Time Series Analysis of GDP and Imports in the Central African Republic

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STA 137: Applied Time Series Analysis

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Introduction

The Central African Republic (CAR) is one of the poorest and least developed landlocked countries in the world (Mercereau, 2004). Despite its hot and humid climate, the country is rich in natural resources, including water, forests, flora and fauna, and minerals. Agriculture is the largest economic sector. Much of the rural population makes a living through livestock breeding, hunting, fishing, farming, and forestry (World Trade Organization, 2013). However, the lack of economic diversification leaves the country vulnerable to climate variability and political instability. Since gaining independence from French colonial rule in 1960, the CAR has encountered conflicts with neighboring countries, weak governance, and economic instability, which have contributed to declining life expectancy. In 2011, the main export products included diamond (60.7%) and wood (27.4%), while imported products were primarily manufactured goods (63.2%), such as transport equipment, along with agricultural raw materials and foodstuffs (33.4%). The CAR's trading partners span Europe, Africa, and Asia.

The current project uses a given dataset containing 58 observations and 9 economic variables, including GDP (Gross Domestic Product), GDP growth rate, CPI (Consumer Price Index), Imports, Exports, and Population. We focus on two variables: annual GDP in current US dollars and Imports as a percentage of GDP (World Bank Group, 2023b, 2023a). Understanding economic trends supports policymaking, international aid, and long-term development. We answer the following research questions:

- What are the best-fitting time series models for GDP and Imports in CAR, respectively?
- What do the forecasts look like over the next five years starting from 2018 to 2022?

Exploratory Data Analysis & Transformations

We plotted the time series, ACF (autocorrelation function), and PACF (partial autocorrelation function) of raw GDP and Imports data to identify unusual observations and visualize general patterns. As shown in Figure 1, both raw time series are non-stationary: while GDP shows an increasing trend, Imports decline initially, followed by a gradual rise. In time series analysis, stationarity is an important property that simplifies modeling and forecasting, assuming constant mean, variance, and autocovariance that only depends on lag. The ACF plots for both time series decay exponentially, suggesting future values depend heavily on past values. The PACF plots display a spike at lag 1, implying a potential AR(1) process.

To address non-stationarity, we applied a log-transformation followed by first-order differencing. First-order differencing helps remove trends and seasonality by computing change between consecutive values, while log transformation helps stabilize variance. Although Box-Cox transformation is another option for variance stabilization, we selected log-differencing due to its interpretability in economic contexts. Specifically, log-differenced GDP represents the GDP growth rate; log-differenced imports represent the growth rate of imports as a percentage of GDP. As shown in Figure 2, the transformed time series appears stationary with a more stable mean and variance. The ACF and PACF plots show no clear cutoff lags, suggesting further model selection is needed based on statistical criteria and residual analysis.

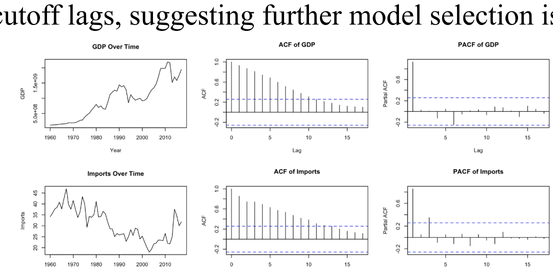


Figure 1: Time Series, ACF, and PACF Plots of Raw Data

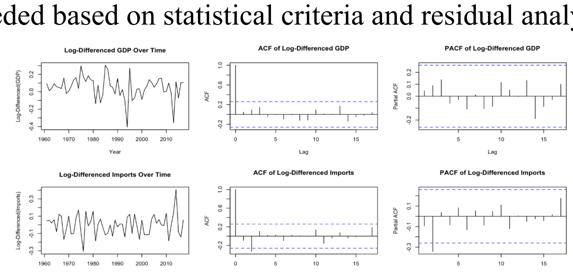


Figure 2: Time Series, ACF, and PACF Plots of Log-Differenced Data

Model Selection

We fitted a total of eight models for each time series, ranging from MA(1) to ARIMA (2,0,2), and evaluated model performance based on coefficient significance, AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), normal QQ plots of residuals, ACF of residuals, and p-values for the Ljung-Box statistic. ARIMA(p,d,q) model stands for AutoRegressive Integrated Moving Average, where AR(p) captures dependence on past values; I(d) represents the number of differencing steps used to achieve stationarity; and MA(q) models dependence on past errors. Because we preprocessed the data through log-differencing, the series is already stationary, and the integrated part I(d) is 0 in our case.

Our model selection prioritized significant coefficients, as statistical insignificance limits the interpretability regardless of performance on other metrics. AIC and BIC values are commonly used to compare statistical model fit: the lower the values, the better the model fits. Compared to BIC, which applies stricter penalties on model complexity, AIC favors more complex models with more parameters. While QQ plots examine the normality assumption of residuals, ACF plots help determine whether residuals resemble white noise. In time series analysis, it is assumed that white noise is normally distributed with a mean of 0 and a constant variance. Lastly, Ljung-Box measures if residual autocorrelation occurs by chance. P-values above the significant threshold suggest residuals are uncorrelated, and the model is adequate.

GDP Model Diagnostics

As shown in Table 1, AIC and BIC values are comparable across all candidate models. Residuals are normally distributed, resembling white noise and exhibiting no significant autocorrelation.

	MA(1)	AR(1)	MA(2)	AR(2)	ARIMA(1,0,1)	ARIMA(1,0,2)	ARIMA(2,0,1)	ARIMA(2,0,2)
Coefficients	Non-significant	Non-significant	Non-significant	Non-significant	Non-significant	Non-significant	Non-significant	All Significant
AIC	-1.16	-1.16	-1.13	-1.13	-1.13	-1.11	-1.11	-1.14
BIC	-1.05	-1.05	-0.99	-0.99	-0.99	-0.93	-0.93	-0.92
Gaussian Noise (QQ-Plot)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
White Noise (ACF)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ljung-Box Statistic	Well-behaved	Well-behaved	Well-behaved	Well-behaved	Well-behaved	Well-behaved	Well-behaved	Well-behaved

Table 1: GDP: Comparison of Model Selection Criteria

As shown in Figure 3, the standardized residuals appear stationary with constant mean and variance. Points in the ACF plot are within the blue confidence bounds, showing no significant lags beyond lag 0. The QQ plot shows that points closely align along the diagonal, suggesting normality. P-values for the Ljung-Box test are above the blue threshold, indicating residual autocorrelation is not significant and the model is adequate. We selected the ARIMA (2,0,2) model as the best fitted time series model, given that the p-values of the coefficients were smaller than the critical value ($\alpha = 0.1$).

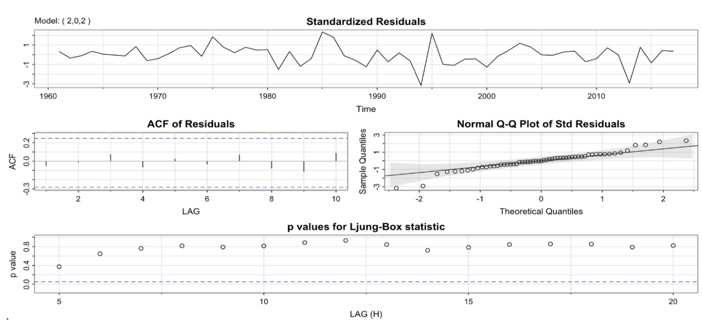


Figure 3: GDP: Final Model Diagnostics ARIMA (2,0,2)

The final model is: $y_t = 0.0508 + 1.5232 \cdot y_{t-1} - 0.9044 \cdot y_{t-2} - 1.5297 \cdot w_{t-1} + 1.0000 \cdot w_{t-2} + w_t$

Imports Model Diagnostics

Model selection for the Imports time series followed the same criteria as for GDP. While AIC and BIC values for the Imports time series are also similar across models, the AR(1) model showed several Ljung-Box p-values below the threshold, suggesting significant residual autocorrelation and inadequate model fit. In contrast, the ARIMA(1,0,1) model has one borderline p-value below the threshold, but all coefficients are statistically significant.

	MA(1)	AR(1)	MA(2)	AR(2)	ARIMA(1,0,1)	ARIMA(1,0,2)	ARIMA(2,0,1)	ARIMA(2,0,2)
Coefficients	Non-significant	Non-significant	Only ma2 significant	Only ar2 significant	All significant	Only ma2 significant	Only ar2 significant	Non-significant
AIC	-1.34	-1.33	-1.42	-1.42	-1.36	-1.41	-1.39	-1.37
BIC	-1.24	-1.22	-1.28	-1.28	-1.22	-1.23	-1.21	-1.16
Gaussian Noise (QQ-Plot)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
White Noise (ACF)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Ljung-Box Statistic	Well-behaved	Some below threshold	Well-behaved	Well-behaved	One below threshold	Well-behaved	Well-behaved	Well-behaved

Table 2: Imports: Comparison of Model Selection Criteria

As shown in Figure 4, residual diagnostics support the adequacy of the ARIMA (1,0,1) model. The residuals resemble white noise, with ACF showing no significant autocorrelation. The QQ plot indicates normality, and most Ljung-Box p-values exceed the blue threshold.

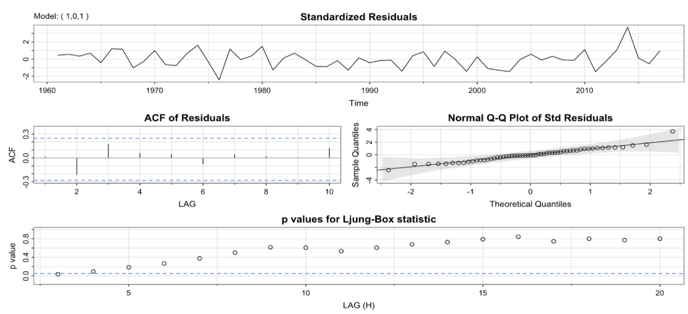


Figure 4: Imports: Final Model Diagnostics ARIMA (1,0,1)

The final model is: $y_t = -0.0075 + 0.7838 \cdot y_{t-1} - 1.0000 \cdot w_{t-1} + w_t$

Forecasting

To assess the short-term outlook of the CAR's economy, we applied the final selected ARIMA models to forecast the log-differenced GDP and Imports over a five year period (2018-2022). Log-differenced values approximate the compounded annual growth rates. Point forecasts, along with 80% and 95% confidence intervals, are presented in the Figures and Tables.

GDP Forecast

Using the ARIMA (2,0,2) model, we forecasted the log-differenced GDP time series. As shown in Figure 5, the forecast suggests a gradual decline in GDP growth rate, with the estimated growth rate decreasing from 7.34% in 2018 to 4.32% in 2022. However, the 95% confidence intervals are relatively wide, spanning over 40% of the growth rate in some years. Such uncertainty in the forecast reflects the economic instability of the CAR.

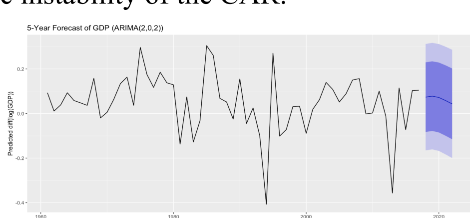


Figure 5: GDP: 5-Year Forecast of Log-Differenced GDP

Year	Point Forecast	80% CI	95% CI
2018	0.07	[-0.08, 0.23]	[-0.17, 0.31]
2019	0.08	[-0.08, 0.23]	[-0.16, 0.32]
2020	0.07	[-0.08, 0.23]	[-0.17, 0.31]
2021	0.06	[-0.10, 0.22]	[-0.18, 0.20]
2022	0.04	[-0.12. 0.20]	[-0.20, 0.29]

Table 3: GDP: 5-Year Forecast of Log-Differenced GDP

Imports Forecast

The log differenced Imports time series were predicted using the ARIMA(1, 0, 1) model. As shown in Figure 6, the resulting five-year forecast suggests that imports as a percentage of GDP decrease at a slower pace, changing from an estimated -6.39% in 2018 to -2.88% in 2022. Although the forecast indicates a moderate upward trend, the 95% confidence intervals remain wide, indicating uncertainty in the model predictions. Compared to GDP, import behavior is highly influenced by external factors, such as trade policy, exchange rate fluctuations, and global commodity prices, so variability is not unexpected. Overall, ARIMA models capture the historical trends and short-term dynamics in economic data, providing insights about economic activity in the CAR.

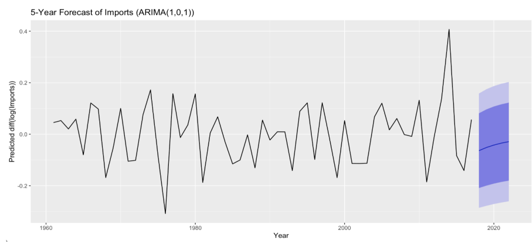


Figure 6: Imports: 5-Year Forecast Log-Differenced Imports

Year	Point Forecast	80% CI	95% CI
2018	-0.06	[-0.21, 0.08]	[-0.29, 0.16]
2019	-0.05	[-0.20, 0.10]	[-0.28, 0.17]
2020	-0.04	[-0.19, 0.11]	[-0.27, 0.19]
2021	-0.03	[-0.19, 0.12]	[-0.27, 0.20]
2022	-0.03	[-0.18. 0.12]	[-0.26, 0.20]

Table 4: Imports: 5-Year Forecast of Log-Differenced Imports

Conclusion

In this project, we analyzed the GDP and Imports time series for the Central African Republic from 1960 to 2017 using ARIMA modeling and addressed the two research questions outlined in the introduction section. First, we selected ARIMA (2,0,2) and ARIMA (1,0,1) as the best fitted models for GDP and Imports, respectively. Prior to model fitting, we applied log transformation and first-order differencing to achieve stationarity. Model selection was based on a variety of criteria, including AIC, BIC, residual diagnostics, and coefficient significance. We chose the two final models given their significant model coefficients and well-behaved residuals resembling white-noise. Second, the five-year forecasts of log-differenced GDP and Imports time series revealed different dynamics for the two variables. While the GDP growth rate showed a gradual decline, Imports exhibited a moderate upward trend. However, the wide confidence intervals highlighted the uncertainty in forecasting and instability in the Central African Republic's economy. Overall, ARIMA models offer valuable information about patterns in GDP and Imports, but are limited in their ability to capture the interdependencies between variables or account for external economic influences. Future studies could enhance model performance by exploring multivariate time series frameworks and incorporating additional economic predictors, such as exports, CPI (Consumer Price Index), and population.

References

Mercereau, B. (2004). *Political Instability and Growth: The Central African Republic*. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=878903

World Bank Group. (2023a). *GDP (current US$)—Central African Republic*. World Bank Group. https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?locations=CF

World Bank Group. (2023b). *Imports of goods and services (% of GDP)—Central African Republic*. World Bank Group. https://data.worldbank.org/indicator/NE.IMP.GNFS.ZS?locations=CF

World Trade Organization. (2013). *Annex 4 Central African Republic* (No. WT/TPR/S/285). World Trade Organization. https://www.wto.org/english/tratop_e/tpr_e/s285-03_e.pdf

Appendix R Code

Jenny Xu

2025-06-05

```
library(astsa)
library(MASS)
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
##
## Attaching package: 'forecast'
```

```
## The following object is masked from 'package:astsa':
##
##     gas
```

```
library(ggplot2)
load("finalproject.Rdata")
```

Exploratory Data Analysis

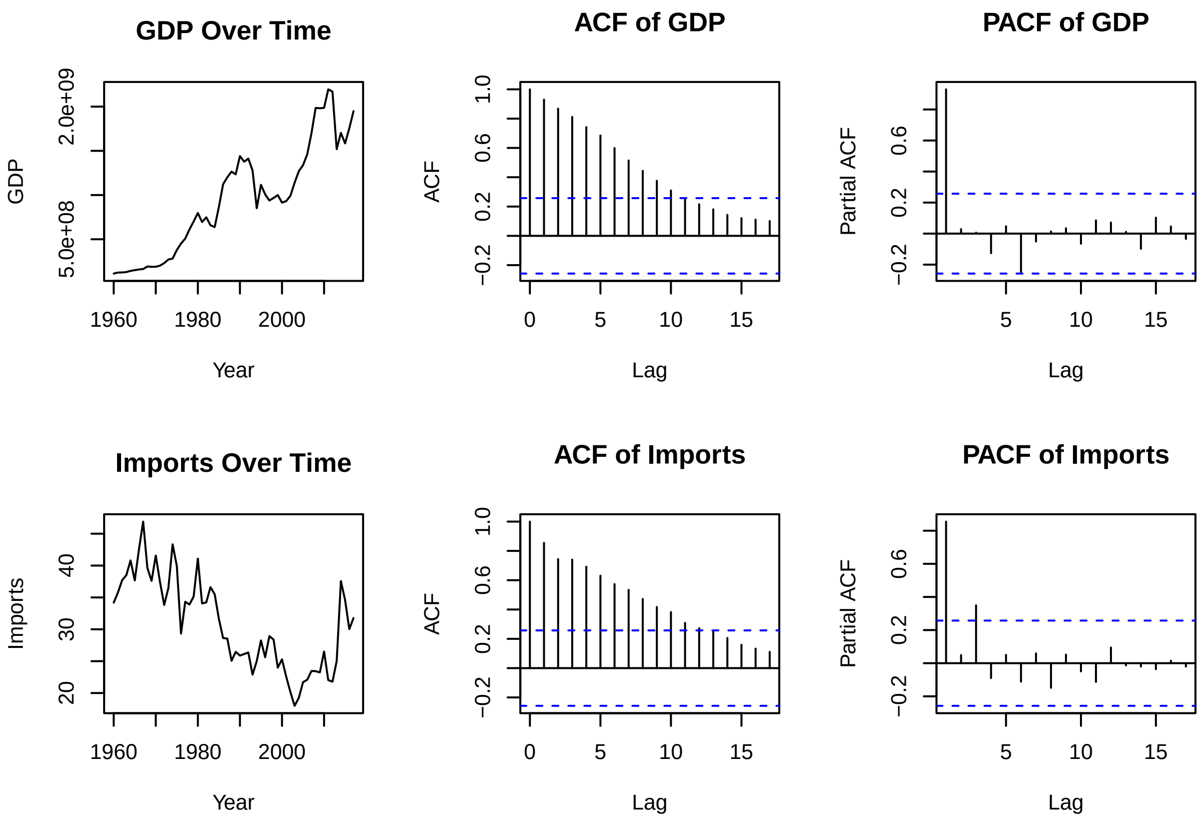
Raw Data

```
# Exploratory Data Analysis
# Raw Data with Year-Based Time Series

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
gdp_ts <- ts(finalPro_data$GDP, start = 1960, frequency = 1)
plot(gdp_ts, main = "GDP Over Time", ylab = "GDP", xlab = "Year")
acf(gdp_ts, main = "ACF of GDP")
pacf(gdp_ts, main = "PACF of GDP")

# 2. Imports
import_ts <- ts(finalPro_data$Imports, start = 1960, frequency = 1)
plot(import_ts, main = "Imports Over Time", ylab = "Imports", xlab = "Year")
acf(import_ts, main = "ACF of Imports")
pacf(import_ts, main = "PACF of Imports")
```



