We have the following linearized equations:

$$AX_{t+1} + BX_t + CY_t + DZ_t = 0 \tag{1}$$

$$FX_{t+2} + GX_{t+1} + HX_t + JY_{t+1} + KY_t + LZ_{t+1} + MZ_t = 0 \tag{2}$$

$$X_{t+1} = PX_t + QZ_t \tag{3}$$

$$Y_t = RX_t + SZ_t \tag{4}$$

Working on first equation:

$$A(PX_t + QZ_t) + BX_t + C(RX_t + SZ_t) + DZ_t = 0$$

$$(AP + B + CR)X_t + (AQ + CS + D)Z_t = 0$$

Working on second equation:

$$F(PX_{t+1} + QZ_{t+1}) + GX_{t+1} + HX_t + J(RX_{t+1} + SZ_{t+1}) + KY_t + LZ_{t+1} + MZ_t = 0$$

$$(FP + G + JR)X_{t+1} + HX_t + KY_t + (FQ + L + JS)Z_{t+1} + MZ_t = 0$$

$$(FP + G + JR)(PX_t + QZ_t) + HX_t + K(RX_t + SZ_t) + (FQ + L + JS)NZ_t + MZ_t = 0$$

$$[(FP + G + JR)P + H + KR]X_t + [(FP + G + JR)Q + KS + (FQ + L + JS)N + M]Z_t = 0 \tag{5}$$

This gives the following conditions which allow us to solve for P, Q, R, and S.

$$AP + B + CR = 0 \tag{6}$$

$$AQ + CS + D = 0 \tag{7}$$

$$(FP + G + JR)P + H + KR = 0 \tag{8}$$

$$(FP + G + JR)Q + KS + (FQ + L + JS)N + M = 0 \tag{9}$$

Solve (6) for R and substitute this into (8). Rearranging gives a matrix quadratic in P. Solving for P then gives R.

$$R = -C^{-1}(AP + B)$$

$$FP^2 + GP + JRP + H + KR = 0$$

$$FP^2 + GP + H + J[-C^{-1}(AP + B)]P + K[-C^{-1}(AP + B)] = 0$$

$$FP^2 + GP + H - JC^{-1}(AP + B)]P - KC^{-1}(AP + B) = 0$$

$$FP^2 + GP + H - JC^{-1}AP^2 - JC^{-1}BP - KC^{-1}AP - KC^{-1}B = 0$$

$$(F - JC^{-1}A)P^2 + (G - JC^{-1}B - KC^{-1}A)P + (H - KC^{-1}B) = 0 \tag{10}$$

$$\mathbf{A}P^2 + \mathbf{B}P + \mathbf{C} = 0$$

$$\mathbf{A} = (F - JC^{-1}A)$$

$$\mathbf{B} = (G - JC^{-1}B - KC^{-1}A)$$

$$\mathbf{C} = (H - KC^{-1}B)$$

Solve (7) for S and substitute this into (9). Rearranging gives a Sylvester equation in Q. Solving for Q then gives S.

$$
\begin{aligned}
&S = -C^{-1}(AQ + D)\\
&FPQ + GQ + JRQ + KS + FQN + LN + JSN + M = 0\\
&FPQ + GQ + J[-C^{-1}(AP + B]Q + K[-C^{-1}(AQ + D)] + FQN + LN\\
&\quad + J[-C^{-1}(AQ + D)]N + M = 0\\
&FPQ + GQ - JC^{-1}APQ + -JC^{-1}BQ - KC^{-1}AQ - KC^{-1}D\\
&\quad + FQN + LN - JC^{-1}AQN - JC^{-1}DN + M = 0\\
&(FP + G - JC^{-1}AP - JC^{-1}B - KC^{-1}A)Q + (F - JC^{-1}A)QN\\
&\quad = KC^{-1}D - LN + JC^{-1}DN - M\\
&(F - JC^{-1}A)^{-1}(FP + G - JC^{-1}AP - JC^{-1}B - KC^{-1}A)Q + QN\\
&\quad = (F - JC^{-1}A)^{-1}(KC^{-1}D - LN + JC^{-1}DN - M)
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&\mathbf{D}Q + Q\mathbf{E} = \mathbf{F}\\
&\mathbf{D} = (F - JC^{-1}A)^{-1}(FP + G - JC^{-1}AP - JC^{-1}B - KC^{-1}A)\\
&\mathbf{E} = N\\
&\mathbf{F} = (F - JC^{-1}A)^{-1}(KC^{-1}D - LN + JC^{-1}DN - M)
\end{aligned}
$$