Chapter 29

Magnetic Fields

A Brief History of Magnetism

13th century BC

- Chinese used a compass
  - Uses a magnetic needle
  - Probably an invention of Arabic or Indian origin

800 BC

- Greeks
  - Discovered magnetite (Fe_3O_4) attracts pieces of iron

A Brief History of Magnetism, cont.

1269

- Pierre de Maricourt found that the direction of a needle near a spherical natural magnet formed lines that encircled the sphere .
- The lines also passed through two points diametrically opposed to each other.
- He called the points poles

1600

- William Gilbert
  - Expanded experiments with magnetism to a variety of materials
  - Suggested the Earth itself was a large permanent magnet

A Brief History of Magnetism, final

1750

- Experimenters showed that magnetic poles exert attractive or repulsive forces on each other.

1819

- Found an electric current deflected a compass needle

1820’s

- Faraday and Henry
  - Further connections between electricity and magnetism
  - A changing magnetic field creates an electric field.
- Maxwell
  - A changing electric field produces a magnetic field.

Hans Christian Oersted

1777 – 1851

Discovered the relationship between electricity and magnetism

An electric current in a wire deflected a nearby compass needle

The first evidence of the connection between electric and magnetic phenomena

Also the first to prepare pure aluminum

Magnetic Poles

Every magnet, regardless of its shape, has two poles.

- Called north and south poles
- Poles exert forces on one another
  - Similar to the way electric charges exert forces on each other
  - Like poles repel each other
    - N-N or S-S
  - Unlike poles attract each other
    - N-S

Magnetic Poles, cont.

The poles received their names due to the way a magnet behaves in the Earth’s magnetic field.

If a bar magnet is suspended so that it can move freely, it will rotate.

- The magnetic north pole points toward the Earth’s north geographic pole.
  - This means the Earth’s north geographic pole is a magnetic south pole.
  - Similarly, the Earth’s south geographic pole is a magnetic north pole.

Magnetic Poles, final

The force between two poles varies as the inverse square of the distance between them.

A single magnetic pole has never been isolated.

- In other words, magnetic poles are always found in pairs.
- All attempts so far to detect an isolated magnetic pole has been unsuccessful.
  - No matter how many times a permanent magnetic is cut in two, each piece always has a north and south pole.

Magnetic Fields

Reminder: an electric field surrounds any electric charge

The region of space surrounding any *moving* electric charge also contains a magnetic field.

A magnetic field also surrounds a magnetic substance making up a permanent magnet.

Magnetic Fields, cont.

A vector quantity

Symbolized by $\vec{\mathbf{B}}$

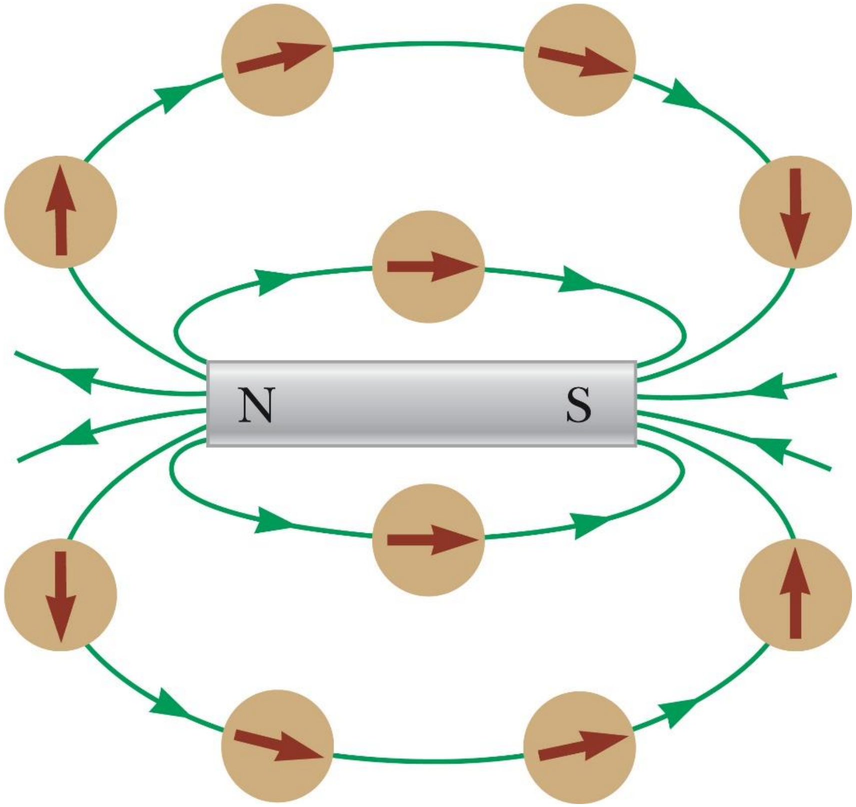
Direction is given by the direction a north pole of a compass needle points in that location

Magnetic field lines can be used to show how the field lines, as traced out by a compass, would look.

Magnetic Field Lines, Bar Magnet Example

The compass can be used to trace the field lines.

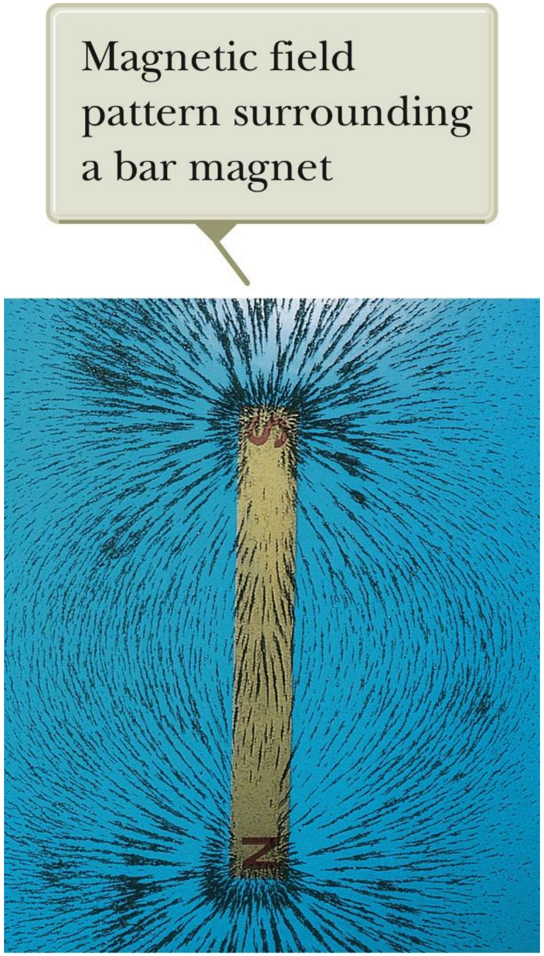
The lines outside the magnet point from the North pole to the South pole.



Magnetic Field Lines, Bar Magnet

Iron filings are used to show the pattern of the electric field lines.

The direction of the field is the direction a north pole would point.



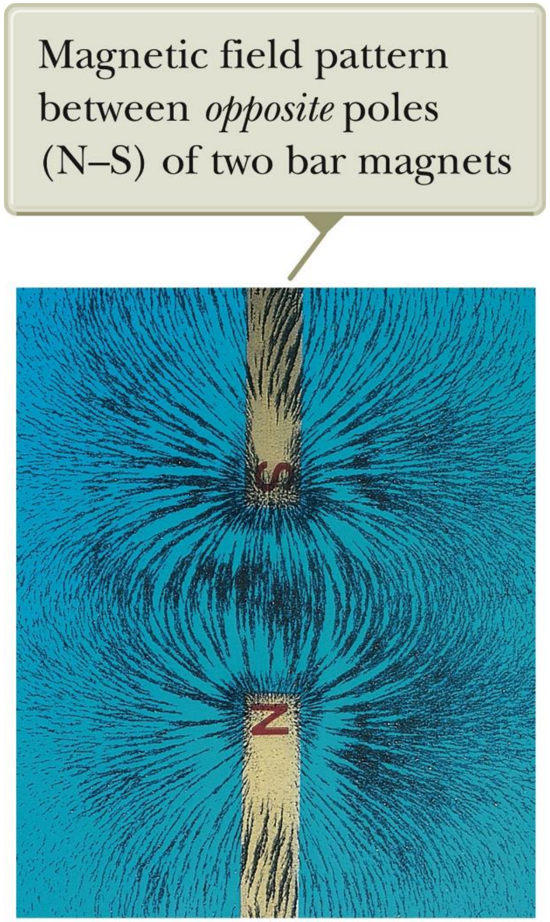
(a)

Magnetic Field Lines, Opposite Poles

Iron filings are used to show the pattern of the electric field lines.

The direction of the field is the direction a north pole would point.

- Compare to the electric field produced by an electric dipole



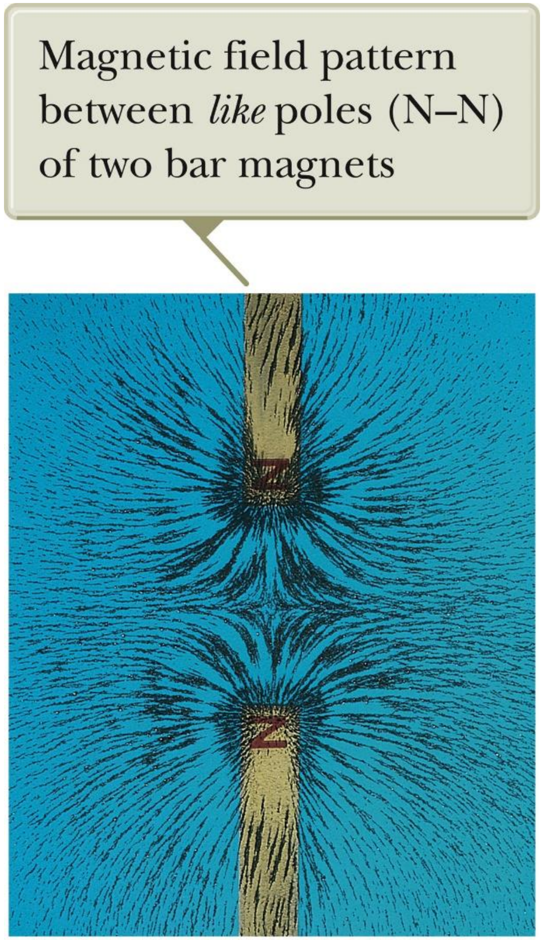
(b)

Magnetic Field Lines, Like Poles

Iron filings are used to show the pattern of the electric field lines.

The direction of the field is the direction a north pole would point.

- Compare to the electric field produced by like charges



(c)

Earth's Magnetic Poles

More proper terminology would be that a magnet has "north-seeking" and "south-seeking" poles.

The north-seeking pole points to the north geographic pole.

- This would correspond to the Earth's south magnetic pole.

The south-seeking pole points to the south geographic pole.

- This would correspond to the Earth's north magnetic pole.

