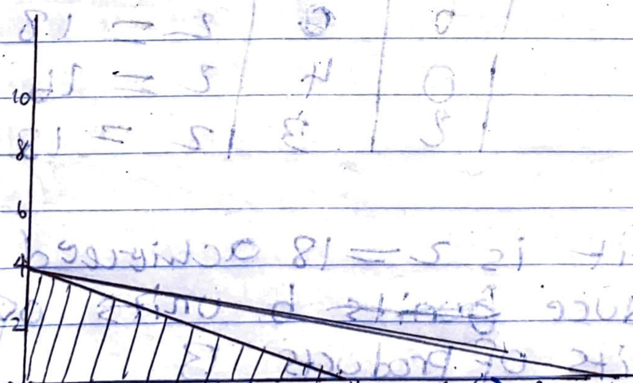
JOTHAM JERRY KATO
Y/UG/CSC/23/9791
CSC 333

23/11/24

(i) x^1 – Number of units of products A produced daily
x^2 – Number of units of products B produced daily
Object function – Maximize Profit ; $Z = 3x_1 + 4x_2$
Constraints : i) Machines time ; $2x_1 + 3x_2 \leq 12$; $2x_1 + 3x_2 = 12$

(ii) How material time ; $x_1 + 2x_2 \leq 8$; $x_1 + 2x_2 = 8$
Non-negativity ; $x_1 \geq 0$, $x_2 \geq 0$
Find intercepts for each ~~time~~ line

- For $2x_1 + 3x^2 = 12$; when $x_1 = 0$; $3x^2 = 12$
$x^2 = $
when $x^2 = 0 : 2x_1 = 12$
$x^1 = 6$
Points : (6,0) and (0, 4)



x^1 (product A)

Identify the corner points of the feasible region
(0,0) – intersection of $x_1 = 0$ and $x_2 = 0$

- (6, 0) -- intersection of ~~$2x_1 +$~~ $2x_1 + 3x_2 = 12$ on x_1 axis
- (0, 4) - intersection of both lines on x^2 axis

<u>Solve for intersection</u>

$2x_1 + 3x_2 = 12$ — eq 1

$x_1 + 2x_2 = 8$ — eq ii

$x^1 = 8 - 2x_2$ sub into eq i

$2(8 - 2x_2) + 3x^2 = 12$

$16 - 4x^2 + 3x_2 = 12$

~~$x^2 = 8 - 2x_2$~~ S. $x^2 = 4 \Rightarrow x^2 = 4$

Sub $x_2 = 4$ into eq ii

$x^1 + 2(4) = 8 \Rightarrow x_1 = 0$

Objective function of each point: $Z = 3x_1 + 4x^2$

Corner point	x_1	x_2	$Z = 3x_1 + 4x_2$
(0, 0)	0	0	$Z = 0$
(6, 0)	6	0	$Z = 18$
(0, 4)	0	4	$Z = 16$
(2, 3)	2	3	$Z = 18$

Max profit is $Z = 18$ achieved at;

(i) (6, 0) - produce ~~units~~ 6 units of product A and 0 units of products B

(ii) (2, 3) - produce 2 units of product A and 3 units of product B.

2.

	labor hour	units of mate	cost per unit
Product x	1	2	#2
product y	2	1	#5

Total labor = 6hrs, total units of materials, 5 units.

Objectives function: minimize cost $C = 2x + 5y$

Constraints

(i) $x + 2y \leq 6$ (Labor constraint)

(ii) $2x + y \leq 5$ (Material constraint)

(iii) $x \geq 0,\ y \geq 0$ (Non-negativity constraint),

Labour constraint:

$y : x + 2y = 6 \Rightarrow y = 3 - x/2$

when $x = 0 : y = 3 - 0/2 = 3 \Rightarrow$ point A $(0,3)$

when $y = 0 : 0 = 3 - x/2 \Rightarrow x/2 = 3 \Rightarrow x = 6$

point B $(6, 0)$

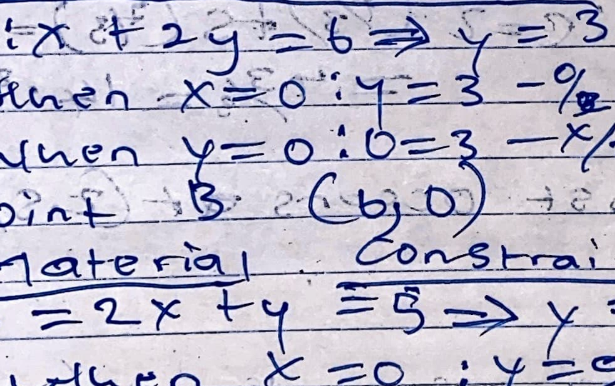
Material constraint:

$y = 2x + y = 5 \Rightarrow y = 5 - 2x$

when $x = 0 ; y = 5 - 2(0) = 5$. point $(0,5)$

when $y = 0 ; 0 = 5 - 2x = 5/2 = 2.5$

point $(2.5, 0)$



Intersection of 2 lines
$x + 2y = 6$ — eq (i)
$2x + y = 5$ — eq (ii)
Solve for y: $y = 5 - 2x$ — eq (iii)
$x + 2(5 - 2x) = 6$
$x + 10 - 4x = 6 \Rightarrow -3x = -4$
$x = 4/3$ sub into eq (iii)

$y = 5 - 2(4/3) \Rightarrow 5 - 8/3 = 7/3$

intersection point is $(4/3, 7/3)$
To minimize cost: $C = 2x + 5y$
Evaluate C at each corner point
At $(0, 3)$: $C = 2(0) + 5(3) = 15$
At $(2.5, 0)$: $C = 2(2.5) + 5(0) = 5$
At $(4/3, 7/3)$: $C = 2(4/3) + 5(7/3) = 8/3 + 35/3$
$= 43/3 = 14.33$

Minimum cost occurs at $(2.5, 0)$

	Hours of Labour	unit of material	Hours of machine time	profit per unit
Product A	2	3	1	5
product B	1	2	2	4

Resource available
- 20hrs of Labour — 30 unit of material
- 18hrs of machine time
Objective function - max total profit: $P = 5A + 4B$
Let A and B represent number of unit produced A & B.

- Labour constraints: $2A + B \leq 20$
- Material constraints: $3A + 2B \leq 30$
- Machine-time constraints: $A + 2B \leq 18$
- Non-negativity constraint: $A \geq 0, B \geq 0$

Labour constraint:

$2A + B = 20$

$B = 20 - 2A$

- when $A = 0$, $B = 20$ $[0, 20]$
- when $B = 0$, $2A = 20 \Rightarrow A = 10$ $[10, 0]$

Material constraint: $3A + 2B = 30$

$B = 30 - 3A/2$

- when $A = 0$, $B = 15$ point $(0, 15)$
- when $B = 0$, $3A = 30 \Rightarrow A = 10$ point $(10, 0)$

Machine-time constraint: $A + 2B = 18$

$B = 30 - 3A/2$

- when $A = 0$: $B = 9$ point $(0, 9)$
- when $B = 0$, $A = 18$, one point is $(18, 0)$

Find the corner points at the intersections between $2A + B = 20$ and $3A + 2B = 30$

