Chapter 25

Electric Potential

Electric Potential

Electromagnetism has been connected to the study of forces in previous chapters.

In this chapter, electromagnetism will be linked to energy.

By using an energy approach, problems could be solved that were insoluble using forces.

The concept of potential energy is of great value in the study of electricity.

Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy.

This will enable the definition of *electric potential.*

Electrical Potential Energy

When a test charge is placed in an electric field, it experiences a force.

- $\vec{\mathbf{F}}_e = q_o\vec{\mathbf{E}}$
- The force is conservative.
- The electric field will be doing work on the charge. This work is *internal* to the system.

If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent.

$d\vec{\mathbf{s}}$ is an infinitesimal displacement vector that is oriented tangent to a path through space.

- The path may be straight or curved and the integral performed along this path is called either a *path integral* or a *line integral.*

Electric Potential Energy, cont

For an infinitesimal displacement $d\vec{\mathbf{s}}$ of a point charge *q* immersed in an electric field, the work done within the charge–field system by the electric field on the charge is

$$W_{\text{int}} = \vec{\mathbf{F}}_e \cdot d\vec{\mathbf{s}} = q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

The internal work done in a system is equal to the negative of the change in the potential energy of the system $W_{\text{int}} = -\Delta U$.

Therefore, as the charge *q* is displaced, the electric potential energy of the charge–field system is changed by an amount

$$dU = -W_{\text{int}} = -q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}.$$

For a finite displacement of the charge from A to B, the change in potential energy of the system is

$$\Delta U = -q\int_{Ⓐ}^{Ⓑ} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Because the force is conservative, the line integral does not depend on the path taken by the charge.

Electric Potential

The potential energy per unit charge, U/q_o, is the **electric potential.**

- The potential is characteristic of the field only.
- The potential energy is characteristic of the charge-field system.

The potential has a value at every point in an electric field and depends only on the source charge distribution.

The electric potential is

$$V = \frac{U}{q_o}$$

Potential and Potential Energy
The *potential is characteristic of the field only*, independent of a charged particle that may be placed in the field. *Potential energy is characteristic of the charge-field system* due to an interaction between the field and a charged particle placed in the field.

The electric potential at any point in an electric field is a scalar quantity.

Electric Potential, cont.

The potential is a scalar quantity.

- Since energy is a scalar

As a charged particle moves in an electric field, it will experience a change in potential or a potential difference given by:

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

The infinitesimal displacement is interpreted as the displacement between two points in space rather than the displacement of a point charge.

Electric Potential, final

The difference in potential is the meaningful quantity.

We often take the value of the potential to be zero at some convenient point in the field.

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

The potential difference between two points exists solely because of a source charge and depends on the source charge distribution.

- For a potential energy to exist, there must be a system of two or more charges.
- The potential energy belongs to the system and changes only if a charge is moved relative to the rest of the system.

Work and Electric Potential

Assume a charge moves in an electric field without any change in its kinetic energy.

The work performed on the charge is

$$W = \Delta U = q\,\Delta V$$

Units: $1\ \mathrm{V} \equiv 1\ \mathrm{J/C}$

- V is a volt.
- It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt.

In addition, $1\ \mathrm{N/C} = 1\ \mathrm{V/m}$

- This indicates we can interpret the electric field as a measure of the rate of change of the electric potential with respect to position.

Electric potential is a measure of potential energy per unit charge

Voltage

Electric potential is described by many terms.

The most common term is *voltage.*

A voltage applied to a device or across a device is the same as the potential difference across the device.

- The voltage is not something that moves through a device.

Electron-Volts

Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt.

One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt.

- 1 eV = 1.60×10^{-19} J

Potential Difference in a Uniform Field

The equations for electric potential between two points A and B can be simplified if the electric field is uniform:

$$V_{Ⓑ} - V_{Ⓐ} = \Delta V = -\int_{Ⓐ}^{Ⓑ} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{Ⓐ}^{Ⓑ} E\, ds\,(\cos 0°) = -\int_{Ⓐ}^{Ⓑ} E\, ds$$

$$\Delta V = -E \int_{Ⓐ}^{Ⓑ} ds$$

$$\Delta V = -Ed$$

The displacement points from A to B and is parallel to the field lines.

The negative sign indicates that the electric potential at point *B* is lower than at point *A.*

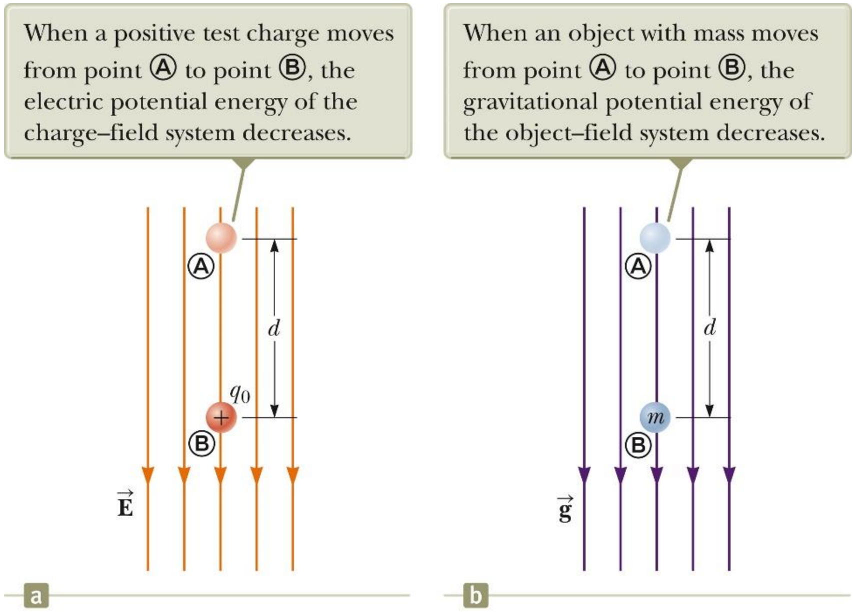
- Electric field lines always point in the direction of decreasing electric potential.

