

## Study on Emergency Relief VRP Based on Clustering and PSO

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**Abstract**—This paper, based on the concern that a large quantity of affected spots, relatively concentrated distribution and a small quantity of emergency materials in the affected area. It addresses an optimized mathematical model and presents a new optimal vehicle routing on emergency relief materials with the goals of the shortest transport time in emergency materials allocation. Firstly the K-means clustering algorithm is adopted to get some of local distribution centers and their distribution scopes, and then particle swarm optimization algorithm is used to design the local optimal allocation routings of emergency relief vehicles inside each scope. Finally, numerical results of the approximately optimal vehicle routings indicate the feasibility and validity of the model and the new algorithm.

**Keywords**—emergency relief; vehicle routing optimization; K-means clustering; particle swarm optimization.

### I. INTRODUCTION

In recent years, various natural disasters have occurred frequently at home and abroad. They are characteristics of sudden strong, wide influence, serious secondary disasters, always cause serious losses to the affected area and a great deal of harm. At the same time, resulting in serious waste because blind rescue and regardless of cost in the post-disaster relief process. After the outbreak of natural disasters, we face with the situation that a large quantity of affected spots, distribution relatively concentrated and a large quantity of emergency materials in the affected area. How to arrange the emergency relief materials vehicle routing? How to fast deliver emergency relief supplies to the affected spots? Its the core to save lives, reduce the loss quickly. And it becomes a key problem for emergency relief.

Vehicle routing problem (VRP) is the core of the optimization of emergency logistics system in the post-disaster, was first proposed by Dantzig and Ramser in 1959[1]. Emergency relief VRP is the key to rapid deliver the emergency relief materials in the post-disaster, the purpose is to design appropriately relief vehicle routing for a series of the affected spots. Based on some certain constraints, the vehicle respectively starts from the distribution spot and return to the same distribution spot after delivering the emergency materials distribution to the affected spots. Over the years, for the research on the emergency relief VRP, the domestic

and foreign scholars have done a lot of significant work. Crainic and Perboli studied the optimal vehicle routing problem of double-layer structure which divided into two storage to transit and transit to customer [2]–[6]; Liu Chunlin and He Jianmin formulated the model of emergency relief VRP with the goals of the least supplying points and the shortest transport time in emergency relief materials allocation [7][8]; Zeng Chuanhua devised the optimization model for the goals of the shortest time and lowest cost of transporting emergency relief materials based on the premise of sufficient transport capacity [9].

Since 1990, more and more scholars have begun to introduce artificial intelligence heuristic algorithm to solve all kinds of VRP research, with the development of artificial intelligence algorithm and its powerful application in the combinatorial optimization. Zhang Bin introduced the vehicle routing problem of emergency logistics, and got the results of the model by using the heuristic algorithm [10]; Liu Fan studied the VRP problems of emergency logistics system, and got the results of the model by using the genetic algorithm [11]; Chen Liang and Wu Gengsheng studied the optimal allocation routings problem that the vehicle does not return to the starting point combined the local search algorithm with ant colony algorithm[12]; Wang Tiejun and Wu Kaijun have solved the optimal vehicle routing problem of multi-depots by improved particle swarm optimization algorithm[13]; Shi Chunyan listed all kinds of vehicle routing problem model and algorithms[14].

To sum up, scholars in domestic and foreign have studied a variety of the far-reaching significant optimal math model and solving algorithm of emergency relief VRP. But, the outbreaks of natural disasters tend to cause the complex and changeable situation that the affected spots and the quantity of relief materials are uncertain. It is difficult to formulate a model and algorithm of emergency relief VRP in general. The innovation of this paper: based on the concern that a large quantity of affected spots, distribution relatively concentrated and a large quantity of emergency materials in the affected area. We set up an optimized mathematical model with the goals of the shortest transport time in emergency materials allocation, using particle swarm

optimization algorithm combined with K-means clustering algorithm. Firstly, K-means clustering algorithm is adopted to get some of local distribution centers and their distribution scope, and then particle swarm optimization algorithm is used to design the local optimal allocation routings of emergency relief vehicles inside each scope.

