Chapter 39

Relativity

Basic Problems of Newtonian Mechanics

Newtonian mechanics fails to describe properly the motion of objects whose speeds approach that of light.

Newtonian mechanics is a limited theory.

- It places no upper limit on speed.
- It is contrary to modern experimental results.
- Newtonian mechanics becomes a specialized case of Einstein's special theory of relativity.
  - When speeds are much less than the speed of light.

Inertial Frames

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

- This is also called the *law of inertia.*
- It defines a special set of reference frames called *inertial frames.*
  - We call this an ***inertial frame of reference.***

Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame.

If you accelerate relative to an object in an inertial frame, you are observing the object from a **non-inertial reference frame.**

A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame.

- We can consider the Earth to be such an inertial frame, although it has a small centripetal acceleration associated with its motion.

Views of an Event

An *event* is some physical phenomenon.

Assume the event occurs and is observed by an observer at rest in an inertial reference frame.

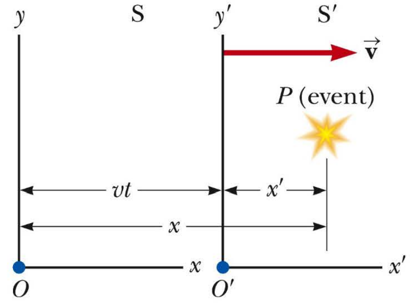
The event's location and time can be specified by the coordinates (x, y, z, t).

Consider two inertial frames, S and S'.

S' moves with constant velocity along the common x and x' axes.

The velocity is measured relative to S.

Assume the origins of S and S' coincide at t = 0.



The relationship among the coordinates are:

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

Albert Einstein

1879 – 1955

1905

- Published four papers
- Most important were about the special theory of relativity

1916

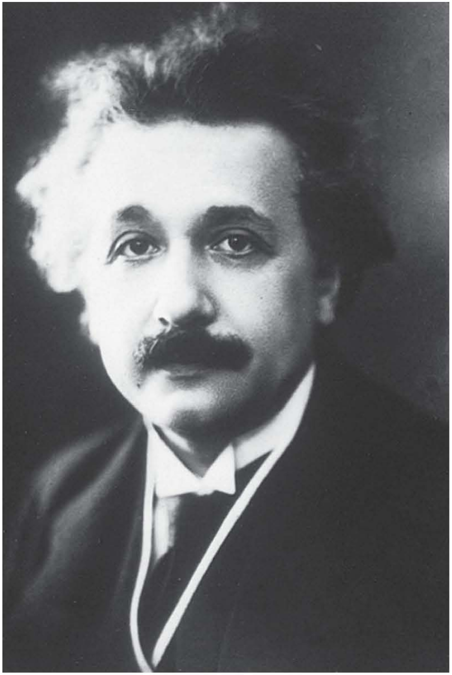
- General relativity
- 1919 – confirmation

1920's

- Didn't accept quantum theory

1940's or so

- Search for unified theory - unsuccessful



Einstein’s Principle of Relativity

Postulates

- **The principle of relativity**: The laws of physics must be the same in all inertial reference frames.
- **The constancy of the speed of light**: the speed of light in a vacuum has the same value, $c = 3.00 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

The Principle of Relativity

The results of any kind of experiment performed in a laboratory at rest must be the same as when performed in a laboratory moving at a constant speed past the first one.

No preferred inertial reference frame exists.

It is impossible to detect absolute motion.

The Constancy of the Speed of Light

This is required by the first postulate.

Confirmed experimentally in many ways.

Relative motion is unimportant when measuring the speed of light.

- We must alter our common-sense notions of space and time.