Energy and the Direction of Electric Field

When the electric field is directed downward, point *B* is at a lower potential than point *A.*

When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy.

Electric field lines always point in the direction of decreasing electric potential.



More About Directions

Now suppose a charge q moves from Ⓐ to Ⓑ. We can calculate the change in the potential energy of the charge–field system from Equations 25.3 and 25.6:

$$\Delta U = q\,\Delta V = -qEd$$

A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field.

- An electric field does work on a positive charge when the charge moves in the direction of the electric field.

The charged particle gains kinetic energy and the potential energy of the charge-field system decreases by an equal amount.

- Another example of Conservation of Energy

Directions, cont.

If q_o is negative, then ΔU is positive.

A system consisting of a negative charge and an electric field *gains* potential energy when the charge moves in the direction of the field.

- In order for a negative charge to move in the direction of the field, an external agent must do positive work on the charge.

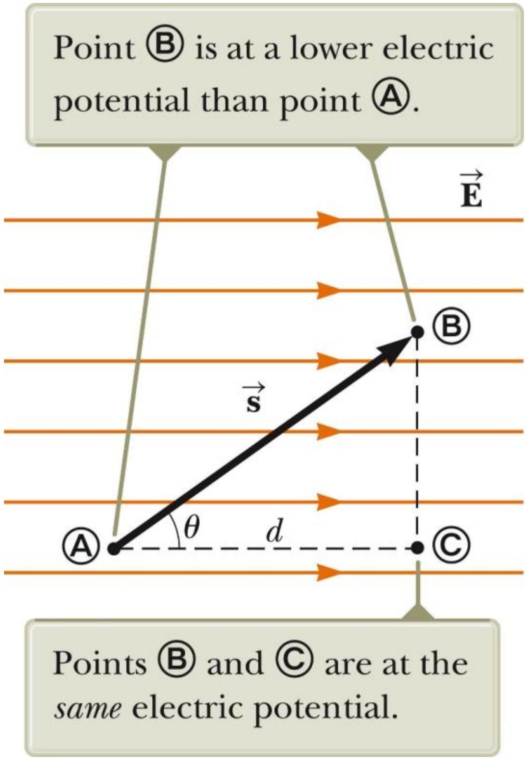
Equipotentials

Point *B* is at a lower potential than point *A.*

Points *B* and *C* are at the same potential.

- All points in a plane perpendicular to a uniform electric field are at the same electric potential.

The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.



Now consider the more general case of a charged particle that moves between Ⓐ and Ⓑ in a uniform electric field such that the vector $\vec{s}$ is *not* parallel to the field lines as shown in Figure. In this case,

$$\Delta V = -\int_{Ⓐ}^{Ⓑ} \vec{E} \cdot d\vec{s} = -\vec{E} \cdot \int_{Ⓐ}^{Ⓑ} d\vec{s} = -\vec{E} \cdot \vec{s} \qquad \Delta U = q\Delta V = -q\vec{E} \cdot \vec{s}$$

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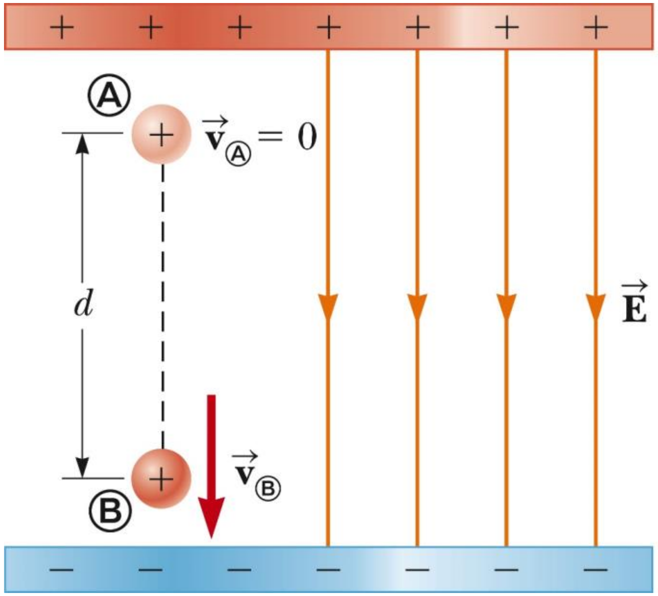
Charged Particle in a Uniform Field, Example

A proton is released from rest at point Ⓐ in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of magnitude $d = 0.50$ m to point Ⓑ in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.



Write the appropriate reduction of Equation 8.2, the conservation of energy equation, for the isolated system of the charge and the electric field:

$$\Delta K + \Delta U = 0$$

Substitute the changes in energy for both terms:

$$\left(\tfrac{1}{2}mv^2 - 0\right) + e\,\Delta V = 0$$

Solve for the final speed of the proton and substitute for ΔV from Equation 25.6:

$$v = \sqrt{\frac{-2e\,\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

Substitute numerical values:

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \text{C})(8.0 \times 10^4\ \text{V})(0.50\ \text{m})}{1.67 \times 10^{-27}\ \text{kg}}}$$