## II. THE MATHEMATICAL OPTIMIZATION MODEL

Emergency relief VRP, based on the actual situation and some certain constraints in the post-disaster, studying vehicle routing problem with  $m$  vehicles respectively start from the distribution spot 0 and return to the same distribution spot 0 after delivering the emergency materials distribution to  $n$  affected spots. It is the key of emergency logistics system in the post-disaster. This paper sets up an optimized mathematical model in order to find the goals of the shortest transport time in emergency materials allocation, according to the reference [15].

The objective function:

$$F = \min \sum_{k=1}^n \sum_{i,j=0,i \neq j}^l d_{ij} x_{ij}^k t + \sum_{i=1}^l \alpha_i \varepsilon$$

Subject to:

$$\begin{aligned} \sum_{j=1}^l x_{0j}^k &= 1, \sum_{i=1}^l x_{i0}^k = 1 \quad \forall k \in (1, 2, \dots, n) \\ \sum_{k=1}^n \sum_{j=0,j \neq i}^l x_{ij}^k &= 1, \sum_{k=1}^n \sum_{i=0,i \neq j}^l x_{ij}^k = 1 \quad \forall i, j \in (1, 2, \dots, l) \\ \sum_{i=1}^l x_{ij}^k - \sum_{i=1}^l x_{ji}^k &= 0 \quad \forall j \in (1, 2, \dots, l), k \in (1, 2, \dots, n) \\ \sum_{i=1}^l (\alpha_i \theta \sum_{j=0,j \neq i}^l x_{ij}^k) &\leq \Omega \quad \forall j \in (1, 2, \dots, l) \\ x_{ij}^k \begin{cases} \geq 0, & d_{ij} \leq D \\ = 0, & d_{ij} \geq D \end{cases} \quad \forall i, j \in (1, 2, \dots, l) \end{aligned}$$

Where:  $d_{ij}$  is the distance of the affected spot  $i$  to  $j$  ( $i = 1, 2, \dots, l; j = 1, 2, \dots, l$ ).  $t$  is the per kilometer distance transportation time of emergency materials transportation vehicle  $k$  ( $k = 1, 2, \dots, n$ ).  $\alpha_i$  is the number of materials that the affected spot  $i$  required ( $i = 1, 2, \dots, l$ ).  $\varepsilon$  (The unit is  $h$ ) for packaging and handling per piece of emergency relief materials time;  $\theta$  (The unit is  $kg$ ) is the quality of per emergency relief material.  $\Omega$  (The unit is  $t$ ) is the maximum transport capacity of emergency relief vehicle ( $k = 1, 2, \dots, n$ ).  $D$  is the maximum scope threshold of the materials distribution spot 0 to the affected spot  $i$  ( $i = 1, 2, \dots, l$ ).

Where:  $x_{ij}^k$  is the decision variable of 0 – 1, when the emergency materials transportation vehicle  $k$  moves from

the affected spot  $i$  to  $j$ ,  $x_{ij}^k = 1$  otherwise  $x_{ij}^k = 0$  ( $k = 1, 2, \dots, n; i = 1, 2, \dots, l; j = 1, 2, \dots, l$ ).

## III. THE ALGORITHM OF DESIGN

### A. K-means clustering algorithm

1) *The theory of the algorithm:* According to the reference [15], K-means is a clustering algorithm which was widely used. The goal is to sample is divided into  $K$  clusters according to certain similarity, so that each cluster sample have high similarity, and find out the corresponding centroid of each cluster sample. The quadratic sum of range clustering criterion is used as evaluation function in K-means clustering algorithm, the definition of quadratic sum of range clustering criteria are defined as follows:

$$J_{(a)} = \sum_{j=1}^K \sum_{i=1}^{N_j} \|Y_{ij} - X_j\|^2, Y_{ij} \in \Pi_j$$

Where:  $a$  in parentheses is the iteration number of clustering process;  $\Pi_j$  is the  $j$  cluster, the cluster centroids are  $X_j$ ;  $N_j$  is the number of objects contained in the  $j$  cluster,  $Y_{ij}$  represents the sample  $i$  which are assigned to the  $j$  cluster.

