Chapter 44

Nuclear Structure

Some Properties of Nuclei

All nuclei are composed of protons and neutrons.

- Exception is ordinary hydrogen with a single proton.

The **atomic number** Z equals the number of protons in the nucleus.

- Sometimes called the charge number.

The **neutron number** N is the number of neutrons in the nucleus.

The **mass number** A is the number of **nucleons** in the nucleus.

- $A = Z + N$
- Nucleon is a generic term used to refer to either a proton or a neutron.
- The mass number is not the same as the mass.

Symbolism

A **nuclide** is a specific combination of atomic number and mass number that represents a nucleus.

$$^{A}_{Z}\mathrm{X}$$

- X is the chemical symbol of the element.

Example:

$$^{27}_{13}\mathrm{Al}$$

- Mass number is 27
- Atomic number is 13
- Contains 13 protons
- Contains 14 (27 – 13) neutrons

- The Z may be omitted since the element can be used to determine Z.

More Properties

The nuclei of all atoms of a particular element must contain the same number of protons.

They may contain varying numbers of neutrons.

- **Isotopes** of an element have the same Z but differing N and A values.
- The natural abundance of isotopes can vary.
- Isotope example:

$$^{11}_{6}C,\ ^{12}_{6}C,\ ^{13}_{6}C,\ ^{14}_{6}C$$

Charge

The proton has a single positive charge, *e.*

The electron has a single negative charge, - *e.*

- $e = 1.6 \times 10^{-19}$ C

The neutron has no charge.

- Made it difficult to detect in early experiments.
- Easy to detect with modern devices.

Mass

It is convenient to use *atomic mass units*, u, to express masses.

- 1 u = 1.660 539 x 10^{-27} kg (almost equal to the mass of a proton/neutron)
- Based on definition that the mass of one atom of ^{12}C is exactly 12 u.

Mass can also be expressed in MeV/c^2.

- From $E_R = mc^2$
- 1 u = 931.494 MeV/c^2
  - Includes conversion 1 eV = 1.602 176 x 10^{-19} J

TABLE 44.1 *Masses of Selected Particles in Various Units*

Particle	Mass kg	Mass u	Mass MeV/c^2
Proton	$1.672\,62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\,93 \times 10^{-27}$	1.008 665	939.57
Electron	$9.109\,38 \times 10^{-31}$	$5.485\,79 \times 10^{-4}$	0.510 999
$^1_1$H atom	$1.673\,53 \times 10^{-27}$	1.007 825	938.783
$^4_2$He nucleus	$6.644\,66 \times 10^{-27}$	4.001 506	3 727.38
$^{12}_6$C atom	$1.992\,65 \times 10^{-27}$	12.000 000	11 177.9

Size of Nucleus

Many experiments have concluded that:

- Most nuclei are approximately spherical.
- Average radius is

$$r = a\,A^{1/3}$$

  - $a = 1.2 \times 10^{-15}$ m
  - A is the mass number

Nuclear Stability

There are very large repulsive electrostatic forces between protons.

- These forces should cause the nucleus to fly apart.

The nuclei are stable because of the presence of another, short-range force, called the **nuclear force.**

- This is an attractive force that acts between all nuclear particles.
- The nuclear attractive force is stronger than the Coulomb repulsive force at the short ranges within the nucleus.

Features of the Nuclear Force

Attractive force that acts between all nuclear particles

Very short range

- It falls to zero when the separation between particles exceeds about several fermis.