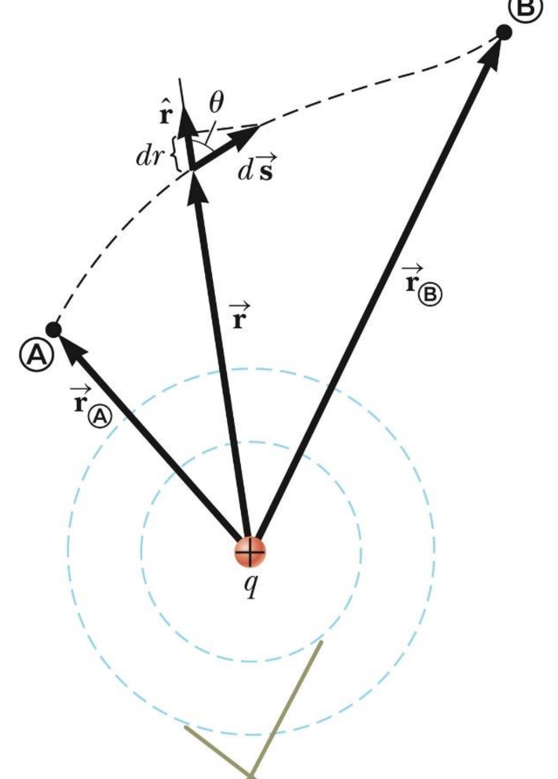
$$= \boxed{2.8 \times 10^6\ \text{m/s}}$$

Potential and Point Charges

An isolated positive point charge produces a field directed radially outward.

The potential difference between points *A* and *B* will be

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$



The two dashed circles represent intersections of spherical equipotential surfaces with the page.

Potential and Point Charges, cont.

The electric potential is independent of the path between points *A* and *B.*

It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$.

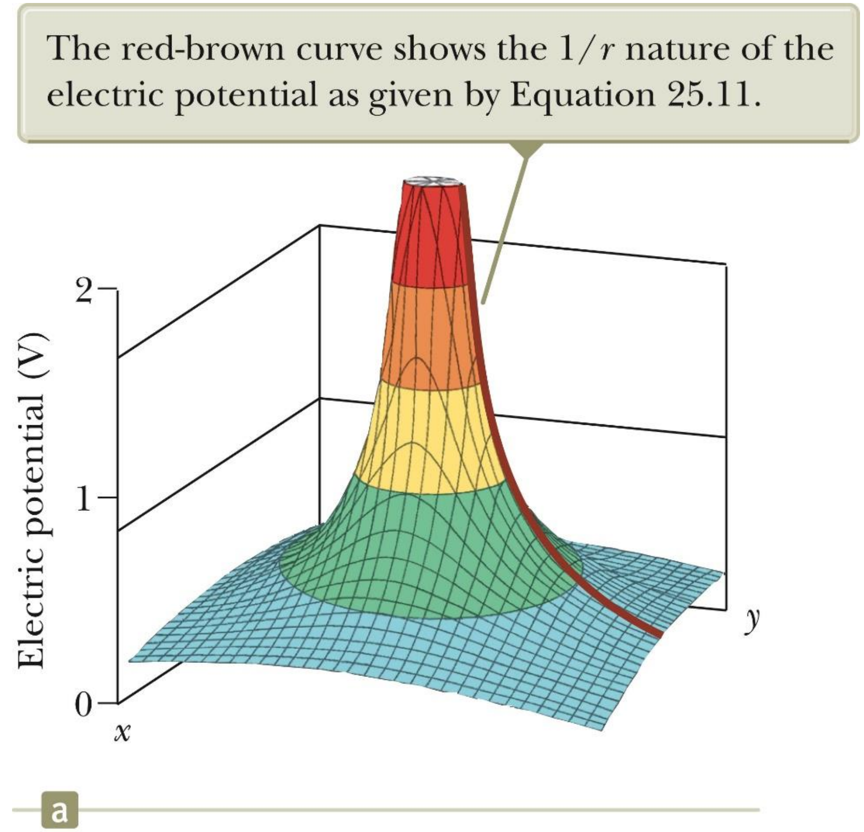
Then the potential due to a point charge at some point *r* is

$$V = k_e \frac{q}{r}$$

Electric Potential of a Point Charge

The electric potential in the plane around a single point charge is shown.

The red line shows the 1/*r* nature of the potential.



Electric Potential with Multiple Charges

The electric potential due to several point charges is the sum of the potentials due to each individual charge.

- This is another example of the superposition principle.
- The sum is the algebraic sum

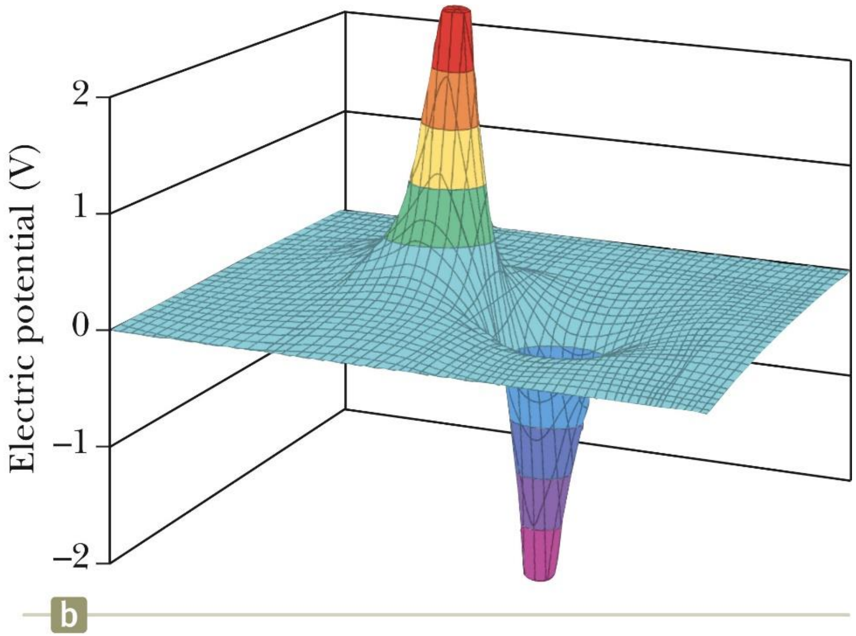
$$V = k_e \sum_i \frac{q_i}{r_i}$$

  - $V = 0$ at $r = \infty$

Electric Potential of a Dipole

The graph shows the potential (y-axis) of an electric dipole.

