Appendix R Code

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2025-06-05

```
library(astsa)
library(MASS)
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
##
## Attaching package: 'forecast'
```

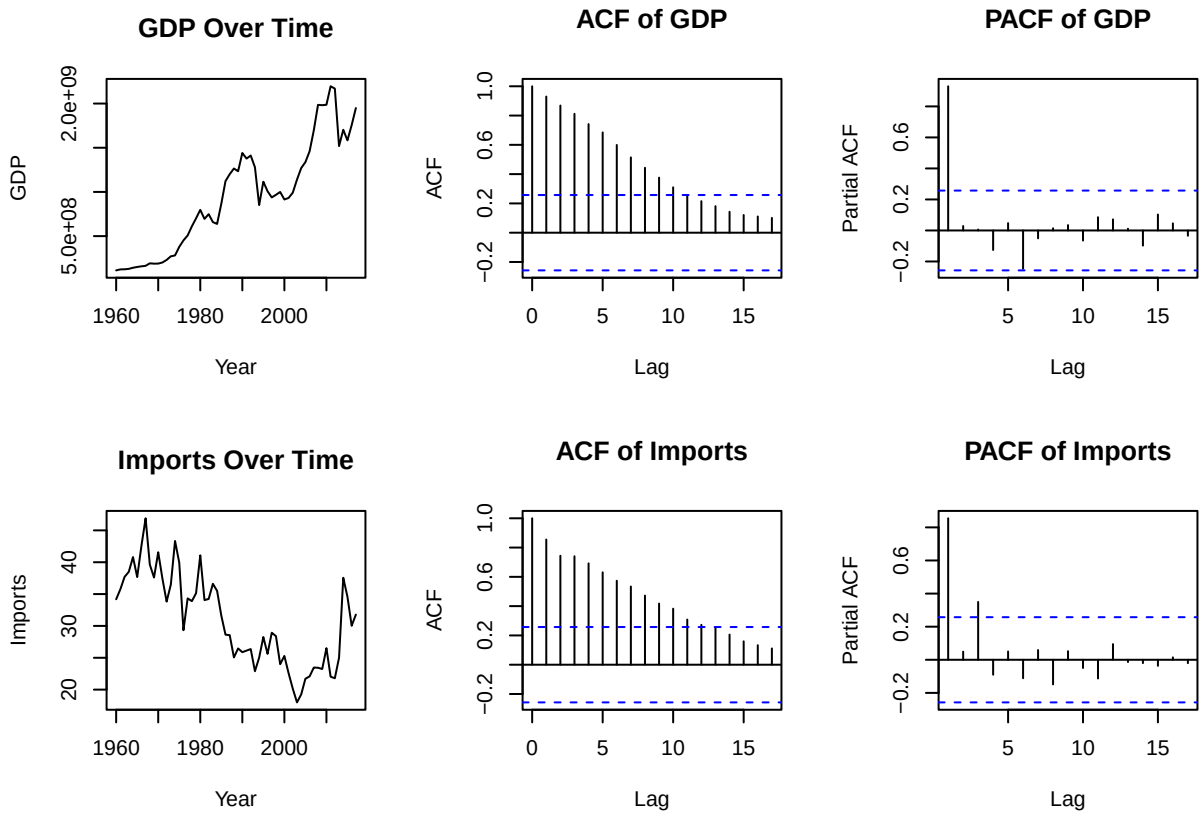
```
## The following object is masked from 'package:astsa':
##
##     gas
```

```
library(ggplot2)
load("finalproject.Rdata")
```

