Chapter 37

Wave Optics

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Wave Optics

Wave optics is a study concerned with phenomena that cannot be adequately explained by geometric (ray) optics.

- Sometimes called *physical optics.*

These phenomena include:

- Interference
- Diffraction
- Polarization

Interference

In ***constructive interference*** the amplitude of the resultant wave is greater than that of either individual wave.

In ***destructive interference*** the amplitude of the resultant wave is less than that of either individual wave.

Young's Double-Slit Experiment: Schematic

Thomas Young first demonstrated interference in light waves from two sources in 1801.

The narrow slits S_1 and S_2 act as sources of waves.

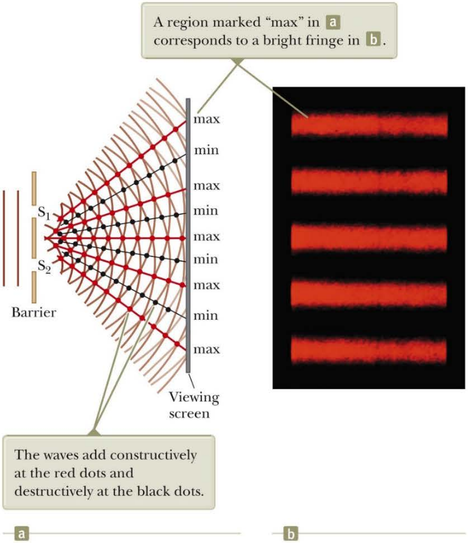
The waves emerging from the slits originate from the same wave front and therefore are always in phase.

The light from the two slits forms a visible pattern on a screen.

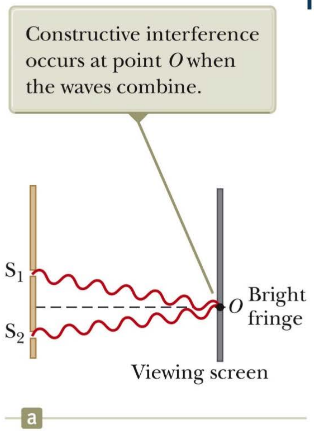
The pattern consists of a series of bright and dark parallel bands called *fringes.*

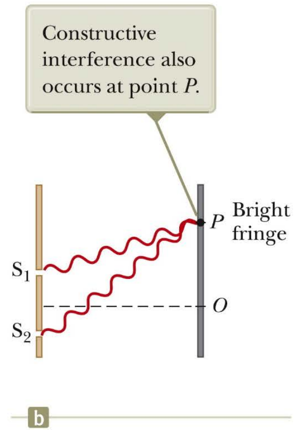
Constructive interference occurs where a bright fringe occurs.

Destructive interference results in a dark fringe.



Interference Patterns, 1





Constructive interference occurs at point O.

The two waves travel the same distance.

- Therefore, they arrive in phase

As a result, constructive interference occurs at this point and a bright fringe is observed.

The lower wave has to travel farther than the upper wave to reach point *P.*

The lower wave travels one wavelength farther.

- Therefore, the waves arrive in phase

A second bright fringe occurs at this position.

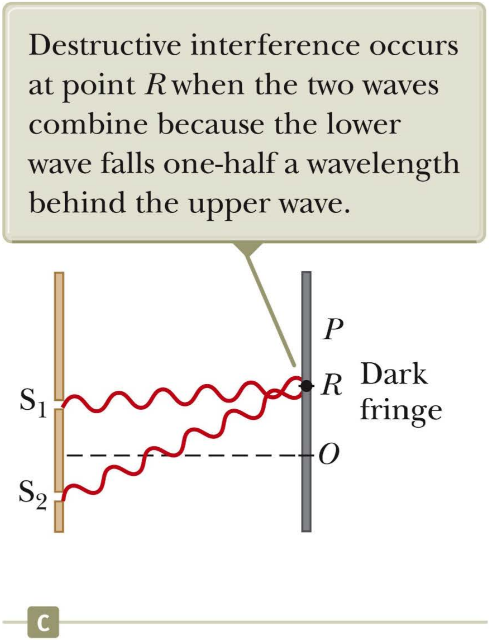
Interference Patterns, 2

The upper wave travels one-half of a wavelength farther than the lower wave to reach point *R.*

The lower wave has fallen half a wavelength behind the upper wave and a crest of the upper wave overlaps a trough of the lower wave.