The steep slope between the charges represents the strong electric field in this region.



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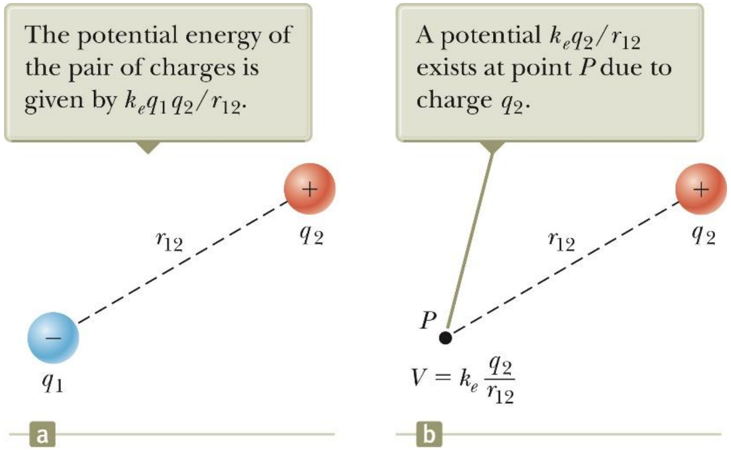
Potential Energy of Multiple Charges



The potential energy of the system is $U = k_e \dfrac{q_1 q_2}{r_{12}}$.

If the two charges are the same sign, U is positive and work must be done to bring the charges together.

If the two charges have opposite signs, U is negative and work is done to keep the charges apart.

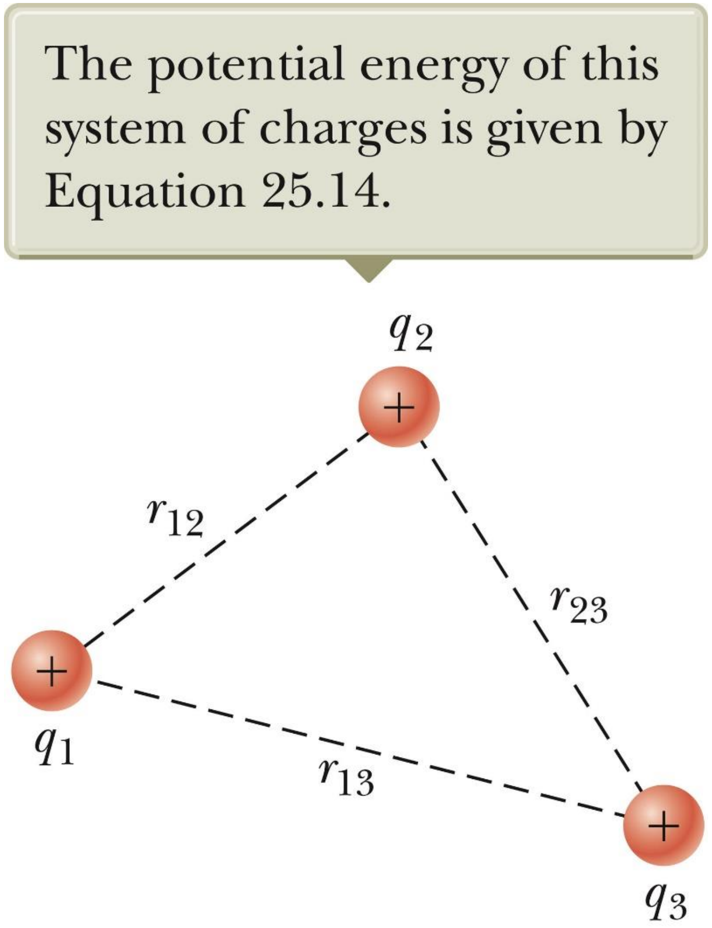
U with Multiple Charges, final

If there are more than two charges, then find *U* for each pair of charges and add them.

For three charges:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

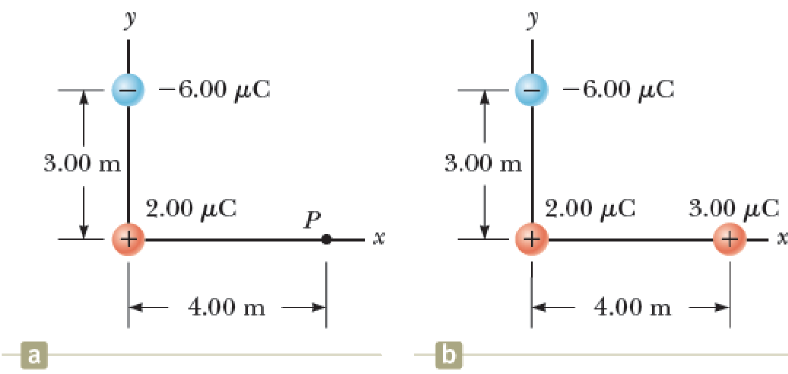
- The result is independent of the order of the charges.



The Electric Potential Due to Two Point Charges, Example

As shown in Figure 25.10a, a charge $q_1 = 2.00\ \mu\text{C}$ is located at the origin and a charge $q_2 = -6.00\ \mu\text{C}$ is located at (0, 3.00) m.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.



$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} + \frac{-6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \text{V}}$$

Finding E From *V*

$$dV = -\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Assume, to start, that the field has only an *x* component. Then, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_x\,dx.$

Therefore,

$$E_x = -\frac{dV}{dx}$$

That is, the *x* component of the electric field is equal to the negative of the derivative of the electric potential with respect to *x.*

Similar statements would apply to the *y* and *z* components.

