



# PHYS143

## Physics for Engineers

### Tutorial - Chapter 44 – Solutions

#### Question 1

Calculate the difference in binding energy per nucleon for the nuclei  $^{23}_{11}\text{Na}$  and  $^{23}_{12}\text{Mg}$  (Mass of Neutral atom: Na: 22.989769, Mg: 22.994124, H: 1.007 825, and 1.008 665 for neutron).

We use

$$E_b (\text{MeV}) = [ZM(\text{H}) + Nm_n - M(^A_Z\text{X})](931.494 \text{ MeV/u})$$

Then, for  $^{23}_{11}\text{Na}$ ,

$$\begin{aligned} E_b(^{23}_{11}\text{Na}) &= [11M(\text{H}) + 12m_n - M(^{23}_{11}\text{Na})](931.494 \text{ MeV/u}) \\ &= [11(1.007 825 \text{ u}) + 12(1.008 665 \text{ u}) - 22.989 769 \text{ u}] \\ &\quad \times (931.494 \text{ MeV/u}) \\ &= 186.565 \text{ MeV} \end{aligned}$$

and  $\frac{E_b}{A} = \frac{186.565 \text{ MeV}}{23} = 8.11 \text{ MeV}$

For  $^{23}_{12}\text{Mg}$ ,

$$\begin{aligned} E_b &= E_b(^{23}_{12}\text{Mg}) \\ &= [12M(\text{H}) + 11m_n - M(^{23}_{12}\text{Mg})](931.494 \text{ MeV/u}) \\ &= [12(1.007 825 \text{ u}) + 11(1.008 665 \text{ u}) - 22.994 124 \text{ u}] \\ &\quad \times (931.494 \text{ MeV/u}) \\ &= 181.726 \text{ MeV} \end{aligned}$$

and  $\frac{E_b}{A} = \frac{181.726 \text{ MeV}}{23} = 7.90 \text{ MeV}$

The difference is

$$\begin{aligned} \frac{\Delta E_b}{A} &= \frac{E_b(^{23}_{11}\text{Na}) - E_b(^{23}_{12}\text{Mg})}{A} \\ &= 8.11 \text{ MeV} - 7.90 \text{ MeV} = 0.210 \text{ MeV} \end{aligned}$$



### Question 2

Calculate the minimum energy required to remove a neutron from the  $^{43}_{20}\text{Ca}$  nucleus. (Mass of Neutral atom:  $^{43}_{20}\text{Ca}$ : 42.958767,  $^{42}_{20}\text{Ca}$ : 41.958618, and 1.008 665 for neutron)

Removal of a neutron from  $^{43}_{20}\text{Ca}$  would result in the residual nucleus,  $^{42}_{20}\text{Ca}$ . If the required separation energy is  $\Delta E_n$ , the overall process can be described by

$$\text{mass}(^{43}_{20}\text{Ca}) + \Delta E_n = \text{mass}(^{42}_{20}\text{Ca}) + \text{mass}(n)$$

$$\Delta E_n = \text{mass}(^{42}_{20}\text{Ca}) + \text{mass}(n) - \text{mass}(^{43}_{20}\text{Ca})$$

$$\begin{aligned}\Delta E_n &= (41.958\,618\,\text{u} + 1.008\,665\,\text{u} - 42.958\,767\,\text{u}) \\ &\quad \times (931.5\,\text{MeV/u}) \\ &= \boxed{7.93\,\text{MeV}}\end{aligned}$$

### Question 3

A freshly prepared sample of a certain radioactive isotope has an activity of 10.0 mCi. After 4.00 h, its activity is 8.00 mCi. Find (a) the decay constant and (b) the half-life. (c) How many atoms of the isotope were contained in the freshly prepared sample? (d) What is the sample's activity 30.0 h after it is prepared?

