Summary

Brownies is very popular in the United States nowadays for its tasty flavor. If you want to bake out high quality of Brownies, it is essential for a housewife to use the proper pan. As we all know, when baking in a rectangular pan heat is concentrated in the four corners and the food gets overcooked at the corners; while baking in a round pan the heat is distributed evenly over the entire outer edge and the food is not overcooked there. Due to lacking efficiency of using the space in an oven, circular pans are not the best choice. We are tasked to solve this dilemma and put forward an optimized design for baking pans.

To illustrate how heat is distributed in a specific baking pan, 3D CFD Simulation of the oven is working out. From the simulation figures, we find out two interesting reality. That is, heat concentrates on the corner of pan, and distributes uneven on the edge of pan. However heat distributes uniformly on the inner area of pan. This is the reason leading to brownie overcooked on the corner of pan and poor-cooked on the central area. And the temperature distribution of high edge amount polygon is better than low edge amount polygon.

For the sake of studying the best shape of pans, the combination optimization model is developed. The model displays the optimal Brownie pan when *W/L* and *P* are changed. When weight p increase from zero to one in step size 0.025 the best shape is from circle to low edge amount polygon. In some step the data is centralization and in some step data is decentralization. The ratio of width and length influence results in a complex way and it is a discrete relationship that can hardly be described. If we want to know the best shape, we must specify parameters by design a good algorithm.

Key words: convection heat flux heat transfer coefficient radiation heat flux Temperature of heater velocity streamline

Contents

目录

The Ultimate Brownie Pan

I. Introduction

Product gets overcooked at the corners when baked in a rectangular pan, but not overcooked at the edges when baked in a round pan. However, using round pan is not efficient with respect to using space in a rectangular oven. So it is necessary to design pans of new shape satisfied for maximizing both even distribution of heat for the pan and number of pans that can fit in the oven.

In order to study better the heat distribution of pans in different shape, the paper develops the complete heat transfer process in the oven, and use CFX to simulate.

For the sake of studying the best shape of pans, the combination optimization model is developed. The model displays the ultimate Brownie pan when W/L and P are changed.

II. Problem Analysis

2.1 Heat Distribution of the Pan

Based on governing equations, that is, equations of mass, momentum and energy, we could analyze the distribution of heat for the whole oven as well as the pan in it. By simulating the thermal process of the whole oven, we can obtain the distribution graphs of heat transfer coefficient, temperature and heat flux and so on when the oven is at steady state. These graphs can show the rough heat transfer coefficient and the temperature of the solid and fluid far from the solid surface of the oven. Then, the heat transfer coefficient and temperature are defined. In the foundation of the previous work, we can easily get the graph of temperature distribution by simulating the transient heat transfer process of the pan. This graph can explain the reason why the corners of product is overcooked in the square pan, and also we can get the distribution of heat across the outer edge of a pan for pans of different shapes. As known, heat is a linear function of temperature, which indicating the distribution of heat is the same as that of the temperature. The non-uniform distribution coefficient I of temperature of the polygon in different shapes can be obtained as well.

2.2 Pan's Tiling problem

Pans are of different shapes ranging from rectangular to circular and other shapes in between. Without loss of generality, this paper would like to think the regular polygonal pans with the same area. Basic mathematics have proved that square and regular hexagonal can splice together seamless on infinite plane, but for regular pentagon, regular heptagon and other regular polygon, space occupancy rate differs from different methods of splicing. And the oven bottom is a limited plane in practical problems, as a result, even square and regular hexagonal pan may also produce voids.

In order to maximize the space occupancy rate F of different splicing polygons, we use MATLAB to make a program. Letting edge number n be unchanged, when

W/L changes, the corresponding optimal space occupancy rate and number of pans can be gained. Therefore the function *F (W/L, n)* can be fitted.

2.3 The Combination Optimization

We must satisfy the three conditions in our combination optimization model with the given assumptions. Firstly, maximize number of pans that can fit in the oven; secondly, maximize even distribution of heat for pan; thirdly, optimize a combination of the above two conditions where weights *p* and (*1-p*) are assigned to illustrate how the results vary with different values of the width to length ratio of *W/L* for the oven and *p*. As the first two metrics are the space occupancy rate *F* of pans and the non-uniform distribution coefficient *I* of temperature respectively, we can construct an objective function to optimize the combination of the above two conditions in terms of *F* and *I,* where *n* is the decision variable. Finally, it is easy for us to illustrate how the results vary with different values of the width to length ratio of *W/L* for the oven and *p*.

III. Assumptions and Symbol Definition

3.1 Assumptions

In order to ensure that our models remain valid under various potential constraints, we make a number of assumptions as follows before stepping into details about our approaches.

1) Initially two racks in the oven, evenly spaced.
2) We only investigate regular polygon and roundness considering the difficulty of actual machining.
3) The pans are arranged in regular rows and columns thinking about the life habit of modern citizens.
4) All the pans are made in cast iron, so they have the same rate of heat conduction.
5) The working medium in the oven is air.

3.2 Symbol Definition

Notation	Definition	Units
L	Length of the oven	m
W	Width of the oven	m
H	Height of the oven	m
n	Number of polygons' edges	
W / L	Weight to length ratio	

p	Assessment weights—parameter to measure importance of F and I	
F	The space occupancy rate	
I	The non-uniform distribution coefficient	

IV. 3D CFD Simulation of the Oven

4.1 Background of CFD

Computational fluid dynamics, usually abbreviated as ***CFD***, is a branch of fluid mechanics that uses numerical methods and algorithms to solve and analyze problems that involve fluid flows. Computers are used to perform the calculations required to simulate the interaction of liquids and gases with surfaces defined by boundary conditions. With high-speed supercomputers, better solutions can be achieved. Ongoing research yields software that improves the accuracy and speed of complex simulation scenarios such as transonic or turbulent flows.

The paper would like to use the 3D CFD simulation method to analyze the heat transfer process of the oven completely.

4.2 Governing Equations and Numerical Methodology

4.2.1 Governing equations

The governing equations for the present purpose are the equations of conservation of mass, momentum and energy. The equation used for conservation of mass (the continuity equation) may be written as:

$$\frac{\partial\rho}{\partial t}+\frac{\partial\left(\rho u_i\right)}{\partial x_i}=0$$

The equation for conservation of momentum in direction i is given by:

$$\frac{\partial\left(\rho u_i\right)}{\partial \mathrm{t}}+\frac{\partial\left(\rho u_i u_j\right)}{\partial x_j}=-\frac{\partial P}{\partial x_i}+\frac{\partial\tau_{ij}}{\partial x_j}+\rho g_i$$

In this balance P is the static pressure, τ_{ij} is the stress tensor. ρg_i is the gravitational body force. The stress tensor τ_{ij} for a Newtonian fluid is defined by:

$$\tau_{ij}=-pS_{ij}+2\mu\left(S_{ij}-\frac{1}{3}S_{kk}\delta_{ij}\right)$$

Here $S_{ij}=\frac{1}{2}\left(\frac{\partial \mathrm{u}_i}{\partial x_j}+\frac{\partial \mathrm{u}_j}{\partial x_i}\right)$. μ is the strain viscosity. The second term on the right hand side of the equation is the effect of volume dilation.

The energy conservation equation is solved in the form:

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial(\rho u_i h)}{\partial x_i}=-\frac{\partial}{\partial x_i}q_i+\frac{\partial p}{\partial t}+\tau_{ik}\frac{\partial u_i}{\partial x_k}+S_h$$

In this equation h is enthalpy and S_h includes heat chemical reaction, any inter-phase exchange of heat, and any other energy sources.

4.2.2 Table governing equations of the compressible Newtonian fluid flow

Table governing equations of the compressible Newtonian fluid flow are the equations of conservation of mass, momentum and energy and the equation of state. The continuity equation in i direction (coordinate) (i =1, 2, 3):

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u_i)}{\partial x_i}=0$$

The momentum equation in i direction (coordinate):

$$\frac{\partial(\rho u_i)}{\partial \mathrm{t}}+\frac{\partial(\rho u_i u_j)}{\partial x_j}=-\frac{\partial \rho}{\partial x_i}+\frac{\partial}{\partial x_j}\left[u\left(\frac{\partial \mathrm{u}_i}{\partial x_j}+\frac{\partial \mathrm{u}_j}{\partial x_i}\right)-\frac{1}{3}u\frac{\partial \mathrm{u}_c}{\partial x_c}S_{ij}\right]+\rho gi$$

The energy equation in i direction (coordinate):

$$\frac{\partial(\rho h)}{\partial t}+\frac{\partial(\rho u_i h)}{\partial x_i}=-\frac{\partial}{\partial x_i}\left(-\lambda\frac{\partial T}{\partial x_i}\right)+\frac{\partial p}{\partial t}+\tau_{ik}\frac{\partial u_i}{\partial x_k}+S_h$$

The equation of state:

$$P=P(\rho,T) \qquad h=h(\rho,T)$$

For perfect air gas: $P=\rho RT \qquad h=C_pT$, where h is enthalpy, C_p is thermal capacity, R is gas constant.

4.2.3 Turbulence model and its' mathematic closed pattern

By estimating Rayleigh number R_a, the flow state can be defined as turbulence or not. For high Rayleigh number flow, the RNG $\kappa-\varepsilon$ model should be employed. This model use κ and ε to define velocity scale υ and length scale l, and represent the large-scale turbulence, where $\upsilon=\kappa^{\frac{1}{2}}$ and $l=\frac{\kappa^{\frac{3}{2}}}{\varepsilon}$. Eddy viscosity is $u_t=\rho C_u\frac{k^2}{\varepsilon}$, where C_u is a dimensionless constant.

The RNG $\kappa-\varepsilon$ governing equations can be solve in the form:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial}{\partial x_i}(\rho u_i k)=\frac{\partial}{\partial x_i}\left[\alpha_k u_{eff}\frac{\partial k}{\partial x_i}\right]+\tau_{kl}S_{kl}-\rho\varepsilon$$

$$\frac{\partial(\rho\varepsilon)}{\partial t}+\frac{\partial}{\partial x_i}(\rho u_i\varepsilon)=\frac{\partial}{\partial x_i}\left[\alpha_k u_{eff}\frac{\partial k}{\partial x_i}\right]+C_{1\varepsilon}^{*}\frac{\varepsilon}{k}\tau_{kl}S_{kl}-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}$$

Where: $\tau_{ij}=-\rho\overline{u_i'\cdot u_j'}=2u_tS_{ij}-\frac{2}{3}\rho kS_{ij}$

$$u_{eff}=u+u_t \qquad u_t=\rho C_u\frac{k^2}{\varepsilon}$$

$$C_u=0.0845 \qquad \alpha_k=\alpha_\varepsilon=1.3$$

$$C_{1\varepsilon}=1.42 \qquad C_{2\varepsilon}=168$$

$$C_{1\varepsilon}^{*}=C_{1\varepsilon}-\frac{\eta\left(1-\frac{\eta}{\eta_0}\right)}{1+\beta\eta^3} \qquad \eta=\frac{k}{\varepsilon}\sqrt{2S_{ij}\cdot S_{i}} \qquad \eta_0=4.37$$

With these equations, this system is mathematically closed and it can be solved with suitable auxiliary boundary and initial conditions.

