Assignment 8

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November 2024

1 Problem 1a: Cycling with Aerodynamic Drag

1.1 Problem Description

In this problem, we modify the equation of motion for a cyclist to include the effects of air drag. Air drag is modeled using the drag force

$$F_{\text{drag}} = \frac{1}{2}\rho C_D A v^2,$$

where ρ is the air density, C_D is the drag coefficient, A is the cross-sectional area, and v is the velocity of the cyclist. Using the provided values $C_D = 0.9$ and $A = 0.33\ \text{m}^2$, we account for the force limiting the cyclist's velocity.

1.2 Results and Discussion

The numerical simulation was performed using the modified equation of motion. The following plot shows the velocity of the cyclist over time with and without aerodynamic drag:

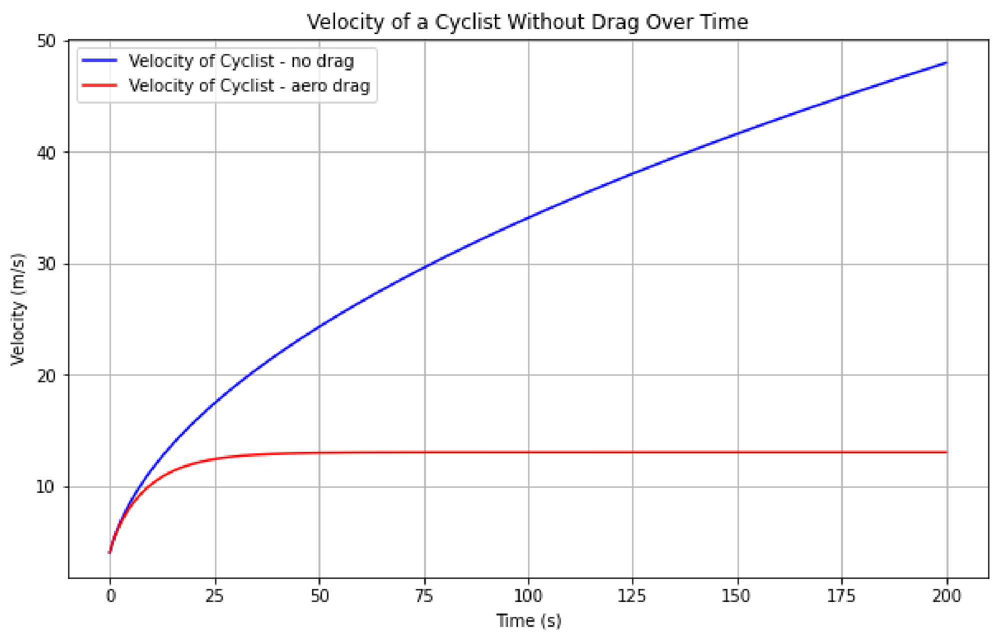

Figure 1: Cyclist's velocity over time with air drag included. The velocity approaches a terminal value due to the drag force limiting further acceleration.

As seen in Figure 1, the velocity of the cyclist increases initially but eventually stabilizes at a terminal velocity, consistent with the drag force balancing the propulsion force.

2 Problem 1b: Cycling with Viscous Drag

2.1 Problem Description

To improve the model, we include the Stokes drag term, given as

$$F_{\text{Stokes}} = 6\pi\eta h v,$$

where $\eta = 2 \times 10^{-5}$ Pa·s is the viscosity of air, and $h = 2$ m is the effective height of the cyclist. This term accounts for viscous interactions at low velocities.

2.2 Results and Discussion

The updated simulation was performed with both the quadratic drag and Stokes drag terms. The following plot illustrates the results, note: the viscous drag doesn't change much:

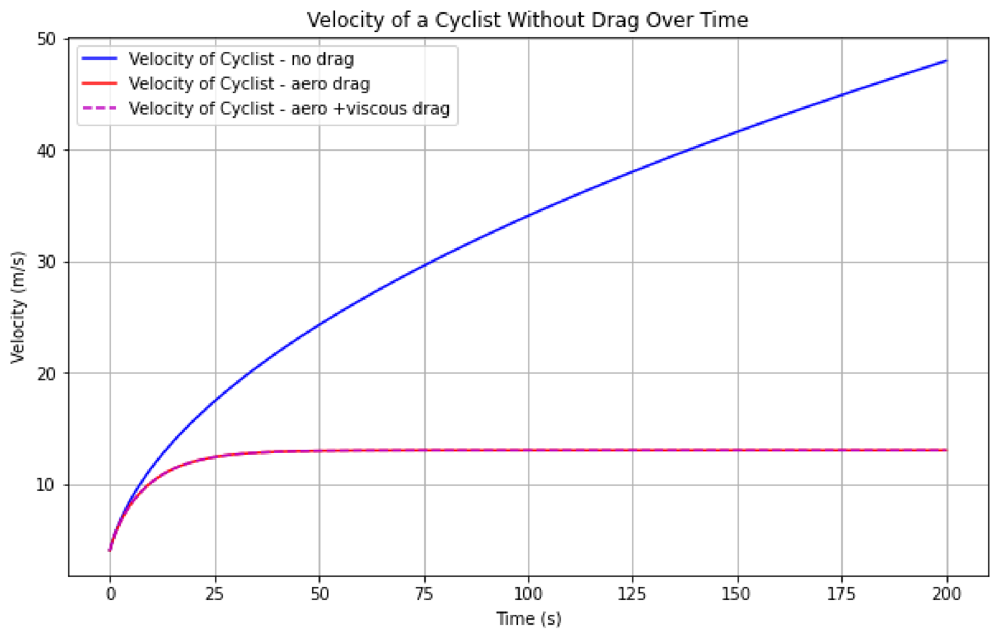

Figure 2: Cyclist's velocity over time with air drag and Stokes drag included. The addition of the Stokes term results in a more realistic velocity curve at lower speeds.

As shown in Figure 2, the inclusion of the Stokes drag should modify the early stages of the motion, providing a more accurate depiction of the cyclist's acceleration phase at low velocities, but since the air isn't too viscous not much changes from the previous plot 1.

3 Cycling on a Hill

3.1 Problem Description

To include the effects of cycling on an incline, we add the force term due to gravity:

$$F_{\text{gravity}} = -mg\sin\theta,$$

where $\sin\theta$ can be derived from the road grade, with $\tan\theta = 0.1$ for a 10% grade. This additional force opposes the cyclist's motion uphill and assists downhill.

3.2 Results and Discussion

The simulation was extended to account for the incline, and the following plot shows the velocity over time:

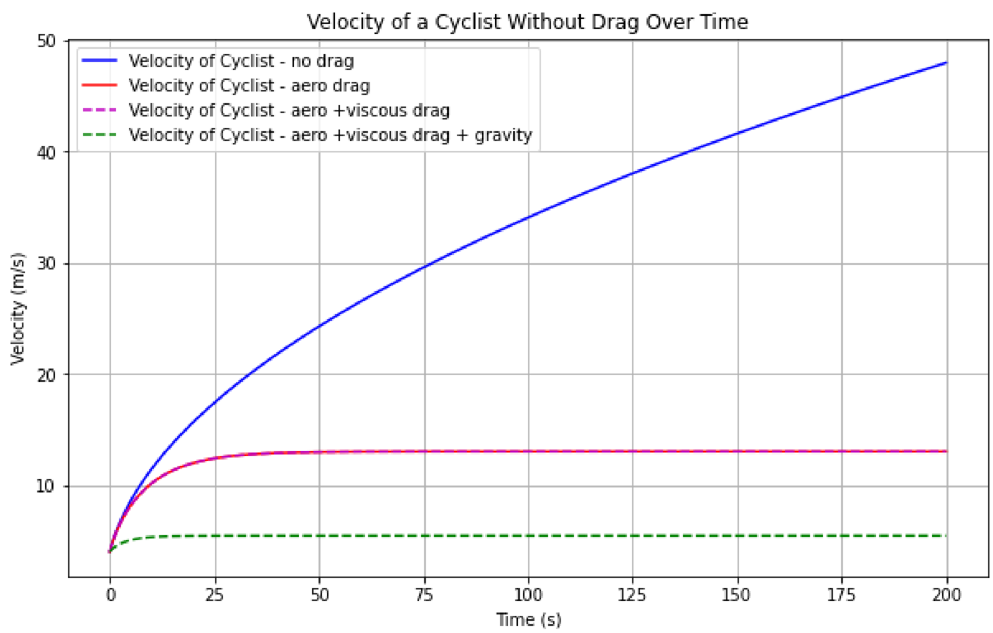

Figure 3: Cyclist's velocity over time on a hill with a 10% grade. Uphill motion reduces terminal velocity, while downhill motion increases it.

Figure 3 demonstrates the impact of the incline. Uphill cycling reduces the terminal velocity due to the opposing gravitational force, while downhill cycling increases it due to the assisting force.

4 Conclusion

This assignment explored the dynamics of cycling with realistic forces, including air drag, Stokes drag, and gravitational effects on slopes. These modifications provide a more comprehensive model of a cyclist's motion, with terminal velocity achieved in flat and uphill scenarios.

5 Task 2: Single Random Walk Simulation

5.1 Problem Description

A random walk simulation was performed with n steps, where each step is of unit length and can move either forward or backward. The walker starts at position 0. Two random walks of 100 steps were simulated, and their paths were plotted.

5.2 Results and Discussion

The random walks were plotted as the position of the walker versus step number. The results are shown in Figures 4 and 5.

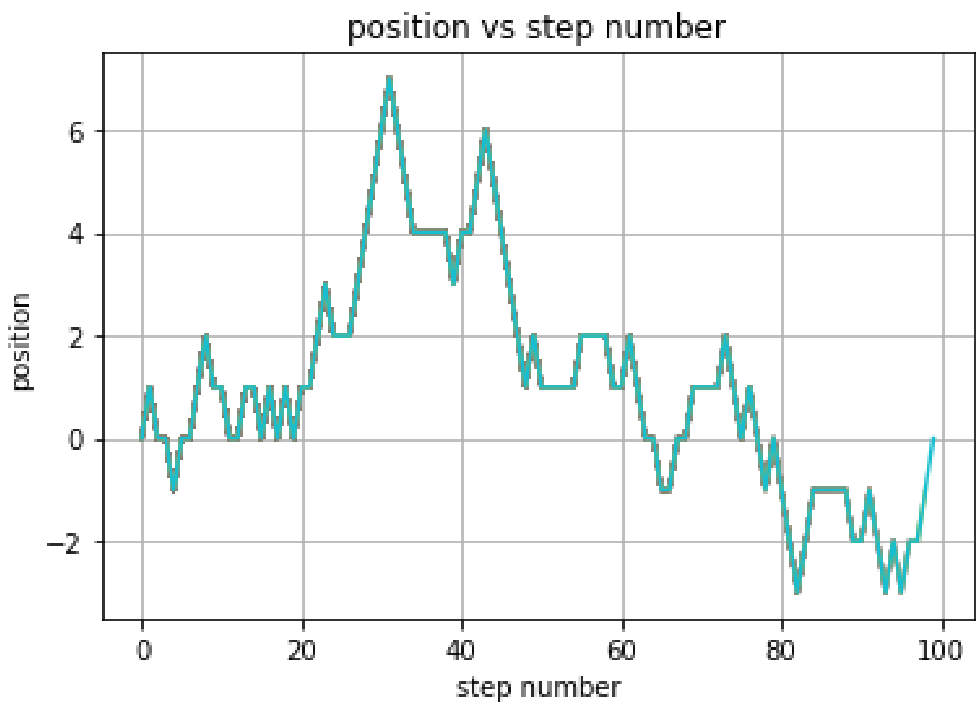

Figure 4: Random walk 1 with 100 steps. The walker starts at position 0, and the trajectory reflects the stochastic nature of the simulation.

Each random walk reflects the inherent randomness of the process, resulting in distinct paths despite identical initial conditions.

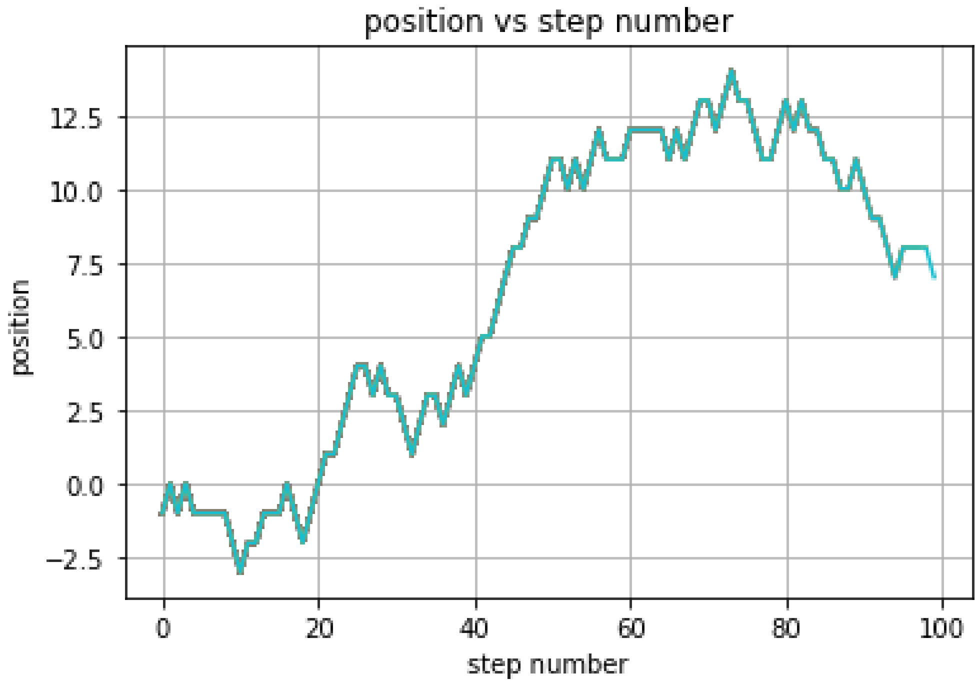

Figure 5: Random walk 2 with 100 steps. A different realization of the random walk shows variation while still adhering to the same underlying stochastic process.

6 Task 3: Mean Displacement and Mean-Squared Displacement

6.1 Problem Description

The random walk program was extended to simulate 500 walkers, each taking n steps. For each step, the mean displacement $\langle x \rangle$ and mean squared displacement $\langle x^2 \rangle$ across all walkers were calculated and plotted.

6.2 Results and Discussion

The mean displacement $\langle x \rangle$ was calculated using the code. As expected, the mean displacement remains approximately zero since the random walk is symmetric.

The mean squared displacement $\langle x^2 \rangle$ was plotted as functions of the step number. The plot is shown in Figure 6.

Mean displacement $\langle x \rangle$ as a function of step number. As expected, the mean displacement remains approximately zero since the random walk is symmetric.

The results are consistent with theoretical predictions:

- The mean displacement $\langle x \rangle$ remains near zero due to the symmetry of forward and backward steps.
- The mean squared displacement $\langle x^2 \rangle$ increases linearly with the step number, as predicted by the equation $\langle x^2 \rangle = n$ for an unbiased random walk.

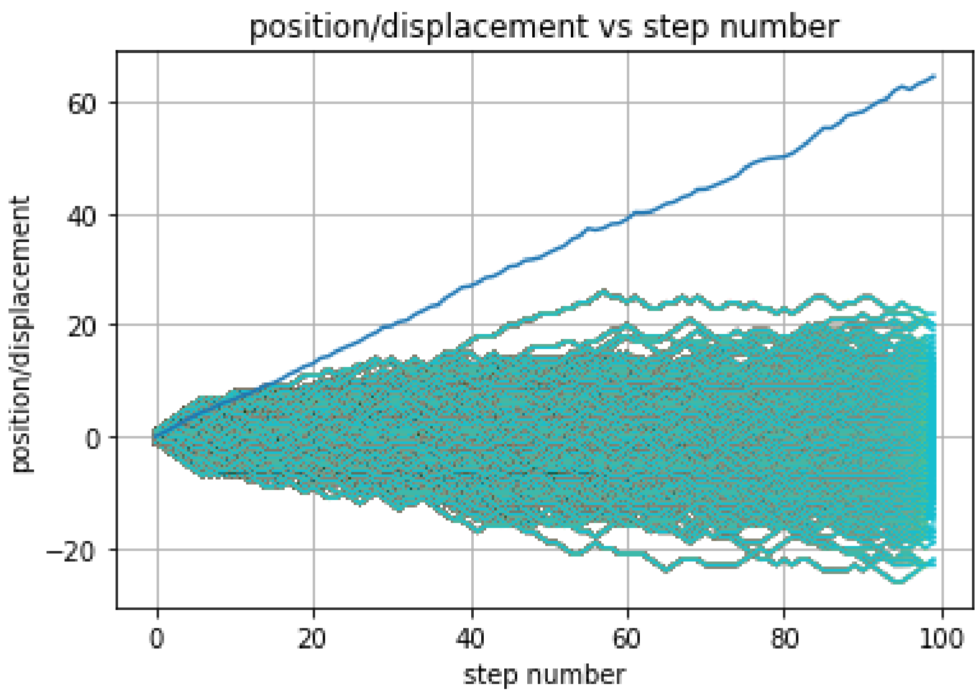

Figure 6: Mean squared displacement $\langle x^2 \rangle$ as a function of step number. The linear growth indicates that $\langle x^2 \rangle \propto n$, consistent with the theoretical expectation for a random walk.