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Vehicle Routing Problems with Simultaneous Pick-up and Delivery Service

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Abstract

Variations of the classical Vehicle Routing Problem (VRP) consider clients that require pick-up and delivery service, simultaneously or not. Each variation corresponds to a specific routing policy, applicable to a given context. Although several routing problems of this nature are defined in the literature, there are relationships among them that define one problem as a particular case of another, which allows transformation among problems. We classify these problems on the basis of the characteristics of the service required by the clients.

We study in particular the Vehicle Routing Problem with Simultaneous Pick-up and Delivery Service. This problem may be solved for example through a tour partitioning heuristic developed for the traveling salesman problem (TSP). The routing in each segment of the partitioned tour is achieved by solving capacitated TSP subproblems with simultaneous pick-up and delivery service. Computational results are given for a set of problems available in the literature.

Key words

Vehicle Routing, Pick-up and Delivery Service, Heuristics.

1. Introduction

One variation of the classical vehicle routing problem considers clients that require simultaneous pick-up and delivery service. This problem is called the Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD). In Brazil this problem was initially studied within the context of personnel transportation by helicopters between the continent and oil exploration and production platforms located in the Campos Basin. The problem consists in determining routes for the fleet of helicopters seeking to minimise transportation costs. Safety requirements impose restrictions related to maximum number of helicopter landings in any given route, and maximum distance flown between two consecutive landings. A heuristic algorithm was developed to solve this problem (see Galvão and Guimarães [9]).

There are few references to VRPSPD in the literature. There exists, however, abundant literature on routing problems in which each client requires either pick-up or delivery service, but not both simultaneously. There exist relationships between these problems and VRPSPD.

This paper is organized in the following manner. Section 2 presents a brief literature review of vehicle routing problems that involve pick-up and delivery service. Section 3 proposes a classification scheme for such problems, showing that VRPSPD can be transformed into other problems of this class and refers to theoretical results used in the development of heuristic methods for these problems. An integer programming formulation for VRPSPD is given in Section 4; which includes possible Lagrangean relaxation schemes for the problem. Heuristics for the solution of VRPSPD are presented in Section 5, which is followed by computational results and conclusions.

2. Brief Literature Review

Several routing problems with pick-up and delivery service are reported in the literature. Savelsbergh and Sol [29] describe a general routing problem in the category, which they call the *General Pick-up and Delivery Problem* so that the majority of pick-up and delivery problems can be stated as particular cases of this general problem.

A classical problem discussed in the literature, the *Dial-a-Ride Problem,* consists of picking up clients in pre-specified locations and transporting them to known delivery locations, using vehicles based in a given depot. Each pick-up location is associated with a delivery location, forming pairs of locations, with priority given to the pick-up activity. The problem is subject to restrictions on vehicle capacity, maximum travel times for clients and time windows. It is sought to minimize the following objectives in a hierarchical fashion: (i) number of vehicles; (ii) total distance traveled; (iii) the difference between effective pick-up and delivery times and those desired by clients.

Exact dynamic programming algorithms were developed by Psaraftis ([24], [25]) to solve the static and dynamic versions of the *Dial-a-Ride Problem,* as well as a variant of the problem with time windows. Desrosiers at al. [5] proposed a more efficient algorithm for the time windows variant, and Psaraftis [26], Sexton and Bodin ([30], [31]) and Sexton and Choi [32] proposed heuristic methods for this problem. More recently Van der Bruggen et al. [35] proposed a simulated annealing algorithm; Madsen et al. [19] used a parallel insertion heuristic for the variant with time windows. The extension of this problem for several vehicles, the *m-Dial-a-Ride problem,* was studied by Roy et al.([27], [28]), Jaw et al. [17], Desrosiers et al. [6] and Ioachim et al. [16], among others. Dumas et al. [8] proposed an exact procedure to solve this problem.