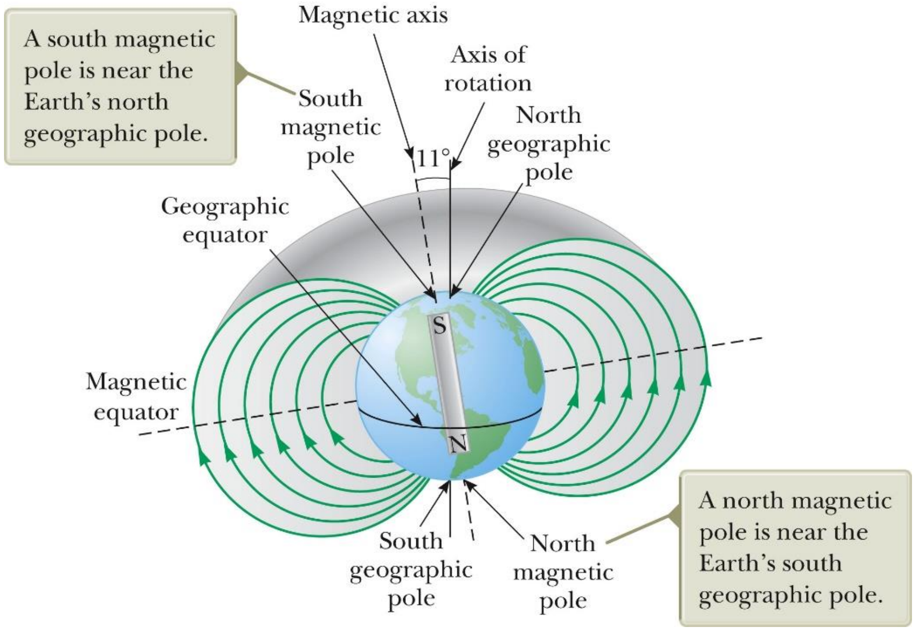
The configuration of the Earth's magnetic field is very much like the one that would be achieved by burying a gigantic bar magnet deep in the Earth's interior.

Earth's Magnetic Field

The source of the Earth's magnetic field is likely convection currents in the Earth's core.

There is strong evidence that the magnitude of a planet's magnetic field is related to its rate of rotation.

The direction of the Earth's magnetic field reverses periodically.



Definition of Magnetic Field

The magnetic field at some point in space can be defined in terms of the magnetic force, $\vec{\mathbf{F}}_B$.

The magnetic force will be exerted on a charged particle moving with a velocity, $\vec{\mathbf{v}}$.

- Assume (for now) there are no gravitational or electric fields present.

Properties of a Force on a Charge Moving in a Magnetic Field

The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge, q, and to the speed, v, of the particle.

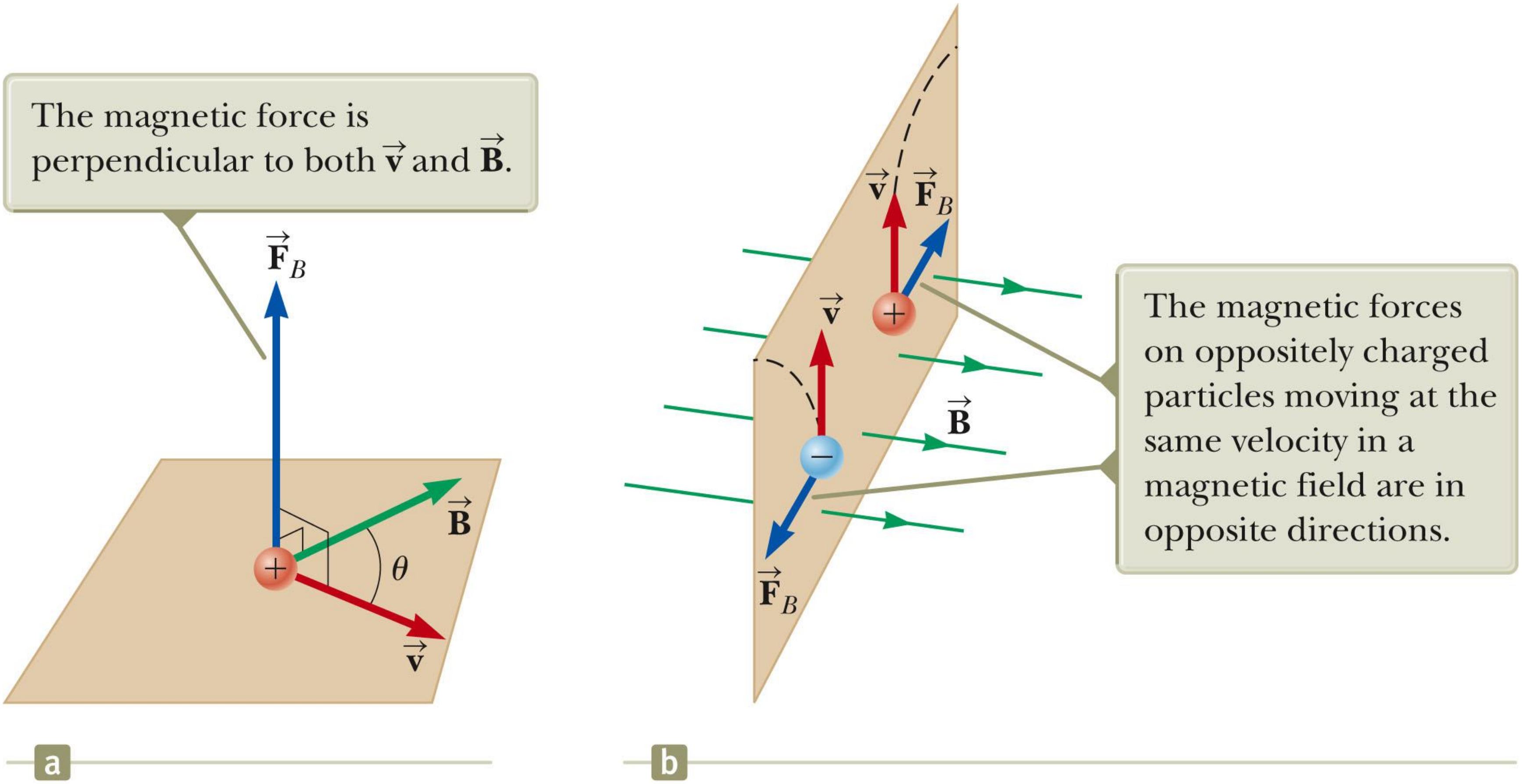
When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.

When the particle's velocity vector makes any angle $\theta \neq 0$ with the field, the force acts in a direction perpendicular to the plane formed by the velocity and the field.

The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.

The magnitude of the magnetic force is proportional to sin θ, where θ is the angle the particle's velocity makes with the direction of the magnetic field.

More About Direction



Force on a Charge Moving in a Magnetic Field, Formula

The properties can be summarized in a vector equation:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

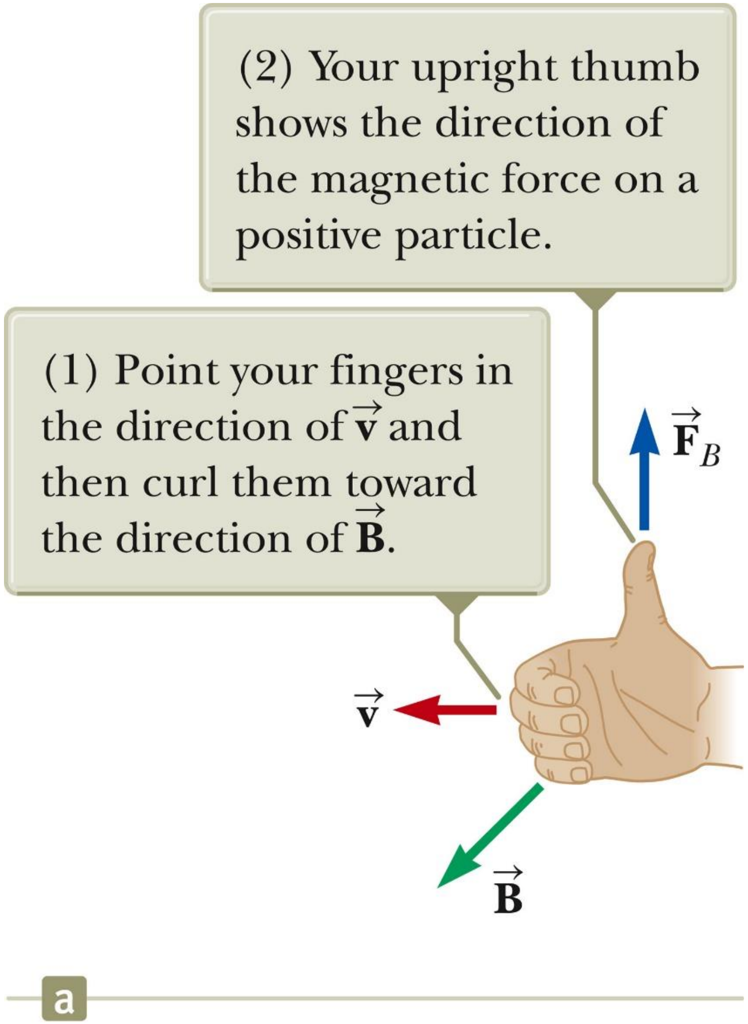
- $\vec{\mathbf{F}}_B$ is the magnetic force
- q is the charge
- $\vec{\mathbf{v}}$ is the velocity of the moving charge
- $\vec{\mathbf{B}}$ is the magnetic field

Direction: Right-Hand Rule #1

This rule is based on the right-hand rule for the cross product.

Your thumb is in the direction of the force if *q* is positive.

The force is in the opposite direction of your thumb if *q* is negative.



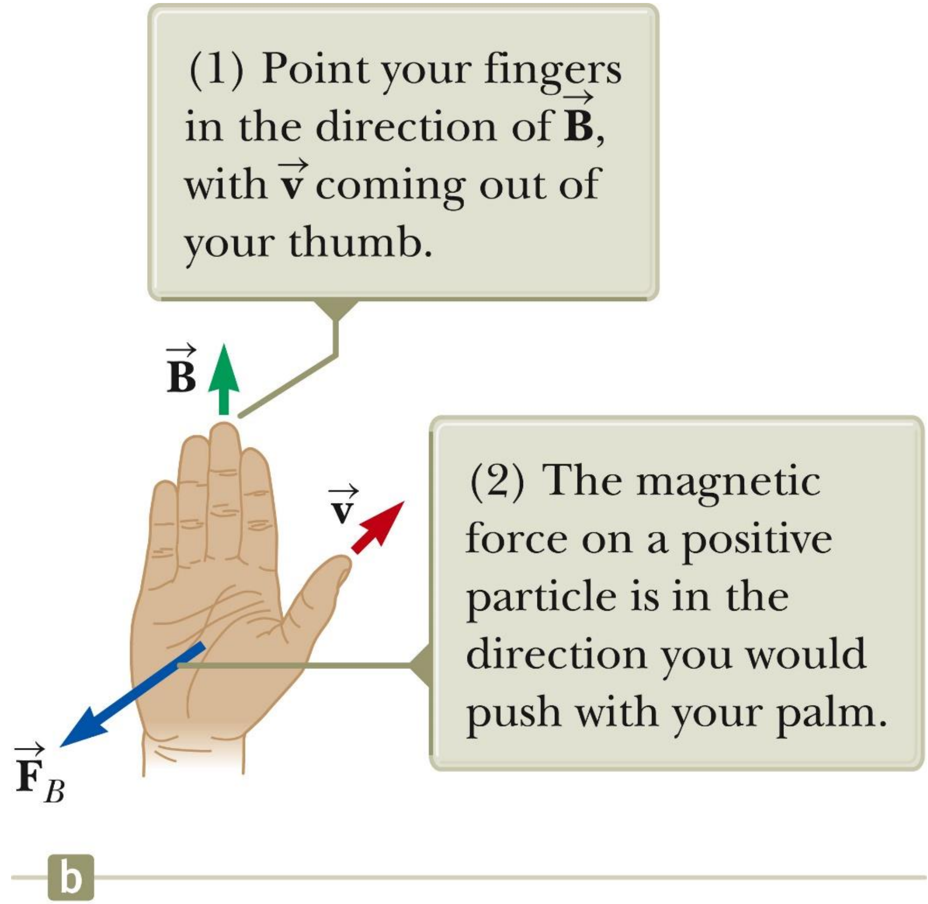
Direction: Right-Hand Rule #2

Alternative to Rule #1

The force on a positive charge extends outward from the palm.