To get the general equation, we conclude all the above equations as diffusion equation. After integrated, the general transport equation can be obtained. The differential form is as follows:

$$\frac{\partial(\rho\phi)}{\partial t}+\nabla\left(\rho\phi\vec{u}\right)=\nabla\left(\Gamma\nabla\phi\right)+S_\phi$$

Meanwhile, the integrated form:

$$\int_{\Delta t}\frac{\partial}{\partial t}\left(\int_{cv}\rho\phi d_v\right)d_t+\int_{\Delta t}\int_{c.s}(\rho\phi\vec{u})d_{\vec{A}}d_t=\int_{\Delta t}\int_{c.s}(\Gamma\nabla\phi)d_{\vec{A}}d_t+\int_{\Delta t}\int_{c.v}S_\phi d_v d_t$$

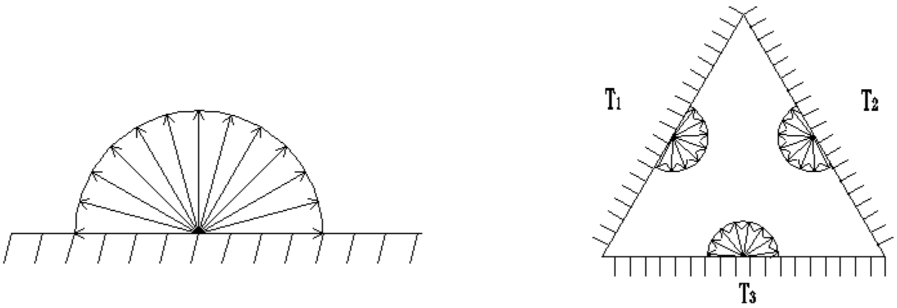
4.2.4 Natural convection

There is no forced velocity in the oven, yet convection current exist within the fluid. According to the expression for ideal gas: $\rho=\frac{P}{R_gT}$, non-uniform temperature causes temperature gradient. At the same time, temperature gradient leads to density gradient. The net effect is a buoyancy force, which leads to natural convection. In the most common case, body force is due to gravitation field. In all governing equations ρ is not a constant but a function of T and P, so natural convection has been described by these equations. And because of the convection, the temperature field in the oven is more uniform than without it.

4.2.5 Radiation convection

The typical radiation heat transfer methods commonly used at present time include: Heat Flux method, Zone method, Monte-Carlo method, PI, Rosseland, DT, etc. Because gas medium is air, and air is almost transparent for heat radiation, the heat radiation pattern is surface to surface. Hence we use Discrete Transfer method (DT) to solve heat radiation problem.

DT method is to concentrate the radiation effect in all directions of the medium into several finite characteristic radiation lines. There are two different radiations, shown in figure 1 (a) and (b). That is, surface radiation and radiation heat transfer in cavity.



(a) (b)

Figure 1 the two different radiations, (a) is surface radiation, (b) is radiation heat transfer in cavity

Radiation ray transport process under surface to surface mod:

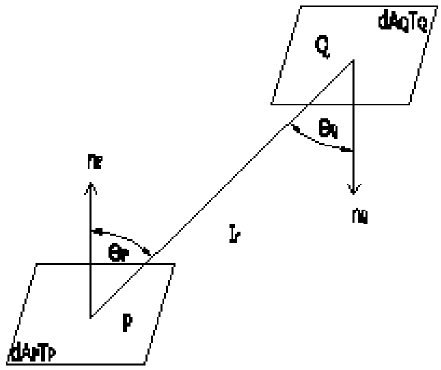


Figure 2 two surface ray transport process

Division Feature-ray on area element:

Considering a boundary grid face whose central point is P and area is dA_p .Then make a hemispherical from this point to calculation area, and divide the longitude angel and latitude angel into $N \times N$ Aliquots. In Another word, we divide hemispherical into $N \times N$ solid angel element. Then leads Feature-ray from each solid angel element.

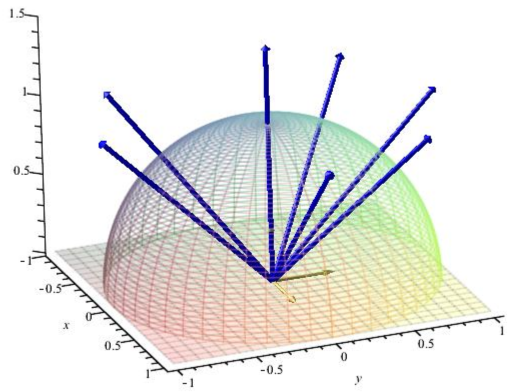


Figure 3 the hemispherical and Feature-rays

How we calculate irradiation Heat Flux and Received radiation Heat Flux?

Considering figure 2, a Feature-ray R leave from area element Q with intensity Ir. The Heat Flux on the P area element is the sum of all Feature-ray from the Top hemispherical space of Parea element central point. The formula of the process:

$$q_\rho^+ = \int_0^\pi I_{\bar{R}} \Omega d_\Omega = \int_0^{2\pi}\int_0^{2\pi} I_{\bar{R}}(\theta,\varphi)\sin\theta\cos\theta d_\theta d_\varphi = \sum_i I_{\bar{R}} i\cos\theta_i$$

The Heat Flux q_p^- leave from area element P was decided by Thermal boundary condition:

Boundary conditions: $q_\rho^+ = \int_0^\pi I_{\bar{R}} \Omega d_\Omega = \int_0^{2\pi}\int_0^{2\pi} I_{\bar{R}}(\theta,\varphi)\sin\theta\cos\theta d_\theta d_\varphi = \sum_i I_{\bar{R}} i\cos\theta_i$

Dilichelet boundary: $q_p^- = (1-\varepsilon_p)q_p^+ + \varepsilon_p \sigma T_p^4$

Neumann Boundary: $q_p^- = q_p^+ - q_p$

If a surface is Diffuse Surface, Then the Intensity of Feature-ray leave from P is:

$$I_p = \frac{q_p}{\pi}$$

Transient Heat Transfer Process:

The Transient Heat Transfer Process can be described by Energy Governing equation we have been given on the step upon. However, because of the process belongs to Transient Heat Transfer process, so we must define Initial condition. The Initial condition is the pan under room temperature when put in the oven. All the boundary condition data was calculated from the step Upon-Steady Oven Heat Transfer numerical simulation with average process.

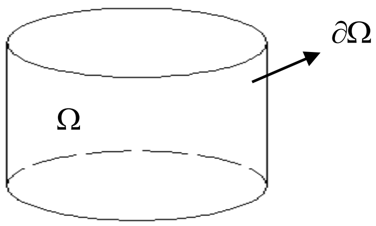


Figure 4 the pan

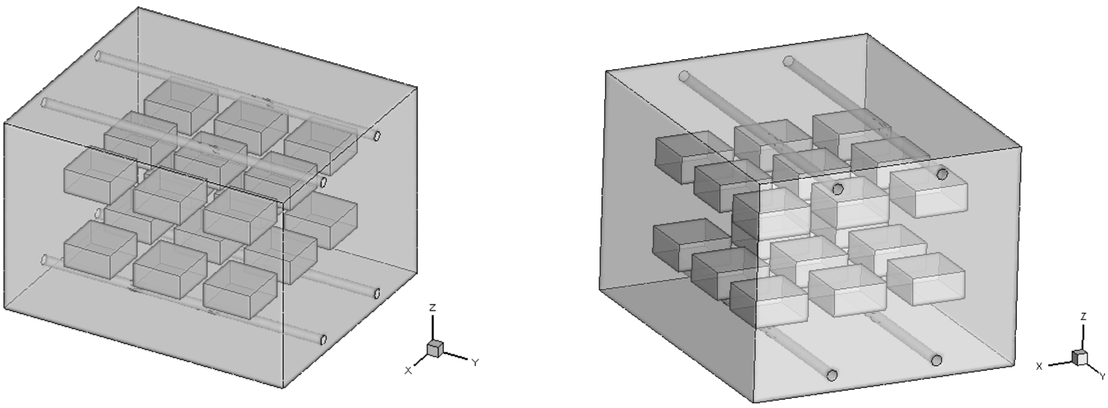
4.2.6 Meshing

This paper implement hybrid meshes using Gambit. The hybrid meshes are with pyramid and tetrahedron cell in fluid domain, also with hexahedron cell in solid domain.

4.3 Result and discussion

4.3.1 Preparation for simulation

1) The oven is $0.6\times0.5\times0.6\,m^3$, the oven model this paper using is shown in figure 5 (a) and (b), which show the oven model in two directions.

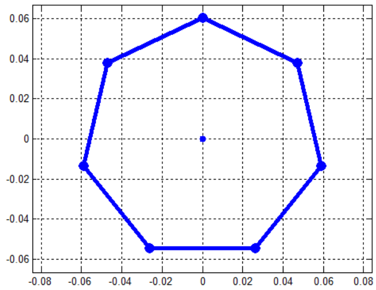


(a) (b)

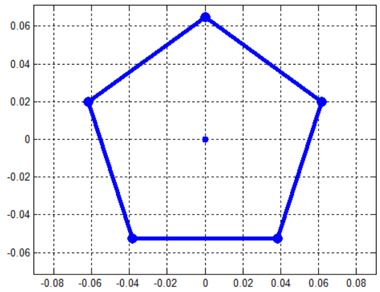
Figure 5 the oven model viewed in two directions

2) The oven must have an area of $A_r = W \times L$, where W is the width of the oven while L is the length of the oven. Thus $A_r = 0.3m^2$. But the width to length ratio W / L can be changed. That is to say, we would like to consider the oven shape as changed.

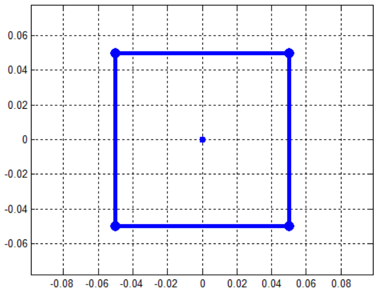
3) Each pan have an area of $A_B = 0.01m^2$, that is, each polygon have the same area A_B. Figure 6 shows polygons with different edge number but the same area.