Another routing problem in this category is the problem with backhauls; where deliveries are given priority and all deliveries must be made before any pick-up can be effected. In the case of a single vehicle the Traveling Salesman Problem with Backhauls is defined; Gendreau et al. [12] developed several two-phase heuristics for this problem. Deif and Bodin [4], Golden et al. [15] and Casco et al. [3] extended the approach to several vehicles. Toth and Vigo [34] and Mingozzi and Giorgi [21] developed mathematical formulations and exact methods to solve this problem. Duhamel et al. [7] developed a tabu search heuristic and Gélinas at al. [11] studied the problem with time windows.

A new class of problems in which the precedence relations are relaxed (it is possible to alternate between pick-ups and deliveries in any given route) has been the object of recent studies. Mosheiov [22] studied the one-vehicle problem. Mosheiov [23] extended the approach to several vehicles for the particular case in which all pick-up and delivery demands are equal to the unity. Min [20] studied the case of several vehicles within the context of a distribution system for public libraries with vehicles of equal capacity.

3. Classification Scheme and Relationship among Problems

In the classical VRP there is only one type of demand (pick-up or delivery). If both types of demand are present, specific routing policies for each particular context must be defined.

3.1 Classification Scheme

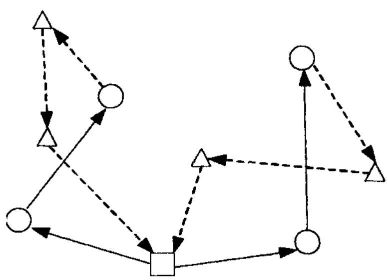
Vehicle routing problems can be classified on the basis of: (i) type of demand and (ii) how this demand is fulfilled. The following classification scheme is proposed.

(i) Demand that requires **either** pick-up **or** delivery service, but not both, in each node with (a) **non-associated** pick-up and delivery service;and (b) **associated** pick-up and delivery service.

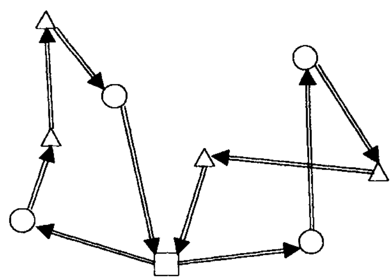
(ii) Demand that potentially requires **both** pick-up **and** delivery service in each node with (a) **non-simultaneous** pick-up and delivery service; and (b) **simultaneous** pick-up and delivery service.

Problems in which each client demands either pick-up or delivery service, but not both, belong to group (i). This group was divided into two sub-groups. In sub-group (a) pick-up and delivery services are not associated. The *Vehicle Routing Problem with Backhauls* (VRPB), for example, belongs to this sub-group and is characterized by the fact that in each route all deliveries must be made before any pick-up service is offered (see Figure 1.1). On the other hand, in the Vehicle Routing Problem with Pick-up and Delivery Service (VRP_PD), which belongs to the same sub-group, there is no priority for either pick-ups or deliveries, which can alternate along the route (see Figure 1.2). In sub-group (b) pick-ups and deliveries are inter-related; in this case there are precedence relations that must be observed. The *Dial-a-Ride* and *m-Dial-a-Ride* problems belong to this subgroup (see Figure 1.3).

In problems belonging to group (ii) each client potentially requires both pick-up and delivery service. This group was also divided into two sub-groups. In sub-group (b) both demands must be satisfied simultaneously, as for example the helicopter routing problem described in the introduction (VRPSPD, see Figure 1.4). Sub-group (a) includes problems in which the two types of demand are satisfied in distinct points in time. The *Express Delivery Problem* (EDP, see Tang et al. [33]) belongs to this sub-group. In this case the fulfilment of demand is divided into two phases: delivery phase and pick-up phase, shown respectively in Figures 1.5 and 1.6. In this problem the pick-up and delivery routes do not necessarily coincide.



