IDEA, Econometrics II, 2022, final exam.

Answer all questions. There are 5 questions total, two of them are on the reverse side of the exam. You have 3 hours.

1. (25) Suppose that we have a rational expectations model. At time t, the representative agent knows the values of all variables indexed $t-1$ and earlier. Suppose the production function is $y_t = k_t^{\alpha} n_t^{1-\alpha} \exp(\epsilon_t)$, where y is output, k is capital, n is labor, and ϵ is a white noise shock. One of the first order conditions tells us that the wage rate, w_t, is equal to the marginal product of labor (recall: MPL=$\partial y/\partial n$). The econometrician has a data set that includes only the variables k_t,n_t and w_t. The econometrician knows the correct specification of the production function, except for the value of α, and also knows that ϵ_t is a white noise shock. At time t, the agent (but not the econometrician) observes the shock ϵ_t, and the capital level, k_t, which is a non-stochastic function of variables that are pre-determined at time t, and then makes choices that determine the values of the variables k_{t+1},n_t, and w_t, among others, using the information that has been given, plus some other unspecified equations.
   (a) Consider the linear regression $\log w_t = c + \alpha(\log k_t - \log n_t) + \epsilon_t$. Will the OLS estimate of α using this regression be consistent or not? Explain.
   (b) Provide at least two moment conditions that can be used to consistently estimate the parameter α by GMM.
   (c) Explain why the proposed moment conditions have mean zero at the true parameter value.
   (d) Explain how you could estimate α using a two step efficient GMM estimator. In particular, explain in how to compute the optimal weight matrix, for your chosen moment conditions.

2. (25) Suppose that $\{y_i\}$ $i = 1,2,...,n$ is an i.i.d. sample of size n from the Poisson density with parameter λ_0. The Poisson density is $f_y(y;\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}$.
   (a) Find the expression for the ML estimator, showing all steps.
   (b) Given that the ML estimator is asymptotically distributed, so that $\sqrt{n}\left(\hat{\lambda} - \lambda_0\right) \xrightarrow{d} N(0, V_\infty)$, find a consistent estimator of V_∞.
   (c) explain how to test $H_0 : \lambda_0 = 2$ versus $H_A : \lambda_0 \neq 2$, using the <u>likelihood ratio test</u>. Give the test statistic, the asymptotic distribution, and the degrees of freedom of the test.

3. (15) Consider the panel data model $y_{it} = \alpha_i + x_{it}'\beta + z_i'\gamma + \epsilon_{it}$, $i = 1,2,...,n$ and $t = 1,2,...,T$. The only observed variables are y_{it} and the x_{it}. The effects α_i and the variables in z_i are unobserved. Explain how one can estimate the β parameters consistently, assuming that the unobserved variables z_i and α_i are correlated with the observed regressors x_{it}, for all t. Assume that x_{is}, z_i and α_i are independent of ϵ_{it}, for all $s,t = 1,2,...,T$.

4. (15) Consider the data generating process

$$Y_t = \alpha t + u_t$$
$$X_t = \gamma t + v_t$$

where $t = 1, 2, ..., T$, and u_t and v_t are two independent $N(0, 1)$ white noise processes. That is, both Y_t and X_t follow time trends, each with a different white noise shock. Consider the regression model *without constant* $Y_t = \beta X_t + \epsilon_t$.

   (a) To what value does the OLS estimator of β tend? Prove your result. Hints: recall that $\frac{1}{T^2} \sum_{t=1}^{T} t \to \frac{1}{2}$ and $\frac{1}{T^3} \sum_{t=1}^{T} t^2 \to \frac{1}{3}$. Also, the formula for the OLS estimator when there is no constant is $\hat{\beta} = \frac{\sum_t Y_t X_t}{\sum_t X_t^2}$.

   (b) Does this regression model exhibit the *spurious regression* problem? Explain why or why not.

5. (20) Consider the linear regression model $y_i = x_i'\beta_0 + \epsilon_i$. Suppose that the n observations satisfy all assumptions of the classical linear regression model, except they are heteroscedastic, so that $V(\epsilon_i|x_i) = \sigma_i^2$. Suppose that σ_i^2 is **known**, for all i. As such, we can compute the generalized least squares (GLS) estimator, which can be expressed as $\hat{\beta}_{GLS} = \left(X'\Sigma^{-1}X\right)^{-1} X'\Sigma^{-1}y$, where Σ is a diagonal matrix with σ_i^2 in position (i, i), $i = 1, 2, ..., n$.

   (a) Define moment conditions which define a GMM estimator that is equivalent to the GLS estimator. Prove that the estimators are equivalent.

   (b) Is your GMM estimator exactly identified, or overidentified?

   (c) For efficient estimation using the moment conditions which you have defined, is single step GMM estimation using an identity matrix as the weight matrix fully efficient, or will two step estimation be more efficient? Explain.