Exploratory Data Analysis

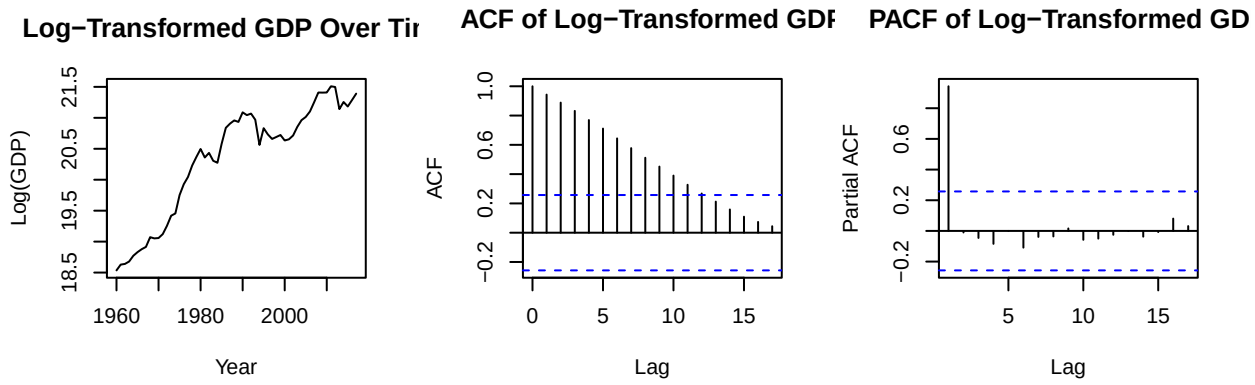
Raw Data

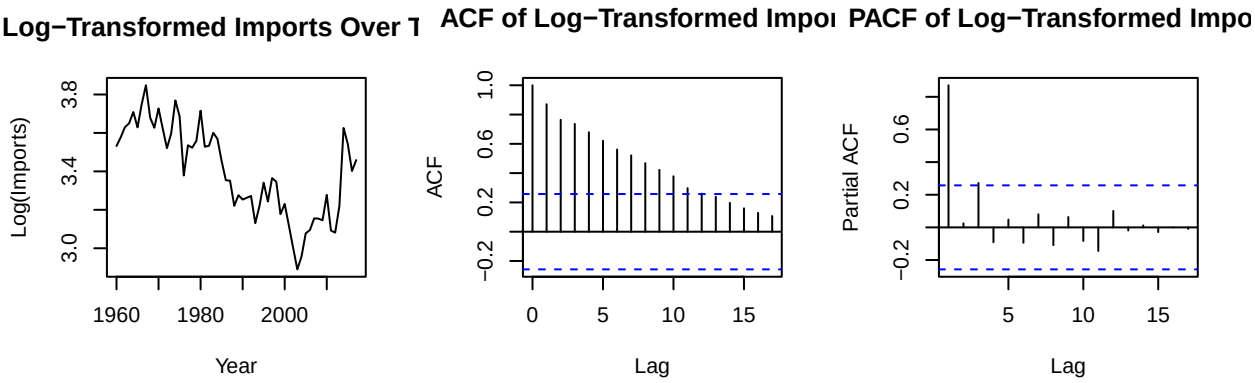
```
# Exploratory Data Analysis
# Raw Data with Year-Based Time Series

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
gdp_ts <- ts(finalPro_data$GDP, start = 1960, frequency = 1)
plot(gdp_ts, main = "GDP Over Time", ylab = "GDP", xlab = "Year")
acf(gdp_ts, main = "ACF of GDP")
pacf(gdp_ts, main = "PACF of GDP")

# 2. Imports
import_ts <- ts(finalPro_data$Imports, start = 1960, frequency = 1)
plot(import_ts, main = "Imports Over Time", ylab = "Imports", xlab = "Year")
acf(import_ts, main = "ACF of Imports")
pacf(import_ts, main = "PACF of Imports")
```



