PHYS143
Physics for Engineers
Tutorial - Chapter 38 – Solutions

Question 1

Light of wavelength 587.5 nm illuminates a slit of width 0.75 mm. (a) At what distance from the slit should a screen be placed if the first minimum in the diffraction pattern is to be 0.85 mm from the central maximum? (b) Calculate the width of the central maximum.

(a) Dark bands (minima) occur where

$$\sin\theta = m\frac{\lambda}{a}$$

For the first minimum, $m = 1$, and the distance from the center of the central maximum is

$$y_1 = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

Thus, the needed distance to the screen is

$$L = y_1\left(\frac{a}{\lambda}\right) = \left(0.85\times10^{-3}\text{ m}\right)\left(\frac{0.75\times10^{-3}\text{ m}}{587.5\times10^{-9}\text{ m}}\right) = \boxed{1.1\text{ m}}$$

(b) The width of the central maximum is

$$2y_1 = 2(0.85\text{ mm}) = \boxed{1.7\text{ mm}}$$

Question 2

A screen is placed 50.0 cm from a single slit, which is illuminated with light of wavelength 690 nm. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

In the equation for single-slit diffraction minima at small angles,

$$\frac{y}{L} \approx \sin\theta_{\text{dark}} = \frac{m\lambda}{a}$$

we take differences between the first and third dark fringes, to see that

$$\frac{\Delta y}{L} = \frac{\Delta m\lambda}{a} \quad \text{with} \quad \Delta y = 3.00\times10^{-3}\text{ m} \quad \text{and} \quad \Delta m = 3-1=2$$

The width of the slit is then

$$a = \frac{\lambda L\Delta m}{\Delta y} = \frac{\left(690\times10^{-9}\text{ m}\right)(0.500\text{ m})(2)}{\left(3.00\times10^{-3}\text{ m}\right)} = \boxed{2.30\times10^{-4}\text{ m}}$$

Question 3

(a) The pupil of a cat's eye narrows to a vertical slit of width 0.500 mm in daylight. Assume the average wavelength of the light is 500 nm. What is the angular resolution for horizontally separated mice?
(b)The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What minimum diameter is required for the telescope's receiving dish?

(a) We assume Rayleigh's criterion applies to the cat's eye with pupil narrowed. For a single slit (not a round aperture), for small angles

$$\theta \approx \sin\theta = \frac{\lambda}{a} = \frac{500\times 10^{-9}\ \text{m}}{0.500\times 10^{-3}\ \text{m}} = \boxed{1.00\times 10^{-3}\ \text{rad}}$$

(b) Using Rayleigh's criterion,

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 0.100°\left(\frac{\pi\ \text{rad}}{180°}\right) = 1.75\times 10^{-3}\ \text{rad}$$

and
$$D = 1.22\left(\frac{\lambda}{\theta_{\min}}\right) = 1.22\left(\frac{3.00\times 10^{-3}\ \text{m}}{\theta_{\min}}\right) = \boxed{2.10\ \text{m}}$$

Question 4

A grating with 250 grooves/mm is used with an incandescent light source. Assume the visible spectrum to range in wavelength from 400 nm to 700 nm. In how many orders can one see (a) the entire visible spectrum and (b) the short-wavelength region of the visible spectrum?

The grating spacing is

$$d = \frac{1.00\times 10^{-3}\ \text{m}}{250} = 4.00\times 10^{-6}\ \text{m} = 4\ 000\ \text{nm}$$

Solving for m gives

$$d\sin\theta = m\lambda \quad \rightarrow \quad m = \frac{d\sin\theta}{\lambda}$$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71$$

or $\boxed{\text{5 orders is the maximum}}$.

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0$$

or $\boxed{\text{10 orders in the short-wavelength region}}$.

Question 5

The first-order diffraction maximum is observed at 12.6° for a crystal having a spacing between planes of atoms of 0.250 nm. (a) What wavelength x-ray is used to observe this first-order pattern? (b) How many orders can be observed for this crystal at this wavelength?

(a) By Bragg's law, $2d\sin\theta = m\lambda$, and $m = 1$:

$$\lambda = 2d\sin\theta = 2(0.250 \text{ nm})\sin 12.6° = \boxed{0.109 \text{ nm}}$$

(b) We obtain the number of orders from

$$\frac{m\lambda}{2d} = \sin\theta \le 1 \rightarrow m \le \frac{2d}{\lambda} = \frac{2(0.250 \text{ nm})}{0.109 \text{ nm}} = 4.59$$

The order-number must be an integer, so the largest value m can have is 4: $\boxed{\text{four}}$ orders can be observed.

Question 6

The angle of incidence of a light beam onto a reflecting surface is continuously variable. The reflected ray in air is completely polarized when the angle of incidence is 48.0°. What is the index of refraction of the reflecting material?

By Brewster's law, for light in air ($n = 1.00$) reflecting off a surface of index n,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{n}{1.00} = n$$

$$n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$$