Chapter 31

Faraday's Law

Induced Fields

Magnetic fields may vary in time.

Experiments conducted in 1831 showed that an emf can be induced in a circuit by a changing magnetic field.

- Experiments were done by Michael Faraday and Joseph Henry.

The results of these experiments led to *Faraday's Law of Induction.*

An *induced current* is produced by a changing magnetic field.

There is an *induced emf* associated with the induced current.

A current can be produced without a battery present in the circuit.

Faraday's law of induction describes the induced emf.

Michael Faraday

1791 – 1867

British physicist and chemist

Great experimental scientist

Contributions to early electricity include:

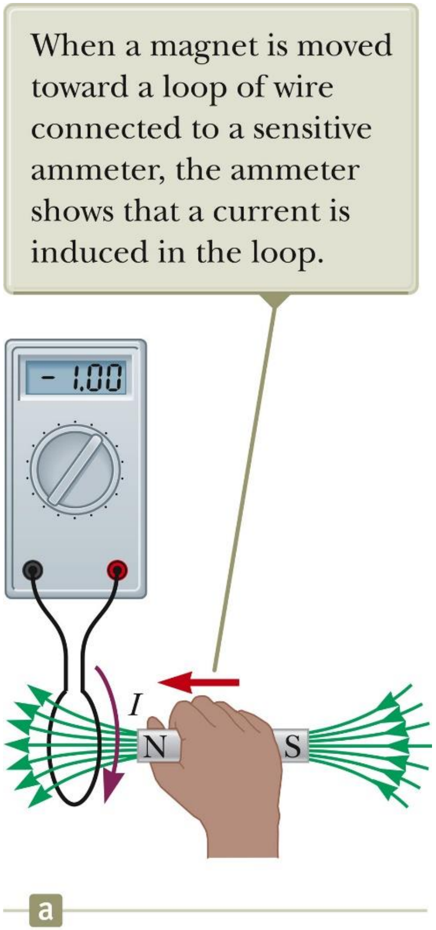
- Invention of motor, generator, and transformer
- Electromagnetic induction
- Laws of electrolysis

EMF Produced by a Changing Magnetic Field, 1

A loop of wire is connected to a sensitive ammeter.

When a magnet is moved toward the loop, the ammeter deflects.

- The direction was arbitrarily chosen to be negative.

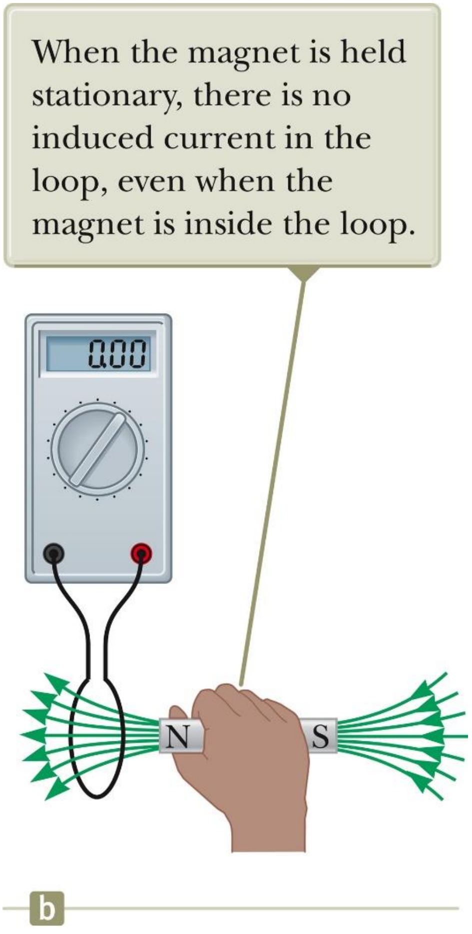


EMF Produced by a Changing Magnetic Field, 2

When the magnet is held stationary, there is no deflection of the ammeter.

Therefore, there is no induced current.

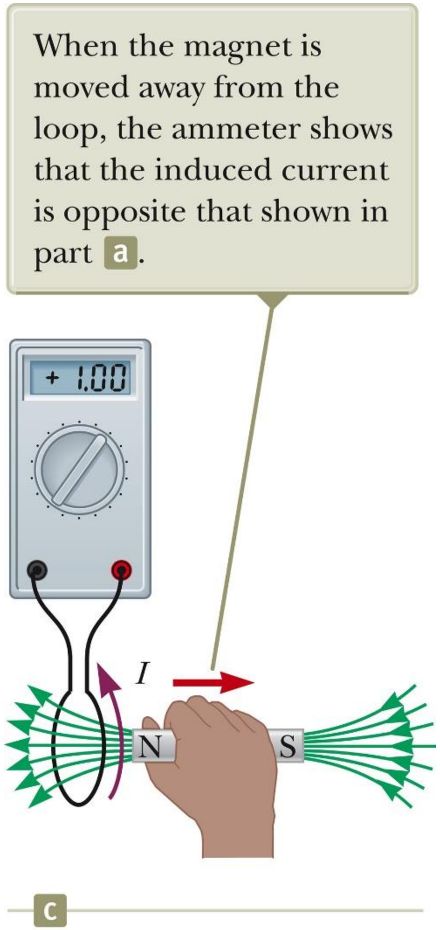
- Even though the magnet is in the loop



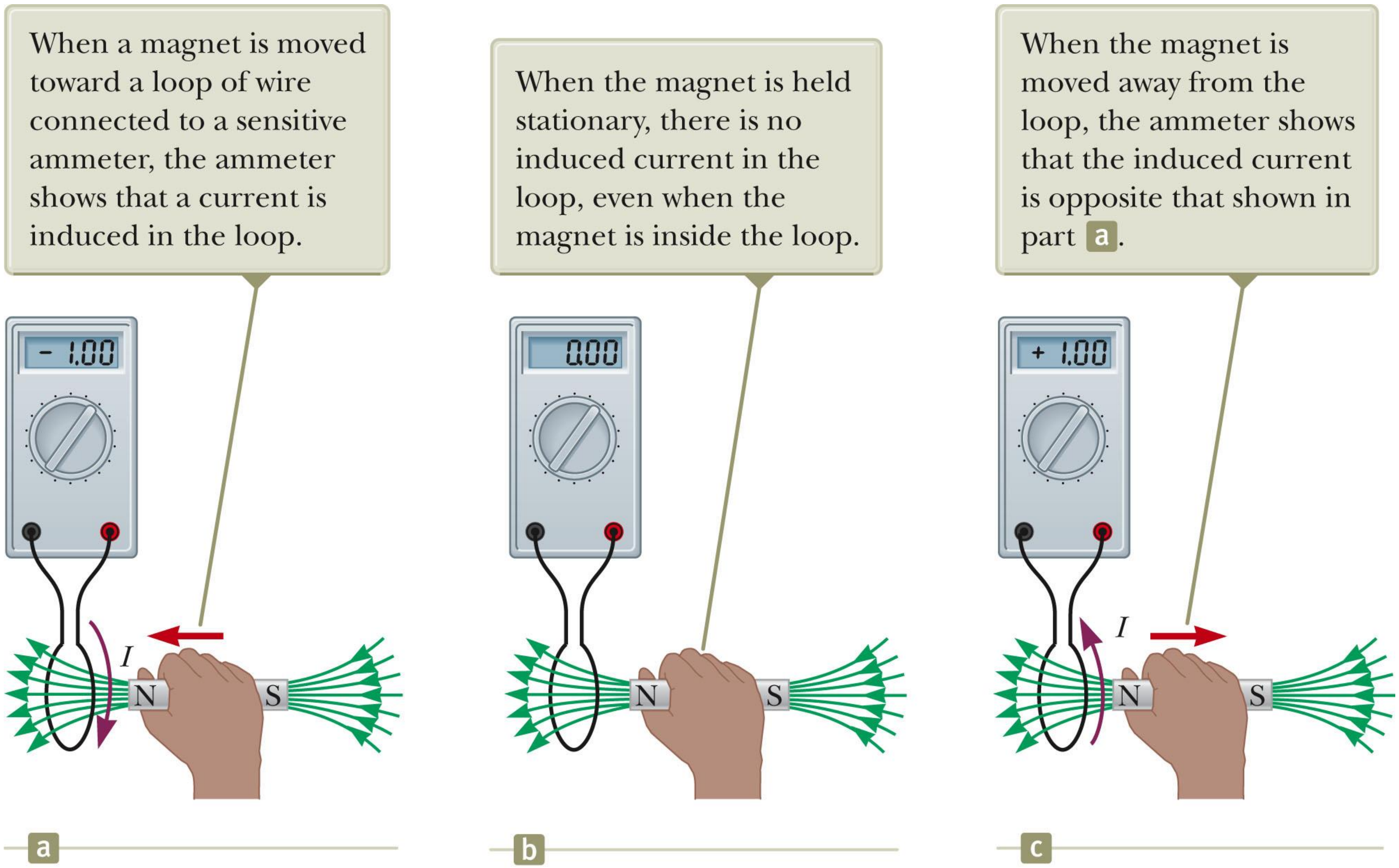
EMF Produced by a Changing Magnetic Field, 3

The magnet is moved away from the loop.