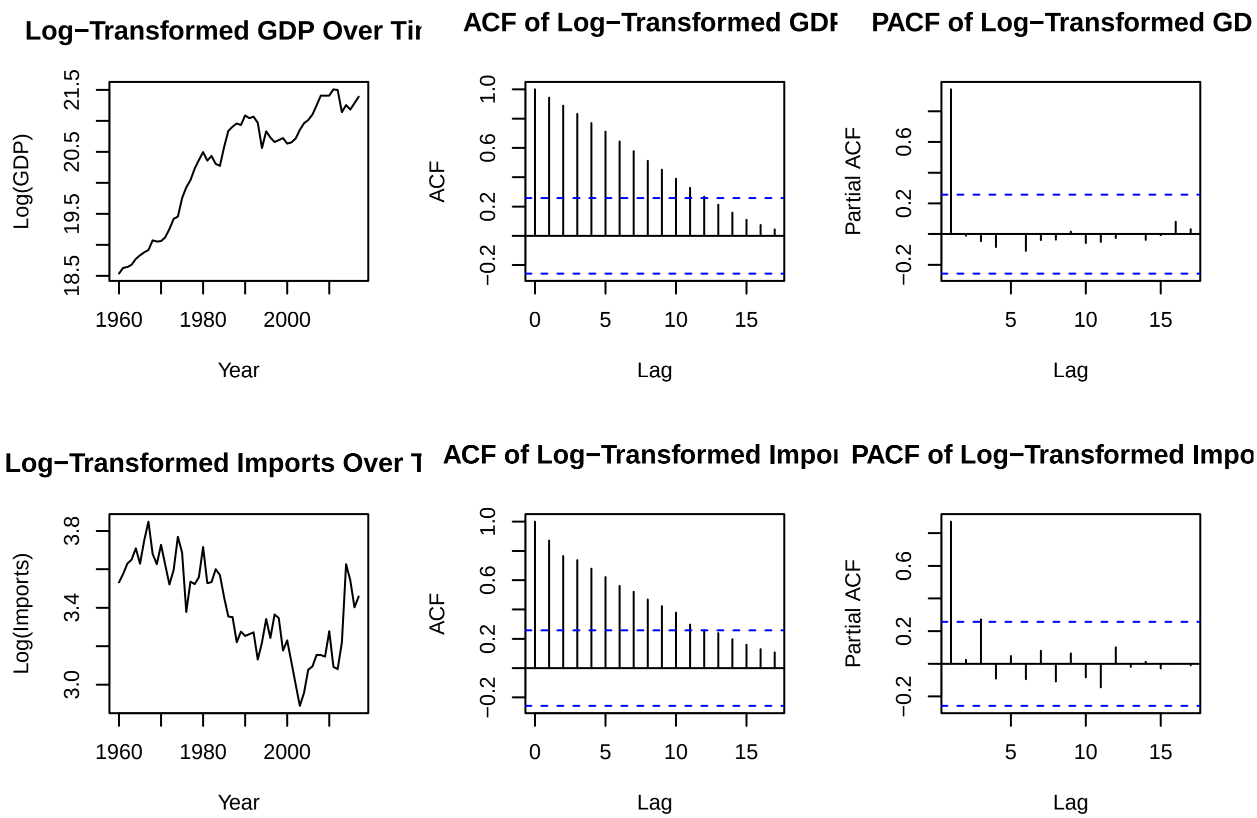
Log-Transformed Data

```r
# Log-Transformed Data
log.gdp.ts <- ts(log(finalPro_data$GDP), start = 1960, frequency = 1)
log.import.ts <- ts(log(finalPro_data$Imports), start = 1960, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(log.gdp.ts, main = "Log-Transformed GDP Over Time", ylab = "Log(GDP)", xlab = "Year")
acf(log.gdp.ts, main = "ACF of Log-Transformed GDP")
pacf(log.gdp.ts, main = "PACF of Log-Transformed GDP")

# 2. Imports
plot(log.import.ts, main = "Log-Transformed Imports Over Time", ylab = "Log(Imports)", xlab = "Year")
acf(log.import.ts, main = "ACF of Log-Transformed Imports")
pacf(log.import.ts, main = "PACF of Log-Transformed Imports")
```



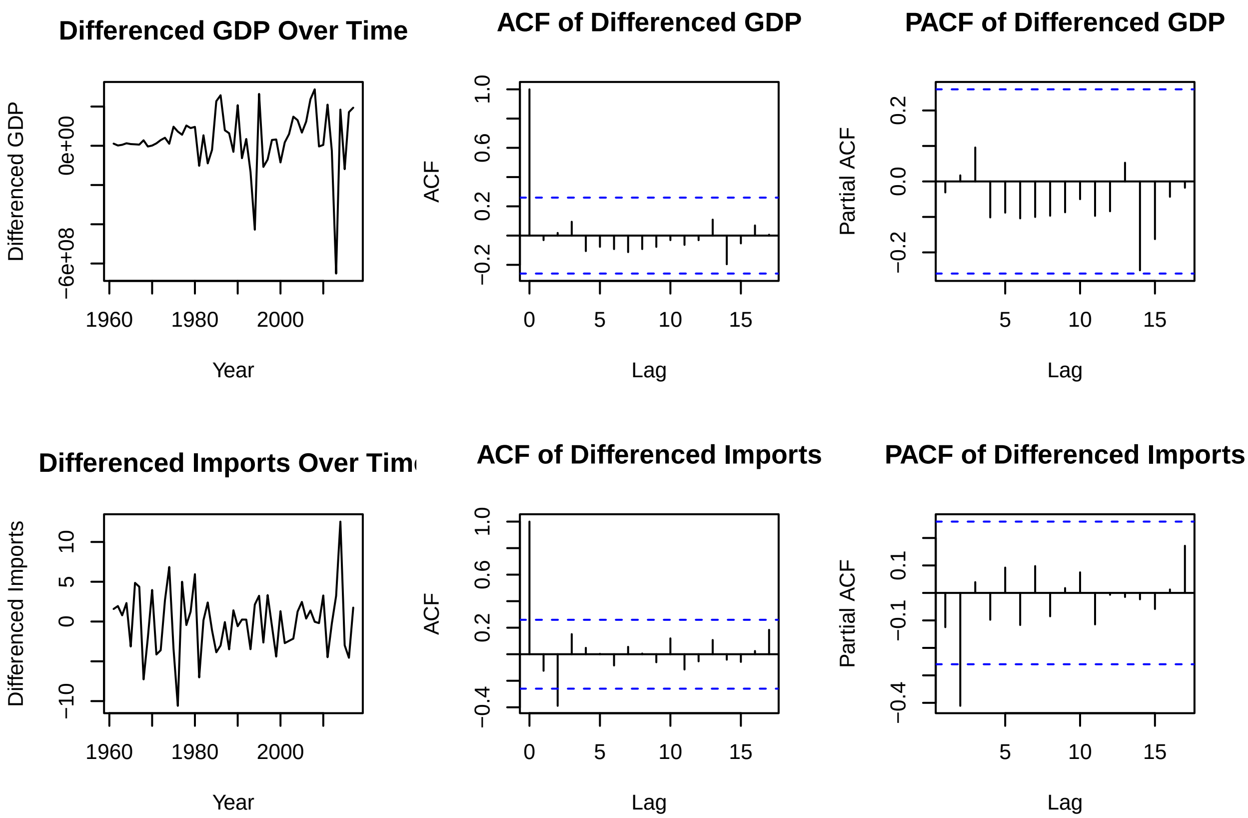
Differencing Data

```r
# Differenced Data (first difference removes first observation, so start at 1961)
diff.gdp.ts <- ts(diff(finalPro_data$GDP), start = 1961, frequency = 1)
diff.import.ts <- ts(diff(finalPro_data$Imports), start = 1961, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(diff.gdp.ts, main = "Differenced GDP Over Time", ylab = "Differenced GDP", xlab = "Year")
acf(diff.gdp.ts, main = "ACF of Differenced GDP")
pacf(diff.gdp.ts, main = "PACF of Differenced GDP")

# 2. Imports
plot(diff.import.ts, main = "Differenced Imports Over Time", ylab = "Differenced Imports", xlab = "Year
acf(diff.import.ts, main = "ACF of Differenced Imports")
pacf(diff.import.ts, main = "PACF of Differenced Imports")
```



Box-cox Transformed Data

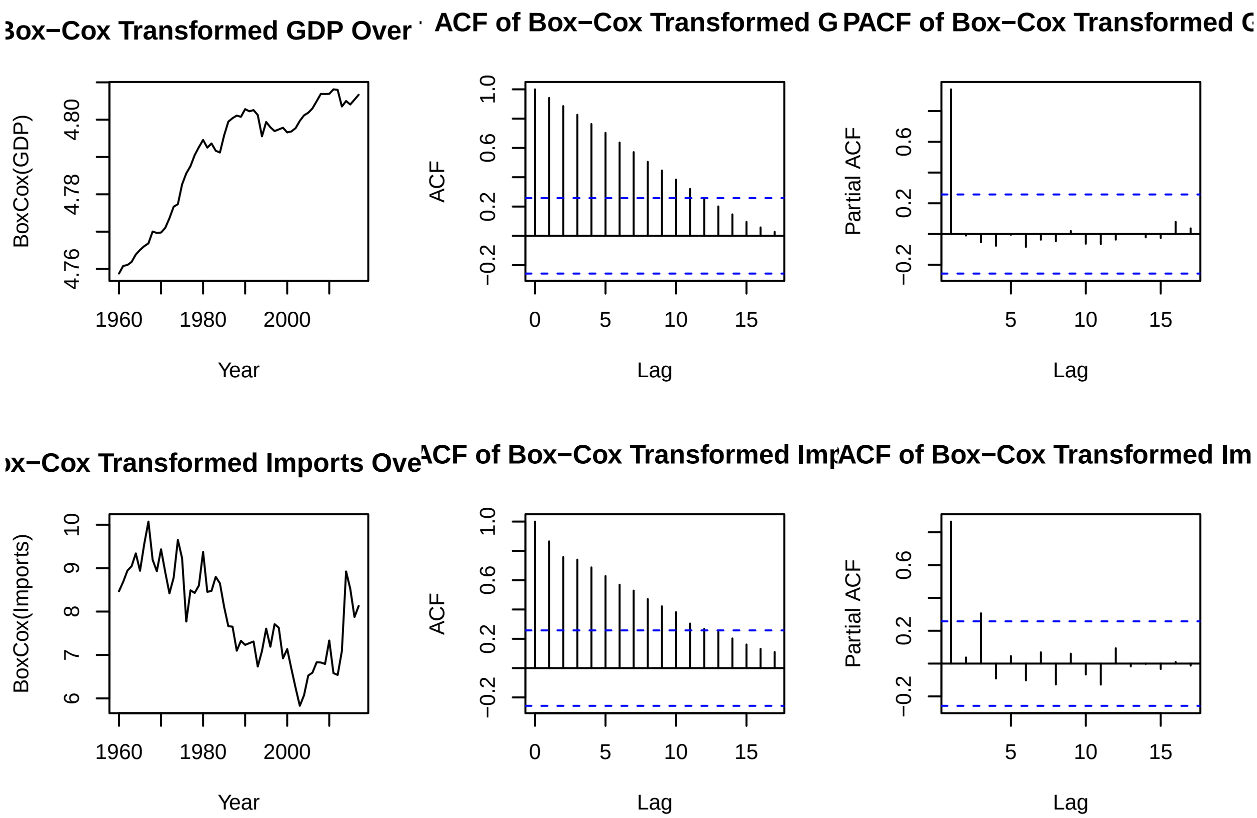
```r
# Box-Cox Transformed Data
gdp.lambda <- BoxCox.lambda(finalPro_data$GDP)
boxcox.gdp.ts <- ts(BoxCox(finalPro_data$GDP, gdp.lambda), start = 1960, frequency = 1)

import.lambda <- BoxCox.lambda(finalPro_data$Imports)
boxcox.import.ts <- ts(BoxCox(finalPro_data$Imports, import.lambda), start = 1960, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(boxcox.gdp.ts, main = "Box-Cox Transformed GDP Over Time", ylab = "BoxCox(GDP)", xlab = "Year")
acf(boxcox.gdp.ts, main = "ACF of Box-Cox Transformed GDP")
pacf(boxcox.gdp.ts, main = "PACF of Box-Cox Transformed GDP")

# 2. Imports
plot(boxcox.import.ts, main = "Box-Cox Transformed Imports Over Time", ylab = "BoxCox(Imports)", xlab =
acf(boxcox.import.ts, main = "ACF of Box-Cox Transformed Imports")
pacf(boxcox.import.ts, main = "PACF of Box-Cox Transformed Imports")
```



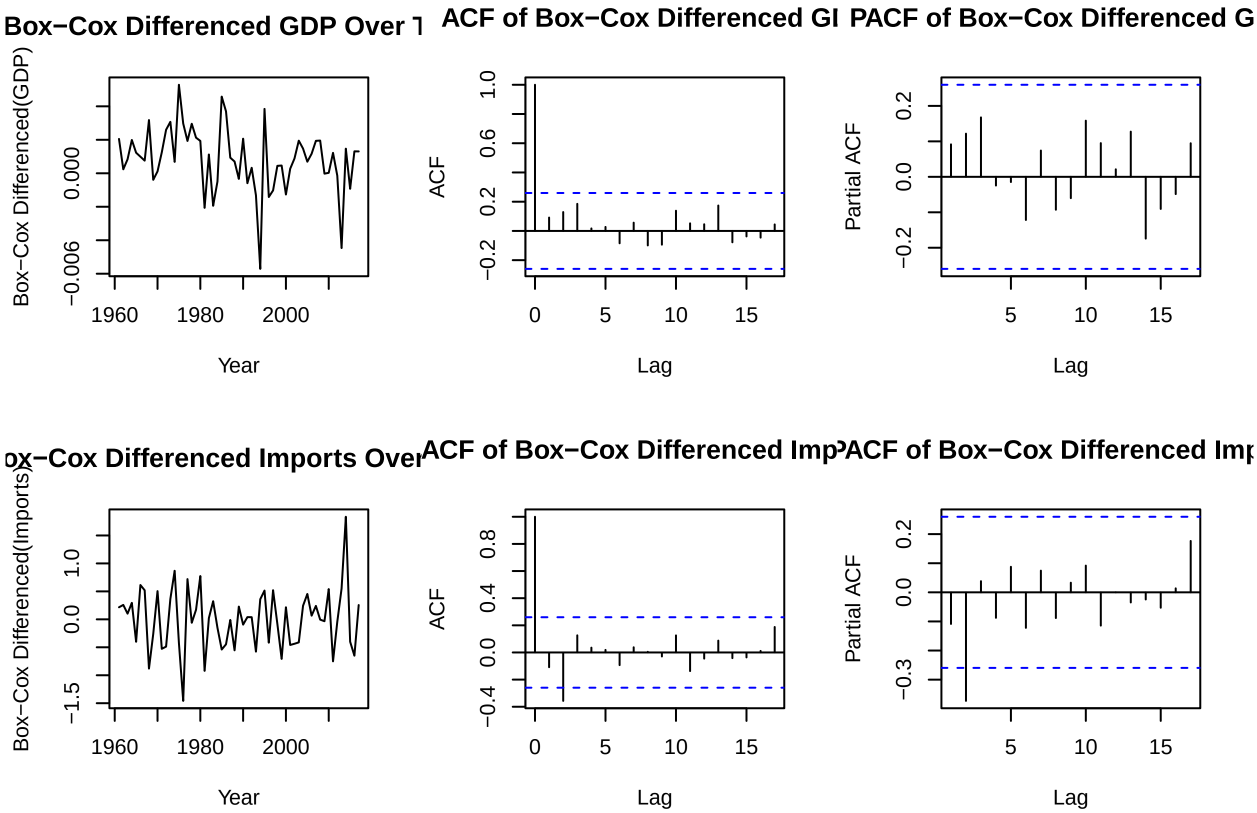
Box-cox Differencing Data

```r
# Box-Cox Differenced Data (first difference removes first year, so start = 1961)
boxcox.diff.gdp.ts <- ts(diff(boxcox.gdp.ts), start = 1961, frequency = 1)
boxcox.diff.import.ts <- ts(diff(boxcox.import.ts), start = 1961, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(boxcox.diff.gdp.ts, main = "Box-Cox Differenced GDP Over Time", ylab = "Box-Cox Differenced(GDP)",
acf(boxcox.diff.gdp.ts, main = "ACF of Box-Cox Differenced GDP")
pacf(boxcox.diff.gdp.ts, main = "PACF of Box-Cox Differenced GDP")

# 2. Imports
plot(boxcox.diff.import.ts, main = "Box-Cox Differenced Imports Over Time", ylab = "Box-Cox Differenced
acf(boxcox.diff.import.ts, main = "ACF of Box-Cox Differenced Imports")
pacf(boxcox.diff.import.ts, main = "PACF of Box-Cox Differenced Imports")
```



