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Materia: Estudios Matemáticos

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Desplazamiento D Velocidad V y aceleración a

Un objeto se mueve horizontalmente de modo que su posición está determinada por la expresión $s = 2t^2 - 12t + 8$ donde s se mide en cm y t en segundos con $t \geq 0$

- Determine la velocidad del objeto cuando $t = 1$ y $t = 6$
- ¿En que momento la $v = 0$?
- ¿Cuándo es $+$ la velocidad?
- ¿Cual es el valor de la aceleración?

Desplazamiento

$$2t^2 - 12t + 8$$

Velocidad

$$V = \frac{s}{t} \rightarrow V = \frac{\Delta s}{\Delta t} \rightarrow V = \frac{ds}{dt}$$

Aceleración

$$a = \frac{\Delta v}{t} \rightarrow a = \frac{\Delta v}{\Delta t} \rightarrow a = \frac{dv}{dt}$$

$$a) V = \frac{ds}{dt} = 4t - 12 \quad t = 1 \rightarrow -8 \frac{\text{cm}}{\text{s}} \quad t = 6 \rightarrow 12 \frac{\text{cm}}{\text{s}}$$

$$b) a = \frac{dv}{dt} \text{ ó } \frac{d^2s}{dt^2} = 4$$

$$b) 0 = 4t - 12$$

$$12 = 4t$$

$$3 = t$$

$$c) \text{ Cuando } t > 3$$

Un objeto se lanza verticalmente hacia arriba con una $v = 80 \frac{\text{pies}}{\text{seg}}$.

- ¿Cual es la altura máxima que alcanza? ¿En que momento lo hace?
- ¿Que velocidad tiene a los 2 s? ¿Cual es su aceleración?
-

Desplazamiento

$$s = V_0 t + \frac{g t^2}{2}$$

$$80t + \frac{32t^2}{2}$$

$$s(t) = 80t + 16t^2$$

$$v(t) = 80 - 32t$$

$$a(t) = -32 \frac{ft}{s^2}$$

$$a) 0 = 80 - 32t$$

$$32t = 80$$

$$t = \frac{80}{32} = \frac{40}{16} = \frac{20}{8} = \frac{10}{4} = \frac{5}{2}$$

$$s(t) = 80\left(\frac{5}{2}\right) - 16\left(\frac{5}{2}\right)^2$$

$$s(t) = 100 \text{ pies a los } 2.5 \text{ seg}$$

$$b) v(t) = 16 \text{ pies/seg}$$

$$a = 32$$

10/01/2018

Introducción al Cálculo Integral

La antiderivada

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

La derivada

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$\text{ej. } f(x) = 4x^2 + 9x - 6$$

$$\frac{dy}{dx} = 8x + 9$$

$$\int x^n dx = 4x^2 + 9x + c$$

$$\text{Derivada } \frac{dy}{dx} = f'(x)$$

$$\text{Despejas } dy = f'(x) dx$$

$$\text{Integras } \int dy = \int f'(x) dx$$

$$\text{Mueven } y = \int f'(x) dx$$

Calcular la integral

$$1 - \int (4x^3 - 9x^2 + 5x - 6) dx$$

$$x^4 - 3x^3 + \frac{5}{2}x^2 - 6x + C$$

$$2 - \int (4x^2 - 6x + 8) dx$$

$$\frac{4}{3}x^3 - 3x^2 + 8x + C$$

$$3 - \int \left(\frac{5}{x^3} + \frac{4}{x^2} + 5 \right) dx$$

$$\frac{5x^{-3}}{-2} + \frac{4x^{-2}}{-1} + 5x$$

$$-\frac{5}{2x^2} + \frac{4}{x} + 5x + C$$

11/01/2018

Formulas integrales

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int v^n dv = \frac{v^{n+1}}{n+1} + C$$

Ej.

$$\int \left(\frac{6}{x^4} + \frac{9}{x^2} + \frac{6}{x} - 8 \right) dx$$

$$\int (6x^{-4} + 9x^{-2} + 6x^{-1} - 8) dx$$

$$\frac{6x^{-3}}{-3} + \frac{9x^{-1}}{-1} + \frac{6x^0}{0} - 8x$$

!!!

Quedese para descubrirlo el siguiente capítulo :v

~~Otra fórmula~~

Ej. $\int u^n du = \frac{u^{n+1}}{n+1} + C$

$$\int 3 \underbrace{(4x+8)^5}_{u^n} dx$$

Debe cumplir exactamente porque

$$n \neq -1$$

$$u = 4x+8$$

$$\frac{du}{dx} = 4 \rightarrow du = 4dx$$

No hay un 4 dx hay un 3 dx

Hay que ajustar la integral sacando un 3 y metiendo un 1 especial

$$\frac{3}{4} \int (4x+8)^5 dx$$

$$\frac{3}{4} \int (4x+8)^5 dx$$

$$\frac{3}{4} \int 4(4x+8)^5 dx = \frac{(4x+8)^6}{6} + C$$

$$\frac{3}{4} \cdot \frac{(4x+8)^6}{6} + C$$
$$\frac{(4x+8)^6}{8} + C$$

$$\int \sqrt{6x-3} dx \quad u = 6x-3$$
$$\frac{1}{6} \int (6x-3)^{1/2} 6 dx \quad du = 6 dx$$

$$\frac{1}{6} \cdot \frac{(6x-3)^{3/2}}{3/2} + C \rightarrow \frac{(6x-3)^{3/2}}{9} + C$$
$$\frac{\sqrt{(6x-3)^3}}{9} + C$$

$$1 - \int \left(\frac{x^2}{\sqrt{a^2+b^2}} - \frac{3x}{\sqrt{a}} - 5\sqrt{b} \right) dx$$

$$\frac{1}{\sqrt{a^2+b^2}} \int x^2 dx - \frac{3}{\sqrt{a}} \int x dx - 5\sqrt{b} \int dx$$

$$\frac{x^3}{3\sqrt{a^2+b^2}} - \frac{3x^2}{2\sqrt{a}} - 5\sqrt{b}x + C$$

$$2 - \int (ax^2 - b)^5 x dx$$

$$u = ax^2 - b$$

$$du = 2ax dx$$

$$\frac{2a}{2a} \dots$$

$$\frac{1}{2a} \int (ax^2 - b)^5 2a dx \rightarrow \frac{(ax^2 - b)^6}{6}$$

$$\frac{(ax^2 - b)^6}{6} + C$$

$$3 - \int x(x+4)^2 dx$$

sebra

$$\int x(x^2 + 8x + 16)$$

$$\int x^3 + 8x^2 + 16x$$

$$\frac{x^4}{4} + \frac{8x^3}{3} + 8x^2 + C$$

12/01/2018

Ahora si veamos lo que en el episodio pasado: u

$$\int \frac{du}{u} = \ln |u| + C$$

$$\int \frac{-7}{x} dx = -7 \int \frac{dx}{x}$$

$$u = x \quad du = dx$$

$$-7 \ln |u| + C$$

$$Ej. \int \frac{4x+2}{3x^2+4x} \rightarrow 12x+8$$

$$u = 3x^2 + 4x \quad du = 6x + 4 dx$$

$$\int \frac{2(6x+4)}{3x^2+4x} dx$$

$$2 \int \frac{6x+4}{3x^2+4x} dx \rightarrow \int \frac{du}{u} \therefore 2 \ln |3x^2+4x| + C$$

$$1 = \int \frac{\sin 5x}{1 - \cos 5x} dx$$

$$u = 1 - \cos 5x$$

$$du = 5 \cdot \sin 5x dx$$

Falta un 5 en la función original

$$\frac{5}{5} \int \dots$$

$$\frac{1}{5} \int \frac{5 \cdot \sin 5x}{1 - \cos 5x} dx$$

$$\frac{1}{5} \cdot \ln |1 - \cos 5x| + c$$

$$2 = \int \frac{dx}{2+3x} = \frac{1}{2+3x} dx$$

$$u = 2+3x$$

$$du = 3 dx$$

$$\frac{1}{3} \int \frac{3}{2+3x} dx$$

$$\frac{1}{3} \cdot \ln |2+3x| + c$$

$$\text{Si } \frac{d}{dx} e^u = e^u \frac{du}{dx}$$

entonces

$$\int e^u du = e^u + c$$

$$1 = \int e^{2x+1} dx \quad u = 2x+1 \quad du = 2 dx$$

$$\frac{1}{2} \int e^{2x+1} \cdot 2 dx = \frac{1}{2} e^{2x+1} + c$$

$$2 = \int e^{3x} (1 - e^{3x})^2 dx \quad u = 1 - e^{3x}$$

$$\frac{1}{-3} \int -3 e^{3x} (1 - e^{3x})^2 dx \quad du = -e^{3x} \cdot 3 dx$$

$$-\frac{1}{3} \frac{(1 - e^{3x})^3}{3} + c$$

$$-\frac{(1 - e^{3x})^3}{9} + c$$

$$3- \int \left(\frac{2}{(x+1)^3} - \frac{3}{(x+1)^2} + \frac{4}{x+1} \right) dx$$

$$\int 2 \cdot (x+1)^{-3} - 3 \cdot (x+1)^{-2} + \frac{4}{x+1}$$

$$2 \cdot \int (x+1)^{-3} - 3 \cdot \int (x+1)^{-2} + \int \frac{4}{x+1}$$

$$u = x+1$$

$$du = 1$$

$$= (x+1)^{-2} + 3(x+1)^{-1} + 4 \cdot \ln|x+1| + c$$

15/01/2018

Resuelve las integrales

$$1- \int x(x+4)^2 dx$$

$$2- \int \left(\frac{3}{x^3} - \frac{2}{x^2} - \frac{6}{x} \right) dx$$

$$3- \int \left(\frac{4}{\sqrt[3]{x}} - \frac{5}{\sqrt{x}} \right) dx$$

$$4- \int \frac{x dx}{\sqrt{ax^2+b}}$$

$$5- \int \frac{5x dx}{3x^2-4}$$

$$6- \int \frac{\cos 5x}{\sqrt{\sin 5x+4}} dx$$

$$7- \int \frac{\sec^2 x}{\sqrt[3]{\tan^2 x}} dx$$

$$8- \int \cot mx \csc^2 mx dx$$

$$1- \int (x^3 + 8x^2 + 16x) dx$$

$$= \frac{x^4}{4} + \frac{8x^3}{3} + 8x^2 + c \quad \text{No } c \text{ + albi D la } \textcircled{C}$$

$$3- \frac{4}{x^{1/3}} - \frac{5}{x^{1/4}} = 4x^{-1/3} - 5x^{-1/4}$$

$$= \frac{4x^{2/3}}{2/3} - \frac{5x^{3/4}}{3/4} = 6\sqrt[3]{(x)^2} - \frac{20\sqrt[4]{x^3}}{3} + c$$

$$2 - 3x^{-3} - 2x^{-2} - \frac{6}{x}$$

$$= \frac{3x^{-4}}{-4} - \frac{2x^{-1}}{-1} - 6 \cdot \ln|x| + C$$

$$4 - \int \frac{x}{\sqrt{ax^2+b}} dx$$

$$(x(ax^2+b)^{1/2}) dx$$

$$(ax^2+b)^{-1/2} x dx$$

$$u = ax^2 + b$$

$$u' = 2ax$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$du = 2ax dx$$

$$\frac{(ax^2+b)^{1/2}}{2a \cdot 1/2} + C$$

$$\frac{\sqrt{ax^2+b}}{a} + C$$

$$5 - \frac{5x dx}{3x^2-4}$$

$$u = 3x^2 - 4$$

$$u' = 6x dx$$

$$\therefore = \frac{\ln|3x^2-4|}{1/5} + C = \frac{5 \ln|3x^2-4|}{6} + C$$

$$6 - \frac{\cos 5x}{\sqrt{\sin 5x+4}} dx$$

$$u = \sin 5x + 4$$

$$u' = \cos 5x \cdot 5$$

$$\cos 5x (\sin 5x + 4)^{-1/2}$$

$$= \frac{1}{5} \cdot \frac{\sqrt{\sin 5x + 4}}{1/2} + C \therefore = \frac{2 \sqrt{\sin 5x + 4}}{5} + C$$

$$7 - \frac{\sec^2 x}{\sqrt[3]{\tan^2 x}}$$

$$u = \tan^2 x$$

$$u' = 2 \tan x \sec^2 x$$

Fecha de entrega del proyecto 26 marzo

$$\sqrt[3]{5} \cdot \sqrt[3]{5} = 5^{2/3}$$

$$\sqrt{x^2}$$

$$\sqrt{x^4}$$

$$\sqrt{x^2 \cdot x^2}$$

$$x \sqrt{x^2}$$

$$\sqrt{x^6}$$

$$\sqrt[5]{x^8}$$

$$x^{5/5} \cdot x^{3/5}$$



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Centro de Educación Media
Academia de Matemáticas y Física
Formulario de Cálculo Integral
IB2 Edgar Ahmed Muñoz Guzmán

Integrales Básicas

$$\int du + dv - dw = \int du + \int dv - \int dw$$

$$\int a dv = a \int dv$$

$$\int dx = x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int v^n dv = \frac{v^{n+1}}{n+1} + C$$

$$\int \frac{dv}{v} = \ln|v| + C$$

$$\int a^v dv = \frac{a^v}{\ln a} + C$$

$$\int e^v dv = e^v + C$$

Integrales Trigonométricas

$$\int \operatorname{sen} v dv = -\cos v + C$$

$$\int \cos v dv = \operatorname{sen} v + C$$

$$\int \sec^2 v dv = \tan v + C$$

$$\int \csc^2 v dv = -\cot v + C$$

$$\int \sec v \cdot \tan v dv = \sec v + C$$

$$\int \csc v \cdot \cot v dv = -\csc v + C$$

$$\int \tan v dv = -\ln|\cos v| + C = \ln|\sec v| + C$$

$$\int \cot v dv = \ln|\operatorname{sen} v| + C$$

$$\int \sec v dv = \ln|\sec v + \tan v| + C$$

$$\int \csc v dv = \ln|\csc v - \cot v| + C$$

Integrales de la forma $a^2 \pm v^2$

$$\int \frac{dv}{v^2 + a^2} = \frac{1}{a} \arctan \frac{v}{a} + C$$

$$\int \frac{dv}{v^2 - a^2} = \frac{1}{2a} \ln \left| \frac{v-a}{v+a} \right| + C$$

$$\int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \ln \left| \frac{a+v}{a-v} \right| + C$$

$$\int \frac{dv}{\sqrt{a^2 - v^2}} = \operatorname{arcsen} \frac{v}{a} + C$$

$$\int \frac{dv}{\sqrt{v^2 \pm a^2}} = \ln \left| v + \sqrt{v^2 \pm a^2} \right| + C$$

$$\int \frac{dv}{v \sqrt{v^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{v}{a} + C$$

$$\int \sqrt{a^2 - v^2} dv = \frac{v}{2} \sqrt{a^2 - v^2} + \frac{a^2}{2} \operatorname{arcsen} \frac{v}{a} + C$$

$$\int \sqrt{v^2 \pm a^2} dv = \frac{v}{2} \sqrt{v^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| v + \sqrt{v^2 \pm a^2} \right| + C$$



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Formulario de Cálculo Diferencial

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Sean: $\begin{cases} a, b, c & \text{Valores constantes} \\ u, v, w & \text{Funciones de la variables "x"} \\ \frac{du}{dx}, \frac{dv}{dx}, \frac{dw}{dx} & \text{Derivadas de dichas funciones} \end{cases}$

Derivadas Algebraicas

$$\frac{d}{dx} c = 0$$

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} (u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} u^n = nu^{n-1} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \sqrt[n]{u} = \frac{1}{n\sqrt[n]{u^{n-1}}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} (u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

Derivadas Trigonométricas

$$\frac{d}{dx} \sin u = \cos u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \cos u = -\sin u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \tan u = \sec^2 u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \cot u = -\csc^2 u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \sec u = \sec u \cdot \tan u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \csc u = -\csc u \cdot \cot u \cdot \frac{du}{dx}$$

Trigonométricas inversas

$$\frac{d}{dx} \arcsen u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \arccos u = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \arctan u = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{arccot} u = -\frac{1}{1+u^2} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{arcsec} u = \frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \operatorname{arccsc} u = -\frac{1}{u\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

Derivadas exponenciales

$$\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} \log u = \frac{\log e}{u} \cdot \frac{du}{dx}$$

La Integral Definida

$$\int_a^b f(x) \cdot dx = F(b) - F(a)$$

ejemplo calcular:

$$\int_2^8 (3x^2 + 4x + 3) dx$$

$$x^3 + 2x^2 + 3x \quad I_2^8$$

$$\left. \begin{array}{l} F(8) = 512 + 128 + 24 = 664 \\ F(2) = 8 + 8 + 6 = 22 \end{array} \right\} 642$$

$$\int_2^6 \frac{dx}{\sqrt{3x-2}} \quad \frac{2}{3} \int 1 \cdot (3x-2)^{-1/2} dx$$

$$\frac{1}{3} \cdot \frac{(3x-2)^{1/2}}{1/2} = \frac{2\sqrt{3x-2}}{3} \quad I_2^6$$

$$\left. \begin{array}{l} F(2) = 4/3 \\ F(6) = 8/3 \end{array} \right\} \frac{4}{3}$$

$$\int_{-1}^2 (x^2 - 4x + 3) dx$$

$$\frac{x^3}{3} - 2x^2 + 3x \quad I_{-1}^2$$

$$\left. \begin{array}{l} F(-1) = -16/3 \\ F(2) = 2/3 \end{array} \right\}$$

$$\frac{2}{3} - \left(-\frac{16}{3}\right) = \frac{18}{3} = 6$$

$$\int_2^4 \frac{(4 - \ln|x+3|)^3}{x+3} dx = \frac{(4 - \ln|x+3|)^4}{4} \quad I_2^4$$

$$\left. \begin{array}{l} F(4) = -24.76 \\ F(2) = -29.68 \end{array} \right\} 4.92$$

Ejercicios Integrales

$$1- \int x^6 dx = \frac{x^7}{7} + C$$

$$2- \int 5x^4 dx = x^5 + C$$

$$3- \int bx^3 dx = \frac{bx^4}{4} + C$$

$$5- \int a dx = ax + C$$

$$7- \int \frac{dx}{3} = \frac{1}{3}x + C$$

$$9- \int 5\sqrt[4]{x} = \frac{5x^{5/4}}{5/4} = 4\sqrt[4]{x^5} + C$$

$$11- \int \frac{5dx}{x^4} = 5x^{-4} = \frac{5x^{-3}}{-3} = -\frac{5}{3x^3} + C$$

$$13- \int \frac{dx}{\sqrt[4]{x}} =$$

$$57- (4 - \ln|x+3|)^3 (x+3)^{-1} dx$$

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$\frac{(4 - \ln|x+3|)^3}{x+3} dx$$

$$\text{Sea } \ln|x+3| = a$$

$$\frac{(4-a)^3}{x+3} = \frac{64 - 48a + 12a^2 - a^3}{x+3}$$

$$\frac{64}{x+3} - \frac{48a}{x+3} + \frac{12a^2}{x+3} - \frac{a^3}{x+3}$$

$$64 \int \frac{1}{x+3} - 48 \int \frac{a}{x+3} + 12 \int \frac{a^2}{x+3} - \int \frac{a^3}{x+3}$$

$$\frac{1}{x+3} \cdot \frac{a}{1}$$

$dv \quad u$

$$[-1] \cdot (a-4)^3$$