The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand.

The force on a negative charge is in the opposite direction.



More About Magnitude of F

The magnitude of the magnetic force on a charged particle is $F_B = |q|\ v\ B \sin\ \theta$.

- θ is the smaller angle between v and B
- F_B is zero when the field and velocity are parallel or antiparallel
  - $\theta = 0$ or 180^o
- F_B is a maximum when the field and velocity are perpendicular
  - $\theta = 90^o$

Differences Between Electric and Magnetic Fields

Direction of force

- The electric force acts along the direction of the electric field.
- The magnetic force acts perpendicular to the magnetic field.

Motion

- The electric force acts on a charged particle regardless of whether the particle is moving.
- The magnetic force acts on a charged particle only when the particle is in motion.

Work

- The electric force does work in displacing a charged particle.
- The magnetic force associated with a steady magnetic field does no work when a particle is displaced.
  - This is because the force is perpendicular to the displacement of its point of application.

Work in Fields, cont.

The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone.

When a charged particle moves with a given velocity through a magnetic field, the field can alter the direction of the velocity, but not the speed or the kinetic energy.

Units of Magnetic Field

The SI unit of magnetic field is the tesla (T).

$$T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

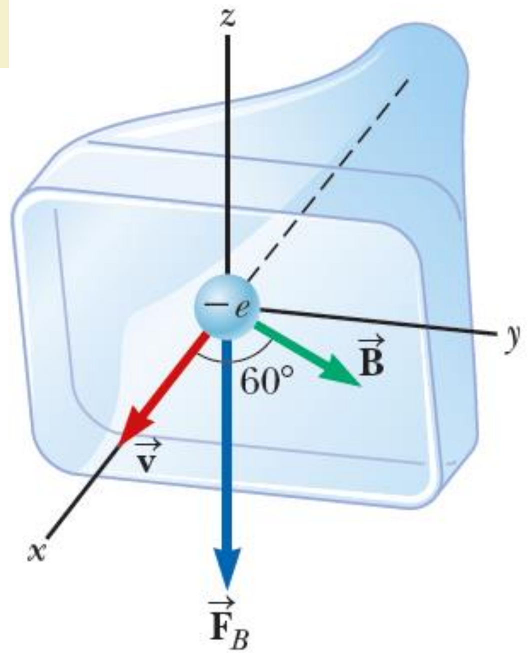
- Wb is a weber

A non-SI commonly used unit is a gauss (G).

- $1\ \text{T} = 10^4\ \text{G}$

An Electron Moving in a Magnetic Field, Example

An electron in an old-style television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the *x* axis (Fig. 29.6). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the *x* axis and lying in the *xy* plane. Calculate the magnetic force on the electron.



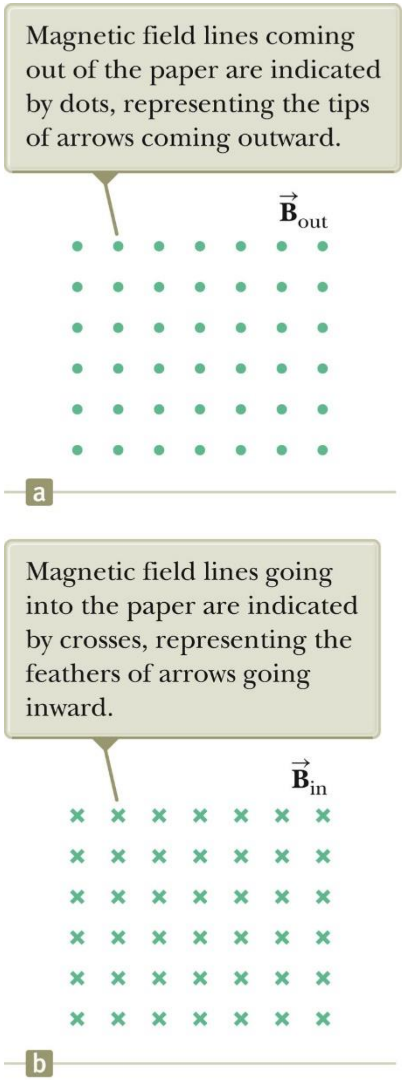
$$
\begin{aligned}
F_B &= |q|vB\sin\theta \\
&= (1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^6\ \text{m/s})(0.025\ \text{T})(\sin 60°) \\
&= \boxed{2.8 \times 10^{-14}\ \text{N}}
\end{aligned}
$$

Notation Notes

When vectors are perpendicular to the page, dots and crosses are used.

- The dots represent the arrows coming out of the page.
- The crosses represent the arrows going into the page.

The same notation applies to other vectors.



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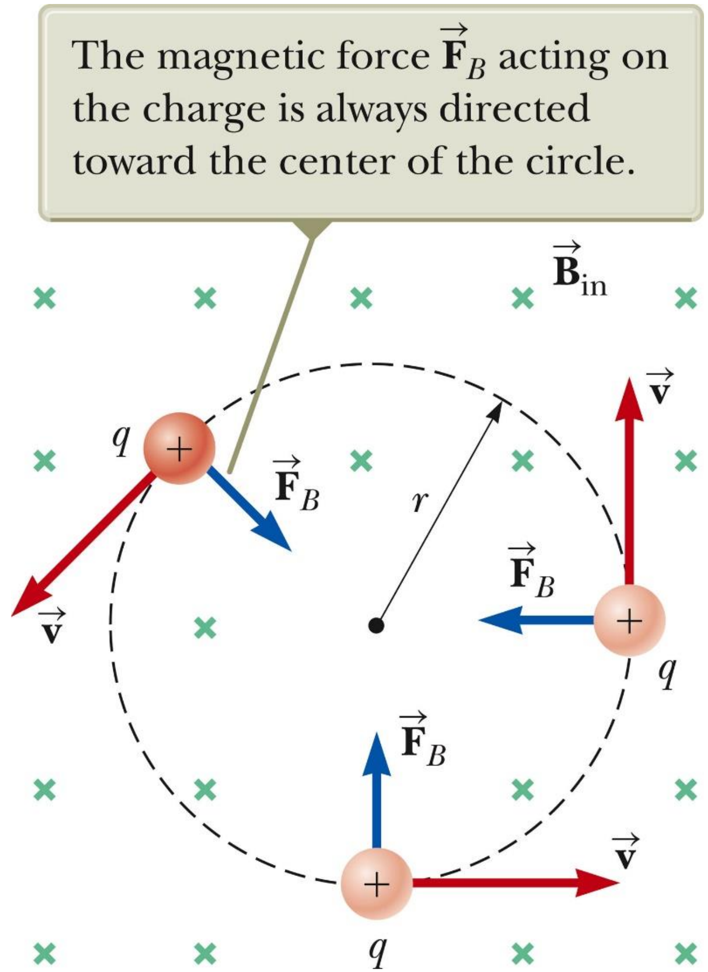
Charged Particle in a Magnetic Field

Consider a particle moving in an external uniform magnetic field with its velocity perpendicular to the field.

The force is always directed toward the center of the circular path.

The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle.

The rotation is counter clockwise for a positive charge in a magnetic field directed into the page. If *q* were negative, the rotation would be clockwise.



Force on a Charged Particle

Use the particle under a net force and a particle in uniform circular motion models.

Equating the magnetic and centripetal forces:

$$F_B = qvB = \frac{mv^2}{r}$$

Solving for r:

$$r = \frac{mv}{qB}$$

- r is proportional to the linear momentum of the particle and inversely proportional to the magnetic field.

More About Motion of Charged Particle

The angular speed of the particle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- The angular speed, ω, is also referred to as the **cyclotron frequency.**

The period of the motion is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

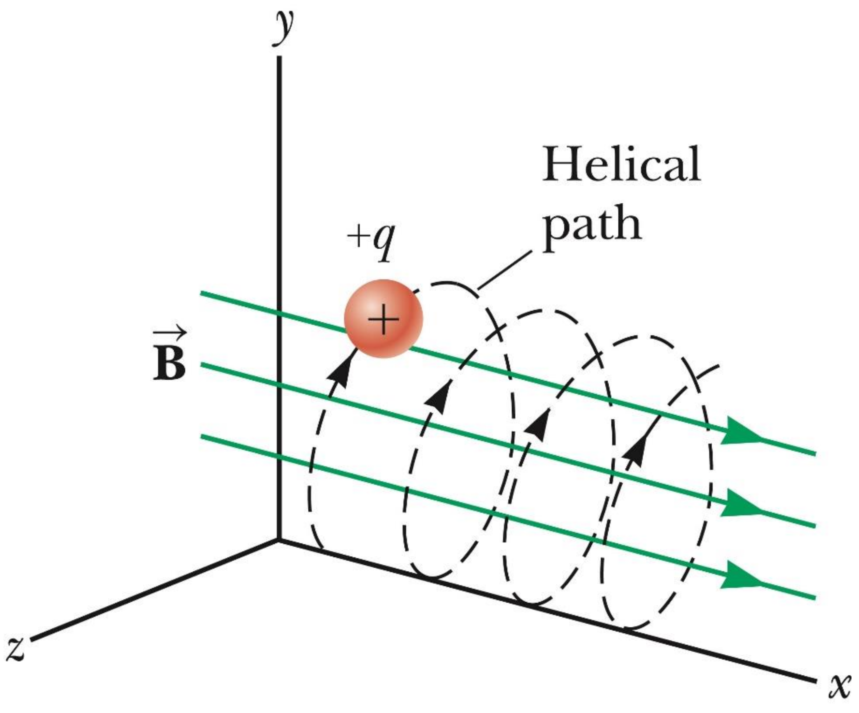
Motion of a Particle, General

If a charged particle moves in a magnetic field at some arbitrary angle with respect to the field, its path is a helix.

Same equations apply, with *v* replaced by

$$v_\perp = \sqrt{v_y^2 + v_z^2}$$

There is no component of force in the *x* direction.



