PHYS143
Physics for Engineers
Tutorial - Chapter 27 - Solutions

Question 1

A metal wire has a resistance of 10.0 Ω at a temperature of 20.0°C. If the same wire has a resistance of 10.6 Ω at 90.0°C, what is the resistance of this wire when its temperature is -20.0°C?

Using $R_0 = 10.0\ \Omega$ at $T = 20.0\ °C$, we have $R = R_0(1+\alpha\Delta T)$ or

$$\alpha = \frac{R/R_0 - 1}{\Delta T} = \frac{10.6/10.0 - 1}{(90.0°C - 20.0°C)} = 8.57\times10^{-4}\ °C^{-1}$$

At $T = -20.0°C$, we have

$$R = R_0(1+\alpha\Delta T)$$
$$= (10.0\ \Omega)\left[1 + 8.57\times10^{-4}\ °C^{-1}(-20.0°C - 20.0°C)\right] = 9.66\ \Omega$$

Question 2

A 200-km-long high-voltage transmission line 2.00 cm in diameter carries a steady current of 1 000 A. If the conductor is copper with a free charge density of 8.50×10^{28} electrons per cubic meter, how many years does it take one electron to travel the full length of the cable?

The drift speed of electrons in the line is

$$v_d = \frac{I}{nqa} = \frac{I}{n|e|(\pi d^2/4)}$$

The time to travel the 200-km length of the line is then

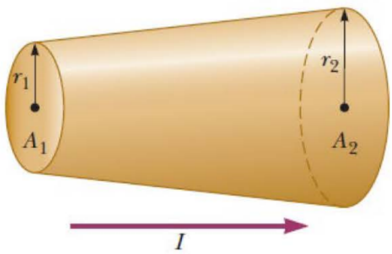
$$\Delta t = \frac{L}{v_d} = \frac{Ln|e|(\pi d^2)}{4I}$$

Substituting numerical values,

$$\Delta t = \frac{(200\times10^3\ \text{m})(8.50\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})\pi(0.02\ \text{m})^2}{4(1\ 000\ \text{A})}$$
$$= (8.55\times10^8\ \text{s})\left(\frac{1\ \text{yr}}{3.156\times10^7\ \text{s}}\right) = \boxed{27.1\ \text{yr}}$$

Question 3

The Figure represents a section of a conductor of nonuniform diameter carrying a current of I = 5.00 A. The radius of cross-section A_1 is $r_1 = 0.400$ cm. (a) What is the magnitude of the current density across A_1? The radius r_2 at A_2 is larger than the radius r_1 at A_1. (b) Is the current at A_2 larger, smaller, or the same? (c) Is the current density at A_2 larger, smaller, or the same? Assume $A_2 = 4A_1$. Specify the (d) radius, (e) current, and (f) current density at A_2.



(a) $J = \frac{I}{A} = \frac{5.00\ \text{A}}{\pi\left(4.00\times10^{-3}\ \text{m}\right)^2} = \boxed{99.5\ \text{kA/m}^2}$

(b) $\boxed{\text{Current is the same.}}$

(c) The cross-sectional area is greater; therefore $\boxed{\text{the current density is smaller.}}$

(d) $A_2 = 4A_1$ or $\pi r_2^2 = 4\pi r_1^2$ so $r_2 = 2r_1 = \boxed{0.800\ \text{cm}}$.

(e) $\boxed{I = 5.00\ \text{A}}$

(f) $J_2 = \frac{1}{4}J_1 = \frac{1}{4}\left(9.95\times10^4\ \text{A/m}^2\right) = \boxed{2.49\times10^4\ \text{A/m}^2}$

Question 4

The electron beam emerging from a certain high energy electron accelerator has a circular cross section of radius 1.00 mm. (a) The beam current is 8.00 µA. Find the current density in the beam assuming it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as 300 Mm/s with negligible error. Find the electron density in the beam. (c) Over what time interval does Avogadro's number of electrons emerge from the accelerator? Avogadro's number = $6.02\times10^{23}\ e/mol$.

(a) $J = \frac{I}{A} = \frac{8.00\times10^{-6}\ \text{A}}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2} = \boxed{2.55\ \text{A/m}^2}$

(b) From $J = nev_d$, we have

$$n = \frac{J}{ev_d} = \frac{2.55\ \text{A/m}^2}{\left(1.60\times10^{-19}\ \text{C}\right)\left(3.00\times10^8\ \text{m/s}\right)} = \boxed{5.31\times10^{10}\ \text{m}^{-3}}$$

(c) From $I = \frac{\Delta Q}{\Delta t}$, we have

$$\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{(6.02 \times 10^{23})(1.60 \times 10^{-19}\ \text{C})}{8.00 \times 10^{-6}\ \text{A}}$$

$$= \boxed{1.20 \times 10^{10}\ \text{s}}$$

(This is about 382 years!)