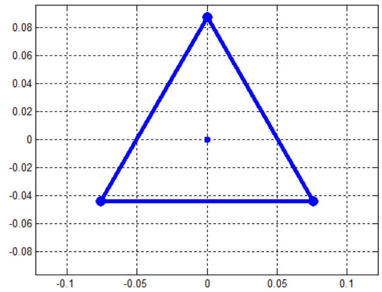
(a) n=6



(b) n=5



(c) n=4



(d) n=3

Figure 6 regular polygons with the same area

4) Far infrared heater tube is the heat source of the oven model. There are 4 far infrared heater tubes shown as figure 3. The total power of the 4 tubes are 1200W and 300W of each tubes respectively.

5) We use Gambit to mesh the space of the oven as well as the pan in it. Figure 7 gives the meshing result. The right hand side picture is amplified of the left.

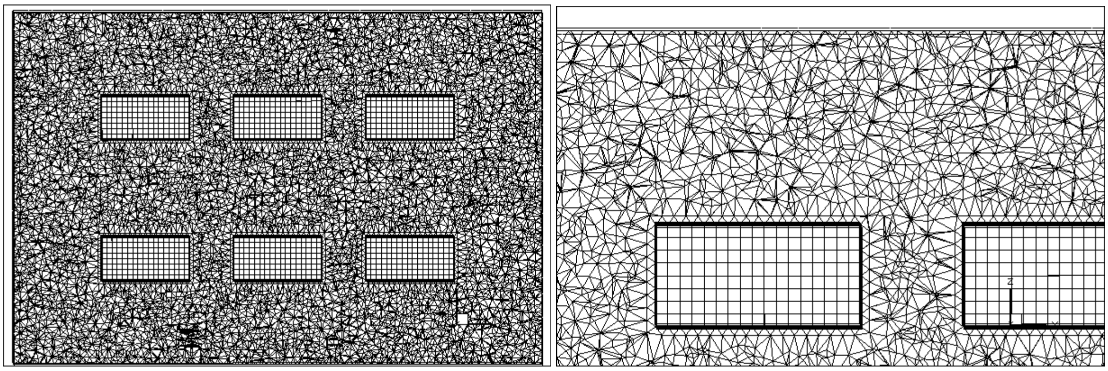


Figure 7 meshing

4.3.2 Complete heat transfer process simulation of the oven

Heat Transfer Process in the oven is a complex heat transfer process, it includes heat conduct natural convection and heat radiation.

After building geometry model and meshing it, we can simulation it on the Professional CFD Software: CFX and Fluent. Then Process Result in Tecplot and

CFD-POST.

The following is some results we get from the simulation.

We find natural convection and heat radiation is the main ways of Heat transfer in the baking oven and heat conduct can almost be neglected. The pictures below show the radiative heat flux and connective heat flux.

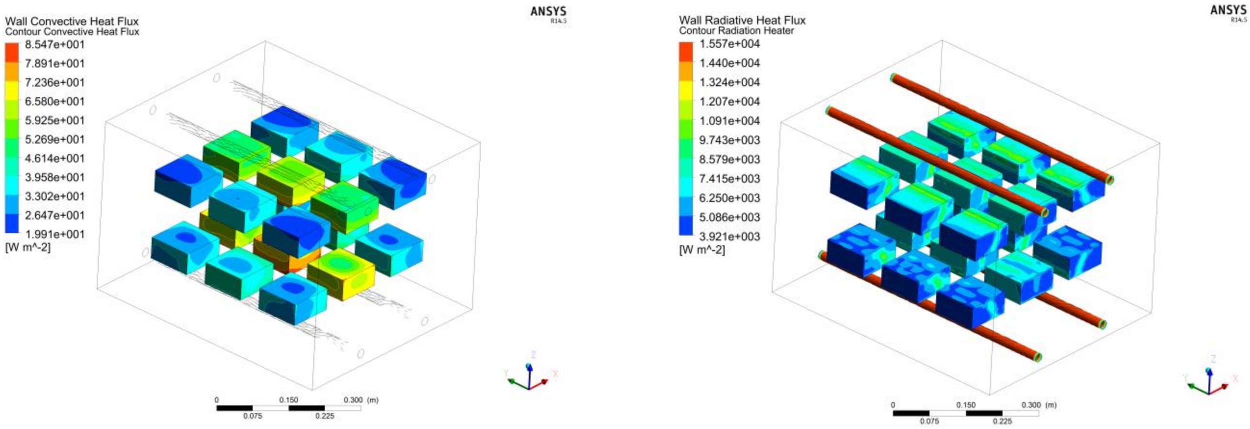


Figure 8 convection heat flux

Wall heat flux has been shown in rainbow color contour, from these two pictures, and we can find the max radiative heat flux is $15570W/m^2$ on the surface of far infrared heater tube and there is two radiative heat flux concentrated area which shape are line in the pans top surface with green color. The wall convective heat flux range from 19.91 to 8.547, and some area convective heat flux are pretty high because of vortex are concentrated in these area leads to highly Heat Transfer Capacity.

The Temperature Field was what we care most:

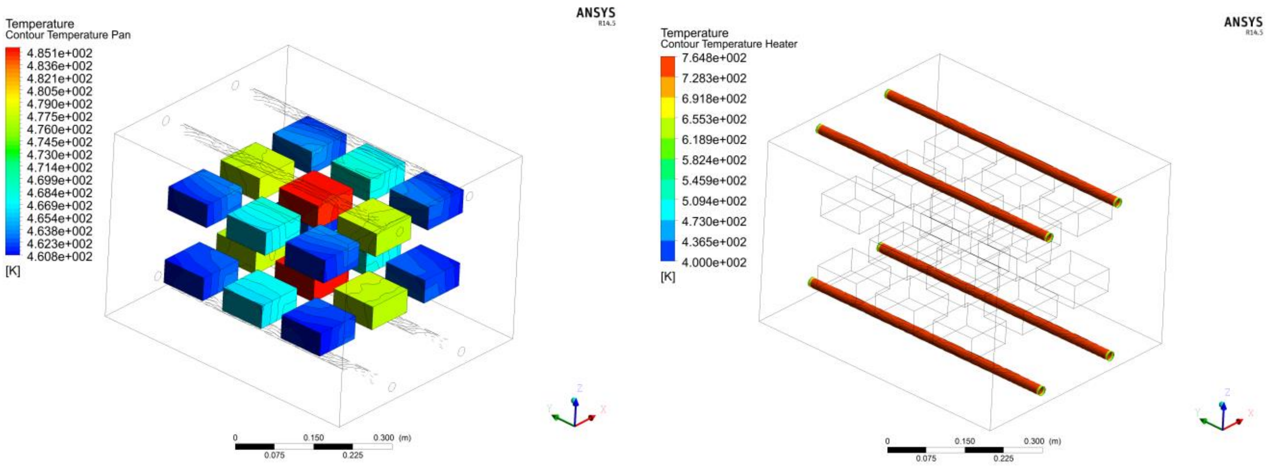
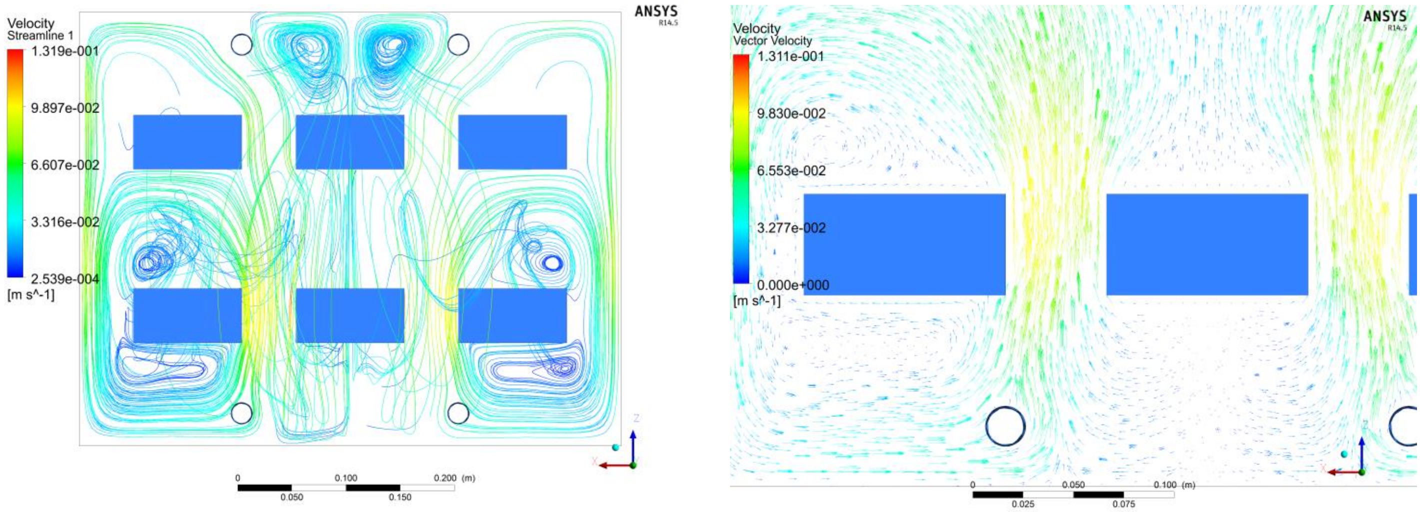


Figure 9 temperature distribution

From these picture we get two important message the {T infinity and Tr}, which we will used for the step below. The temperature of far infrared heater tube is 764.8K and the average temperature of surrounding is 423.15K so these two parameter we has been get and then use them to calculate Transient Heat Transfer Process of Baking Brownie.

Another important parameter in heat transfer process is Convective Heat Transfer Coefficient, In order to get it we must know the fluid flow condition on the oven. The fluid flow on the oven is quite difference form normal situation: it was driven by buoyancy, because of density gradient in the gravity field. These picture below has shown how fluid flow in the oven



(a) Velocity stream line (b) velocity vector

Figure 10 the velocity stream line and velocity vector

Fluid flow in the oven was driven by buoyancy heat fluid flow up and cold fluid flow down. In the Velocity Vector Plot fluid was heated by the Heater tube and flow to the up space. Fluid Velocity magnitude increase because of compressed when it closed to solid surface interspace. At the same time the Local Rayleigh number in the solid surface is pretty high, so boundary layer separate and leads laminar flow transient to turbulence flow. We can see this phenomenon in the Velocity Steam Line by notice vortex concentrate area. Convection heat transfer coefficient on these vortex concentrate area is pretty high than other area because of the boundary layer on the solid surface was turbulence boundary layer and fluid scour surface in a high frequency, at the same time transpose large mount heat.

Convective Heat Transfer Coefficient has been shown in the {Fig} and its value is range from 1.18W/m^2 to 2.058W/m^2. So we use 2W/m^2 as Heat Transfer Coefficient in the Transient Heat Transfer Process of baking brownie.