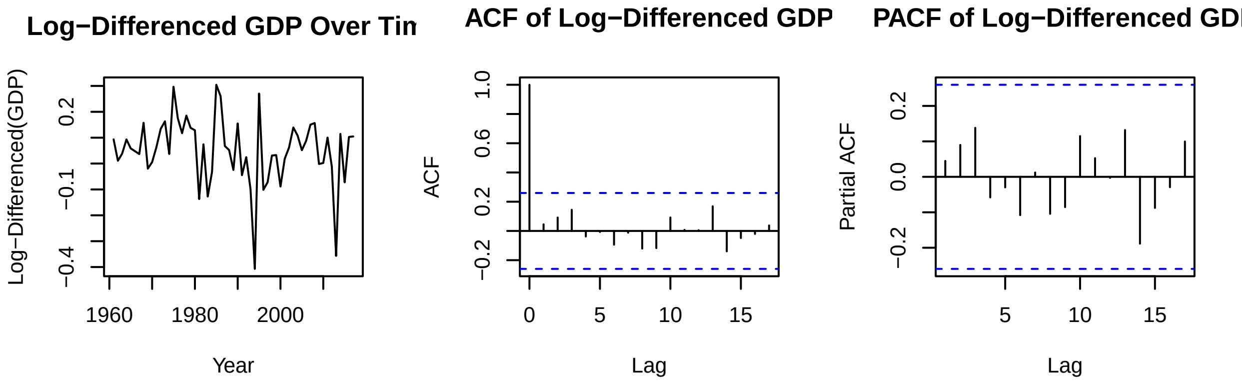
Log-Differencing Data (Final)

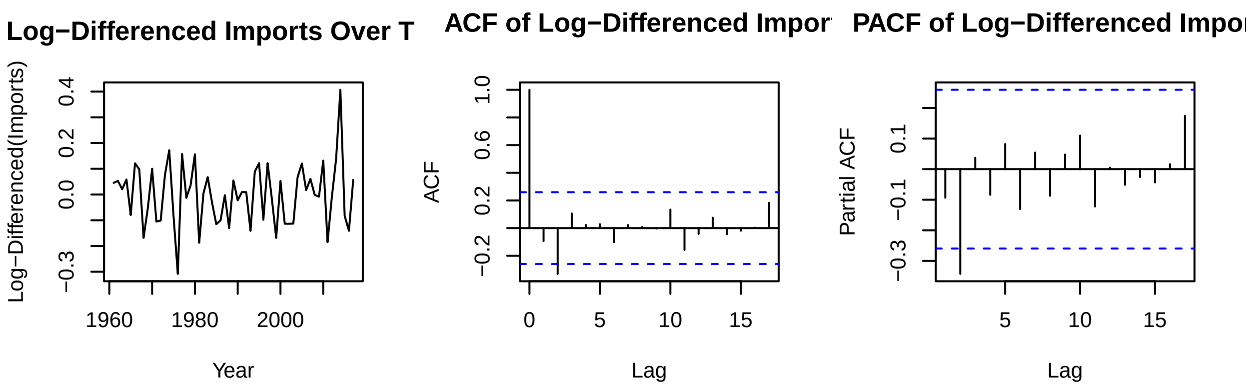
```
# Log-Differenced Data (Final)
log.diff.gdp.ts <- ts(diff(log.gdp.ts), start = 1961, frequency = 1)
log.diff.import.ts <- ts(diff(log.import.ts), start = 1961, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(log.diff.gdp.ts, main = "Log-Differenced GDP Over Time", ylab = "Log-Differenced(GDP)", xlab = "Yea
acf(log.diff.gdp.ts, main = "ACF of Log-Differenced GDP")
pacf(log.diff.gdp.ts, main = "PACF of Log-Differenced GDP")

# 2. Imports
plot(log.diff.import.ts, main = "Log-Differenced Imports Over Time", ylab = "Log-Differenced(Imports)",
acf(log.diff.import.ts, main = "ACF of Log-Differenced Imports")
pacf(log.diff.import.ts, main = "PACF of Log-Differenced Imports")
```





Model Selection

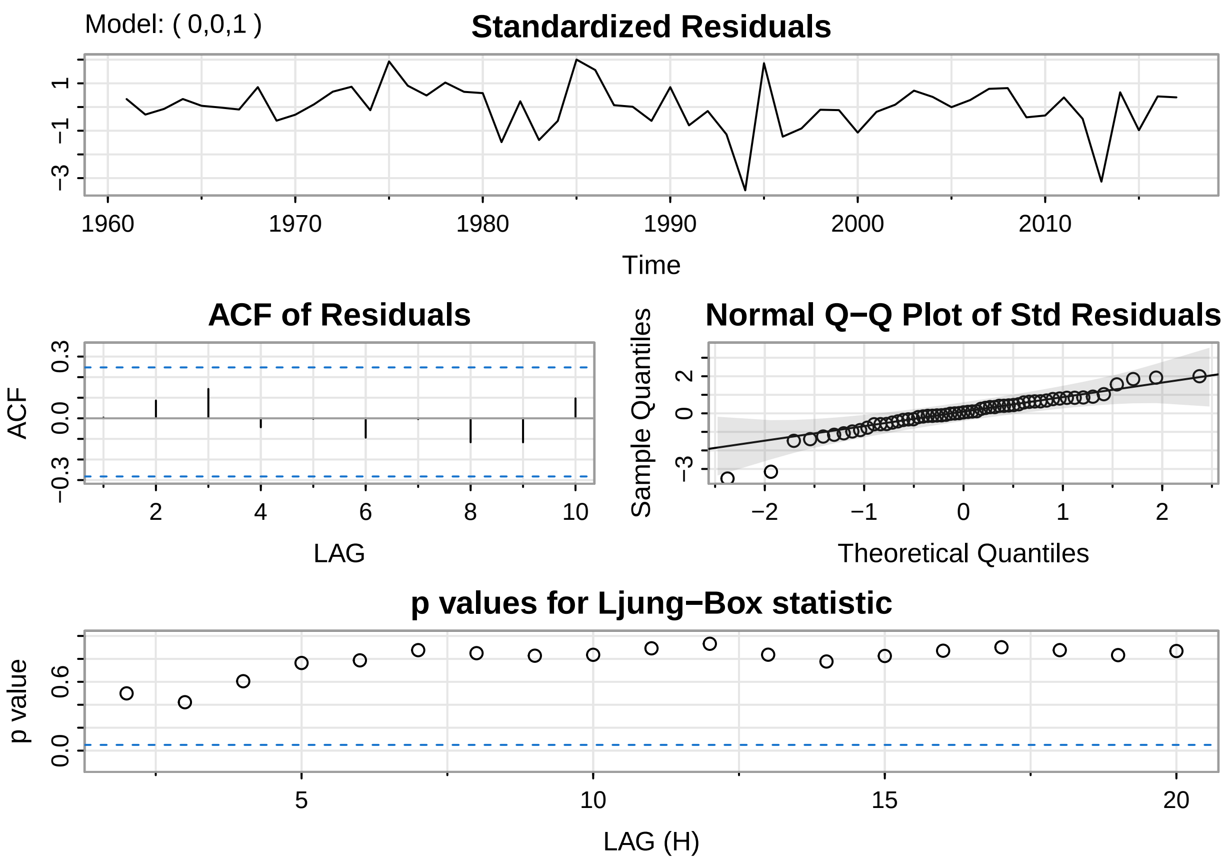
GDP

```r
# Model Selection on Log-Differenced GDP (stationary)
# GDP

# Model 1: MA(1)
log.diff.gdp.ma1 <- arima(log.diff.gdp.ts, order = c(0,0,1))
log.diff.gdp.ma1.diagnostics <- sarima(log.diff.gdp.ts, 0, 0, 1)
```

```
## initial  value -2.051458
## iter   2 value -2.052302
## iter   3 value -2.052325
## iter   4 value -2.052325
## iter   4 value -2.052325
## iter   4 value -2.052325
## final  value -2.052325
## converged
## initial  value -2.052314
## iter   2 value -2.052314
## iter   2 value -2.052314
## iter   2 value -2.052314
## final  value -2.052314
## converged
## <><><><><><><><><><><><><><>
##
```

```
## Coefficients:
##       Estimate     SE t.value p.value
## ma1     0.0382 0.1228  0.3113  0.7568
## xmean   0.0502 0.0177  2.8416  0.0063
##
## sigma^2 estimated as 0.01649574 on 55 degrees of freedom
##
## AIC = -1.161487  AICc = -1.157589  BIC = -1.053958
##
```




```
log.diff.gdp.ma1
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(0, 0, 1))
##
## Coefficients:
##          ma1  intercept
##       0.0382     0.0502
## s.e.  0.1228     0.0177
##
## sigma^2 estimated as 0.0165:  log likelihood = 36.1,  aic = -66.2
```

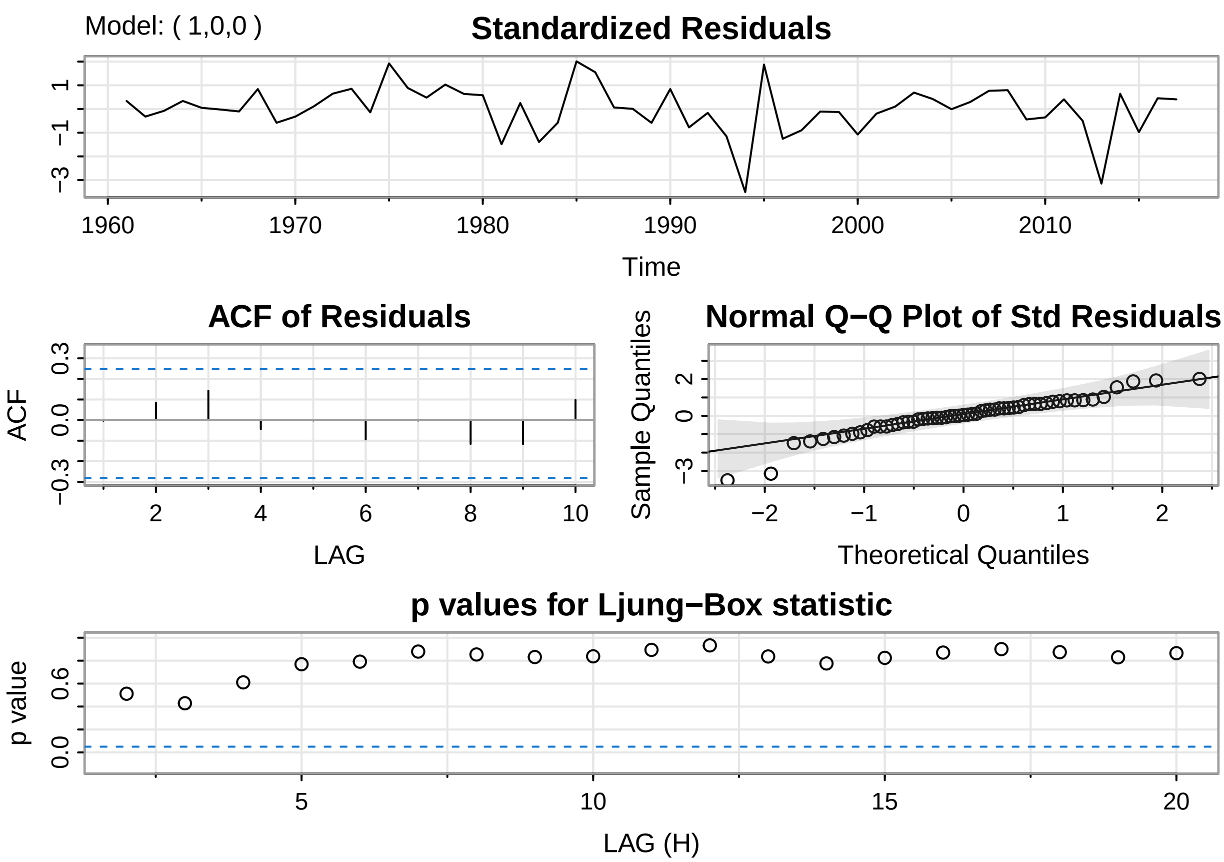
```
log.diff.gdp.ma1.diagnostics
```

```
## $fit
##
## Call:
```

```
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ma1   xmean
##       0.0382  0.0502
## s.e.  0.1228  0.0177
##
## sigma^2 estimated as 0.0165:  log likelihood = 36.1,  aic = -66.2
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ma1     0.0382 0.1228  0.3113  0.7568
## xmean   0.0502 0.0177  2.8416  0.0063
##
## $ICs
##       AIC      AICc       BIC
## -1.161487 -1.157589 -1.053958
```

```r
# Model 2: AR(1)
log.diff.gdp.ar1 <- arima(log.diff.gdp.ts, order = c(1,0,0))
log.diff.gdp.ar1.diagnostics <- sarima(log.diff.gdp.ts, 1, 0, 0)
```

```
## initial  value -2.043604
## iter   2 value -2.044628
## iter   3 value -2.044635
## iter   4 value -2.044638
## iter   4 value -2.044638
## iter   4 value -2.044638
## final  value -2.044638
## converged
## initial  value -2.052438
## iter   2 value -2.052450
## iter   3 value -2.052458
## iter   3 value -2.052458
## iter   3 value -2.052458
## final  value -2.052458
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.0444 0.1315  0.3380  0.7367
## xmean   0.0502 0.0178  2.8207  0.0067
##
## sigma^2 estimated as 0.01649082 on 55 degrees of freedom
##
## AIC = -1.161776  AICc = -1.157878  BIC = -1.054247
##
```




```
log.diff.gdp.ar1
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.0444     0.0502
## s.e.  0.1315     0.0178
##
## sigma^2 estimated as 0.01649:  log likelihood = 36.11,  aic = -66.22
```

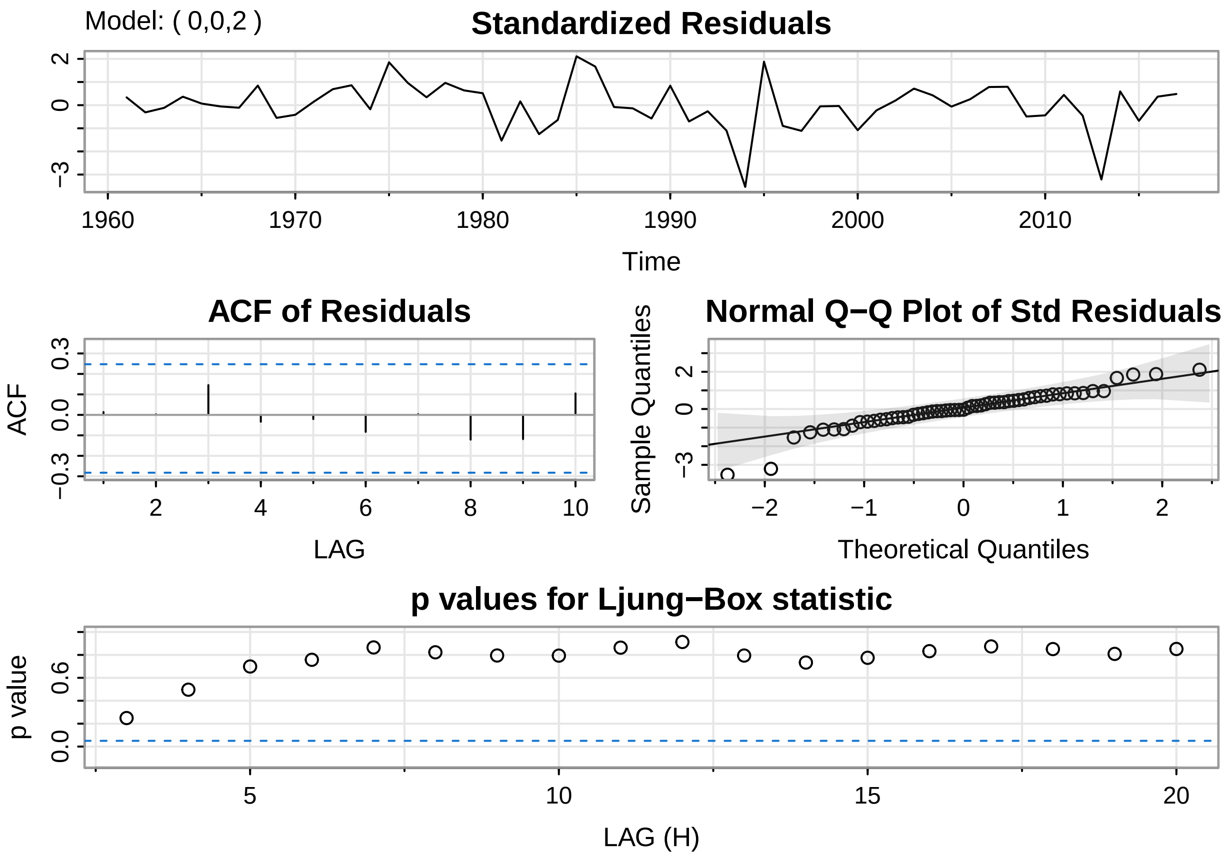
```
log.diff.gdp.ar1.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1   xmean
##       0.0444  0.0502
## s.e.  0.1315  0.0178
##
## sigma^2 estimated as 0.01649:  log likelihood = 36.11,  aic = -66.22
```

```
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.0444 0.1315  0.3380  0.7367
## xmean   0.0502 0.0178  2.8207  0.0067
##
## $ICs
##       AIC      AICc       BIC
## -1.161776 -1.157878 -1.054247
```

```r
# Model 3: MA(2)
log.diff.gdp.ma2 <- arima(log.diff.gdp.ts, order = c(0,0,2))
log.diff.gdp.ma2.diagnostics <- sarima(log.diff.gdp.ts, 0, 0, 2)
```

```
## initial  value -2.051458
## iter   2 value -2.055718
## iter   3 value -2.056120
## iter   4 value -2.056142
## iter   5 value -2.056143
## iter   5 value -2.056143
## iter   5 value -2.056143
## final  value -2.056143
## converged
## initial  value -2.056002
## iter   2 value -2.056006
## iter   3 value -2.056006
## iter   3 value -2.056006
## iter   3 value -2.056006
## final  value -2.056006
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ma1     0.0159 0.1367  0.1162  0.9079
## ma2     0.0908 0.1388  0.6540  0.5159
## xmean   0.0503 0.0187  2.6901  0.0095
##
## sigma^2 estimated as 0.01636998 on 54 degrees of freedom
##
## AIC = -1.133784  AICc = -1.12584  BIC = -0.9904121
##
```




```
log.diff.gdp.ma2
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(0, 0, 2))
##
## Coefficients:
##          ma1     ma2  intercept
##       0.0159  0.0908     0.0503
## s.e.  0.1367  0.1388     0.0187
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.31,  aic = -64.63
```

