

Econometrics, second part, 2022, questions for final exam.

1. In the context of GMM estimation with *overidentification*, let the moment conditions be $m_n(\theta) = \frac{1}{n} \sum_{t=1}^n m_t(\theta)$, and we assume that $E(m_t(\theta_0)) = 0$ (where the 0 is a vector of zeros). Assume that the covariance of $m_t(\theta_0)$ is Γ_0 . The moment contributions $m_t(\theta)$ are covariance stationary. Assume that moment contributions are not autocorrelated over time: $E[m_t(\theta_0)m'_{t-s}(\theta_0)] = 0, s > 0$.
 - (a) What is meant by “overidentification”?
 - (b) What is meant by “covariance stationary”?
 - (c) What is the form of Ω_n , the covariance matrix of $\sqrt{n}m_n(\theta_0)$? Give the steps used to find your answer.
 - (d) For the purpose of obtaining a consistent GMM estimator, is it strictly necessary to estimate Ω_n ? Explain why or why not.
 - (e) Carefully explain how Ω_n may be estimated consistently, taking into account the specific information given in this problem.
 - (f) Carefully explain how to implement the two step efficient GMM estimator.
2. Suppose that $\{y_i\} i = 1, 2, \dots, n$ is an i.i.d. sample of size n from the Poisson density with parameter λ_0 . The Poisson density is $f_y(y; \lambda) = \frac{e^{-\lambda} \lambda^y}{y!}$.
 - (a) Find the expression for the ML estimator, showing all steps.
 - (b) Verify that the ML estimator is asymptotically distributed as $\sqrt{n}(\hat{\lambda} - \lambda_0) \xrightarrow{d} N(0, \lambda_0)$, where λ_0 is the true parameter value. Hint: compute the asymptotic variance using $-\mathcal{J}_\infty(\lambda_0)^{-1}$.
 - (c) give a consistent estimator of the asymptotic variance of $\sqrt{n}(\hat{\lambda} - \lambda_0)$.
 - (d) explain how to test $H_0 : \lambda_0 = 2$ versus $H_A : \lambda_0 \neq 2$, using the likelihood ratio test.
3. Suppose we have the model

$$\begin{aligned}y_t &= \alpha_1 + \alpha_2 x_t + \epsilon_t \\ \epsilon_t &= u_t + \phi u_{t-1}\end{aligned}$$

where the shocks u_t are independent and identically distributed with mean zero and variance σ^2 . Suppose that $x_t = 2w_t + 1z_t + u_t$. Suppose the vector (w_t, z_t, u_t) is i.i.d. $N(0, I_3)$.

- (a) show that the regressor x_t is correlated with the error ϵ_t .
- (b) show that the regressor x_t is correlated with w_t and z_t .
- (c) explain in detail how to estimate the parameters α_1 and α_2 consistently and efficiently, using a GMM estimator that is based on the given information. Your answer should include a detailed explanation of:
 - i. the specific moment conditions you propose

- ii. a description of how to compute your proposed weight matrix
 - iii. a detailed description of the method of estimation
- (d) explain in detail how one may test the hypothesis $H_0 : \alpha_2 = 0$ versus $H_A : \alpha_2 \neq 0$.
4. In the context of GMM estimation with *exact identification*, let the moment conditions be $m_n(\theta) = \frac{1}{n} \sum_{t=1}^n m_t(\theta)$, and we assume that $E(m_t(\theta_0)) = 0$. Consider the possibility that moment function contributions may be correlated: $E(m_t m_{t-s}') = \Gamma_s$.
- (a) for the purpose of simply computing the GMM estimator, does the potential correlation of moment function contributions matter? Explain why or why not.
 - (b) for the purpose of testing hypotheses regarding θ_0 , does the potential correlation of moment function contributions matter? Explain why or why not.
5. A particular way of parameterizing the Pareto density is

$$f_X(x) = \begin{cases} \frac{\alpha}{x^{\alpha+1}}, & x \geq 1 \\ 0, & x < 1 \end{cases}$$

Suppose we have an independently and identically distributed sample of size n , $\{x_i\}, i = 1, 2, \dots, n$, where each x_i follows this Pareto distribution.

- (a) write the log likelihood function
 - (b) compute the maximum likelihood estimator of the parameter α .
 - (c) give a consistent estimator of the asymptotic variance of $\sqrt{n}(\hat{\alpha} - \alpha_0)$.
6. Are the following statements TRUE or FALSE?
- (a) the maximum likelihood estimator is unbiased
 - (b) the maximum likelihood estimator is asymptotically unbiased
 - (c) the maximum likelihood estimator is always more efficient than the GMM estimator
 - (d) the parameters of a stationary GARCH(1,1) model must satisfy some restrictions
 - (e) the parameters of a stationary ARCH(4) model must satisfy some restrictions
 - (f) for a correctly specified model estimated by ML, the individual observations' contributions to the score vector (the first derivative of the log-likelihood function) may be correlated with one another
 - (g) if we compute the Bayes information criterion (BIC) for a set of models, and the set includes the correctly specified model, the BIC will always be lowest for the correctly specified model
 - (h) maximizing the likelihood function gives the same result as maximizing the log-likelihood function

- (i) the Matlab function 'fminunc' will always find the global minimum of a function to be minimized
- (j) given a particular restriction on a parameter, it is possible to consistently estimate the parameters of an ARCH(1,1) model by ordinary least squares
- (k) For a linear regression model, if we estimate by instrumental variables estimation, in order for the estimator to be consistent, every instrument must be correlated with every regressor.
- (l) For a linear regression model, if we estimate by instrumental variables estimation, in order for the estimator to be consistent, every instrument must be uncorrelated with the error term.