A proton Moving Perpendicular to a Uniform Magnetic Field, Example

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

$$v = \frac{qBr}{m_p}$$

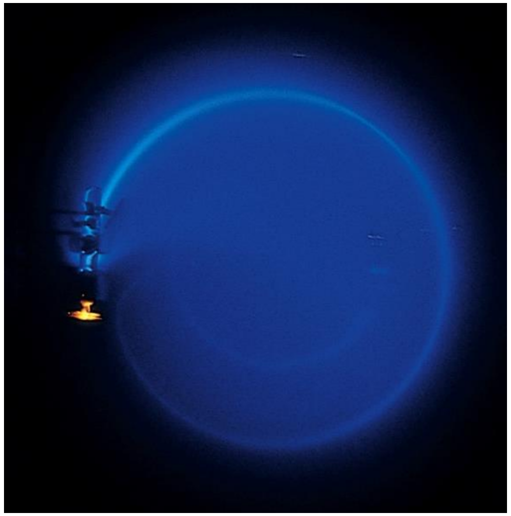
$$v = \frac{(1.60 \times 10^{-19}\ \mathrm{C})(0.35\ \mathrm{T})(0.14\ \mathrm{m})}{1.67 \times 10^{-27}\ \mathrm{kg}}$$

$$= \boxed{4.7 \times 10^{6}\ \mathrm{m/s}}$$

Bending of an Electron Beam, Example

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm. (Such a curved beam of electrons is shown in Fig. 29.10.)

(A) What is the magnitude of the magnetic field?



$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}m_e v^2 - 0\right) + (q\,\Delta V) = 0$$

$$v = \sqrt{\frac{-2q\,\Delta V}{m_e}}$$

$$v = \sqrt{\frac{-2(-1.60 \times 10^{-19}\ \mathrm{C})(350\ \mathrm{V})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 1.11 \times 10^{7}\ \mathrm{m/s}$$

$$B = \frac{m_e v}{er}$$

$$B = \frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.11 \times 10^{7}\ \mathrm{m/s})}{(1.60 \times 10^{-19}\ \mathrm{C})(0.075\ \mathrm{m})} = \boxed{8.4 \times 10^{-4}\ \mathrm{T}}$$

(B) What is the angular speed of the electrons?

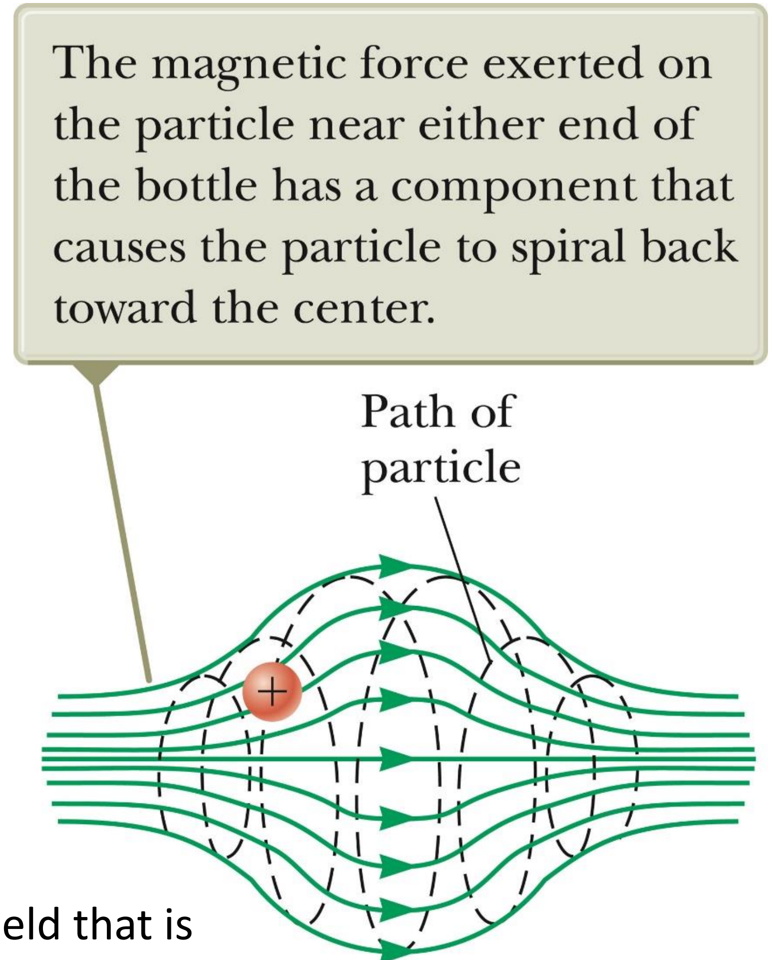
$$\omega = \frac{v}{r} = \frac{1.11 \times 10^{7}\ \mathrm{m/s}}{0.075\ \mathrm{m}} = \boxed{1.5 \times 10^{8}\ \mathrm{rad/s}}$$

Particle in a Nonuniform Magnetic Field

The motion is complex.

For example, the particles can oscillate back and forth between two positions.

This configuration is known as a *magnetic bottle.*



A magnetic field that is strong at the ends and weak in the middle

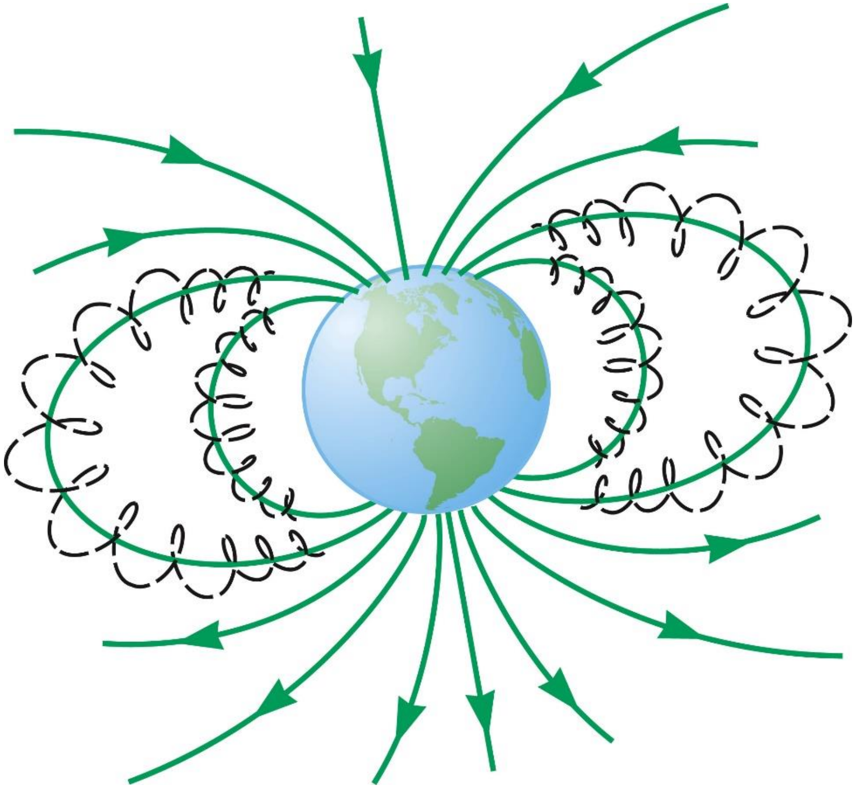
Van Allen Radiation Belts

The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions.

The particles are trapped by the Earth's nonuniform magnetic field.

The particles spiral from pole to pole.

- May result in auroras



Charged Particles Moving in Electric and Magnetic Fields

In many applications, charged particles will move in the presence of both magnetic and electric fields.

In that case, the total force is the sum of the forces due to the individual fields.

- The total force is called the Lorentz force.

In general:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Velocity Selector

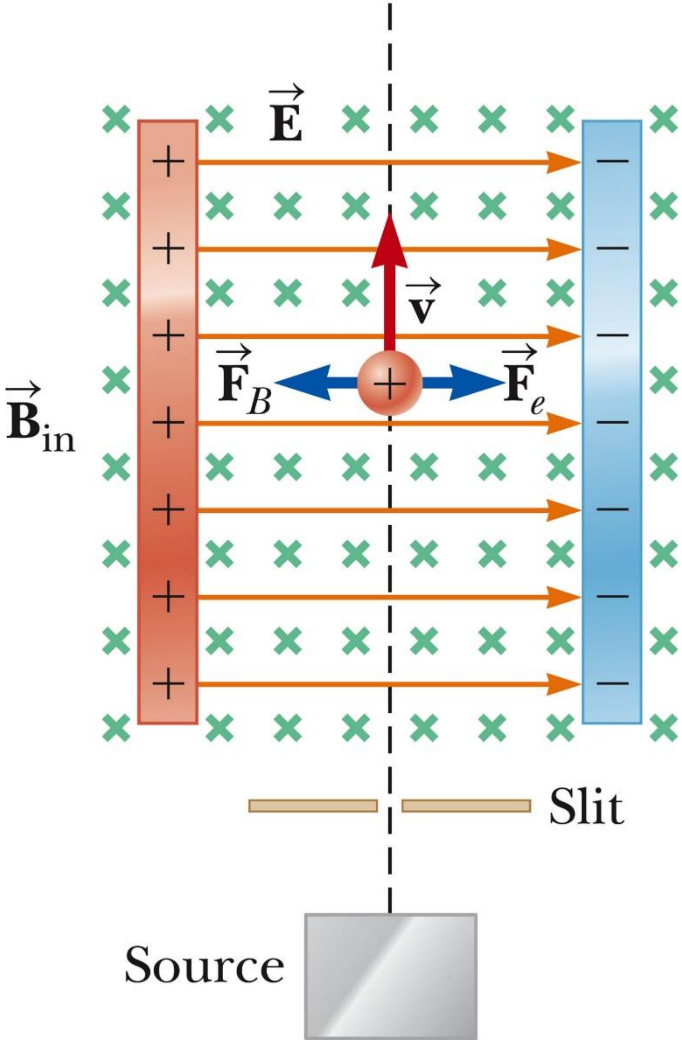
Used when all the particles need to move with the same velocity.

A uniform electric field is perpendicular to a uniform magnetic field.

When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line.

From the expression $qE = qvB$, we find that

$$v = \frac{E}{B}$$



Velocity Selector, cont.

Only those particles with the given speed will pass through the two fields undeflected.

The magnetic force exerted on particles moving at a speed greater than this is stronger than the electric field and the particles will be deflected to the left.

Those moving more slowly will be deflected to the right.

Mass Spectrometer

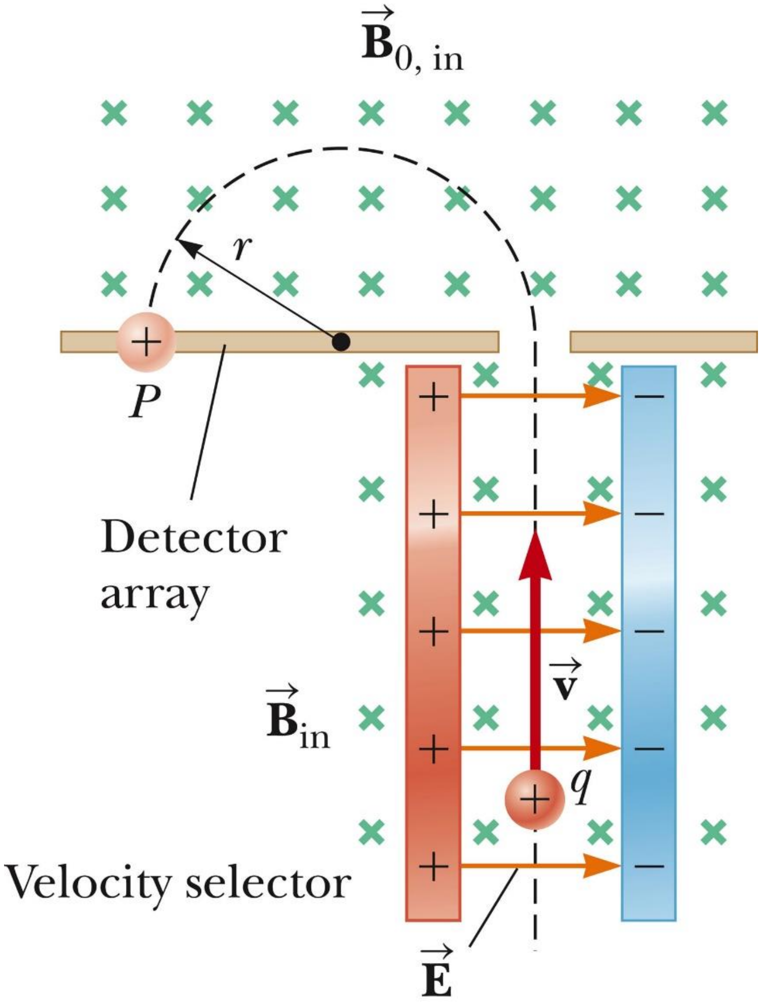
A mass spectrometer separates ions according to their mass-to-charge ratio.

