CSC376 - ASSIGNMENT 1

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Each question is listed below.

Question 1:

1. - First, let us consider the displacement from w to c, which, looking at the visual, is F in the x direction, D in the y direction, and E in the z direction. Looking at the axis, it can be seen that the x and y axis are both opposite by π radians on the z axis, using the common rotation matrices, we can solve:

$$
\begin{aligned}
T_{wc} &= \begin{bmatrix} R & p \\ 0 & b \end{bmatrix} && (1) \\
&= \begin{bmatrix} cos(\pi) & -sin(\pi) & 0 & F \\ sin(\pi) & cos(\pi) & 0 & D \\ 0 & 0 & 1 & E \\ 0 & 0 & 0 & 1 \end{bmatrix} && (2) \\
&= \begin{bmatrix} -1 & 0 & 0 & F \\ 0 & -1 & 0 & D \\ 0 & 0 & 1 & E \\ 0 & 0 & 0 & 1 \end{bmatrix} && (3)
\end{aligned}
$$

2. Note that both p_1 and p_2 have no rotation on the axis, and are simply displaced on the y axis by a total of $4L + A$ units, using this, let us compute: the z axis, using the common rotation matrices, we can solve:

$$
T_{p_1p_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(A+4L) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)
$$

3. Let us simply apply the properties of matrices:

$$
\begin{aligned}
T_{bc} &= T_{bs}T_{sc} && (5) \\
&= (T_{sb})^{-1}T_{sw}T_{wc} && (6) \\
&= (T_{sb})^{-1}(T_{ws})^{-1}T_{wc} && (7)
\end{aligned}
$$

From the question, the matrice T_{ws} is known, alongside T_{sb}, while T_{wc} is known from part a.

4. First, notice the displacement, which is $5L$ in the x direction, nothing in the y direction, and H in the z direction. Also, notice that the axis is rotated by $\pi/2$ around the z axis, putting this together:

$$\begin{aligned} T_{p_1 box_1} &= \begin{bmatrix} R & p \\ 0 & b \end{bmatrix} && (8) \\ &= \begin{bmatrix} cos(\pi/2) & -sin(\pi/2) & 0 & 5L \\ sin(\pi/2) & cos(\pi/2) & 0 & 0 \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} && (9) \\ &= \begin{bmatrix} 0 & -1 & 0 & 5L \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} && (10) \end{aligned}$$

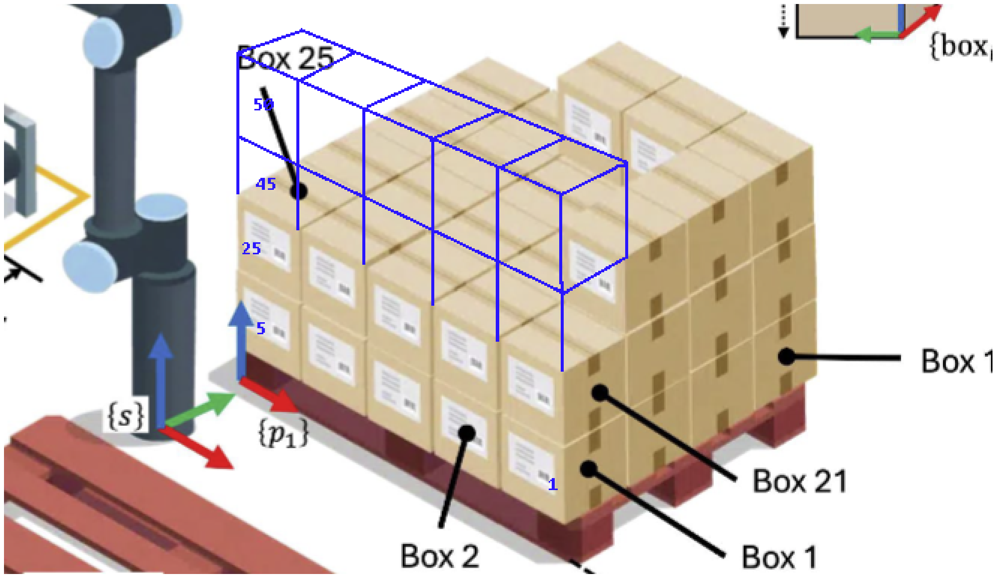
5. Note how from p_1, box 16 is $5L$ on the x axis, $3L$ on the Y axis, and H on the Z axis, while keeping the same rotations as before:

$$\begin{aligned} T_{p_1 box_{16}} &= \begin{bmatrix} R & p \\ 0 & b \end{bmatrix} && (11) \\ &= \begin{bmatrix} cos(\pi/2) & -sin(\pi/2) & 0 & 5L \\ sin(\pi/2) & cos(\pi/2) & 0 & 3L \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} && (12) \\ &= \begin{bmatrix} 0 & -1 & 0 & 5L \\ 1 & 0 & 0 & 3L \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} && (13) \end{aligned}$$

6. Consider the path from c to p_{50}, let us break it down in terms of more known matrices:

$$\begin{aligned} T_{cp_{50}} &= T_{cw} T_{wp_{50}} && (14) \\ &= (T_{wc})^{-1} T_{wp_1} T_{p_1 p_{50}} && (15) \end{aligned}$$

Now, from the previous questions and information, T_{wc} and T_{wp_1} are known, so all that needs to be solved is $T_{p_1 p_5 0}$, consider the following drawing:



The reference for the $50th$ box is $H + 3L$ in the z direction, L in the x direction, and nothing in the y direction, while still applying the same rotation in the previous box questions:

$$T_{p_1 p_5 0} = \begin{bmatrix} R & p \\ 0 & b \end{bmatrix} \tag{16}$$

$$= \begin{bmatrix} cos(\pi/2) & -sin(\pi/2) & 0 & L \\ sin(\pi/2) & cos(\pi/2) & 0 & 0 \\ 0 & 0 & 1 & H+3L \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$= \begin{bmatrix} 0 & -1 & 0 & L \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & H+3L \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

Now, all three matrices are known, and the matrix can be solved for such that:

$$T_{cp_5 0} = (T_{wc})^{-1} T_{wp_1} T_{p_1 p_{50}} \tag{19}$$

$$= (T_{wc})^{-1} T_{wp_1} \begin{bmatrix} 0 & -1 & 0 & L \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & H+3L \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

Now, T_{ws} is given in the question, T_{wp_1} as well, while $T_{p_1 box_1}$ was determined in above.

7. Let us simplify this matrix further:

$$\begin{aligned} T_{sbox_1} &= T_{sp_1} T_{p_1 box_1} && (21)\\ &= T_{sw} T_{wp_1} T_{p_1 box_1} && (22)\\ &= (T_{ws})^{-1} T_{wp_1} T_{p_1 box_1} && (23) \end{aligned}$$

These are all known from previous questions or given to be known in the question.

Question 2:

	α_{i-1}	a_{i-1}	d_i	ϕ_i
1	0	0	$\theta_1 + L_0$	0
2	$\pi/2$	L_2	0	$\pi/2 + \theta_2$
3	$\pi/2$	0	L_2	$\pi/2$
4	0	0	$L_3 + \theta_3$	θ_4

(24)

Question 3:
Note that I used "DH parameters", either was accepted and we had this checked over with the professor and TA office hours, here's our diagram below, alongside the code: