Computer Architecture and Programming Languages   Course: CO20-320241
Jacobs University Bremen   23, 09, 2019
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Homework 2

Problem 2.1

Solution:

(a) $777_8 + 1_8 = 1000_8$ (b) $888_{16} + 1_{16} = 889_{16}$ (c) $32007_8 + 1_8 = 32010_8$

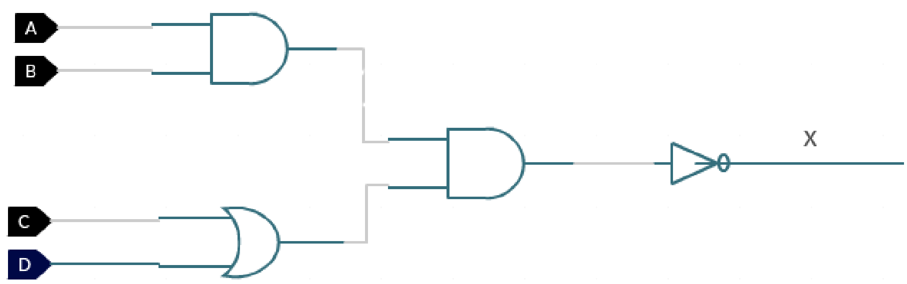
(d) $32108_{16} + 1_{16} = 32109_{16}$ (e) $8BFF_{16} + 1_{16} = 8C00_{16}$

(f) $1219_{16} + 1_{16} = 121A_{16}$

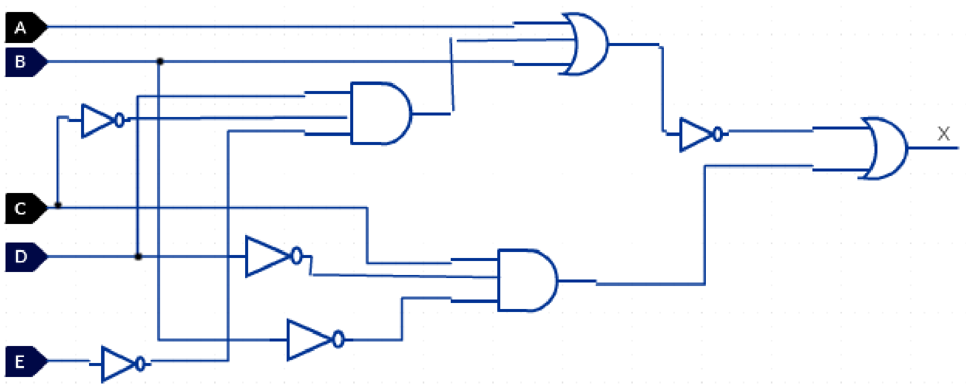
Problem 2.2

Solution:

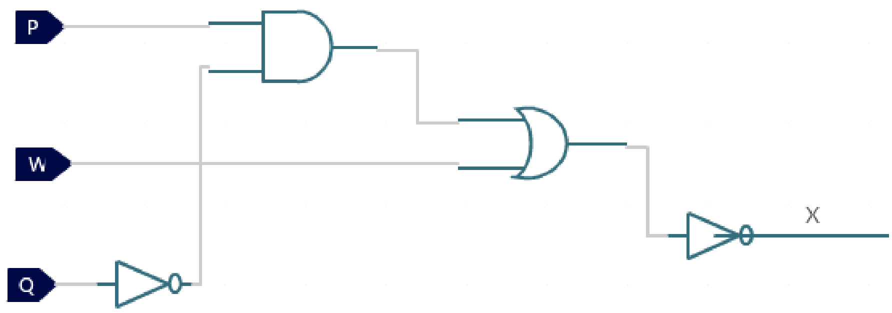
(a) For x = $\overline{A \cdot B \cdot (C + D)}$



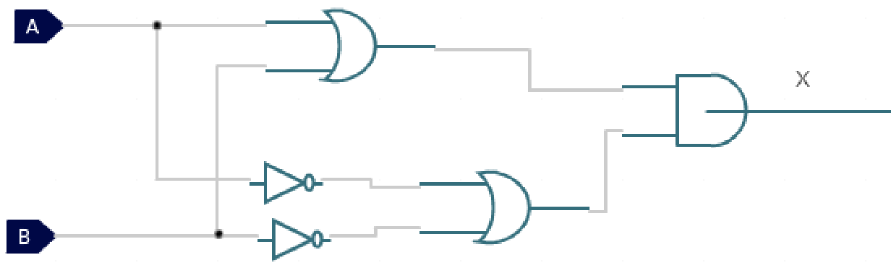
(b) For x = $\overline{A + B + \overline{C} \cdot D \cdot \overline{E}} + \overline{B} \cdot C \cdot \overline{D}$



(c) For x = $\overline{W + P \cdot \overline{Q}}$



(d) For x = $(A + B) \cdot (\overline{A} + \overline{B})$



Problem 2.3

Solution:

(a) After performing calculations, the truth table of the given logic circuit is as follows:

M	N	Q	x
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	0
1	1	0	0
1	0	1	1
0	1	1	1
1	1	1	1

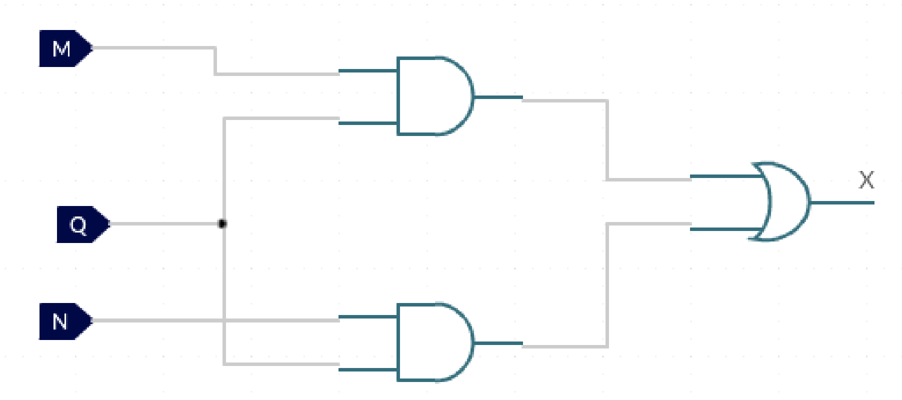
Now from the truth table we easily derive the DNF (disjunctive normal form), which is: $x = \overline{M}NQ + M\overline{N}Q + MNQ$

Before simplifying, I'll first write down the rules that I will be using (a small mapping)

$R1 \longrightarrow$ Commutativity
$R2 \longrightarrow x + \overline{x} = 1$
$R3 \longrightarrow x \cdot y + x \cdot z = x \cdot (y + z)$
$R4 \longrightarrow x = x + x$
Now lets simplify the expression:

$x = \overline{M}NQ + M\overline{N}Q + MNQ = \overline{M}NQ + M\overline{N}Q + MNQ + MNQ \qquad (R4)$
$= \overline{M}NQ + MNQ + M\overline{N}Q + MNQ \qquad (R1)$
$= NQ(\overline{M} + M) + MQ(\overline{N} + N) \qquad (R3)$
$= NQ + MQ \qquad (R2)$

So after simplifying our circuit becomes:



Problem 2.4

Solution:

(a) $X + \overline{X} \cdot Y = X + Y$ We calculate the Truth table below:

X	Y	$\overline{X}$	$\overline{X} \cdot Y$	$X + \overline{X} \cdot Y$	$X + Y$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

As we clearly see from the table, the last two columns are equal, which proves the statement.

(b) $\overline{X} + X \cdot Y = \overline{X} + Y$ We calculate the Truth table below:

X	Y	$\overline{X}$	$X \cdot Y$	$\overline{X} + X \cdot Y$	$\overline{X} + Y$
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	1	1

As we clearly see from the table, the last two columns are equal, which proves the statement.

Problem 2.5

Solution:

a) $A + 1 = 1$ b) $A \cdot A = A$ c) $B \cdot \overline{B} = 0$ d) $C + C = C$
e) $x \cdot 0 = 0$ f) $D \cdot 1 = D$ g) $D + 0 = D$ h) $C + \overline{C} = 1$
i) $G + G \cdot F = G(1 + F) = G$ j) $y + \overline{w} \cdot y = y(1 + \overline{w}) = y$

Problem 2.6

Solution:

The following 2 Truth tables prove De Morgan's first and second theorem respectively. De Morgan's first theorem states that . The following truth table shows the results:

1) $\overline{X+Y} = \overline{X} \cdot \overline{Y}$

X	Y	$\overline{X}$	$\overline{Y}$	$X+Y$	$\overline{X+Y}$	$\overline{X} \cdot \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

From the last 2 columns we see the theorem is true.

2) $\overline{X \cdot Y} = \overline{X} + \overline{Y}$

X	Y	$\overline{X}$	$\overline{Y}$	$X \cdot Y$	$\overline{X \cdot Y}$	$\overline{X} + \overline{Y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

From the last 2 columns we see the theorem is true.

Problem 2.7

Solution:
I will use the exact same rules used in problem 2.3 (from R1 to R4)

Constructing the DNF through the truth table and using our 4 rules we get:

$$x = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + ABCD$$
$$= \bar{A}\bar{B}D(\bar{C}+C) + \bar{A}BD(\bar{C}+C) + A\bar{B}\bar{C}\bar{D} + ABD(\bar{C}+C) \qquad (R3)$$
$$= \bar{A}\bar{B}D + \bar{A}BD + ABD + A\bar{B}\bar{C}\bar{D} \qquad (R1, R2)$$
$$= \bar{A}\bar{B}D + \bar{A}BD + \bar{A}BD + ABD + A\bar{B}\bar{C}\bar{D} \qquad (R4)$$
$$= \bar{A}D(\bar{B}+B) + BD(\bar{A}+A) + A\bar{B}\bar{C}\bar{D} \qquad (R3)$$
$$= \bar{A}D + BD + A\bar{B}\bar{C}\bar{D} \qquad (R2)$$

Problem 2.8

Solution:
First we construct the K map (for sake of simplicity as the negation lines overlap with the table horizontal lines, I use AB(01) instead of $\bar{A}B$):

	$\bar{C}\bar{D}(00)$	$\bar{C}D(01)$	$CD(11)$	$CD(10)$
$AB(00)$	0	1	1	0
$AB(01)$	0	1	1	0
$AB(11)$	0	1	1	0
$AB(10)$	1	0	0	0

Counting the cells of the table from 1 to 16 (counting horizontally), we can see that there is one quad of 1s which consists of cells 2,3,6,7 (resulting in $\bar{A}D$), another quad on cells 6, 7, 10,11 (resulting in BD) and also the lonely 1 on cell 13 which results in $A\bar{B}\bar{C}\bar{D}$. Therefore we reach the same result from above:

$$x = \bar{A}D + BD + A\bar{B}\bar{C}\bar{D}$$