In one design, a beam of ions passes through a velocity selector and enters a second magnetic field.

After entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector at P.

If the ions are positively charged, they deflect to the left.

If the ions are negatively charged, they deflect to the right.



Mass Spectrometer, cont.

The mass to charge (m/q) ratio can be determined by measuring the radius of curvature and knowing the magnetic and electric field magnitudes.

Using equation $v = \frac{E}{B}$

$$\frac{m}{q} = \frac{rB_o}{v} = \frac{rB_oB}{E}$$

In practice, one usually measures the masses of various isotopes of a given ion, with the ions all carrying the same charge *q.*

The mass ratios can be determined even if the charge is unknown.

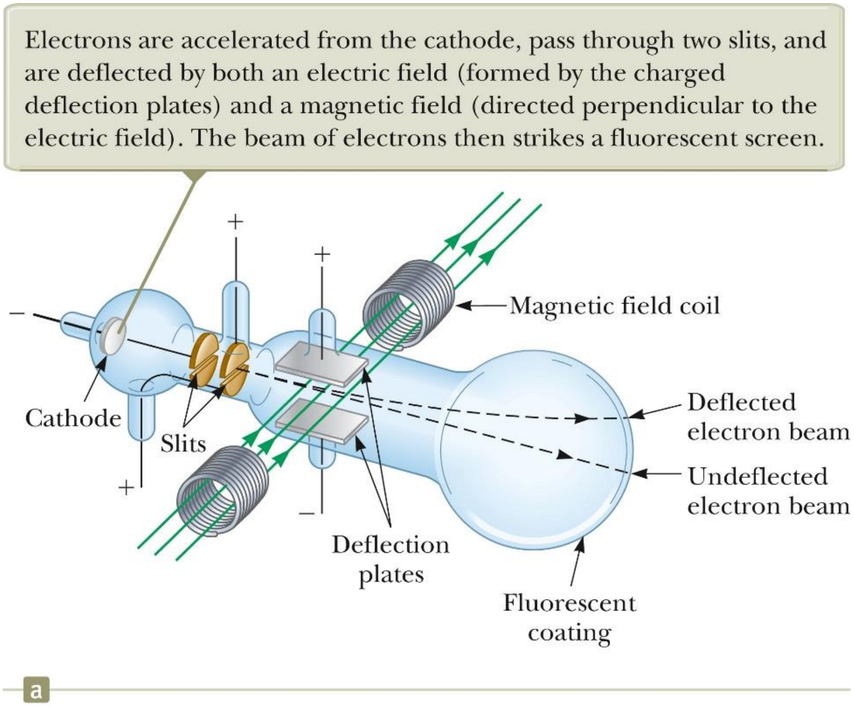
Thomson's *e/m* Experiment

Electrons are accelerated from the cathode.

They are deflected by electric and magnetic fields.

The beam of electrons strikes a fluorescent screen.

e/m was measured

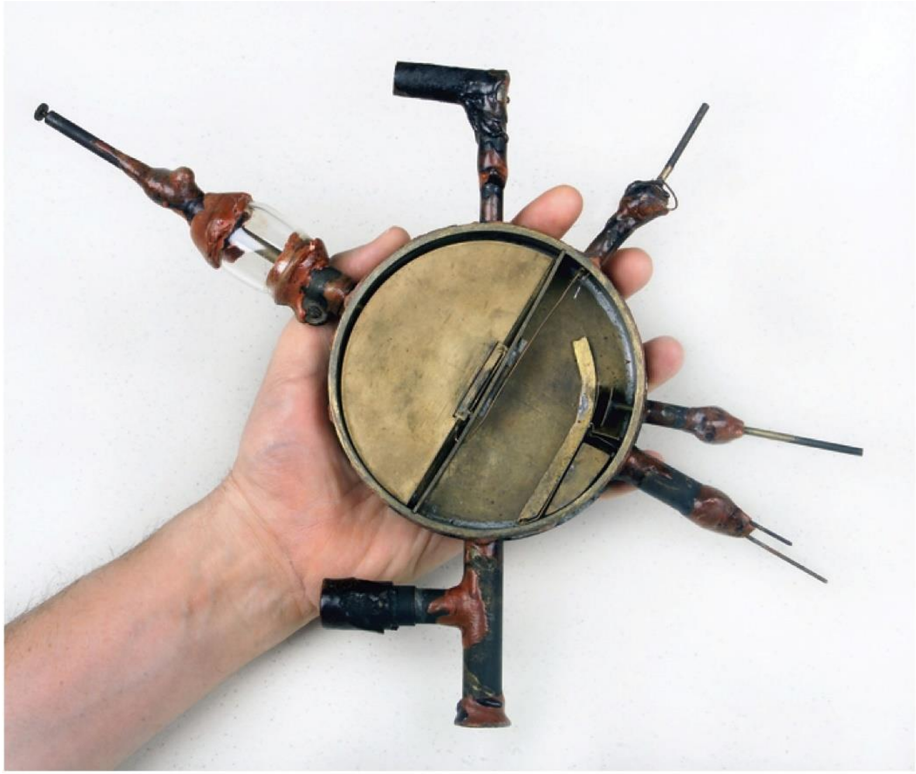


Cyclotron

A **cyclotron** is a device that can accelerate charged particles to very high speeds.

The energetic particles produced are used to bombard atomic nuclei and thereby produce nuclear reactions.

These reactions can be analyzed by researchers.



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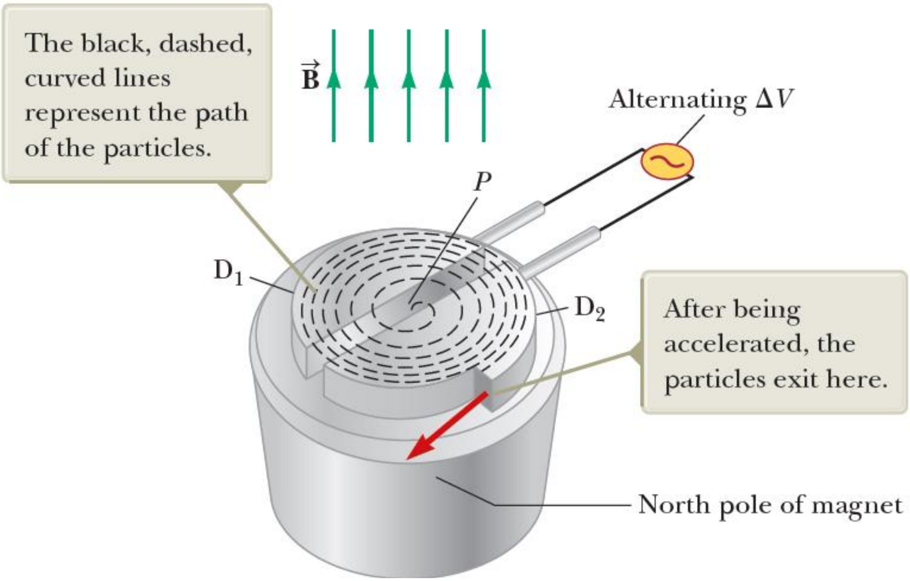
Cyclotron, cont.

The charges move inside two semicircular containers D1 and D2, referred to as dees because of their shape like the letter D.

A high frequency alternating potential is applied to the dees and a uniform magnetic field is perpendicular to them.

A positive ion is released near the center and moves in a semicircular path.



Section 29.3

Cyclotron, final

The potential difference is adjusted so that the polarity of the dees is reversed in the same time interval as the particle travels around one dee.

This ensures the kinetic energy of the particle increases each trip.

The cyclotron's operation is based on the fact that T is independent of the speed of the particles and of the radius of their path.

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m} \qquad v = qBR/m$$

When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play.

Magnetic Force on a Current Carrying Conductor

A force is exerted on a current-carrying wire placed in a magnetic field.

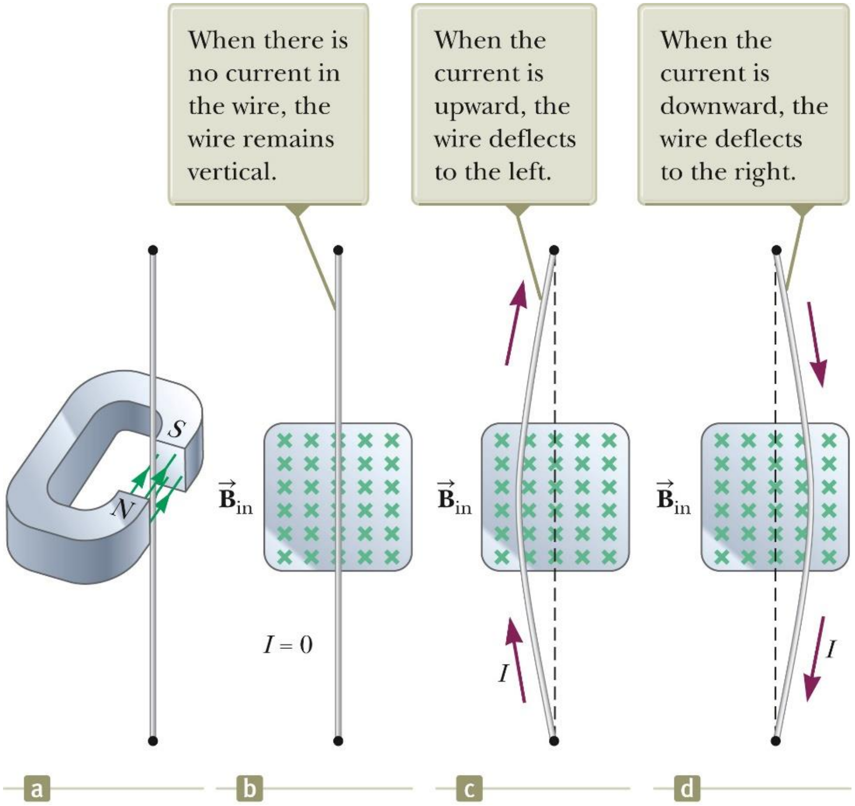
- The current is a collection of many charged particles in motion.

The direction of the force is given by the right-hand rule.