Consequences of Special Relativity

In relativistic mechanics

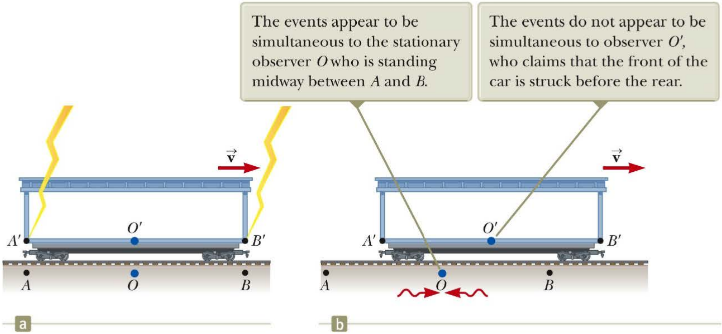
- There is no such thing as absolute length.
- There is no such thing as absolute time.
- Events at different locations that are observed to occur simultaneously in one frame are not observed to be simultaneous in another frame moving uniformly past the first.

Simultaneity

In special relativity, Einstein abandoned the assumption of simultaneity.

Thought experiment to show this

- A boxcar moves with uniform velocity.
- Two lightning bolts strike the ends.
- The lightning bolts leave marks (A' and B') on the car and (A and B) on the ground.
- Two observers are present: O' in the boxcar and O on the ground



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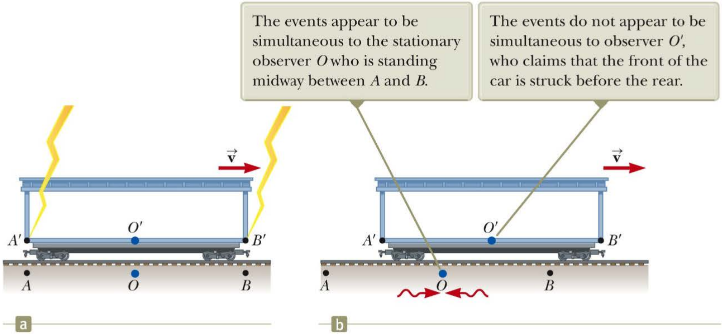
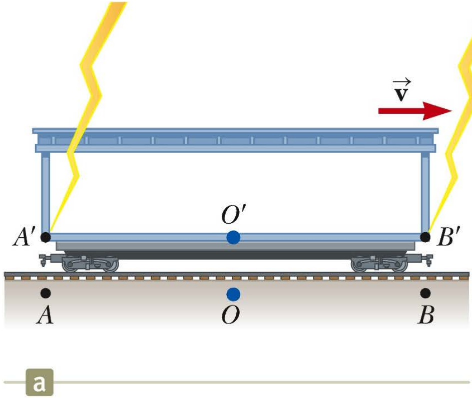
Simultaneity – Thought Experiment Set-up



Observer O is midway between the points of lightning strikes on the ground, A and B.

Observer O' is midway between the points of lightning strikes on the boxcar, A' and B'.

Simultaneity – Thought Experiment Results



The light reaches observer O at the same time.

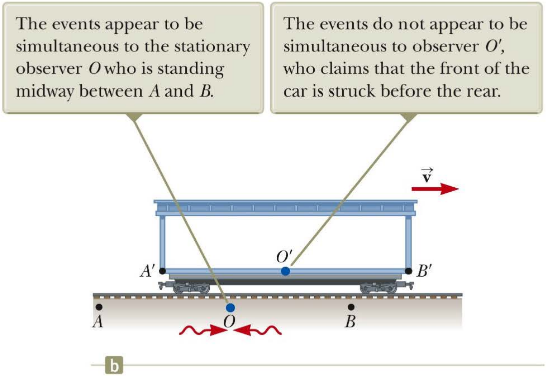
- He concludes the light has traveled at the same speed over equal distances.
- Observer O concludes the lightning bolts occurred simultaneously.

Simultaneity – Thought Experiment Results, cont.

By the time the light has reached observer *O*, observer *O*' has moved.

The signal from *B*' has already swept past *O*', but the signal from *A*' has not yet reached him.

