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RESEARCH PAPER

Review on Traffic Flow Phenomena and Theory

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Abstract: Aiming at solving severe traffic congestion problem in metropolitan, in the last several decades, a number of traffic flow phenomena were discovered on freeways all over the world. Recent researches on traffic flow focused on developing traffic modes and methodology which could explain and reproduce all these phenomena. In this paper, observed traffic phenomena such as traffic breakdown, hysteresis, stop-and-go traffic and synchronized flow are introduced. Then traffic flow models are classified by their equations of speed, acceleration and second-order acceleration in macroscopic/microscopic or continuous/discontinuous forms. Features of these traffic models and their explanation on traffic phenomena are also discussed.

Key Words: intelligent transportation; traffic flow model; traffic phenomenon; freeway; traffic flow theory

1 Introduction

Recent developments on the transportation system mainly focus on possible explanations for the traffic phenomena observed on freeways. Since the first parabolic flow-density relation proposed by Greenshield in 1936[1], many other phenomena have been observed since the past several decades, such as traffic breakdown, hysteresis effect[2], and widely scattered traffic flow data on the flow-density plane[3,4]. Traffic breakdown and hysteresis effect can be explained as being the results of drivers' different behaviors in different traffic states[5–7] or of vehicles changing between different types of lanes[8]. The widely scattered states on the flow-density plane can be categorized as uncertain driving behavior, an unstable traffic state[9], different types of drivers[10], or the drivers' acceptance of a range of speed with a unique gap[11]. Apart from the static traffic features just mentioned, research conducted on a series of adjacent detection sections shows complex and dynamic traffic patterns. Patterns such as stop-and-go traffic, wide moving jam, and upstream propagation of a synchronized region are widely accepted as common traffic features.

Stop-and-go traffic is usually regarded as a result of the amplification during the propagation of small downstream perturbations[2], superposition of multiple downstream perturbations[8,12], or spontaneous slowdown within the pinch region at bottleneck upstream[13]. Perturbations in the stop-and-go traffic may merge with the wide moving jam during propagation, which is viewed as an independent phase in Kerner's three-phase traffic theory. Synchronized flow and wide moving jam are often observed at bottleneck upstream, which is related to the capacity drop of the bottleneck. A significant feature observed in synchronized flow is the speed synchronization among lanes[14–16]. Such a balance is mainly caused by lane changes that are triggered by speed differences among lanes. Recently, indepth investigations conducted on the dynamic traffic bottleneck features depict more traffic phenomena such as bottleneck capacity drop[17], pinch effect at bottleneck upstream, short time headway at on-ramp downstream[18], moving bottleneck[19,20], and so on. These traffic phenomena are believed to be associated with bottleneck vehicle slowdown.

The urban freeway traffic has received a lot of attention that is attributed to it being a particular kind of freeway. Recent research conducted on the urban freeway shows different phenomena compared with those on the highways because of the freeway's low speed limit and dense ramps. Such that a new phase named coherent-moving phase proposed in Ref. [14]–[16] appears on the flow-density plane.

Based on the explanation on traffic phenomena, researchers proposed various types of traffic flow models. The first macroscopic traffic flow model is the LWR model proposed by Lighthill, Whitham, and Richard[21,22] in 1955. Correspondingly, early microscopic models[23,24], which are

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also called following-the-leader models, were developed in the 1960s. In the 1980s, another type of microscopic model, cellular automata (CA) model[7,25], was developed. Different from deterministic macroscopic models and following-the-leader models, stochastic driving characteristics are depicted well in CA models. The LWR models and the LWR-extended models[8,26] are able to reproduce some traffic phenomena such as traffic waves and breakdown, but display a lack for the mechanics of producing stop-and-go traffic. To overcome this problem, in some high-order macroscopic models[27-31], pressure terms and relaxation terms are added to the dynamic velocity equation. As a result, the high-order macroscopic models as well as most of the other microscopic models[32] can reproduce stop-and-go traffic. Another well-known phenomenon, the existence of synchronized flow, also has various possible explanations. Driving errors, different driving patterns, or the drivers' acceptance of various speeds at the same gap are all reasonable assumptions for the formation of synchronized flow[38–42]. Apart from the fact that all these general traffic phenomena exist along a freeway, traffic bottleneck features are simulated through a certain mechanism[19, 26,32-37].