Force on a Wire, 1

In this case, there is no current, so there is no force.

Therefore, the wire remains vertical.



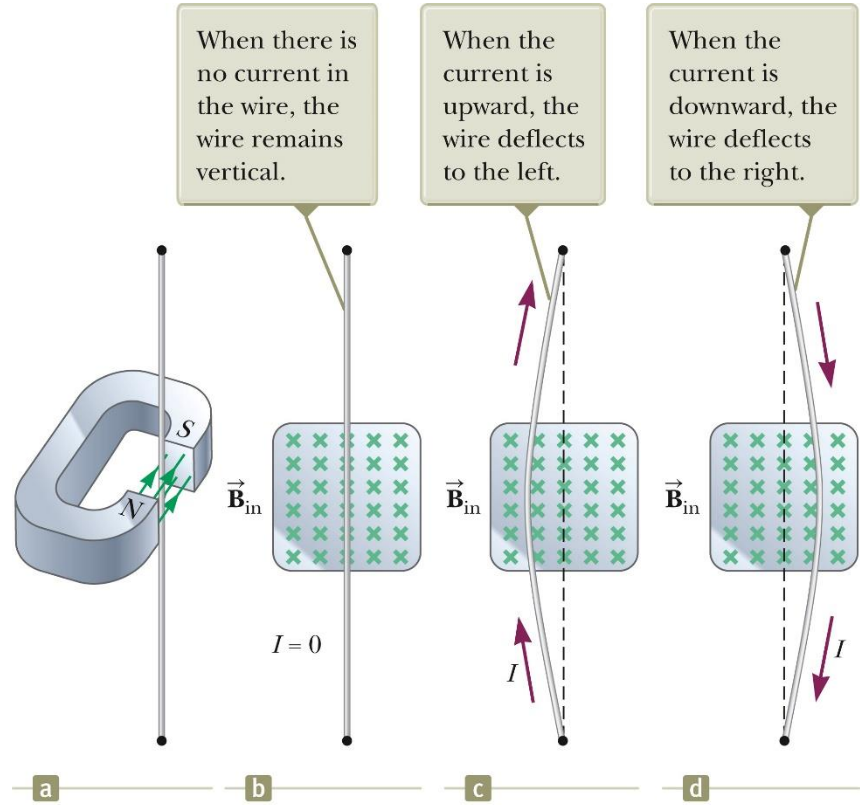
Force on a Wire, 2

The magnetic field is into the page

The current is up the page

The force is to the left

The wire deflects to the left



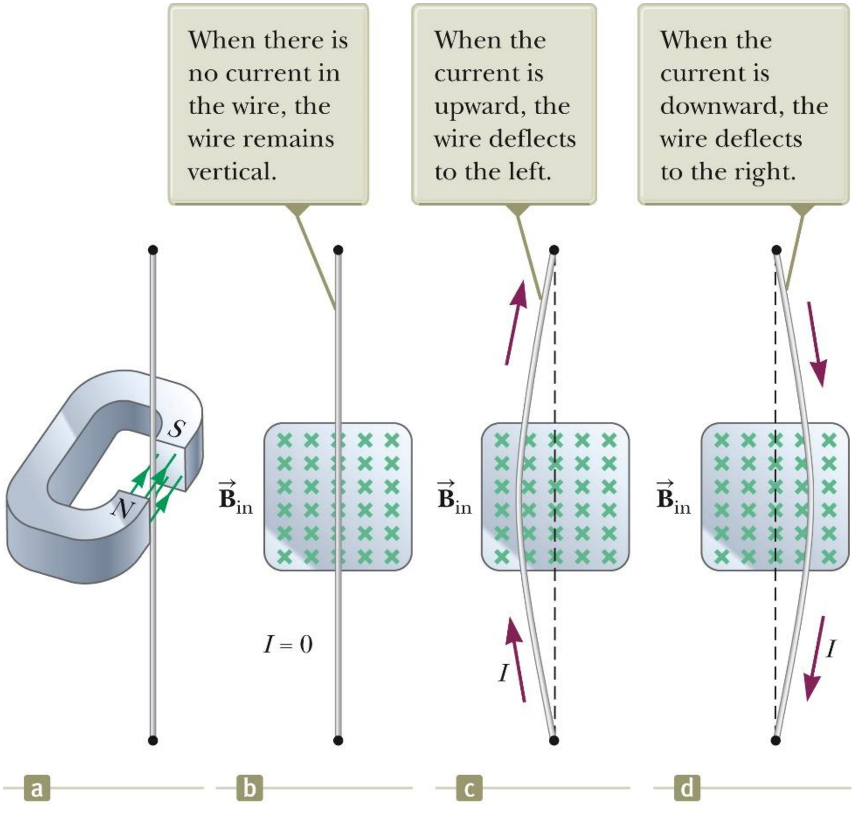
Force on a Wire, 3

The magnetic field is into the page

The current is down the page

The force is to the right

The wire deflects to the right



Force on a Wire, equation

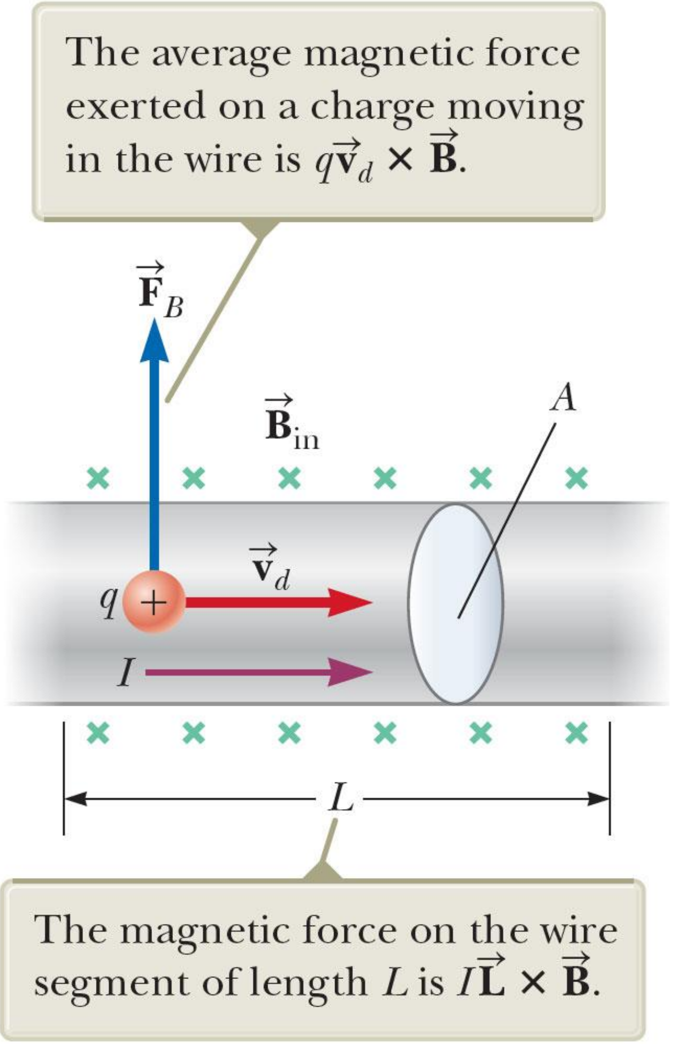
The magnetic force is exerted on each moving charge in the wire.

- $\vec{\mathbf{F}} = q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}$

The total force is the product of the force on one charge and the number of charges.

$$\vec{\mathbf{F}} = \left(q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}}\right) nAL$$

- The volume of the segment is *AL*
- *n* is the number of mobile charge carriers per unit volume.



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Force on a Wire, Equation cont.

In terms of the current, this becomes

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}} \qquad I = nqv_d A$$

- I is the current.
- $\vec{\mathbf{L}}$ is a vector that points in the direction of the current.
  - Its magnitude is the length L of the segment.
- $\vec{\mathbf{B}}$ is the magnetic field.

This expression applies only to a straight segment of wire in a uniform magnetic field.

Force on a Wire, Arbitrary Shape

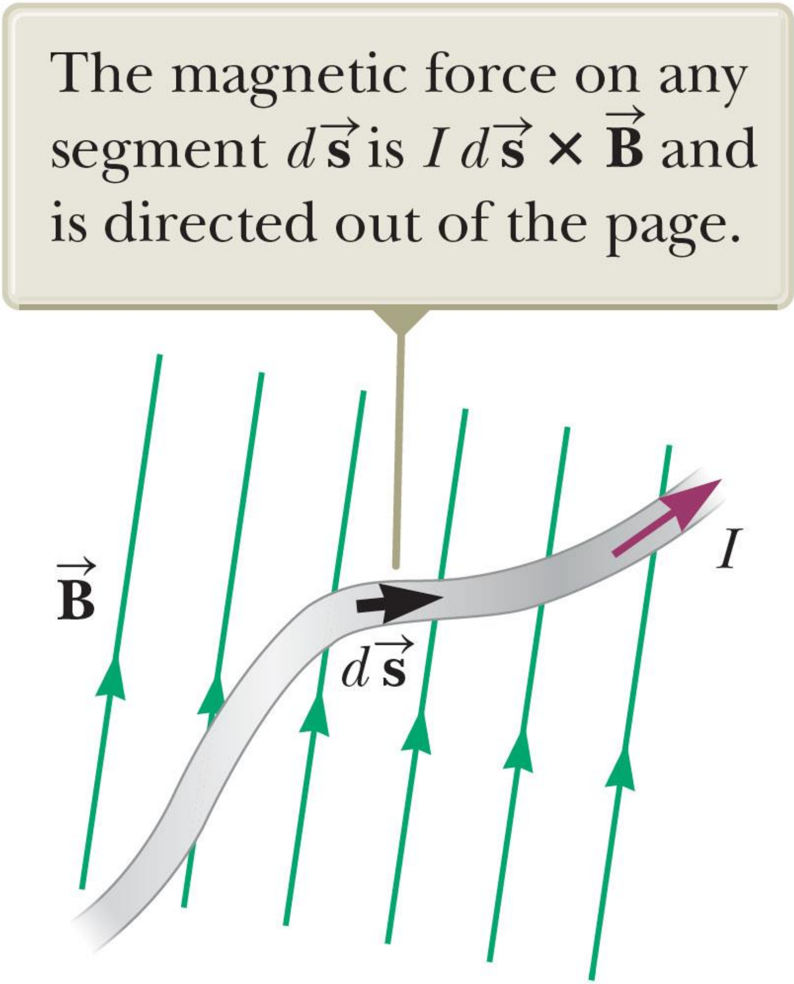
Consider a small segment of the wire, $d\vec{\mathbf{s}}$

The force exerted on this segment is

$$d\vec{\mathbf{F}}_B = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

The total force is

$$\vec{\mathbf{F}}_B = I \int_{\mathbf{a}}^{b} d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$



Force on a Semicircular Conductor, Example

A wire bent into a semicircle of radius R forms a closed circuit and carries a current I. The wire lies in the xy plane, and a uniform magnetic field is directed along the positive y axis as in Figure 29.20. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.

