Simulation of Orbital Motions in the Solar System: A Study of the Two-Body and Three-Body Problems

Joud Bamehriz

December 8, 2024

1 Introduction

In this assignment, we simulate the motion of objects in space under gravitational influences. The first part involves the motion of Earth and Sun in a two-body system, while the second part extends the system to the restricted three-body problem by adding Jupiter. The final part incorporates the full three-body problem, where the Sun's motion is also considered. We solve the equations of motion using numerical methods and analyze the effects of varying Jupiter's mass on the orbits of Earth and Jupiter, as well as on the Sun's position.

2 Problem 1: Two-Body Problem

In the two-body problem, we consider the Sun and Earth as two objects orbiting each other. Since the Sun is much more massive than the Earth, we can assume that the Sun remains fixed at the origin, and the Earth's motion is governed by the gravitational force between the Earth and the Sun. The gravitational force acting on Earth is given by:

$$F_G = \frac{GM_S M_E}{r^2}$$

where M_S and M_E are the masses of the Sun and Earth, respectively, and r is the distance between them. The equations of motion for Earth in the x-y plane are:

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_E}, \quad \frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_E}.$$

We can convert these second-order differential equations into a system of first-order equations:

$$\frac{dv_x}{dt} = \frac{F_{G,x}}{M_E}, \quad \frac{dx}{dt} = v_x, \quad \frac{dv_y}{dt} = \frac{F_{G,y}}{M_E}, \quad \frac{dy}{dt} = v_y.$$

The Euler-Cromer method is used to solve these equations numerically, where the velocities are updated first, followed by the positions. The result of this simulation is shown in Figure 1.

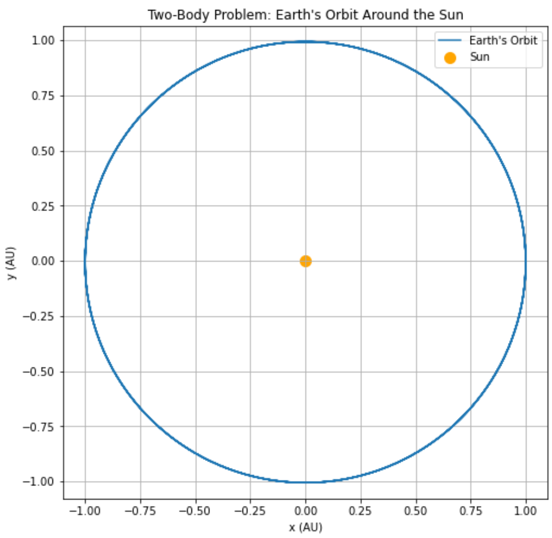

Figure 1: Orbit of Earth around the Sun in the two-body problem.

3 Problem 2: The Restricted Three-Body Problem

Next, we add Jupiter to the system. The equations of motion for Jupiter are similar to those for Earth. Since Jupiter has a significant mass, it will exert a gravitational force on Earth, which must be accounted for in Earth's equations of motion. The force between Earth and Jupiter is given by:

$$F_{EJ,x} = -\frac{GM_J M_E}{r_{EJ}^2} \cos\theta_{EJ} = -\frac{GM_J M_E (x_E - x_J)}{r_{EJ}^3}.$$

We solve the system of equations for both Earth and Jupiter, using the same approach as in Problem 1, with the additional term for the gravitational force between Earth and Jupiter. We simulate the system for different values of Jupiter's mass, including multiplications by factors of 10 and 1000. The results for the restricted three-body problem are shown in Figures 2, 3, and 4.

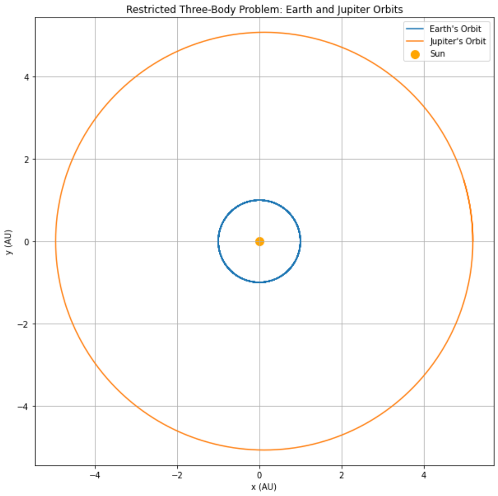

Figure 2: Orbits of Earth and Jupiter with no mass changes.

