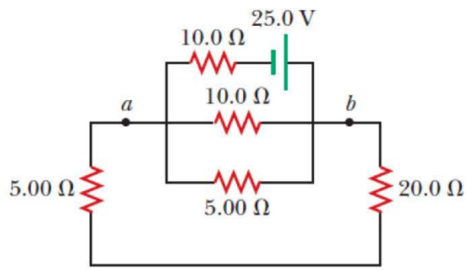
PHYS143
Physics for Engineers
Tutorial - Chapter 28 - Solutions

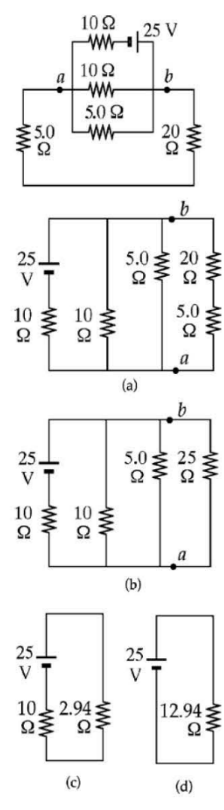
Question 1

Consider the circuit shown in Figure. Find (a) the current in the 20.0-Ω resistor and (b) the potential difference between points a and b.



If we turn the given diagram on its side and change the lengths of the wires, we find that it is the same as ANS. FIG. P28.9(a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is shown in ANS. FIG. P28.9(b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$\frac{1}{R_{eq}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega} \rightarrow R_{eq} = 2.94\ \Omega$$



This is shown in ANS. FIG. P28.9(c), which in turn reduces to the circuit shown in ANS. FIG. P28.9(d), from which we see that the total resistance of the circuit is 12.94 Ω.

Next, we work backwards through the diagrams applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The total 12.94-Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In ANS. FIG. P28.9(c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\text{ A})(2.94\ \Omega) = 5.68\text{ V}$$

From ANS. FIG. P28.9(b), we see that this potential difference is the same as the potential difference ΔV_{ab} across the 10-Ω resistor and the 5.00-Ω resistor. Thus we have first found the answer to part (b), which is

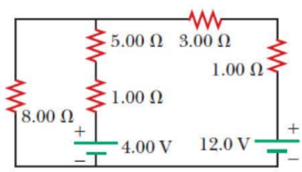
$$\Delta V_{ab} = \boxed{5.68\text{ V}}$$

Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

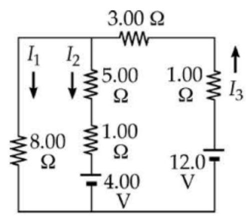
$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\text{ V}}{25.0\ \Omega} = 0.227\text{ A} = \boxed{227\text{ mA}}$$

Question 2

The circuit shown in Figure is connected for 2.00 min. (a) Determine the current in each branch of the circuit. (b) Find the energy delivered by each battery.



We name currents I_1, I_2, and I_3 as shown in ANS. FIG. P28.23. From Kirchhoff's current rule, $I_3 = I_1 + I_2$.



ANS. FIG.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\text{ V} - (4.00\ \Omega)I_3 - (6.00\ \Omega)I_2 - 4.00\text{ V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00\ \Omega)I_2 - 4.00\text{ V} + (8.00\ \Omega)I_1 = 0$$

or $$(8.00\ \Omega)I_1 = 4.00 + (6.00\ \Omega)I_2$$

Solving the above linear system (by substituting $I_1 + I_2$ for I_3), we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = \dfrac{4}{3}I_1 - \dfrac{2}{3} \end{cases}$$

and to the single equation

$$8 = 4I_1 + 10\left(\frac{4}{3}I_1 - \frac{2}{3}\right) = \frac{52}{3}I_1 - \frac{20}{3}$$

which gives

$$I_1 = \frac{3}{52}\left(8 + \frac{20}{3}\right) = 0.846 \text{ A}$$

Then $\quad I_2 = I_2 = \frac{4}{3}(0.846) - \frac{2}{3} = 0.462$

and $\quad I_3 = I_1 + I_2 = 1.31 \text{ A}$

give $\quad \boxed{I_1 = 846 \text{ mA}, I_2 = 462 \text{ mA}, I_3 = 1.31 \text{ A}}$

(a) The results are: 0.846 A down in the 8.00-Ω resistor; 0.462 A down in the middle branch; 1.31 A up in the right-hand branch.