Local Convective Heat Transfer Coefficient is pretty high in some area of pan because of vortex concentrate and turbulence boundary layer.

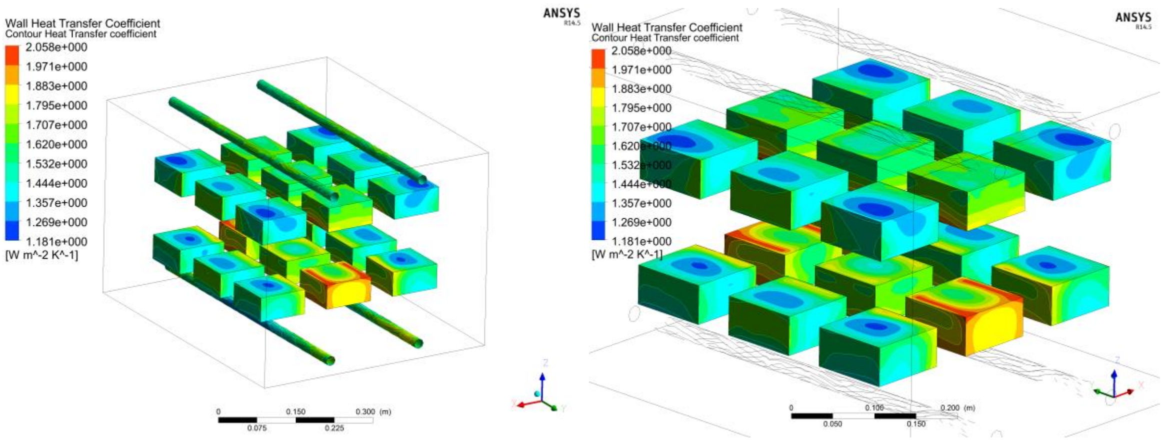


Figure 11 heat transfer coefficient

Result analysis of Transient Heat Transfer when baking brownie:

We simulation the transient heat transfer process and get temperature field when heating time is 25 minute on multi-shape pans. Then isolated boundary temperature distribute on the baking pan, use these data plot and compare differences boundary temperature distribute between multi-shape pans. The Temperature Variance can describe temperature distribute uniform level, so plot boundary temperature vs edge amount of pans can show the relationship of shape and boundary temperature distribute.

The picture below is temperature field of different shape pans:

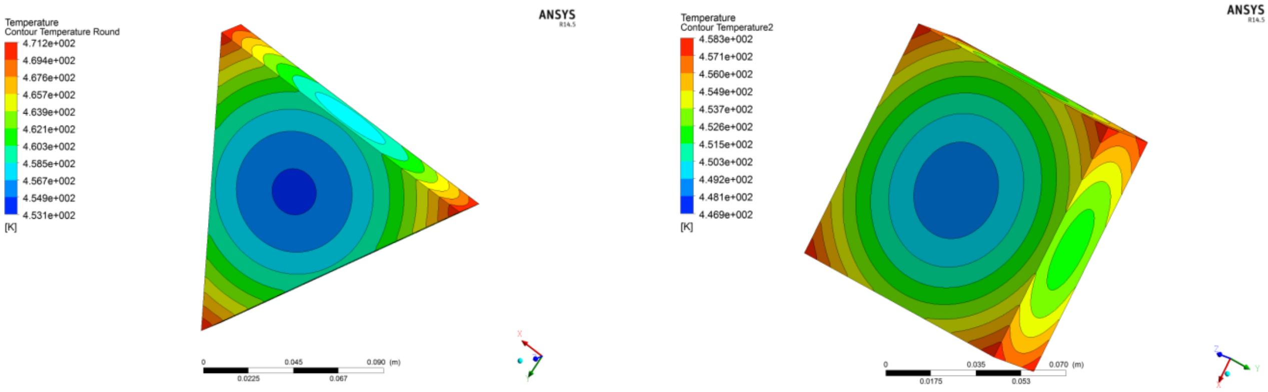


Figure 11 temperature contour

Heat concentrate on the corner of pan heat distributed uneven on the edge of pan, However heat distributed uniform on the inner area of pan. That leads to brownie overcooked on the corner of pan and poor-cooked on the central area. All of these lead to Energy waste. So temperature field optimize is necessary in this condition.

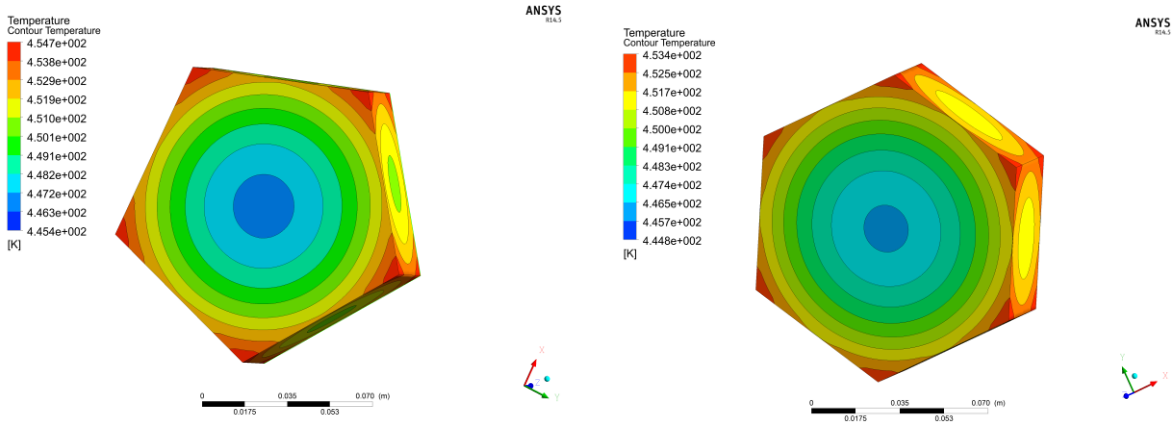


Figure 12 temperature contour

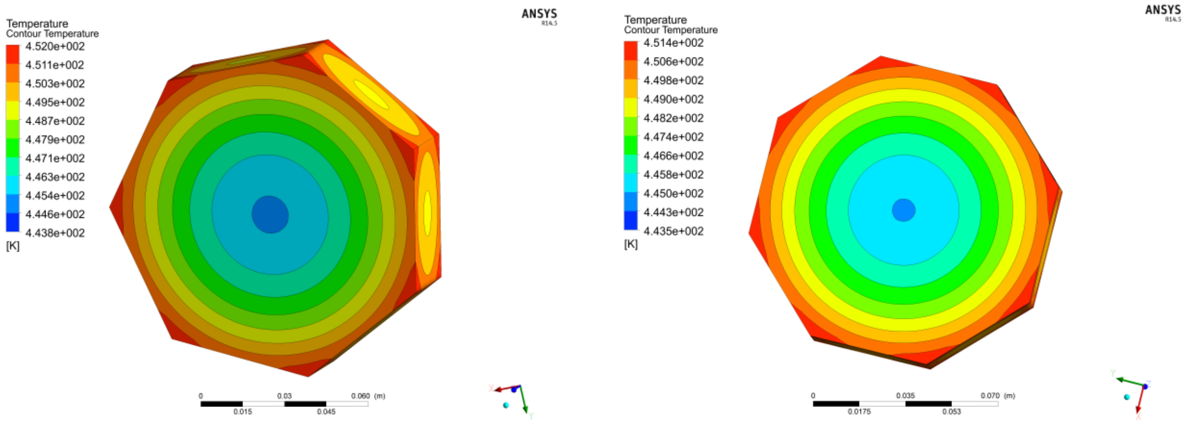


Figure 13 temperature contour

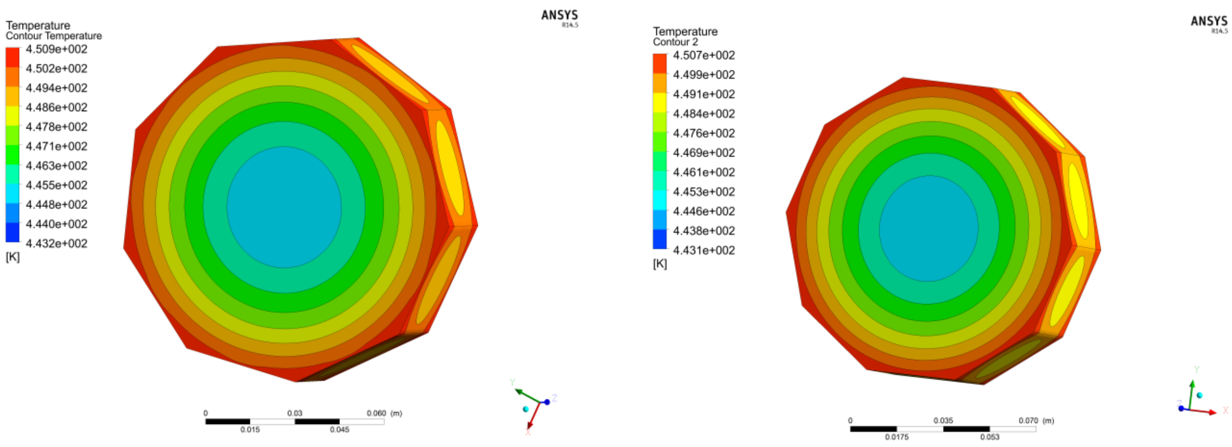


Figure 14 temperature contour

Analysis temperature field on the polygon we can get the qualitative result: The temperature distributed more and more uniform from triangle pan to decagon pan, and average temperature also more and more low. In another word the temperature distribution of high edge amount polygon is better than low edge amount polygon.

The temperature distribution of cylinder is quite better, with low average temperature and uniform boundary temperature distribution.

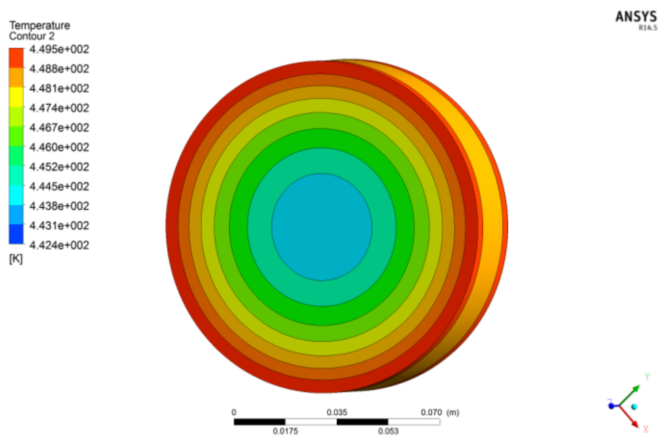


Figure 15 temperature contour

We get an interesting result: heat more like concentrate on the High Geometry Curvature area.