Log-Transformed Data

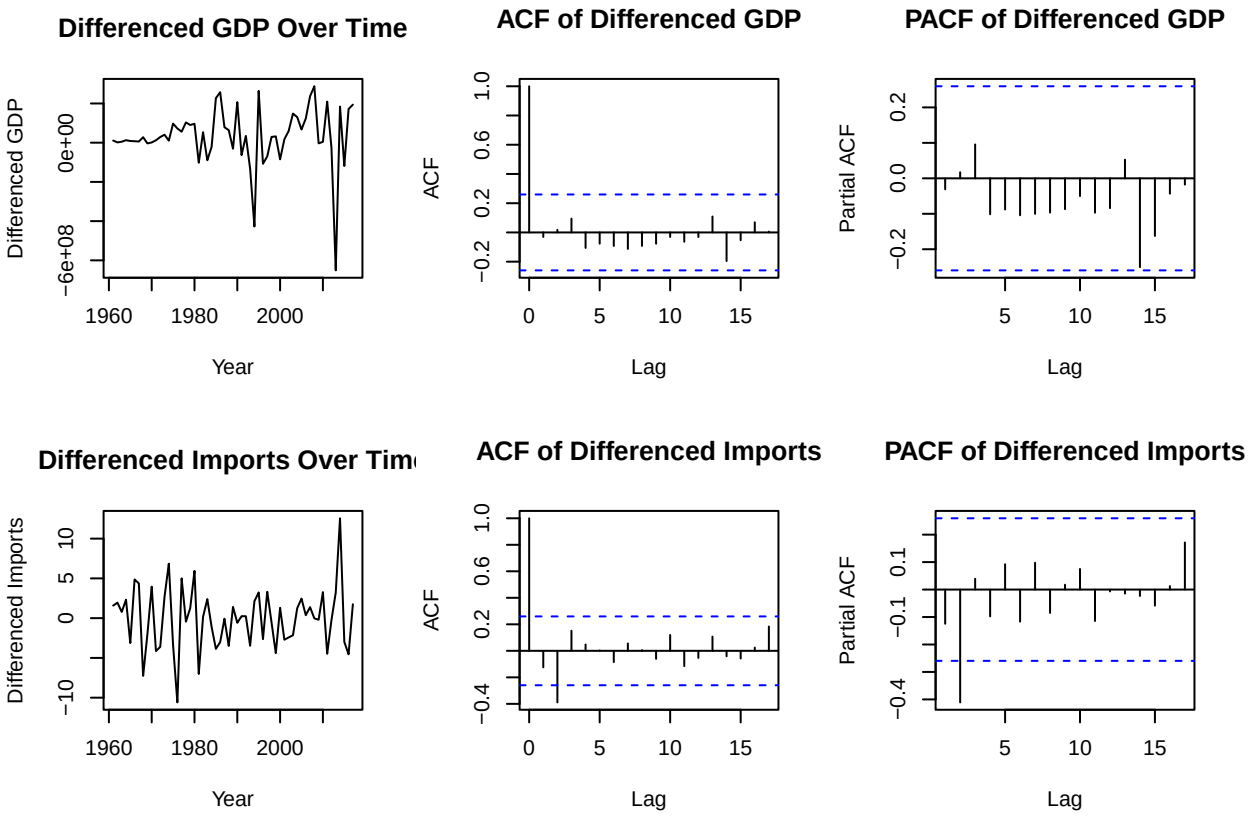
```r
# Log-Transformed Data
log.gdp.ts <- ts(log(finalPro_data$GDP), start = 1960, frequency = 1)
log.import.ts <- ts(log(finalPro_data$Imports), start = 1960, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(log.gdp.ts, main = "Log-Transformed GDP Over Time", ylab = "Log(GDP)", xlab = "Year")
acf(log.gdp.ts, main = "ACF of Log-Transformed GDP")
pacf(log.gdp.ts, main = "PACF of Log-Transformed GDP")

# 2. Imports
plot(log.import.ts, main = "Log-Transformed Imports Over Time", ylab = "Log(Imports)", xlab = "Year")
acf(log.import.ts, main = "ACF of Log-Transformed Imports")
pacf(log.import.ts, main = "PACF of Log-Transformed Imports")
```





