1 Starting from Dave's Exchange notes

Start from Dave's exchange economy notes, but add back heterogeneity in agent parameters $(\rho, \alpha, \beta, \omega, \sigma)$. Combine the FOC in a_1 and a_2 to obtain

$$\frac{w_1}{w_2} = \frac{(1-\beta_2)c_2^{\rho_2-\sigma_2}\omega_2 a_2^{\sigma_2-1}}{(1-\beta_1)c_1^{\rho_1-\sigma_1}(1-\omega_1)a_1^{\sigma_1-1}} \tag{1}$$

Also combine FOC in b_1 and b_2:

$$\frac{w_1}{w_2} = \frac{(1-\beta_2)c_2^{\rho_2-\sigma_2}(1-\omega_2) b_2^{\sigma_2-1}}{(1-\beta_1)c_1^{\rho_1-\sigma_1}\omega_1 b_1^{\sigma_1-1}} \tag{2}$$

Combine these two to obtain the equation he has in the middle of the page:

$$\frac{\omega_2 a_2^{\sigma_2-1}}{(1-\omega_1)a_1^{\sigma_1-1}} = \frac{(1-\omega_2)b_2^{\sigma_2-1}}{\omega_1 b_1^{\sigma_1-1}}$$

For now let's assume that $\sigma_1 = \sigma_2 = \sigma$ and define $s_a := a_1/y_1$ and $s_b = b_1/y_2$. We can now derive

$$\left(\frac{s_b}{1-s_b}\right)^{\sigma-1} = \frac{(1-\omega_1)(1-\omega_2)}{\omega_1\omega_2}\left(\frac{s_a}{1-s_a}\right)^{\sigma-1},$$

which can be solved in closed form for

$$s_b = \frac{\left(\frac{(1-\omega_1)(1-\omega_2)}{\omega_1\omega_2}\right)^{\frac{1}{\sigma-1}}\frac{s_a}{1-s_a}}{1+\left(\frac{(1-\omega_1)(1-\omega_2)}{\omega_1\omega_2}\right)^{\frac{1}{\sigma-1}}\frac{s_a}{1-s_a}}. \tag{3}$$

Then, for a given $w_1/w_2 := \spadesuit$, $y_1 = 1$ and $y_2 = z_2/z_1$ we can use (1) as a residual to search over s_a and we can use (2) as the needed residual to form the RHS of the regression that updates $\spadesuit_t$ to $\spadesuit_{t+1}$.

2 Algorithm

Now the computation algorithm will be presented. As an initialization phase, do the following:

- Choose a large simulation length T
- Choose an initial state $\spadesuit_0$ and $z_{2,0}/z_{1,0}$.

- Simulate the exogenous process forward to obtain $\{z_{2,t}/z_{1,t}\}_{t=1}^{T}$
- Choose an initial coefficient vector $b = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix}$ in the equation

$$\spadesuit_{t+1} = b_0 + b_1 \spadesuit_t + b_2(\log z_{2,t} - \log z_{1,t})$$

Then, one iteration of the simulation algorithm is carried out as follows:

1. Use the coefficients b, simulated history for z_2/z_1, and the initial condition $\spadesuit_0 = 1$ to simulate $\{\spadesuit_t\}_{t=1}^{T}$
2. Using this history, use equations (1) and (3) so solve for $\{s_{a,t}, s_{b,t}\}_{t=1}^{T}$
3. Given $s_{a,t}, s_{b,t}$, and $z_{2,t}/z_{1,t}$ we know $y_{1,t} = 1$, $y_{2,t} = z_{2,t}/z_{1,t}$, $a_{1,t} = s_{a,t}y_{1,t}$, $a_{2,t} = (1 - s_{a,t})y_{1,t}$, $b_{1,t} = s_{b,t}y_{2,t}$, $b_{2,t} = (1 - s_{b,t})y_{2,t}$, $c_{1,t} = h_1(a_{1,t}, b_{1,t})$, and $c_{2,t} = h_2(a_{2,t}, b_{2,t})$. We can use (2) to form an updated guess for what $\spadesuit_t$ should be.
4. Given this updated guess we can now run the following regression:

$$\hat{\spadesuit}_{t+1} = \hat{b}_0 + \hat{b}_1 \tilde{\spadesuit}_t + \hat{b}_2(\log z_{2,t} - \log z_{1,t})$$

   where $\hat{\spadesuit}_{t+1}$ is obtained in step 1 from using the old coefficient vector and $\tilde{spadesuit}_t$ is the lefthand side of (2) as computed in step 3.

5. Then say that the new coefficient vector is

$$b = (1 - \xi)\hat{b} + \xi b,$$

   where $\hat{b}$ is the new coefficient vector from the previous step and b is the coefficient vector in hand when you simulated forward in step 1.

6. Finally, check convergence of the time series of $\{\spadesuit_t\}$ computed on successive iterations