The ammeter deflects in the opposite direction.



Induced Current Experiment, Summary



EMF Produced by a Changing Magnetic Field, Summary

The ammeter deflects when the magnet is moving toward or away from the loop.

The ammeter also deflects when the loop is moved toward or away from the magnet.

Therefore, the loop detects that the magnet is moving relative to it.

- We relate this detection to a change in the magnetic field.
- This is the induced current that is produced by an induced emf.

Faraday’s Experiment – Set Up

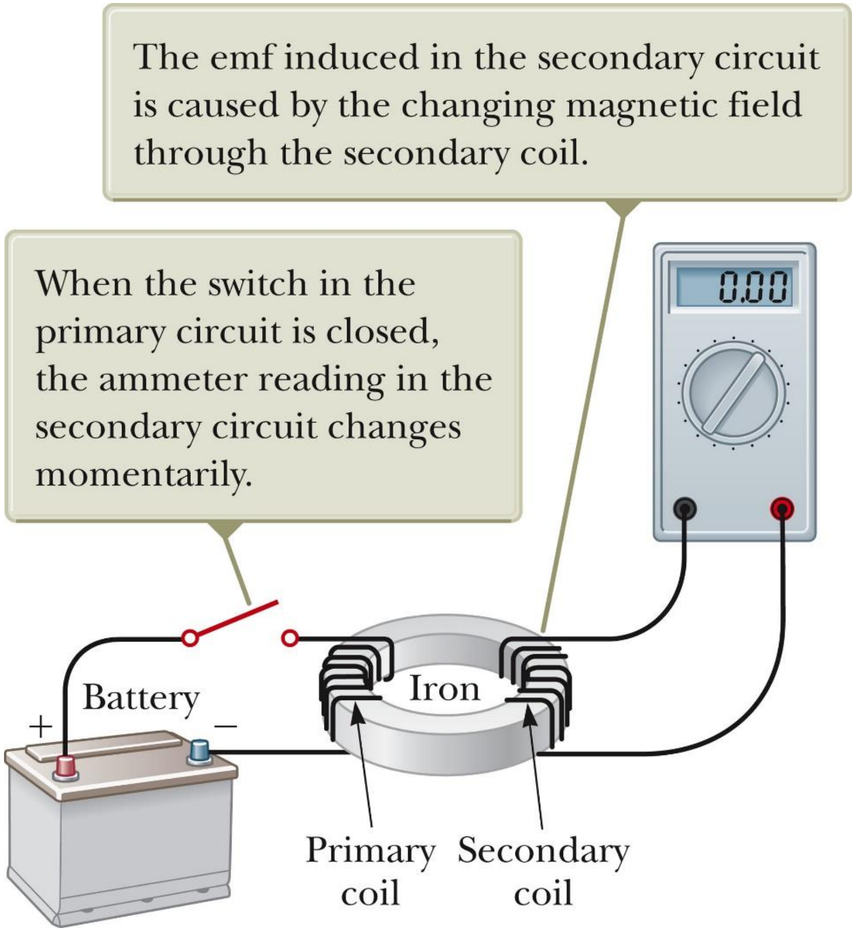
A primary coil is connected to a switch and a battery.

The wire is wrapped around an iron ring.

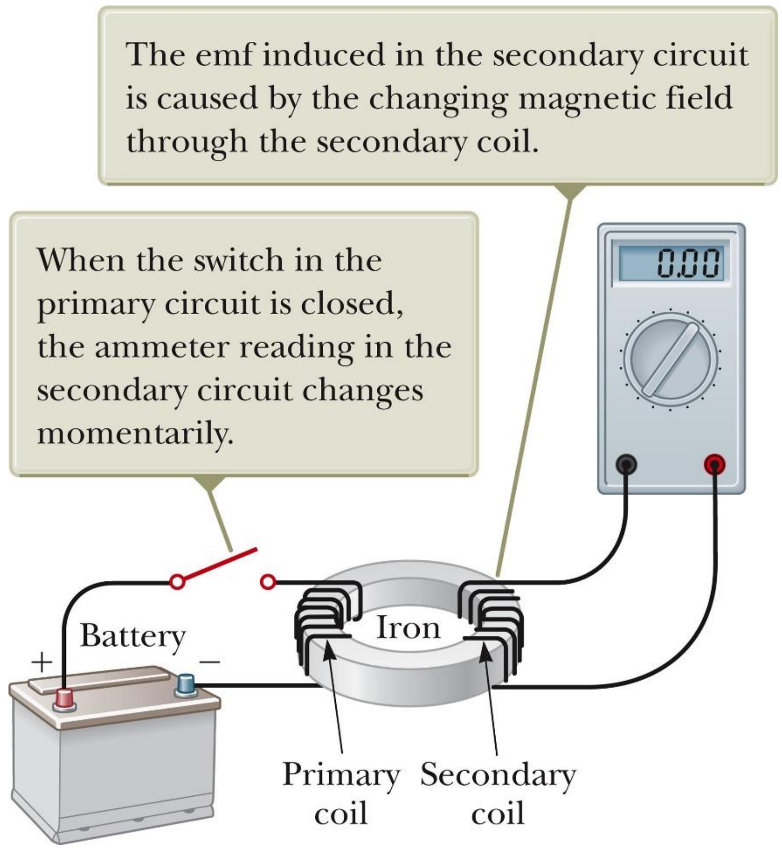
A secondary coil is also wrapped around the iron ring.

There is no battery present in the secondary coil.

The secondary coil is not directly connected to the primary coil.



Faraday's Experiment



A current in the primary coil produces a magnetic field when the switch is closed.

Close the switch and observe the current readings given by the ammeter.

Faraday's Experiment – Findings

At the instant the switch is closed, the ammeter changes from zero in one direction and then returns to zero.

When the switch is opened, the ammeter changes in the opposite direction and then returns to zero.

The ammeter reads zero when there is a steady current or when there is no current in the primary circuit.

Faraday's Experiment – Conclusions

An electric current can be induced in a loop by a changing magnetic field.

- This would be the current in the secondary circuit of this experimental set-up.

The induced current exists only while the magnetic field through the loop is changing.

This is generally expressed as: ***an induced emf is produced in the loop by the changing magnetic field.***

- The actual existence of the magnetic flux is not sufficient to produce the induced emf, the flux must be changing.

Faraday's Law of Induction – Statements

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

Mathematically,

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Remember Φ_B is the magnetic flux through the circuit and is found by

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

If the circuit consists of N loops, all of the same area, and if Φ_B is the flux through one loop, an emf is induced in every loop and Faraday's law becomes

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

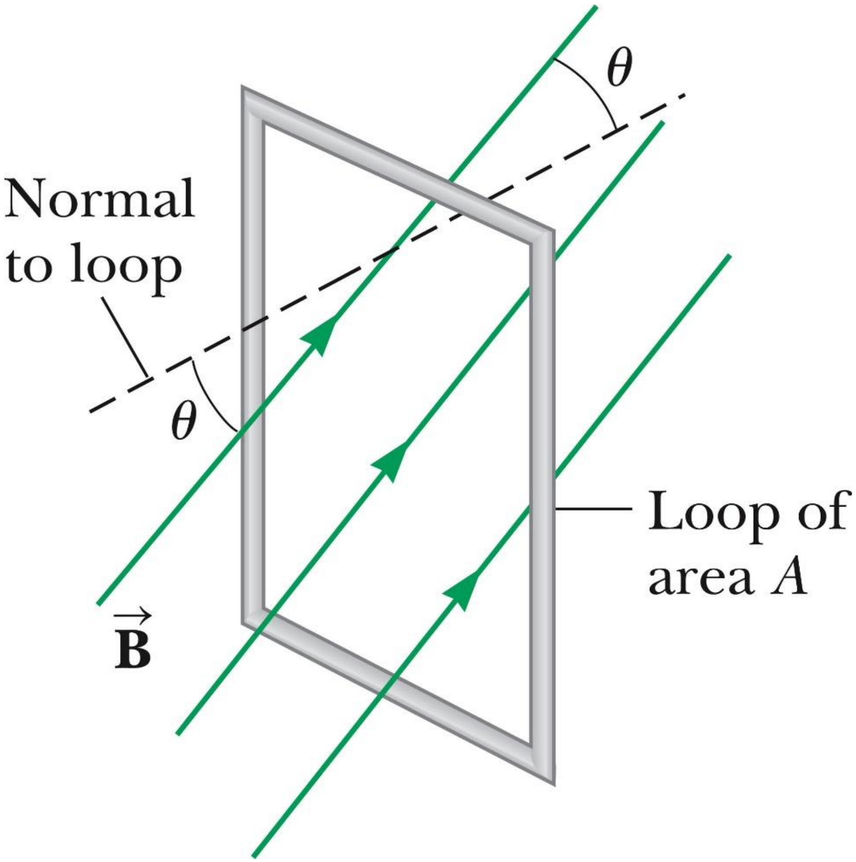
Faraday's Law – Example

Assume a loop enclosing an area A lies in a uniform magnetic field.

The magnetic flux through the loop is $\Phi_B = BA \cos \theta$.

The induced emf is

$$\mathcal{E} = -\frac{d}{dt}(BA \cos \theta)$$



Ways of Inducing an emf

- The magnitude of $\vec{\mathbf{B}}$ can change with time.
- The area enclosed by the loop can change with time.
- The angle θ between $\vec{\mathbf{B}}$ and the normal to the loop can change with time.
- Any combination of the above can occur.

Applications of Faraday's Law – GFCI

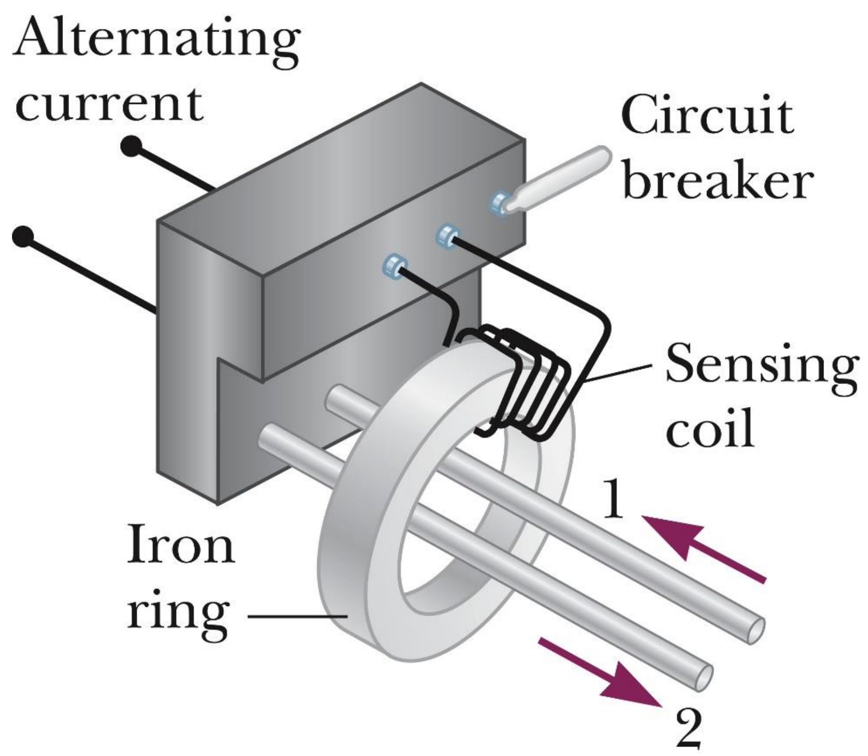
A GFCI (ground fault circuit interrupter) protects users of electrical appliances against electric shock.

Wire 1 leads from the wall outlet to the appliance to be protected and wire 2 leads from the appliance back to the wall outlet.

When the currents in the wires are in opposite directions, the flux is zero.

When the return current in wire 2 changes, the flux is no longer zero.

The resulting induced emf can be used to trigger a circuit breaker.



Applications of Faraday's Law – Pickup Coil

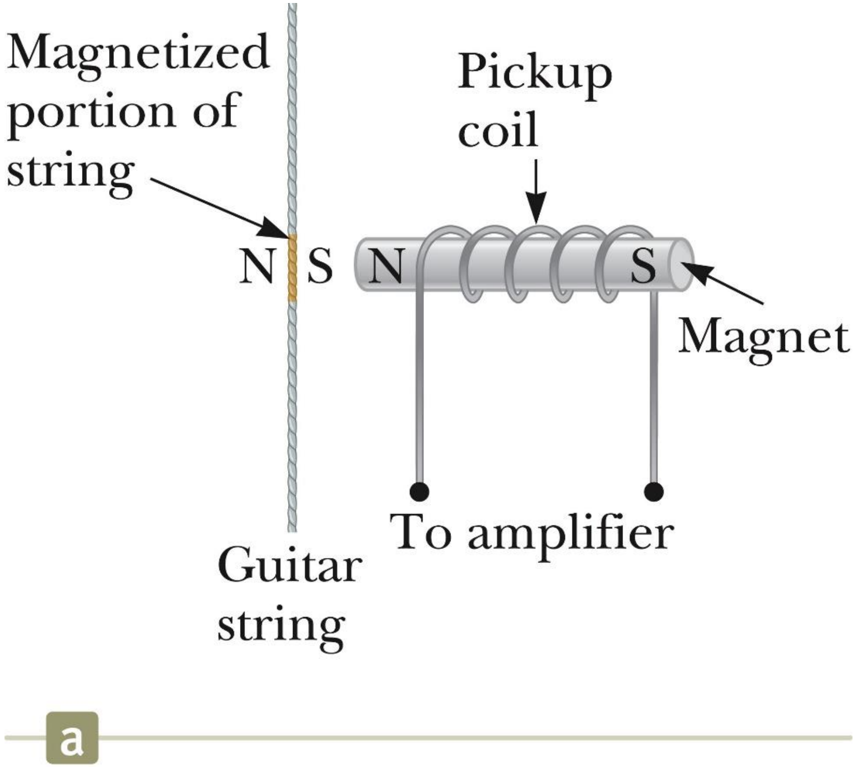
Another interesting application of Faraday's law is the production of sound in an electric guitar.

The pickup coil of an electric guitar uses Faraday's law.

The coil is placed near the vibrating string and causes a portion of the string to become magnetized.

When the string vibrates at some frequency, the magnetized segment produces a changing flux through the coil.

The induced emf is fed to an amplifier.



The output of the amplifier is sent to the loudspeakers, which produce the sound waves we hear.