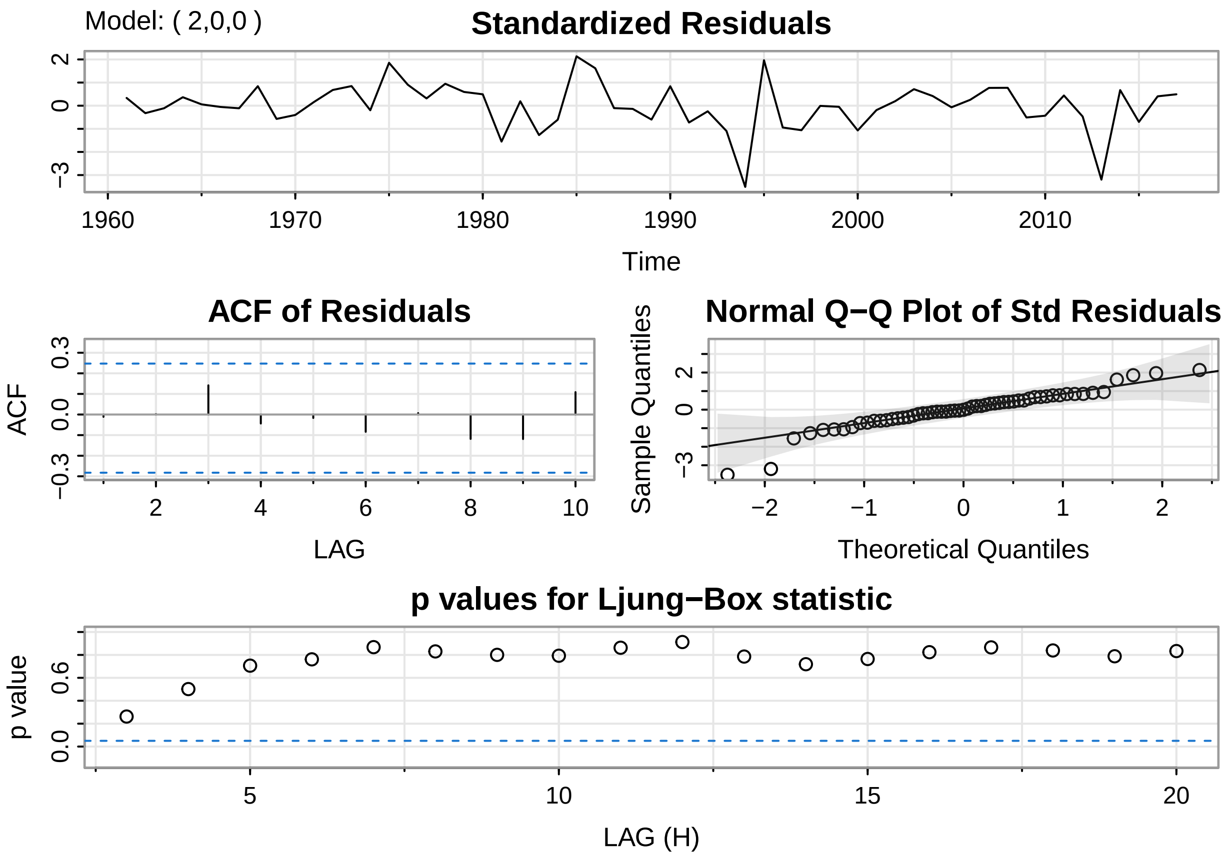
```
log.diff.gdp.ma2.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ma1     ma2   xmean
##       0.0159  0.0908  0.0503
## s.e.  0.1367  0.1388  0.0187
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.31,  aic = -64.63
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ma1     0.0159 0.1367  0.1162  0.9079
## ma2     0.0908 0.1388  0.6540  0.5159
## xmean   0.0503 0.0187  2.6901  0.0095
##
## $ICs
##        AIC       AICc        BIC
## -1.1337841 -1.1258397 -0.9904121
```

```
# Model 4: AR(2)
log.diff.gdp.ar2 <- arima(log.diff.gdp.ts, order = c(2,0,0))
log.diff.gdp.ar2.diagnostics <- sarima(log.diff.gdp.ts, 2, 0, 0)
```

```
## initial  value -2.035413
## iter   2 value -2.040611
## iter   3 value -2.040622
## iter   4 value -2.040622
## iter   4 value -2.040622
## iter   4 value -2.040622
## final  value -2.040622
## converged
## initial  value -2.056414
## iter   2 value -2.056422
## iter   3 value -2.056422
## iter   3 value -2.056422
## iter   3 value -2.056422
## final  value -2.056422
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.0405 0.1309  0.3097  0.7580
## ar2     0.0878 0.1302  0.6738  0.5033
## xmean   0.0504 0.0194  2.6026  0.0119
##
## sigma^2 estimated as 0.01635617 on 54 degrees of freedom
##
## AIC = -1.134616  AICc = -1.126672  BIC = -0.991244
##
```




```
log.diff.gdp.ar2
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(2, 0, 0))
##
## Coefficients:
##          ar1     ar2  intercept
##       0.0405  0.0878     0.0504
## s.e.  0.1309  0.1302     0.0194
##
## sigma^2 estimated as 0.01636:  log likelihood = 36.34,  aic = -64.67
```

```
log.diff.gdp.ar2.diagnostics
```

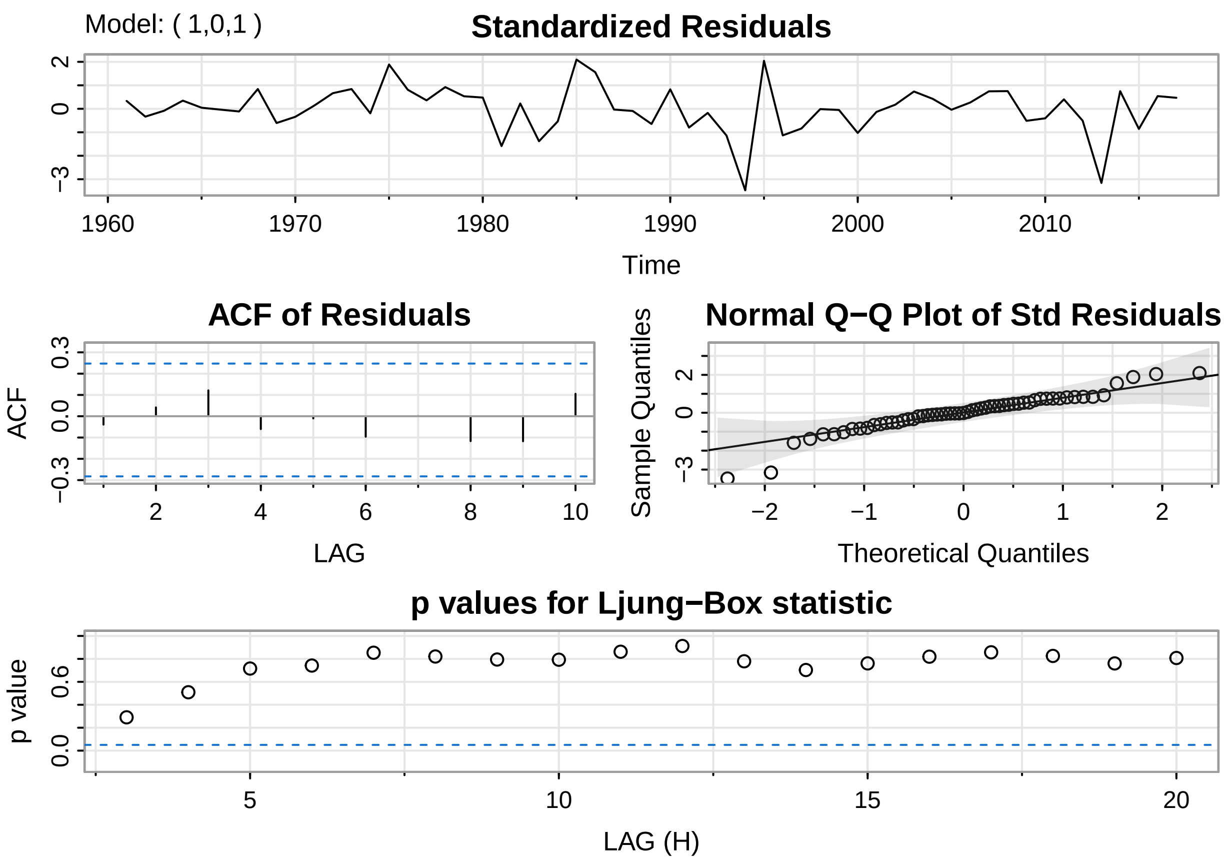
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1     ar2   xmean
##       0.0405  0.0878  0.0504
## s.e.  0.1309  0.1302  0.0194
##
## sigma^2 estimated as 0.01636:  log likelihood = 36.34,  aic = -64.67
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.0405 0.1309  0.3097  0.7580
## ar2     0.0878 0.1302  0.6738  0.5033
## xmean   0.0504 0.0194  2.6026  0.0119
##
## $ICs
##       AIC      AICc       BIC
## -1.134616 -1.126672 -0.991244
```

```r
# Model 5: ARMA(1,1)
log.diff.gdp.arma11 <- arima(log.diff.gdp.ts, order = c(1,0,1))
log.diff.gdp.arma11.diagnostics <- sarima(log.diff.gdp.ts, 1, 0, 1)
```

```
## initial  value -2.043604
## iter   2 value -2.044276
## iter   3 value -2.044596
## iter   4 value -2.044598
## iter   5 value -2.044625
## iter   6 value -2.044786
## iter   7 value -2.044823
## iter   8 value -2.044926
## iter   9 value -2.044934
## iter  10 value -2.044947
## iter  11 value -2.045576
## iter  12 value -2.046165
## iter  13 value -2.046254
## iter  14 value -2.047052
## iter  15 value -2.047177
## iter  16 value -2.047178
## iter  16 value -2.047178
## iter  16 value -2.047178
## final  value -2.047178
## converged
## initial  value -2.056080
## iter   2 value -2.056116
## iter   3 value -2.056124
## iter   4 value -2.056126
## iter   5 value -2.056128
## iter   6 value -2.056139
## iter   7 value -2.056145
## iter   8 value -2.056149
## iter   9 value -2.056149
## iter  10 value -2.056149
## iter  10 value -2.056149
## iter  10 value -2.056149
## final  value -2.056149
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
```

```
##       Estimate     SE t.value p.value
## ar1     0.6353 0.4481  1.4177  0.1620
## ma1    -0.5615 0.4727 -1.1879  0.2401
## xmean   0.0502 0.0202  2.4854  0.0161
##
## sigma^2 estimated as 0.0163663 on 54 degrees of freedom
##
## AIC = -1.134071  AICc = -1.126126  BIC = -0.9906985
##
```




```
log.diff.gdp.arma11
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(1, 0, 1))
##
## Coefficients:
##          ar1      ma1  intercept
##       0.6353  -0.5615     0.0502
## s.e.  0.4481   0.4727     0.0202
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.32,  aic = -64.64
```

```
log.diff.gdp.arma11.diagnostics
```

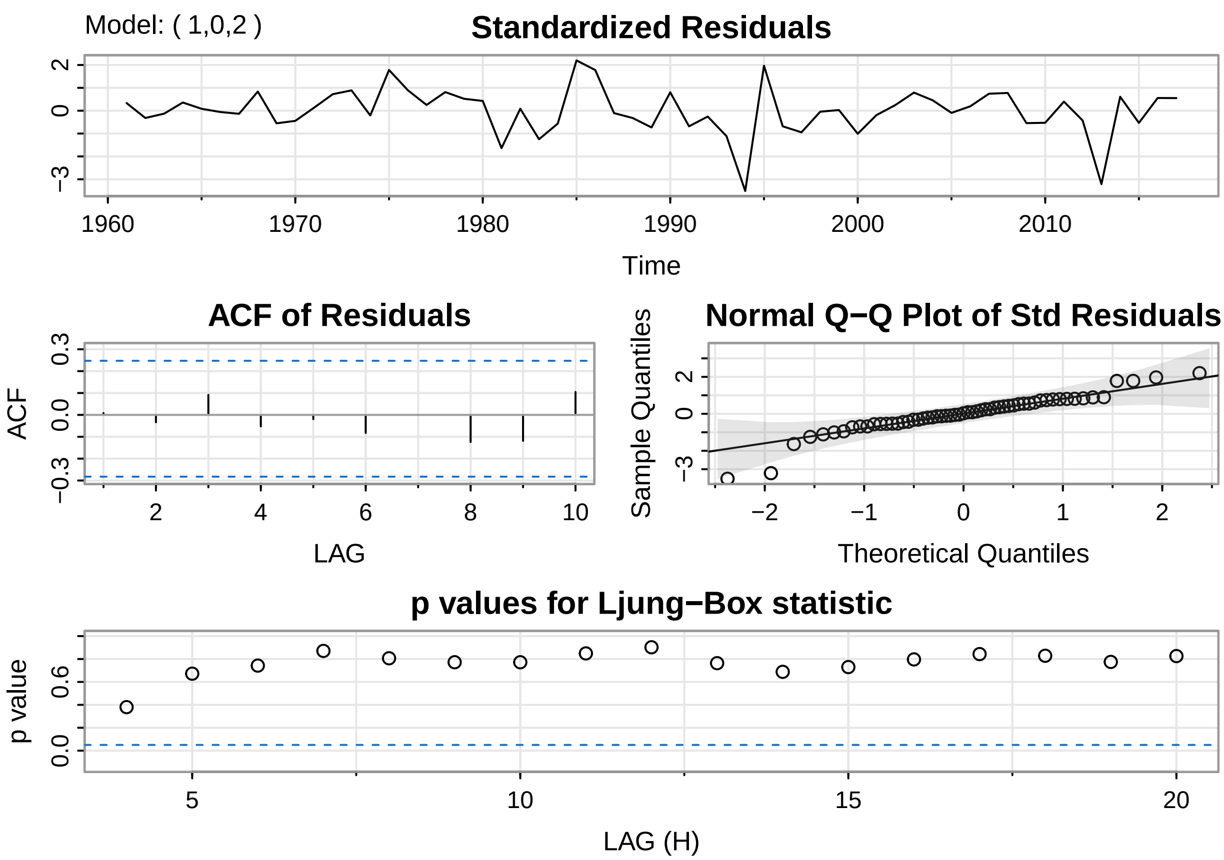
```
## $fit
##
## Call:
```

```
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1   xmean
##       0.6353  -0.5615  0.0502
## s.e.  0.4481   0.4727  0.0202
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.32,  aic = -64.64
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.6353 0.4481  1.4177  0.1620
## ma1    -0.5615 0.4727 -1.1879  0.2401
## xmean   0.0502 0.0202  2.4854  0.0161
##
## $ICs
##        AIC       AICc        BIC
## -1.1340705 -1.1261261 -0.9906985
```

```r
# Model 6: ARMA(1,2)
log.diff.gdp.arma12 <- arima(log.diff.gdp.ts, order = c(1,0,2))
log.diff.gdp.arma12.diagnostics <- sarima(log.diff.gdp.ts, 1, 0, 2)
```