Although current traffic models can explain almost all the traffic phenomena observed, there is no unique and well-accepted traffic flow theory[43,44].

2 Conservation of traffic flow

The traffic flow model generally consists of one flow conservation equation and one or more dynamic variable equations. The macroscopic and microscopic models are different in terms of their system equations.

If there is no vehicle entering or leaving along a road, then the flow conservation equation can be written in the following macroscopic form:

$$\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=\varphi(x,t) \qquad (1),$$

where $\rho(x,\ t)$, $q(x,\ t)$ and $\varphi(x,\ t)$are density, flow rate, and lane-changing rate, respectively, at a space-time point $(x,\ t)$. The flow conservation equation Eq. (1) can be written identically in the following microscopic form:

$$\frac{\partial s(l,n,t)}{\partial t}=v(l,t)-v(n,t) \qquad (2)$$

where l and n are vehicle ids. Vehicle ids upstream are larger than the downstream ones. Let $x(l,\ t)$ and $x(n,\ t)$ denote the positions of vehicle l and vehicle n at time t; the $s(l,n,t)=x(l,t)-x(n,t)$ is the distance between vehicle l and vehicle n. Equation (2) is an approximation of

$$\frac{s(l,n,t+\Delta t)-s(l,n,t)}{\Delta t}=v(l,t)-v(n,t) \qquad (3)$$

When $\Delta t\to 0$.

Let vehicle $l=L(n,t)$ be the leading vehicle of vehicle n in Eq. (3); then, the space gap of vehicle n can be written by $g(n,t)=s(l,n,t)$. If there is no lane change, then l will be the leading vehicle of vehicle n at time $t+\Delta t$, that is, $g(n,t+\Delta t)=s(l,n,t+\Delta t)$. If a lane change occurs, then the leading vehicle of vehicle n at time $t+\Delta t$ can be denoted as $l'=L(n,t+\Delta t)$. Then, $s(l,n,t+\Delta t)$ is written by

$$\begin{aligned}s(l,n,t+\Delta t)&=s(l',n,t+\Delta t)+s(l,l',t+\Delta t)\\&=g(n,t+\Delta t)+s(l,l',t+\Delta t)\end{aligned} \qquad (4)$$

Substituting (4) into (3), we get

$$\frac{g(n,t+\Delta t)-g(n,t)}{\Delta t}+\frac{s(l,l',t+\Delta t)}{\Delta t}=v(l,t)-v(n,t) \qquad (5)$$

When $\Delta t\to 0$, then Eq. (5) is approximated by

$$\frac{\partial g(n,t)}{\partial t}+\frac{1}{n-l}\frac{\partial v(n,t)}{\partial n}=\delta(n,t) \qquad (6)$$

where $\delta(n,t)=-\lim_{\Delta t\to 0}(s(l,l',t+\Delta t)/\Delta t)$, $l=L(n,t)$, $l'=L(n,t+\Delta t)$. If $L(n,t)\equiv n-1$, Eq. (6) can be written in a concise form:

$$\frac{\partial g(n,t)}{\partial t}+\frac{\partial v(n,t)}{\partial n}=\delta(n,t) \qquad (7)$$

According to Eq. (6) and (7), it is obvious that $l'=l$ and $\delta=0$ when there is no lane change; whereas $l'>l$ and $\delta<0$ if a vehicle enters between l and n at time $t+\Delta t$. Similarly, there are $l'<l$ and $\delta>0$ if a vehicle l leaves the lane.

We can denote $v(n,t)$, $g(n,t)$, and $\delta(n,t)$ as v_n, g_n, and δ_n, respectively; $v(x,t)$, $\rho(x,t)$, and $\varphi(x,t)$ as v, ρ and φ, respectively. Eqs. (6) and (1), are identical with the micro-macroscopic link $\rho=1/g_n$, $v=v_n=\partial x(n,t)/\partial t$, and $\rho\leftrightarrow-(n-L(n,t))\partial n(x,t)/\partial x$.