Inducing an emf in a Coil, Example

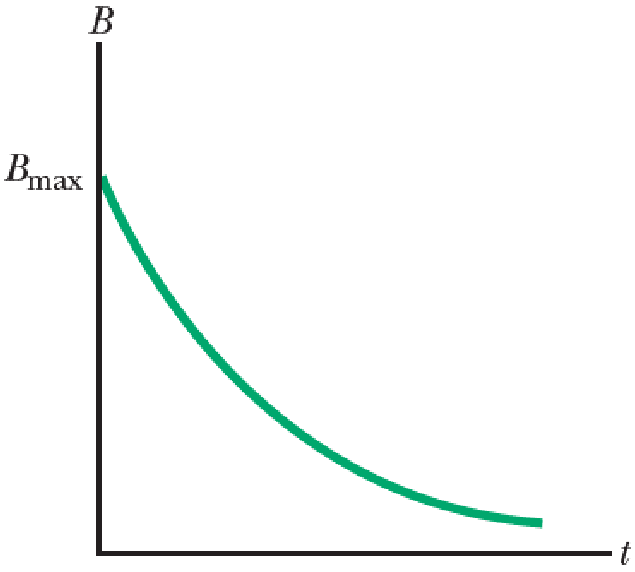
A coil consists of 200 turns of wire. Each turn is a square of side $d = 18$ cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\frac{B_f - B_i}{\Delta t}$$

$$|\mathcal{E}| = (200)(0.18\ \text{m})^2\frac{(0.50\ \text{T} - 0)}{0.80\ \text{s}} = \boxed{4.0\ \text{V}}$$

Inducing an emf in a Coil, Example

A loop of wire enclosing an area A is placed in a region where the magnetic field is perpendicular to the plane of the loop. The magnitude of $\vec{\mathbf{B}}$ varies in time according to the expression $B = B_{\max} e^{-at}$, where a is some constant. That is, at $t = 0$, the field is $B_{\max}$, and for $t > 0$, the field decreases exponentially (Fig. 31.6). Find the induced emf in the loop as a function of time.



$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(AB_{\max}\, e^{-at}\right) = -AB_{\max}\frac{d}{dt}e^{-at} = \boxed{aAB_{\max}\, e^{-at}}$$

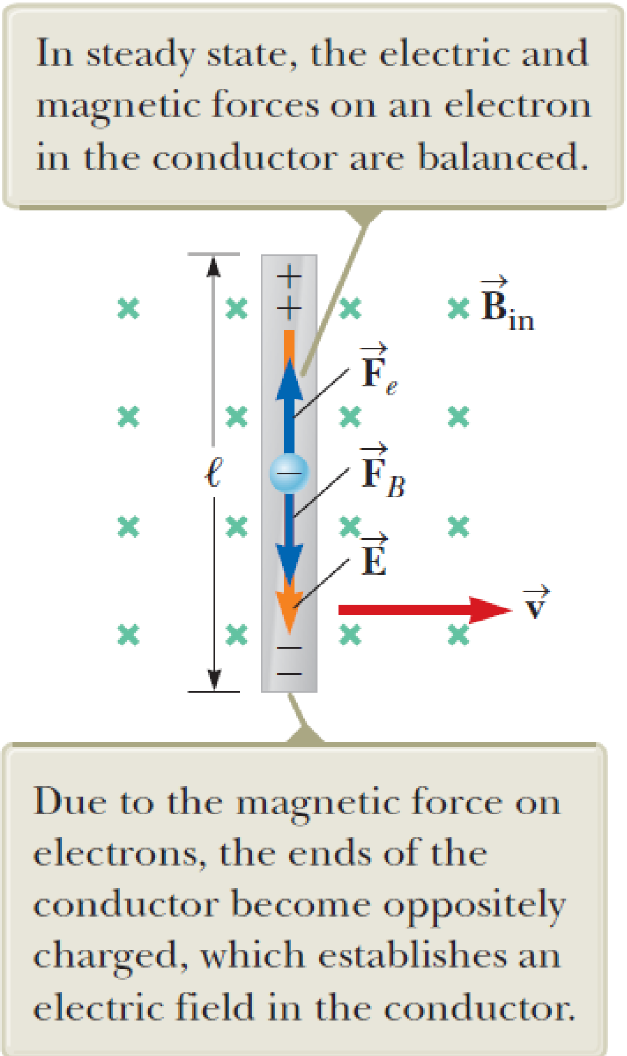
Motional emf

A motional emf is the emf induced in a conductor moving through a constant magnetic field.

The straight conductor of length ℓ is moving through a uniform magnetic field directed into the page.

The conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

The electrons in the conductor experience a force, that is directed along ℓ . $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$



Motional emf, cont.

Under the influence of the force, the electrons move to the lower end of the conductor and accumulate there.

As a result of the charge separation, an electric field is produced inside the conductor.

The charges accumulate at both ends of the conductor until they are in equilibrium with regard to the electric and magnetic forces.

For equilibrium, $qE = qvB$ or $E = vB$.

The electric field is related to the potential difference across the ends of the conductor: $\Delta V = E\,\ell = B\,\ell\,v$.

A potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

If the direction of the motion is reversed, the polarity of the potential difference is also reversed.

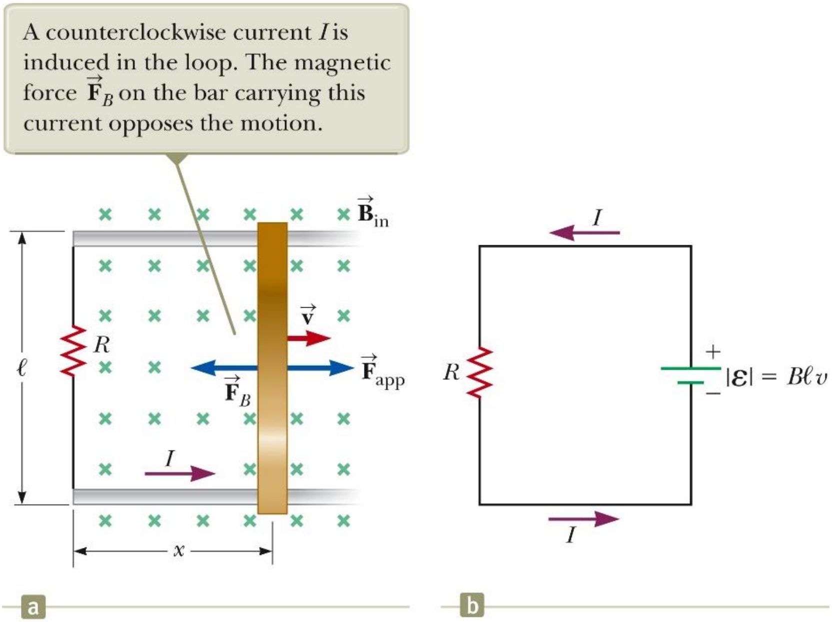
Sliding Conducting Bar

A more interesting situation occurs when the moving conductor is part of a closed conducting path.

Assume the bar has zero resistance.

The stationary part of the circuit has a resistance R.

A uniform and constant magnetic field $\vec{B}$ is applied perpendicular to the plane of the circuit.



As the bar is pulled to the right with a velocity $\vec{V}$ under the influence of an applied force $\vec{F}_{app}$, free charges in the bar are moving particles in a magnetic field that experience a magnetic force directed along the length of the bar.

This force sets up an induced current because the charges are free to move in the closed conducting path.

Sliding Conducting Bar, Cont.

The rate of change of magnetic flux through the circuit and the corresponding induced motional emf across the moving bar are proportional to the change in area of the circuit.

Because the area enclosed by the circuit at any instant is ℓx, where x is the position of the bar, the magnetic flux through that area is

$$\Phi_B = B\ell x$$

Using Faraday's law and noting that x changes with time at a rate $dx/dt = v$, we find that the induced motional emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt} \quad \Rightarrow \quad I = \frac{|\varepsilon|}{R} = \frac{B\ell v}{R}$$

$$\varepsilon = -B\ell v$$

Sliding Conducting Bar, Energy Considerations

The applied force does work on the conducting bar.

- Model the circuit as a nonisolated system.

The movement of the bar through the field causes charges to move along the bar with some average drift velocity; hence, a current is established.

The change in energy of the system during some time interval must be equal to the transfer of energy into the system by work.

The power input is equal to the rate at which energy is delivered to the resistor.