Independent of charge

- The nuclear force on p-p, p-n, n-n are all the same
- Does not affect electrons

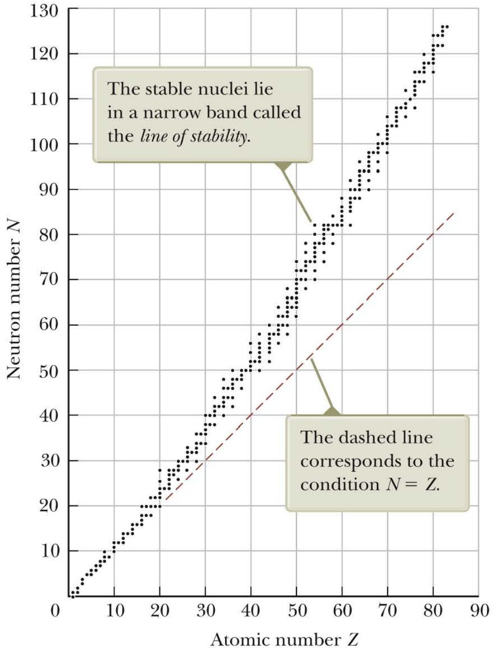
Nuclear Stability, cont.

Light nuclei are most stable if $N = Z$.

Heavy nuclei are most stable when $N > Z$.

- Above about $Z = 20$
- As the number of protons increases, the Coulomb force increases and so more neutrons are needed to keep the nucleus stable.

No nuclei are stable when $Z > 83$.



Binding Energy

The total energy of the bound system (the nucleus) is less than the combined energy of the separated nucleons.

- This difference in energy is called the **binding energy** of the nucleus. It can be thought of as the amount of energy you need to add to the nucleus to break it apart into its components.

The binding energy can be calculated from conservation of energy and the Einstein mass-energy equivalence principle:

$$E_b = [ZM(\mathrm{H}) + Nm_n - M(^A_Z\mathrm{X})] \times 931.494\ \mathrm{MeV/u}$$

  - $M(\mathrm{H})$ is the atomic mass of the neutral hydrogen atom
  - $M\ (^A_Z\mathrm{X})$ represents the atomic mass of an atom of the isotope $(^A_Z\mathrm{X})$
  - m_n is the mass of the neutron
- The masses are expressed in atomic mass units and E_b will be in MeV.
- An increase in binding energy corresponds to a decrease in the energy of the system.

Marie Curie

1867 – 1934

Polish scientist

Shared Nobel Prize in Physics in 1903 for studies in radioactive substances

- Shared with Pierre Curie and Becquerel

Won Nobel Prize in Chemistry in 1911 for discovery of radium and polonium

Radioactivity

Radioactivity is the spontaneous emission of radiation.

- Discovered by Becquerel in 1896
- Many experiments were conducted by Becquerel and the Curies.

Experiments suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei.

Radioactivity – Types of Decay

Three types of radiation can be emitted.

- Alpha particles
  - The particles are ^{4}He nuclei. (2 protons, 2 neutrons)
- Beta particles
  - The particles are either electrons or positrons.
    - A **positron** is the antiparticle of the electron.
    - It is similar to the electron except its charge is +e.
- Gamma rays
  - The “rays” are high energy photons.

Distinguishing Types of Radiation

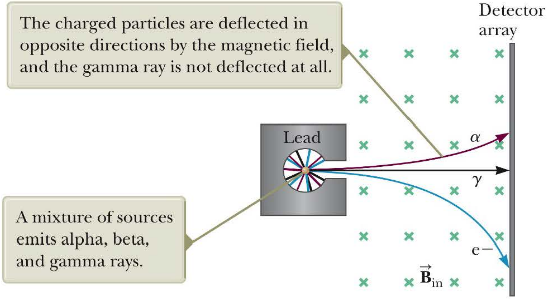
All three types of radiation enter a region where there is a magnetic field.

The gamma particles carry no charge.

The alpha particles are deflected upward.

The beta particles are deflected downward.

- A positron would be deflected upward, but would follow a different trajectory than the α due to its mass.



Penetrating Ability of Particles

Alpha particles

- Barely penetrate a piece of paper

Beta particles

- Can penetrate a few mm of aluminum

Gamma rays

- Can penetrate several cm of lead

The Decay Constant

The number of particles that decay in a given time is proportional to the total number of particles in a radioactive sample.

$$\frac{dN}{dt} = -\lambda N \text{ gives } N = N_o e^{-\lambda t}$$

- λ is called the **decay constant** and determines the probability of decay per nucleus per second.
- N is the number of undecayed radioactive nuclei present.
- N_o is the number of undecayed nuclei at time t = 0.