1.1



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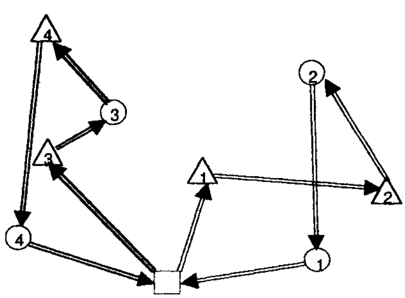

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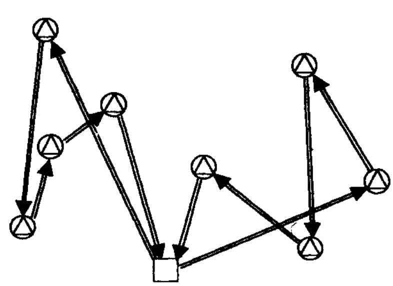

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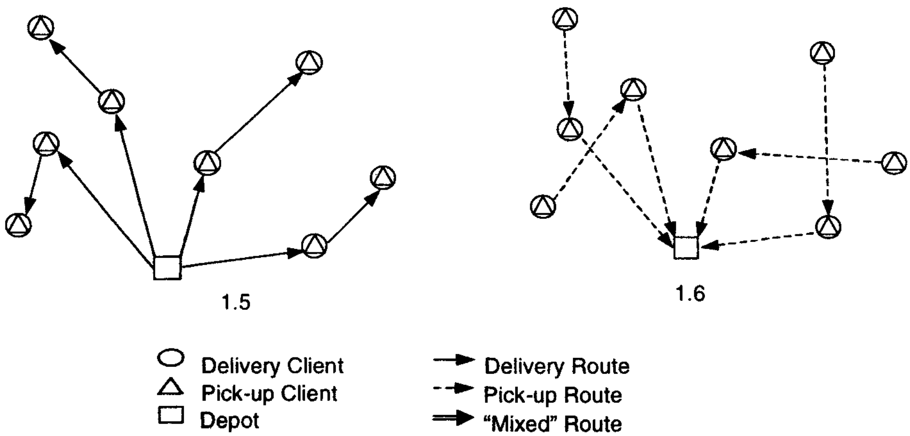


(1.1)VRPB; (1.2) VRP_PD; (1.3) m-Dial-a-Ride Problem;
(1.4) VRPSPD; (1.5)-(1.6) Delivery and Pick-up Phases of EDP

Figure 1 - VRP's with Pick-up and Delivery Service

3.2 Equivalence of Problems

Although simple transformations can be used to convert VRPSPD into other routing problems with pick-up and delivery service, this will not be discussed here in detail. It is easy to show that any particular instance of VRPSPD can be transformed into an instance of VRP_PD, since this problem is a particular case of VRPSPD in which only one of the demands (pick-up or delivery) is different from zero in each node. It is also easy to show that any instance of VRPSPD can be transformed into an instance of the *m-Dial-a-Ride Problem*, of which it is a particular case.

Given the equivalences mentioned above, several theoretical results, used in the development of heuristic procedures to solve VRPSPD, are reported in the literature. These are mainly results obtained for VRP_PD by Mosheiov [22] and Gendreau et al. [13] which, because of the equivalence reported above, can be used in heuristics for VRPSPD.

4. A Mathematical Formulation for VRPSPD

The mathematical formulation proposed by Mosheiov [23] for VRP_PD is extended here for VRPSPD. This formulation belongs to a set of formulations based on commodity flows developed for vehicle routing problems (see Gavish and Graves [10]). In the formulation below each client has both pick-up and delivery demands. Even though this formulation is not used in the present paper, it formalizes the model and may be used, for example, in the search of lower bounds for the problem.

The following notation will be used :

V	:	set of clients;
V_0	:	set of clients plus depot (client 0) : $V_0 = V \cup \{0\}$;
$V_P(V_D)$	:	set of pick-up (delivery) clients : $V = V_P = V_D$
N	:	total number of clients : $N = \lvert V \rvert$;
$N_P(N_D)$	:	number of pick-up (delivery) clients : $N_P = \lvert V_P \rvert$, $N_D = \lvert V_D \rvert$ and $N = N_P = N_D$;
c_{ij}	:	distance between clients i and j;
p_i	:	pick-up demand of client i, i=1,...,N;
d_i	:	delivery demand of client i, i=1,...,N;
Q	:	vehicle capacity;
NV	:	maximum number of vehicles.