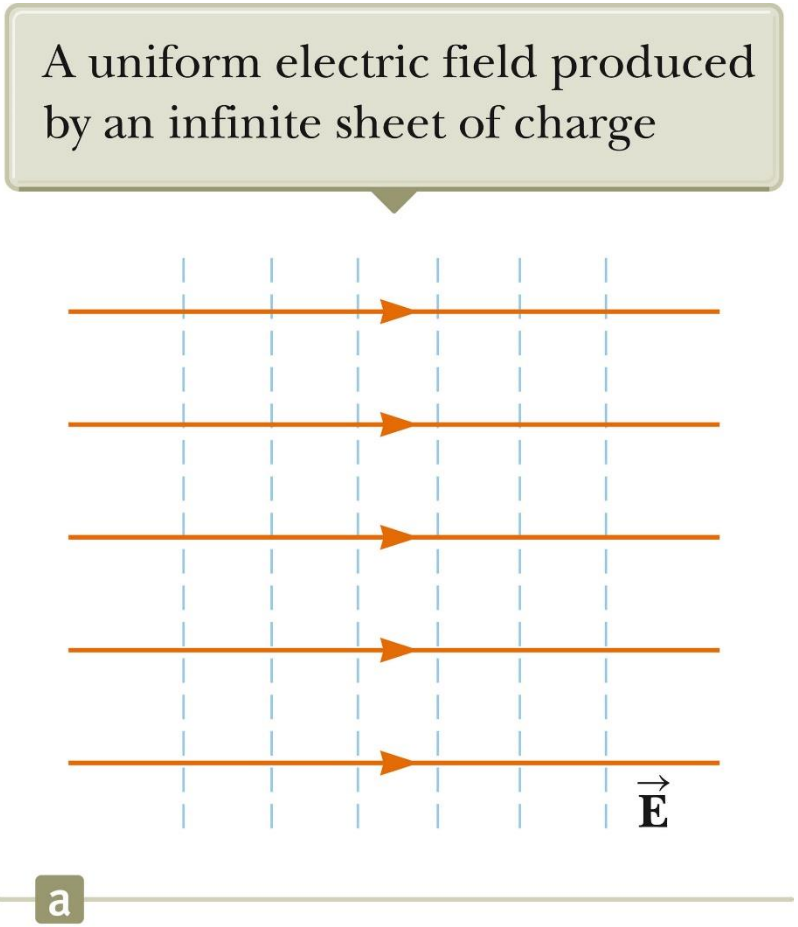
Equipotential surfaces must always be perpendicular to the electric field lines passing through them.

E and V for an Infinite Sheet of Charge

The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The equipotential lines are everywhere perpendicular to the field lines.



E and *V* for a Point Charge

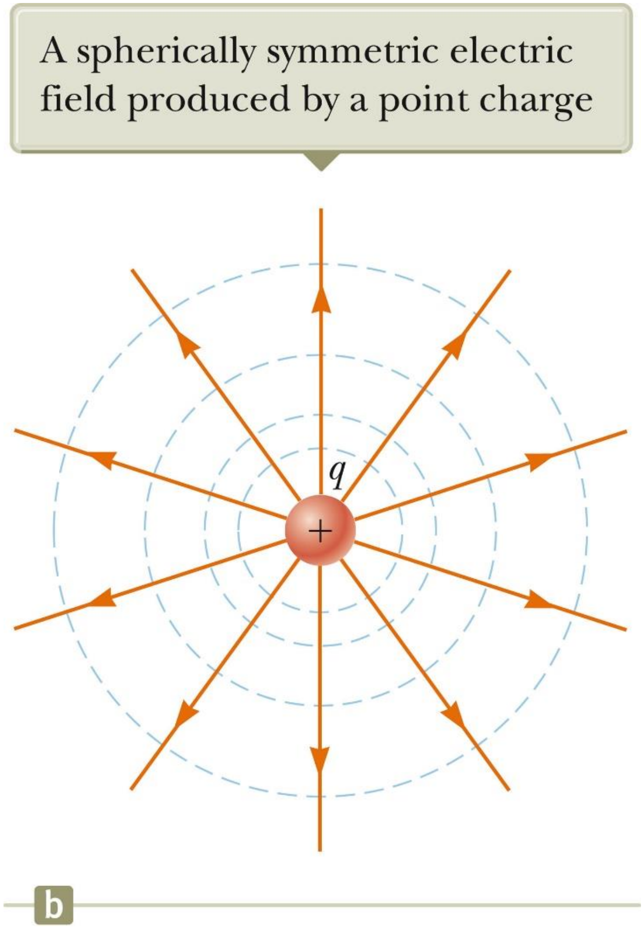
The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The electric field is radial.

$E_r = - dV / dr$

The equipotential lines are everywhere perpendicular to the field lines.

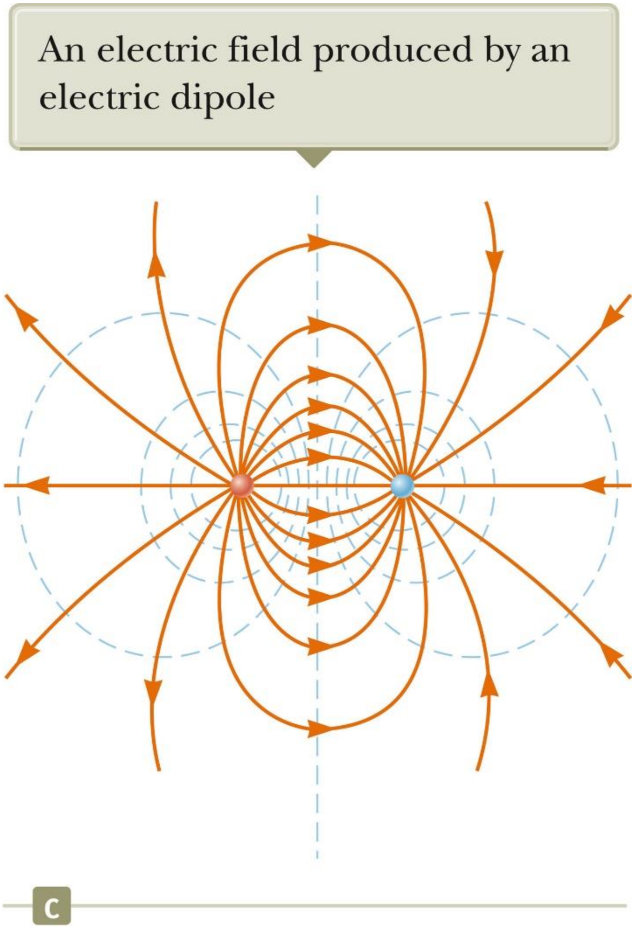


E and V for a Dipole

The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The equipotential lines are everywhere perpendicular to the field lines.