Question 5

A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross sectional area of 0.600 mm^2. What is the current in the wire? ($\rho = 5.60 \times 10^{-8} \Omega.m$)

$\Delta V = IR$ and $R = \dfrac{\rho \ell}{A}$. The area is

$$A = (0.600\ \text{mm}^2)\left(\frac{1.00\ \text{m}}{1\,000\ \text{mm}}\right)^2 = 6.00 \times 10^{-7}\ \text{m}^2$$

From the potential difference, we can solve for the current, which gives

$$\Delta V = \frac{I\rho\ell}{A} \quad \rightarrow \quad I = \frac{\Delta V A}{\rho \ell} = \frac{(0.900\ \text{V})(6.00 \times 10^{-7}\ \text{m}^2)}{(5.60 \times 10^{-8}\ \Omega \cdot \text{m})(1.50\ \text{m})}$$

$$I = \boxed{6.43\ \text{A}}$$

Question 6

An aluminum wire with a diameter of 0.100 mm has a uniform electric field of 0.200 V/m imposed along its entire length. The temperature of the wire is 50.0°C. Assume one free electron per atom. (a) Determine the resistivity of aluminum at this temperature. (b) What is the current density in the wire? (c) What is the total current in the wire? (d) What potential difference must exist between the ends of a 2.00-m length of the wire to produce the stated electric field? ($\alpha = 3.9 \times 10^{-3}\ {}^\circ\text{C}^{-1}, \rho_0 = 2.82 \times 10^{-8}\ \Omega.m$ at $T_0 = 20\ {}^\circ\text{C}$).

(a) The resistivity is computed from $\rho = \rho_0\left[1 + \alpha(T - T_0)\right]$:

$$\rho = (2.82 \times 10^{-8}\ \Omega \cdot \text{m})\left[1 + (3.90 \times 10^{-3}\ {}^\circ\text{C}^{-1})(30.0^\circ\text{C})\right]$$

$$= \boxed{3.15 \times 10^{-8}\ \Omega \cdot \text{m}}$$

(b) The current density is

$$J = \sigma E = \frac{E}{\rho} = \left(\frac{0.200\ \text{V/m}}{3.15 \times 10^{-8}\ \Omega \cdot \text{m}}\right)\left(\frac{1\ \Omega \cdot \text{A}}{\text{V}}\right) = \boxed{6.35 \times 10^6\ \text{A/m}^2}$$

(c) The current density is related to the current by $J = \dfrac{I}{A} = \dfrac{I}{\pi r^2}$.

$$I = J(\pi r^2) = (6.35 \times 10^6\ \text{A/m}^2)\left[\pi(5.00 \times 10^{-5}\ \text{m})^2\right] = \boxed{49.9\ \text{mA}}$$

(d) The applied voltage is $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$.

Question 7

A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the magnitude of the electric field in the wire and (b) the power delivered to it? (c) What If? If the temperature is increased to 340°C and the potential difference across the wire remains constant, what is the power delivered? ($\rho = 1.50 \times 10^{-6}\ \Omega.m, \alpha = 0.4 \times 10^{-3}\ ^{\circ}\mathrm{C}^{-1}$).

First, we compute the resistance of the wire:

$$R = \frac{\rho \ell}{A} = \frac{\left(1.50 \times 10^{-6}\ \Omega \cdot \mathrm{m}\right) 25.0\ \mathrm{m}}{\pi \left(0.200 \times 10^{-3}\ \mathrm{m}\right)^2} = 298\ \Omega$$

The potential drop across the wire is then

$$\Delta V = IR = (0.500\ \mathrm{A})(298\ \Omega) = 149\ \mathrm{V}$$

(a) The magnitude of the electric field in the wire is

$$E = \frac{\Delta V}{\ell} = \frac{149\ \mathrm{V}}{25.0\ \mathrm{m}} = \boxed{5.97\ \mathrm{V/m}}$$

(b) The power delivered to the wire is

$$P = (\Delta V) I = (149\ \mathrm{V})(0.500\ \mathrm{A}) = \boxed{74.6\ \mathrm{W}}$$

(c) We use:

$$R = R_0\left[1 + \alpha (T - T_0)\right] = (298\ \Omega)\left[1 + \left(0.400 \times 10^{-3}/^{\circ}\mathrm{C}\right) 320^{\circ}\mathrm{C}\right]$$
$$= 337\ \Omega$$

To find the power delivered, we first compute the current flowing through the wire:

$$I = \frac{\Delta V}{R} = \frac{149\ \mathrm{V}}{337\ \Omega} = 0.443\ \mathrm{A}$$

then,

$$P = (\Delta V) I = (149\ \mathrm{V})(0.443\ \mathrm{A}) = \boxed{66.1\ \mathrm{W}}$$