Computer Architecture and Programming Languages Course: CO20-320241
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Homework 3

Problem 3.1

Solution:
These are the rules I will be using (same as in lecture 3-4 slides) :

$R1 \longrightarrow$ Commutative
$R2 \longrightarrow$ Associative
$R3 \longrightarrow$ Distributive
$R4 \longrightarrow$ Idempotent
$R5 \longrightarrow$ Involution
$R6 \longrightarrow$ Complement
$R7 \longrightarrow$ De Morgan
$R8 \longrightarrow 1 \cdot x = x$ and $0 + x = x$
$R9 \longrightarrow 0w = 0$ and $1 + w = 1$

(a) $x = (M+N)(\overline{M}+P)(\overline{N}+\overline{P})$
$= \big((M+N)\overline{M} + (M+N)P\big)(\overline{N}+\overline{P}) \qquad (R3)$
$= (M\overline{M} + N\overline{M} + MP + NP)(\overline{N}+\overline{P}) \qquad (R3)$
$= (0 + N\overline{M} + MP + NP)(\overline{N}+\overline{P}) \qquad (R6)$
$= (N\overline{M} + MP + NP)\overline{N} + (N\overline{M} + MP + NP)\overline{P} \qquad (R3, R8)$
$= N\bar{M}\overline{N} + MP\overline{N} + NP\overline{N} + N\bar{M}\bar{P} + MP\overline{P} + NP\overline{P} \qquad (R3, R1)$
$= 0\overline{M} + MP\overline{N} + 0 + N\bar{M}\bar{P} + 0 + 0 \qquad (R6)$
$= MP\overline{N} + N\bar{M}\bar{P} \qquad (R8, R9)$

(b) $z = \overline{A}B\overline{C} + AB\overline{C} + B\overline{C}D = B\bar{C}(\bar{A} + A + D) \qquad (R1, R2, R3)$
$= B\bar{C}(1+D) \quad (R6) = B\overline{C} \cdot 1 \quad (R9) = B\overline{C} \quad (R8)$

(c) $x = \overline{(M+N+P)Q} = \overline{(M+N+P)} + \overline{Q} \quad (R7) = \bar{M}\bar{N}\bar{P} + \bar{Q} \quad (R7)$

(d) $z = \overline{ABC + DEF} = \overline{ABC} \cdot \overline{DEF} \qquad (R7)$
$= (\bar{A} + \bar{B} + \bar{C})(\bar{D} + \bar{E} + \bar{F}) \qquad (R7)$
$= \bar{A}(\bar{D} + \bar{E} + \bar{F}) + \bar{B}(\bar{D} + \bar{E} + \bar{F}) + \bar{C}(\bar{D} + \bar{E} + \bar{F}) \qquad (R3)$
$= \bar{A}\bar{D} + \bar{A}\bar{E} + \bar{A}\bar{F} + \bar{B}\bar{D} + \bar{B}\bar{E} + \bar{B}\bar{F} + \bar{C}\bar{D} + \bar{C}\bar{E} + \bar{C}\bar{F} \qquad (R3)$

(e) $z = \overline{A\overline{B} + C\overline{D} + EF} = \overline{A\overline{B}} \cdot \overline{C\overline{D}} \cdot \overline{EF} \qquad (R7)$
$= (\overline{A} + \overline{\overline{B}})(\overline{C} + \overline{\overline{D}})(\overline{E} + \overline{F}) \qquad (R7) = (\overline{A} + B)(\overline{C} + D)(\overline{E} + \overline{F}) \qquad (R5)$
$= \big((\overline{A} + B)\overline{C} + (\overline{A} + B)D\big)(\overline{E} + \overline{F}) \qquad (R3)$
$= \big(\overline{A} \cdot \overline{C} + B\overline{C} + \overline{A}D + BD\big)(\overline{E} + \overline{F}) \qquad (R3)$
$= (\overline{A} \cdot \overline{C} + B\overline{C} + \overline{A}D + BD)\overline{E} + (\overline{AC} + B\overline{C} + \overline{A}D + BD)\overline{F} \qquad (R3)$
$= \bar{A}\bar{C}\bar{E} + B\bar{C}\bar{E} + \overline{A}D\overline{E} + BD\overline{E} + \bar{A}\bar{C}\bar{F} + B\bar{C}\bar{F} + \overline{A}D\overline{F} + BD\overline{F} \qquad (R3)$