- The two observers must find that light travels at the same speed.
- Observer *O*' concludes the lightning struck the front of the boxcar before it struck the back (they were not simultaneous events).



Simultaneity – Thought Experiment, Summary

Two events that are simultaneous in one reference frame are in general not simultaneous in a second reference frame moving relative to the first.

That is, simultaneity is not an absolute concept, but rather one that depends on the state of motion of the observer.

- In the thought experiment, both observers are correct, because there is no preferred inertial reference frame.

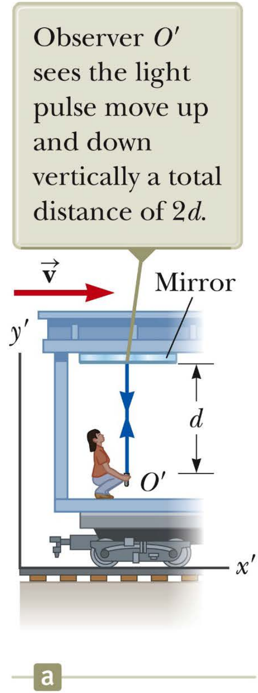
Time Dilation

A mirror is fixed to the ceiling of a vehicle.

The vehicle is moving to the right with speed $\vec{\mathbf{v}}$.

An observer, *O*', at rest in the frame attached to the vehicle holds a flashlight a distance *d* below the mirror.

The flashlight emits a pulse of light directed at the mirror (event 1) and the pulse arrives back after being reflected (event 2).



Time Dilation, Moving Observer

Observer *O*' carries a clock.

She uses it to measure the time between the events (Δt_p).

Model the pulse of light as a particle under constant speed.

- The observer sees the events to occur at the same place.
- Δt_p = distance/speed = $(2d)/c$

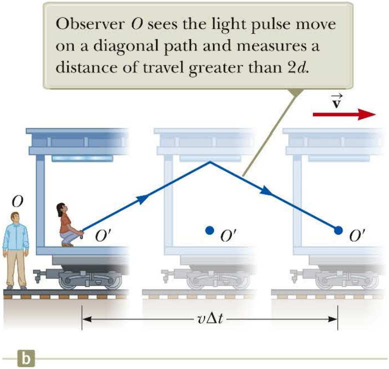
Time Dilation, Stationary Observer

Observer O is in a second frame at rest with respect to the ground.

He observes the mirror and O' to move with speed *v.*

By the time the light from the flashlight reaches the mirror, the mirror has moved to the right.

The light must travel farther with respect to O than with respect to O'.



Time Dilation, Observations

Both observers must measure the speed of the light to be *c.*

The light travels farther for O.

The time interval, Δt, for O is longer than the time interval for O', Δt_p.

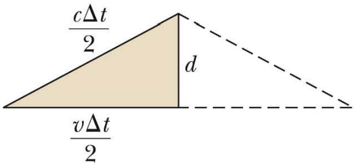
Time Dilation, Time Comparisons

The time interval Δt is longer than the time interval Δt_p

$$\Delta t = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma \Delta t_p \qquad \Delta t_p = 2d/c$$

$$\text{where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

γ is always greater than unity



C

$$\left(\frac{c\,\Delta t}{2}\right)^2 = \left(\frac{v\,\Delta t}{2}\right)^2 + d^2$$

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

The time interval Δt between two events measured by an observer moving with respect to a clock is longer than the time interval Δtp between the same two events measured by an observer at rest with respect to the clock.

- This effect is known as **time dilation.**

γ Factor

Time dilation is not observed in our everyday lives.

For slow speeds, the factor of γ is so small that no time dilation occurs.

As the speed approaches the speed of light, γ increases rapidly.