```
## initial  value -2.043604
## iter   2 value -2.046177
## iter   3 value -2.048533
## iter   4 value -2.048624
## iter   5 value -2.048815
## iter   6 value -2.049700
## iter   7 value -2.051473
## iter   8 value -2.051869
## iter   9 value -2.052374
## iter  10 value -2.052542
## iter  11 value -2.052542
## iter  11 value -2.052543
## iter  11 value -2.052543
## final  value -2.052543
## converged
## initial  value -2.061088
## iter   2 value -2.061098
## iter   3 value -2.061118
## iter   4 value -2.061120
## iter   5 value -2.061122
## iter   6 value -2.061133
## iter   7 value -2.061138
## iter   8 value -2.061139
## iter   9 value -2.061139
## iter   9 value -2.061139
## iter   9 value -2.061139
## final  value -2.061139
```

```
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##        Estimate     SE t.value p.value
## ar1      0.4806 0.4350  1.1048  0.2742
## ma1     -0.4623 0.4343 -1.0644  0.2920
## ma2      0.1225 0.1510  0.8116  0.4206
## xmean    0.0505 0.0212  2.3799  0.0209
##
## sigma^2 estimated as 0.01619399 on 53 degrees of freedom
##
## AIC = -1.108962  AICc = -1.095467  BIC = -0.9297474
##
```




```
log.diff.gdp.arma12
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(1, 0, 2))
##
## Coefficients:
##          ar1      ma1     ma2  intercept
##       0.4806  -0.4623  0.1225     0.0505
## s.e.  0.4350   0.4343  0.1510     0.0212
##
## sigma^2 estimated as 0.01619:  log likelihood = 36.61,  aic = -63.21
```

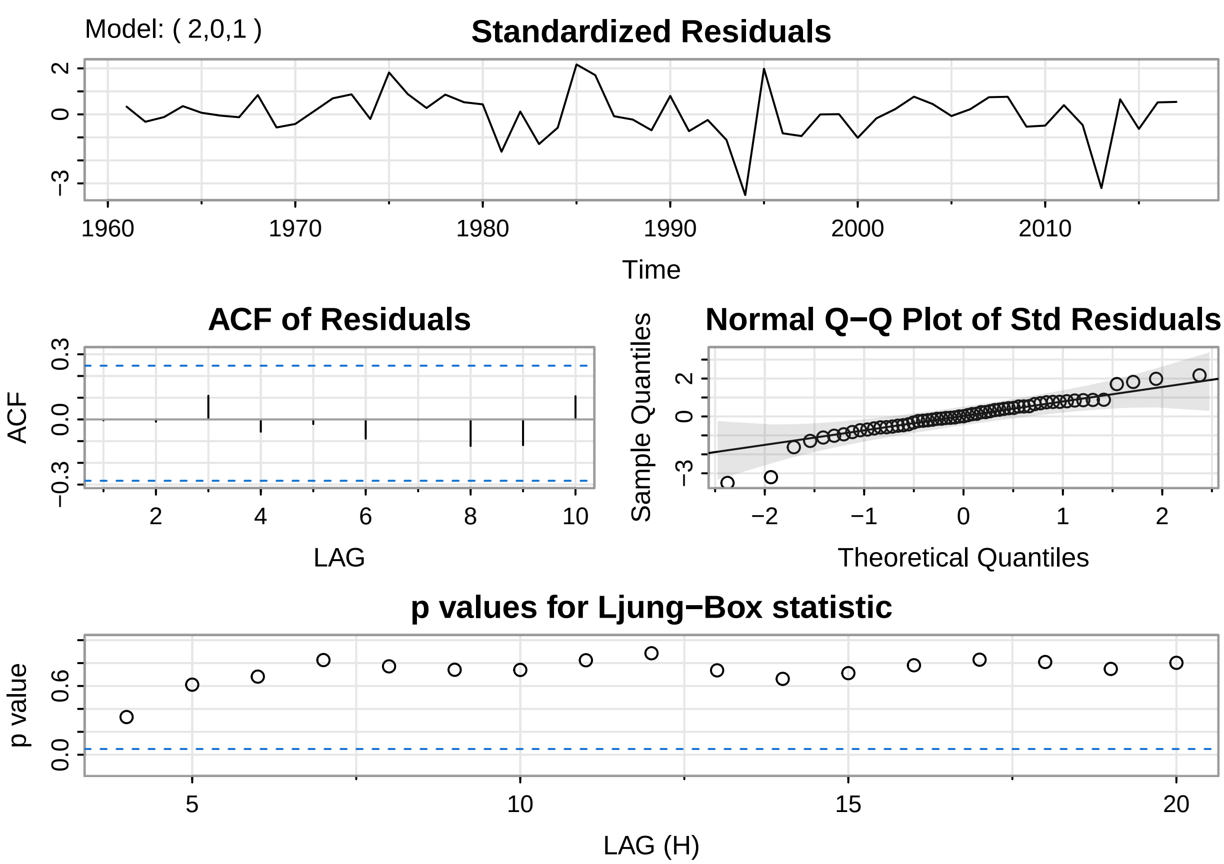
```r
log.diff.gdp.arma12.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1     ma2   xmean
##       0.4806  -0.4623  0.1225  0.0505
## s.e.  0.4350   0.4343  0.1510  0.0212
##
## sigma^2 estimated as 0.01619:  log likelihood = 36.61,  aic = -63.21
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.4806 0.4350  1.1048  0.2742
## ma1    -0.4623 0.4343 -1.0644  0.2920
## ma2     0.1225 0.1510  0.8116  0.4206
## xmean   0.0505 0.0212  2.3799  0.0209
##
## $ICs
##        AIC       AICc        BIC
## -1.1089624 -1.0954671 -0.9297474
```

```r
# Model 7: ARMA(2,1)
log.diff.gdp.arma21 <- arima(log.diff.gdp.ts, order = c(2,0,1))
log.diff.gdp.arma21.diagnostics <- sarima(log.diff.gdp.ts, 2, 0, 1)
```

```
## initial  value -2.035413
## iter   2 value -2.038759
## iter   3 value -2.040329
## iter   4 value -2.040504
## iter   5 value -2.040649
## iter   6 value -2.041805
## iter   7 value -2.042863
## iter   8 value -2.043277
## iter   9 value -2.043499
## iter  10 value -2.043624
## iter  11 value -2.043626
## iter  12 value -2.043627
## iter  13 value -2.043627
## iter  14 value -2.043629
## iter  15 value -2.043629
## iter  16 value -2.043630
## iter  16 value -2.043630
## iter  16 value -2.043630
## final  value -2.043630
## converged
```

```
## initial  value -2.059446
## iter   2 value -2.059459
## iter   3 value -2.059466
## iter   4 value -2.059467
## iter   5 value -2.059470
## iter   6 value -2.059475
## iter   7 value -2.059479
## iter   8 value -2.059480
## iter   9 value -2.059480
## iter   9 value -2.059480
## iter   9 value -2.059480
## final  value -2.059480
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.4057 0.5161  0.7862  0.4353
## ar2     0.0918 0.1378  0.6661  0.5082
## ma1    -0.3718 0.5064 -0.7343  0.4660
## xmean   0.0504 0.0209  2.4092  0.0195
##
## sigma^2 estimated as 0.01625255 on 53 degrees of freedom
##
## AIC = -1.105644  AICc = -1.092148  BIC = -0.9264285
##
```



```r
log.diff.gdp.arma21
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(2, 0, 1))
##
## Coefficients:
##          ar1     ar2      ma1  intercept
##       0.4057  0.0918  -0.3718     0.0504
## s.e.  0.5161  0.1378   0.5064     0.0209
##
## sigma^2 estimated as 0.01625:  log likelihood = 36.51,  aic = -63.02
```

```r
log.diff.gdp.arma21.diagnostics
```

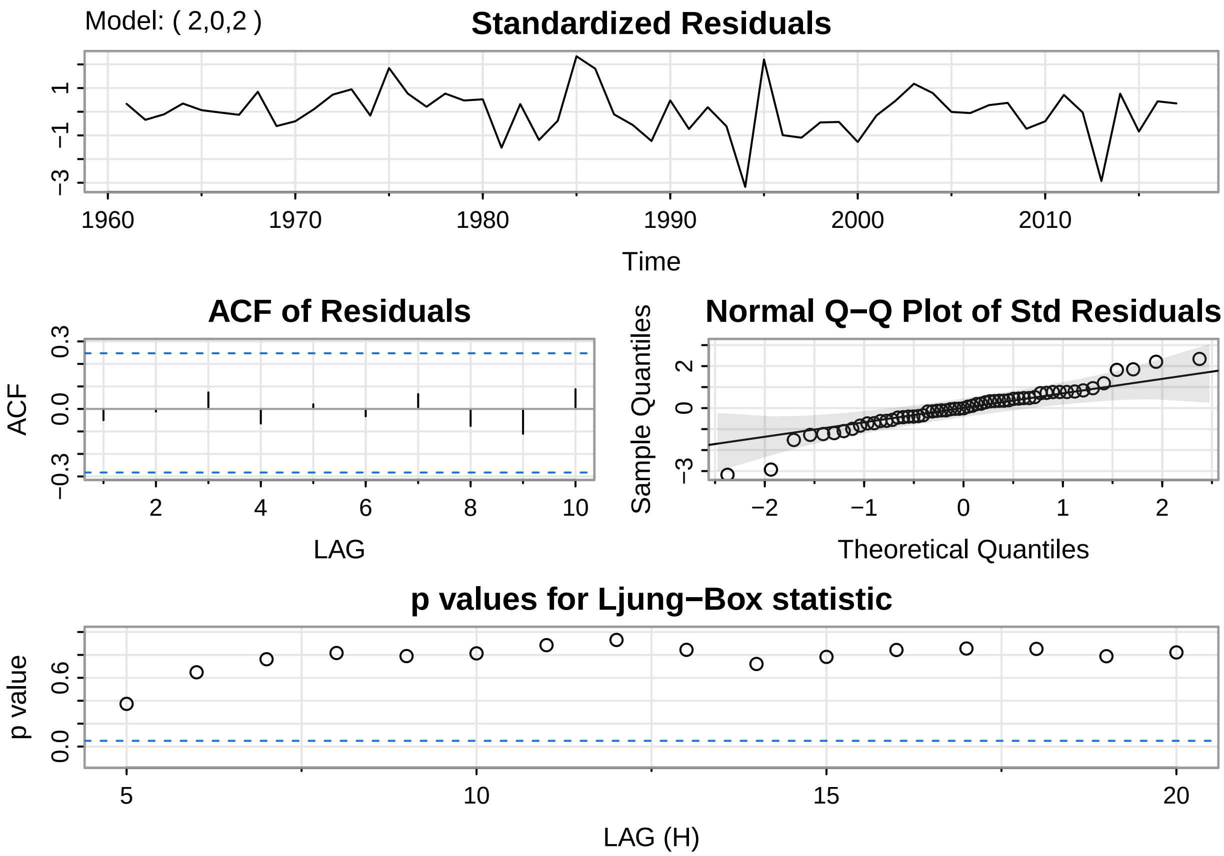
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1     ar2      ma1   xmean
##       0.4057  0.0918  -0.3718  0.0504
## s.e.  0.5161  0.1378   0.5064  0.0209
##
## sigma^2 estimated as 0.01625:  log likelihood = 36.51,  aic = -63.02
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.4057 0.5161  0.7862  0.4353
## ar2     0.0918 0.1378  0.6661  0.5082
## ma1    -0.3718 0.5064 -0.7343  0.4660
## xmean   0.0504 0.0209  2.4092  0.0195
##
## $ICs
##        AIC       AICc        BIC
## -1.1056435 -1.0921482 -0.9264285
```

```r
# Model 8: ARMA(2,2) (final)
log.diff.gdp.arma22 <- arima(log.diff.gdp.ts, order = c(2,0,2))
log.diff.gdp.arma22.diagnostics <- sarima(log.diff.gdp.ts, 2, 0, 2)
```

```
## initial  value -2.035413
## iter   2 value -2.038802
## iter   3 value -2.040478
## iter   4 value -2.040641
## iter   5 value -2.040791
## iter   6 value -2.042896
## iter   7 value -2.044919
## iter   8 value -2.044997
```

```
## iter   9 value -2.045682
## iter  10 value -2.045871
## iter  11 value -2.046747
## iter  12 value -2.047273
## iter  13 value -2.047581
## iter  14 value -2.047650
## iter  15 value -2.047702
## iter  16 value -2.047784
## iter  17 value -2.047792
## iter  18 value -2.047798
## iter  19 value -2.047815
## iter  20 value -2.047853
## iter  21 value -2.047961
## iter  22 value -2.048189
## iter  23 value -2.048265
## iter  24 value -2.048319
## iter  25 value -2.048351
## iter  26 value -2.048358
## iter  27 value -2.048491
## iter  28 value -2.048729
## iter  29 value -2.049412
## iter  30 value -2.050278
## iter  31 value -2.050624
## iter  32 value -2.050668
## iter  33 value -2.051024
## iter  34 value -2.051982
## iter  35 value -2.056774
## iter  36 value -2.059606
## iter  37 value -2.062725
## iter  38 value -2.067098
## iter  39 value -2.072222
## iter  40 value -2.088772
## iter  41 value -2.094336
## iter  42 value -2.097030
## iter  43 value -2.099760
## iter  44 value -2.099886
## iter  45 value -2.100899
## iter  46 value -2.101130
## iter  47 value -2.101203
## iter  48 value -2.101207
## iter  49 value -2.101215
## iter  50 value -2.101240
## iter  51 value -2.101241
## iter  51 value -2.101241
## iter  51 value -2.101241
## final  value -2.101241
## converged
## initial  value -2.050397
## iter   2 value -2.056026
## iter   3 value -2.081594
## iter   4 value -2.084062
## iter   5 value -2.085851
## iter   6 value -2.086765
## iter   7 value -2.087484
```

```
## iter   8 value -2.089790
## iter   9 value -2.091474
## iter  10 value -2.092000
## iter  11 value -2.093082
## iter  12 value -2.093234
## iter  13 value -2.093239
## iter  14 value -2.093261
## iter  15 value -2.093270
## iter  16 value -2.093279
## iter  17 value -2.093280
## iter  18 value -2.093280
## iter  19 value -2.093280
## iter  20 value -2.093280
## iter  21 value -2.093280
## iter  21 value -2.093280
## iter  21 value -2.093280
## final  value -2.093280
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE  t.value p.value
## ar1     1.5232 0.1236  12.3269  0.0000
## ar2    -0.9044 0.0924  -9.7861  0.0000
## ma1    -1.5297 0.1006 -15.2047  0.0000
## ma2     1.0000 0.1000  10.0004  0.0000
## xmean   0.0508 0.0196   2.5979  0.0122
##
## sigma^2 estimated as 0.01443647 on 52 degrees of freedom
##
## AIC = -1.138156  AICc = -1.117517  BIC = -0.9230984
##
```




```r
log.diff.gdp.arma22
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(2, 0, 2))
##
## Coefficients:
##          ar1      ar2      ma1  ma2  intercept
##       1.5232  -0.9044  -1.5297  1.0     0.0508
## s.e.  0.1236   0.0924   0.1006  0.1     0.0196
##
## sigma^2 estimated as 0.01444:  log likelihood = 38.44,  aic = -64.87
```

```r
log.diff.gdp.arma22.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ar2      ma1  ma2   xmean
##       1.5232  -0.9044  -1.5297  1.0  0.0508
## s.e.  0.1236   0.0924   0.1006  0.1  0.0196
##
## sigma^2 estimated as 0.01444:  log likelihood = 38.44,  aic = -64.87
```

```
##
## $degrees_of_freedom
## [1] 52
##
## $ttable
##       Estimate     SE  t.value p.value
## ar1     1.5232 0.1236  12.3269  0.0000
## ar2    -0.9044 0.0924  -9.7861  0.0000
## ma1    -1.5297 0.1006 -15.2047  0.0000
## ma2     1.0000 0.1000  10.0004  0.0000
## xmean   0.0508 0.0196   2.5979  0.0122
##
## $ICs
##        AIC       AICc        BIC
## -1.1381564 -1.1175166 -0.9230984
```

GDP Final Model: ARMA(2,2)