(b) For 4.00-V battery:

$$\Delta U = P\Delta t = (\Delta V)I\Delta t = (4.00 \text{ V})(-0.462 \text{ A})(120 \text{ s}) = -222 \text{ J}$$

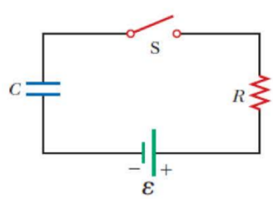
For 12.0-V battery:

$$\Delta U = (12.0 \text{ V})(1.31 \text{ A})(120 \text{ s}) = 1.88 \text{ kJ}$$

The results are: –222 J by the 4.00-V battery and 1.88 kJ by the 12.0-V battery.

Question 3

Consider a series RC circuit as in Figure for which R = 1.00 MΩ, C = 5.00 μF, and ε = 30.0 V. Find (a) the time constant of the circuit and (b) the maximum charge on the capacitor after the switch is thrown closed. (c) Find the current in the resistor 10.0 s after the switch is closed.



(a) The time constant is

$$RC = (1.00 \times 10^6 \ \Omega)(5.00 \times 10^{-6} \text{ F}) = \boxed{5.00 \text{ s}}$$

(b) After a long time interval, the capacitor is "charged to thirty volts," separating charges of

$$Q = C\mathcal{E} = (5.00 \times 10^{-6} \text{ C})(30.0 \text{ V}) = \boxed{150 \ \mu\text{C}}$$

$$I(t)=\frac{\varepsilon}{R}e^{-t/RC}=\left(\frac{30.0\ \text{V}}{1.00\times10^{6}\ \Omega}\right)\exp\left[\frac{-10.0\ \text{s}}{(1.00\times10^{6}\ \Omega)(5.00\times10^{-6}\ \text{F})}\right]$$

(c)
$$=\boxed{4.06\ \mu\text{A}}$$

Question 4

A 10.0-μF capacitor is charged by a 10.0-V battery through a resistance R. The capacitor reaches a potential difference of 4.00 V in a time interval of 3.00 s after charging begins. Find R.

The potential difference across the capacitor is

$$\Delta V(t)=\Delta V_{\max}\left(1-e^{-t/RC}\right)$$

Using 1 farad = 1 s/Ω,

$$4.00\ \text{V}=(10.0\ \text{V})\left[1-e^{-(3.00\ \text{s})/\left[R(10.0\times10^{-6}\ \text{s}/\Omega)\right]}\right]$$

Therefore,

$$0.400=1.00-e^{-(3.00\times10^{5}\ \Omega)/R}$$

or
$$e^{-(3.00\times10^{5}\ \Omega)/R}=0.600.$$

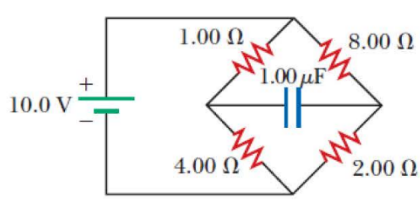
Taking the natural logarithm of both sides,

$$-\frac{3.00\times10^{5}\ \Omega}{R}=\ln(0.600)$$

and
$$R=-\frac{3.00\times10^{5}\ \Omega}{\ln(0.600)}=+5.87\times10^{5}\ \Omega=\boxed{587\ \text{k}\Omega}.$$

Question 5

The circuit in Figure has been connected for a long time. (a) What is the potential difference across the capacitor? (b) If the battery is disconnected from the circuit, over what time interval does the capacitor discharge to one-tenth its initial voltage?