Electric Field from Potential, General

In general, the electric potential is a function of all three dimensions.

Given $V(x, y, z)$ you can find E_x, E_y and E_z as partial derivatives:

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Electric Potential for a Continuous Charge Distribution

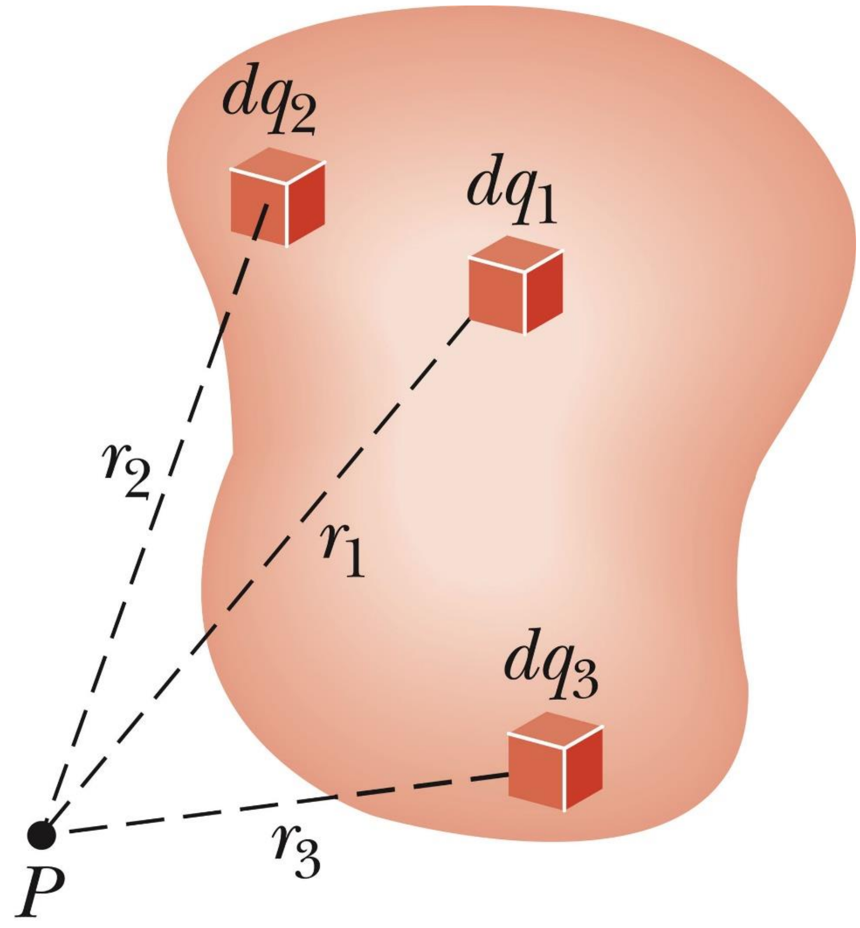
Method 1: The charge distribution is known.

Consider a small charge element *dq*

- Treat it as a point charge.

The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$



V for a Continuous Charge Distribution, cont.

To find the total potential, you need to integrate to include the contributions from all the elements.

$$V = k_e \int \frac{dq}{r}$$

- This value for V uses the reference of $V = 0$ when P is infinitely far away from the charge distributions.

V for a Continuous Charge Distribution, final

Method 2: If the electric field is already known from other considerations such as Gauss's Law, the potential can be calculated using the original approach:

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- If the charge distribution has sufficient symmetry, first find the field from Gauss' Law and then find the potential difference between any two points,
  - Choose V = 0 at some convenient point

Problem-Solving Strategies

Conceptualize

- Think about the individual charges or the charge distribution.
- Imagine the type of potential that would be created.
- Appeal to any symmetry in the arrangement of the charges.

Categorize

- Group of individual charges or a continuous distribution?
- The answer will determine the procedure to follow in the analysis step.

Problem-Solving Strategies, 2

Analyze

- General
  - Scalar quantity, so no components
  - Use algebraic sum in the superposition principle
    - Keep track of signs
  - Only changes in electric potential are significant
  - Define V = 0 at a point infinitely far away from the charges.
    - If the charge distribution extends to infinity, then choose some other arbitrary point as a reference point.

Problem-Solving Strategies, 3

Analyze, cont

- If a group of individual charges is given
  - Use the superposition principle and the algebraic sum.
- If a continuous charge distribution is given
  - Use integrals for evaluating the total potential at some point.
  - Each element of the charge distribution is treated as a point charge.
- If the electric field is given
  - Start with the definition of the electric potential.
  - Find the field from Gauss' Law (or some other process) if needed.