Decision variables are as follows :

$$x_{ij} = \begin{cases} 1, \text{ if arc } (i,j) \text{ belongs to the optimal set of routes;} \\ 0, \text{ otherwise;} \end{cases}$$

y_{ij} : demand picked-up in clients routed up to node i (including node i) and transported in arc (i,j);

z_{ij} : demand to be delivered to clients routed after node i and transported in arc (i,j).

The mathematical formulation of VRPSPD is then given by :

$$Minimize \sum_{i=0}^{N} \sum_{j=0}^{N} c_{ij}\, x_{ij} \qquad (1)$$

subject to

$$\sum_{i=0}^{N} x_{ij} = 1, \qquad j=1,\ldots, N \; ; \qquad (2)$$

$$\sum_{j=0}^{N} x_{ij} = 1, \qquad i=1,\ldots, N \; ; \qquad (3)$$

$$\sum_{j=0}^{N} x_{ij} \leq NV; \qquad (4)$$

$$\sum_{i=0}^{N} y_{ji} - \sum_{i=0}^{N} y_{ij} = pj, \qquad \forall\, j \neq 0; \qquad (5)$$

$$\sum_{i=0}^{N} z_{ij} - \sum_{i=0}^{N} z_{ji} = dj, \qquad \forall\, j \neq 0; \qquad (6)$$

$$y_{ij} + z_{ij} \leq Qx_{ij}, \qquad i,j=0,\ldots N \; ; \qquad (7)$$

$$x_{ij} \in \{0,1\}, \qquad i,j=0,\ldots N \; ; \qquad (8)$$

$$y_{ij} \geq 0, \qquad i,j=0,\ldots N \; ; \qquad (9)$$

$$z_{ij} \geq 0, \qquad i,j=0,\ldots N \; ; \qquad (10)$$

The objective function seeks to minimize total distance traveled. Constraints (2) and (3) assure that each client is visited by exactly one vehicle. Restriction (4) sets a limit on the number of vehicles used. Constraints (5) and (6) are flow equations for pick-up and delivery demands, respectively; they guarantee that both demands are satisfied for each client. Restrictions (7) establish that pick-up and delivery demands will only be transported using arcs included in the solution; they further impose an upper limit on the total volume transported by a vehicle in any given section of the route. Finally, constraints (8)-(10) define the nature of the decision variables. In a broader sense the problem constraints guarantee that each vehicle leaves the depot with a volume equivalent to the sum of the delivery demands of the clients in the route serviced by that vehicle, and that each vehicle returns to the depot with a volume equivalent to the sum of the pick-up demands of the clients in the same route.

5. Heuristic Procedures for VRPSPD

Two heuristic procedures developed for the classical VRP have been extended to solve VRPSPD. These are a tour partitioning heuristic and an adaptation of the algorithm of Gillet and Miller [14].

In the tour partitioning heuristic the grouping of clients is made based on a traveling salesman tour, which is partitioned in a sequencial manner. Once the groups of clients are formed, the routing of each group is performed in such a way that the restriction on pick-up and delivery capacity is satisfied, by solving a *Traveling Salesman Problem with Simultaneous Pick-up and Delivery* (TSPSPD). The tour partitioning is repeated for different starting nodes, with the objective of improving the quality of the solution.

In Gillet and Miller's algorithm [14] the clients are grouped according to their polar coordinates. The routes are built sequentially and clients may be added or removed from the current route if this reduces the value of the objective function. In the present case an algorithm that is an adaptation of Gillet and Miller's algorithm forms the groups of clients and each group routing is performed by solving a TSPSPD. Given that TSPSPD is central to both heuristic procedures, approximate solution methods for this problem are described below.

