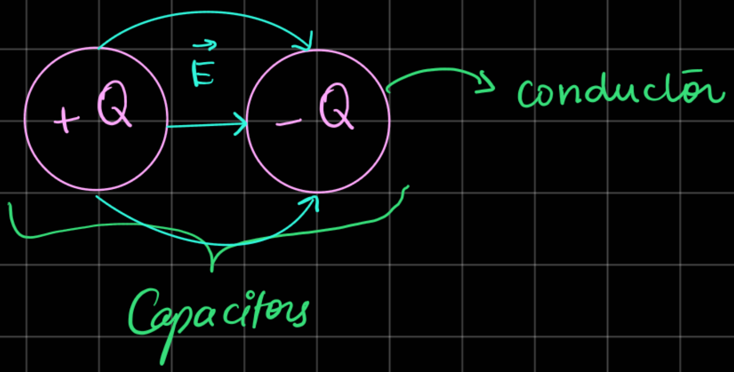
Capacitance & Dielectrics



$C = \dfrac{Q}{\Delta V}$

$Q \longrightarrow$ Charge of conductors

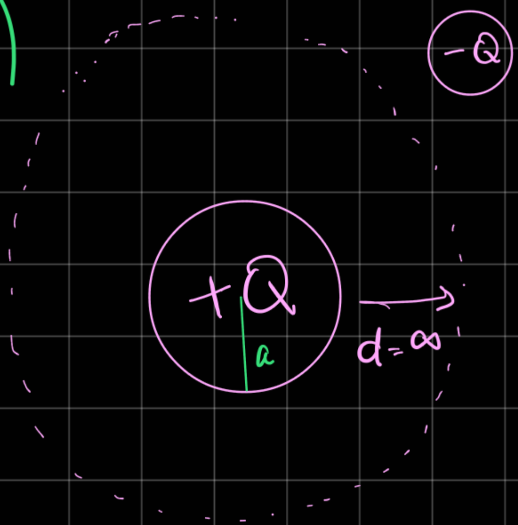
$\Delta V \longrightarrow$ Voltage created by electric field

$\Delta V = \vec{E} \cdot \vec{d}$
applies for all capacitor types

Capacitance
always +ve
always constant, regardless of charge on conductor

Unit $\Rightarrow$ Farad (F) $= \dfrac{\text{Coulomb}}{\text{Volt}}$

Isolated Sphere
(-ve charge at infinity)



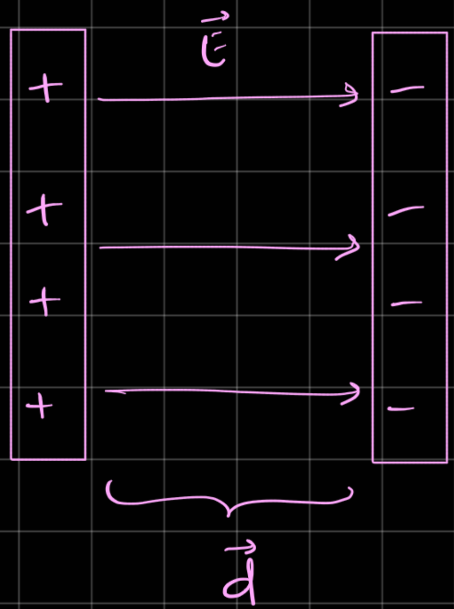
$C = Q$ Q a capacitance of

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/a} = \frac{a}{k_e}$$

Capacitance of isolated sphere

$k_e \to 8.85 \times 10^{9}$

Parallel Plates



$$C = \frac{Q}{\Delta V} = \frac{Q}{\vec{E}\cdot\vec{d}}$$

$$= \frac{Q}{\frac{\sigma}{\epsilon_0} d}$$

$$= \frac{Q}{\frac{Q}{A}\cdot\frac{1}{\epsilon_0} d}$$

$$= \frac{A\epsilon_0}{d}$$

$\epsilon_0 \to$ Permittivity of space 8.5×10^{-12}

$d \to$ distance b/w plates

$A \to$ area of plate

Electric Symbols

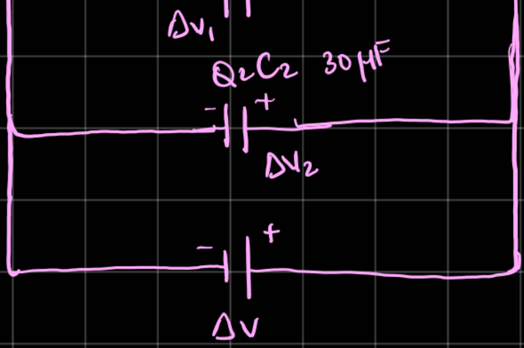
Battery　　Capacitor　　Open Switch　　Closed Switch