Decay Rate

The **decay rate** R of a sample is defined as the number of decays per second.

$$R = \left|\frac{dN}{dt}\right| = \lambda N = R_o e^{-\lambda t}$$

- $R_o = N_o\lambda$ is the decay rate at $t = 0$.
- The decay rate is often referred to as the activity of the sample.

Units

The unit of activity, R, is the **curie** (Ci)

- $1\ \text{Ci} \equiv 3.7 \times 10^{10}$ decays/s

The SI unit of activity is the **becquerel** (Bq)

- $1\ \text{Bq} \equiv 1$ decay/s
  - Therefore, $1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq}$

Decay Curve and Half-Life

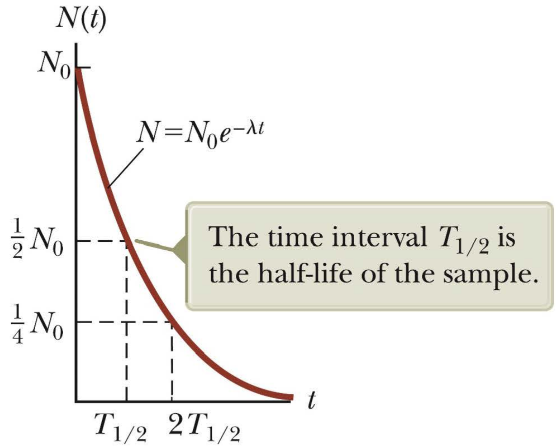
The decay curve follows the equation

$N = N_o e^{-\lambda t}$.

The **half-life** is also a useful parameter.

- The half-life is defined as the time interval during which half of a given number of radioactive nuclei decay.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$



Half-Life, cont.

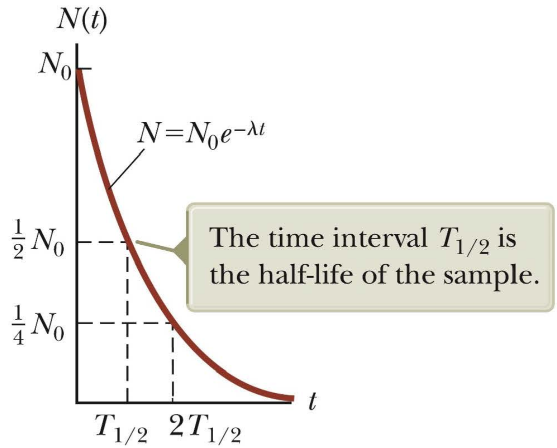
During the first half-life, ½ of the original material will decay.

During the second half-life, ½ of the remaining material will decay, leaving ¼ of the original material remaining.

Summarizing, the number of undecayed radioactive nuclei remaining after *n* half-lives is

$$N = N_o \left(\tfrac{1}{2}\right)^n$$

- *n* can be an integer or a noninteger.



Example

The isotope carbon-14, ${}^{14}_{6}\mathrm{C}$, is radioactive and has a half-life of 5 730 years. If you start with a sample of 1000 carbon-14 nuclei, how many nuclei will still be undecayed in 25000 years?

Divide the time interval by the half-life to determine the number of half-lives:

$$n = \frac{25\ 000\ \mathrm{yr}}{5\ 730\ \mathrm{yr}} = 4.363$$

Determine how many undecayed nuclei are left after this many half-lives

$$N = N_0\left(\tfrac{1}{2}\right)^n = 1\ 000\left(\tfrac{1}{2}\right)^{4.363} = \boxed{49}$$

Example

At time t = 0, a radioactive sample contains 3.50 μg of pure ${}^{11}_{6}C$, which has a half-life of 20.4 min. If the number of nuclei in sample at t = 0 is 1.92×10^{17}. What is the activity of the sample initially and after 8.00 h?