First of all, data  $K$  were randomly selected as the initial cluster centroids from the  $N$  sample data; and then we calculate the quadratic sum of range from the rest of sample data to the  $K$  cluster centroids. The lower the value of quadratic sum of range, the greater the similarity of sample data, the sample data is assigned to cluster of the largest similarity with cluster centroids. Next, in order to obtain the cluster centroid we calculate the mean value of all samples in each cluster. Keep repeating this process, until the clustering criterion function converges.

#### 2) The steps of K-means clustering algorithm:

Step1) When  $a = 1, X_1(a), X_2(a), \dots, X_K(a)$  randomly selected as initial cluster centroids.

Step2) According to the principle of minimum distance, sample data  $Y_i$  are assigned to the  $K$  cluster centroids. That is: if  $\min \|Y_i - X_j(a)\| = \|Y_i - X_j(a)\|$ , then  $Y_i \in \Pi_j$ ,  $Y_i \leftarrow Y_{ij}$  ( $i = 1, 2, \dots, N; j = 1, 2, \dots, K$ ).

Step3) Recalculate cluster centroid of each cluster:

$$X_j(a) = \frac{1}{N_j} \sum_{i=1}^{N_j} Y_{ij} \quad \forall k \in 1, 2, \dots, K$$

Step4) If  $|J_{(a)} - J_{(a-1)}| < \xi$  (where  $\xi$  is the infinitesimal value), the algorithm ends; otherwise it returns to Step2), The sample data will be reclassified, and iteratively recalculate  $a \leftarrow a + 1$ .

### B. Particle Swarm Optimization (PSO)

1) *The theory of the algorithm:* According to the reference [16] [17], PSO is a global search algorithm that simulates the biological activities of the group in the natural

world, each individual particle represents a candidate solution of the problem. The particle comprises two vectors: the velocity vector  $v_i = [v_i^1, v_i^2, \dots, v_i^d]$  and the position vector  $x_i = [x_i^1, x_i^2, \dots, x_i^d]$  in the process of evolution, where:  $i$  is the number of individual particle,  $d$  is the dimension of problem solution.

$v_i$  determines the direction and velocity of the particle  $i$  in a current movement,  $x_i$  determines the position of the solution that the particle  $i$  representd in the current solution space. In addition, the position of individual particle is affected by two aspects which themselves-cognition and social environment, that is affected by the historical optimal position of the individual particle in the process of movement and the optimal particle of the swarm in the global scope ( $pBest$  is the historical optimal position of individual particle;  $gBest$  is optimal particle in the global scope).

Velocity and position of a particle is updated with the formula as follows:

$$v_i^d = \omega \times v_i^d + c_1 \times r_1 \times (pBest_i^d - x_i^d) + c_2 \times r_2 \times (gBest^d - x_i^d)$$

$$x_i^d = x_i^d + v_i^d$$

Where:  $\omega$  is the inertia weight, 0 represents the degree that the current position impact on the next position of particles;  $c_1, c_2$  is the learning factor, respectively represent the degree that the individual particle influenced by itself-cognition and social environment;  $r_1, r_2$  are two random numbers on the interval  $[0, 1]$ .

2) *The particle coding*: This paper, according to the characteristics of emergency relief VRP, adopts the integer coding. is the emergency relief materials distribution spot;  $1, 2, \dots, l$  are all the affected spots. We assume that there are  $k$  vehicles at the same distribution spot, there are  $k$  distribution routings, and each routing begins with distribution spot, and terminates at the distribution spot.

Here, PSO is used to increase  $k$  virtual distribution spots, namely increase  $k$  distribution spots 0 coding. Therefore, there are distribution  $k + 1$  spots and  $1, 2, \dots, l$  which constitute  $k + l + 1$  integers. Its beginning is 0 and its ending is the same as the beginning, discontinuously appear 0, corresponding one emergency relief vehicle routing schemes. Such as: the thirteen integers 0 1 4 3 0 6 4 0 5 8 7 9 0 represent that the arrangement of emergency relief vehicle routings of three vehicles corresponding to nine affected spots at the same distribution spot 0.

