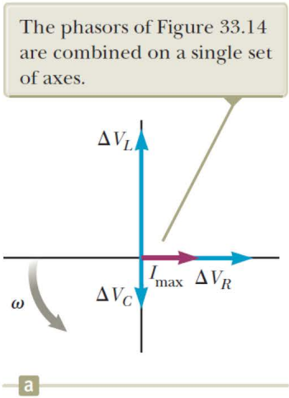


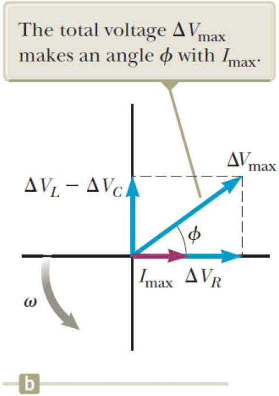
To account for the different phases of the voltage drops, vector techniques are used.

Remember the phasors are rotating vectors

The phasors for the individual elements are shown.

Resulting Phasor Diagram





The individual phasor diagrams can be combined.

Here a single phasor I_{max} is used to represent the current in each element.

- In series, the current is the same in each element.

Vector addition is used to combine the voltage phasors.

ΔV_L and ΔV_C are in opposite directions, so they can be combined.

Their resultant is perpendicular to $\Delta V_{R.}$

The resultant of all the individual voltages across the individual elements is $\Delta v_{max.}$

- This resultant makes an angle of φ with the current phasor $I_{max.}$

Section 33.5

Total Voltage in RLC Circuits

From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} = \sqrt{(I_{\max}R)^2 + (I_{\max}X_L - I_{\max}X_C)^2}$$

$$\Delta V_{\max} = I_{\max}\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance

The current in an *RLC* circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

Z is called the **impedance** of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

- Impedance has units of ohms

Phase Angle

The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The phase angle can be positive or negative and determines the nature of the circuit.

Determining the Nature of the Circuit

If ϕ is positive

- $X_L > X_C$ (which occurs at high frequencies)
- The current lags the applied voltage.
- The circuit is *more inductive than capacitive.*

If ϕ is negative

- $X_L < X_C$ (which occurs at low frequencies)
- The current leads the applied voltage.
- The circuit is *more capacitive than inductive.*

If ϕ is zero

- $X_L = X_C$
- The circuit is *purely resistive.*

Example

At what frequency does the inductive reactance of a 57.0-μH inductor equal the capacitive reactance of a 57.0-μF capacitor?

From the definitions of inductive and capacitive reactance, $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$.

Setting these equal and solving for the angular frequency gives

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(57.0\times10^{-6}\right)\left(57.0\times10^{-6}\right)}} = 1.75\times10^{4}\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \boxed{2.79\ \text{kHz}}$$

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Example

An RLC circuit consists of a 150-Ω resistor, a 21.0-μF capacitor, and a 460-mH inductor connected in series with a 120-V, 60.0-Hz power supply. (a) What is the phase angle between the current and the applied voltage? (b) Which reaches its maximum earlier, the current or the voltage?

The reactance of the inductor is

$$X_L = 2\pi f L = 2\pi\left(60.0\ \mathrm{s}^{-1}\right)(0.460\ \mathrm{H}) = 173\ \Omega$$

The reactance of the capacitor is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi\left(60.0\ \mathrm{s}^{-1}\right)\left(21.0 \times 10^{-6}\ \mathrm{F}\right)} = 126\ \Omega$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{173\ \Omega - 126\ \Omega}{150\ \Omega}\right) = \boxed{17.4^\circ}$$

Since $X_L > X_C$, ϕ is positive, so $\boxed{\text{voltage leads the current}}$.

Example

An inductor (L = 400 mH), a capacitor (C = 4.43 μF), and a resistor (R = 500 Ω) are connected in series. A 50.0-Hz AC source produces a peak current of 250 mA in the circuit. (a) Calculate the required peak voltage ΔV_{max}. (b) Determine the phase angle by which the current leads or lags the applied voltage.

(a) We first find the impedances of the inductor and the capacitor:

$$X_L = \omega L = 2\pi(50.0)\left(400\times10^{-3}\right) = 126\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)\left(4.43\times10^{-6}\right)} = 719\ \Omega$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{500^2 + (126-719)^2} = 776\ \Omega$$

$$\Delta V_{\max} = I_{\max} Z = \left(250\times10^{-3}\right)(776) = \boxed{194\ \text{V}}$$

(b) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{126-719}{500}\right) = \boxed{-49.9^\circ}$ $\boxed{\text{current leads the voltage}}$

Power in an AC Circuit

The average power delivered by the AC source is converted to internal energy in the resistor.