This is destructive interference.

- A dark fringe occurs.



Conditions for Interference

To observe interference in light waves, the following two conditions must be met:

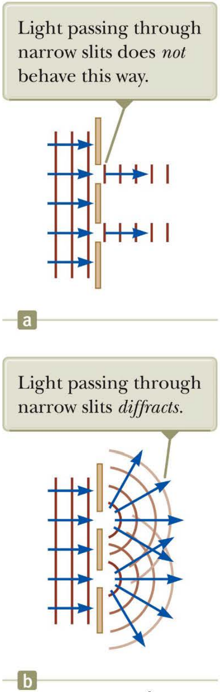
- The sources must be **coherent.**
  - They must maintain a constant phase with respect to each other.
- The sources should be **monochromatic.**
  - Monochromatic means they have a single wavelength.

Diffraction

If the light traveled in a straight line after passing through the slits, no interference pattern would be observed.

However, light passing through narrow slits does not behave this way. The waves spread out from the slits.

This divergence of light from its initial line of travel is called **diffraction.**



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Young's Double-Slit Experiment: Geometry

The viewing screen is located a perpendicular distance *L* from the barrier containing two slits, S_1 and S_2.

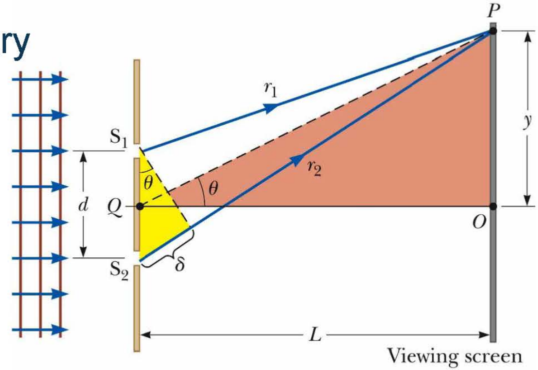
These slits are separated by a distance *d*, and the source is monochromatic.

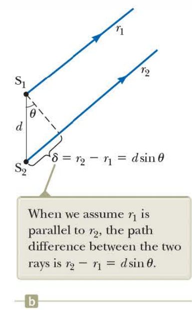
To reach any arbitrary point *P* in the upper half of the screen, a wave from the lower slit must travel farther than a wave from the upper slit. The extra distance traveled from the lower slit is the **path difference δ.**

The path difference, δ, is found from geometry.

$\delta = r_2 - r_1 = d \sin\theta$

- This assumes the paths are parallel.
- Not exactly true, but a very good approximation if *L* is much greater than *d*





Interference Equations

For a bright fringe produced by constructive interference, the path difference must be either zero or some integer multiple of the wavelength.

$$d \sin \theta_{\text{bright}} = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

- m = 0, ±1, ±2, …
- m is called the order number
  - When m = 0, it is the zeroth-order maximum
  - When m = ±1, it is called the first-order maximum

When destructive interference occurs, a dark fringe is observed.

This needs a path difference of an odd half wavelength.

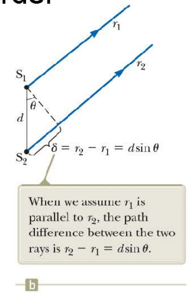
$$d \sin \theta_{\text{dark}} = (m + \tfrac{1}{2})\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

- m = 0, ±1, ±2, …

Interference Equations, cont.

The positions of the fringes can be measured vertically from the zeroth-order maximum. Using the large triangle, *OPQ*,

- $y_{bright} = L \tan \theta_{bright}$ $y_{dark} = L \tan \theta_{dark}$



Assumptions in a Young's Double Slit Experiment:

- L >> d and d >> λ

Approximation:

- θ is small and therefore the small angle approximation tan θ ~ sin θ can be used

$$y = L \tan\theta \approx L \sin\theta$$

For small angles,

$$y_{bright} = L\frac{m\lambda}{d} \text{ and } y_{dark} = L\frac{(m+\frac{1}{2})\lambda}{d}$$

This result shows that y_{bright} is linear in the order number *m*, so the fringes are equally spaced for small angles.