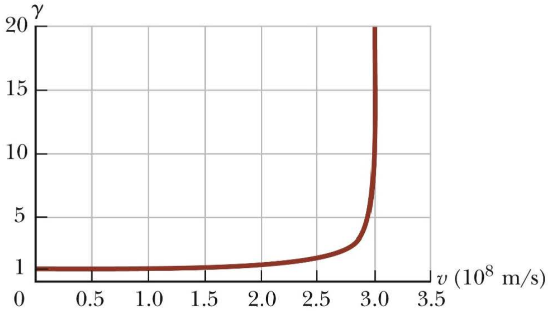


TABLE 39.1

Approximate Values for γ at Various Speeds

v/c	γ
0	1
0.001 0	1.000 000 5
0.010	1.000 05
0.10	1.005
0.20	1.021
0.30	1.048
0.40	1.091
0.50	1.155
0.60	1.250
0.70	1.400
0.80	1.667
0.90	2.294
0.92	2.552
0.94	2.931
0.96	3.571
0.98	5.025
0.99	7.089
0.995	10.01
0.999	22.37

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Identifying Proper Time

The time interval Δt_p is called the proper time interval.

- The proper time interval is the time interval between events as measured by an observer who sees the events occur at the same point in space.
  - You must be able to correctly identify the observer who measures the proper time interval.

Time Dilation – Generalization

If a clock is moving with respect to you, the time interval between ticks of the moving clock is observed to be longer than the time interval between ticks of an identical clock in your reference frame.

All physical processes are measured to slow down when these processes occur in a frame moving with respect to the observer.

- These processes can be chemical and biological as well as physical.

Example

The period of a pendulum is measured to be 3.00 s in the reference frame of the pendulum. What is the period when measured by an observer moving at a speed of 0.960*c* relative to the pendulum?

The proper time interval, measured in the rest frame of the pendulum, is $\Delta t_p = 3.00$ s.

$$\Delta t = \gamma \, \Delta t_p = \frac{1}{\sqrt{1 - \frac{(0.960c)^2}{c^2}}} \Delta t_p = \frac{1}{\sqrt{1 - 0.921\,6}} \Delta t_p$$

$$= 3.57(3.00 \text{ s}) = \boxed{10.7 \text{ s}}$$

This result shows that a moving pendulum is indeed measured to take longer to complete a period than a pendulum at rest does. The period increases by a factor of $\gamma = 3.57$.

Length Contraction

The measured distance between two points depends on the frame of reference of the observer.

The proper length, L_p, of an object is the length of the object measured by someone at rest relative to the object.

The length of an object measured in a reference frame that is moving with respect to the object is always less than the proper length.

- This effect is known as length contraction.

More About Proper Length

Very important to correctly identify the observer who measures proper length.

The proper length is always the length measured by the observer at rest with respect to the points.

Often the proper time interval and the proper length are *not* measured by the same observer.

Length Contraction – Equations

To understand length contraction, consider a spacecraft traveling with a speed v from one star to another.

There are two observers: one on the Earth and the other in the spacecraft.

The observer at rest on the Earth (and also assumed to be at rest with respect to the two stars) measures the distance between the stars to be the proper length L_p. According to this observer, the time interval required for the spacecraft to complete the voyage is given by the particle under constant velocity model as $\Delta t = L_p/v$.

The passages of the two stars by the spacecraft occur at the same position for the space traveler. Therefore, the space traveler measures the proper time interval Δt_p.

Because of time dilation, the proper time interval is related to the Earth-measured time interval by $\Delta t_p = \Delta t/\gamma$. Because the space traveler reaches the second star in the time Δt_p, he or she concludes that the distance L between the stars is

$$L = v\,\Delta t_p = v\,\frac{\Delta t}{\gamma} \quad \xrightarrow{L_p = v\,\Delta t} \quad L = \frac{L_P}{\gamma} = L_P\sqrt{1 - \frac{v^2}{c^2}}$$

Section 39.4

Length Contraction – Example

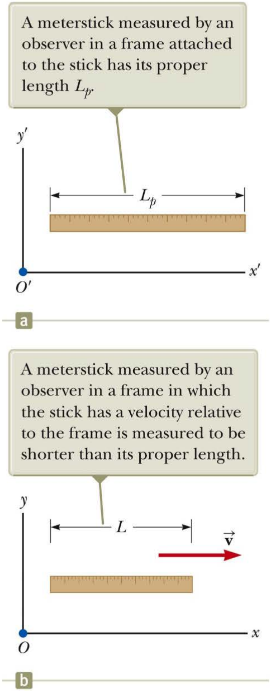
The length when an object moves with a speed *v* in a direction parallel to its length is measured to be shorter than its length measured by an observer at rest with respect to the object.