5.1 Heuristic Methods for TSPSPD

The heuristics described below have been conceived to solve TSP_PD but can be easily adapted to solve the simultaneous problem (TSPSPD). Let $\delta(i) = p_i - d_i$. Consider delivery clients those clients for which $\delta(i) \leq 0$, and pick-up clients those clients for which $\delta(i) \geq 0$.

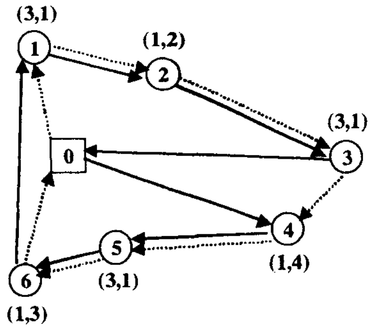
The <u>initial node heuristic</u> was initially *proposed* by Mosheiov [22] and consists of the following main steps :

Step 1. Obtain a TSP tour including all clients except the depot;

Step 2. Determine the client i_k in which the vehicle reachs the maximum load $L_{Max.}$ If $L_{Max} \leq Q$ the tour is feasible. Otherwise define a new tour in which client i_{k+1} occupies the first position and client i_k the last one, maintaining the relative positions of the remaining clients;

Step 3. Insert the depot between the first and last clients of the tour.

Figure 2 illustrates this procedure. The initial tour is indicated using discontinuous lines, whereas the feasible tour corresponds to the continuous lines. The table besides the figure shows the initial tour, the net demand of each client, the corresponding vehicle load (VL), the feasible tour and the corresponding vehicle load in each node of the tour.



i	δ(i)	VL(i)	j	VL(j)
0	-	12	0	12
1	2	14	4	9
2	-1	13	5	11
3	2	<u>15</u>	6	9
4	-3	12	1	11
5	2	14	2	10
6	-2	12	3	12
Q=12				

Figure 2 - Initial Node Heuristic

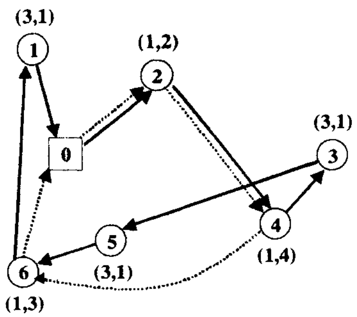
The <u>cheapest feasible insertion</u> heuristic was also proposed by Mosheiov [22] as an extension of the classical VRP cheapest insertion heuristic developed for the TSP_PD. The initial TSP tour includes all delivery clients and the depot. Pick-up clients are then inserted into the tour, in the best feasible points regarding the capacity constraints. The heuristic procedure is defined by the following steps:

Step 1. Obtain a TSP tour including the depot and all clients i for which $\delta(i) \leq 0$;

Step 2. Choose a client i for which $\delta(i) > 0$ (that does not belong to the tour) and an insertion position in the tour such that (a) the capacity constraint is satisfied; and (b) the position of insertion produces the least possible increase in the tour length;

Step 3. Repeat Step 2 until all clients for which $\delta(i) > 0$ are included in the tour.

Figure 3 illustrates this procedure. The initial tour is shown as discontinuous lines, and feasible tours as continuous lines. The table besides the figure shows the three iterations necessary to obtain a feasible tour.



i	VL(i)	j	VL(j)	k	VL(k)	l	VL(l)
0	12	0	12	0	12	0	12
2	11	2	11	2	11	2	11
4	8	4	8	4	8	4	8
6	6	<u>5</u>	<u>10</u>	<u>3</u>	<u>10</u>	3	10
-	-	6	8	5	12	5	12
-	-	-	-	6	10	6	10
-	-	-	-	-	-	<u>1</u>	<u>12</u>
Q=12							

Figure 3 - Cheapest Feasible Insertion Heuristic

The minimum spanning tree heuristic as proposed by Anily and Mosheiov [1] consists of building a directed tree such that all arcs are oriented in a direction away from the depot. The tree is initially used as a means of visiting the clients and calculating a load concentration index for each client. Clients are visited once more following the tree, preference being given this time to those clients with the smallest indices. The details of the algorithm are omitted.