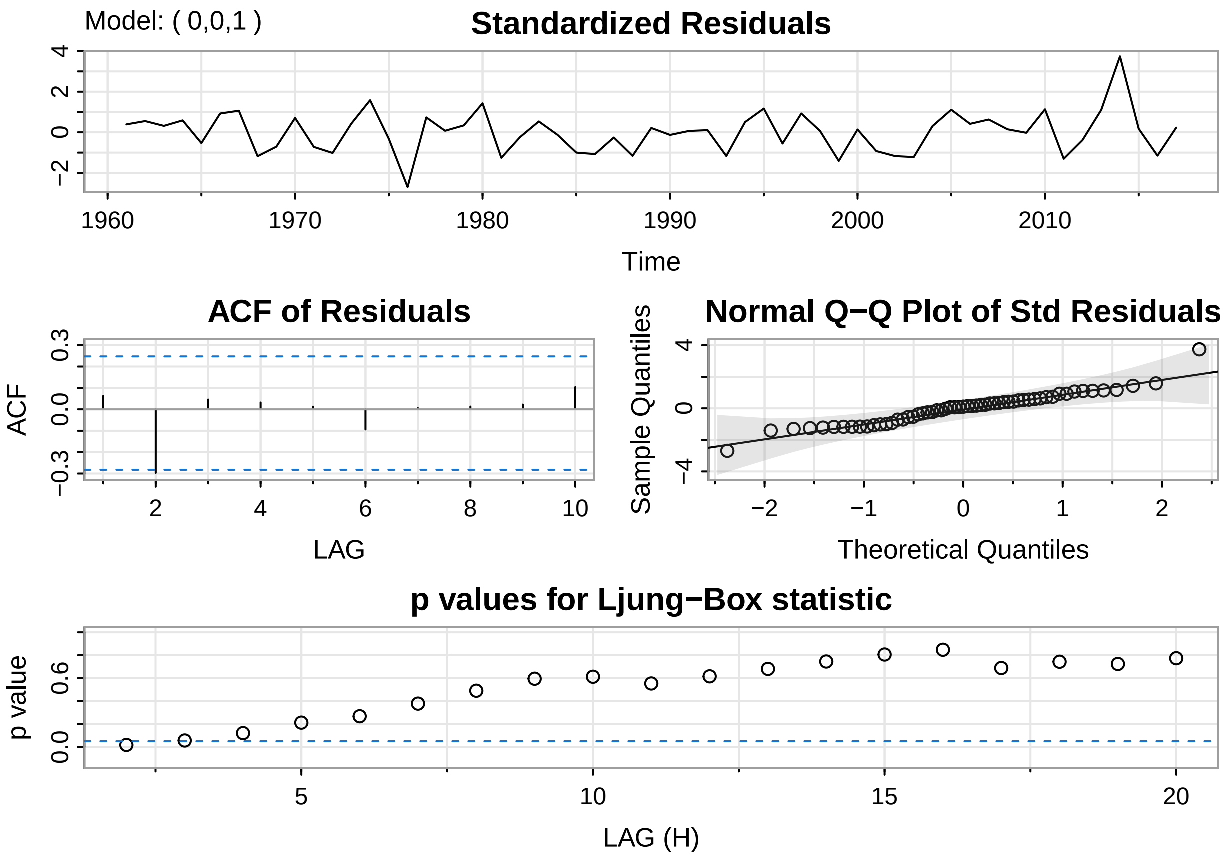
Imports

```r
# Model Selection on Log-Differenced Imports (stationary)
# Imports

# Model 1: MA(1)
log.diff.import.ma1 <- arima(log.diff.import.ts, order = c(0,0,1))
log.diff.import.ma1.diagnostics <- sarima(log.diff.import.ts, 0, 0, 1)
```

```
## initial  value -2.132354
## iter   2 value -2.139659
## iter   3 value -2.143882
## iter   4 value -2.144278
## iter   5 value -2.144320
## iter   6 value -2.144360
## iter   7 value -2.144361
## iter   8 value -2.144361
## iter   9 value -2.144361
## iter   9 value -2.144361
## iter   9 value -2.144361
## final  value -2.144361
## converged
## initial  value -2.143999
## iter   2 value -2.144005
## iter   3 value -2.144005
## iter   4 value -2.144007
## iter   4 value -2.144007
## iter   4 value -2.144007
## final  value -2.144007
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ma1    -0.2314 0.1836 -1.2604  0.2128
## xmean  -0.0016 0.0120 -0.1367  0.8917
```

```
##
## sigma^2 estimated as 0.01371892 on 55 degrees of freedom
##
## AIC = -1.344874  AICc = -1.340975  BIC = -1.237345
##
```




```
log.diff.import.ma1
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(0, 0, 1))
##
## Coefficients:
##           ma1  intercept
##       -0.2314    -0.0016
## s.e.   0.1836     0.0120
##
## sigma^2 estimated as 0.01372:  log likelihood = 41.33,  aic = -76.66
```

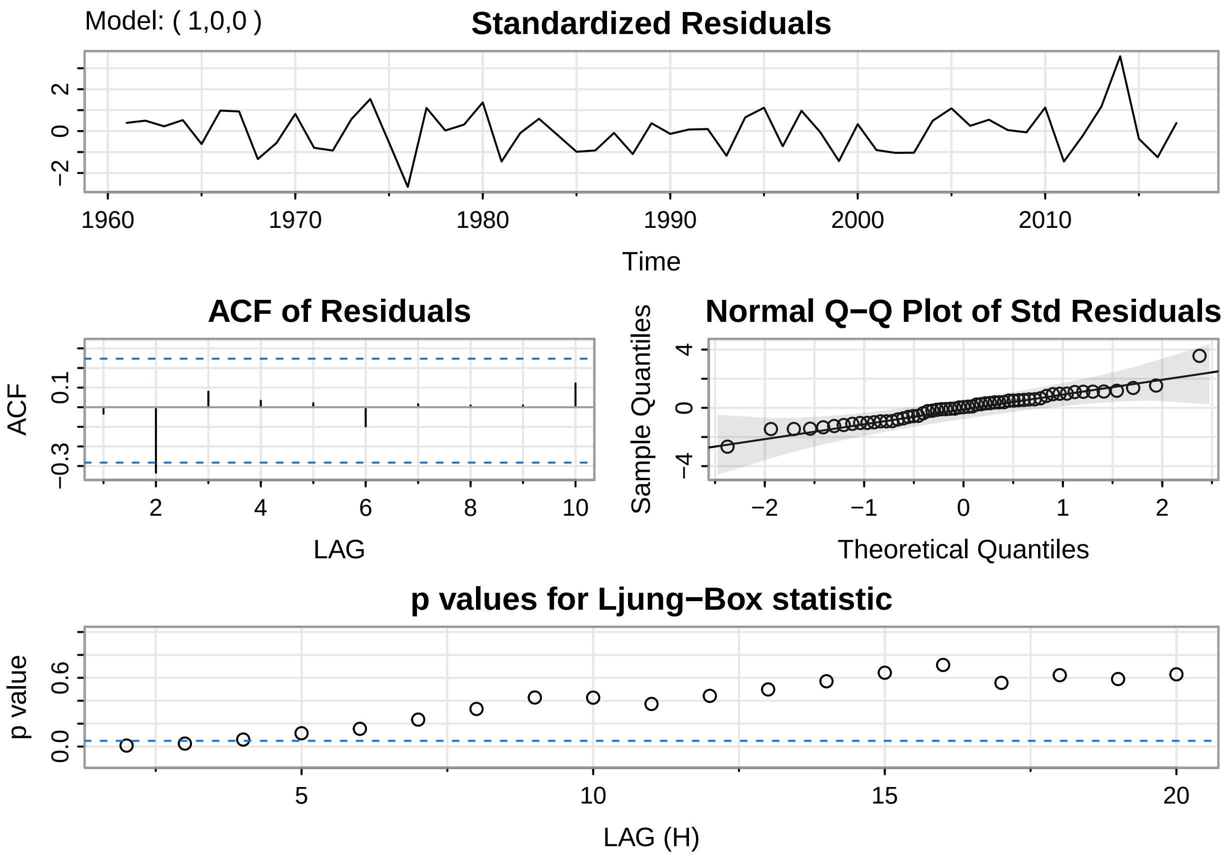
```
log.diff.import.ma1.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
```

```
## Coefficients:
##           ma1    xmean
##       -0.2314  -0.0016
## s.e.   0.1836   0.0120
##
## sigma^2 estimated as 0.01372:  log likelihood = 41.33,  aic = -76.66
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ma1    -0.2314 0.1836 -1.2604  0.2128
## xmean  -0.0016 0.0120 -0.1367  0.8917
##
## $ICs
##       AIC      AICc       BIC
## -1.344874 -1.340975 -1.237345
```

```r
# Model 2: AR(1)
log.diff.import.ar1 <- arima(log.diff.import.ts, order = c(1,0,0))
log.diff.import.ar1.diagnostics <- sarima(log.diff.import.ts, 1, 0, 0)
```

```
## initial  value -2.124853
## iter   2 value -2.129291
## iter   3 value -2.129301
## iter   4 value -2.129306
## iter   4 value -2.129306
## iter   4 value -2.129306
## final  value -2.129306
## converged
## initial  value -2.136680
## iter   2 value -2.136694
## iter   3 value -2.136701
## iter   3 value -2.136701
## iter   3 value -2.136701
## final  value -2.136701
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.0926 0.1312 -0.7056  0.4834
## xmean  -0.0014 0.0143 -0.1002  0.9205
##
## sigma^2 estimated as 0.01393219 on 55 degrees of freedom
##
## AIC = -1.330262  AICc = -1.326364  BIC = -1.222733
##
```




```
log.diff.import.ar1
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(1, 0, 0))
##
## Coefficients:
##           ar1  intercept
##       -0.0926    -0.0014
## s.e.   0.1312     0.0143
##
## sigma^2 estimated as 0.01393:  log likelihood = 40.91,  aic = -75.82
```

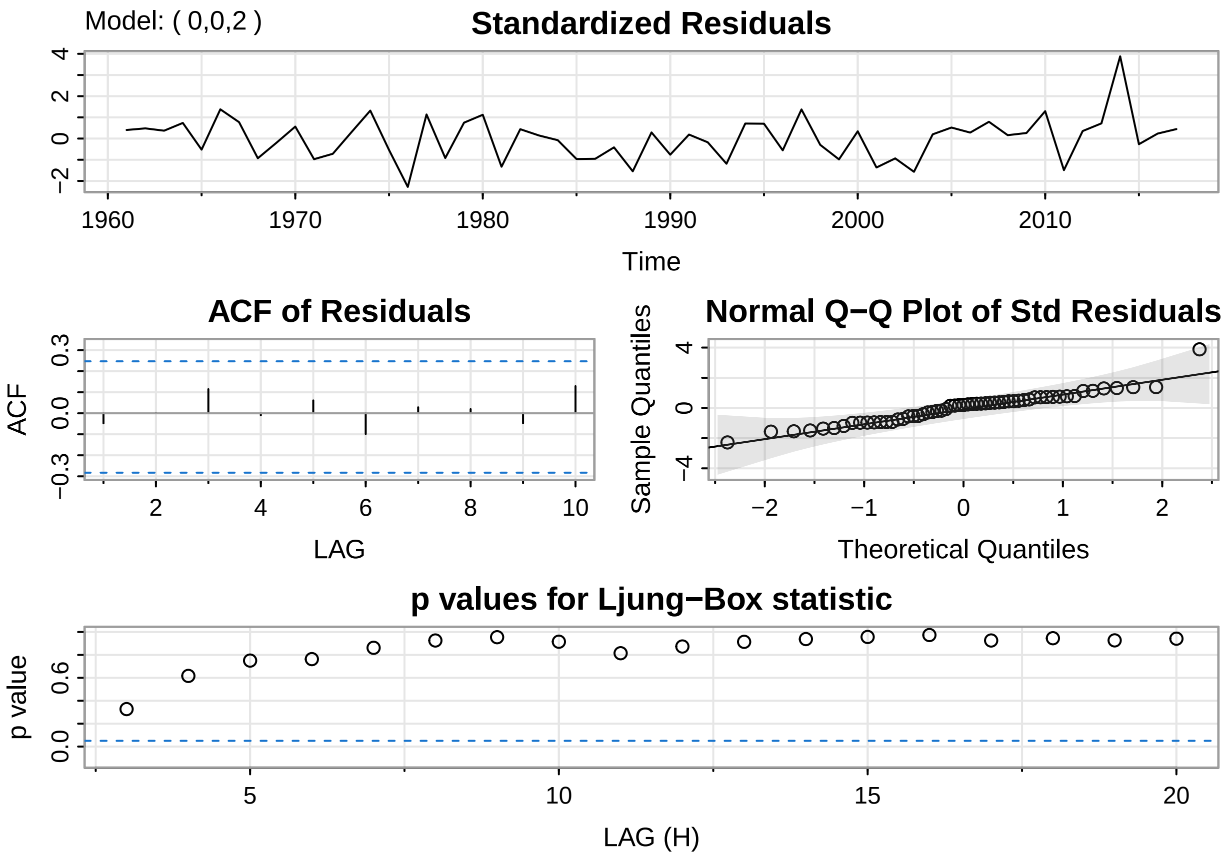
```
log.diff.import.ar1.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1    xmean
##       -0.0926  -0.0014
## s.e.   0.1312   0.0143
##
## sigma^2 estimated as 0.01393:  log likelihood = 40.91,  aic = -75.82
```

```
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.0926 0.1312 -0.7056  0.4834
## xmean  -0.0014 0.0143 -0.1002  0.9205
##
## $ICs
##       AIC      AICc       BIC
## -1.330262 -1.326364 -1.222733
```

```r
# Model 3: MA(2)
log.diff.import.ma2 <- arima(log.diff.import.ts, order = c(0,0,2))
log.diff.import.ma2.diagnostics <- sarima(log.diff.import.ts, 0, 0, 2)
```

```
## initial  value -2.132354
## iter   2 value -2.199743
## iter   3 value -2.200444
## iter   4 value -2.201470
## iter   5 value -2.202071
## iter   6 value -2.202083
## iter   7 value -2.202083
## iter   7 value -2.202083
## iter   7 value -2.202083
## final  value -2.202083
## converged
## initial  value -2.200774
## iter   2 value -2.200797
## iter   3 value -2.200822
## iter   4 value -2.200824
## iter   5 value -2.200824
## iter   5 value -2.200824
## iter   5 value -2.200824
## final  value -2.200824
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ma1    -0.0465 0.1315 -0.3538  0.7249
## ma2    -0.3865 0.1414 -2.7324  0.0085
## xmean  -0.0029 0.0086 -0.3335  0.7401
##
## sigma^2 estimated as 0.01218653 on 54 degrees of freedom
##
## AIC = -1.423419  AICc = -1.415475  BIC = -1.280047
##
```




```
log.diff.import.ma2
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(0, 0, 2))
##
## Coefficients:
##           ma1      ma2  intercept
##       -0.0465  -0.3865    -0.0029
## s.e.   0.1315   0.1414     0.0086
##
## sigma^2 estimated as 0.01219:  log likelihood = 44.57,  aic = -81.13
```

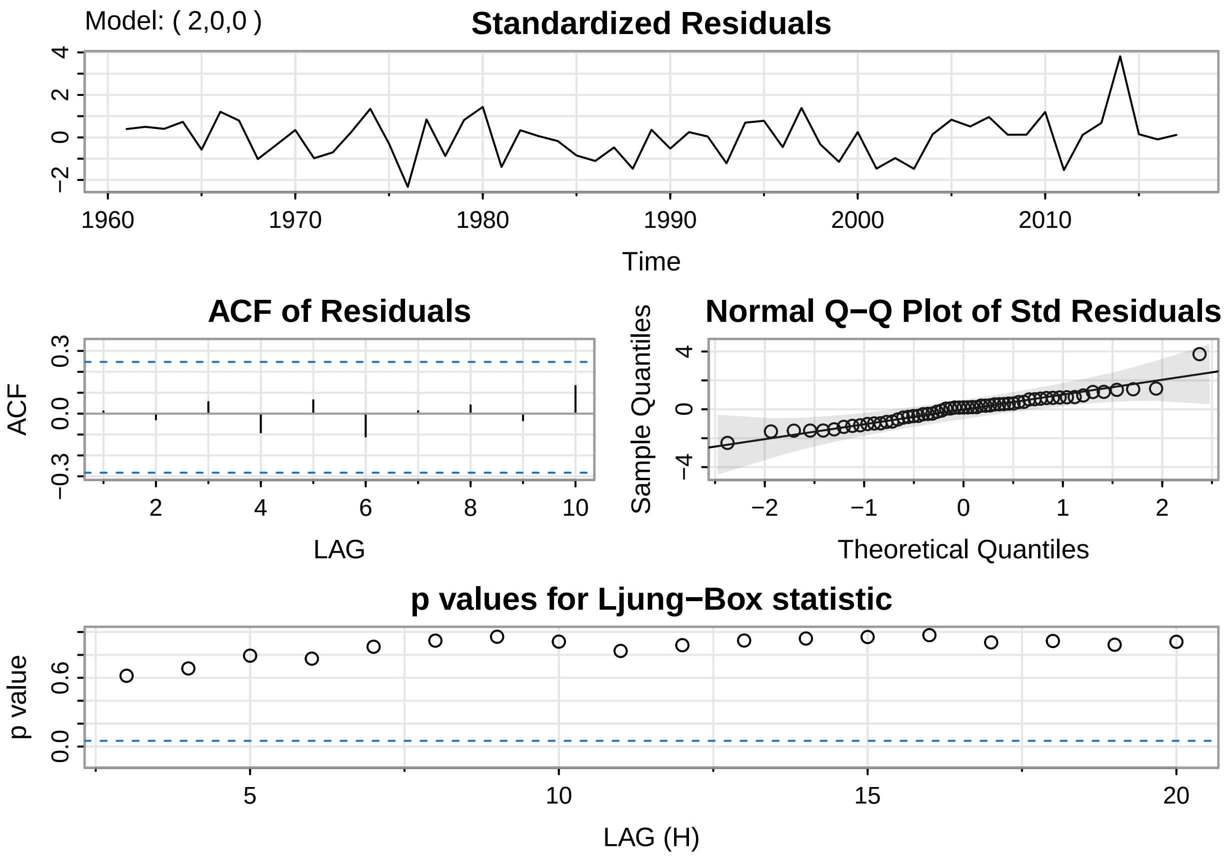
```
log.diff.import.ma2.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ma1      ma2    xmean
##       -0.0465  -0.3865  -0.0029
## s.e.   0.1315   0.1414   0.0086
##
## sigma^2 estimated as 0.01219:  log likelihood = 44.57,  aic = -81.13
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ma1    -0.0465 0.1315 -0.3538  0.7249
## ma2    -0.3865 0.1414 -2.7324  0.0085
## xmean  -0.0029 0.0086 -0.3335  0.7401
##
## $ICs
##       AIC      AICc       BIC
## -1.423419 -1.415475 -1.280047
```

```r
# Model 4: AR(2)
log.diff.import.ar2 <- arima(log.diff.import.ts, order = c(2,0,0))
log.diff.import.ar2.diagnostics <- sarima(log.diff.import.ts, 2, 0, 0)
```

```
## initial  value -2.117685
## iter   2 value -2.186025
## iter   3 value -2.186679
## iter   4 value -2.186969
## iter   5 value -2.187031
## iter   6 value -2.187032
## iter   6 value -2.187032
## iter   6 value -2.187032
## final  value -2.187032
## converged
## initial  value -2.198846
## iter   2 value -2.198923
## iter   3 value -2.198958
## iter   4 value -2.198960
## iter   5 value -2.198962
## iter   5 value -2.198962
## iter   5 value -2.198962
## final  value -2.198962
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.1229 0.1240 -0.9912  0.3260
## ar2    -0.3417 0.1236 -2.7637  0.0078
## xmean  -0.0015 0.0101 -0.1499  0.8814
##
## sigma^2 estimated as 0.01224758 on 54 degrees of freedom
##
## AIC = -1.419696  AICc = -1.411751  BIC = -1.276324
##
```




```
log.diff.import.ar2
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(2, 0, 0))
##
## Coefficients:
##           ar1      ar2  intercept
##       -0.1229  -0.3417    -0.0015
## s.e.   0.1240   0.1236     0.0101
##
## sigma^2 estimated as 0.01225:  log likelihood = 44.46,  aic = -80.92
```

```
log.diff.import.ar2.diagnostics
```