$$R_0 = \lambda N_0 = \frac{0.693}{T_{1/2}} N_0 = \frac{0.693}{20.4\ \text{min}} \left(\frac{1\ \text{min}}{60\ \text{s}}\right)(1.92 \times 10^{17})$$

$$= (5.66 \times 10^{-4}\ \text{s}^{-1})(1.92 \times 10^{17}) = \boxed{1.09 \times 10^{14}\ \text{Bq}}$$

$$R = R_0 e^{-\lambda t} = (1.09 \times 10^{14}\ \text{Bq}) e^{-(5.66 \times 10^{-4}\ \text{s}^{-1})(2.88 \times 10^{4}\ \text{s})} = \boxed{8.96 \times 10^{6}\ \text{Bq}}$$

Example

A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5.0 mCi at the time of shipment. Upon receipt of the sample at a medical laboratory, the activity is 2.1 mCi. How much time has elapsed between the two measurements?

$$\frac{R}{R_0} = e^{-\lambda t}$$

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

$$(1) \quad t = -\frac{1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right)$$

$$t = -\frac{8.04\ \mathrm{d}}{0.693}\ln\left(\frac{2.1\ \mathrm{mCi}}{5.0\ \mathrm{mCi}}\right) = \boxed{10\ \mathrm{d}}$$

Decay Processes

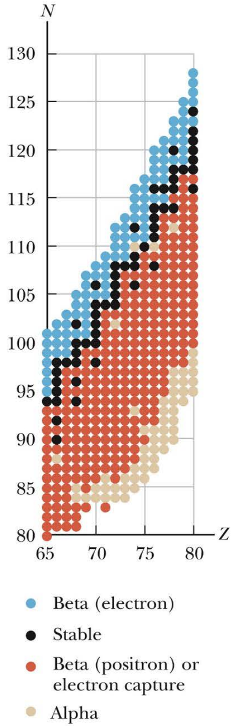
A radioactive nucleus spontaneously decays by one of three processes: alpha decay, beta decay, or gamma decay.

The black circles are the stable nuclei.

Above the line the nuclei are neutron rich and undergo beta decay (blue), in which an electron is emitted.

Just below the line are proton rich nuclei that undergo beta decay, in which a positron is emitted or a competing process called electron capture (red).

Farther below the line the nuclei are very proton rich and undergo alpha decay (tan).



Alpha Decay

When a nucleus emits an alpha particle it loses two protons and two neutrons.

- N decreases by 2
- Z decreases by 2
- A decreases by 4

Symbolically $^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$

- X is called the **parent nucleus.**
- Y is called the **daughter nucleus.**

Decay – General Rules

The sum of the mass numbers A must be the same on both sides of the equation.

The sum of the atomic numbers Z must be the same on both sides of the equation.

When one element changes into another element, the process is called **spontaneous decay** or transmutation.

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

Disintegration Energy

The disintegration energy Q of a system is defined as

$$Q = (M_X - M_Y - M_\alpha)c^2$$

M_X the mass of the parent nucleus, M_Y the mass of the daughter nucleus, and M_α the mass of the alpha particle.

The energy Q is in joules when the masses are in kilograms and c is the speed of light.

The disintegration energy Q is the amount of rest energy transformed and appears in the form of kinetic energy in the daughter nucleus and the alpha particle.

It is sometimes referred to as the Q value of the nuclear decay.

A negative Q value indicates that such a proposed decay does not occur spontaneously.

