Energy

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1 Energy, What is it?

Energy can be understood in two different forms. Imagine holding a heavy weight at a height if accidentally you drop it, the weight can fall and *potentially* damage something.

The word potentially means that, as you hold the weight, there is a "potential" stored in the weight that can turn to an action. Such potential is an example of what *energy* feels like. There are other ways to store or embed energy that can be potentially turned to some sort of action. For example some explosive material. In this case *chemical energy* is behind what can turn to an explosion. Similarly the food you eat everyday has some chemical potential energy that turn to kinetic of our bodies after absorptions and transformations inside the body.

A different form of energy is called the *Kinetic* energy. This energy appears when some potential energies somewhere is turned to action. We say a potential energy is *transformed to Kineteic energy.*

Energies can transform to each other and this is called *energy transformation*. We can sum all the possible energies in a system: A moving car has different types of energies: Kinetic (since it is moving), chemical energy in the gas, if it is moving on top of a hill for example you could also say the car has some *gravitational energy* (which is a potential energy just like the weight example above).

The sum of all these energies in the car is called the *total energy of the system* and is denoted by E.

As a car is moving its chemical energies turn to some mechanical energies in the engine. Eventually, all these forms of energies are either turn into kinetic of the car or some *thermal energies* that are added to the atmosphere.

The Kinetic energy is always denoted by K. For potential energies we may use subscripts to indicate the type. For example for gravitational energy, the notation will be U_g, g standing for gravitational.

2 Work

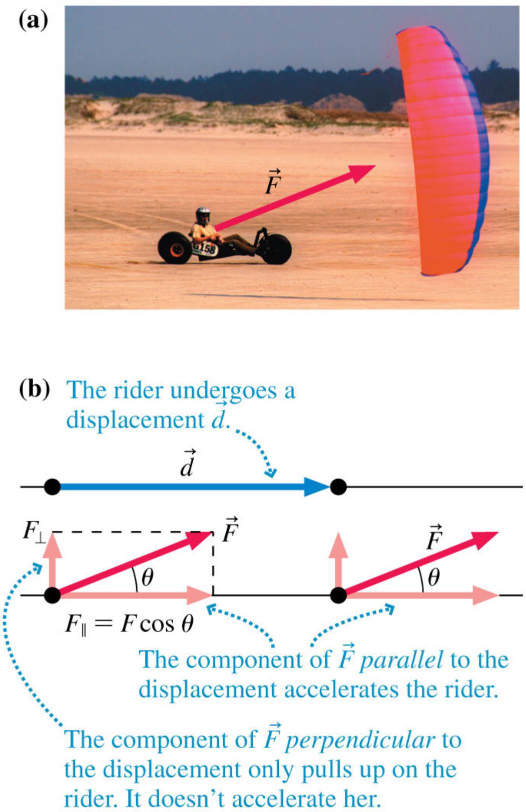
A closely related concept is *Work*. The Work we talk about here has a slightly different meaning that the daily life meaning of the word work. Below, we consider only simple cases in which, mostly, the force is constant.

A work done by a force F by definition $W = F\,d$, where d is the displacement of the force. What does this mean ? Assume there is a force F applied on a box and the box is also displaced by a value d.

Force as well as the displacement (see chapter one and two) are vectors. There is no reason for F (to be precise $\vec{F}$) to be in the same direction as the displacement d. So a more general definition of work is:

$$W = F_{||} d = F d \cos\theta,$$

$F_{||}$ means the component of force that is parallel to the direction of d.



Notice that here F and d are the magnitude of the vectors $\vec{F}$ and $\vec{d}$ and θ is the angle between the two vectors.

Then the work is the product of Fd. Notice the followings:

- If the $d = 0$ there is no work no matter ho much you push or pull (you may get tired but do some work you must move the box !)
- the force applied may not be the reason for the displacement: example if you are holding a heavy box and slowly putting it on the ground. In this case, intuitively, it is the gravity that pull the box down. You just slow it down. Yet, you have done some works... good job !
- if the direction of the force $\vec{F}$ is perpendicular to the direction of the displacement $\vec{d}$ the force F has done *no Work*! Even though the object is moved.

Question: You are holding a bag and walking.Are you doing any work? What if you are stepping up on a staircase ?

Question: if the bag your are holding is $1Kg$ and you walk for $1m$ horizontally, What is the work done by you? What if you go $1m$ up a ladder while holding the same bag? Assume the ladder has an angle of $60°$ with the ground.

If you push chair in the classroom for a distance d, the expectation is that the chair starts to speed up. It means your work (pushing the chair) must turn to some sort of kinematics for the chair. In fact this is true and is an example of Work transformed to *Kinetic Energy*.

3 Kinetic Energy

3.1 Linear

Kinetic energy is the energy you can assign to an object that is moving. Formally this is how we can assign a value to this kinematic energy...

You remember

$$v_f^2 = v_i^2 + 2a\Delta x \tag{1}$$

if I replace a with $\frac{F}{m}$ where m is the mass of the moving object,

$$v_f^2 = v_i^2 + 2\frac{F}{m}(\Delta x) \tag{2}$$

then you can recognize $F\Delta x$ as the work. Rearranging the equation:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = F(\Delta x) = W \tag{3}$$

Let's now assume $K = \frac{1}{2}mv^2$ to be the *(linear) kinetic energy* of an object that is moving with a velocity v and has a mass m, then:

$$K_f - K_i = W \tag{4}$$

That is the work transferred to the object causes a change in the Kinematic energy.

3.2 Rotational

What if the object is rotated as a result of some work being done on it? Consider a particle that rotates around a circle, with a radius r, by an applied force F parallel to the direction of the motion (tangent to the circle).

According to the definition of work, after a short travelled arc s the work is $W = F\, s$.

Ok now, separately, we know that

$$2\alpha\Delta\theta = \omega_f^2 - \omega_i^2 \tag{5}$$

replacing $\alpha = \frac{a}{r} = \frac{F}{mr}$ and $\Delta\theta = s/r$, we have

$$2\frac{F\, s}{m\, r^2} = \omega_f^2 - \omega_i^2 \tag{6}$$

That is

$$\begin{aligned} F\, s = W &= \frac{mr^2}{2}\omega_f^2 - \frac{mr^2}{2}\omega_i^2 \\ &= \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 \end{aligned} \tag{7}$$

Let's call $\frac{mr^2}{2}\omega^2$ to be the *rotational kinetic energy*, the work is then transferred to the object and its effect is to change the rotational kinetic energy.

4 Potential Energy

4.1 Gravitational

In the Kinetic energies, work was transferred to some sort of motions in the object. Sometimes you do a work, but the object is not having any motion at the end... then what happens to the work you did? Example is when you lift a heavy bag from the ground. You have done work since your force is parallel to the displacement of the object, in this case vertical. At the end however you just have a bag at rest in your hand above the ground.

The work you have done here is actually transferred. It is transferred to *a potential energy* (gravitational in nature).

Since to hold the bag you need to apply a force equal to the weight of the bag mg, where the mass is m, the amount of work you did is calculated as

$$W = mg\Delta y \tag{8}$$

where $\Delta y = y_f - y_i$. y_f and y_i are the final and initial heights of the object respectively: Basically the level you are currently holding the bag y_f and the ground level y_i. Notice the measurement of these values is with respect to a y-axis that you pre-define !

Now if we call $mgy = U$ the equation above is simply saying that your work is spared on changing the potential energy of the object:

$$W = mg\Delta y = U_f - U_i = \Delta U \tag{9}$$

4.2 Spring (elastic)

similarly you can show that the work you do to compress a a spring is calculated by

$$W = f\,\Delta x \tag{10}$$

This can be simplified by assuming an average value for the force as $f_{avg} = (f_f + f_i)/2 = \frac{k}{2}(x_f - x_i)$. k is the spring constant and x_i and x_f are the position of the spring before and after compression (again measured with respect to a coordinate that you choose in the problem).

Replacing thes in the equation above:

$$\begin{aligned} W &= \frac{1}{2}k(x_f + x_i)(x_f - x_i) \\ &= \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \\ &= U_f - U_i \end{aligned} \tag{11}$$

where we define $U = \frac{k}{2}x^2$ as the *potential energy of the spring*.

4.3 Summary:

Energies can be classified as Kinetic (K) and Potential (U). We saw some examples of both.

5 Total Energy, conservation of energy

Now that we learned about some examples of energy and their calculations, let's combine everything. For a generic system we can sum all the energies involved and call it the total energy of the system. Since the change in the energies are ultimately related to the work don on objects, the change in the total energy should be equal to the work done on the system.

$$\Delta E = \Delta K + \Delta U = W \tag{12}$$

5.1 Conservation of Energy

So if there is no work on a system The change in the total energy is zero, but the total energy is sum of kinetic and potential. Either these two are separately do not change, or if one changed the other must be changed an amount that is negative in value in order to ensure the total ensergy is not changed:

$$\Delta E = 0 = \Delta K + \Delta U = 0 \tag{13}$$

Thus at any moment for a system that is *isolated*:

$$K_i + U_i = K_f + U_f \tag{14}$$

To make this equation more elegant we could also take into account the change in the *thermal energy*. See the book for more details.

6 Elastic Collision

From the momentum and conservation of momentum we learned that the total momentum of a system that is isolated is conserved and this allowed us to figure out how to find some information about the velocities of particles involved in a collision. We encounter a situation (The billiard balls) where the conservation of momentum was not enough to get the final value of all the final velocities.

Here we can add the conservation of momentum. With the two of them there are a lot of problems that can be solved... you will see in the problems and home-works.

Consrv. of Momentum:

$$\vec{p}_{1,i} + \vec{p}_{2,i} + \cdots \quad = \quad \vec{p}_{1,f} + \vec{p}_{2,f} + \cdots \tag{15}$$

Consrv. of Energy:

$$(K_{1,i} + U_{1,i}) + (K_{2,i} + U_{2,i}) \cdots \quad = \quad (K_{1,f} + U_{1,f}) + (K_{2,f} + U_{2,f}) \cdots \tag{16}$$

7 Power

The last concept in this chapter is *power* and is the amount of energy that is spared or consumed during a time interval Δt:

$$P = \frac{\Delta E}{\Delta t} \tag{17}$$

If the energy change is a result of a work :

$$P = \frac{W}{\Delta t} = \frac{f\,\Delta x}{\Delta t} = F\,v \tag{18}$$

where the velocity is the velocity of the object. Obviously this velocity changes so the equation is an approximation.

8 Examples

You will see many examples in the class. And probably later added to this section.