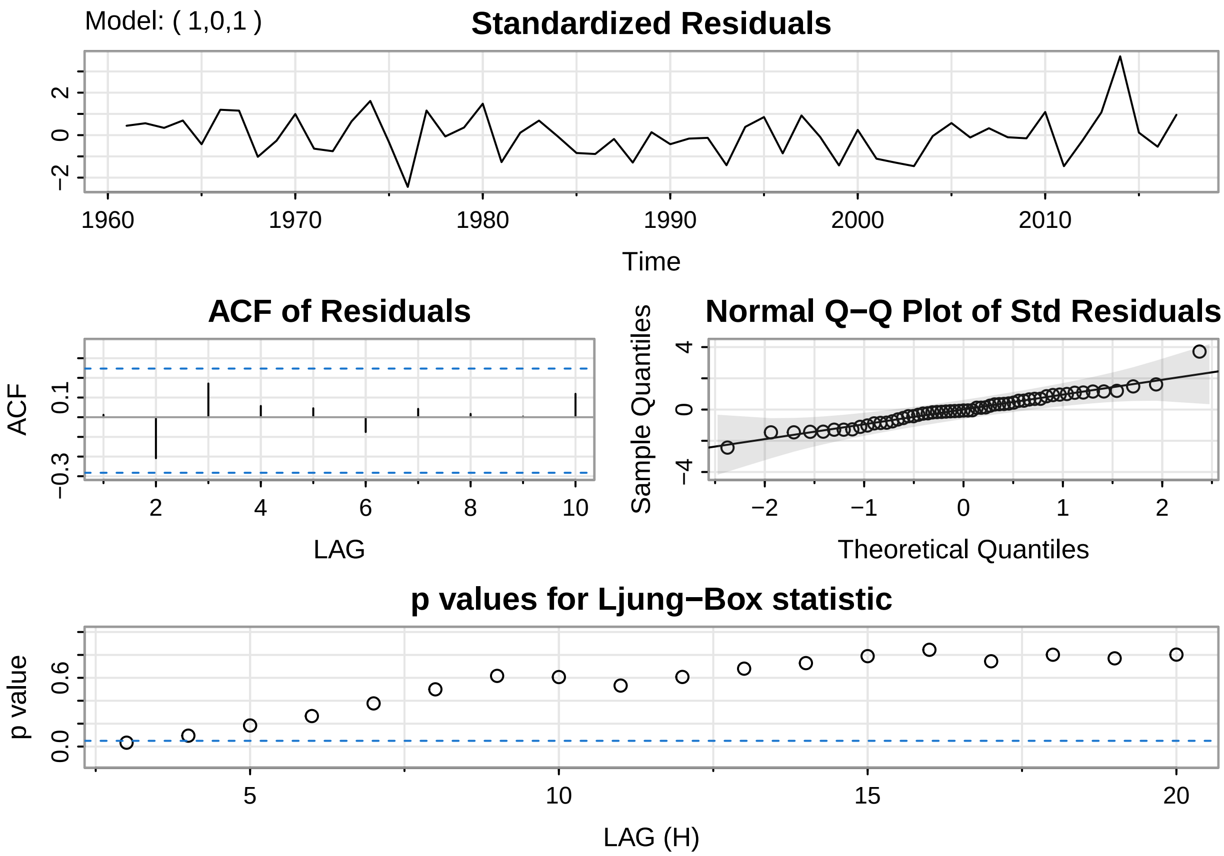
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ar2    xmean
##       -0.1229  -0.3417  -0.0015
## s.e.   0.1240   0.1236   0.0101
##
## sigma^2 estimated as 0.01225:  log likelihood = 44.46,  aic = -80.92
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.1229 0.1240 -0.9912  0.3260
## ar2    -0.3417 0.1236 -2.7637  0.0078
## xmean  -0.0015 0.0101 -0.1499  0.8814
##
## $ICs
##       AIC      AICc       BIC
## -1.419696 -1.411751 -1.276324
```

```r
# Model 5: ARMA(1,1) (final)
log.diff.import.arma11 <- arima(log.diff.import.ts, order = c(1,0,1))
log.diff.import.arma11.diagnostics <- sarima(log.diff.import.ts, 1, 0, 1)
```

```
## initial  value -2.124853
## iter   2 value -2.131198
## iter   3 value -2.133231
## iter   4 value -2.137431
## iter   5 value -2.149287
## iter   6 value -2.160505
## iter   7 value -2.163489
## iter   8 value -2.164322
## iter   9 value -2.164496
## iter  10 value -2.164541
## iter  11 value -2.164592
## iter  12 value -2.164651
## iter  13 value -2.164733
## iter  14 value -2.164745
## iter  15 value -2.164927
## iter  16 value -2.165021
## iter  17 value -2.165204
## iter  18 value -2.166863
## iter  19 value -2.167436
## iter  20 value -2.176018
## iter  21 value -2.178987
## iter  22 value -2.192394
## iter  23 value -2.194201
## iter  24 value -2.200608
## iter  25 value -2.203958
## iter  26 value -2.210785
## iter  27 value -2.211433
## iter  28 value -2.219517
## iter  29 value -2.220264
## iter  30 value -2.226721
## iter  31 value -2.239600
## iter  32 value -2.253113
## iter  33 value -2.260094
## iter  34 value -2.260647
## iter  35 value -2.263222
## iter  36 value -2.264100
## iter  36 value -2.264100
```

```
## iter  37 value -2.264274
## iter  37 value -2.264274
## iter  38 value -2.264308
## iter  38 value -2.264308
## iter  39 value -2.264316
## iter  39 value -2.264316
## iter  40 value -2.264317
## iter  40 value -2.264317
## iter  41 value -2.264318
## iter  41 value -2.264318
## iter  42 value -2.264318
## iter  42 value -2.264318
## iter  42 value -2.264318
## final  value -2.264318
## converged
## initial  value -2.151965
## iter   2 value -2.159581
## iter   3 value -2.165801
## iter   4 value -2.167893
## iter   5 value -2.168446
## iter   6 value -2.168480
## iter   7 value -2.168526
## iter   8 value -2.168538
## iter   9 value -2.168538
## iter  10 value -2.168539
## iter  11 value -2.168542
## iter  12 value -2.168542
## iter  12 value -2.168542
## iter  12 value -2.168542
## final  value -2.168542
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE  t.value p.value
## ar1     0.7838 0.0991   7.9080   0.000
## ma1    -1.0000 0.0583 -17.1426   0.000
## xmean  -0.0075 0.0036  -2.0729   0.043
##
## sigma^2 estimated as 0.0126088 on 54 degrees of freedom
##
## AIC = -1.358857  AICc = -1.350913  BIC = -1.215485
##
```




```
log.diff.import.arma11
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(1, 0, 1))
##
## Coefficients:
##          ar1      ma1  intercept
##       0.7838  -1.0000    -0.0075
## s.e.  0.0991   0.0583     0.0036
##
## sigma^2 estimated as 0.01261:  log likelihood = 42.73,  aic = -77.45
```

```
log.diff.import.arma11.diagnostics
```

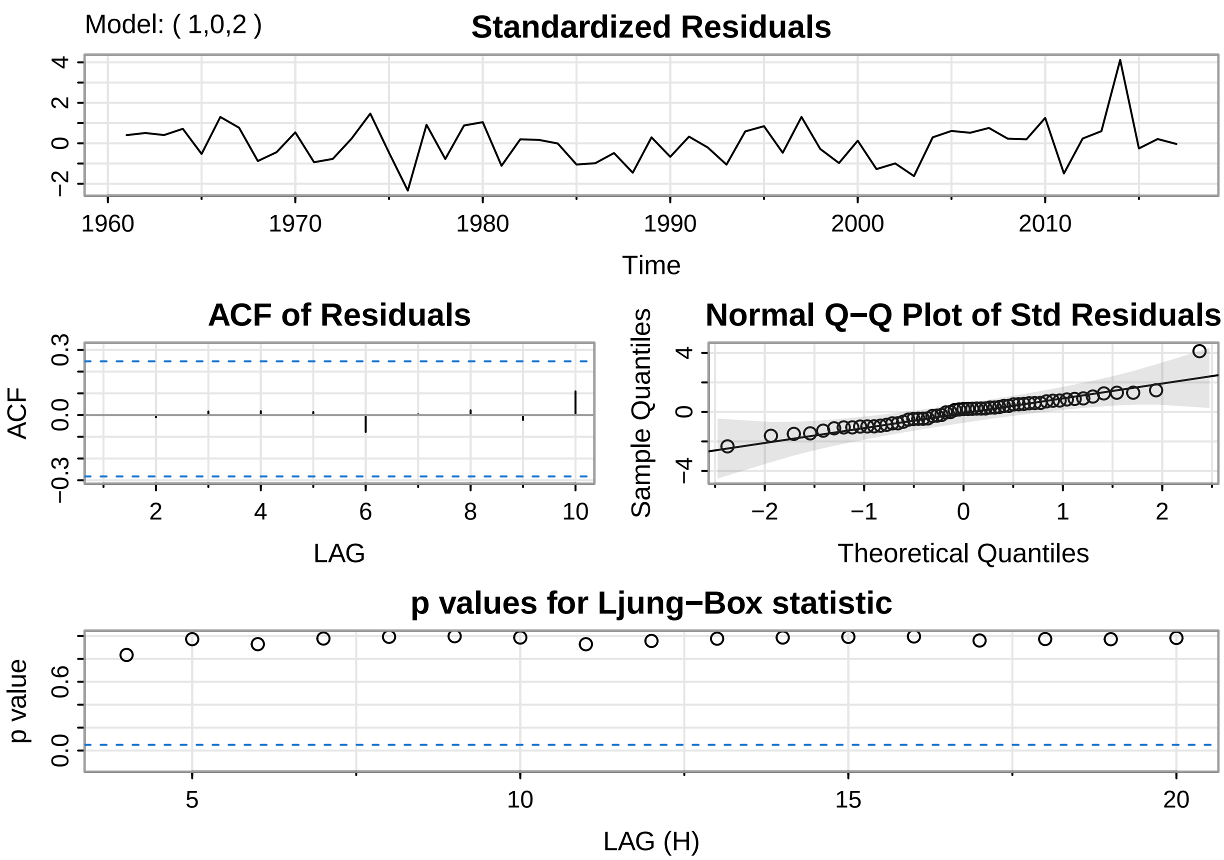
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1    xmean
##       0.7838  -1.0000  -0.0075
## s.e.  0.0991   0.0583   0.0036
##
## sigma^2 estimated as 0.01261:  log likelihood = 42.73,  aic = -77.45
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE  t.value p.value
## ar1     0.7838 0.0991   7.9080   0.000
## ma1    -1.0000 0.0583 -17.1426   0.000
## xmean  -0.0075 0.0036  -2.0729   0.043
##
## $ICs
##       AIC      AICc       BIC
## -1.358857 -1.350913 -1.215485
```

```r
# Model 6: ARMA(1,2)
log.diff.import.arma12 <- arima(log.diff.import.ts, order = c(1,0,2))
log.diff.import.arma12.diagnostics <- sarima(log.diff.import.ts, 1, 0, 2)
```

```
## initial  value -2.124853
## iter   2 value -2.190526
## iter   3 value -2.196281
## iter   4 value -2.197046
## iter   5 value -2.198253
## iter   6 value -2.199359
## iter   7 value -2.201100
## iter   8 value -2.202005
## iter   9 value -2.202359
## iter  10 value -2.202363
## iter  11 value -2.202364
## iter  12 value -2.202364
## iter  13 value -2.202364
## iter  14 value -2.202364
## iter  15 value -2.202364
## iter  15 value -2.202364
## iter  15 value -2.202364
## final  value -2.202364
## converged
## initial  value -2.209843
## iter   2 value -2.209920
## iter   3 value -2.210058
## iter   4 value -2.210177
## iter   5 value -2.210240
## iter   6 value -2.210268
## iter   7 value -2.210269
## iter   7 value -2.210269
## iter   7 value -2.210269
## final  value -2.210269
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.3312 0.2902 -1.1412  0.2589
## ma1     0.2433 0.2720  0.8944  0.3751
## ma2    -0.4033 0.1267 -3.1820  0.0024
```

```
## xmean  -0.0025 0.0093 -0.2631  0.7935
##
## sigma^2 estimated as 0.01194652 on 53 degrees of freedom
##
## AIC = -1.407221  AICc = -1.393726  BIC = -1.228006
##
```




```
log.diff.import.arma12
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(1, 0, 2))
##
## Coefficients:
##           ar1     ma1      ma2  intercept
##       -0.3312  0.2433  -0.4033    -0.0025
## s.e.   0.2902  0.2720   0.1267     0.0093
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -80.21
```

```
log.diff.import.arma12.diagnostics
```

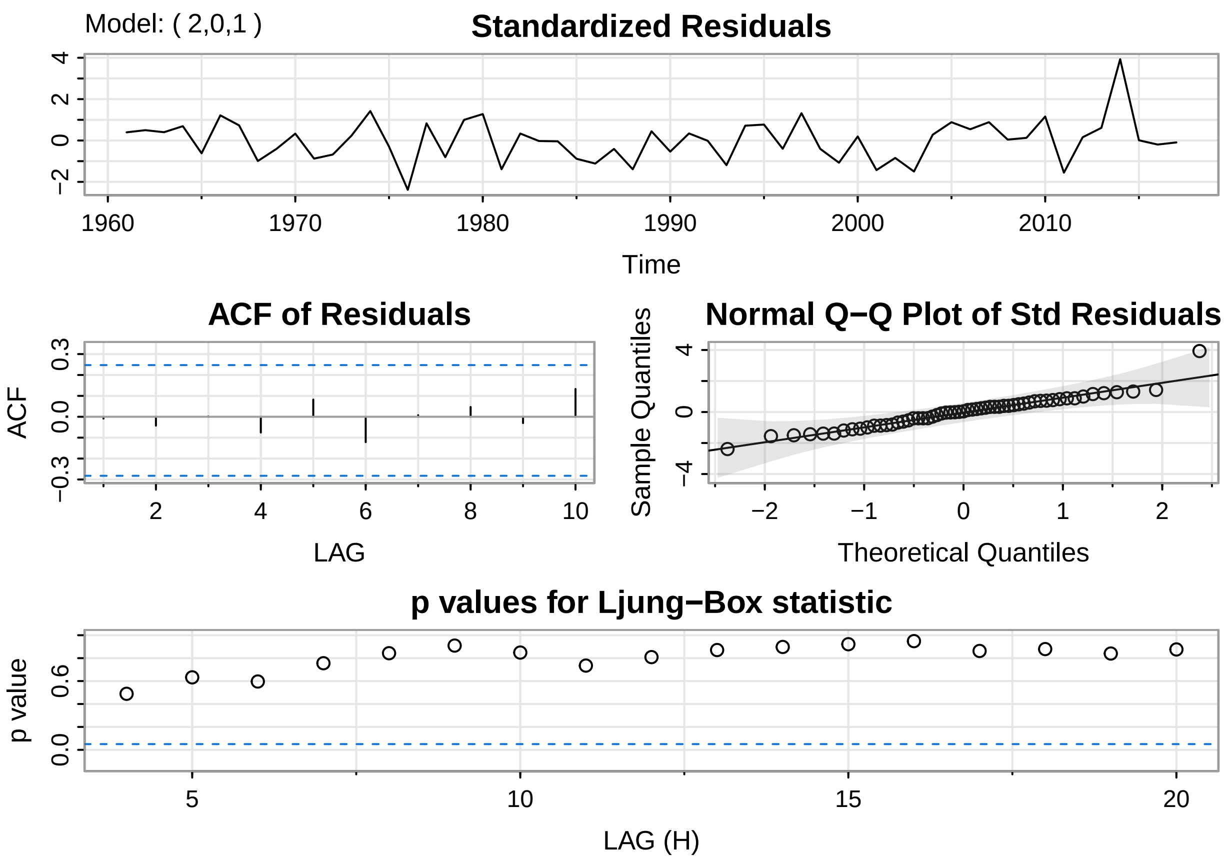
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
```

```
##
## Coefficients:
##           ar1     ma1      ma2    xmean
##       -0.3312  0.2433  -0.4033  -0.0025
## s.e.   0.2902  0.2720   0.1267   0.0093
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -80.21
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.3312 0.2902 -1.1412  0.2589
## ma1     0.2433 0.2720  0.8944  0.3751
## ma2    -0.4033 0.1267 -3.1820  0.0024
## xmean  -0.0025 0.0093 -0.2631  0.7935
##
## $ICs
##       AIC      AICc       BIC
## -1.407221 -1.393726 -1.228006
```

