Appendix 3: Extension of Derivation of Target Wealth to Include Unemployment Insurance

The model described in Carroll and Toche (2009) assumes that income for unemployed/retired households is zero. A step in the direction of realism is to recognize the existence (in most countries) of an unemployment insurance system that guarantees some level of income to unemployed persons. The implications of such a system are straightforward to model if we assume that the unemployment insurance benefit is a constant proportion of the labor income earned in the first year of unemployment if not losing job.

In the perfect foresight context, receiving a constant payment with perfect certainty is equivalent to receiving a lump sum "severance" payment whose value is equal to the PDV of the stream of future UI payments. Thus, for simplicity, we assume $\mathcal{S} = \varsigma \cdot \ell \mathsf{W}$, which means individuals will receive one-period severance payment $\mathcal{S}$ in the amount of a certain ratio ς to labor income of the period when they first lose their jobs. After that, they will not receive any unemployment insurance benefit.

The only modifications of the decision problem are to add the severance payment and a corresponding lump-sum tax into the dynamic budget constraint (DBC) of employed consumers in Carroll and Toche (2009),

$$m_{t+1} = \begin{cases} b_{t+1} + \ell_{t+1}\mathsf{W}_{t+1} - \tau_{t+1} & \text{w.p. } \mho \\ b_{t+1} + \mathcal{S}_{t+1} & \text{w.p. } 1-\mho, \end{cases}$$

where m, b and $\ell\mathsf{W}$ denote market resources, assets and labor income respectively. We let $\tau = \mho \times \mathcal{S}$ to ensure a balanced budget for the unemployment insurance system.[1]

Following Carroll and Toche (2009), we have the following condition derived from the Euler equation,

$$1 = \Gamma^{-\rho}\mathsf{R}\beta \left\{ (1-\mho)\left(\frac{c^e_{t+1}}{c^e_t}\right)^{-\rho} + \mho\left(\frac{c^u_{t+1}}{c^e_t}\right)^{-\rho} \right\}, \tag{1}$$

where nonbold variables represent the bold variables normalized by labor income $\ell\mathsf{W}$. Superscripts e and u represent the two possible states.

To find the $\Delta c^e = 0$ and $\Delta m^e = 0$ loci, we let $c^e_{t+1} = c^e_t \equiv c^e$ and $m^e_{t+1} = m^e_t \equiv m^e$. Given $c^u_{t+1} = m^u_{t+1}\kappa^u$ (κ^u is the MPC of an unemployed consumer), combined with

[1]Each period, proportion $\mho$ of employed consumers lose their jobs, i.e., the "exit rate" in the current labor market is $\mho$. In order to raise a corresponding amount of revenues, we need to assume that there is a "birth rate" of $\mho$ of new employed consumers, which means a same proportion of consumers are entering the labor market each period. Combined with the assumption that $\tau = \mho \times \mathcal{S}$, the severance payment and the severance payment are balanced.

the modified DBC above, we have

$$1 = \Gamma^{-\rho}\mathsf{R}\beta\left\{(1-\mho)+\mho\left(\frac{\kappa^u(\mathcal{R}(m^e-c^e)+\varsigma)}{c^e}\right)^{-\rho}\right\}.$$

Rearranging terms, the $\Delta c^e = 0$ locus can be characterized as

$$\overbrace{\left(\frac{\Gamma^{\rho}(\mathsf{R}\beta)^{-1}-\cancel{\mho}}{\mho}\right)^{1/\rho}}^{\Pi} = \left(\frac{c^e}{(\mathcal{R}(m^e-c^e)+\varsigma)\kappa^u}\right). \tag{2}$$

Given the modified DBC of employed consumers, the $\Delta m^e = 0$ locus becomes

$$m^e = \mathcal{R}(m^e-c^e)+(1-\mho\varsigma). \tag{3}$$

Given the two equations above, we are able to obtain the exact formula for target wealth $\check{m}$, which is the steady state value of m^e. Following Carroll and Toche (2009), define $\eta \equiv \mathcal{R}\kappa^u\Pi$. We have

$$\begin{aligned}
\frac{\eta\check{m}+\frac{\eta\varsigma}{\mathcal{R}}}{\eta+1} &= (1-\mathcal{R}^{-1})\check{m}+\frac{1-\mho\varsigma}{\mathcal{R}} = \check{c} \\
\left(\frac{1}{\mathcal{R}}-\frac{1}{\eta+1}\right)\check{m} &= \frac{1}{\mathcal{R}}\left(1-\mho\varsigma-\frac{\eta\varsigma}{\eta+1}\right) \\
\check{m} &= \frac{(\eta+1)(1-\mho\varsigma)-\eta\varsigma}{\eta+1-\mathcal{R}}. \qquad (4)
\end{aligned}$$

Clearly, target wealth decreases when the severance payment becomes more generous and it can even be negative if we make the severance ratio ς large enough.

References

Carroll, Christopher D., and Patrick Toche (2009): "A Tractable Model of Buffer Stock Saving," *NBER Working Paper Number 15265*, http://econ.jhu.edu/people/ccarroll/papers/ctDiscrete.