$2A + B = 20$ — equ 1

$3A + 2B = 30$ — equ 2

$B = 20 - 2A$ — Sub into equ 2

$3A + 2(20 - 2A) = 30$

$3A + 40 - 4A = 30 \Rightarrow -A = -10 \Rightarrow A = 10$

Sub $A = 10$ into $B = 20 - 2A$

$B = 20 - 2(10) = 0$

(11) Intersection of $2A + B = 20$ and $A + 2B = 18$

$2A + B = 20$ --- equ 1

$A + 2B = 18$ --- equ 3

Sub $B = 20 - 2A$ into equ 3

$A + 2(20 - 2A) = 18$

$A + 40 - 4A = 18 \Rightarrow 3A - 22 \Rightarrow A = 22/3$

Sub $A = (22/3) = 20 - 44/3 = 60/3 - 44/3$

$= (16/3 = 5.33)$

Intersection of $3A + 2B = 30$ and $A + 2B = 18$

$3A + 2B = 30$ --- equ 2

$A + 2B = 18$ --- equ 3

$A = 18 - 2B$ sub into equ 2

$3(18 - 2B) + 2B = 30$

$54 - 6B + 2B = 30 \Rightarrow -4B = -24 \Rightarrow B = 6$

Sub $B = 6$ into equ 3

$A = 18 - 2(6) - 6$

$\therefore$ Intersection point $(6, 6)$

Objective function : $P = 5A + 4B$

At $(10, 0)$ $P = 5(10) + 4(0) = 50$

At $(7.33, 5.33)$: $P = 5(7.33) + 4(5.33) = 59.97$

At $(6, 6)$: $P = 5(6) + 4(6) = 54$

$\therefore$ Optimal solution : Max profit occurs at $(7.33, 5.33)$ with profit of ~~H~~ 58

		Budget	
(4) Product A	4	1	1
Product B	5	2	2

Available resources

Advertising budget	#20 total
production budget	15

Objective function:

Maximize total revenue: $R = 4A + 5B$

Let A and B represent the number of units of products A and B

Constraints

- Advertising budget constraint: $A + 2B \leq 20$
- production capacity constraint: $A + 2B \leq 15$
- Non-negativity constraint: $A \geq 0, B \geq 0$

Graphical solution:

- Advertising budget constraint: $A + 2B \leq 20$

$B = 20 - A/2$

plot points

$A = 0, B = 10$

$A = 20, B = 0$

production capacity: $A + 2B \leq 15$

$B = 15 - A/2$ Non-negativity = A

$A = 0, B = 7.5$ $A \geq 0, B \geq 0$

$A = 15, B = 0$

Graphical representation

Corner points

① Intersection of $A+2B=20$ and $A+2B=15$

The lines are parallel so no intersection

(ii) Intersection of $A+2B=20$ with $B=0$; $A=20$, $B=0$

(iii) Intersection of $A+2B=15$ with $B=0$; $A=15$, $B=0$

The revenue function is

$R=4A+5B$

- At $(0,10)$: $R=4(0)+5(10)=50$
- At $(15,0)$: $R=4(15)+5(0)=60$
- At $(0,7.5)$: $R=4(0)+5(7.5)=37.5$

