Homework 1

Problem 1.1

Solution:

For the first 4 values, since they are with base 2, in order to find their decimal equivalent (base 10), we have to find the sum of the digits multiplied by 2 to the power of the position of the digit starting from 0 from the right side:

a) $10100_2 \implies 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 16 + 0 + 4 + 0 + 0 = 20_{10}$

b) $11011011_2 \implies 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 128 + 64 + 16 + 8 + 2 + 1 = 219_{10}$

c) $001001001_2 \implies 0 + 1 \cdot 2^6 + 0 + 0 + 1 \cdot 2^3 + 0 + 0 + 1 \cdot 2^0 = 64 + 8 + 1 = 73_{10}$

d) 111111111111_2 $\implies 1 \cdot 2^{11} + 1 \cdot 2^{10} + 1 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 2048 + 1024 + 512 + 256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 4095_{10}$

For the other examples, the procedure is the same, but instead of base 2, we are converting from bases 8,3,7, and 5 respectively

e) $75077_8 \implies +7 \cdot 8^4 + 5 \cdot 8^3 + 0 \cdot 8^2 + 7 \cdot 8^1 + 7 \cdot 8^0 = 28672 + 2560 + 56 + 7 = 31295_{10}$

f) $12101_3 \implies +1 \cdot 3^4 + 2 \cdot 3^3 + 1 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0 = 81 + 54 + 9 + 1 = 145_{10}$

g) $26601_7 \implies +2 \cdot 7^4 + 6 \cdot 7^3 + 6 \cdot 7^2 + 0 \cdot 7^1 + 1 \cdot 7^0 = 4802 + 2058 + 294 + 1 = 7155_{10}$

h) $431021_5 \implies +4 \cdot 5^5 + 3 \cdot 5^4 + 1 \cdot 5^3 + 0 \cdot 5^2 + 2 \cdot 5^1 + 1 \cdot 5^0 = 12500 + 1875 + 125 + 10 + 1 = 14511_{10}$

Problem 1.2

Solution:

a) One possible way to find the binary equivalent of a decimal number is to keep dividing the number by 2 and keeping tract of the remainder until we reach 0. Then we get the binary equivalent of our number by writing together the remainders knowing that the least significant digit is the result of the first division. So we have:

$4272_{10} \xrightarrow{/2} 2136(0) \xrightarrow{/2} 1068(0) \xrightarrow{/2} 534(0) \xrightarrow{/2} 267(0) \xrightarrow{/2} 133(1) \xrightarrow{/2} 66(1) \xrightarrow{/2} 33(0) \xrightarrow{/2} 16(1)$

$\xrightarrow{/2} 8(0) \xrightarrow{/2} 4(0) \xrightarrow{/2} 2(0) \xrightarrow{/2} 1(0) \xrightarrow{/2} 0(1)$

So we get the binary equivalent 1000010110000_2

b) To convert hexadecimal number to a binary number there is a simple method that we have proved and used in Introduction to Computer Science. We first find the binary representation of each of the digits then merge them together to get the binary representation of the whole hexadecimal number:

$C \longrightarrow 1100$ $\quad$ $B \longrightarrow 1011$ $\quad$ $A \longrightarrow 1010$

$\implies$ $\quad$ CBA$_{16} \equiv 110010111010_2$

c) We use the same approach as in problem 1.1 but in this case for base 16:

$B8C \implies 11 \cdot 16^2 + 8 \cdot 16^1 + 12 \cdot 16^0 = 2956_{10}$

d) Same approach as in point c:

$29D8 \implies 2 \cdot 16^3 + 9 \cdot 16^2 + 13 \cdot 16^1 + 8 \cdot 16^0 = 8192 + 2304 + 208 + 8 = 10712_{10}$

e) We now that to find a following number we just have to add one to the current number, and also considering the 'overflow' as we are in base 16 and any digit can be at most F, we can write:
$8CE_{16} \implies 8CF_{16} \longrightarrow 8D1_{16} \longrightarrow 8D2_{16} \longrightarrow 8D3_{16} \longrightarrow 8D4_{16}$

Problem 1.3

Solution:
a) Using BCD we can write:
$7 \rightarrow 0111_2$ $3 \rightarrow 0011_2$ $2 \rightarrow 0010_2$

So 732_{10} is converted to 011100110010_{BCD}

b) Invalid BCD codes are those which represent a decimal value from 10 to 15, namely:

$$10_{10} \longrightarrow 1010_2$$
$$11_{10} \longrightarrow 1011_2$$
$$12_{10} \longrightarrow 1100_2$$
$$13_{10} \longrightarrow 1101_2$$
$$14_{10} \longrightarrow 1110_2$$
$$15_{10} \longrightarrow 1111_2$$

c) Splitting it into parts of 4 digits and find their decimal equivalents we get:

$$1001 \longrightarrow 9_{10} \qquad 0101 \longrightarrow 5_{10} \qquad 0110 \longrightarrow 6_{10}$$

So the result of the conversion: 956_{10}

d) Using the same procedure as in problem 2:
$77_{10} \xrightarrow{/2} 38(1) \xrightarrow{/2} 19(0) \xrightarrow{/2} 9(1) \xrightarrow{/2} 4(1) \xrightarrow{/2} 2(0) \xrightarrow{/2} 1(0) \xrightarrow{/2} 0(1)$
and since $0100 \longrightarrow 4_{16}$ and $1101 \longrightarrow D_{16}$ we get that the binary value is 01001101_2 and the hexadecimal one is $4D_{16}$

e) Using the same procedure as above we have:
$109_{10} \xrightarrow{/2} 54(1) \xrightarrow{/2} 27(0) \xrightarrow{/2} 13(1) \xrightarrow{/2} 6(1) \xrightarrow{/2} 3(0) \xrightarrow{/2} 1(1) \xrightarrow{/2} 0(1)$
and since $0110 \longrightarrow 6_{16}$ and $1101 \longrightarrow D_{16}$ we get that the binary value is 01101101_2 and the hexadecimal one is $6D_{16}$

Problem 1.4

Solution:
a) The NOT gate is not correct in this case as when a NOT gate's input is low the output is high and vice versa. Taking a look at the truth tables for AND and OR gates we can say that:

OR Gate

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

AND Gate

A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1

We can see that the AND Gate has 3 cases where at least 1 of inputs are low the output will be low. On the other side, 1 case for the OR Gate when this requirement is fulfilled. Both gates fulfill the requirements.
b) From the truth tables on the question above it is obvious that on both the OR or AND Gates there is 1 case respectively when the requirement is met.

Problem 1.5

Solution:

AND Gate

A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Problem 1.6

Solution:

OR Gate

A	B	C	D	Output
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1