(f) $z = \overline{\overline{A + B\overline{\overline{C}} + D(\overline{E + \overline{\overline{F}}})}} = \overline{\overline{A + B\overline{\overline{C}}}} \cdot \overline{D(\overline{E + \overline{\overline{F}}})} \quad (R7)$
$= (A + B\overline{C})(\overline{D} + (\overline{E + \overline{\overline{F}}}) \quad (R5, R7) = (A + B\overline{C})(\overline{D} + \bar{E}F) \quad (R5, R7)$
$= (A + B\overline{C})\overline{D} + (A + B\overline{C})\bar{E}F \quad (R3)$
$= A\overline{D} + B\overline{C} \cdot \overline{D} + A\overline{E}F + B\overline{C} \cdot \overline{E}F \quad (R3)$

Problem 3.2

Solution:
We first construct the truth table for the circuit

A	B	C	D	x
0	0	0	0	1
1	0	0	0	1
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1
0	0	1	1	1
0	1	0	1	0
1	0	0	1	1
1	1	0	0	0
0	1	1	0	0
1	0	1	0	0
0	1	1	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

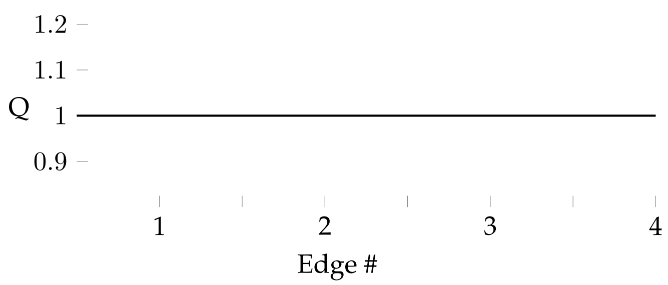
So we get the following K-map (same notation as last time):

	$CD(00)$	$CD(01)$	$CD(11)$	$CD(10)$
$AB(00)$	1	1	1	0
$AB(01)$	0	0	0	0
$AB(11)$	0	0	0	0
$AB(10)$	1	1	0	0

Using K-map to simplify we see there is a quad 1, 2, 13, 14 which results in $\bar{B}\bar{C}$ and a pair 2,3 which results in $\bar{A}\bar{B}D$. Therefore the result is:
$x = \bar{B}\bar{C} + \bar{A}\bar{B}D$

Problem 3.3

Solution:
It is given that $Q_0 = 1$. For the given CLK signal, there are 4 positive going edges so we have to check these 4 cases for the value of the output. On the first one, S is 1 and R is 0, which results in a set, so the value of Q stays one. On the second one we have again same values for S and R so Q still stays 1. On the third one the value of S is 0 and the value of R is also 0, which results in the previous state (unchanged) so Q still remains 1. Finally, on the fourth case, S is 1 and R is 0 which results in a set, so Q stays 1. So the waveform for Q will be a straight horizontal line at y=4. So the waveform of Q will be:



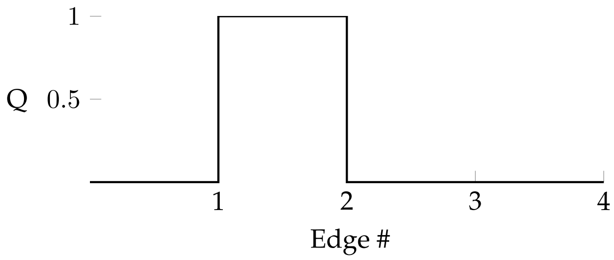
So the state table is:

S	R	Output	Edge #	State
1	0	1	$1^{st}\uparrow$	SET
1	0	1	$2^{nd}\uparrow$	SET
0	0	1	$3^{rd}\uparrow$	(no change)
1	0	1	$4^{th}\uparrow$	SET

Problem 3.4

Solution:

We are given the initial state $Q_0 = 0$. As in the previous case, there are 4 negative going edges so we will see what happens to Q on all 4 of them. On the first one S has a value of 1 and R a value of 0 which as in 3.3 result in a SET so Q becomes 1. On the second one S is 0 and R 1 so we have a RESET therefore Q becomes 0. On the third we see that both S and R are 0 so no change happens and Q still remains 0. Finally on the last one they are again both 0 so the state of Q does not change so it still remains 0. Thus we see a waveform as drawn below:



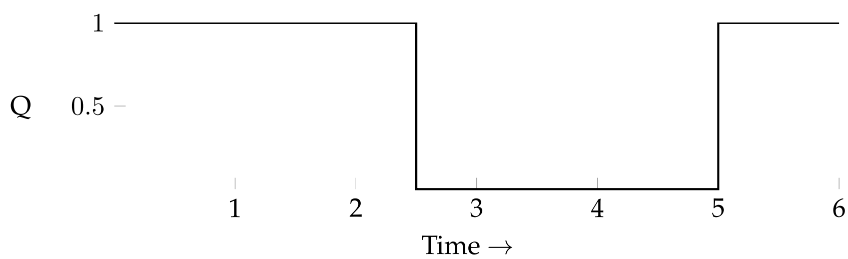
Now drawing the state table:

S	R	Output	Edge #	State
1	0	1	$1^{st} \downarrow$	SET
0	1	0	$2^{nd} \downarrow$	RESET
0	0	0	$3^{rd} \downarrow$	(no change)
0	0	0	$4^{th} \downarrow$	(no change)

Problem 3.5

Solution:

We are given the initial condition of $Q_0 = 1$. We first assume that the hold time is infinitely close to 0. Since for the given CLK waveform there are 6 positive going edges, we need to consider 6 cases. We first notice that for both the first and second case, both J and K are 0, which results in no change, thus Q remains still 1. In the third case, J and K are both 1, which results in a TOGGLE, so Q now becomes 0. In the fourth triggering edge J is 0 and K is 1, which results in a RESET, so Q still remains 0. In the fifth one, J is 1 and K is 0, which results in a SET, so now Q becomes 1. Same case also for the sixth one, J is 1 and K is 0, which results in SET action so Q remains 1.



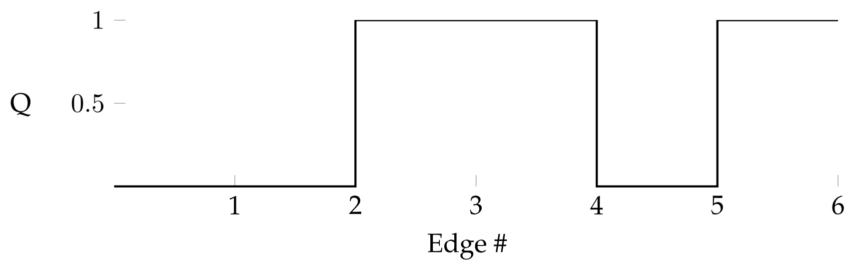
We calculate the state table as below:

J	K	Output	Edge #	State
0	0	1	$1^{st} \uparrow$	(no change)
0	0	1	$2^{nd} \uparrow$	(no change)
1	1	0	$3^{rd} \uparrow$	$TOOGLE$
0	1	0	$4^{th} \uparrow$	$RESET$
1	0	1	$5^{th} \uparrow$	SET
1	0	1	$6^{th} \uparrow$	SET

Problem 3.6

Solution:

We are given the initial condition of $Q_0 = 0$. As above we first assume that the hold time is infinitely close to 0. Also here we need to consider 6 effective edges in total. For the first one, both J and K are 0 so no change occurs. For both second and third J is 1 and K is 0, which results in SET action, so Q becomes and remains 1. In both fourth and fifth one, both J and K are 1, which results in a TOGGLE action(state), so Q is first toggled to 0 and then again. Finally at the sixth one both J and K are 0, which results in no change at all, so Q still remains 1. So we get the following waveform for Q:



From the reasoning done above the state table is:

J	K	Output	Edge #	State
0	0	0	$1^{st}\uparrow$	(no change)
1	0	1	$2^{nd}\uparrow$	SET
1	0	1	$3^{rd}\uparrow$	SET
1	1	0	$4^{th}\uparrow$	$TOOGLE$
1	1	1	$5^{th}\uparrow$	$TOOGLE$
0	0	1	$6^{th}\uparrow$	(no change)

Problem 3.7

Solution:

(a) We want to make $Y = 1$. Since K is connected to the ground it is 0, therefore the only way to achieve what we want is by a SET action, which means we need J to be 1 in the second flip-flop. This means that X has to be 1 and since the first K is also connected to the ground, we want J to also be 1 in the first J-K flip flop so A has to be 1. We also need B and C to be one so that we can see the change, therefore also considering the possible delays of each element, the sequence would be A, B and C.

(b) The initial START pulse is needed to make sure that both X and Y are LOW at the start (It is an asynch. input)

(c) We can use D flip-flops to get a similar circuit (I did my best in the drawing):