- $P_{avg} = \frac{1}{2} I_{max} \Delta V_{max} \cos \phi = I_{rms} \Delta V_{rms} \cos \phi$
- $\cos \phi$ is called the **power factor** of the circuit

We can also find the average power in terms of R.

- $P_{avg} = I^2_{rms} R$

When the load is purely resistive, $\phi = 0$ and $\cos \phi = 1$

- $P_{avg} = I_{rms} \Delta V_{rms}$

The average power delivered by the source is converted to internal energy in the resistor.

No power losses are associated with pure capacitors and pure inductors in an AC circuit.

Example

A series RLC circuit has a resistance of 45.0 Ω and an impedance of 75.0 Ω. What average power is delivered to this circuit when ΔV_{rms} = 210 V?

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}, \quad \Rightarrow \quad \left(X_L - X_C\right) = \sqrt{Z^2 - R^2}$$

$$\left(X_L - X_C\right) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1^\circ$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\ \text{V}}{75.0\ \Omega} = 2.80\ \text{A}$$

$$P = \left(\Delta V_{\text{rms}}\right) I_{\text{rms}} \cos\phi = (210\ \text{V})(2.80\ \text{A})\cos(53.1^\circ) = \boxed{353\ \text{W}}$$

Example

An AC voltage of the form Δv = 100 sin 1000t, where Δv is in volts and t is in seconds, is applied to a series RLC circuit. Assume the resistance is 400 Ω, the capacitance is 5.00 μF, and the inductance is 0.500 H. Find the average power delivered to the circuit.

$$\Delta V_{\text{rms}} = \frac{100\ \text{V}}{\sqrt{2}} = 70.7\ \text{V}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(1\,000\ \text{s}^{-1})(5.00 \times 10^{-6}\ \text{F})} = 200\ \Omega$$

X_L = ω L = (1 000 s–1)(0.500 H) = 500 Ω

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(400\ \Omega)^2 + (500\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

$$\Rightarrow \quad I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{70.7\ \text{V}}{500\ \Omega} = 0.141\ \text{A}$$

$$P_{\text{avg}} = I_{\text{rms}}^2 R = (0.141\ \text{A})^2(400\ \Omega) = \boxed{8.00\ \text{W}}$$

Resonance in an AC Circuit

Resonance occurs at the frequency ω_o where the rms current has its maximum value.

- To achieve maximum current, the impedance must have a minimum value.
- This occurs when $X_L = X_C$
- Solving for the frequency gives

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$I_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.

The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.

At the resonance frequency, the current is in phase with the applied voltage.

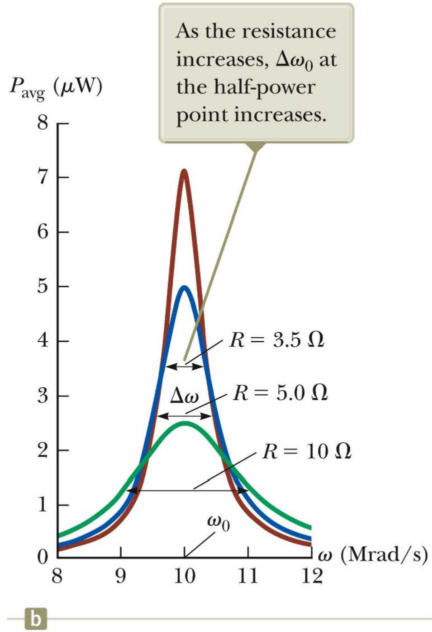
Power as a Function of Frequency

Power can be expressed as a function of frequency in an *RLC* circuit.

$$P_{av} = \frac{(\Delta V_{rms})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_o^2)^2}$$

This shows that at resonance, the average power is a maximum with a value of

$$(\Delta V_{\mathrm{rms}})^2/R$$



Quality Factor

The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q.

$Q = \omega_o / \Delta\omega = (\omega_o L) / R$

- $\Delta\omega$ is the width of the curve, measured between the two values of ω for which P_{avg} has half its maximum value.
    - These points are called the *half-power points.*

A high-*Q* circuit responds only to a narrow range of frequencies.

- Narrow peak

A low-*Q* circuit can detect a much broader range of frequencies.

A radio's receiving circuit is an important application of a resonant circuit.

A Resonating Series RLC Circuit, Example

Consider a series *RLC* circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $\Delta V_{rms} = 20.0$ V, and $\omega = 5\,000\ \text{s}^{-1}$. Determine the value of the capacitance for which the current is a maximum.