Optimal solution

Maximum revenue occurs at $(15,0)$

where maximum revenue = ₦60

⑤

	Labour hours	capital	profit per unit
P_1	3	2	8
P_2	4	1	7

Total labour hours = 12

Available capital = 6

Constraint

Let x represent units of Project P1

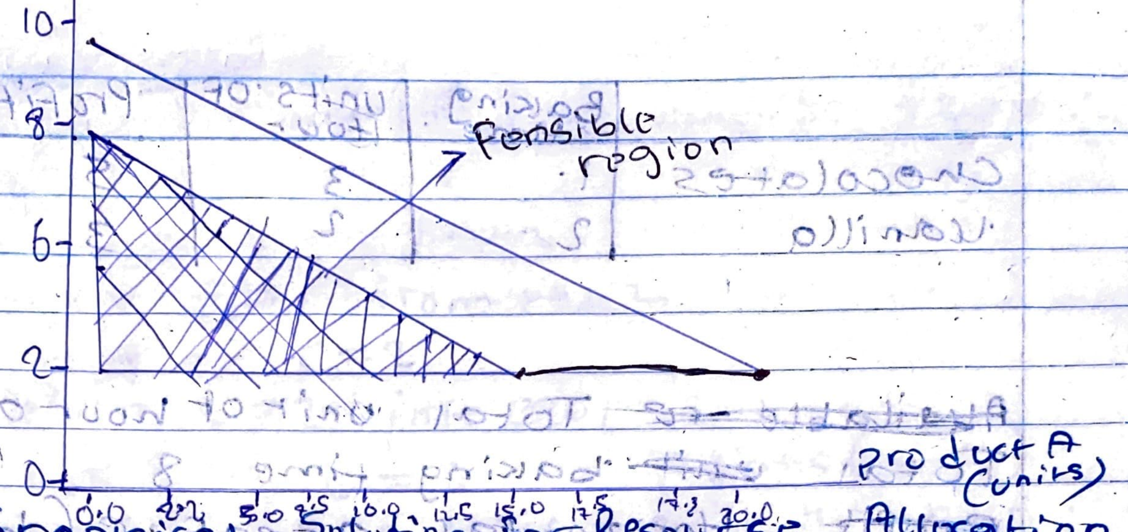
Let y represent units of Project P2

Labour hours constraint

$3x+4y \leq 12$

Capital unit constraint: $2x+y \leq 6$

$P=42.0$ Non-negativity $x, y \geq 0$



Graphical Solution for Resource Allocation

Corner points using $Z = 8x + 7y$

- At $(0,0)$; profits $= 0$
- At $(0,6)$: $Z = 8(0) + 7(6) = 42$
- At $(4,0)$: $Z = 8(4) + 7(0) = 32$
- At $(2,2)$: $Z = 8(2) + 8(2) = 30$

Optimal solution

Max profit $Z = 42$ is achieved at corner points $(0,6)$. Company should allocate all resources to project P2 and produce 6 units of it.

(6) Battery production planning

Let c = number of chocolate cakes produced

Let v = number of vanilla cakes produced

Objective function:

Maximize profit $P = 5c + 3v$; P: total profit

Constraints

Baking time: $c + 2v \leq 8$

flour: $3c + 2v \leq 12$

Non-negativity: $c \geq 0$; $v \geq 0$

	Baking time	units of Hour	Profit
Chocolates	1	3	5
vanilla	2	2	3

~~Available~~ Total unit of hour available 12
Total ~~unit~~ baking time 8
~~profit~~ Horizontal areas: C (Chocolate cake
vertical axis; v (vanilla cakes)

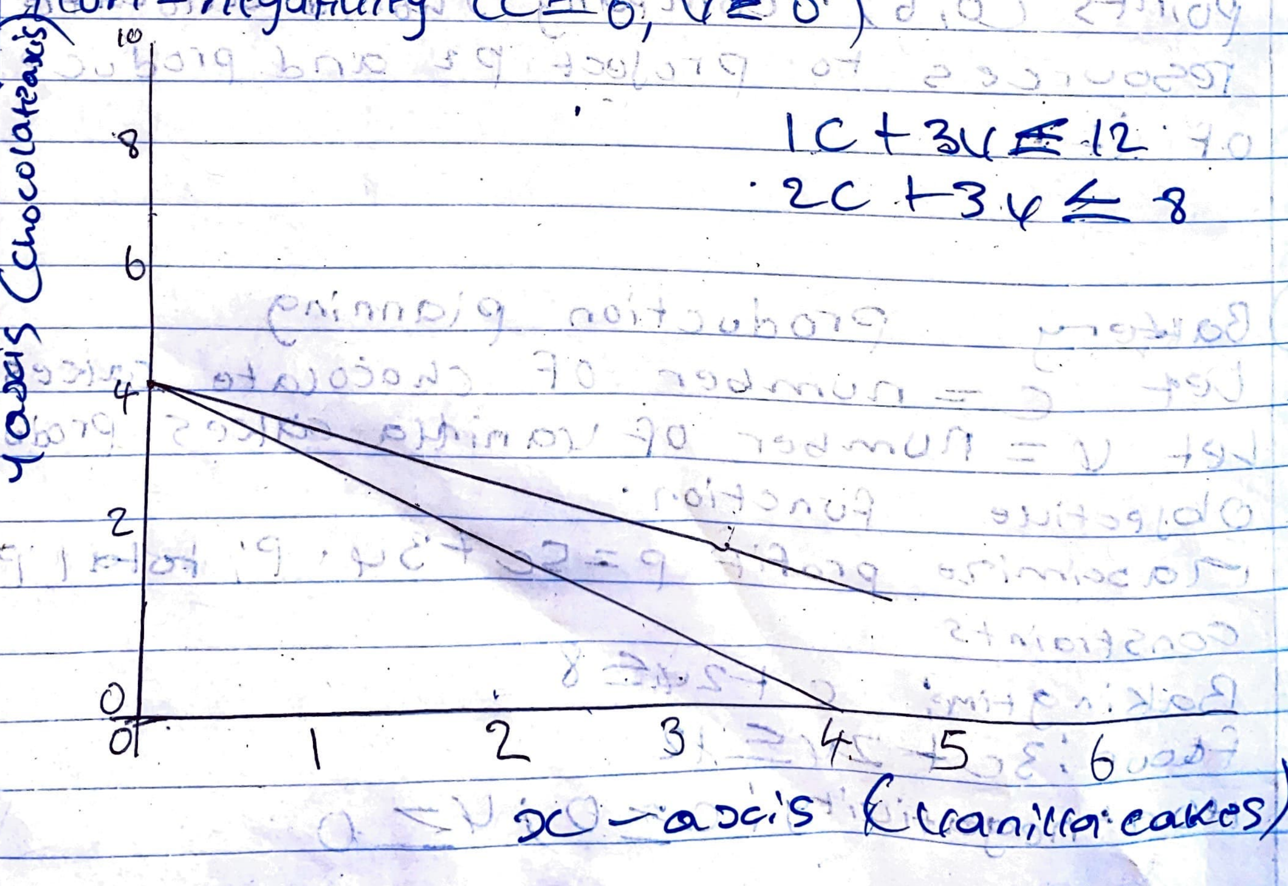
For $c + 2v \le 8$
- when $c = 0$, $v = 4$ point $(0, 4)$
- when $v = 0$, $c = 8$ = point $(8, 0)$

