Numerical Analysis Homework #7

due 2020 MAY 26, 9:50 a.m.

1 Assignments

Caution:

- To get full credit, *you must write down sufficient intermediate steps*, only giving the final answer earns you no credit!
- Please make sure that your handwriting is recognizable, otherwise you only get partial credit for the recognizable part.

I. If the bisection method is used in single precision FPNs of IEEE 754 starting with the interval $[128, 129]$, can we compute the root with absolute accuracy $< 10^{-6}$? Why?

II. What are the condition numbers of the following functions? Where are they large?

- $(x-1)^{\alpha}$,
- $\ln x$,
- e^x,
- $\arccos x$.

III. The last Exercise in Section 1.3.5 in the notes.

IV. Consider the function $f(x) = 1 - e^{-x}$ for $x \in [0, 1]$.

- Show that $\mathrm{cond}_f(x) \le 1$ for $x \in [0, 1]$.
- Let A be the algorithm that evaluates $f(x)$ for the machine number $x \in \mathbb{F}$. Assume that the exponential function is computed with relative error within machine roundoff. Estimate $\mathrm{cond}_A(x)$ for $x \in [0, 1]$.
- Use `C++` to plot $\mathrm{cond}_f(x)$ and $\mathrm{cond}_A(x)$ as a function of x on $[0, 1]$. Discuss your results.

V. The math problem of root finding for a polynomial

$$q(x) = \sum_{i=0}^{n} a_i x^i, \qquad a_n = 1, a_0 \neq 0, a_i \in \mathbb{R} \tag{1}$$

can be considered as a vector function $f : \mathbb{R}^n \to \mathbb{C}$:

$$r = f(a_0, a_1, \ldots, a_{n-1}).$$

Derive the componentwise condition number of f based on the 1-norm. For the Wilkinson example, compute your condition number, and compare your result with that in the Wilkinson Example. What does the comparison tell you?

The above eight questions weigh 5, 8, 5, 12, 10 points, respectively, totaling 40 points.

2 Extra credits

Additional 10% credits will be given to you if you typeset your solutions in LaTeX. You are welcome to use the LaTeX template available on my webpage. You can also get partial extra credit for typesetting solutions of *some* problems.

Note: If you choose to typeset your solutions in LaTeX, you still need to turn in a hard copy in class. In addition, please upload your latex source (.tex), supporting files, and `C++` program in a single zip file (**format**: `YourName_Homework7.zip`) to the course email `NumApproximation@163.com`.