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_0^2 L}$$

$$C = \frac{1}{(5.00 \times 10^3\ \text{s}^{-1})^2(20.0 \times 10^{-3}\ \text{H})} = \boxed{2.00\ \mu\text{F}}$$

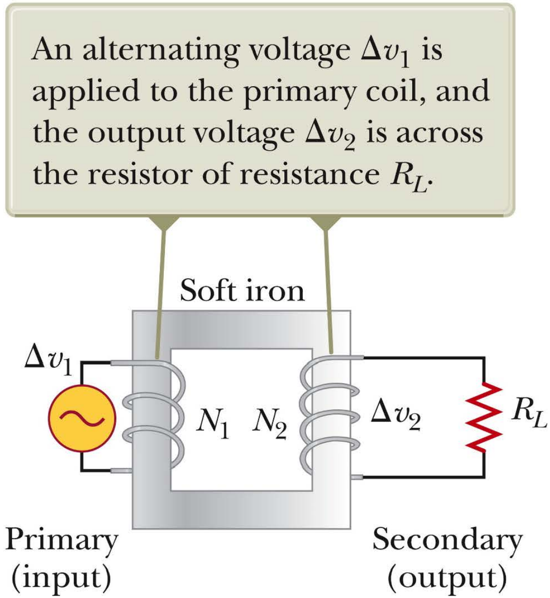
Transformers

An **AC transformer** consists of two coils of wire wound around a core of iron.

The side connected to the input AC voltage source is called the *primary* and has N_1 turns.

The other side, called the secondary, is connected to a resistor and has N_2 turns.

The core is used to increase the magnetic flux and to provide a medium for the flux to pass from one coil to the other.



Transformers, cont.

Assume an ideal transformer, one in which the energy losses in the windings and the core are zero.

- Typical transformers have power efficiencies of 90% to 99%.

In the primary,

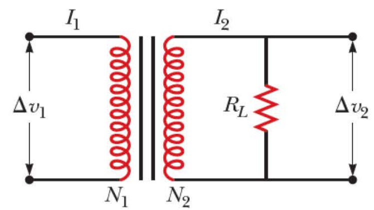
$$\Delta v_1 = -N_1 \frac{d\Phi_B}{dt}$$

The rate of change of the flux is the same for both coils.

The voltage across the secondary is

$$\Delta v_2 = -N_2 \frac{d\Phi_B}{dt}$$

Transformers – Step-up and Step-down



The voltages are related by

$$\Delta v_2 = \frac{N_2}{N_1}\Delta v_1$$

When $N_2 > N_1$, the transformer is referred to as a **step-up** transformer.

When $N_2 < N_1$, the transformer is referred to as a **step-down** transformer.

The power input into the primary equals the power output at the secondary.

$$I_1 \Delta V_1 = I_2 \Delta V_2$$

The equivalent resistance of the load resistance when viewed from the primary is

$$R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

Example

The primary coil of a transformer has N_1 = 350 turns, and the secondary coil has N_2 = 2 000 turns. If the input voltage across the primary coil is Δv = 170 cos ωt, where Δv is in volts and t is in seconds, what rms voltage is developed across the secondary coil?

The rms primary voltage is

$$\Delta V_{1,\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$$

The rms voltage across the bigger coil is

$$\Delta V_{2,\text{rms}} = \left(\frac{N_2}{N_1}\right)\Delta V_{1,\text{rms}} = \left(\frac{2\,000}{350}\right)(120\text{ V}) = \boxed{687\text{ V}}$$

Rectifier

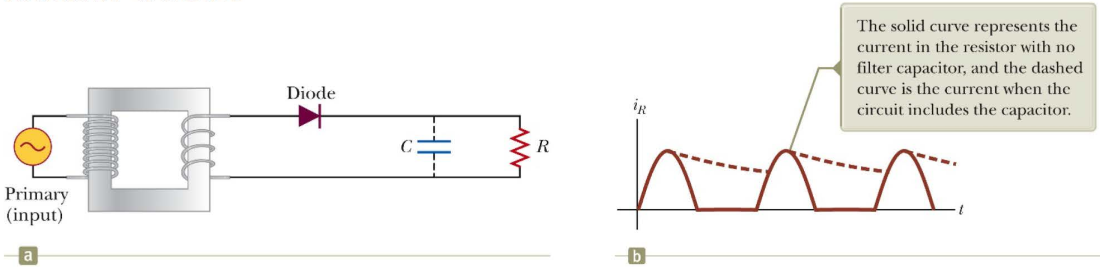
The process of converting alternating current to direct current is called **rectification.**

A **rectifier** is the converting device.

The most important element in a rectifier circuit is the diode.

- A diode is a circuit element that conducts current in one direction but not the other.

Rectifier Circuit



The arrow on the diode (—▶|—) indicates the direction of the current in the diode.