Measuring the Wavelength of a Light Source, Example

A viewing screen is separated from a double slit by 4.80 m. The distance between the two slits is 0.030 0 mm. Monochromatic light is directed toward the double slit and forms an interference pattern on the screen. The first dark fringe is 4.50 cm from the center line on the screen.

(A) Determine the wavelength of the light.

$$\lambda = \frac{y_{\text{dark}} d}{(m + \frac{1}{2})L} = \frac{(4.50 \times 10^{-2}\text{ m})(3.00 \times 10^{-5}\text{ m})}{(0 + \frac{1}{2})(4.80\text{ m})}$$

$$= 5.62 \times 10^{-7}\text{ m} = \boxed{562\text{ nm}}$$

(B) Calculate the distance between adjacent bright fringes.

$$y_{m+1} - y_m = L\frac{(m+1)\lambda}{d} - L\frac{m\lambda}{d}$$

$$= L\frac{\lambda}{d} = 4.80\text{ m}\left(\frac{5.62 \times 10^{-7}\text{ m}}{3.00 \times 10^{-5}\text{ m}}\right)$$

$$= 9.00 \times 10^{-2}\text{ m} = \boxed{9.00\text{ cm}}$$

Separating Double-Slit Fringes of Two Wavelengths, Example

A light source emits visible light of two wavelengths: $\lambda = 430$ nm and $\lambda' = 510$ nm. The source is used in a double-slit interference experiment in which $L = 1.50$ m and $d = 0.025\,0$ mm. Find the separation distance between the third-order bright fringes for the two wavelengths.

$$\Delta y = y'_{\text{bright}} - y_{\text{bright}} = L\frac{m\lambda'}{d} - L\frac{m\lambda}{d} = \frac{Lm}{d}(\lambda' - \lambda)$$

$$\Delta y = \frac{(1.50\text{ m})(3)}{0.025\,0 \times 10^{-3}\text{ m}}(510 \times 10^{-9}\text{ m} - 430 \times 10^{-9}\text{ m})$$

$$= 0.014\,4\text{ m} = \boxed{1.44\text{ cm}}$$

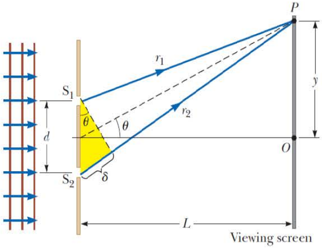
Example

In the double-slit arrangement of Figure, d = 0.150 mm, L = 140 cm, λ = 643 nm, and y = 1.80 cm. (a) What is the path difference d for the rays from the two slits arriving at P ? (b) Express this path difference in terms of λ. (c) Does P correspond to a maximum, a minimum, or an intermediate condition? Give evidence for your answer.

(a) The path difference $\delta = d\sin\theta$, and when $L >> y$

$$\delta = \frac{yd}{L} = \frac{(1.80\times10^{-2}\text{ m})(1.50\times10^{-4}\text{ m})}{1.40\text{ m}}$$

$$= 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$



(b) $$\frac{\delta}{\lambda} = \frac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00 \qquad \boxed{\delta = 3.00\lambda}$$

(c) Point P will be a $\boxed{\text{maximum}}$ because the path difference is an integer multiple of the wavelength.

Example

A laser beam is incident on two slits with a separation of 0.200 mm, and a screen is placed 5.00 m from the slits. An interference pattern appears on the screen. If the angle from the center fringe to the first bright fringe to the side is 0.181° , what is the wavelength of the laser light?

The location of the bright fringe of order *m* (measured from the position of the central maximum) is

$$d \sin\theta = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

For first bright fringe to the side, $m = 1$. Thus, the wavelength of the laser light must be

$$\lambda = d \sin\theta = (0.200 \times 10^{-3}\,\text{m}) \sin 0.181°$$

$$= 6.32 \times 10^{-7}\ \text{m} = \boxed{632\ \text{nm}}$$

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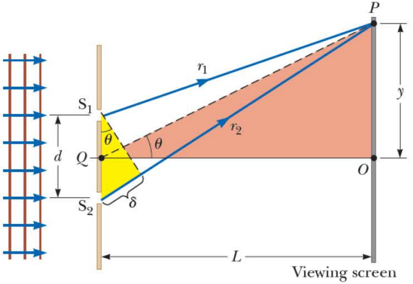
Intensity Distribution: Double-Slit Interference Pattern

The bright fringes in the interference pattern do not have sharp edges.