The cycle heuristic was proposed by Gendreau at al. [13] for the TSPSPD where the graph that defines the problem forms a cycle. The heuristic can be adapted for the case where the defining graph is a complete graph. The following procedure defines the algorithm:

Step 1. Obtain a TSP tour including all clients and the depot;

Step 2. Follow the nodes in the order given by the TSP tour. For every node **i**, compute VL(**i**), the load accumulated in the vehicle after visiting node (**i**). If the capacity constraint is violated (VL(**i**)>Q) make v(**i**)=1, otherwise make v(**i**)=0. Make **i** = 1;

Step 3. Include clients in the TSP tour. This is done following steps (a)-(c) :

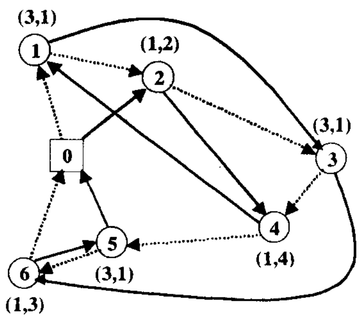
(a) While v(**i**)=0, include clients following the order given by the TSP tour. Call **i*** the first client with v(**i**)=1;

(b) While v(**i**)=1, include clients with $\delta(i) \le 0$. Also include the first client with v(**i**)=0;

(a) Return to the position of client **i***. While v(**i**)=1 include the remaining clients with $\delta(i) > 0$;

Step 4. Make i = i+1. Repeat *Step 3* until all clients are included in the tour.

Figure 4 illustrates the procedure. The initial tour is shown as discontinuous lines and the feasible tour as continuous lines. The table besides the figure shows parameters of interest for initial and final tours.



i	δ(i)	VL(i)	v(i)	j	VL(j)
0	-	12	-	0	12
1	2	14	1	2	11
2	-1	13	1	4	8
3	2	15	1	1	10
4	-3	12	0	3	12
5	2	14	1	6	10
6	-2	12	0	5	12
Q=12					

Figure 4 - Cycle Heuristic

6. Computational Results

The heuristic procedures given in Section 5 for TSPSPD were incorporated into the tour partitioning heuristic and the adaptation of Gillet and Miller's algorithm developed for VRPSPD. In the former case, the initial tour containing all clients and the initial tours of TSPSPD were sometimes obtained using the the 3-optimal heuristic developed for the TSP (see Lin [18]); and in other cases the 2-optimal heuristic was used. When the heuristics based on Gillet and Miller's algorithm were used, the initial tour containing all clients was obtained by a "sweeping" heuristic, whereas the initial tours for TSPSPD were generated by the 3-optimal heuristic. The nomenclature used to identify each heuristic procedure is shown below :

PCY3 - Tour Partitioning/3-opt for TSP/Cycle Heuristic for TSPSPD;

PST3 - Tour Partitioning/3-opt for TSP/Minimum Spanning Tree for TSPSPD;

PIN3 - Tour Partitioning/3-opt for TSP/Initial Node for TSPSPD;

PFI3 - Tour Partitioning/3-opt for TSP/Cheapest Feasible Insertion for TSPSPD;

PCY2 - Tour Partitioning/2-opt for TSP/Cycle Heuristic for TSPSPD;

PFI2 - Tour Partitioning/2-opt for TSP/Cheapest Feasible Insertion for TSPSPD;

SCY - Adapted Gillet & Miller/"Sweeping"+3-opt for TSP/Cycle Heuristic for TSPSPD;

SFI - Adapted Gillet & Miller/"Sweeping"+3-opt for TSP/Cheapest Feasible Insertion for TSPSPD.