(a) From  $R = R_0 e^{-\lambda t}$ , the decay constant is

$$\begin{aligned}\lambda &= \frac{1}{t} \ln \left( \frac{R_0}{R} \right) = \left( \frac{1}{4.00\,\text{h}} \right) \ln \left( \frac{10.0\,\text{mCi}}{8.00\,\text{mCi}} \right) = 5.58 \times 10^{-2}\,\text{h}^{-1} \\ &= \boxed{1.55 \times 10^{-5}\,\text{s}^{-1}}\end{aligned}$$

(b) The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\,\text{h}}$$

(c) The number of original atoms can be found if we convert the initial activity from curies into becquerels (decays per second):

$$1\,\text{Ci} \equiv 3.70 \times 10^{10}\,\text{Bq}.$$

$$\begin{aligned}R_0 &= 10.0\,\text{mCi} = (10.0 \times 10^{-3}\,\text{Ci})(3.70 \times 10^{10}\,\text{Bq/Ci}) \\ &= 3.70 \times 10^8\,\text{Bq}\end{aligned}$$

Since  $R_0 = \lambda N_0$ , the original number of nuclei is

$$N_0 = \frac{R_0}{\lambda} = \frac{3.70 \times 10^8\,\text{decays/s}}{1.55 \times 10^{-5}\,\text{s}} = \boxed{2.39 \times 10^{13}\,\text{atoms}}$$

(d) The decay rate after thirty hours is



$$R = R_0 e^{-\lambda t} = (10.0 \text{ mCi}) \exp \left[ (-5.58 \times 10^{-2} \text{ h}^{-1})(30.0 \text{ h}) \right]$$

$$= \boxed{1.88 \text{ mCi}}$$

#### Question 4

The radioactive isotope  $^{198}\text{Au}$  has a half-life of 64.8 h. A sample containing this isotope has an initial activity ( $t = 0$ ) of 40.0  $\mu\text{Ci}$ . Calculate the number of nuclei that decay in the time interval between  $t_1 = 10.0 \text{ h}$  and  $t_2 = 12.0 \text{ h}$ .

The number of nuclei that decay during the interval will be

$$\Delta N = N_1 - N_2 = N_0 (e^{-\lambda t_1} - e^{-\lambda t_2})$$

First we find the decay constant  $\lambda$ :

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{64.8 \text{ h}} = 0.0107 \text{ h}^{-1} = 2.97 \times 10^{-6} \text{ s}^{-1}$$

Now we find  $N_0$ :

$$N_0 = \frac{R_0}{\lambda} = \frac{(40.0 \mu\text{Ci})}{2.97 \times 10^{-6} \text{ s}^{-1}} \left( \frac{3.70 \times 10^4 \text{ s}^{-1}}{\mu\text{Ci}} \right)$$

$$= 4.98 \times 10^{11} \text{ nuclei}$$

Substituting in these values,

$$N_1 - N_2 = (4.98 \times 10^{11}) \left[ e^{-(\ln 2/64.8 \text{ h})(10.0 \text{ h})} - e^{-(\ln 2/64.8 \text{ h})(12.0 \text{ h})} \right]$$

$$N_1 - N_2 = \boxed{9.47 \times 10^9 \text{ nuclei}}$$

#### Question 5

Determine which decays can occur spontaneously.

- (a)  $^{40}_{20}\text{Ca} \rightarrow e^+ + ^{40}_{19}\text{K}$       (b)  $^{98}_{44}\text{Ru} \rightarrow ^4_2\text{He} + ^{94}_{42}\text{Mo}$   
 (c)  $^{144}_{60}\text{Nd} \rightarrow ^4_2\text{He} + ^{140}_{58}\text{Ce}$

Atomic Masses:  $^{40}\text{Ca}$  (39.962 591),  $^{40}\text{K}$  (39.963 998),  $^{98}\text{Ru}$  (97.905 287),  $^{94}\text{Mo}$  (93.905 088),  $^{144}\text{Nd}$  (143.910 087),  $^{140}\text{Ce}$  (139.905 439),  $m_e$  (0.000549),  $M_\alpha$  (4.002603).

- (a) For this  $e^+$  decay,

$$Q = (M_X - M_Y - 2m_e)c^2$$

$$= [39.962 591 \text{ u} - 39.963 999 \text{ u} - 2(0.000 549 \text{ u})]$$

$$\times (931.5 \text{ MeV/u})$$

$$Q = -2.33 \text{ MeV}$$

Since  $Q < 0$ , the decay cannot occur spontaneously.

- (b) For this alpha decay,



$$Q = (M_X - M_Y - M_\alpha)c^2$$

$$= [97.905\,287\,\text{u} - 4.002\,603\,\text{u} - 93.905\,088\,\text{u}]$$

$$\times (931.5\,\text{MeV/u})$$

$$Q = -2.24\,\text{MeV}$$

Since  $Q < 0$ , the decay cannot occur spontaneously.