- The equations developed give the location of only the centers of the bright and dark fringes.

Direct our attention to the intensity of the light at other points between the positions of maximum constructive and destructive interference.

- In other words, calculate the distribution of light intensity associated with the double-slit interference pattern.



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Light Intensity

The interference pattern consists of equally spaced fringes of equal intensity

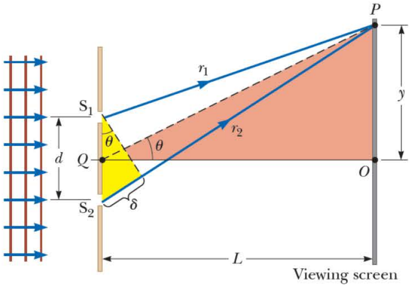
$$I = I_{\max}\cos^2\left(\frac{\phi}{2}\right) \quad \Rightarrow \quad \boxed{I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)}$$

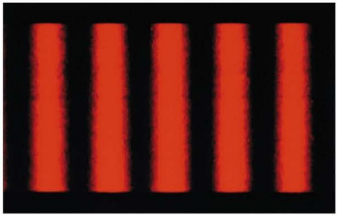
$$\phi = \frac{2\pi}{\lambda}\,\delta = \frac{2\pi}{\lambda}\,d\sin\theta$$

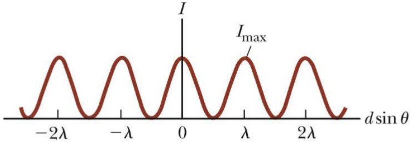
ϕ *is* the phase difference between the two waves at P

because sin θ ≈ *y*/*L* for small values of θ

$$I = I_{\max}\cos^2\left(\frac{\pi d}{\lambda L}\,y\right) \quad \text{(small angles)}$$







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Example

Two slits are separated by 0.180 mm. An interference pattern is formed on a screen 80.0 cm away by 656.3-nm light. Calculate the fraction of the maximum intensity a distance y = 0.600 cm away from the central maximum.

We use

$$I = I_{\max} \cos^2\left(\frac{\pi y d}{\lambda L}\right)$$

Solving and substituting then gives

$$\frac{I}{I_{\max}} = \cos^2\left[\frac{\pi\left(6.00\times 10^{-3}\text{ m}\right)\left(1.80\times 10^{-4}\text{ m}\right)}{\left(656.3\times 10^{-9}\text{ m}\right)\left(0.800\text{ m}\right)}\right] = \boxed{0.968}$$

Lloyd's Mirror

An arrangement for producing an interference pattern with a single light source.

Waves reach point *P* either by a direct path or by reflection.

The reflected ray can be treated as a ray from the source S' behind the mirror.

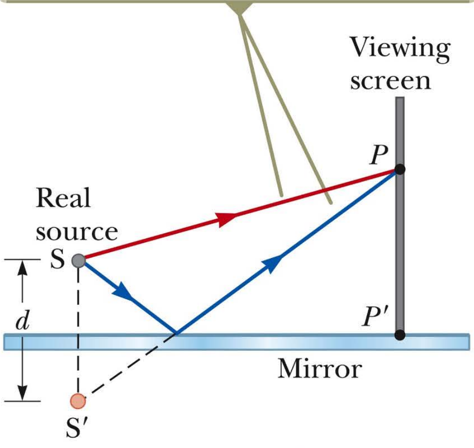
This arrangement can be thought of as a double-slit source with the distance between points S and S' comparable to length *d*.

An interference pattern is formed.

The positions of the dark and bright fringes are reversed relative to the pattern of two real sources.

This is because there is a 180° phase change produced by the reflection.

An interference pattern is produced on the screen as a result of the combination of the direct ray (red) and the reflected ray (blue).