```r
# Model 7: ARMA(2,1)
log.diff.import.arma21 <- arima(log.diff.import.ts, order = c(2,0,1))
log.diff.import.arma21.diagnostics <- sarima(log.diff.import.ts, 2, 0, 1)
```

```
## initial  value -2.117685
## iter   2 value -2.183588
## iter   3 value -2.185259
## iter   4 value -2.185930
## iter   5 value -2.186463
## iter   6 value -2.186556
## iter   7 value -2.187485
## iter   8 value -2.187673
## iter   9 value -2.187720
## iter  10 value -2.187726
## iter  11 value -2.187727
## iter  11 value -2.187727
## iter  11 value -2.187727
## final  value -2.187727
## converged
## initial  value -2.199962
## iter   2 value -2.200019
## iter   3 value -2.200062
## iter   4 value -2.200062
## iter   5 value -2.200063
## iter   6 value -2.200064
## iter   7 value -2.200069
## iter   8 value -2.200072
## iter   9 value -2.200074
## iter  10 value -2.200074
## iter  10 value -2.200074
## final  value -2.200074
## converged
## <><><><><><><><><><><><><><>
```

```
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.3207 0.7160 -0.4479  0.6561
## ar2    -0.3531 0.1222 -2.8890  0.0056
## ma1     0.2253 0.8057  0.2797  0.7808
## xmean  -0.0016 0.0108 -0.1486  0.8824
##
## sigma^2 estimated as 0.01221868 on 53 degrees of freedom
##
## AIC = -1.386832  AICc = -1.373337  BIC = -1.207617
##
```




```
log.diff.import.arma21
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(2, 0, 1))
##
## Coefficients:
##           ar1      ar2     ma1  intercept
##       -0.3207  -0.3531  0.2253    -0.0016
## s.e.   0.7160   0.1222  0.8057     0.0108
##
## sigma^2 estimated as 0.01222:  log likelihood = 44.52,  aic = -79.05
```

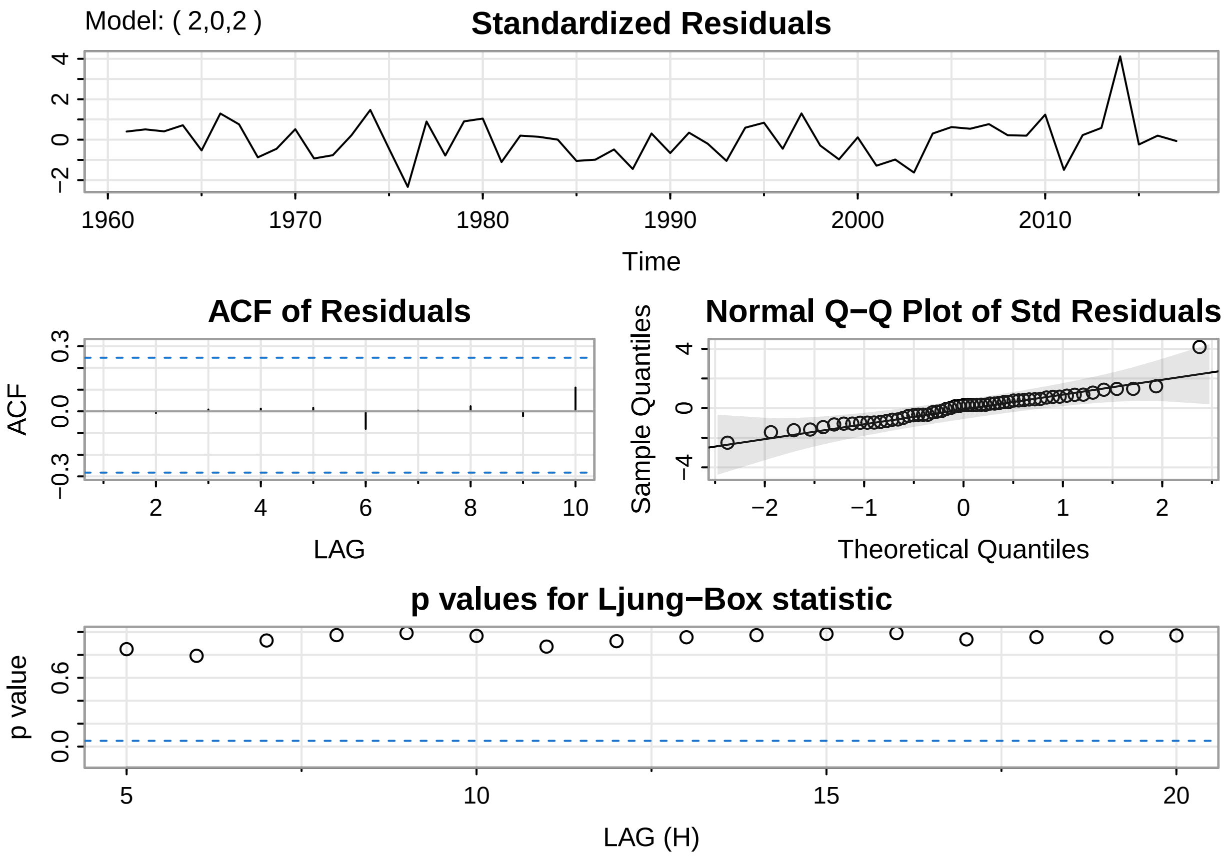
```r
log.diff.import.arma21.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ar2     ma1    xmean
##       -0.3207  -0.3531  0.2253  -0.0016
## s.e.   0.7160   0.1222  0.8057   0.0108
##
## sigma^2 estimated as 0.01222:  log likelihood = 44.52,  aic = -79.05
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.3207 0.7160 -0.4479  0.6561
## ar2    -0.3531 0.1222 -2.8890  0.0056
## ma1     0.2253 0.8057  0.2797  0.7808
## xmean  -0.0016 0.0108 -0.1486  0.8824
##
## $ICs
##       AIC      AICc       BIC
## -1.386832 -1.373337 -1.207617
```

```r
# Model 8: ARMA(2,2)
log.diff.import.arma22 <- arima(log.diff.import.ts, order = c(2,0,2))
log.diff.import.arma22.diagnostics <- sarima(log.diff.import.ts, 2, 0, 2)
```

```
## initial  value -2.117685
## iter   2 value -2.146070
## iter   3 value -2.189558
## iter   4 value -2.191196
## iter   5 value -2.191658
## iter   6 value -2.194157
## iter   7 value -2.196115
## iter   8 value -2.199075
## iter   9 value -2.200842
## iter  10 value -2.202596
## iter  11 value -2.203318
## iter  12 value -2.203457
## iter  13 value -2.203461
## iter  14 value -2.203464
## iter  15 value -2.203467
## iter  16 value -2.203469
## iter  17 value -2.203470
## iter  18 value -2.203472
## iter  19 value -2.203476
## iter  20 value -2.203477
```

```
## iter  21 value -2.203477
## iter  22 value -2.203478
## iter  23 value -2.203478
## iter  23 value -2.203478
## final  value -2.203478
## converged
## initial  value -2.209480
## iter   2 value -2.209834
## iter   3 value -2.209908
## iter   4 value -2.210012
## iter   5 value -2.210118
## iter   6 value -2.210246
## iter   7 value -2.210304
## iter   8 value -2.210312
## iter   9 value -2.210329
## iter  10 value -2.210339
## iter  11 value -2.210345
## iter  12 value -2.210345
## iter  12 value -2.210345
## final  value -2.210345
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.3480 0.3419 -1.0178  0.3135
## ar2    -0.0325 0.3434 -0.0947  0.9249
## ma1     0.2586 0.3225  0.8019  0.4263
## ma2    -0.3743 0.3343 -1.1197  0.2680
## xmean  -0.0024 0.0095 -0.2494  0.8040
##
## sigma^2 estimated as 0.0119451 on 52 degrees of freedom
##
## AIC = -1.372287  AICc = -1.351647  BIC = -1.157229
##
```




```
log.diff.import.arma22
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(2, 0, 2))
##
## Coefficients:
##           ar1      ar2     ma1      ma2  intercept
##       -0.3480  -0.0325  0.2586  -0.3743    -0.0024
## s.e.   0.3419   0.3434  0.3225   0.3343     0.0095
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -78.22
```

```
log.diff.import.arma22.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ar2     ma1      ma2    xmean
##       -0.3480  -0.0325  0.2586  -0.3743  -0.0024
## s.e.   0.3419   0.3434  0.3225   0.3343   0.0095
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -78.22
```

```
##
## $degrees_of_freedom
## [1] 52
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.3480 0.3419 -1.0178  0.3135
## ar2    -0.0325 0.3434 -0.0947  0.9249
## ma1     0.2586 0.3225  0.8019  0.4263
## ma2    -0.3743 0.3343 -1.1197  0.2680
## xmean  -0.0024 0.0095 -0.2494  0.8040
##
## $ICs
##       AIC      AICc       BIC
## -1.372287 -1.351647 -1.157229
```

Imports Final Model: ARMA(1,1)

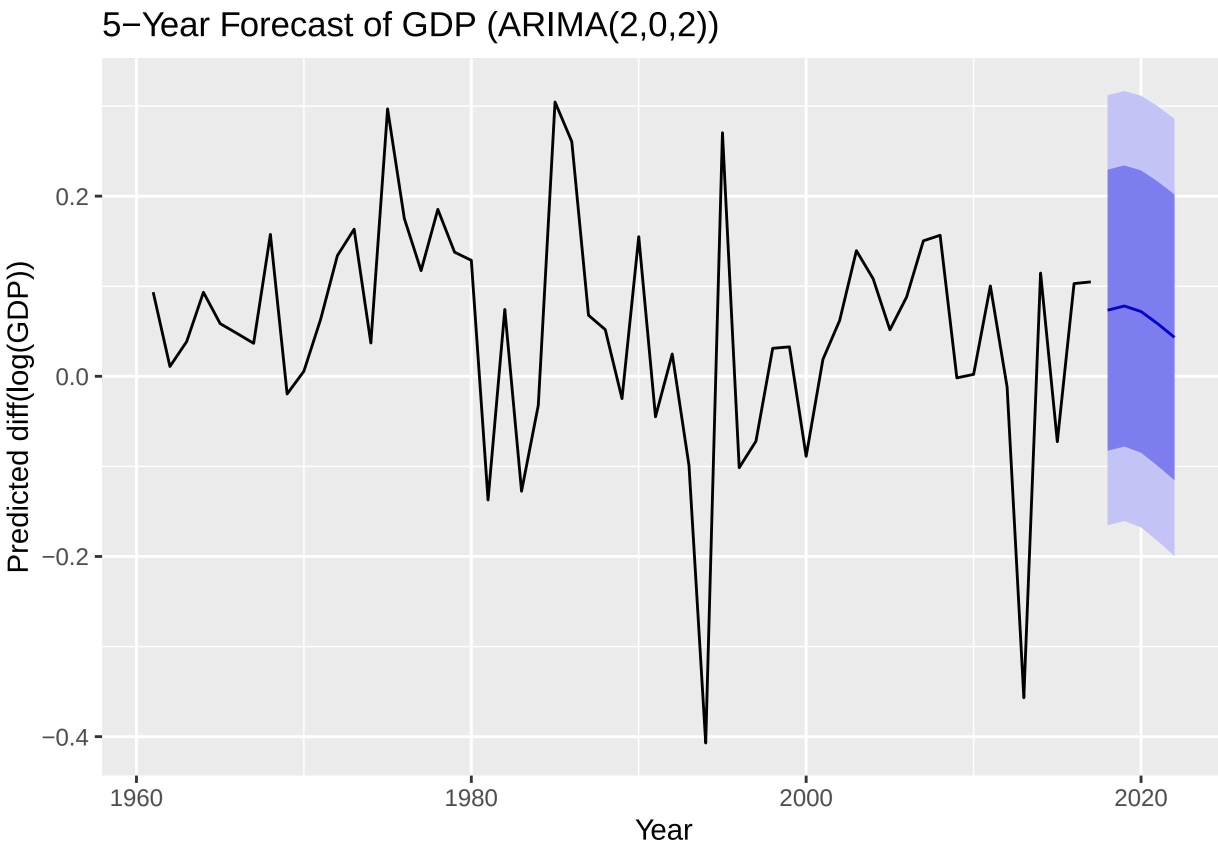
Forecast

GDP

```r
# Forecast
# GDP
gdp_forecast <- forecast(log.diff.gdp.arma22, h = 5)
gdp_forecast
```

```
##      Point Forecast       Lo 80     Hi 80      Lo 95     Hi 95
## 2018     0.07335514 -0.08268150 0.2293918 -0.1652823 0.3119926
## 2019     0.07804164 -0.07800850 0.2340918 -0.1606165 0.3166997
## 2020     0.07190733 -0.08476349 0.2285781 -0.1677000 0.3115147
## 2021     0.05832534 -0.09951227 0.2161630 -0.1830665 0.2997171
## 2022     0.04318533 -0.11551828 0.2018889 -0.1995309 0.2859015
```

```r
autoplot(gdp_forecast) +
  ggtitle("5-Year Forecast of GDP (ARIMA(2,0,2))") +
  xlab("Year") +
  ylab("Predicted diff(log(GDP))")
```

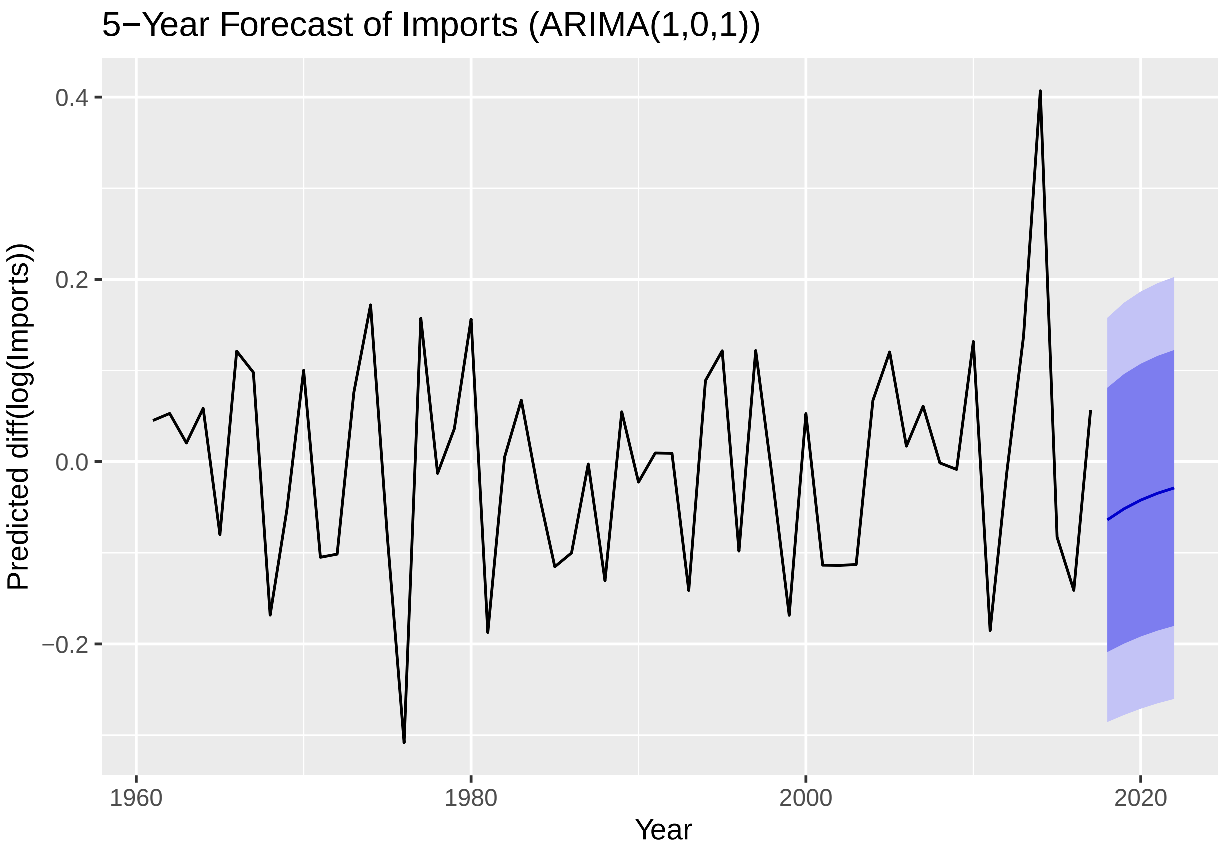


Imports

```r
# Imports
import_forecast <- forecast(log.diff.import.arma11, h = 5)
import_forecast
```

```
##      Point Forecast      Lo 80      Hi 80      Lo 95     Hi 95
## 2018    -0.06390127 -0.2089035 0.08110099 -0.2856631 0.1578605
## 2019    -0.05171036 -0.1995994 0.09617871 -0.2778872 0.1744665
## 2020    -0.04215508 -0.1917900 0.10747989 -0.2710020 0.1866919
## 2021    -0.03466560 -0.1853631 0.11603193 -0.2651376 0.1958064
## 2022    -0.02879531 -0.1801419 0.12255130 -0.2602600 0.2026694
```

```r
autoplot(import_forecast) +
  ggtitle("5-Year Forecast of Imports (ARIMA(1,0,1))") +
  xlab("Year") +
  ylab("Predicted diff(log(Imports))")
```


5−Year Forecast of Imports (ARIMA(1,0,1))
0.4
0.2
0.0
−0.2
Predicted diff(log(Imports))
1960
1980
2000
2020
Year