Problem-Solving Strategies, final

Finalize

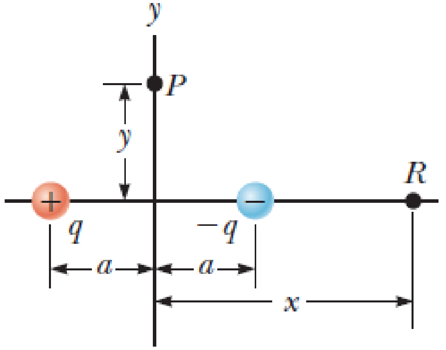
- Check to see if the expression for the electric potential is consistent with your mental representation.
- Does the final expression reflect any symmetry?
- Image varying parameters to see if the mathematical results change in a reasonable way.

The Electric Potential Due to a Dipole, Example

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in Figure 25.13. The dipole is along the x axis and is centered at the origin.

(A) Calculate the electric potential at point P on the y axis.

$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q}{\sqrt{a^2+y^2}} + \frac{-q}{\sqrt{a^2+y^2}}\right) = \boxed{0}$$



(B) Calculate the electric potential at point R on the positive x axis.

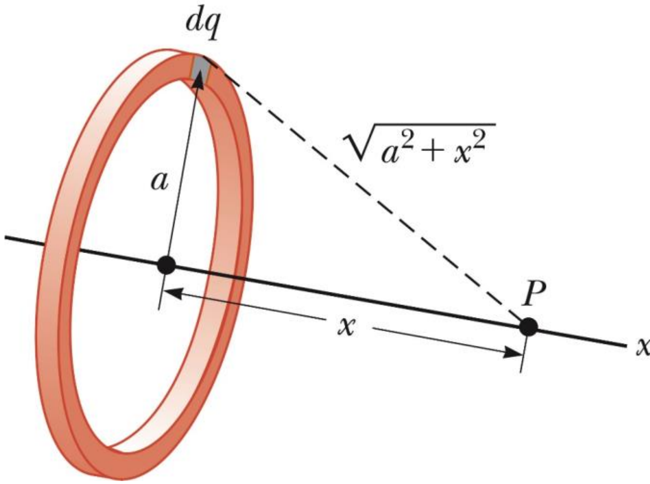
$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{-q}{x-a} + \frac{q}{x+a}\right) = \boxed{-\frac{2k_e q a}{x^2 - a^2}}$$

Electric Potential Due to a Uniformly Charged Ring, Example

(A) Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$

$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{a^2 + x^2}}}$$



(B) Find an expression for the magnitude of the electric field at point P.

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}(a^2 + x^2)^{-1/2}$$

$$= -k_e Q(-\tfrac{1}{2})(a^2 + x^2)^{-3/2}(2x)$$

$$E_x = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}} Q}$$

Electric Potential Due to a Uniformly Charged Disk, Example

A uniformly charged disk has radius R and surface charge density σ.

(A) Find the electric potential at a point P along the perpendicular central axis of the disk.

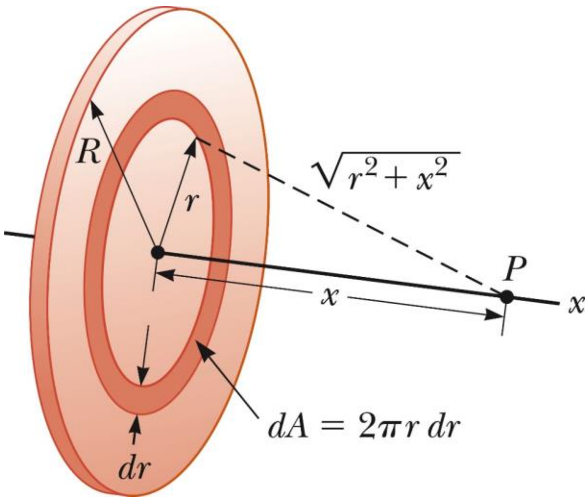
$$dq = \sigma\, dA = \sigma(2\pi r\, dr) = 2\pi\sigma r\, dr$$

$$dV = \frac{k_e\, dq}{\sqrt{r^2 + x^2}} = \frac{k_e 2\pi\sigma r\, dr}{\sqrt{r^2 + x^2}}$$

(From previous slide)

$$V = \pi k_e \sigma \int_0^R \frac{2r\, dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^R (r^2 + x^2)^{-1/2}\, 2r\, dr$$

$$V = \boxed{2\pi k_e \sigma[(R^2 + x^2)^{1/2} - x]}$$



(B) Find the x component of the electric field at a point P along the perpendicular central axis of the disk.

$$E_x = -\frac{dV}{dx} = \boxed{2\pi k_e \sigma\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right]}$$

Electric Potential Due to a Finite Line of Charge, Example

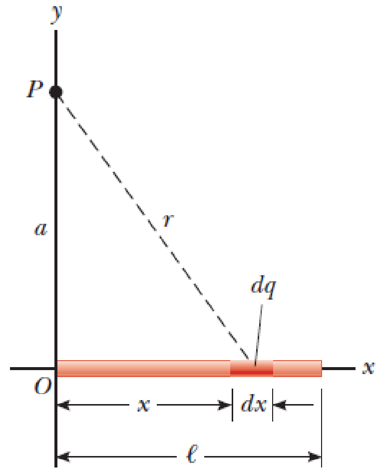
A rod of length ℓ located along the x axis has a total charge Q and a uniform linear charge density λ. Find the electric potential at a point P located on the y axis a distance a from the origin (Fig. 25.16).