3) *Particle evaluation*: In order to evaluate the merits of the solution, firstly the individual particle was decoded to obtain the solution which correspond vehicle routing scheme. Secondly, we judge whether the vehicle routing scheme meet the constraint conditions of emergency relief. Third, the fitness value was calculated by the fitness function:

$$F = \sum_{k=1}^n \sum_{i,j=0, i \neq j}^l d_{ij} x_{ij}^k t + \sum_{i=1}^l \alpha_i \varepsilon$$

Based on all the certain constraints, the smaller the fitness function values, the higher the quality of solution is the better the emergency relief vehicle routing schemes.

#### 4) The steps of PSO:

Step1) Initializing all kinds of parameters, such as learning factor  $c_1, c_2$ ; inertia weight  $\omega$ ; the size of particle swarm  $Psize$ ; the maximum number of iterations  $Mgen$ .

Step2) Randomly initializing the particle swarm, namely initializing and coding all kinds of emergency relief vehicle numbers and the vehicle routings, we can get the particle's initial position and velocity. We put the historical optimal position  $pBest$  of individual particle as current position, the optimal particle in the global scope as the current  $gBest$ .

Step3) Calculating all particles fitness value and judging the individual particles according to constraint conditions.

Step4) Comparing all individual particles compared with  $pBest$  and  $gBest$ , to decide whether to update the  $pBest$  and  $gBest$  according to the results of comparison.

Step5) Updating the velocity and position of each individual particle.

Step6) If the iteration is not bigger than the maximum number of iterations  $Mgen$ , it returns to Step3); otherwise, the algorithm ends, it outputs  $gBest$ .

Step7) Decoding the optimal individual particle position, we will get the optimal emergency relief vehicle routing scheme.

#### C. The Algorithm flow chart based on K-means clustering and PSO

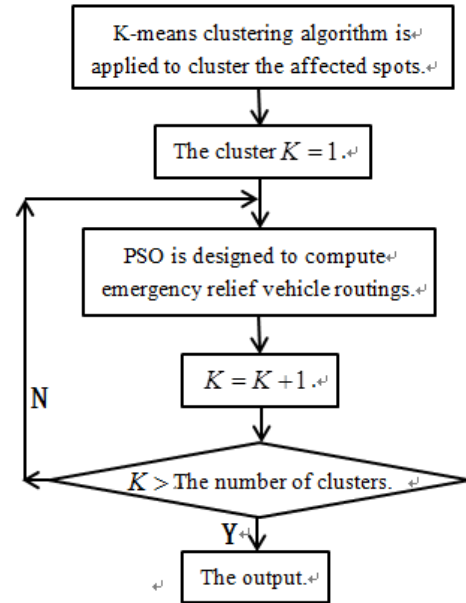


Figure 1. The Algorithm flow chart based on K-means clustering and PSO

Table I  
THE COORDINATES AND THE QUANTITY OF REQUIRED MATERIALS OF AFFECTED SPOTS

The NO. of affected spot	The coordinates	The quantity of materials	The NO. of affected spot	The coordinates	The quantity of materials
1	(40,60)	200	21	(80,290)	300
2	(20,80)	400	22	(90,210)	200
3	(70,180)	200	23	(130,100)	200
4	(100,90)	300	24	(280,140)	300
5	(80,250)	300	25	(210,80)	200
6	(90,70)	600	26	(60,100)	300
7	(50,20)	400	27	(270,60)	300
8	(40,270)	300	28	(90,100)	400
9	(120,60)	300	29	(240,40)	400
10	(110,40)	200	30	(120,140)	200
11	(200,90)	400	31	(250,110)	300
12	(250,150)	300	32	(80,120)	400
13	(70,60)	200	33	(60,250)	300
14	(40,20)	300	34	(85,80)	300
15	(270,90)	300	35	(50,40)	200
16	(60,30)	300	36	(70,270)	300
17	(40,120)	200	37	(260,190)	300
18	(80,40)	300	38	(50,80)	500
19	(60,210)	400	39	(190,120)	300
20	(230,100)	400	40	(80,180)	200