Notice that $d\vec{s}$ is perpendicular to $\vec{B}$ everywhere on the straight portion of the wire.

$$\vec{\mathbf{F}}_1 = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}} = I\int_{-R}^{R} B\,dx\,\hat{\mathbf{k}} = \boxed{2IRB\,\hat{\mathbf{k}}}$$

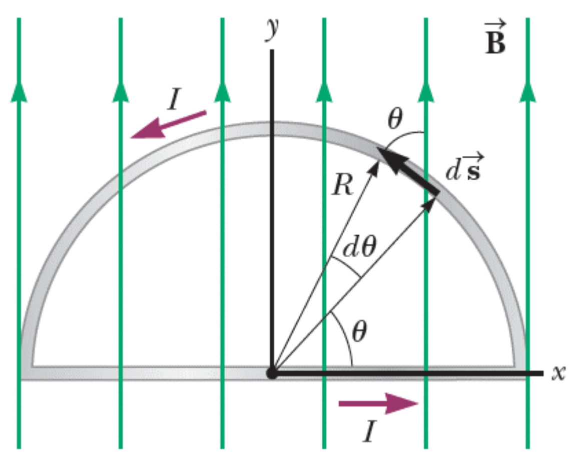
To find the magnetic force on the curved part, first write an expression for the magnetic force $d\vec{F}$ on the element $d\vec{s}$

$$(1)\quad d\vec{\mathbf{F}}_2 = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}} = -IB\sin\theta\; ds\,\hat{\mathbf{k}}$$

$$(2)\quad ds = R\,d\theta$$

$$\vec{\mathbf{F}}_2 = -\int_0^{\pi} IRB\sin\theta\,d\theta\,\hat{\mathbf{k}} = -IRB\int_0^{\pi}\sin\theta\,d\theta\,\hat{\mathbf{k}} = -IRB[-\cos\theta]_0^{\pi}\,\hat{\mathbf{k}}$$

$$= IRB(\cos\pi - \cos 0)\hat{\mathbf{k}} = IRB(-1-1)\hat{\mathbf{k}} = \boxed{-2IRB\,\hat{\mathbf{k}}}$$



$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

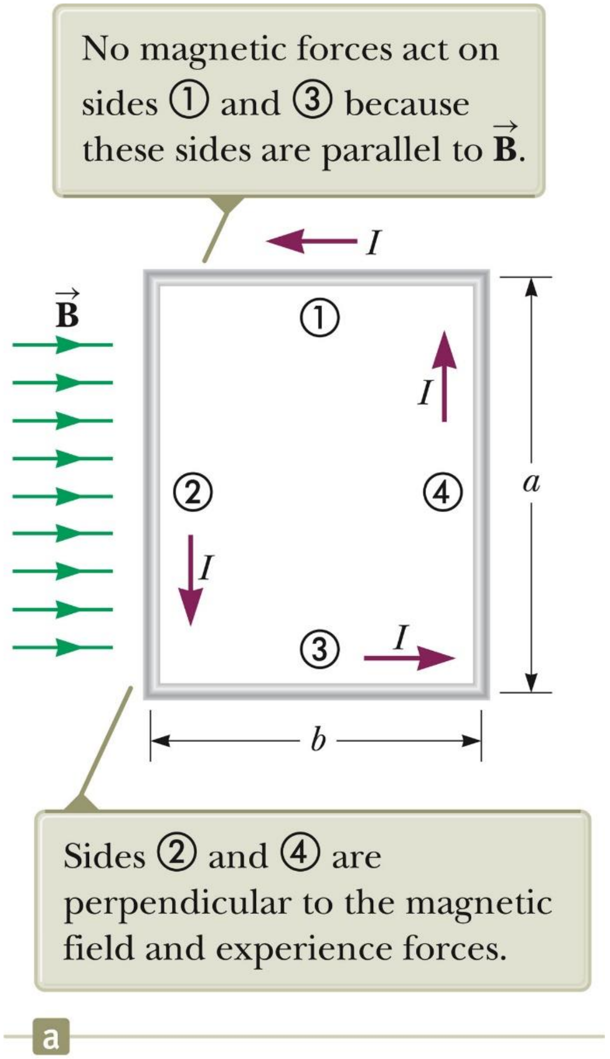
The net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

Torque on a Current Loop

The rectangular loop carries a current I in a uniform magnetic field.

No magnetic force acts on sides 1 & 3.

- The wires are parallel to the field and $\vec{\mathbf{L}} \times \vec{\mathbf{B}} = 0$



Torque on a Current Loop, 2

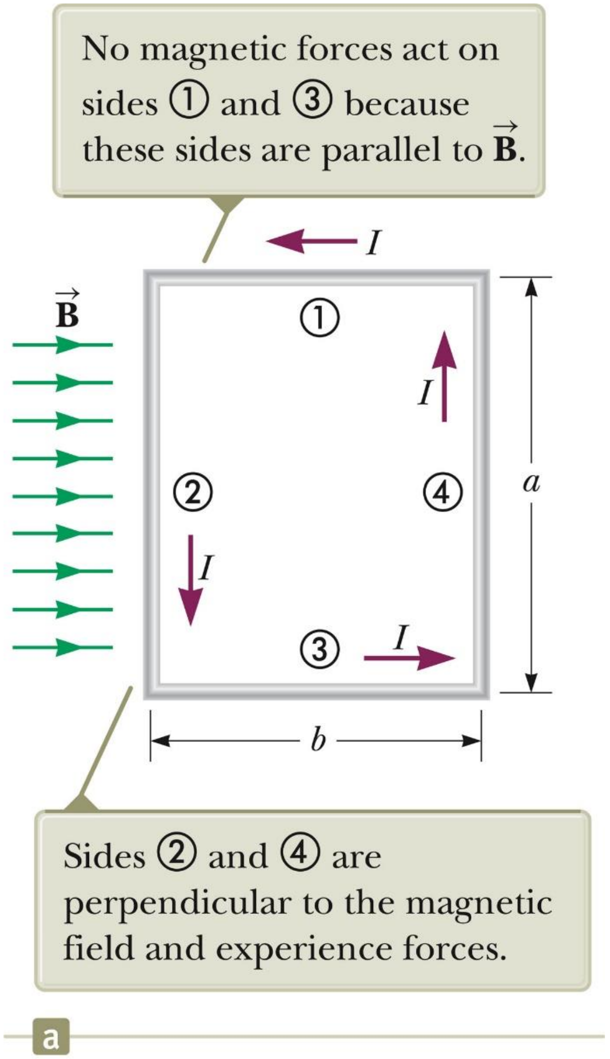
There is a force on sides 2 & 4 since they are perpendicular to the field.

The magnitude of the magnetic force on these sides will be:

- $F_2 = F_4 = I\,a\,B$

The direction of F_2 is out of the page.

The direction of F_4 is into the page.

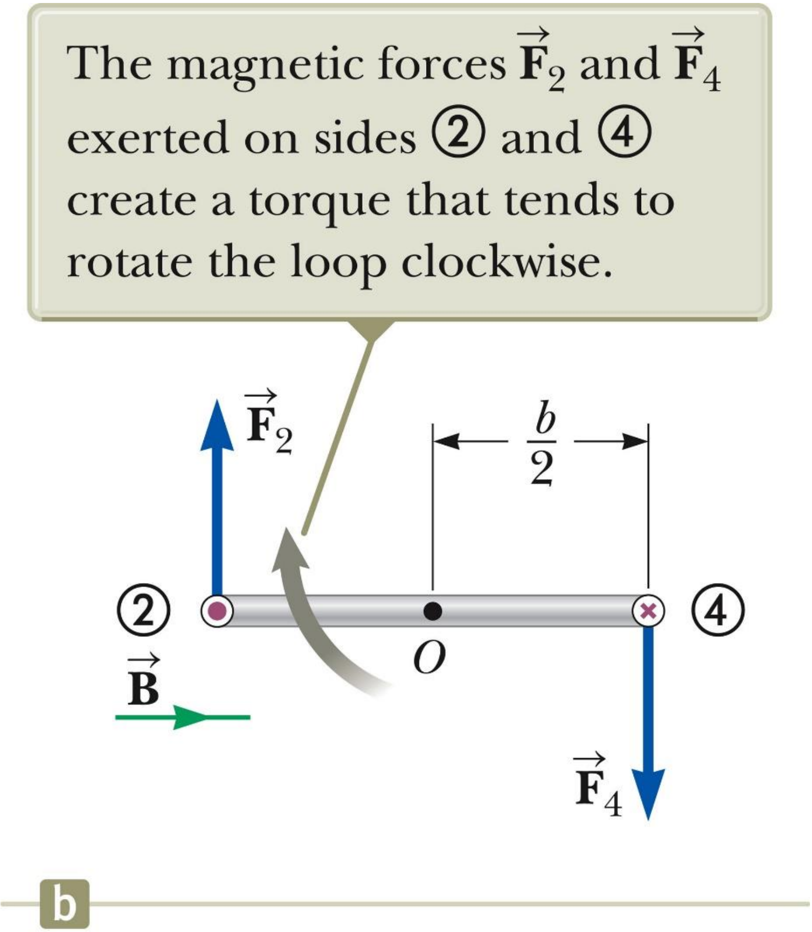


Torque on a Current Loop, 3

The forces are equal and in opposite directions, but not along the same line of action.

The forces produce a torque around point *O.*

If the loop is pivoted so that it can rotate about point O, these two forces produce about O a torque that rotates the loop clockwise.



Torque on a Current Loop, Equation

The maximum torque is found by:

$$\tau_{max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (I\,aB)\frac{b}{2} + (I\,aB)\frac{b}{2}$$
$$= I\,abB$$

The area enclosed by the loop is *ab*, so $\tau_{\max} = IAB$.

- This maximum value occurs only when the field is parallel to the plane of the loop.

Torque on a Current Loop, General

Assume the magnetic field makes an angle of $\theta < 90^{\circ}$ with a line perpendicular to the plane of the loop.

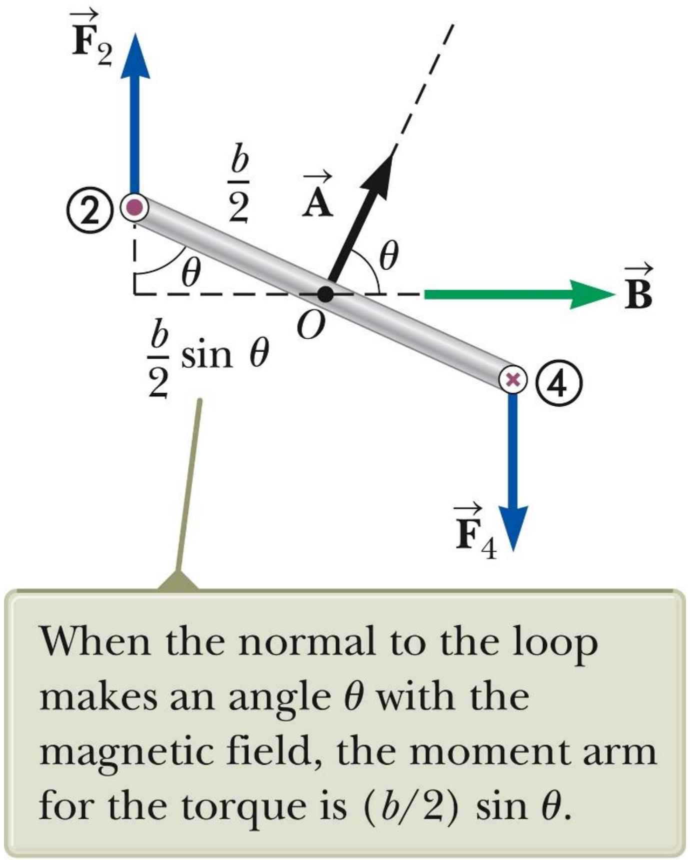
The net torque about point *O* will be

$$\tau = F_2 \frac{b}{2} \sin\theta + F_4 \frac{b}{2} \sin\theta$$

$$= IaB\left(\frac{b}{2} \sin\theta\right) + IaB\left(\frac{b}{2} \sin\theta\right) = IabB \sin\theta$$

$$= IAB \sin\theta$$

$$A = ab$$



Torque on a Current Loop, Summary

The torque has a maximum value when the field is perpendicular to the normal to the plane of the loop.