Draw boundary temperature distribution curve to compare boundary temperature trend between different pan shapes:

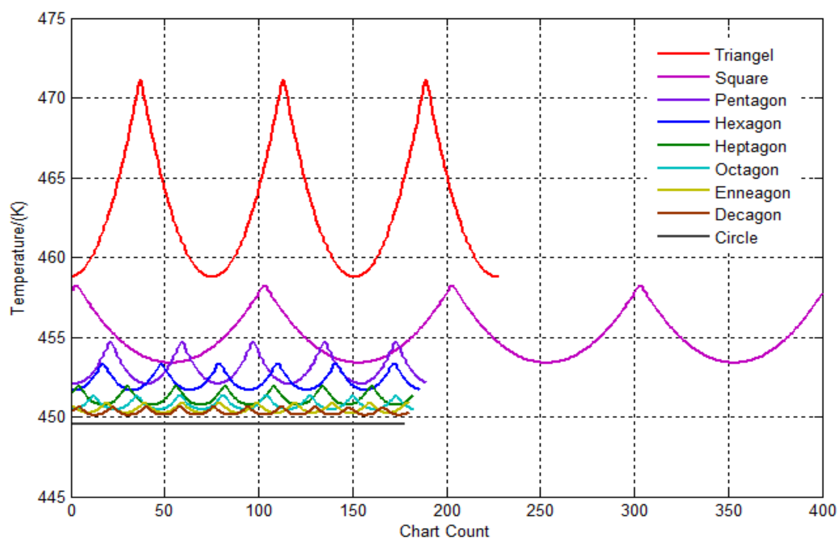


Figure 16 temperature distribution curve

The boundary temperature distribution curve reflected the trend which has been observed on the temperature contour plot.

Variance is a useful standard to evaluate temperature fluctuation, the figure below shown the relationship between temperature variance and edge amount of polygons. The temperature variance is attenuation while n increase. And this Feature like exponential attenuation pattern. In order to making optimization on the problem two more easily regress analysis is necessary. The figure below shows the regress result:

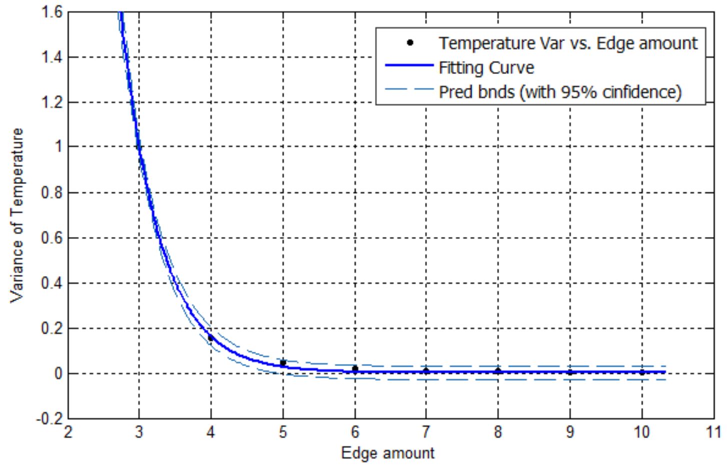


Figure 17 regress result

Regress result is displayed by these blanks below:

Table 1 regress result

Regress model	$f(x) = ae^{bx} + \varepsilon_i$	
Coefficients (with 95% confidence bounds)	a = 237.5	(110.6, 364.4)
	b = -1.824	(-2, -1.647)

Table 2 regress result

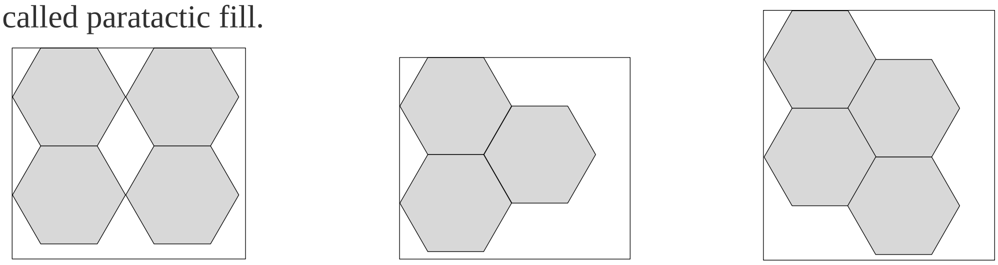
SSE	8.7738e-04
R-Square	0.9989
DFE	6
AdjRsquare	0.9988
RMSE	0.0121

V. Pan's Tiling model

5.1 The Foundation of Pan's Tiling Model

In order to maximize the space occupancy rate F of different splicing polygons, we use MATLAB to make a program. Letting edge number n be unchanged, when W/L changes, the corresponding optimal space occupancy rate and number of pans can be gained. Therefore the function $F(W/L, n)$ can be obtained.

Taking hexagon for example to explain the program flow process. Assuming the area of the hexagon is A, and the area of the oven is S, edge length of the hexagon is v, and the maximum number of pans is defined by Result. The paper only consider two regular arrangements shown in figure 18 (a) and (b). (a) is called scattering fill, (b) is called paratactic fill.



(a) Scattering fill con1 (b) Paratactic fill (c) Scattering fill con2

Figure 18 two regular arrangements

As the width to length ratio W/L can be changed, there are two conditions of scattering fill as figure (a) and (c), where con1 means condition one, while con2 means condition two. Figure (a) is called mode one, figure (c) is called mode two. The group of three hexagon is named as one group. When scattering fill is mode two, scattering fill rate is higher than that of paratactic fill, so we only consider scattering fill. However, when scattering fill is mode one, we should to consider whether transfer scattering fill to paratactic fill or not. Figure 19 is two different methods to fill the racks with hexagonal pans. Calculating the max number of groups in X direction, the result is represented by the symbol *num_group*. At the same time, calculating the max number of hexagon in Y direction, the result is called *n*.

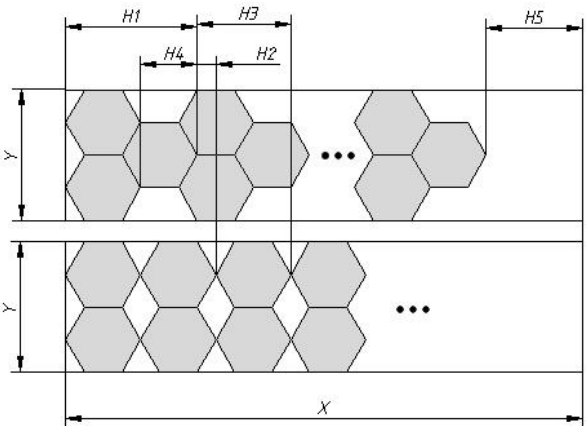


Figure 19 two different methods filling the racks

To find out the maximum space occupancy rate *F* of different splicing polygons, the progress should satisfy the following conditions:

1) $X \geq H1 + H3 \cdot (num\_group - 1)$
2) If H5 > H4, adding *n* number of hexagons in a column, then updating by H5=H5-H4.
3) If H5>K*H2, rearranging min(num_group,K) groups of scattering fill to paratactic fill. That is, adding a hexagon.

Then we can define the fill rate: $\text{Fill rate} = \text{Result} \cdot A / S$, and the program flow chart will help us on an easy understanding of the program.

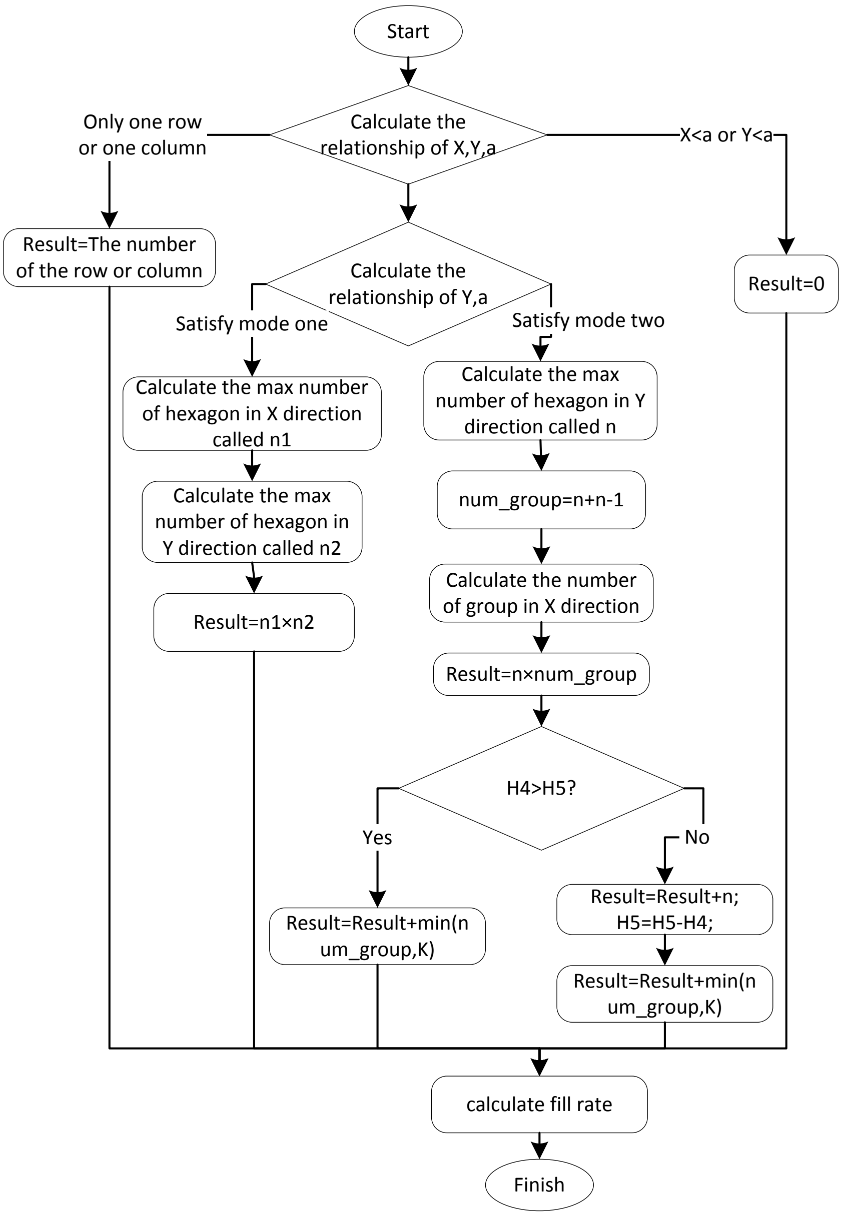
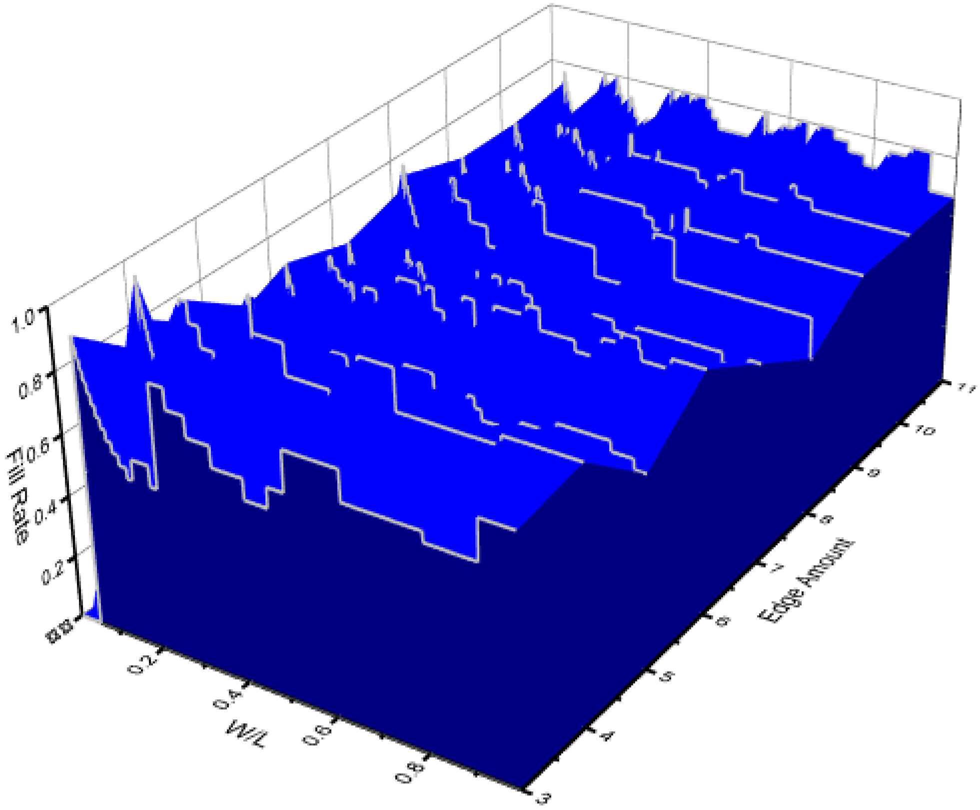