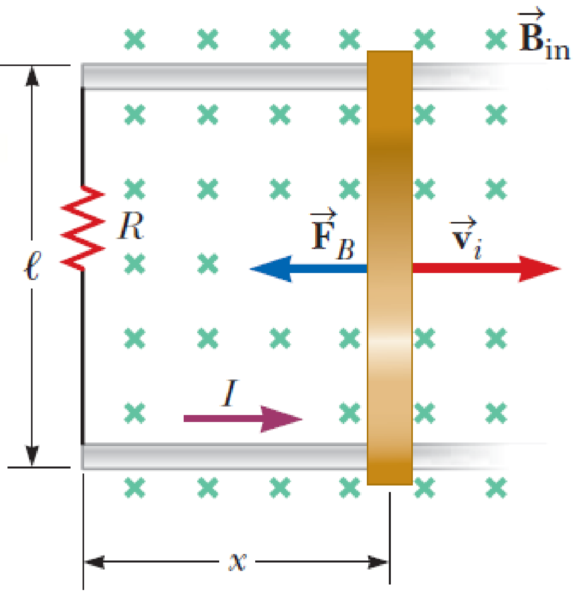
In equilibrium: $F_{\text{app}} = F_B = I\ell B$ and the power delivered by the applied force is:

$$P = F_{\text{app}} v = (I\ell B)v = \frac{B^2\ell^2v^2}{R} = \frac{\mathcal{E}^2}{R}$$

Magnetic Force Acting on a Sliding Bar, Example

The conducting bar illustrated in Figure 31.9 (page 942) moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass m, and its length is ℓ. The bar is given an initial velocity $\vec{\mathbf{v}}_i$ to the right and is released at $t = 0$.

(A) Using Newton's laws, find the velocity of the bar as a function of time.



Using the particle under a net force model, apply Newton's second law to the bar in the horizontal direction:

$$F_x = ma \rightarrow -I\ell B = m\frac{dv}{dt}$$

Substitute $I = B\ell v/R$

$$m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v$$

Rearrange the equation so that all occurrences of the variable v are on the left and those of t are on the right:

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$

Integrate this equation using the initial condition that $v = v_i$ at $t = 0$ and noting that $(B^2\ell^2/mR)$ is a constant:

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int_0^t dt$$

$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{B^2\ell^2}{mR}\right)t$$

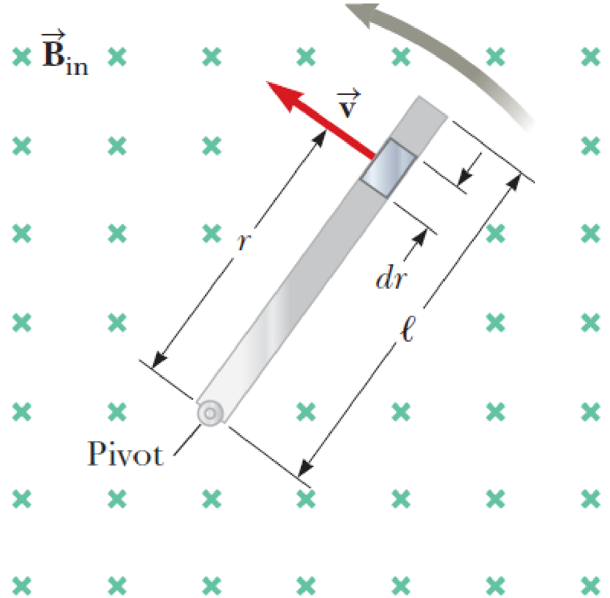
Define the constant $\tau = mR/B^2\ell^2$ and solve for the velocity:

$$(1)\quad v = \boxed{v_i e^{-t/\tau}}$$

The negative sign for the force indicates that the force is to the left.

Motional emf Induced in a Rotating Bar, Example

A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field $\vec{\mathbf{B}}$ is directed perpendicular to the plane of rotation as shown in Figure 31.10. Find the motional emf induced between the ends of the bar.



$$d\mathcal{E} = Bv\,dr$$

$$v = r\omega$$

$$\mathcal{E} = \int Bv\,dr$$

$$\mathcal{E} = B\int v\,dr = B\omega\int_0^{\ell} r\,dr = \boxed{\tfrac{1}{2}B\omega\ell^2}$$

Lenz’s Law

Faraday’s law indicates that the induced emf and the change in flux have opposite algebraic signs.

This has a physical interpretation that has come to be known as Lenz’s law.

Developed by German physicist Heinrich Lenz

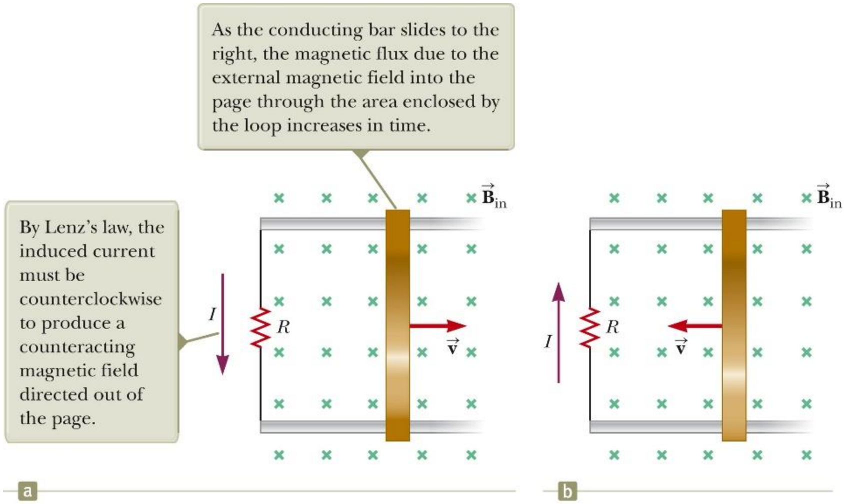
Lenz’s law: *the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.*

The induced current tends to keep the original magnetic flux through the circuit from changing.

Lenz' Law, Example

As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases.

Lenz's law states that the induced current must be directed so that the magnetic field it produces opposes the change in the external magnetic flux.



Because the magnetic flux due to an external field directed into the page is increasing, the induced current—if it is to oppose this change—must produce a field directed out of the page. Hence, the induced current must be directed counterclockwise when the bar moves to the right.

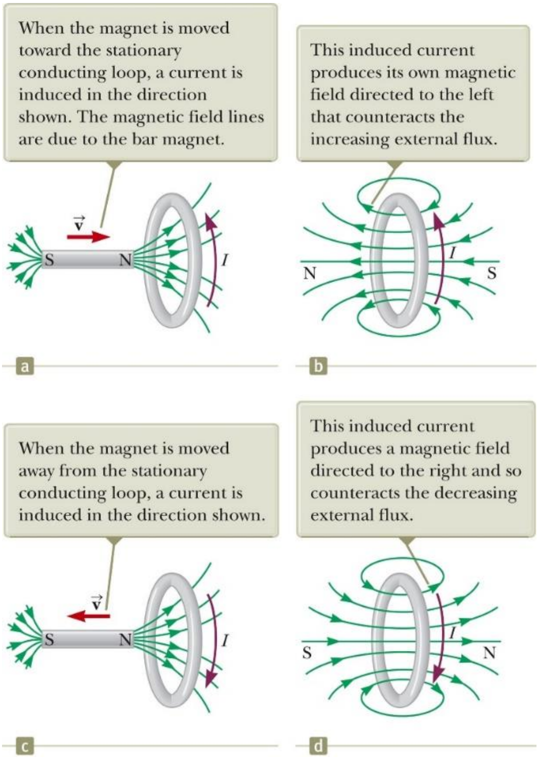
If the bar is moving to the left external magnetic flux through the area enclosed by the loop decreases with time. Because the field is directed into the page, the direction of the induced current must be clockwise if it is to produce a field that also is directed into the page.

Application of Lenz's Law, Example

A magnet is placed near a metal loop as shown in Figure 31.13a (page 946).

(A) Find the direction of the induced current in the loop when the magnet is pushed toward the loop.