- (c) For this alpha decay,

$$Q = (M_X - M_Y - M_\alpha)c^2$$

$$= [143.910\,083\,\text{u} - 4.002\,603\,\text{u} - 139.905\,434\,\text{u}]$$

$$\times (931.5\,\text{MeV/u})$$

$$Q = 1.91\,\text{MeV}$$

Since  $Q > 0$ , the decay can occur spontaneously.

### Question 6

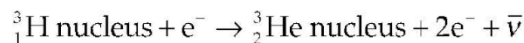
A  ${}^3_1\text{H}$  nucleus beta decays into  ${}^3_2\text{He}$  by creating an electron and an antineutrino according to the reaction



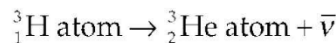
Determine the total energy released in this decay. Atomic Masses:  ${}^3\text{H}$  (3.016 049),  ${}^3\text{He}$  (3.016 029).

- (a) The reaction is  ${}^3_1\text{H} \rightarrow {}^3_2\text{He} + e^- + \bar{\nu}$ .

Adding one electron, the reaction becomes



Ignoring the slight difference in ionization energies, we have



- (b) The total energy released is the  $Q$  value:

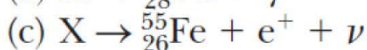
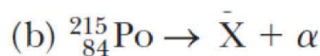
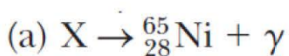
$$Q = (M_{{}^3\text{H}} - M_{{}^3\text{He}})c^2$$

$$Q = (3.016\,049\,\text{u} - 3.016\,029\,\text{u})(931.5\,\text{MeV/u})$$

$$= 0.018\,6\,\text{MeV} = \boxed{18.6\,\text{keV}}$$

### Question 7

Identify the unknown nuclide or particle (X).





Total  $Z$  and  $A$  are conserved.

- (a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray  $Z = 0$  and  $A = 0$ . Keeping the total values of  $Z$  and  $A$  for the system conserved requires  $Z = 28$  and  $A = 65$  for  $X$ . With this atomic number it must be nickel, and the nucleus must be in an excited state, so  $X$  is



- (b) An alpha particle,  $\alpha = {}^4_2\text{He}$ , has  $Z = 2$  and  $A = 4$ . Total initial  $Z$  is 84, and total initial  $A$  is 215, so for  $X$  we require

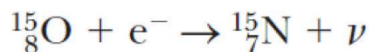
$$Z = 84 = Z_X + 2 \rightarrow Z_X = 82 \rightarrow \text{Pb, and}$$

$$A = 215 = A_X + 4 \rightarrow A_X = 211, \rightarrow X \text{ is } \boxed{{}^{211}_{82}\text{Pb}}.$$

- (c) A positron,  $e^+ = {}^0_1e$ , has charge the same as a nucleus with  $Z = 1$ . A neutrino,  ${}^0_0\nu$ , has no charge. Neither contains any protons or neutrons. So  $X$  must have by conservation  $Z = 26 + 1 + 0 = 27$ ; so,  $X$  is Co. And  $A = 55 + 0 + 0 = 55$ ;  $X$  is  $\boxed{{}^{55}_{27}\text{Co}}$ .

### Question 8

The nucleus  ${}^{15}_8\text{O}$  decays by electron capture. The nuclear reaction is written

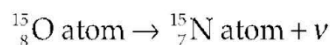


(a) Write the process going on for a single particle within the nucleus. (b) Disregarding the daughter's recoil, determine the energy of the neutrino. Atomic masses:  ${}^{15}\text{O}$  (15.003 066),  ${}^{15}\text{N}$  (15.000 109)

- (a) The reaction for one particle is  $\boxed{e^- + p \rightarrow n + \nu}$ .

- (b) For nuclei,  ${}^{15}_8\text{O} + e^- \rightarrow {}^{15}_7\text{N} + \nu$ .

Add seven electrons to both sides to obtain



$$\begin{aligned} Q &= (15.003\,065\,\text{u} - 15.000\,109\,\text{u})(931.5\,\text{MeV/u}) \\ &= \boxed{2.75\,\text{MeV}} \end{aligned}$$