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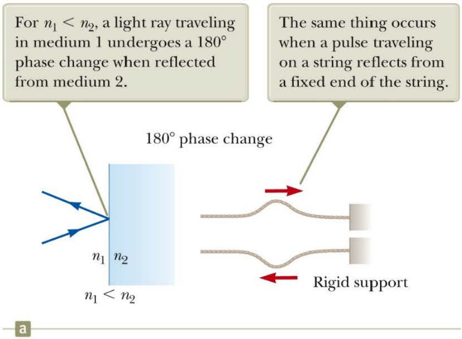
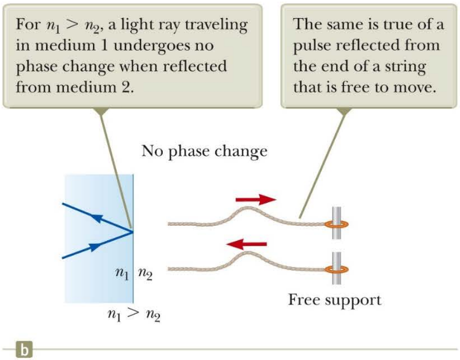
Phase Changes Due To Reflection





An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling.

There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction.

Interference in Thin Films

Interference effects are commonly observed in thin films.

- Examples include thin layers of oil in water or the thin surface of a soap bubble.

The various colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Facts to remember:

- An electromagnetic wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change on reflection when $n_2 > n_1$.
  - There is no phase change in the reflected wave if $n_2 < n_1$.
- The wavelength of light λ_n in a medium with index of refraction n is $\lambda_n = \lambda/n$ where λ is the wavelength of light in vacuum.

Interference in Thin Films, 2

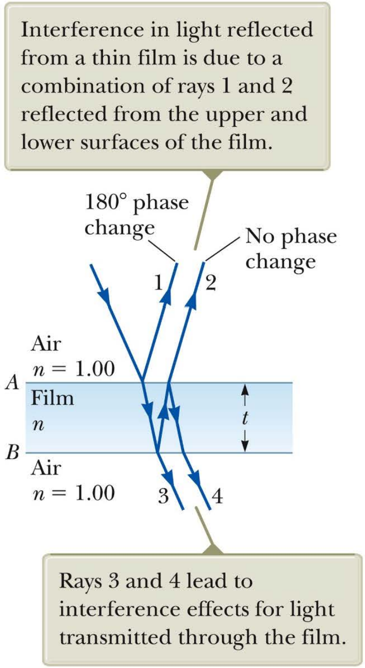
Assume the light rays are traveling in air nearly normal to the two surfaces of the film.

Ray 1, reflected from the upper surface (A), undergoes a phase change of 180° with respect to the incident ray.

Ray 2, which is reflected from the lower surface (B), undergoes no phase change with respect to the incident wave.

- Because it is reflected from a medium (air) that has a lower index of refraction.

Therefore, ray 1 is 180° out of phase with ray 2, which is equivalent to a path difference of $\lambda_n/2$.



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Interference in Thin Films, 3

Ray 2 also travels an additional distance of 2t before the waves recombine in the air above surface *A.*

If $2t = \lambda_n/2$, rays 1 and 2 recombine in phase and the result is constructive interference.

For constructive interference:

- $2nt = (m + ½)\lambda$ (m = 0, 1, 2 …)
  - This takes into account both the difference in optical path length for the two rays and the 180° phase change.

If the extra distance $2t$ traveled by ray 2 corresponds to a multiple of λ_n, the two waves combine out of phase and the result is destructive interference.

For destructive interference

- $2nt = m\lambda$ (m = 0, 1, 2 …)

Interference in Thin Films, 4

Two factors influence interference

- Possible phase reversals on reflection.
- Differences in travel distance.

The conditions are valid if the medium above the top surface is the same as the medium below the bottom surface.

- If there are different media, these conditions are valid as long as the index of refraction for both is less than n.

Interference in Thin Films, 5

If the thin film is between two different media, one of lower index than the film and one of higher index, the conditions for constructive and destructive interference are reversed.

- In that case, either there is a phase change of 180° for both ray 1 reflecting from surface *A* and ray 2 reflecting from surface *B* or there is no phase change for either ray; hence, the net change in relative phase due to the reflections is zero.

With different materials on either side of the film, you may have a situation in which there is a 180° phase change at both surfaces or at neither surface.

- Be sure to check both the path length and the phase change.

Interference in a Soap Film, Example

Calculate the minimum thickness of a soap-bubble film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm. The index of refraction of the soap film is 1.33.

$$t = \frac{(0 + \frac{1}{2})\lambda}{2n} = \frac{\lambda}{4n} = \frac{(600 \text{ nm})}{4(1.33)} = \boxed{113 \text{ nm}}$$