For $3c + 2v \le 12$
- when $c = 0$, $v = 6$ point $(0, 6)$
- when $c = 4$, $v = 0$ point $(4, 0)$

Non-negativity $(c \ge 0, v \ge 0)$

$1c + 3v \le 12$

$2c + 3v \le 8$



Cornor points

(I) Intersection of $c+2v=8$ and $3c+2v=12$

$c+2v=8$ — equ 1

$3c+2v=12$ — equ 2

Substract equ 1 from equ 2

$2c=4 \Rightarrow c=2$

Substitute $c=2$ into equ 1

$2+2v=8 \Rightarrow v=3$ points $(2,3)$

(II) Intersection of $c+2v=8$ with $v=0$;

$c=8$ point: $(8;0)$

(III) Intersection of $3c+2v=12$ with $v=0$

$3c=12 \Rightarrow c=4$

(IV) Intersection of $c=0$ with $3c+2v=12$;

$2v=12 \Rightarrow v=6$ point: $(0,6)$

<u>Objective function of corner point</u>

$p=5c+3v$

i At point $(2,3)$: $p=5(2)+3(3)=19$

ii At $(8,0$; $p=5(8)+3(0)=40$

iii At $(4,0)$: $p=3(4)+3(0)=20$

IV At $(0,6)$: $p=3(0)+3(6)=18$

<u>Optimal solutions</u>

Max profit is $p=40$ at $(8,0)$. Produce 8 chocolate cake and 0 vanilla cakes

(7)

	Hours of fuel	Hours of drive time	cost per trip
~~x~~	3	2	6
y	4	1	7

Available resources	
Total hours of fuel	18
Total hours of driver	10

Constraints

Fuel Constraints: $3x + 4y \leq 18$

Driver time constraints: $2x + y \leq 10$

Non-negativity = $x \geq 0$, $y \geq 0$

Objective function: Minimize the cost function:

$C = 6x + 7y$

constraints: $3x + 4y \leq 18$

$3x + 4y = 18$

$y = 18 - 3x/4$

Intercepts

- If $x = 0$: $y = 18/4 = 4.5$
- If $y = 0$: $x = 18/3 = 6$

Constraints 2: $2x + y \leq 10$

$\therefore 2x + y = 10$

$y = 10 - 2x$

intercepts

i) If $x = 0$; $y = 10$

ii) If $y = 0$; $x = 5$

Non-negativity $x \geq 0$ - $y \geq 0$

Find the intersections of constraints

(i) $3x + 4y = 18$ and $2x + y = 10$

(ii) $2x + y = 10$

$y = 10 - 2x$ Sub into $3x + 4y = 18$

$3x + 4(10 - 2x) = 18$

$3x + 4(0 - 8x) = 18$

$-5x + 40 = 18$

$-5x = -22$

$x = 4.4$

Sub $x = 4.4$ into $y = 10 - 2x$

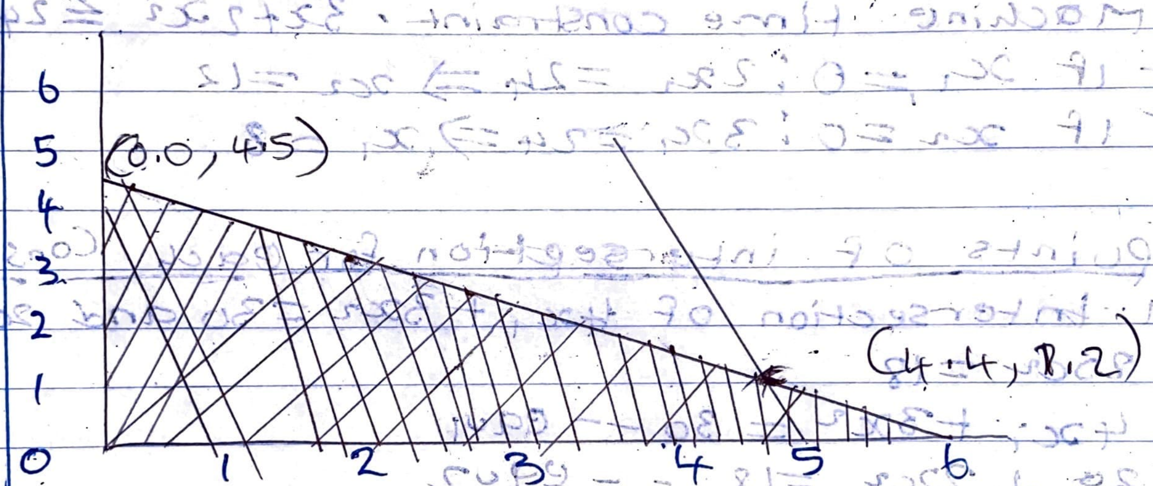
$y = 10 - 2(4.4) = 10 - 8.8$

$= 1.2$

Intersection point $(4.4, 1.2)$

corner point	X	y	$C = 6x + 7y$
(0,0)	0	0	C = 0
(0, 4.5)	0	4.5	C = 31.5
(3, 0)	5	0	C = 30
(4.4) 1.2	4.4	1.2	C = 34.8

Optimal Solution
Use 5 trips with vehicle X and 0 trips with vehicle Y. The smallest value of C is at (5,0) where C = 30



(8)	labour hours	Raw materials	machine Time	Revenue
P1	4	1	3	10
P2	3	2	2	12
Total	30	18	24	

Constraints.
Labour Constraint: $4x_1\ 3x_2 \leq 30$
Raw - material constraints: $x_1 + 3x_2 \leq 18$
Machine time constraint: $3x_1 + 2x_2 \leq 24$
Non-negativity constraints: $x_1 \geq 0,\ x_2 \geq 0$

Objective function: maximize the revenue
$Z = 10x_1 + 12x^2$

Intercepts of each constraints

Labour constraint: $4x_1 + 3x^2 \leq 30$

- if $x_1 = 0$; $2x^2 = 30 \Rightarrow x^2 = 10$
- if $x^2 = 0$; $4x = 30 \Rightarrow x_1 = 7.528$

(ii) Raw material constraints; $x_1 + 2x_2 \leq 18$

- if $x_1 = 0$; $2x_2 = 18 \Rightarrow x^2 = 9$
- if $x^2 = 0$; $x_1 = 18$

(iii) Machine time constraint. $3x + 2x^2 \leq 24$

- if $x_1 = 0$; $2x_2 = 24 \Rightarrow x_2 = 12$
- if $x_2 = 0$; $3x_1 = 24 \Rightarrow x_1 = 8$

points of intersection for each Constraint

1. Intersection of $4x_1 + 3x_2 = 30$ and $x_1 + 2x_2 = 18$

$4x_1 + 3x^2 = 30$ -- equ 1

$2x + 2x_2 = 18$ -- equ 2

from equ 2

$x_1 + 2x_2 = 18$

$x1 = 18 - 2x_2$ -- equ 3

Substitute equ 3 into equ 1