Capacitors in Parallel

Q_1 C_1 $10\ \mu F$

Q_{eq}

ΔV $C_{eq} = 40\ \mu F$

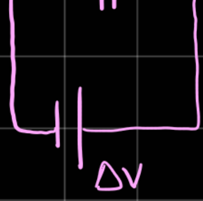


$C_{eq} > C_1, C_2$

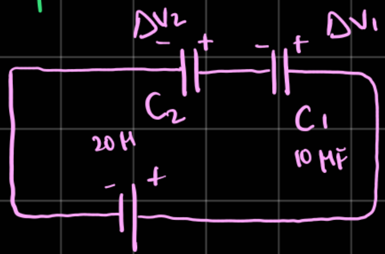
$C_{eq} = C_1 + C_2$

$\Delta V_1 = \Delta V_2$

$Q_{eq} = Q_1 + Q_2$



Capacitors in Series



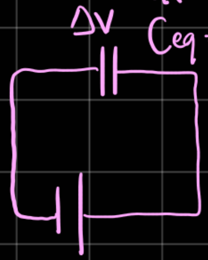
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{10}$$

$$= \frac{3}{20}$$

$$C_{eq} = \frac{20}{3}\ \mu F$$

$Q_1 = Q_2$



$C_{eq} = \frac{20}{3}\ \mu F$

$= 6.67\ \mu F$

$C_{eq} < C_1, C_2$

$\Delta V = \Delta V_1 + \Delta V_2$

$Q_1 = Q_2$

Example

$C_{eq_1} = 4\ \mu F$

$C_{eq_2} = \frac{1}{4} + \frac{1}{4} = 2\ \mu F$

$C_{eq_3} = 8\ \mu F$

$C_{eq_4} = \frac{1}{8} + \frac{1}{8} = 4\ \mu F$

$C_{eq} = 4 + 2 = 6\ \mu F$

Example

a) $C_{eq} = 17\ \mu F$

b) $\Delta V = 9\ V$

c) $Q_1 = C_1 \cdot \Delta V$ $\qquad$ $Q_2 = C_2\, \Delta V$

$= 5 \times 9$ $\qquad$ $= 12 \times 9$

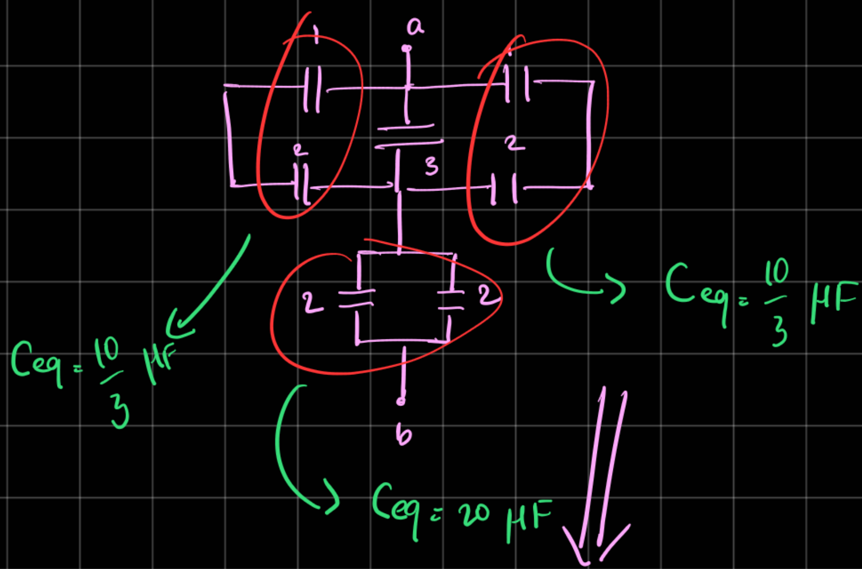
$= 45\ \mu C$ $\qquad$ $= 108\ \mu C$

Example

$C_1 = 5\ \mu F$

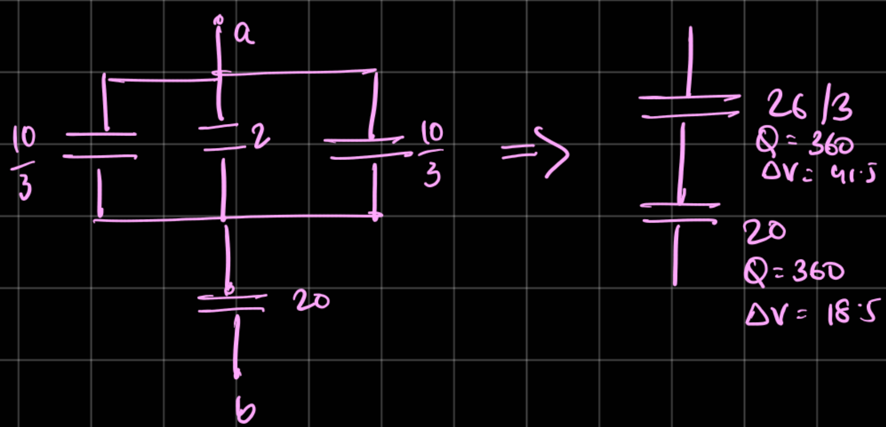
$C_2 = 10\ \mu F$

$C_3 = 2\ \mu F$



$\frac{10}{3} + \frac{10}{3} + 2$

$= \frac{26}{3}$



$\frac{1}{C_{eq}} = \frac{3}{26} + \frac{1}{20}$

$= \frac{60 + 26}{20 \times 26}$

$= 86$

$$= \frac{\quad}{20\times 26}$$

$$= \frac{20\times 26}{86}$$

$$= 6.05\ \mu F$$

$V = 60\ V$

$$C = \frac{Q}{\Delta V} \qquad Q_{eq} = C \cdot \Delta V$$

$$= 6 \times 60$$

$$= 360\ \mu C$$

$$Q_3 = C_3 \Delta V_3$$

$$= 2 \times 41.5$$

$$= 83\ \mu C$$

Energy stored on capacitors

$$U_E = \frac{Q^2}{2C} = \frac{Q^2}{2\,Q/\Delta V} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