Example

The ^{226}Ra nucleus undergoes alpha decay according to:

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

Calculate the *Q* value for this process. The masses are 226.025 410 u for ^{226}Ra, 222.017 578 u for ^{222}Rn, and 4.002 603 u for ^{4}He.

$$\begin{aligned} Q &= (M_X - M_Y - M_\alpha) \times 931.494\ \mathrm{MeV/u} \\ &= (226.025\,410\ \mathrm{u} - 222.017\,578\ \mathrm{u} - 4.002\,603\ \mathrm{u}) \times 931.494\ \mathrm{MeV/u} \\ &= (0.005\,229\ \mathrm{u}) \times 931.494\ \mathrm{MeV/u} = \boxed{4.87\ \mathrm{MeV}} \end{aligned}$$

Alpha Decay, Example

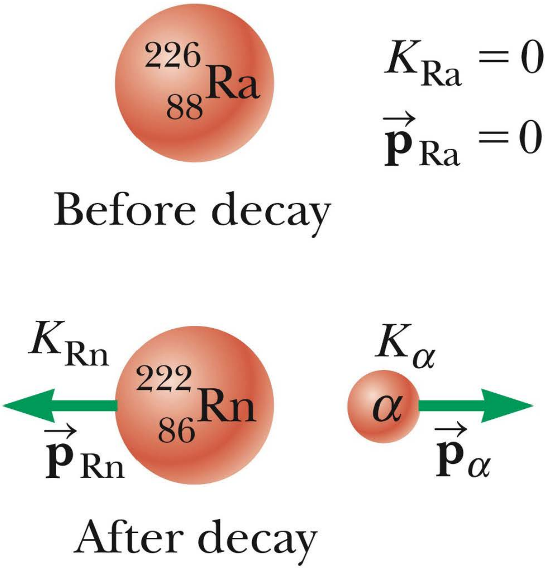
Decay of ^{226}Ra

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$$

If the parent is at rest before the decay, the total kinetic energy of the products is 4.87 MeV.

Most of this kinetic energy is associated with the alpha particle because this particle is much less massive than the daughter nucleus ^{222}Rn.

Generally, less massive particles carry off most of the energy in nuclear decays.



Beta Decay

During beta decay, the daughter nucleus has the same number of nucleons as the parent, but the atomic number is changed by one.

Symbolically

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + \mathrm{e}^{-}$$

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \mathrm{e}^{+}$$

- e^- designates an electron and e^+ designates a positron, with *beta particle* being the general term referring to either.
- Because *A* does not change but *Z* does, we conclude that in beta decay, either a neutron changes to a proton or a proton changes to a neutron.
- Note that the electron or positron emitted in these decays is not present beforehand in the nucleus; it is created in the process of the decay from the rest energy of the decaying nucleus.
- Beta decay is not completely described by these equations.

Neutrino

In1930 Pauli proposed the existence of another particle.

Enrico Fermi later named this particle the neutrino.

Properties of the neutrino:

- Zero electrical charge
- Mass much smaller than the electron, probably not zero.
- Spin of ½
- Very weak interaction with matter and so is difficult to detect

The neutrino was detected experimentally in 1956.

Beta Decay – Completed

Symbolically

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + \mathrm{e}^{-} + \bar{v} $$

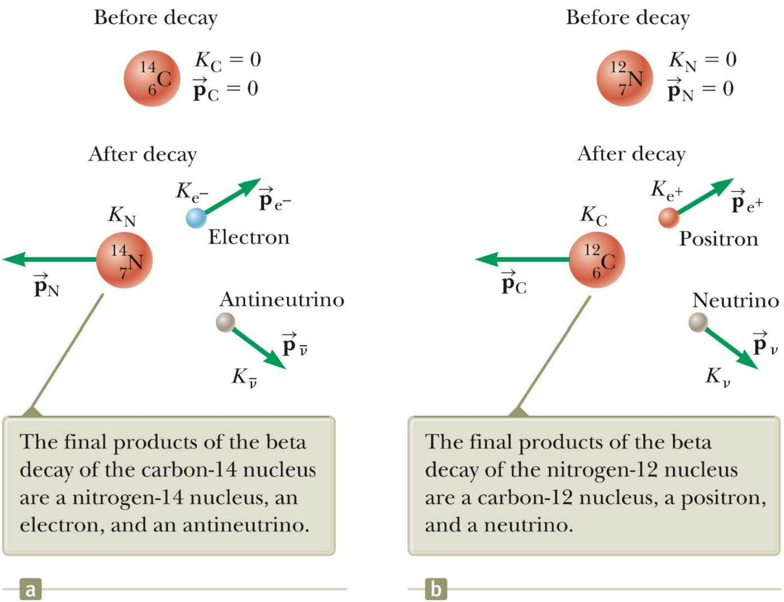
$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \mathrm{e}^{+} + v $$

- v is the symbol for the neutrino.
- $\bar{v}$ is the symbol for the antineutrino.

