CSC376 - ASSIGNMENT 3

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November 2024

Resources used: Lecture slides, and the Modern Robotics textbook.

Question 1:

Starting with p_z, by looking at the diagram provided for this assignment, we can see that:

$$p_z = L + (\sqrt(2)L + \theta_2)cos(\pi/4) + L = 3L + \frac{\theta_2}{\sqrt(2)}$$

Solving for θ_2:

$$\theta_2 = \sqrt(2)(p_z - 3L)$$

Thus we have

$$\frac{\theta_2}{\sqrt{2}} = p_z - 3L$$

In addition, given that $0 <= \theta_2 <= \sqrt{2}L$, we can see that $3L <= p_z <= 4L$.

Now, for the solutions to θ_3 and θ_1, we must analyze a number of different cases based on the values of p_x, p_y and p_z. If a given p_x, p_y and p_z does not fit one of these cases, there is no solution for it.

Case 1: $0 < p_z - 3L < \sqrt{p_x^2 + p_y^2} < p_z - L < 3L$

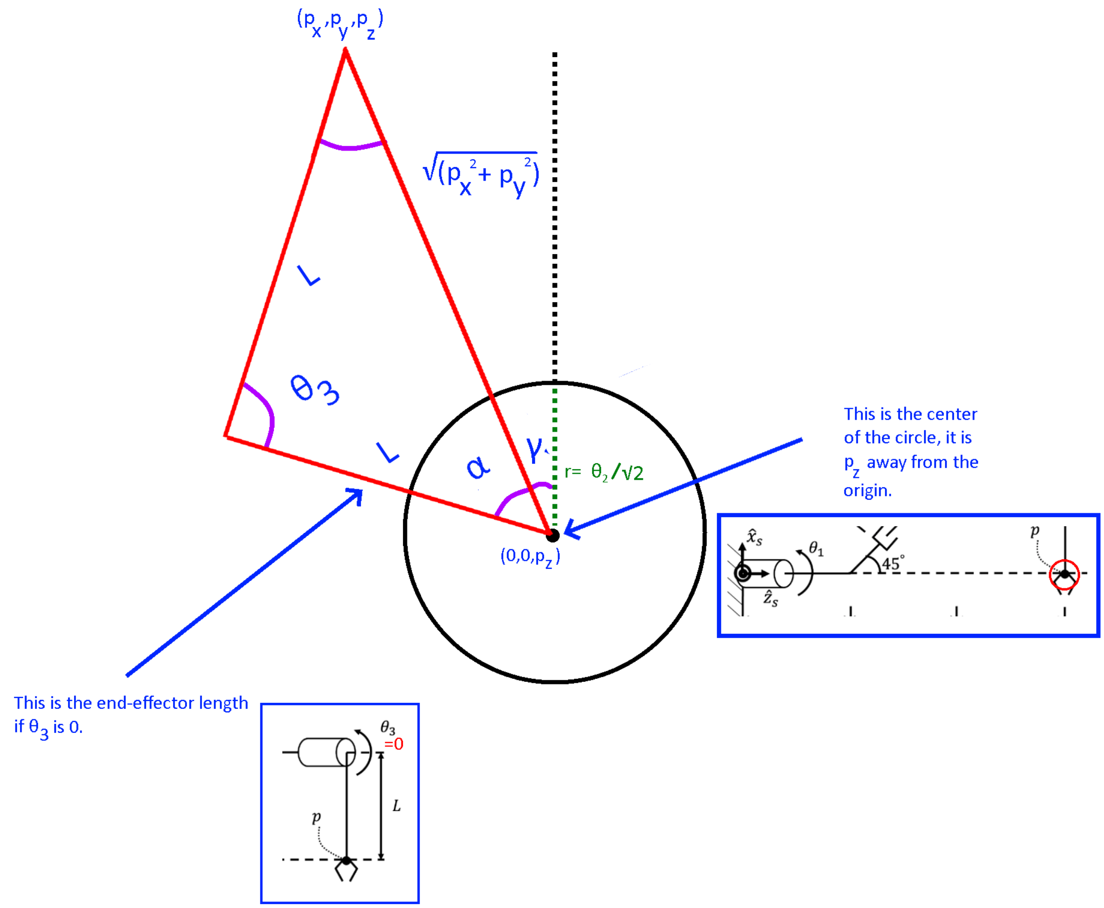


Figure 1: Diagram to find θ_3 and θ_1 in Case 1

Using the cosine law based on what is seen in Figure 1, we can derive:

$$p_x^2 + p_y^2 = (p_z - 2L)^2 + L^2 - 2(L)(p_z - 2L)cos(\theta_3)$$

Rearranging this to solve for θ_3:

$$\theta_3 = cos^{-1}(\frac{(p_z - 2L)^2 + L^2}{2(L)(p_z - 2L)cos(\theta_3)})$$

This is the only solution for θ_3 in this case due to the joint limits of θ_3 being $[0, \pi]$.

