Our empirical modeling strategy differs deliberately from both the VAR (and SVAR) literature and the approach based on an attempt to estimate a cointegrating relationship and then to analyze the dynamics of the residuals from that relationship. Although both of those literatures have made important contributions, for our purposes they may obscure as much as they reveal about the key economic relationships we are interested in.

In this appendix, we begin by showing that, under the assumptions of our model, a particular cointegrating relationship should in fact exist if all elements of the economic environment are perpetually unchanging. However, in an economy with slow but continuous change in many of the 'deep parameters' that determine the cointegrating vector, serious errors of inference can be made by assuming that the cointegrating vector is constant when it is not. In particular, we show that the usual procedure of first estimating a cointegrating vector (assumed to be constant over the sample), then analyzing the dynamics of its residuals, leads to serious errors if the actual data are generated by our structural model. The nature of the problems of the cointegrating approach is not specific to our model, but generic. Specifically, the problem is that the time span of the available data is insufficient to reliably identify the cointegrating vector when there are omitted influences (like the increasing availability of credit over our sample period). The problem is that in a finite sample, the first stage cointegrating regression inevitably will be biased in the presence of an important omitted variable, and that bias then contaminates the residuals, imparting to them distorted dynamics that do not reflect the true characteristics of the model.

We then show that, using the same data (generated by the model), our estimation methodology succeeds where the cointegration approach fails. That is, our procedure is able to successfully uncover the 'true' characteristics of the model generating the data from the simulated data from the model itself.

0.1 Cointegrating Vector Implied By the Model

Carroll (1992) shows that a model of the class considered here, in which a target wealth-to-permanent-income ratio exists, will satisfy the equation

$$s \;\approx\; \gamma(\check{m} - 1) \tag{1}$$

where γ is the 'permanent' growth rate of income. In the usual cointegrating methodology, the knowledge that such a cointegrating vector in principle exists would lead to the following approach to understanding saving dynamics:

1. Estimate $s_t = \alpha_0 + \alpha_1(m_t - 1)$
2. Construct residuals $\epsilon_t = s_t - \hat{\alpha}_0 - \hat{\alpha}_1(m_t - 1)$

3. Analyze the dynamics of the residuals ϵ_t and the $\hat{\alpha}$ to discover answers to the questions of interest motivating the analysis.

We have conducted exactly this exercise on the data generated by the estimated version of our model. (The model predictions for the saving rate that we use for this analysis are those depicted, for example, in Figure 12 in the paper).

The first test of the success of the strategy is whether estimation of the cointegrating vector correctly uncovers the true cointegrating vector. In this case, that vector is $\alpha_0 = 0$ and $\alpha_1 = \gamma$. In fact, the coefficient estimates are highly statistically significantly different from the 'truth' that we know because we constructed the model ourselves.

The second test of the success of the strategy is whether the dynamics of the ϵ_t residuals are a reasonable approximation to the 'true' dynamics of saving residuals in the model. Some judgment is required to determine how to answer this question, but a plausible method would be to identify the 'true' dynamics of the saving residual as reflecting the serial correlation of the saving rate predicted by the model over the sample period. That is, again referring to figure 12, we can calculate the serial correlation of $s_t - \hat{s}_t$ where $\hat{s}_t$ is the model-predicted saving rate.

Again, the results from the cointegrating approach do not resemble the 'truth' as interpreted in the model. As hinted above, the reason for this is that the first-stage regression misidentifies the residuals because the sample size is small relative to the time frame needed to reestablish the long-run relationship between saving and wealth.

In contrast, when we apply our SMM estimation procedure to the data simulated by our model, we correctly recover what we know are the 'true' coefficient estimates (becuase they are the coefficient estimates used in simulating and constructing the data).

It might be objected that this treatment sets up and then knocks down a 'straw man' version of the cointegrating approach; and indeed the best of the practitioners of that approach (like Duca, Muellbauer, and Murphy (2010) and Muellbauer in a number of papers) have attempted to adjust the approach to account (for example) for movements in credit availability over time. It is possible that sufficiently careful (and properly specified) cointegration approaches might remedy the deficiencies we articulate above.

But our method has the appeal of great simplicity, and is at least able to reproduce the correct structural coefficients with confronted with data in which those coefficients are embedded, and in a context in which the standard cointegration methodology fails. Further research may uncover ways to integrate our approach better with the cointegration methodology, but until that research is conducted the answer is unclear (at least to us).

References

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