3 Dynamic variable equations

Besides the conservation equation, one or more dynamic variable equations are also necessary in system equations. Most models contain a dynamic equation that relates to the evolution of velocity, acceleration, or second-order acceleration with time. Sometimes, additional dynamic variable equations are also developed as a supplement to the dynamic velocity equation.

3.1 Density-dependent velocity equation

In the LWR model, velocity is a function of ρ, such that the LWR model can be solved by

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho V_e(\rho)}{\partial x}=\varphi \qquad (8)$$

The LWR model is the first macroscopic model that reproduces traffic breakdown, but it fails to reproduce stop-and-go traffic. For a more realistic description of traffic breakdown, some models introduce a viscosity term that avoids the appearance of shock waves[27–30].

The LWR model can be extended to models where velocity is defined as a function of density and other traffic parameters. In some inhomogeneous models, velocity is defined as $v=V_\rho(\rho,$

α) where α is an inhomogeneous factor only dependent on road structure[45,46]. Since $\alpha$is an independent variable in the system, a dynamic equation for α should be provided:

$$\frac{\partial \alpha}{\partial t} = 0 \tag{9}$$

In ref. [47], a lane-changing intensity$\varepsilon$is introduced in the definition of velocity $v=V_\rho(\rho, \varepsilon)$. Here, ε is also a time-independent variable that

$$\frac{\partial \varepsilon}{\partial t} = 0 \tag{10}$$

In the model proposed in Ref. [48], drivers are classified into a different class I. In each class I, the vehicle velocity is obtained by $v=V_e(\rho, I)$. Since I is a driver-dependent variable that does not change with movement, the dynamic equation for I is

$$\frac{\partial I}{\partial t} + v\frac{\partial I}{\partial x} = 0 \tag{11}$$

In the microscopic driver perception model[40], drivers are classified by a critical gap d_n which is also called perception factor. This model provides both a velocity equation $v = V_e^*(g_n, d_n)$ and a dynamic equation for d_n:

$$\frac{\partial d_n}{\partial t} = \frac{\partial v_n}{\partial n}\lambda(v_n, g_n) \tag{12}$$

where λ is a deterministic function with v_n and g_n.

Models with a velocity equation dependent on more than one parameters give two-dimensional steady state on the flow-density plane. This is also a possible explanation for the formation of synchronized flow.

3.2 Definition of acceleration

To reproduce stop-and-go traffic, a typical method involves specifying the acceleration function instead of the velocity function as a system equation.

In microscopic models, acceleration a is obtained by $a(n,t) = \partial v(n,t)/\partial t$. Since

$$\frac{\partial v(n,t)}{\partial t} = \frac{\partial v(x,t)}{\partial t} + v(x,t)\frac{\partial v(x,t)}{\partial x} \tag{13}$$

the acceleration in macroscopic models can be expressed as $a(x,t) = \partial v/\partial t + v\partial v/\partial x$.

In early microscopic traffic flow models[49], acceleration is defined as a function of velocity difference Δv_n:

$$\frac{\partial v_n}{\partial t} = \frac{\Delta v_n}{\tau} \tag{14}$$

These type of models can reproduce traffic waves and thus prevent collision. However, they lack the acceleration mechanism in sparse traffic. To solve this problem, Bando proposes the optimal velocity model (OVM) [32], where acceleration is calculated by

$$\frac{\partial v_n}{\partial t} = \frac{V_e^*(g_n) - v_n}{\tau} \tag{15}$$

However, it is often assumed that the velocity differenceΔv_n has a significant impact on driving behavior. In IDM (intelligent driver model)[36], acceleration is denoted as a function of v_n, Δv_n, and g_n:

$$\frac{\partial v_n}{\partial t} = a_{\max}\left[1 - \left(\frac{v_n}{v_{\max}}\right)^b - \left(\frac{g^*(v_n, \Delta v_n)}{g_n}\right)^2\right] \tag{16}$$

where $a_{\max}$ is the maximum acceleration and $g^*>0$ the “effective desired distance” related to the velocity difference Δv_n. g^* reflects the drivers’ anticipation of traffic situations ahead. Moreover, g^* with $\Delta v_n>0$ is less than g^* with $\Delta v_n<0$.