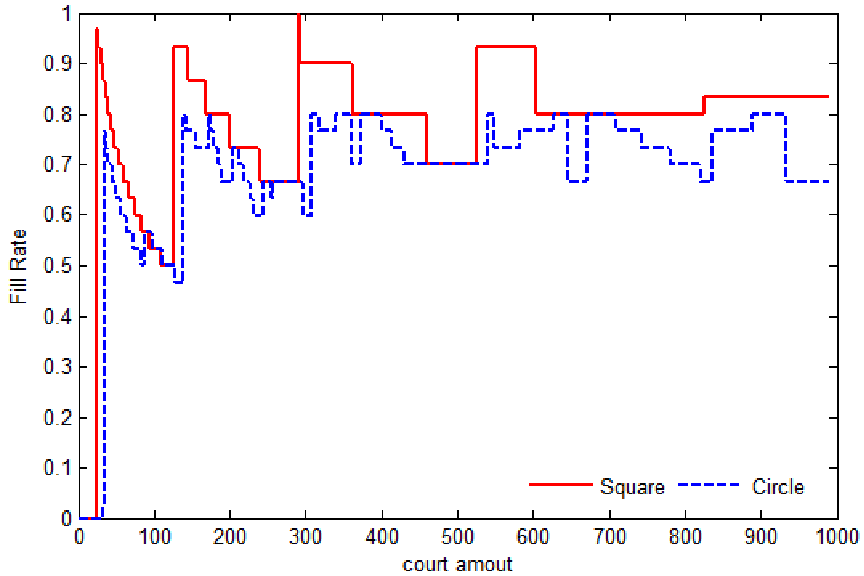
Figure 20 program flow chart

5.2 Result and Analysis of the problem

The Problem is a discrete problem so the result is discrete result is also a discrete result and display the result in 3D graph can describe the relationship between Fill Rate and the ratio of Weight and length and Edge amount.



The result tell the trend Fill Rate increase while the Edge amount of polygon is decrease. To show the trend we plot triangle pan's fill rate and circle pan's fill rate:



The Fill Rate of square is higher than the fill rate of circle because of square can make full use of rooms in the rack. However circle pan has a uniform temperature distribution. The Fill rate is change while court amount which refer to the ratio of width and length on the rack is change.

The result data of this problem will be used to solve the next problem: Optimization Pan's shape with difference ratio of width and length and difference weight p.

VI. Discrete Optimization model

5.1 The Foundation of Discrete Optimization Model

Optimize a combination of the above two conditions where weights *p* and (*1-p*) are assigned to illustrate how the results vary with different values of the width to length ratio of *W/L* for the oven and *p*. As the first two metrics are the space occupancy rate *F* of pans and the non-uniform distribution coefficient *I* of temperature respectively, we can construct an objective function to optimize the combination of the above two conditions in terms of *F* and *I,* where *n* is the decision variable. Finally, it is easy for us to illustrate how the results vary with different values of the width to length ratio of *W/L* for the oven and *p.*

5.2 Solution to the Problem

This discrete optimize problem can be describe in the standard mathematic language:

$$\min \quad f = P \cdot F\left(\frac{W}{L}, n\right) + \left(1-P\right)\left(I\right)$$

$$s.t$$

$$3 \le n \le 11$$

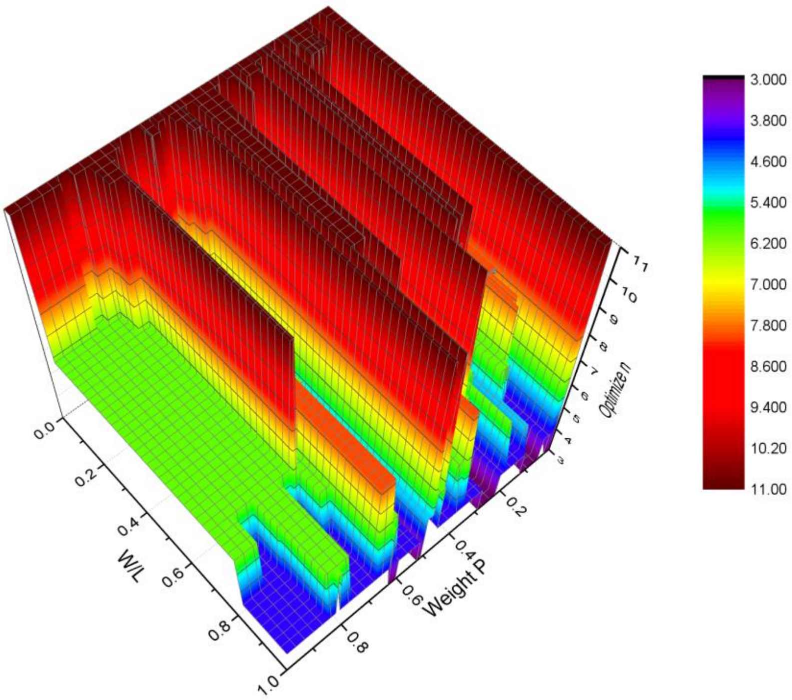
$$n \in Z$$

P is the weight of maximize pan's amount and maximize uniform of temperature distribution. Function F is discrete function. In another word data structure is a matrix with 9 rows and 991 columns. Function $I\left(n\right)$ is a continuity function with the expression $I\left(n\right) = a \cdot e^{-bn}$ and these two const a and b was given in the Transient heat transfer model of baking Brownie.

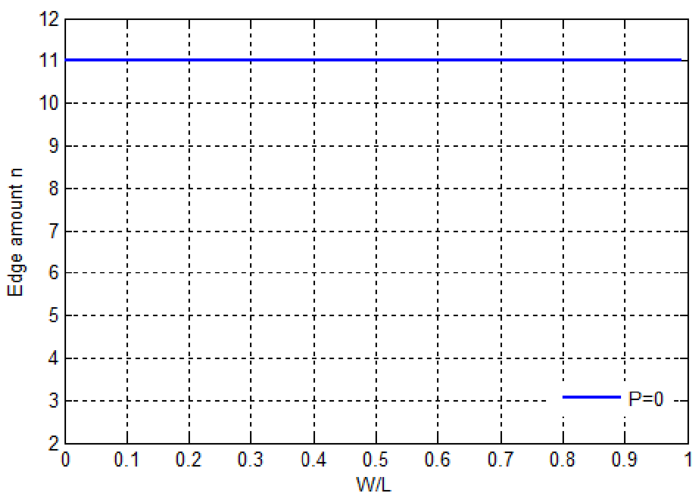
5.3 Result and Analysis of the problem

The Result of this problem is the optimization n while parameter $\frac{W}{L}$ and parameter

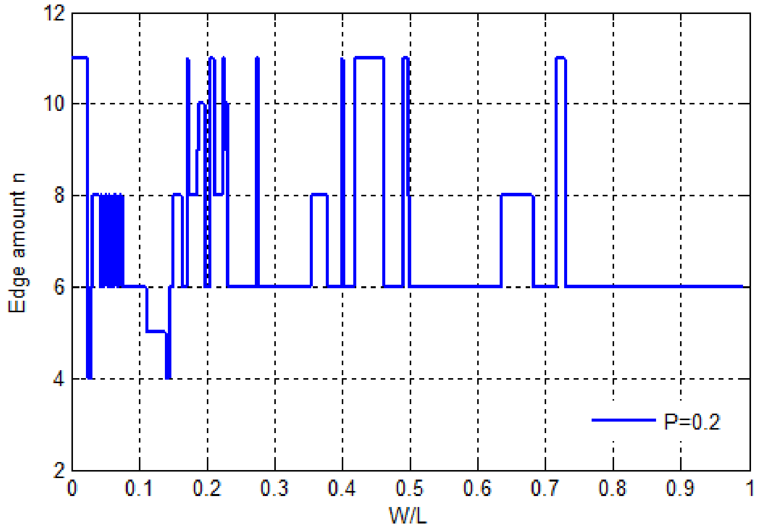
P changed. Because of the step size of $\frac{W}{L}$ is 0.01 and amount of step is 991 and the step size of P is 0.025 and amount of step is 41,so the result is a discrete function: $n = f(\frac{W}{L}, P)$ and it data structure is a matrix with 40 rows and 991 columns. To illustrate the data in a direct way we plot it in a 3D graph below.



