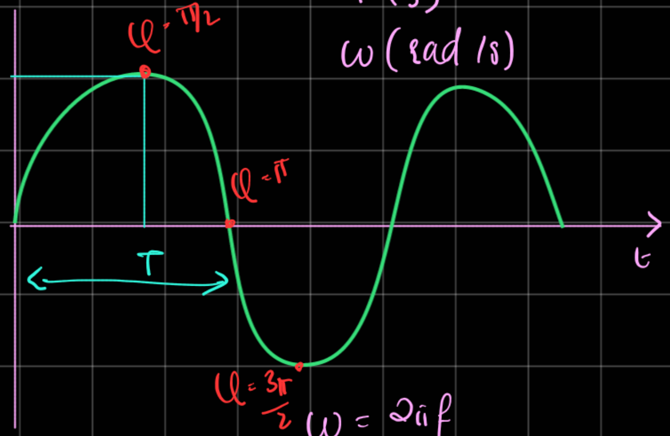
Alternating Current Circuits

f (Hz)
T (s)
ω (rad/s)

Time domain form

ΔV_{max} → amplitude



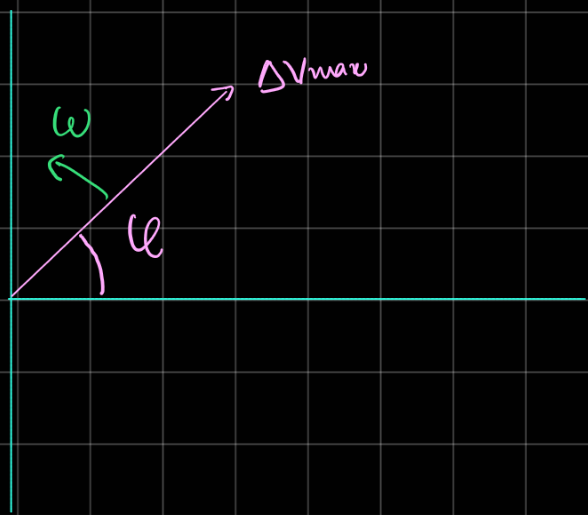
$$\Delta v = \Delta V_{max} \sin(\omega t - \varphi)$$

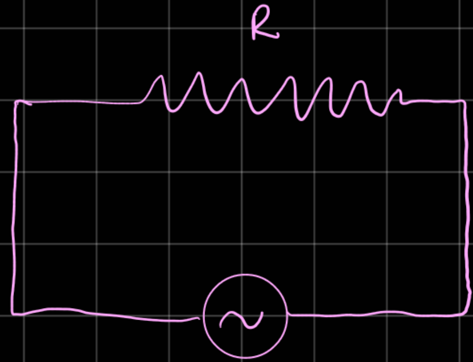
ΔV_{max} → amplitude, ωt → frequency, φ → phase

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Phasers

Length = Amplitude
Angle = Phase
Speed of rotation = ω





$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{max}}{R} \sin(\omega t)$$

$$= I_{max} \sin(\omega t) \quad \varphi = 0$$

$$\Delta V_R = i_R R = I_{max} R \sin(\omega t) \quad \varphi\, 0$$

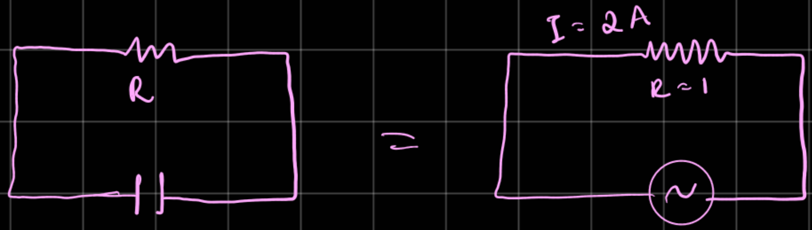
$$\Delta v = \Delta V_{max} \sin(\omega t)$$

Current & voltage for a resistor are always in phase

$P_{avg} = I_{rms}^2 R$

$rms \downarrow$ root mean square

$I_{rms} = \dfrac{I_{max}}{\sqrt{2}}$ $\Delta V_{rms} = \dfrac{\Delta V_{max}}{\sqrt{2}}$