- The diode has low resistance to current flow in this direction.
- It has high resistance to current flow in the opposite direction.

Because of the diode, the alternating current in the load resistor is reduced to the positive portion of the cycle.

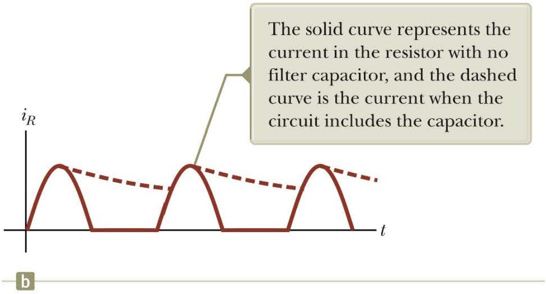
The transformer reduces the 120 V AC to the voltage needed by the device.

- Typically 6 V or 9 V

Half-Wave Rectifier

The solid line in the graph is the result through the resistor.

It is called a ***half-wave rectifier*** because current is present in the circuit during only half of each cycle.



Half-Wave Rectifier, Modification

A capacitor can be added to the circuit.

The circuit is now a simple DC power supply.

The time variation in the circuit is close to zero.

- It is determined by the RC time constant of the circuit.
- This is represented by the dotted lines in the graph.

Filter Circuit, Example

A **filter circuit** is one used to smooth out or eliminate a time-varying signal.

After rectification, a signal may still contain a small AC component.

- This component is often called a ripple.

By filtering, the ripple can be reduced.

Filters can also be built to respond differently to different frequencies.

High-Pass Filter

The circuit shown is one example of a **high-pass filter.**

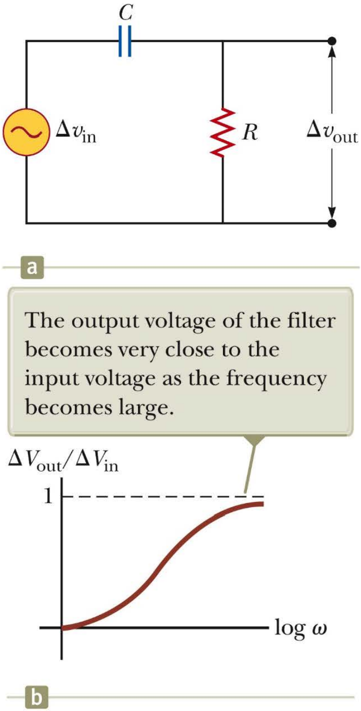
A high-pass filter is designed to preferentially pass signals of higher frequency and block lower frequency signals.

At low frequencies, ΔV_{out} is much smaller than $\Delta v_{in.}$

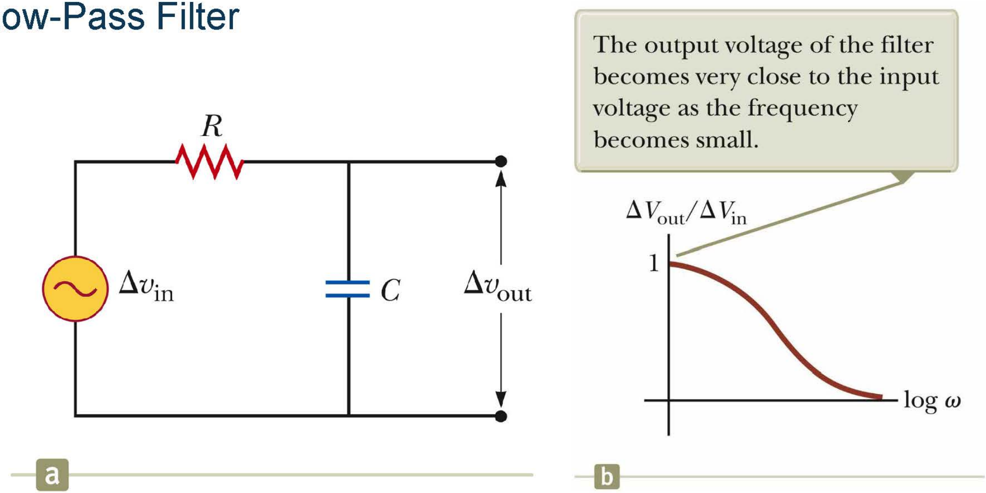
- At low frequencies, the capacitor has high reactance and much of the applied voltage appears across the capacitor.

At high frequencies, the two voltages are equal.

- At high frequencies, the capacitive reactance is small and the voltage appears across the resistor.



Low-Pass Filter



At low frequencies, the reactance and voltage across the capacitor are high.

As the frequency increases, the reactance and voltage decrease.

This is an example of a low-pass filter.