Length contraction takes place only along the direction of motion.

For example, suppose a meterstick moves past a stationary Earth-based observer with speed *v* as in Figure.

The length of the meterstick as measured by an observer in a frame attached to the stick is the proper length L_p shown in Figure a.

The length of the stick L measured by the Earth observer is shorter than L_p by the factor $(1 - v^2/c^2)^{1/2}$ as suggested in Figure b.



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Proper Length vs. Proper Time

The proper length and proper time interval are defined differently.

The proper length is measured by an observer for whom the end points of the length remained fixed in space.

The proper time interval is measured by someone for whom the two events take place at the same position in space.

Relativistic Doppler Effect

If a light source and an observer approach each other with a relative speed, *v*, the frequency measured by the observer is

$$f' = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}\, f$$

f is the frequency of the source measured in the rest frame.

The shift depends only on the relative velocity, v, of the source and observer.

$f' > f$ when the source and the observer approach each other.

We obtain the expression for the case in which the source and observer recede from each other by substituting negative values for *v*.

Lorentz Transformation Equations, Set-Up

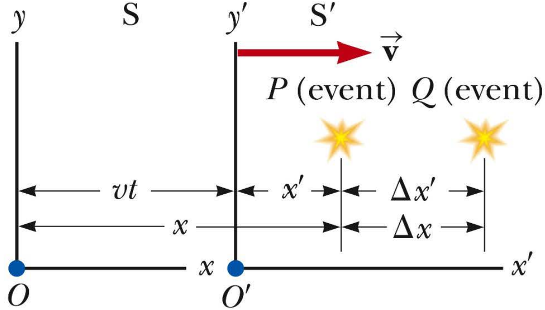
Assume the events at points *P* and *Q* are reported by two observers.

One observer is at rest in frame S.

The other observer is in frame S' moving to the right with speed *v.*

The observer in frame S reports the event with space-time coordinates of (x, y, z, t).

The observer in frame S' reports the same event with space-time coordinates of (x', y', z', t').



Lorentz Transformations, Equations

To transform coordinates from S to S' use

$$x' = \gamma(x - vt) \qquad y' = y \qquad z' = z \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

These show that in relativity, space and time are not separate concepts but rather closely interwoven with each other.

To transform coordinates from S' to S use

$$x = \gamma(x' + vt') \qquad y = y' \qquad z = z' \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

Lorentz Transformations, Pairs of Events

The Lorentz transformations can be written in a form suitable for describing pairs of events.

$$\left.\begin{aligned} \Delta x' &= \gamma(\Delta x - v\,\Delta t) \\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) \end{aligned}\right\} \text{S} \rightarrow \text{S}'$$

$$\left.\begin{aligned} \Delta x &= \gamma(\Delta x' + v\,\Delta t') \\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) \end{aligned}\right\} \text{S}' \rightarrow \text{S}$$

In the preceding equations, observer O' measures $\Delta x' = x'_2 - x'_1$ and $\Delta t' = t'_2 - t'_1$

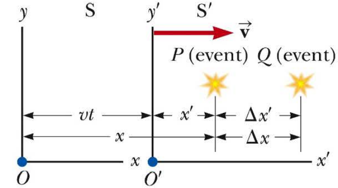
Also, observer O measures $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$

The y and z coordinates are unaffected by the motion along the x direction.

Lorentz Velocity Transformation, 1

The "event" is the motion of the object.