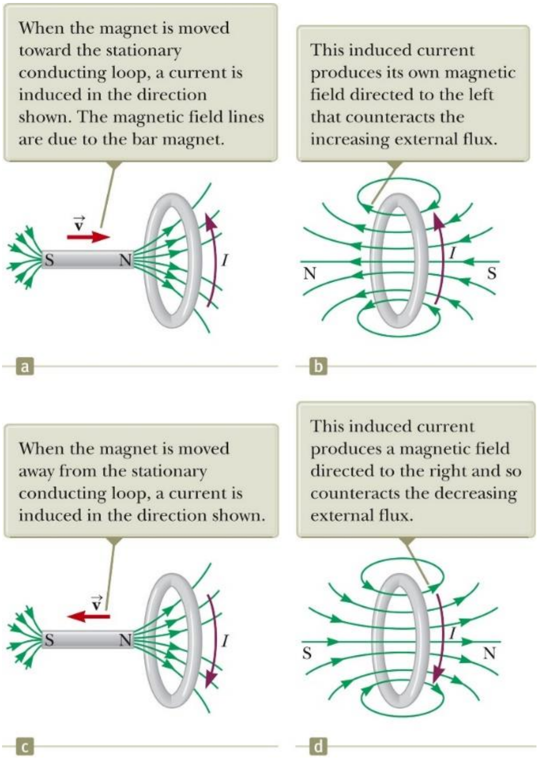
As the magnet moves to the right toward the loop, the external magnetic flux through the loop increases with time. To counteract this increase in flux due to a field toward the right, the induced current produces its own magnetic field to the left as illustrated in Figure b; hence, the induced current is in the direction shown. Knowing that like magnetic poles repel each other, we conclude that the left face of the current loop acts like a north pole and the right face acts like a south pole.



Application of Lenz's Law, Example

(B) Find the direction of the induced current in the loop when the magnet is pulled away from the loop.

If the magnet moves to the left as in Figure c, its flux through the area enclosed by the loop decreases in time. Now the induced current in the loop is in the direction shown in Figure d because this current direction produces a magnetic field in the same direction as the external field. In this case, the left face of the loop is a south pole and the right face is a north pole.



Induced emf and Electric Fields

An electric field is created in the conductor as a result of the changing magnetic flux.

Even in the absence of a conducting loop, a changing magnetic field will generate an electric field in empty space.

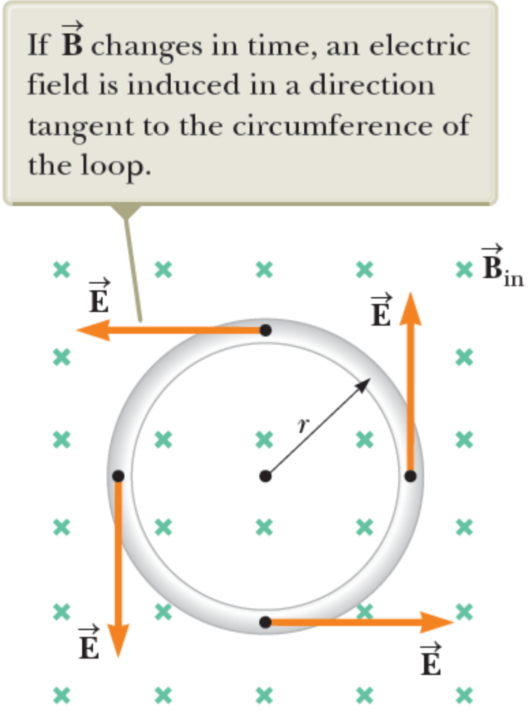
Induced emf and Electric Fields, Cont

The induced electric field is nonconservative. Unlike the electric field produced by stationary charges

Consider a conducting loop of radius *r* situated in a uniform magnetic field that is perpendicular to the plane of the loop.

If the magnetic field changes with time, an emf, according to Faraday's law, induced in the loop.

The induction of a current in the loop implies the presence of an induced electric field, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force.



The work done by the electric field in moving a charge *q* once around the loop is equal to $q\varepsilon$. Because the electric force acting on the charge is $q\vec{\mathbf{E}}$, the work done by the electric field in moving the charge once around the loop is $qE(2\pi r)$.

$$q\mathcal{E} = qE(2\pi r) \Rightarrow E = \frac{\mathcal{E}}{2\pi r} \Rightarrow E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt}$$

Induced emf and Electric Fields, cont.

The emf for any closed path can be expressed as the line integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ over the path.

Faraday's law can be written in a general form:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

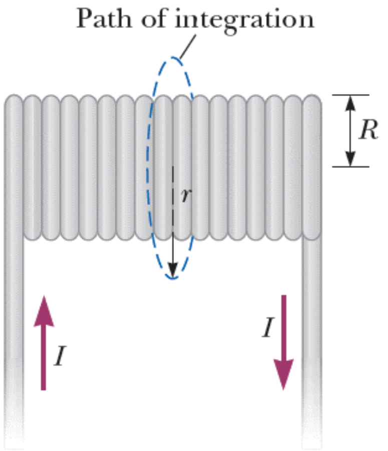
The induced electric field is a nonconservative field that is generated by a changing magnetic field.

The field cannot be an electrostatic field because if the field were electrostatic, and hence conservative, the line integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ over a closed loop would be zero and it isn't.

Electric Field Induced by a Changing Magnetic Field in a Solenoid, Example

A long solenoid of radius R has n turns of wire per unit length and carries a time-varying current that varies sinusoidally as $I = I_{max} \cos \omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (Fig. 31.16).

(A) Determine the magnitude of the induced electric field outside the solenoid at a distance $r > R$ from its long central axis.



First consider an external point and take the path for the line integral to be a circle of radius *r* centered on the solenoid as illustrated in Figure.

$$(1) \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi R^2) = -\pi R^2 \frac{dB}{dt}$$

$$(2) \quad B = \mu_0 n I = \mu_0 n I_{max} \cos \omega t$$ (Chapter 30)

$$(3) \quad -\frac{d\Phi_B}{dt} = -\pi R^2 \mu_0 n I_{max} \frac{d}{dt}(\cos \omega t) = \pi R^2 \mu_0 n I_{max} \omega \sin \omega t$$

$$(4) \quad \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E(2\pi r) \Rightarrow E(2\pi r) = \pi R^2 \mu_0 n I_{max} \omega \sin \omega t \Rightarrow E = \boxed{\frac{\mu_0 n I_{max} \omega R^2}{2r} \sin \omega t} \quad (\text{for } r > R)$$

Electric Field Induced by a Changing Magnetic Field in a Solenoid, Example

(B) What is the magnitude of the induced electric field inside the solenoid, a distance r from its axis?

$$(5)\quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\pi r^2) = -\pi r^2 \frac{dB}{dt}$$

$$(6)\quad -\frac{d\Phi_B}{dt} = -\pi r^2 \mu_0 n I_{\max} \frac{d}{dt}(\cos \omega t) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E(2\pi r) = \pi r^2 \mu_0 n I_{\max} \omega \sin \omega t$$

$$E = \boxed{\frac{\mu_0 n I_{\max} \omega}{2} r \sin \omega t} \quad (\text{for } r < R)$$

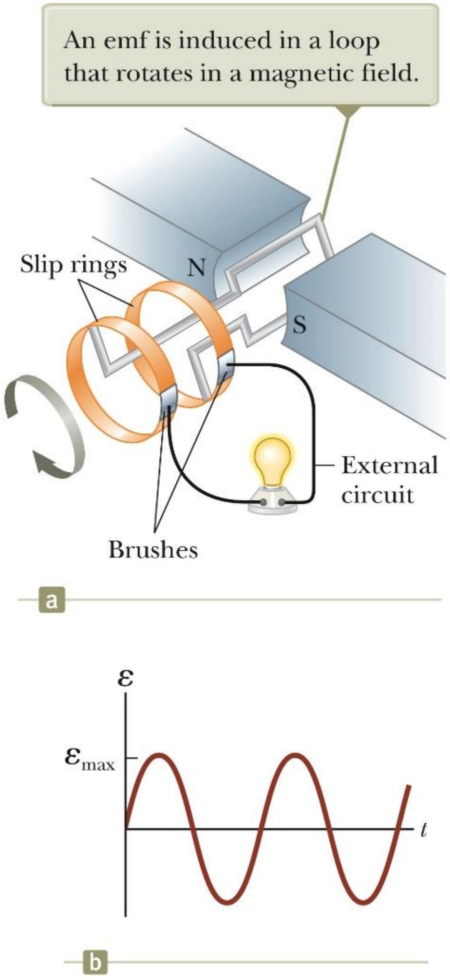
Generators

Electric generators take in energy by work and transfer it out by electrical transmission.

The AC generator consists of a loop of wire rotated by some external means in a magnetic field.