(a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.

$$I_L = \frac{10.0 \text{ V}}{5.00\ \Omega} = 2.00 \text{ A}$$

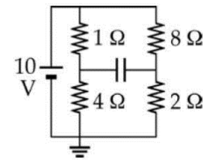
$$V_L = 10.0 \text{ V} - (2.00 \text{ A})(1.00\ \Omega) = 8.00 \text{ V}$$

$$I_R = \frac{10.0 \text{ V}}{10.0\ \Omega} = 1.00 \text{ A}$$

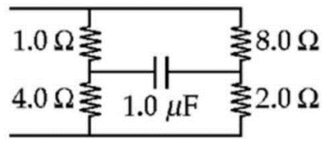
$$V_R = (10.0 \text{ V}) - (8.00\ \Omega)(1.00 \text{ A}) = 2.00 \text{ V}$$

Therefore,

$$\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00 \text{ V}}$$



ANS. FIG. P28.43(a)



ANS. FIG. P28.4

(b) We suppose the battery is pulled out leaving an open circuit. We are left with ANS. FIG. P28.43(b), which can be reduced to the equivalent circuits shown in ANS. FIG. P28.43(c) and ANS.

FIG. P28.43(d). From ANS. FIG. P28.43(d), we can see that the capacitor discharges through a 3.60-Ω equivalent resistance.

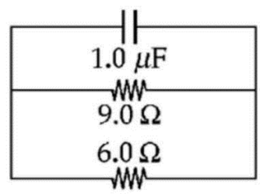
According to $q = Q_i e^{-t/RC}$,

we calculate that $q/C = (Q_i e^{-t/RC})/C$

and $\Delta V = \Delta V_i e^{-t/RC}$.

We proceed to solve for t:

$$\frac{\Delta V}{\Delta V_i} = e^{-t/RC} \quad \text{or} \quad \frac{\Delta V_i}{\Delta V} = e^{+t/RC}$$

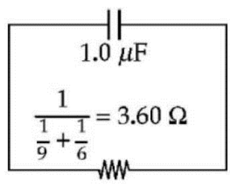


ANS. FIG. P28.43(c)

Take natural logarithms of both sides:

$$\ln\left(\frac{\Delta V_i}{\Delta V}\right) = t/RC$$

$$t = RC \ln\left(\frac{\Delta V_i}{\Delta V}\right)$$



ANS. FIG. P28.43(d)

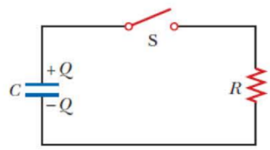
so

$$= (3.60\ \Omega)(1.00 \times 10^{-6} \text{ F}) \ln\left(\frac{\Delta V_i}{0.100 \Delta V_i}\right) = (3.60 \times 10^{-6} \text{ s}) \ln 10$$

$$= \boxed{8.29\ \mu\text{s}}$$

Question 6

A charged capacitor is connected to a resistor and switch as in Figure. The circuit has a time constant of 1.50 s. Soon after the switch is closed, the charge on the capacitor is 75.0% of its initial charge. (a) Find the time interval required for the capacitor to reach this charge. (b) If R = 250 kΩ, what is the value of C?



(a) The charge remaining on the capacitor after time t is $q = Qe^{-t/\tau}$.

Thus, if $q = 0.750Q$, then

$$0.750Q = Qe^{-t/\tau}$$

$$e^{-t/\tau} = 0.750$$

$$t = -\tau \ln(0.750) = -(1.50\text{ s})\ln(0.750) = \boxed{0.432\text{ s}}$$

(b) $\tau = RC$, so

$$C = \frac{\tau}{R} = \frac{1.50\text{ s}}{250 \times 10^{3}\ \Omega} = 6.00 \times 10^{-6}\text{ F} = \boxed{6.00\ \mu\text{F}}$$