S' is the frame moving at v relative to S. Suppose an object has a velocity component u'_x measured in the S' frame, where



$$u'_x = \frac{u_x - v}{1 - \dfrac{u_x v}{c^2}}$$

$$dt' = \gamma\left(dt - \frac{v}{c^2}\,dx\right)$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \dfrac{u_x v}{c^2}\right)} \quad \text{and} \quad u'_z = \frac{u_z}{\gamma\left(1 - \dfrac{u_x v}{c^2}\right)}$$

The term v does not appear in the u'_y and u'_z equations since the relative motion is in the x direction.

If $u_x = c$, $u'_x = c$ and the speed of light is shown to be independent of the relative motion of the frame.

Lorentz Velocity Transformation, 2

When v is much smaller than c (the nonrelativistic case), the denominator of

$$u'_x = \frac{dx'}{dt'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

approaches unity and so $u'_x \approx u_x - v,$

In another extreme, when $u_x = c$, $u'_x = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{c\left(1 - \frac{v}{c}\right)}{1 - \frac{v}{c}} = c$

To obtain u_x in terms of u'_x, use $u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$

Measurements Observers Do Not Agree On

Two observers *O* and *O'* do not agree on:

- The time interval between events that take place in the same position in one reference frame
- The distance between two points that remain fixed in one of their frames
- The velocity components of a moving particle
- Whether two events occurring at different locations in both frames are simultaneous or not

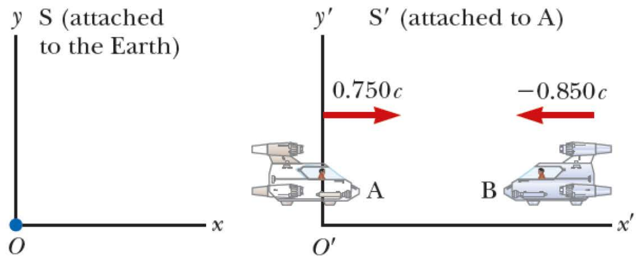
Measurements Observers Do Agree On

Two observers *O* and *O'* can agree on:

- Their relative speed of motion *v* with respect to each other
- The speed c of any ray of light
- The simultaneity of two events which take place at the same position and time in some frame.

Example

Two spacecraft A and B are moving in opposite directions as shown in Figure. An observer on the Earth measures the speed of spacecraft A to be 0.750*c* and the speed of spacecraft B to be 0.850*c*. Find the velocity of spacecraft B as observed by the crew on spacecraft A.

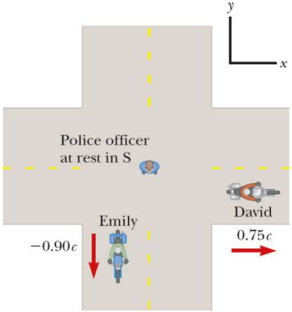


Obtain the velocity u'_x of spacecraft B relative to spacecraft A

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = \boxed{-0.977c}$$

Example

Two motorcycle pack leaders named David and Emily are racing at relativistic speeds along perpendicular paths as shown in Figure. How fast does Emily recede as seen by David over his right shoulder?



Identify the velocity components for David and Emily according to the police officer:

David: $v_x = v = 0.75c \quad v_y = 0$

Emily: $u_x = 0 \quad u_y = -0.90c$

Calculate u'_x and u'_y for Emily as measured by David:

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0 - 0.75c}{1 - \frac{(0)(0.75c)}{c^2}} = -0.75c$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} = \frac{\sqrt{1 - \frac{(0.75c)^2}{c^2}}\,(-0.90c)}{1 - \frac{(0)(0.75c)}{c^2}} = -0.60c$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Finally, find the speed of Emily as measured by David:

$$u' = \sqrt{(u'_x)^2 + (u'_y)^2}$$

$$= \sqrt{(-0.75c)^2 + (-0.60c)^2} = \boxed{0.96c}$$

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