As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time, and this change induces an emf and a current in the loop according to Faraday's law.

The ends of the loop are connected to slip rings that rotate with the loop. Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary metallic brushes in contact with the slip rings.

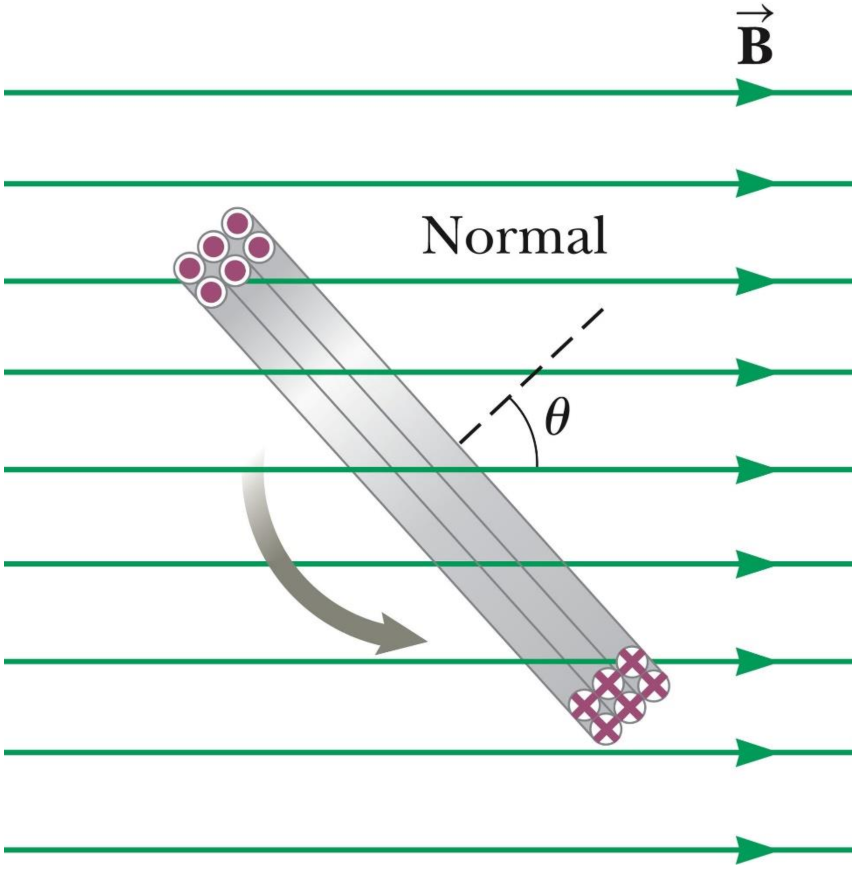


Rotating Loop

Assume a loop with *N* turns, all of the same area rotating in a magnetic field with a constant angular speed ω.

The flux through the loop at any time *t* is $\Phi_B = BA \cos \theta = BA \cos \omega t$

where θ is the angle between the magnetic field and the normal to the plane of the coil

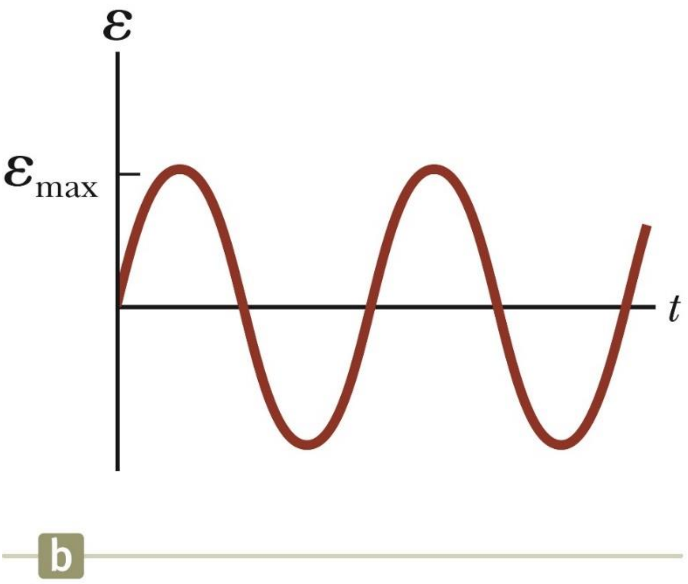


Induced emf in a Rotating Loop

The induced emf in the loop is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\,\frac{d}{dt}(\cos\omega t) = NBA\omega\sin\omega t$$

This is sinusoidal, with $\mathcal{E}_{max} = NAB\omega$



Induced emf in a Rotating Loop, cont.

ε_{max} occurs when ωt = 90º or 270º

- This occurs when the magnetic field is in the plane of the coil and the time rate of change of flux is a maximum.

ε = 0 when ωt = 0º or 180º

- This occurs when the magnetic field is perpendicular to the plane of the coil and the time rate of change of flux is zero.

emf Induced in a Generator, Example

The coil in an AC generator consists of 8 turns of wire, each of area $A = 0.090\,0\ \text{m}^2$, and the total resistance of the wire is 12.0 Ω. The coil rotates in a 0.500-T magnetic field at a constant frequency of 60.0 Hz.

(A) Find the maximum induced emf in the coil.

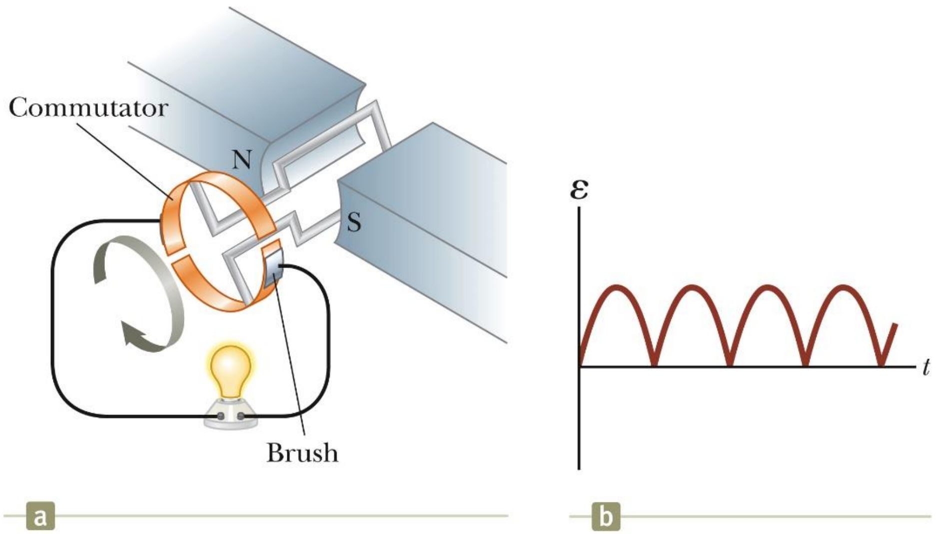
$$\mathcal{E}_{\text{max}} = NBA\omega = NBA(2\pi f)$$

$$\mathcal{E}_{\text{max}} = 8(0.500\ \text{T})(0.090\,0\ \text{m}^2)(2\pi)(60.0\ \text{Hz}) = \boxed{136\ \text{V}}$$

(B) What is the maximum induced current in the coil when the output terminals are connected to a low-resistance conductor?

$$I_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{136\ \text{V}}{12.0\ \Omega} = \boxed{11.3\ \text{A}}$$

DC Generators



The DC (direct current) generator has essentially the same components as the AC generator.