$$C = \frac{Q}{\Delta V} \qquad Q = C\Delta V$$

Parallel plate

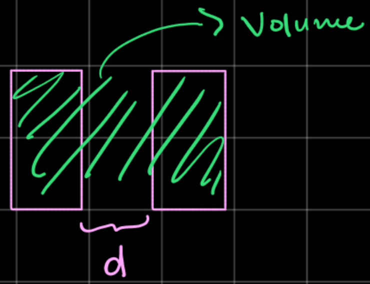
$$C = \frac{A\epsilon_0}{d} \qquad U_E = \frac{Q^2}{2C} = \frac{dQ^2}{2A\epsilon_0} = \frac{1}{2}\frac{A\epsilon_0}{\cancel{d}}(\vec{E}\cdot\vec{d})^2$$

$$= \frac{1}{2} AE^2 d\,\epsilon_0$$

Energy Density

$$\frac{U_E}{\text{Volume}} = \frac{\frac{1}{2}\epsilon_0 \cancel{Ad} E^2}{\cancel{Ad}}$$

$$= \frac{1}{2}\epsilon_0 E^2 \quad J/m^3$$



Dielectric

↳ insulator

inserted between +ve and negative plate

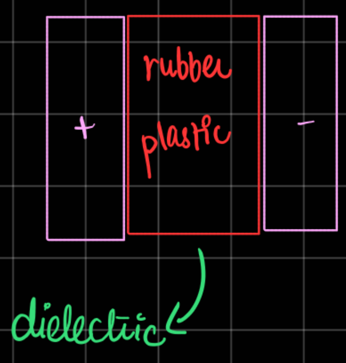
increases capacitance

makes capacitor more solid & rigid

$$C = \frac{k \varepsilon_0 A}{d}$$

↳ Dielectric constant

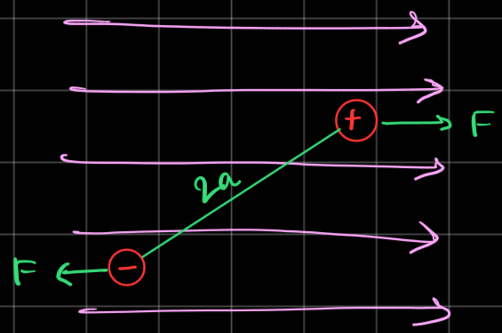
max. electric field that can be handled by insulator before it becomes a conductor



Dipole

dipole starts rotating in electric field

generates torque



net F = 0

$\vec{\tau} = \vec{P} \times \vec{E} = PE \sin\theta$

↳ cross product

↳ Electric dipole moment

$\vec{P} = 2aQ$

$U_E = -\vec{P} \cdot \vec{E} = -PE\cos\theta$

dot product

Example

$$U_E = \frac{Q^2}{2C}$$

$$= \frac{Q^2}{2Q/\Delta V}$$

$= \frac{1}{2} Q \Delta V$

$= \frac{1}{2} \times 54 \times 12$

$= 324 \text{ J}$

Example

$$\overset{\uparrow}{U_E} = \frac{Q^2}{\underset{\downarrow}{2C}}$$

b) parallel

Example

$C \uparrow \quad Q \uparrow \quad \Delta V = \quad U_E \uparrow$

Example

$$C = \frac{k \epsilon_0 A}{d}$$

$$= \frac{100 \times 8.85 \times 10^{-12} \times 1 \times 10^{-4}}{1 \times 10^{-3}}$$

$= 8.85 \times 10^{-11} \text{ F}$

Example

$U_0 = \frac{Q^2}{2C} \longrightarrow$ before

$U_0 = \frac{Q^2}{2kC} \longrightarrow$ after