Microscopic models define the relationships among vehicle acceleration $\partial v_n/\partial t$, velocity v_n, velocity difference Δv_n, and gap g_n. Hence, various driving behaviors, such as keeping a safe space gap, traffic situation anticipation, drivers’ over-reactions, and limited acceleration/deceleration abilities, are simulated[50–53].

To conclude, the acceleration functions in Eqs. (14), (15), and (16) in the microscopic models can be written in a general form:

$$\frac{\partial v_n}{\partial t} = A(g_n, v_n, \Delta v_n) \tag{17}$$

In many macroscopic models[27-31], acceleration is written in the following form:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{D}{\rho}\frac{\partial \rho}{\partial x} + A\frac{\partial v}{\partial x} + \nu\frac{\partial^2 v}{\partial x^2} + \frac{1}{\tau}\left(v - V_e(\rho)\right) \tag{18}$$

The first two terms used on the right side in Eq. (18) are anticipation terms, indicating the drivers’ anticipation to the changes in the density and velocity ahead. Coefficient D compares to the diffusion coefficient while ν the viscosity coefficient in fluid dynamics. The corresponding diffusion term and viscosity term on the right side in Eq. (18) are used to avoid the appearance of shock waves in model solutions. The coefficient A relates to driving anticipation. The last term is the relaxation term, and τ is the relaxation time, indicating that the velocity will change according to the equilibrium velocity $V_e(\rho)$ within time τ. Compared with the LWR models and the LWR-extended models, this type of model can reproduce stop-and-go traffic.

3.3 Definition of velocity in the next time step

In discrete time models such as cellular automata models, velocity at the next time step $t + \Delta t$ is defined as follows:

$$v_n(t + \Delta t) = V(v_n, v_{n-1}, g_n) \tag{19}$$

where a vehicle with id $n-1$ is the preceding vehicle of vehicle n.

The function $V(v_n, v_{n-1}, g_n)$ has the following general form:

$$V(v_n, v_{n-1}, g_n) = \max\left[0, \min\left(v_{\max}, v_{safe}, v_{des}\right)\right] \tag{20}$$

In the NS model (Nagel–Schreckenberg model), the definition for v_{safe} and v_{des} is

$$v_{safe} = \frac{g_n}{\Delta t} - \frac{\Delta x}{\Delta t} - \xi_n^{(p)}, \quad v_{des} = v_n + \frac{\Delta x}{\Delta t} - \xi_n^{(p)} \tag{21}$$

where $\xi_n^{(p)}$ is a stochastic number that equals 1 with probability p and 0 with probability $1-p$; p is also called *slowdown probability*. $v_{\max}$ is the allowed maximum velocity, v_{safe} is the safe velocity related to the current traffic situation, and v_{des} is the drivers' desired velocity at the next time step. According to Eq. (21), both the velocity and acceleration are discontinuous in the NS model.

Various types of slowdown probability are used in other CA models. In some models, p at v_n=0 is either larger than that that at v_n>0[54], or lower than that at a lower density[55]. Based on the NS model, continuous-time and -space CA models are also developed.

The CA models, based on Kerner's three-phase traffic theory, develop a new mechanism that simulates the drivers' behavior in a synchronized flow[39,42,56]. In Ref. [42], when the space gap g_n is less than a synchronized gap $G(v_n)$, then the following update function is triggered:

$$v_n(t+\Delta t) = v_n + \frac{\Delta x}{\Delta t}\operatorname{sgn}(v_{n-1} - v_n) \tag{22}$$

Eq. (22) indicates that the change in velocity is indifferent with the space gap but is related to the velocity of the preceding vehicle in a synchronized flow. This rule satisfies the hypothesis of the two-dimensional steady states in the synchronized phase on the flow-density plane.

3.4 Definition of acceleration in the next time step

To investigate the impact of the drivers' delays on traffic flow, delay time Δt is introduced in some traffic models. In early car-following models[23], acceleration at time $t+\Delta t$ is

$$\frac{\partial v_n(t+\Delta t)}{\partial t} = \frac{\Delta v_n}{T_0}\frac{[v_n(t+\Delta t)]^{m_1}}{g_n{}^{m_2}} \tag{23}$$

where m_1 and m_2 are suitable model parameters that are used to reproduce realistic model solutions. When Δt is large enough, then the traffic model in Eq. (23) can reproduce stop-and-go traffic and collisions. By discretizing Eq. (23) and rewriting it in a logistic form, a chaotic solution is discovered when the leading vehicle moves with a cycled velocity[57].