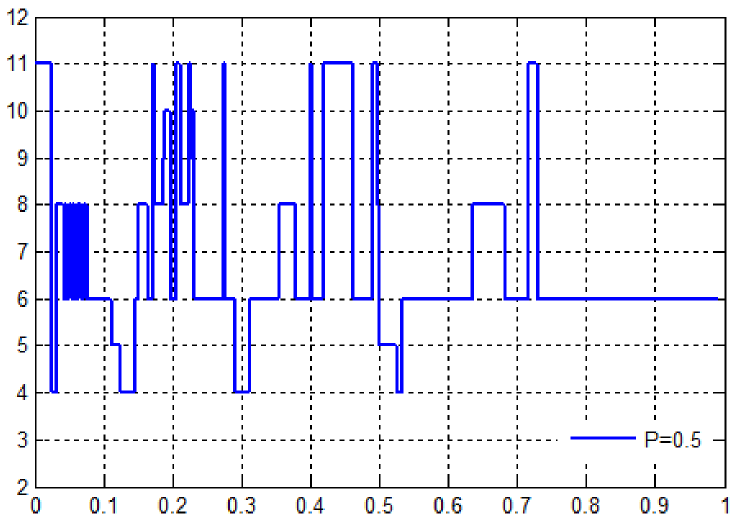
There is not too much laws in the result because of the discrete. However This 3D graph can alos explain some interesting phenomenon: When weigh P is low the Optimization shape most time is circle and when weight P is high the Optimization shape is square. To illustrate result more clearly then plot 2D graph while P is difference and the ratio of width and length is difference.



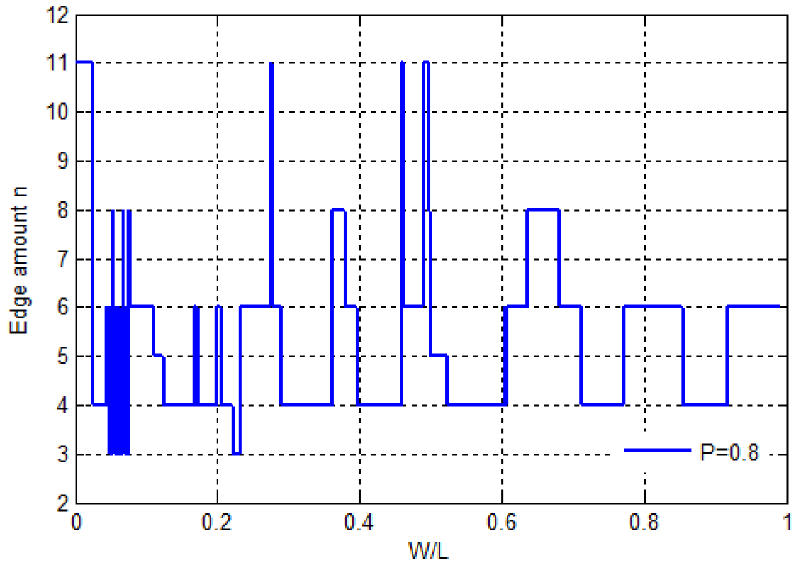
It's really an interesting phenomenon that when the weight p is zero the optimize shape is circle because at that condition we only considering the Temperature distribution on the boundary of pan, So naturally the best shape of pan is circle.



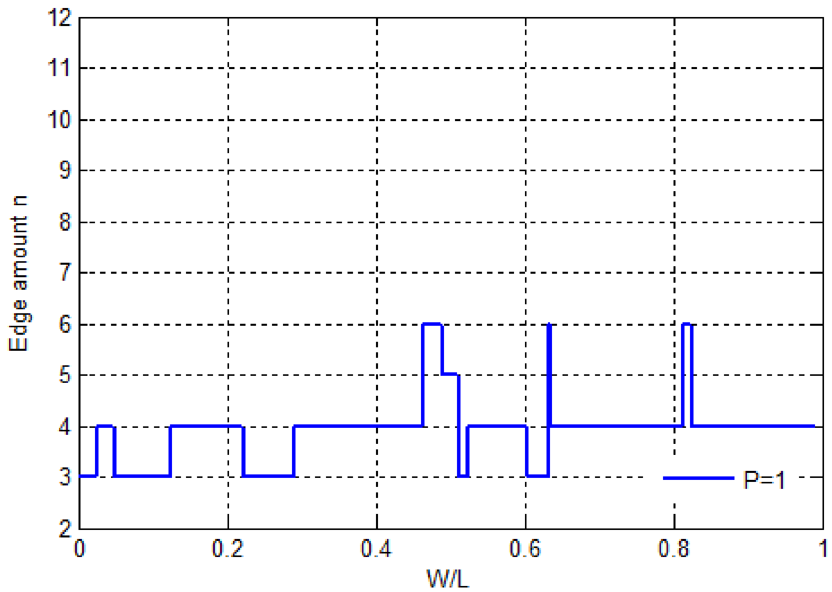
When P is greater than zero the situation become more complex best shape may vary because of difference ratio of width and length.



The weight of considering Pan's amount and Temperature distribution is the same when p=0.5 and the result is also very complex difference ratio of width and length leads to difference Optimization Pan's Shape. However this graph show all this trend in a clear way.

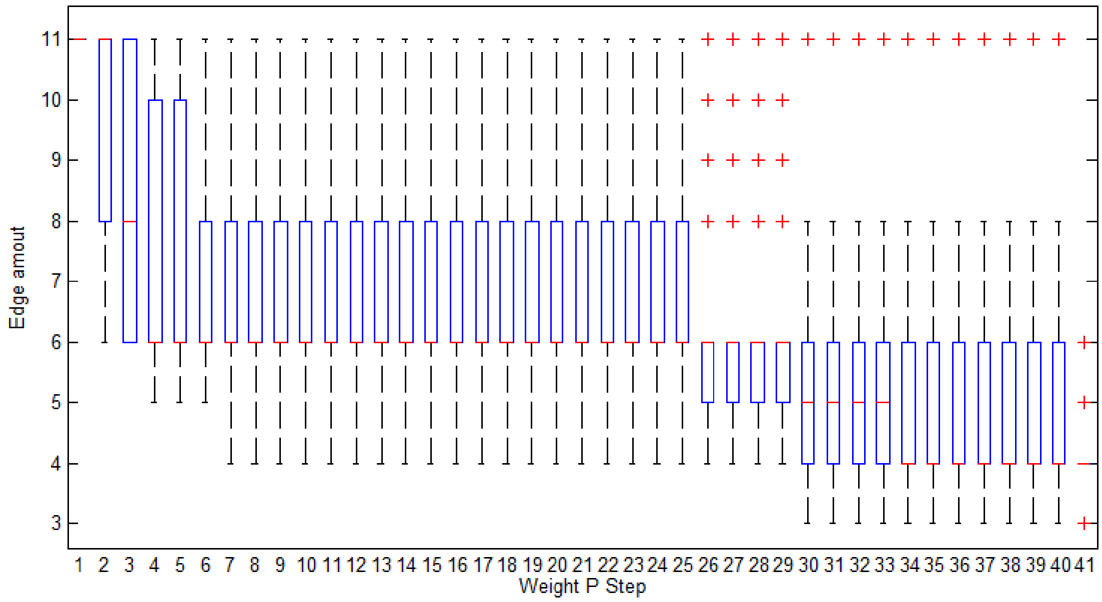


The situation When P=0.8 is difference with the situation when P=0.5.



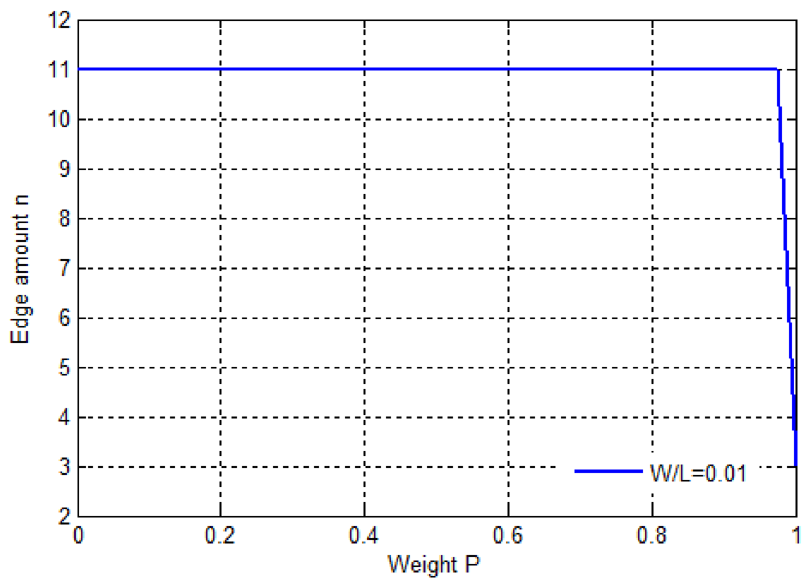
The situation that P=1 means we only considering the amount of pans, but the result also complex the best shape may be triangle square and hexagon. Yet most time the best shape was square.

A box plot can show the trend of result while we looked p as index and $\frac{W}{L}$ as sample.

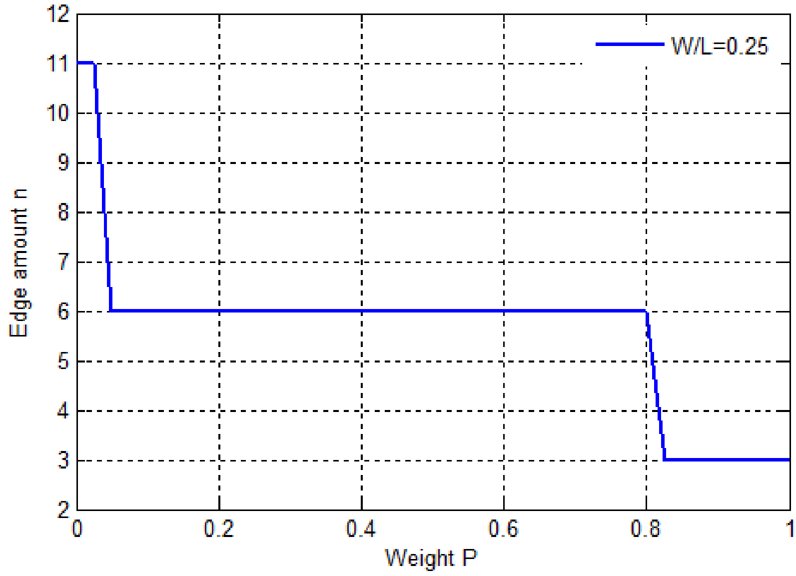


When weight p increase from zero to one in step size 0.025 the best shape is from circle to low Edge amount polygon in some step the data is centralization and in some step data is decentralization.