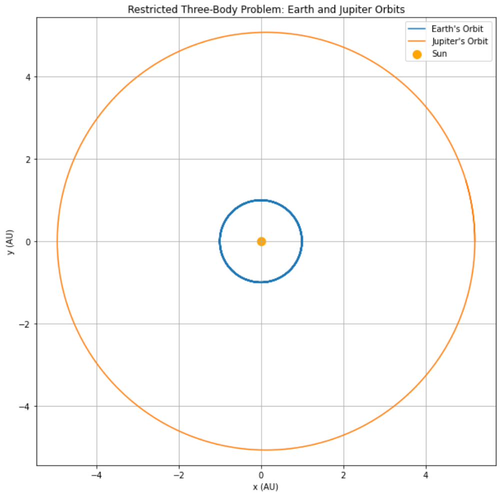

Figure 3: Jupiter's mass 10 times larger.

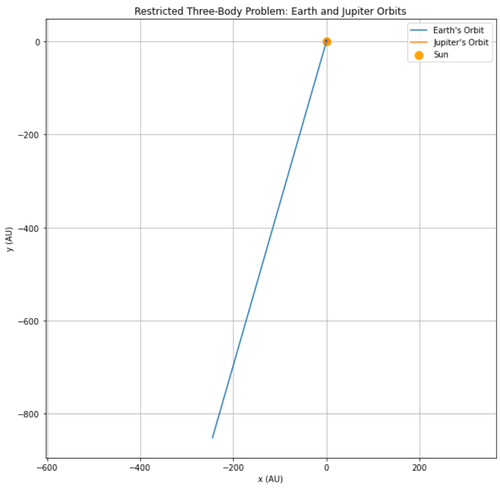

Figure 4: Jupiter's mass 1000 times larger.

4 Problem 3: Full Three-Body Problem

In the full three-body problem, we also consider the motion of the Sun due to the gravitational forces from Earth and Jupiter. The center of mass of the system is calculated as:

$$x_{\mathrm{cm}} = \frac{M_S x_S + M_E x_E + M_J x_J}{M_S + M_E + M_J}.$$

The initial positions are adjusted to ensure that the center of mass remains at the origin. The velocity of the Sun is determined using the conservation of momentum, ensuring that the system's momentum remains constant. The equations of motion for the Sun, Earth, and Jupiter are solved using the Euler-Cromer method. The result of this simulation for the full three-body problem is shown in Figures 5, 6, and 7.

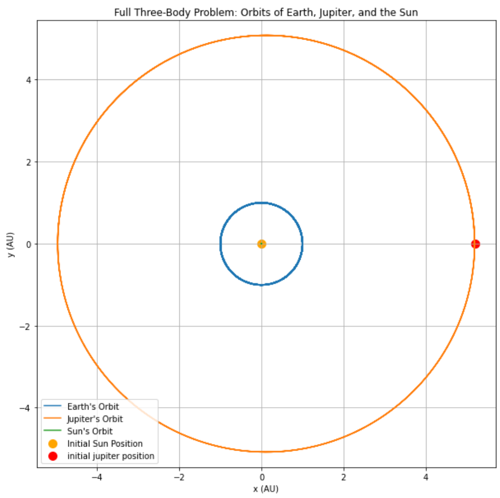

Figure 5: Orbits of Earth, Jupiter, and sun with no mass changes.

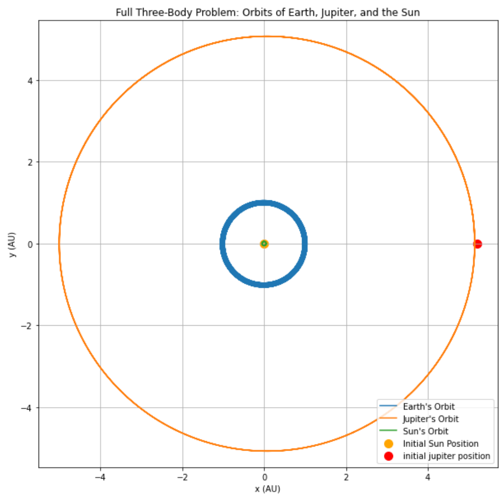

Figure 6: Jupiter's mass 10 times larger.