To summarize, in beta decay, the following pairs of particles are emitted.

- An electron and an antineutrino
- A positron and a neutrino

Beta Decay – Examples



Section 44.5

Beta Decay, Final Notes

The fundamental process of e^- decay is a neutron changing into a proton, an electron and an antineutrino.

$$n \rightarrow p + e^- + \bar{\nu}$$

In e^+, the proton changes into a neutron, positron and neutrino.

- This can only occur within a nucleus.
- It cannot occur for an isolated proton since its mass is less than the mass of the neutron.

Electron Capture

Electron capture is a process that competes with e+ decay.

In this case, a parent nucleus captures one of its own orbital electrons and emits a neutrino:

$$ {}^{A}_{Z}\mathrm{X} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{\;\;\;A}_{Z-1}\mathrm{Y} + \nu $$

Q Values for Beta Decay

For e^- decay and electron capture, the Q value is

$$Q = (M_X - M_Y)c^2.$$

For e^+ decay, the Q value is

$$Q = (M_X - M_Y - 2m_e)c^2.$$

If Q is negative, the decay will not occur.

Gamma Decay

Gamma rays are given off when an excited nucleus decays to a lower energy state.

The decay occurs by emitting a high-energy photon called gamma-ray photons.

$$ {}^{A}_{Z}\mathrm{X}^* \rightarrow {}^{A}_{Z}\mathrm{X} + \gamma $$

- The X* indicates a nucleus in an excited state.

Typical half-life is 10^{-10} s

The only change in the nucleus is that it ends up in a lower energy state.

- No changes in *Z*, *N* or *A* occur

Gamma Decay – Example

Example of a decay sequence:

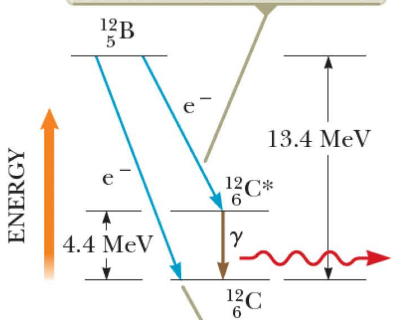
- The first decay is a beta emission.
- The second step is a gamma emission.

$$^{12}_{5}\text{B} \rightarrow {}^{12}_{6}\text{C}* + \text{e}^{-} + \bar{\nu}$$

$$^{12}_{6}\text{C}* \rightarrow {}^{12}_{6}\text{C} + \gamma$$

- Gamma emission doesn't change *Z, N*, or *A*
- The emitted photon has an energy of h*f* equal to ΔE between the two nuclear energy levels.

In this decay process, the daughter nucleus is in an excited state, denoted by $^{12}_{6}\text{C}*$, and the beta decay is followed by a gamma decay.



In this decay process, the daughter nucleus $^{12}_{6}\text{C}$ is left in the ground state.

Summary of Decays

TABLE 44.3 *Various Decay Pathways*

Alpha decay	${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$
Beta decay (e^-)	${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z+1}^{A}\mathrm{Y} + e^- + \overline{\nu}$
Beta decay (e^+)	${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z-1}^{A}\mathrm{Y} + e^+ + \nu$
Electron capture	${}^{A}_{Z}\mathrm{X} + e^- \rightarrow {}_{Z-1}^{A}\mathrm{Y} + \nu$
Gamma decay	${}^{A}_{Z}\mathrm{X}^* \rightarrow {}^{A}_{Z}\mathrm{X} + \gamma$