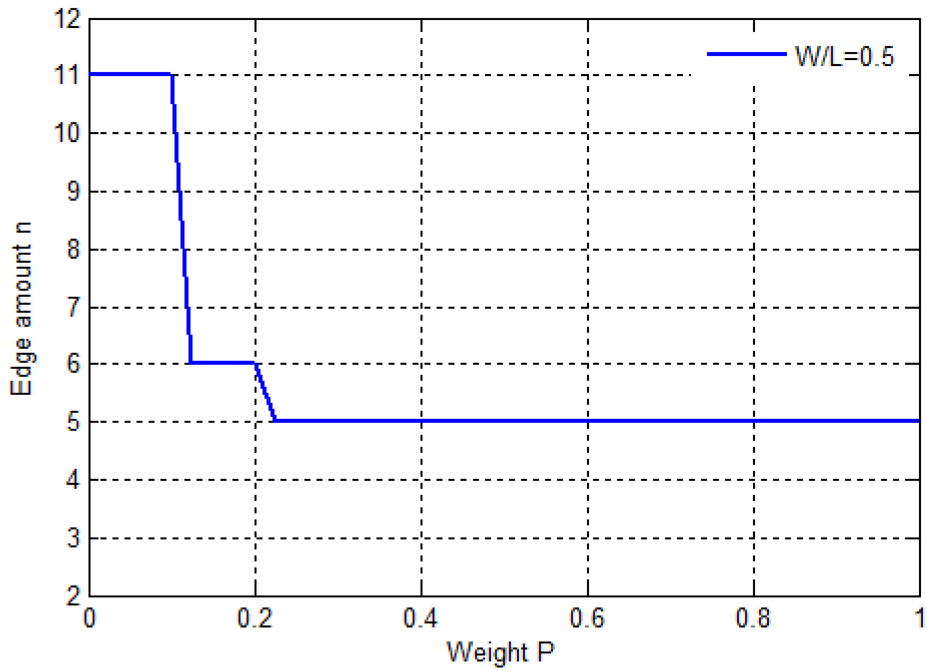
Next we compare Best shape while $\frac{W}{L}$ is changed



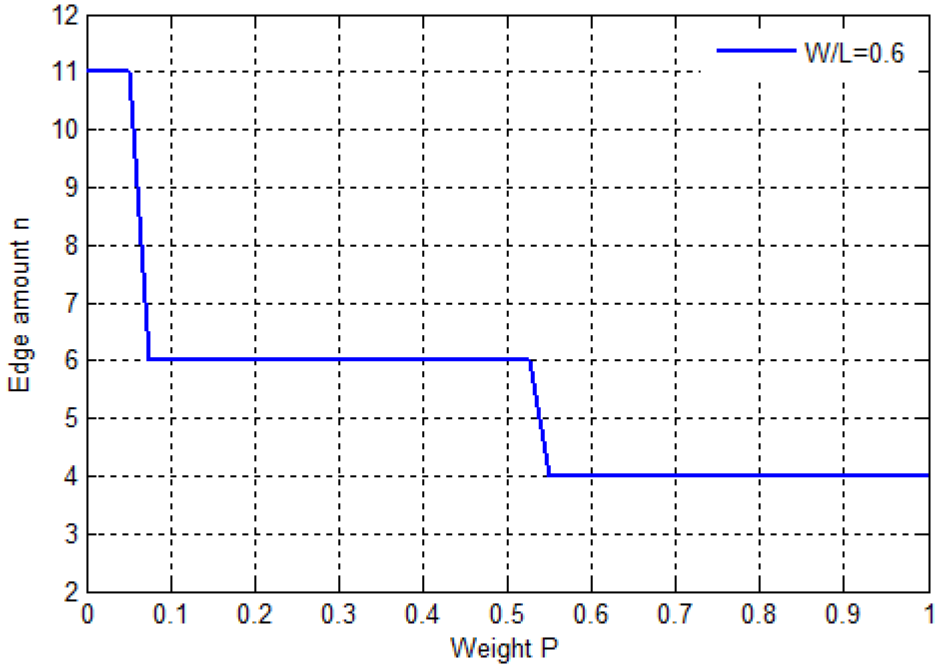
When the ratio is very small the best shape is circle in the most time.



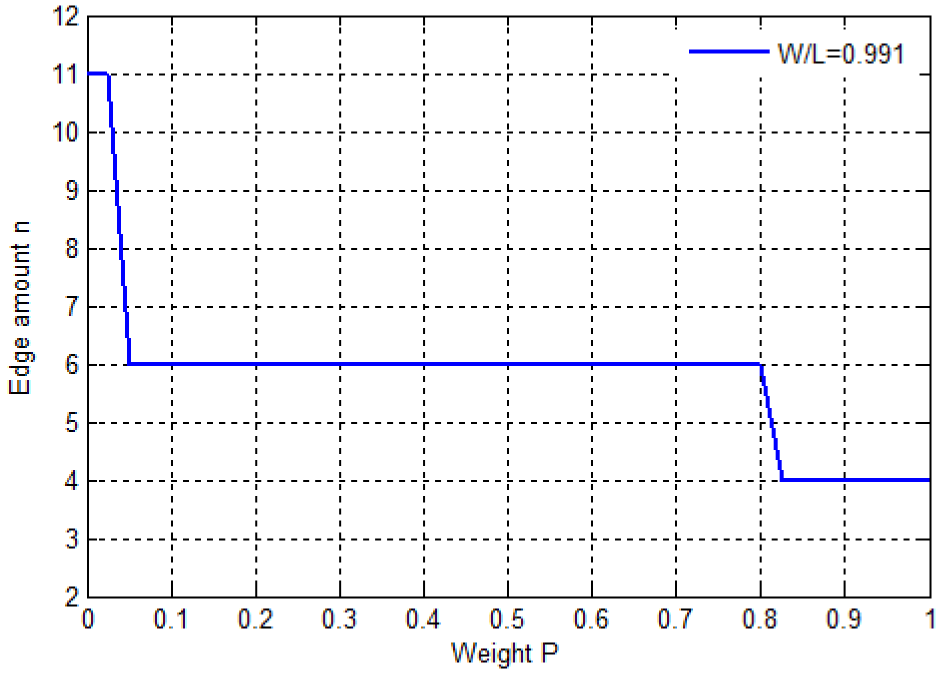
When the ratio become large the situation become complex, but in this situation ratio=0.25 the best shape in the most time is hexagon.



When ratio=0.5 the best shape in the most time is pentagon.

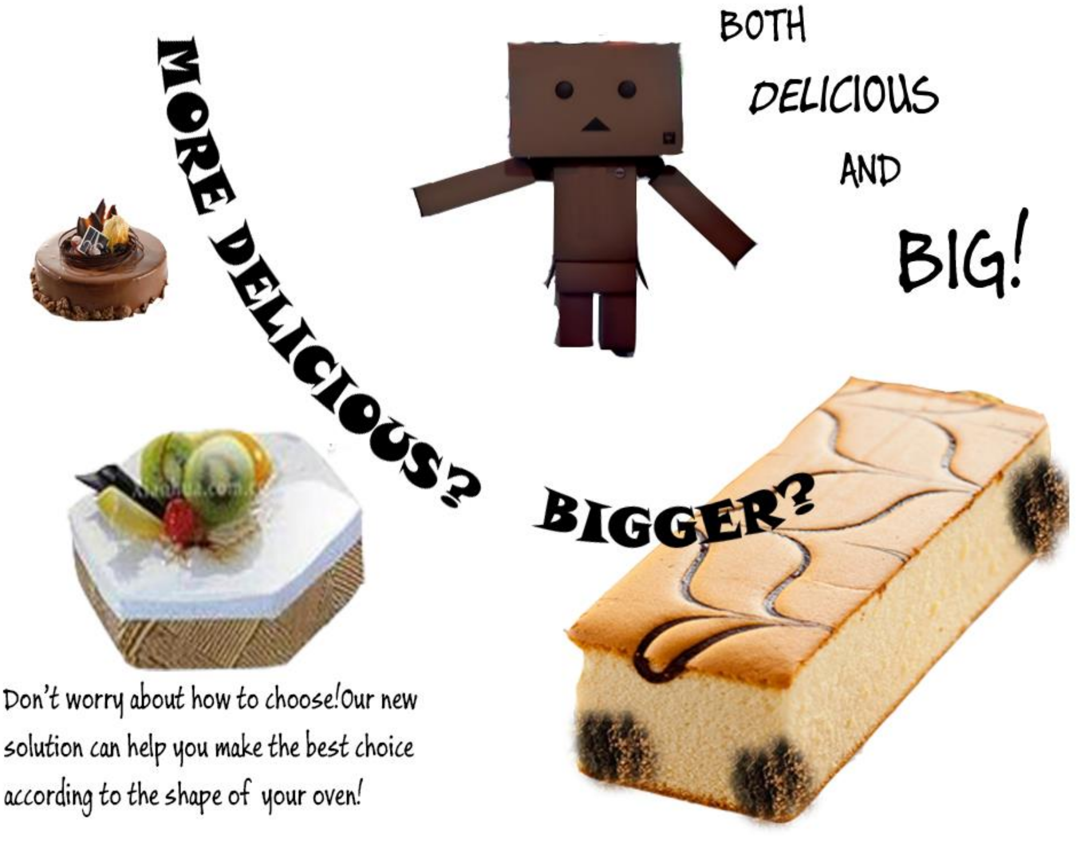


When ratio=0.6 the best shape in the most time is square and hexagon.



When ratio is pretty large the best shape in the most time is Hexagon.

In one word the ratio of width and length influence result in a complex way and it is a discrete relationship that can hard be describe easily. If we want to know the best shape we must specific parameters by design a good algorithm. The problem of decided the best pan's shape is a complex problem depends on many conditions


MORE DELICIOUS?
BIGGER?
BOTH
DELICIOUS
AND
BIG!
Don't worry about how to choose!Our new
solution can help you make the best choice
according to the shape of your oven!

References

[1] Dixon A G, Taskin M E, Nijemeisland M, et al. Systematic mesh development for 3D CFD simulation of fixed beds: Single sphere study [J]. Computers and Chemical Engineering, 2011, 35(7): 1171-1185.

[2] Saravanan S, Sivaraj C. Coupled thermal radiation and natural convection [J].International Journal of Heat and Fluid Flow, 2013, 40: 54–64.

[3] Wen L, Gao L, Li X Y, et al. Free pattern search for global optimization [J]. Applied Soft Computing, 2013, 13: 3853–3863.

[4] Turkyilmazoglua M, Pop I. Heat and mass transfer of unsteady natural convection flow of some nanofluids past a vertical infinite flat plate with radiation effect [J]. International Journal of Heat and Mass Transfer, 2013, 59: 167–171.