The torque is zero when the field is parallel to the normal to the plane of the loop.

A convenient vector expression for the torque exerted on a loop placed in a uniform magnetic field

$$\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

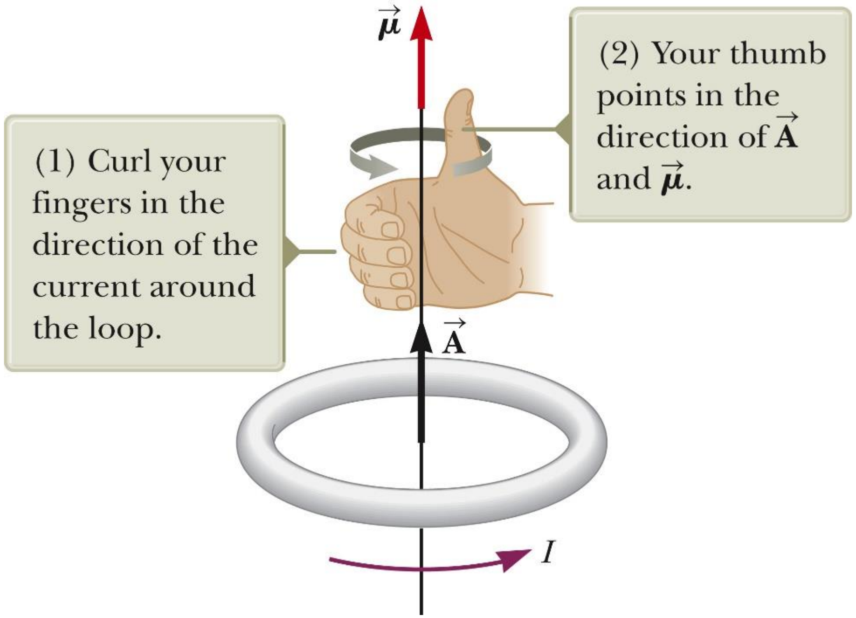
- $\vec{\mathbf{A}}$ is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop.

Direction

The right-hand rule can be used to determine the direction of $\vec{\mathbf{A}}$

Curl your fingers in the direction of the current in the loop.

Your thumb points in the direction of $\vec{\mathbf{A}}$.



Magnetic Dipole Moment

The product $I\vec{\mathbf{A}}$ is defined as the **magnetic dipole moment**, $\vec{\mu}$, of the loop.

- Often called the magnetic moment

SI units: A · m^2

If a coil of wire contains *N* loops of the same area, the magnetic moment of the coil is

$$\vec{\boldsymbol{\mu}}_{\text{coil}} = NI\vec{\mathbf{A}}$$

Torque in terms of magnetic moment: $\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$

- Analogous to $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ for electric dipole
- Valid for any orientation of the field and the loop
- Valid for a loop of any shape

Potential Energy

The potential energy of the system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field given by

$$U_B = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

- $U_{min} = -\mu B$ and occurs when the dipole moment is in the same direction as the field.
- $U_{max} = +\mu B$ and occurs when the dipole moment is in the direction opposite the field.

The Magnetic Dipole Moment of a Coil, Example

A rectangular coil of dimensions 5.40 cm × 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. A 0.350-T magnetic field is applied parallel to the plane of the coil.

(A) Calculate the magnitude of the magnetic dipole moment of the coil.

$$\mu_{\text{coil}} = NIA = (25)(15.0 \times 10^{-3}\text{ A})(0.054\,0\text{ m})(0.085\,0\text{ m})$$

$$= \boxed{1.72 \times 10^{-3}\text{ A} \cdot \text{m}^2}$$

(B) What is the magnitude of the torque acting on the loop?

$$\tau = \mu_{\text{coil}} B = (1.72 \times 10^{-3}\text{ A} \cdot \text{m}^2)(0.350\text{ T})$$

$$= \boxed{6.02 \times 10^{-4}\text{ N} \cdot \text{m}}$$

Hall Effect

When a current carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field.

This phenomena is known as the *Hall effect.*

It arises from the deflection of charge carriers to one side of the conductor as a result of the magnetic forces they experience.

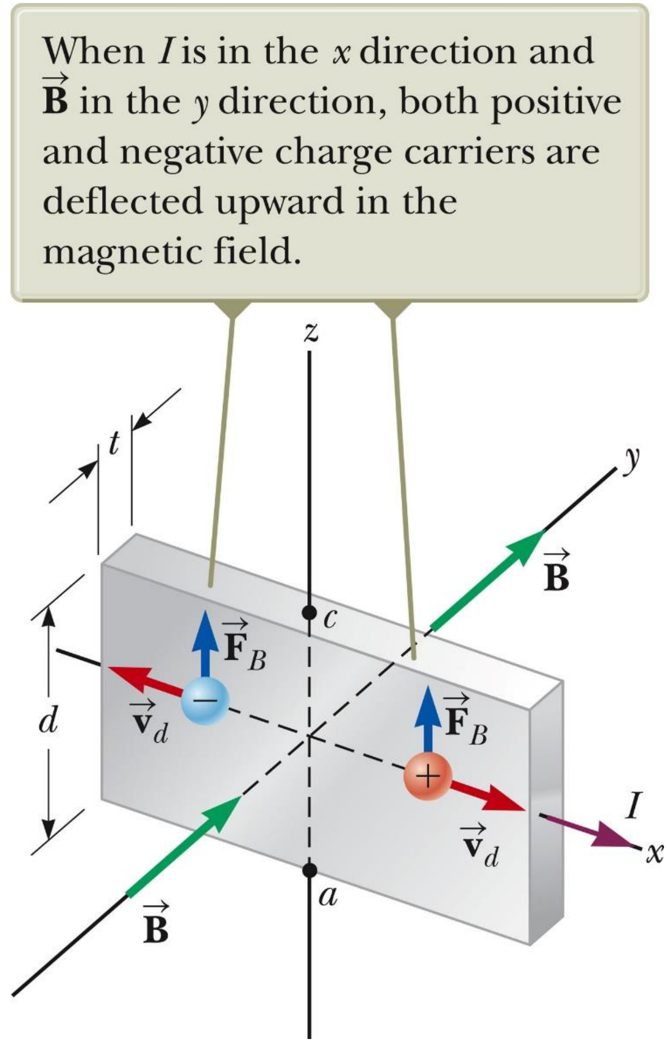
The Hall effect gives information regarding the sign of the charge carriers and their density.

It can also be used to measure magnetic fields.

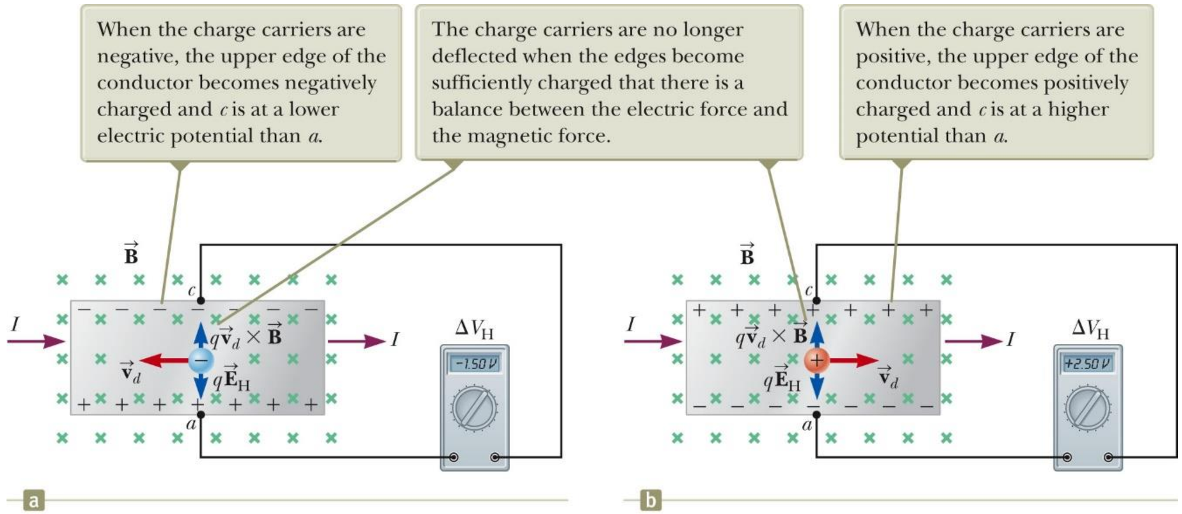
Hall Voltage

This shows an arrangement for observing the Hall effect.

The Hall voltage is measured between points *a* and *c.*



Hall Voltage, cont.



When the charge carriers are negative, they experience an upward magnetic force, they are deflected upward, an excess of positive charge is left at the lower edge.

This accumulation of charge establishes an electric field in the conductor.

It increases until the electric force balances the magnetic force.

If the charge carriers are positive, an excess of negative charges accumulates on the lower edge.

Hall Voltage, final

in equilibrium:

$$qv_d B = qE_{\mathrm{H}}$$

$$E_{\mathrm{H}} = v_d B$$

$$\Delta V_{\mathrm{H}} = E_{\mathrm{H}} d = v_d B d$$

- d is the width of the conductor
- v_d is the drift velocity
- If B and d are known, v_d can be found

$$v_d = \frac{I}{nqA} \Rightarrow \Delta V_{\mathrm{H}} = \frac{IBd}{nqA} \Rightarrow \Delta V_{\mathrm{H}} = \frac{IB}{nqt} = \frac{R_{\mathrm{H}} IB}{t}$$

- *A* = *td* , where *t* is the thickness of the conductor
- R_H = 1 / nq is called the **Hall coefficient.**

The sign and magnitude of R_H give the sign of the charge carriers and their number density.

The Hall Effect of a Copper, Example

A rectangular copper strip 1.5 cm wide and 0.10 cm thick carries a current of 5.0 A. Find the Hall voltage for a 1.2-T magnetic field applied in a direction perpendicular to the strip.

$$n = \frac{N_A}{V} = \frac{N_A \rho}{M}$$

The molar mass *M* and density ρ of copper

$$\Delta V_H = \frac{IB}{nqt} = \frac{MIB}{N_A \rho q t}$$

$$\Delta V_H = \frac{(0.063\,5 \text{ kg/mol})(5.0 \text{ A})(1.2 \text{ T})}{(6.02 \times 10^{23} \text{ mol}^{-1})(8\,920 \text{ kg/m}^3)(1.60 \times 10^{-19} \text{ C})(0.001\,0 \text{ m})}$$

$$= \boxed{0.44\ \mu\text{V}}$$