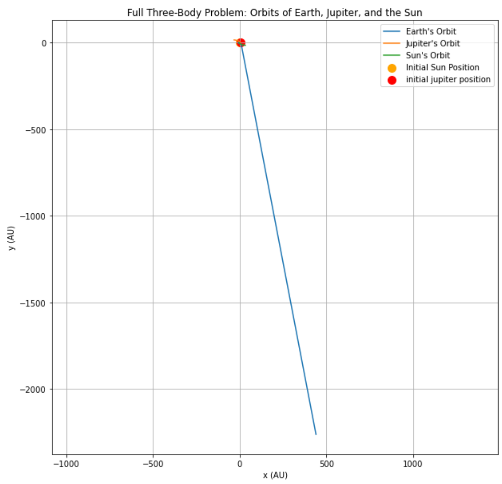

Figure 7: Jupiter's mass 1000 times larger.

5 Analysis of Results

In the simulation for the restricted three-body problem, we observe that the orbits of Earth and Jupiter are elliptical, and the Sun exhibits a small motion due to the gravitational influence of both planets. When Jupiter's mass is multiplied by 10 and 1000, the influence on Earth's orbit becomes more pronounced. The Earth's orbit becomes more perturbed, and Jupiter's motion is more noticeable.

For the full three-body problem, the Sun's motion is also evident, and it moves in response to the gravitational forces from Earth and Jupiter. The center of mass of the system remains near the origin, and the Sun moves in a small orbit around this center.

When Jupiter's mass is increased by a factor of 1000, the Sun's motion becomes more significant, and the orbits of Earth and Jupiter are noticeably altered. Additionally, the time step of the simulation can affect the results. Reducing the time step improves the accuracy of the simulation, especially for highly perturbed systems such as when Jupiter's mass is increased significantly as shown in figure 8.

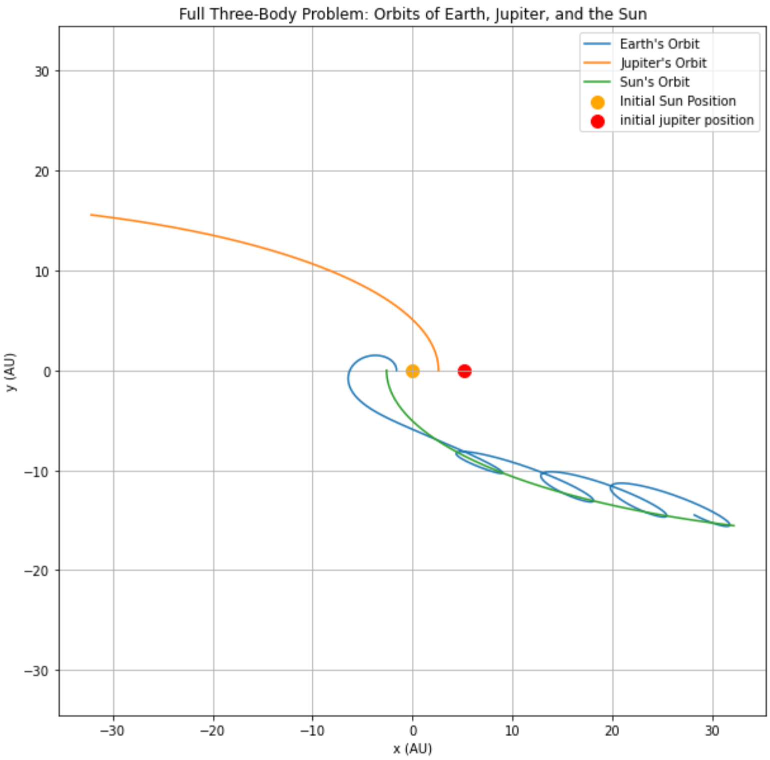

Figure 8: Orbits of Earth, Jupiter, and the Sun in the full three-body problem with Jupiter a 1000 times more massive, but with a much smaller timestep.

6 Conclusion

The simulation results agree with theoretical expectations. In the restricted three-body problem, the Sun's motion is small but noticeable, and the orbits of Earth and Jupiter are influenced by the gravitational forces between them. In the full three-body problem, the Sun's motion becomes more pronounced, and the system's behavior is more complex. Increasing Jupiter's mass results in greater perturbations in Earth's orbit and a more significant motion of the Sun.