e) Using the master method

for $a = 3$ $b = 2$ and $f(n) = \theta(n)$

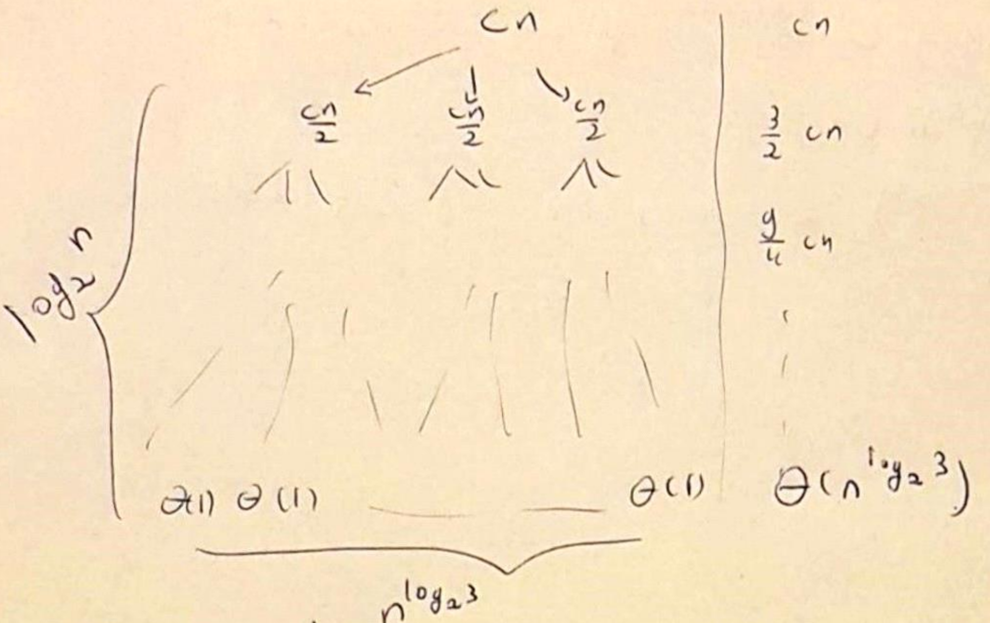
we get $f(n) = \Theta(n^{\log_2 3 - \epsilon})$ for $\epsilon \approx 0.6$ thus we are

in case one of the theorem so

$T(n) = \theta(n^{\log_2 3})$ which is the same result we got

in point d)

d) we have $T(n) = 3T(n/2) + \Theta(n)$. for simplicity write $\Theta(n)$ as cn. Then we have



$$\text{So } T(n) = \sum_{i=0}^{\log_2 n - 1} \left(\tfrac{3}{2}\right)^i cn + \Theta(n^{\log_2 3})$$

$$= cn\left(\frac{(3/2)^{\log_2 n} - 1}{3/2 - 1}\right) + \Theta(n^{\log_2 3})$$

$$= 2c(3^{\log_2 n} - 1) + \Theta(n^{\log_2 3})$$

we will show that $T(n) = \Theta(n^{\log_2 3})$, it sufficces to show

$$3^{\log_2 n} = O(n^{\log_2 3})$$

or $\exists c$ s.t $3^{\log_2 n} \le c \cdot n^{\log_2 3}$ letting $d = (c)^{1/\log_2 3}$ and taking $\ln$ of both sides

$$\log_2 n \ln(3) \le \log_2 3 \ln(dn)$$

or $\log_3 n \le \log_3 dn$ and thus true for d sufficcenty large

So $T(n) = \Theta(n^{\log_2 3})$