The delayed model in Eq. (23) does not consider the impact of space gap on velocity. An improved delayed model is proposed in Ref. [58]:

$$\frac{\partial v_n(t+\Delta t)}{\partial t} = \frac{\Delta v_n}{T_0}\frac{[v_n(t+\Delta t)]^{m_1}}{g_n^{m_2}} + b(g_n - g_{\min})^3 \tag{24}$$

where $g_{\min}$ is the minimum space gap.

Another different form of delayed function is written as

$$\frac{\partial v_n(t+\Delta t)}{\partial t} = B\left(1-\frac{v_nT+g_{\min}}{g_n}\right) - \frac{Z^2(-\Delta v_n)}{2(g_n-g_{\min})} - kZ(v_n - v_{\max}) + \eta \tag{25}$$

where T is the safe time gap, B and k are constants, and η is a random factor. $Z(x)=x$ when x>0, whereas $Z(x)=0$ when $x\le 0$. Solving Eq. (25), when Δt is large enough, a multifractal chaotic attractor is discovered.

3.5 Definition of higher-order acceleration

Another type of delayed model involves the definition of the second-order acceleration $\partial^2 v_n/\partial t^2$.

$$\frac{\partial^2 v_n}{\partial t^2} = \frac{1}{\tau_a}\left[A(v_n, g_n, \Delta v_n) - \frac{\partial v_n}{\partial t}\right] \tag{26}$$

where τ_a is the delay time compared with Eq. (24) and (25). Function A is the desired acceleration related to v_n, g_n, and Δv_n.

In Ref. [59], A is written in an OVM-like form:

$$A(v_n, g_n, \Delta v_n) = (V_e(g_n) - v_n)/\tau_b \tag{27}$$

where τ_b is constant delay time similar to τ_a. Eq. (27) also depicts chaotic solutions in congested flow.

In the ATD (acceleration time delay) model[6], based on the three-phase traffic theory, the second-order acceleration is written in the form of Eq. (26). However, in the ATD model, τ_a is different with regard to acceleration and deceleration, while the desired acceleration A has a different form with regard to free flow, synchronized flow, and traffic jam:

$$A(v_n, g_n, \Delta v_n) = (\min(\max(\tilde{A}^{(\text{phase})}, a_{\min}), a_{\max}, a_s) \tag{28}$$

where $\tilde{A}^{(\text{phase})}$ is the desired acceleration in the corresponding phase:

$$\begin{aligned} \tilde{A}^{(free)} &= c(V^{(\text{free})}(g_n) - v_n) + K(v_n, \Delta v_n)\Delta v_n \\ \tilde{A}^{(\text{syn})} &= c\min(V_{\max}^{(\text{syn})}(g_n) - v_n, 0) + K(v_n, \Delta v_n)\Delta v_n \\ \tilde{A}^{(\text{syn})} &= -K^{(\text{jam})}v_n \end{aligned} \tag{29}$$

In Eq. (28), the superscript phase indicates free flow (free), synchronized flow (syn), and traffic jam (jam). $a_{\min}$, $a_{\max}$ and a_s are the maximum, minimum, and safe acceleration, respectively. According to Eq. (29), two-dimensional steady states exist on the flow-density plane when $V_{\max}^{(\text{syn})}(g_n) \ge v_n$. The traffic phenomena discovered in the three-phase traffic theory can be reproduced by using the ATD model.

4 Conclusions

This article illustrates traffic phenomena and the reproducibility of models in their mathematical forms. Now, many traffic phenomena that are observed can be well explained. On the other hand, the same phenomena can be reproduced by different models based on different traffic flow theories. Hence, there are controversies with regard to many traffic problems, and there is no unique traffic theory in the traffic flow field. With the development of intelligent transportation and traffic measurement technology, it is expected that more traffic phenomena will be discovered and analyzed, thus resulting in new problems and challenges in the area of traffic flow research.

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