Next, with Figure 1, we define Γ as the following:

$$\Gamma = atan2(p_y, p_x)$$

Looking at L^2 with the cosine law we can finally solve:

$$L^2 = (L + \frac{\theta_2}{\sqrt{2}})^2 + p_y^2 + p_x^2 - 2(L + \frac{\theta_2}{\sqrt{2}})(\sqrt{p_x^2 + p_y^2})cos(\alpha)$$

Using inverse cosine, we can solve for α:

$$\alpha = cos^{-1}[\frac{(L + \frac{\theta_2}{\sqrt{2}})^2 + p_y^2 + p_x^2}{2(L + \frac{\theta_2}{\sqrt{2}})(\sqrt{p_x^2 + p_y^2})}]$$

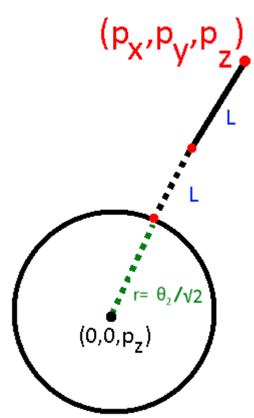
Plugging in the equation solved at the start where $\theta_2/\sqrt{2} = p_z - 3L$:

$$\alpha = cos^{-1}[\frac{(p_z - 2L)^2 + p_y^2 + p_x^2}{2(p_z - 2L)(\sqrt{p_x^2 + p_y^2})}]$$

Given the restrictions on θ_3 and looking at Figure 1, we can see that the only solution for θ_1 in this case is:

$$\theta_1 = \Gamma + \alpha$$

Case 2: $2L < p_z - L = \sqrt{p_x^2 + p_y^2} <= 3L$



Case where
θ3= 0

Figure 2: Diagram to find θ_3 and θ_1 in Case 2

We can see from Figure 2 that in this case, $\theta_3 = 0$ must be true, and $\theta_1 = atan2(p_y, p_x)$.

Case 3: $0 < p_z - 3L = \sqrt{p_x^2 + p_y^2} <= L$

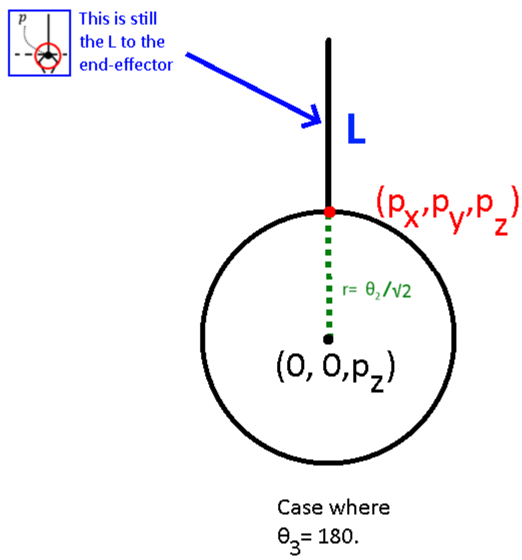


Figure 3: Diagram to find θ_3 and θ_1 in Case 3

We can see from Figure 3 that in this case, $\theta_3 = \pi$ must be true, and $\theta_1 = atan2(p_y, p_x)$.

Case 4: $p_x = 0, p_y = 0, p_z = 3L$

This is p when the robot is in its zero position. We can easily see from the diagram provided for this assignment that $\theta_3 = 0$ and $\theta_2 = 0$ must both be true. Finally, given that p in this case is on the same axis of rotation as θ_1, there are infinite solutions to θ_1 for this position.

Question 2: All the comments are in the code itself.
Question 3: **Part a** :
For each of the graphs:

- A: Fifth-order acceleration
- B: Third order velocity

- C: Trapezoid acceleration
- D: Trapezoid velocity
- E: Third-order acceleration
- F: fifth order velocity

Discussing the advantages and disadvantages of each in comparison:

- Fifth order:
  - Advantage: Motion is smoother b/c acceleration at the beginning and end are constrained to 0, resulting in smoother motion, no vibrations, it is much smoother then the third-order polynomial.
  - Disadvantage: More complex than trapezoidal, does not accelerate as fast
- Trapezoid:
  - Advantage:Reaches max straight line motion velocity the fastest, it has the largest v and a satifsying the equations over the limit.
  - Disadvantage: It is not as smooth as cubic or fifth order, it is much more "snappy"
- Third order:
  - Advantage: Simpler than quintic, smoother than trapezoidal
  - Disadvantage: Not as fast a trapezoidal, the acceleration jumps at the start at the end, compared to the trapezoidal and fifth-order accelerations, the acceleration jumps discontinuously at the start and end of the time.

Part b :
From the description of the problem, we can see that:

$$\theta_{end} = 300 \tag{1}$$
$$\theta_{start} = 50 \tag{2}$$
$$\dot{\theta}_{limit} = 25 \tag{3}$$
$$\ddot{\theta}_{limit} = 50 \tag{4}$$
$$\tag{5}$$

We wish to solve for maximum velocity, recall that from the lectures, it is known that we can solve for the largest v and a, in the fastest straight line possible[minimum time]:

$$|\theta_{end} - \theta_{start} v| \leq \dot{\theta}_{limit} \tag{6}$$
$$|\theta_{end} - \theta_{start} a| \leq \ddot{\theta}_{limit} \tag{7}$$
$$\tag{8}$$

Plugging in the values we have gotten and solving:

$$|(300 - 50)v| \leq 25 \tag{9}$$
$$|250v| \leq 25 \tag{10}$$
$$|v| \leq 25/250 \tag{11}$$
$$|v| \leq 0.1 \tag{12}$$
$$|(300 - 50)a| \leq 50 \tag{13}$$
$$|250a| \leq 50 \tag{14}$$
$$|a| \leq 50/250 \tag{15}$$
$$|a| \leq 0.2 \tag{16}$$

Setting the acceleration and velocity to the max, now have the maximum acceleration and velocity known to solve, now, from lectures, it is known that a "bang-bang" motion is reached if $v^2/a > 1$, fortunately, $v^2/a = 0.1^2/0.2 - 0.05$, therefore, we have a normal trapezoidal motion. In chapter 9.2, page 334 of the modern robotics textbook, it was shown that for any $v^2/a \leq 1$, the following yields the minimum time T:

$$T = \frac{a + v^2}{va}$$

Plugging in the values, we finally obtain:

$$T = \frac{0.2 + 0.1^2}{0.2 \times 0.1} = 10.5$$

Therefore, the minimum time is 10.5.