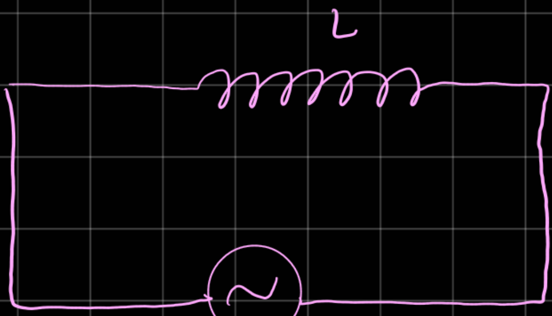


$I = 1.414\,A = 2/\sqrt{2}$

$\Delta V_{max} = 2V$

$\Delta v_L = \Delta V_{max} \sin(\omega t)$ $\varphi = 0$

$i_L = \dfrac{\Delta V_{max}}{X_L} \sin(\omega t - \pi/2)$ $\varphi = -\pi/2$



$\Delta v_L = \Delta V_{max} \sin(\omega t)$

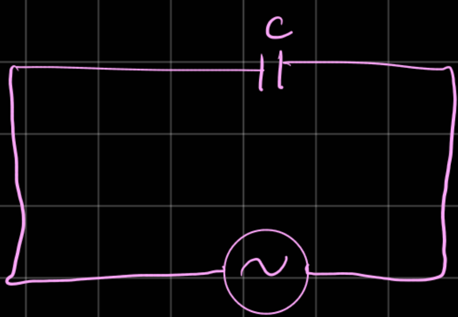
The current 'lags' behind the voltage by $\pi/2$

$X_L = \omega L = 2\pi f L \Rightarrow$ inductive reactance (Ω)

$f \uparrow$ $f \downarrow$

$X_L \uparrow$ $X_L \downarrow$

$L \to$ open circuit $L \to$ short circuit



$\Delta v_C = \Delta V_{max} \sin(\omega t)$

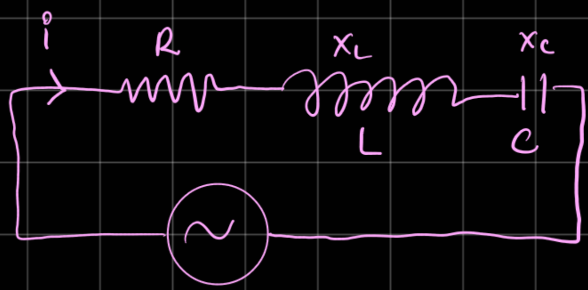
$i_C = \dfrac{\Delta V_{max}}{X_C} \sin(\omega t + \pi/2)$

The current 'leads' the voltage by $\pi/2$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$f \uparrow$ $\quad$ $X_C \downarrow$ $\quad$ C → short circuit

$f \downarrow$ $\quad$ $X_C \uparrow$ $\quad$ C → open circuit



$\Delta V = \Delta V_{max} \sin(\omega t)$

$$i = \frac{\Delta V}{Z} = \frac{\Delta V_{max} \sin(\omega t - \varphi)}{Z}$$

$$i = I_{max} \sin(\omega t - \varphi)$$

Z ⇒ impedance (Ω)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

φ phase angle (degree) $\tan^{-1}\left[\frac{X_L - X_C}{R}\right]$

$X_L = X_C$ ⇒ i & ΔV are in phase

$Z = \sqrt{R^2 + 0^2} = R$

Circuit is purely resistive

$\varphi = 0$

$X_L > X_C$ ⇒ more inductive than capacitive

φ is positive

current lags behind voltage

$X_L < X_C$ ⇒ more capacitive than inductive

φ is negative

current leads the voltage

$X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

$$\omega = \frac{1}{\sqrt{LC}}$$

resonance

resonant frequency

maximum current

In RLC Circuit

$$P_{avg} = \frac{1}{2} I_{max} V_{max} \cos\varphi$$

$$= I_{rms} V_{rms} \cos\varphi$$

$$= I_{rms}^2 R$$

$\cos\varphi \Rightarrow$ power factor

Power as function of frequency

$$Q = \frac{\omega_0}{\Delta\omega} = \omega_0 \frac{L}{R}$$

half power point