1 Introduction

This repository holds the solutions to a general set of growth models that incorporate the following explore the differences between

- One Agent vs Two Agent
- One Good vs Two Goods
- Time Additive Preferences vs Recursive Preferences
- Constant Volatility vs Stochastic Volatility

By solve the 16 possible versions of this model, we are able to isolate the effects of each bell and whistle that we include in the model.

2 One Agent One Good General Model

We will write our one agent general unscaled model as the following:

$$
\begin{aligned}
J_t(k_t, x_t, v_t) &= \max_{k_{t+1}} \left[(1-\beta)c_t^{\rho} + \beta\mu(J_{t+1}(k_{t+1}, x_{t+1}, v_{t+1}))^{\rho}\right]^{\frac{1}{\rho}} \\
&\text{Subject To} \\
c_t &= \left[\eta k_t^{\nu} + (1-\eta)z_t^{\nu}\right]^{\frac{1}{\nu}} + (1-\delta)k_t - k_{t+1} \\
x_{t+1} &= Ax_t + Bv_t^{\frac{1}{2}}\varepsilon_{1,t+1} \\
v_{t+1} &= (1-\phi_v)\bar{v} + \phi_v v_t + \tau\varepsilon_{2,t+1} \\
\log(z_t) &= \log(z_{t-1}) + \log(\bar{g}) + x_t
\end{aligned}
$$

We could then scale the model by dividing by z_t to give us (will not both writing tildes for now – everything here is divided by z_t)

$$
\begin{aligned}
J_t(k_t, x_t, v_t) &= \max_{\chi_t} \left[(1-\beta)c_t^{\rho} + \beta\mu(J_{t+1}(k_{t+1}, x_{t+1}, v_{t+1})g_{t+1})^{\rho}\right]^{\frac{1}{\rho}} \\
&\text{Subject To} \\
c_t &= \left[\eta k_t^{\nu} + (1-\eta)\right]^{\frac{1}{\nu}} + (1-\delta)k_t - \chi_t \\
\chi_t &= k_{t+1}g_{t+1} \\
x_{t+1} &= Ax_t + Bv_t^{\frac{1}{2}}\varepsilon_{1,t+1} \\
v_{t+1} &= (1-\phi_v)\bar{v} + \phi_v v_t + \tau\varepsilon_{2,t+1} \\
\log(g_t) &= \log(z_t) - \log(z_{t-1}) = \log(\bar{g}) + x_t
\end{aligned}
$$

Notice we can get constant volatility by allowing $\tau = \phi_v = 0$ or time separable preferences by $\rho = \alpha$.

3 One Agent Two Goods General Model

4 Two Agent One Good General Model

5 Two Agent Two Goods General Model