The eight heuristic procedures described above were implemented and tested using a 350 Mz. Pentium II microcomputer. The test problem set was obtained by adapting a problem set created by Augerat et al. [2] to the classical VRP. This set consists of 27 problems with 32 to 80 nodes. Data related to the pick-up demand of each node was added to these problems; these demands being obtained from a uniform distribution defined in the interval [1,30].

Computational times for the three heuristic sets (3-opt Tour Partitioning, 2-opt Tour Partitioning and Gillet & Miller Adaptation) were very similar, as shown in figure 5. Computational times for the 3-opt Tour Partitioning heuristics grow exponentially with the number of clients but remain practically invariant for the two other groups.

The % Gap for each proposed heuristic procedures in relation to the best heuristic solution is calculated as %Gap = (Heuristic Solution - Best Heuristic Solution)/Best Heuristic Solution * 100%. Tables 1 and 2 show minimum, average and maximum % Gap for the 3-opt Tour Partitioning and the six heuristic procedures.

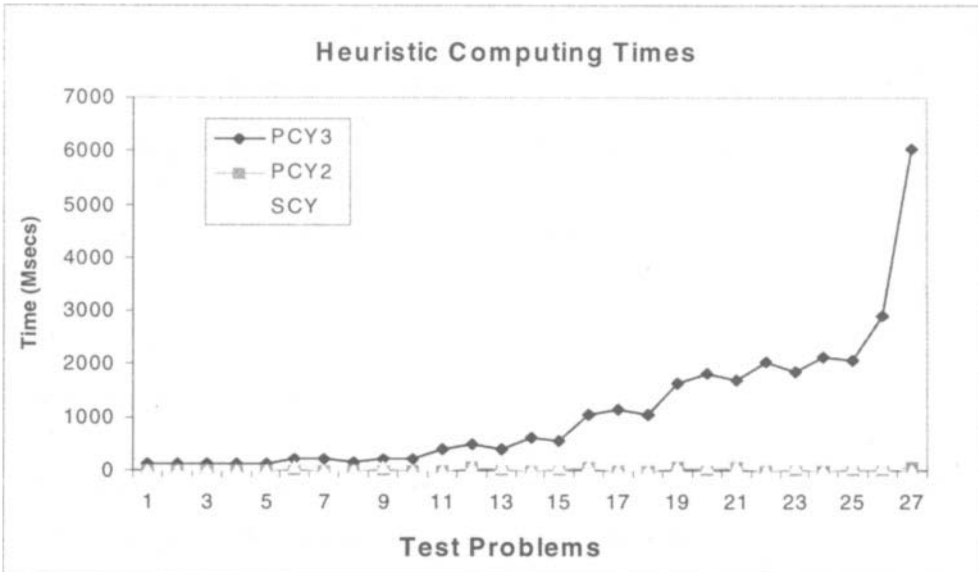


Figure 5 - Computing Times for Three Groups of Heuristics

Table 1 - %Gap Among 3-opt Tour Partitioning Heuristics

	PCY3	PST3	PIN3	PFI3
Minumum	0.00	1.26	0.00	0.00
Average	0.90	5.07	5.87	0.92
Maximum	3.86	9.06	14.33	4.10

Table 2 - %Gap Among Selected Heuristic Procedures

	PCY3	PFI3	PCY2	PFI2	SCY	SFI
Minimum	0.00	0.00	0.00	0.00	6.05	5.85
Average	1.51	1.51	3.29	3.24	15.50	16.75
Maximum	12.91	9.44	10.06	9.37	22.32	35.12

7. Conclusions

Heuristic procedures PCY3 and PFI3 produced better results than the other tour partitioning heuristics. Tour partitioning heuristics results were much better than the results produced by the "sweeping" heuristics on the average (see Table 2). As for computational times, heuristics using 3-opt substitution to solve TSP's consumed considerably more time than other heuristics, with the corresponding times increasing exponentially with problem size.

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