Table II  
THE COORDINATES AND THE QUANTITY OF REQUIRED MATERIALS OF AFFECTED SPOTS

The NO. of vehicle	Vehicle Routing	Transport time	The NO. of vehicle	Vehicle Routing	Transport time
NO.1	A0-28-30-23-4-34-A0	6.424h	NO.6	B0-19-3-40-22-5-B0	5.914h
NO.2	A0-9-10-18-A0	5.557h	NO.7	B0-33-8-36-21-B0	5.038h
NO.3	A0-13-16-7-14-35-A0	5.734h	NO.8	C0-31-12-37-24-C0	6.302h
NO.4	A0-1-2-38-A0	4.637h	NO.9	C0-15-27-29-C0	4.793h
NO.5	A0-26-17-32-A0	4.654h	NO.10	C0-25-11-39-20-C0	5.304h

#### IV. THE EXAMPLE SIMULATION

The example simulation supposes that there are  $n = 10$  vehicles to transport emergency relief materials in all the affected spots. The maximum transport capacity of each vehicle  $\Omega = 15t$ . The maximum scope threshold value  $D = 60km$ . Per kilometer distance transportation time of each vehicle  $t = 0.02h/km$ . The quality of per emergency relief material / item. Packaging and handling per piece of emergency relief supplies time  $\theta = 10kg/item$ . In the rectangular plane coordinate system of  $300km \times 300km$ , randomly generating the coordinates of 40 affected spots and the corresponding quantity of required emergency relief materials are listed in the table I.

The formulation is programmed by MATLAB, K-means clustering algorithm is designed to write the corresponding procedures based on the above , to get coordinates of distribution spots in the three affected areas cluster-s:A0(75,75); B0(70,250);C0(250,100). And then, PSO is designed respectively to write the corresponding procedures in all the affected areas. We can approximately get optimal schemes of example: the transport time of affected areas A is 27.006h, the transport time of affected areas B is 10.952h, the transport time of affected areas C is 16.399h, the transport time of all emergency relief materials is 54.36h. The optimal emergency vehicle routings are listed in the table II and the figure 2.

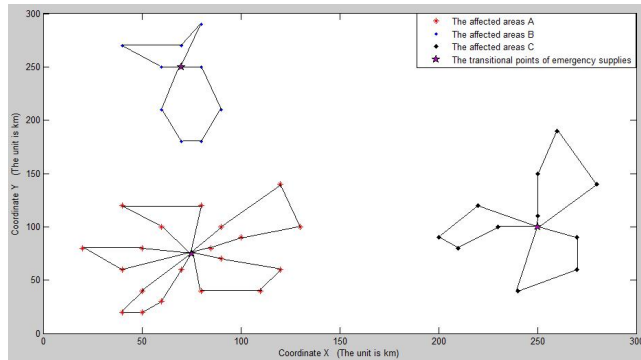


Figure 2. The optimal scheme of emergency relief VRP

## V. CONCLUSION

Emergency relief materials distribution is an important part of optimizing the emergency logistics system in the post-disaster. The accuracy of the relief vehicle routing plays an important role in saving the loss. The innovation of this paper, based on the concern that a large quantity of affected spots, distribution relatively concentrated and a small quantity of emergency relief materials in the affected area, we try to figure out an optimized mathematical model with the goals of the shortest transport time in emergency materials allocation and study a new type of emergency relief materials vehicle routing optimization scheme with the combination of K-means clustering and PSO. First of all, K-means clustering algorithm is adopted to get some of local distribution centers and their distribution scope. And then, we can get approximately optimal schemes which the PSO is designed respectively to write the corresponding procedures in all the affected areas. Finally, the computational results of optimal scheme of emergency relief VRP indicates the feasibility and validity of the model and new algorithm.

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