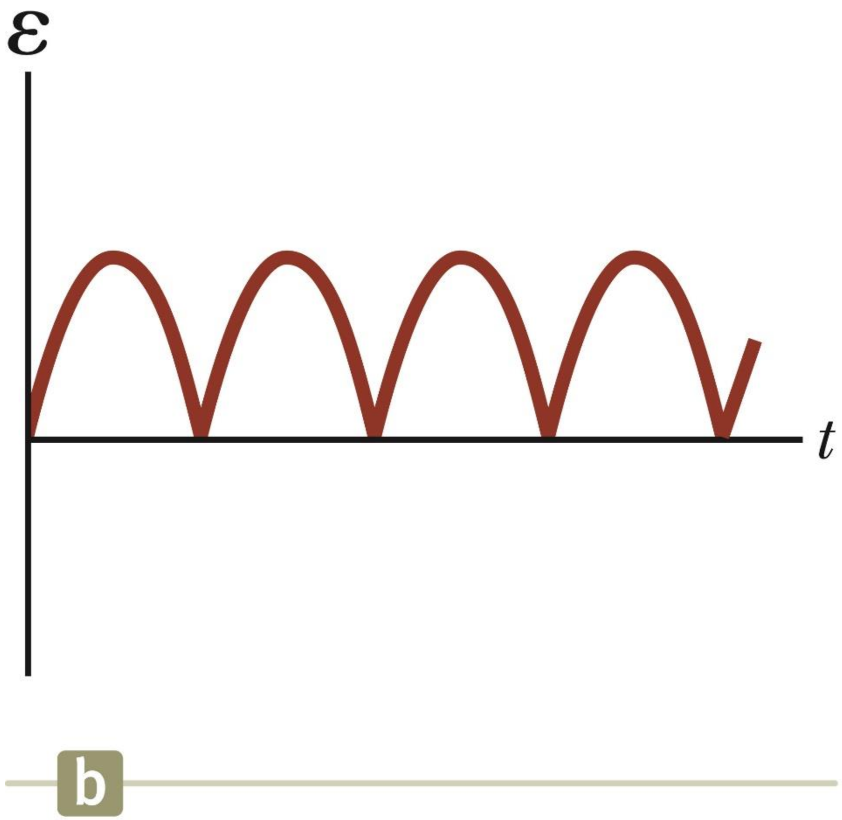
The main difference is that the contacts to the rotating loop are made using a split ring called a *commutator.*

DC Generators, cont.

In this configuration, the output voltage always has the same polarity.

It also pulsates with time.

To obtain a steadier DC current, commercial DC generators use many coils and commutators distributed so that the sinusoidal pulses from the various coils are out of phase. When these pulses are superimposed, the DC output is almost free of fluctuations.



Motors

Motors are devices into which energy is transferred by electrical transmission while energy is transferred out by work.

A motor is a generator operating in reverse.

A current is supplied to the coil by a battery and the torque acting on the current-carrying coil causes it to rotate.

Useful mechanical work can be done by attaching the rotating coil to some external device.

However, as the coil rotates in a magnetic field, an emf is induced, consistent with Lenz's law.

- This induced emf always acts to reduce the current in the coil.
- The back emf increases in magnitude as the rotational speed of the coil increases.

Motors, cont.

Because the voltage available to supply current equals the difference between the supply voltage and the back emf, the current in the rotating coil is limited by the back emf.

- The term *back emf* is commonly used to indicate an emf that tends to reduce the supplied current.

Motors, cont.

When a motor is turned on, there is initially no back emf, and the current is very large because it is limited only by the resistance of the coil.

As the coil begins to rotate, the induced back emf opposes the applied voltage and the current in the coil decreases.

If the mechanical load increases, the motor slows down, which causes the back emf to decrease. This reduction in the back emf increases the current in the coil and therefore also increases the power needed from the external voltage source.

For this reason, the power requirements for running a motor are greater for heavy loads than for light ones.

If the motor is allowed to run under no mechanical load, the back emf reduces the current to a value just large enough to overcome energy losses due to internal energy and friction.

If a very heavy load jams the motor so that it cannot rotate, the lack of a back emf can lead to dangerously high current in the motor's wire.

Hybrid Drive Systems

In an automobile with a hybrid drive system, a gasoline engine and an electric motor are combined to increase the fuel economy of the vehicle and reduce its emissions.

Power to the wheels can come from either the gasoline engine or the electric motor.

In normal driving, the electric motor accelerates the vehicle from rest until it is moving at a speed of about 15 mph.

During the acceleration periods, the engine is not running, so gasoline is not used and there is no emission.

At higher speeds, the motor and engine work together so that the engine always operates at or near its most efficient speed.

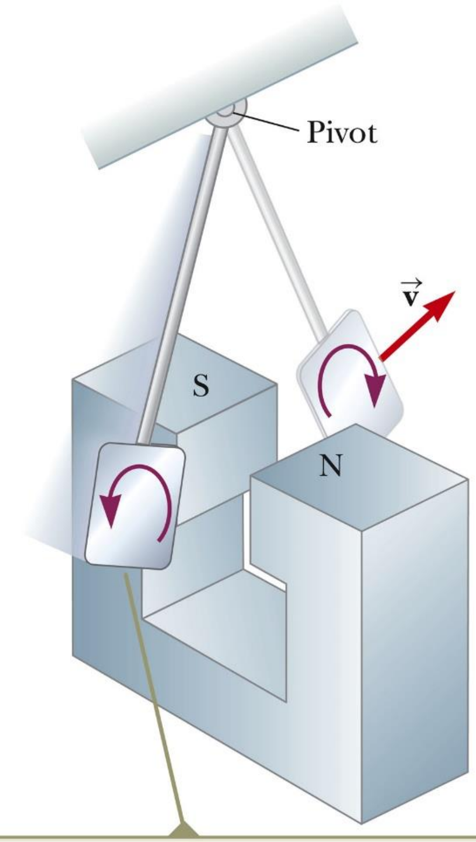
The result is significantly higher gas mileage than a traditional gasoline-powered automobile.

Eddy Currents

Circulating currents called eddy currents are induced in bulk pieces of metal moving through a magnetic field.

As the plate enters the field, the changing magnetic flux induces an emf in the plate, which in turn causes the free electrons in the plate to move, producing the swirling eddy currents.

The eddy currents are in opposite directions as the plate enters or leaves the field.



As the plate enters or leaves the field, the changing magnetic flux induces an emf, which causes eddy currents in the plate.

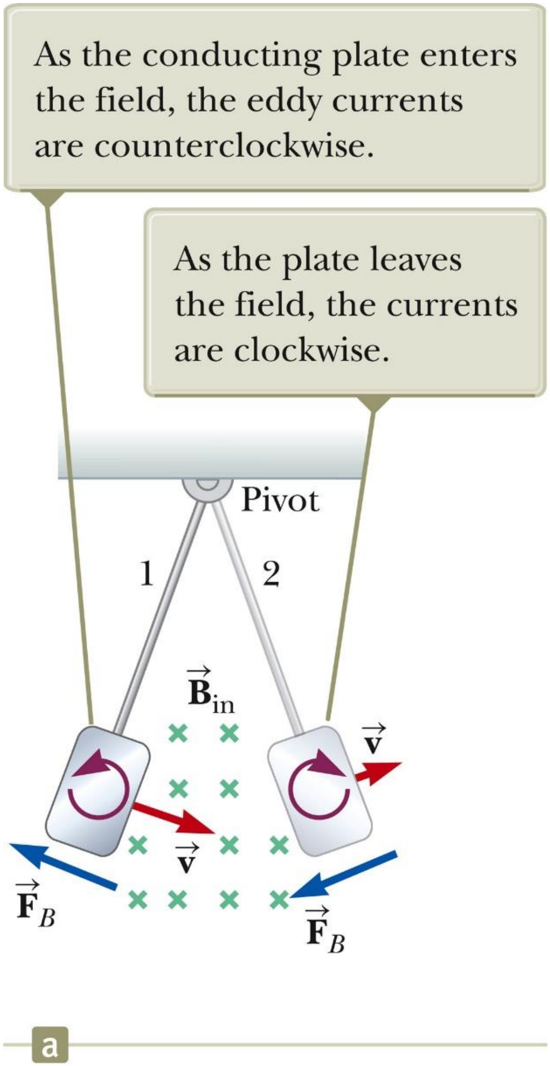
Eddy Currents, Example

The magnetic field is directed into the page.

The induced eddy current is counterclockwise as the plate enters the field.

It is opposite when the plate leaves the field.

The induced eddy currents produce a magnetic retarding force and the swinging plate eventually comes to rest.



Eddy Currents, Final

Eddy currents are often undesirable because they represent a transformation of mechanical energy into internal energy.

To reduce energy loses by the eddy currents, the conducting parts can.

- Be built up in thin layers separated by a nonconducting material
- Have slots cut in the conducting plate

Both prevent large current loops and increase the efficiency of the device.