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = \int_0^{\ell} k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}} = k_e \frac{Q}{\ell} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^{\ell}$$

$$V = k_e \frac{Q}{\ell}\left[\ln\left(\ell + \sqrt{a^2 + \ell^2}\right) - \ln a\right] = \boxed{k_e \frac{Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)}$$



V Due to a Charged Conductor

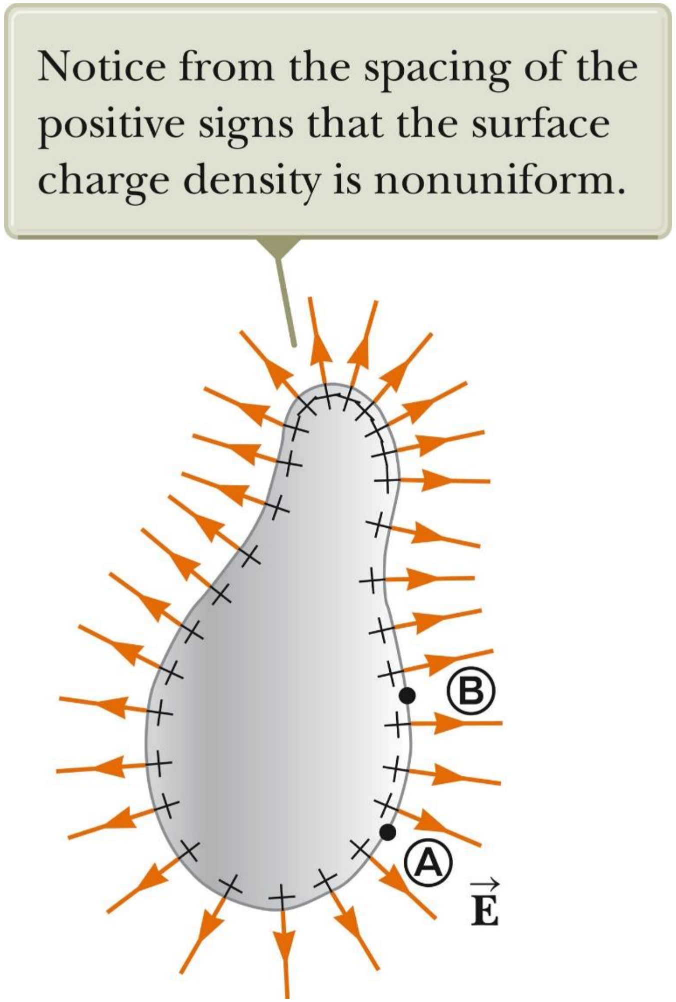
Consider two points on the surface of the charged conductor as shown.

$\vec{\mathbf{E}}$ is always perpendicular to the displacement $d\vec{\mathbf{s}}$.

Therefore, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$

Therefore, the potential difference between A and B is also zero.

Because of the constant value of the potential, no work is required to move a charge from the interior of a charged conductor to its surface.



V Due to a Charged Conductor, cont.

V is constant everywhere on the surface of a charged conductor in equilibrium.

- $\Delta V = 0$ between any two points on the surface

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface.

Every point on the surface of a charge conductor in equilibrium is at the same electric potential.

Because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to the value at the surface.

Irregularly Shaped Objects

The charge density is high where the radius of curvature is small.

- And low where the radius of curvature is large

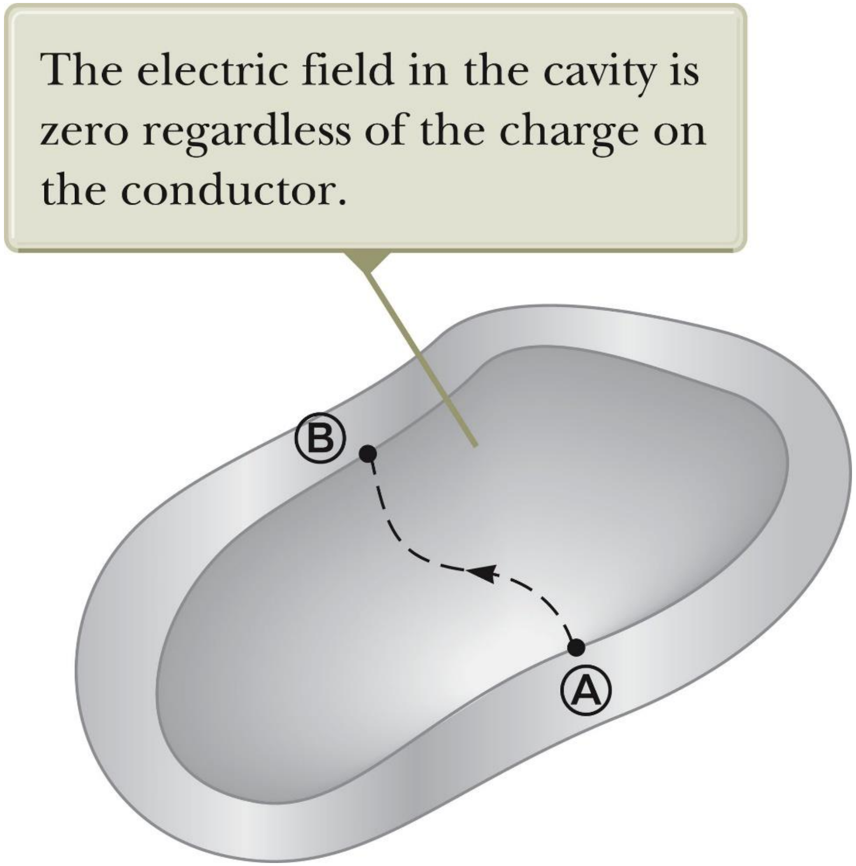
The electric field is large near the convex points having small radii of curvature and reaches very high values at sharp points.

Cavity in a Conductor

Assume an irregularly shaped cavity is inside a conductor.

Assume no charges are inside the cavity.

The electric field inside the conductor must be zero.



Cavity in a Conductor, cont

The electric field inside does not depend on the charge distribution on the outside surface of the conductor.

For all paths between A and B,

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

A cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

Corona Discharge

If the electric field near a conductor is sufficiently strong, electrons resulting from random ionizations of air molecules near the conductor accelerate away from their parent molecules.

These electrons can ionize additional molecules near the conductor.

This creates more free electrons.

The **corona discharge** is the glow that results from the recombination of these free electrons with the ionized air molecules.

The ionization and corona discharge are most likely to occur near very sharp points.