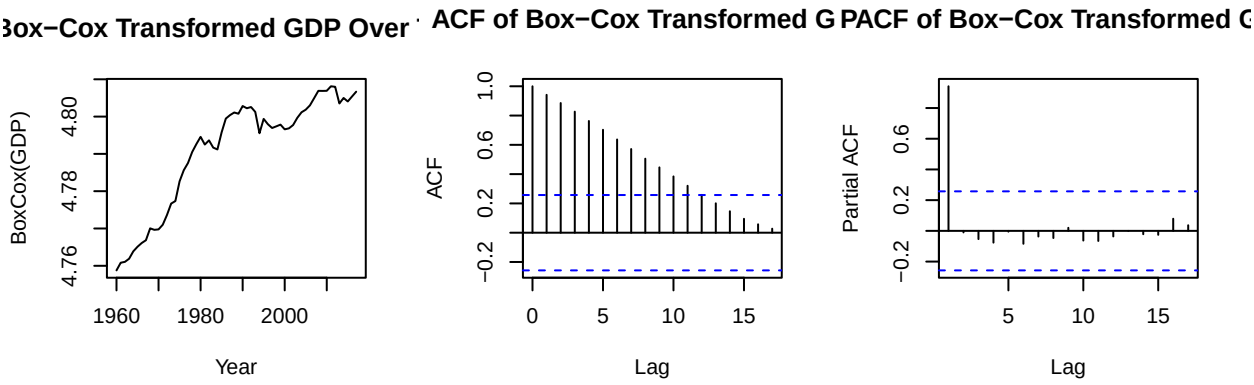
Differencing Data

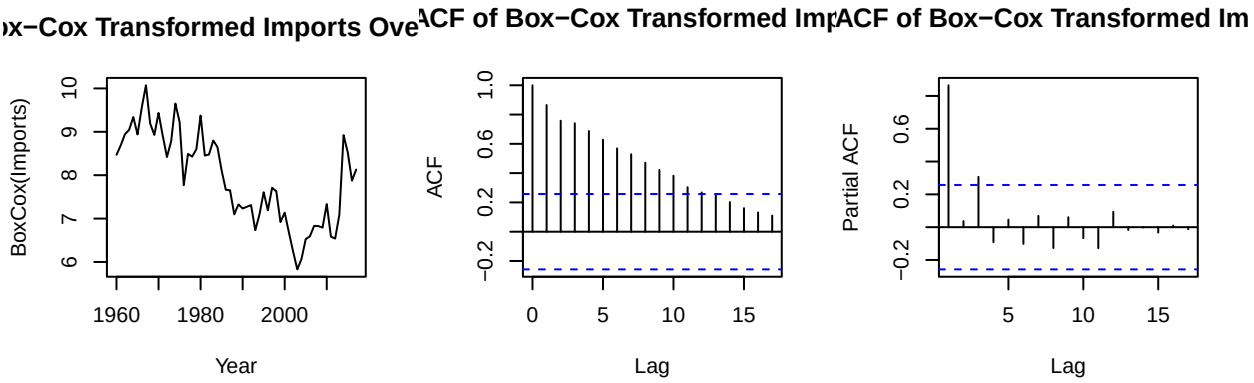
```r
# Differenced Data (first difference removes first observation, so start at 1961)
diff.gdp.ts <- ts(diff(finalPro_data$GDP), start = 1961, frequency = 1)
diff.import.ts <- ts(diff(finalPro_data$Imports), start = 1961, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(diff.gdp.ts, main = "Differenced GDP Over Time", ylab = "Differenced GDP", xlab = "Year")
acf(diff.gdp.ts, main = "ACF of Differenced GDP")
pacf(diff.gdp.ts, main = "PACF of Differenced GDP")

# 2. Imports
plot(diff.import.ts, main = "Differenced Imports Over Time", ylab = "Differenced Imports", xlab = "Year
acf(diff.import.ts, main = "ACF of Differenced Imports")
pacf(diff.import.ts, main = "PACF of Differenced Imports")
```



Box-cox Transformed Data

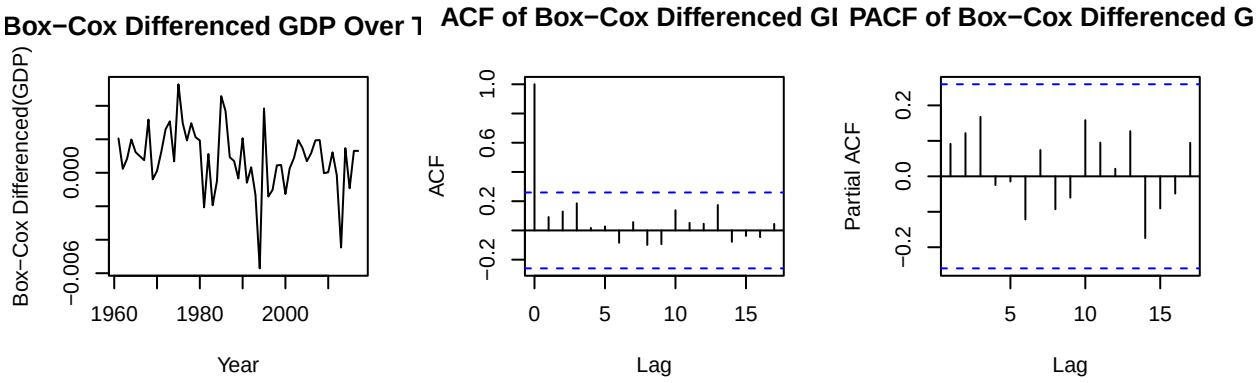
```
# Box-Cox Transformed Data
gdp.lambda <- BoxCox.lambda(finalPro_data$GDP)
boxcox.gdp.ts <- ts(BoxCox(finalPro_data$GDP, gdp.lambda), start = 1960, frequency = 1)

import.lambda <- BoxCox.lambda(finalPro_data$Imports)
boxcox.import.ts <- ts(BoxCox(finalPro_data$Imports, import.lambda), start = 1960, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(boxcox.gdp.ts, main = "Box-Cox Transformed GDP Over Time", ylab = "BoxCox(GDP)", xlab = "Year")
acf(boxcox.gdp.ts, main = "ACF of Box-Cox Transformed GDP")
pacf(boxcox.gdp.ts, main = "PACF of Box-Cox Transformed GDP")

# 2. Imports
plot(boxcox.import.ts, main = "Box-Cox Transformed Imports Over Time", ylab = "BoxCox(Imports)", xlab =
acf(boxcox.import.ts, main = "ACF of Box-Cox Transformed Imports")
pacf(boxcox.import.ts, main = "PACF of Box-Cox Transformed Imports")
```

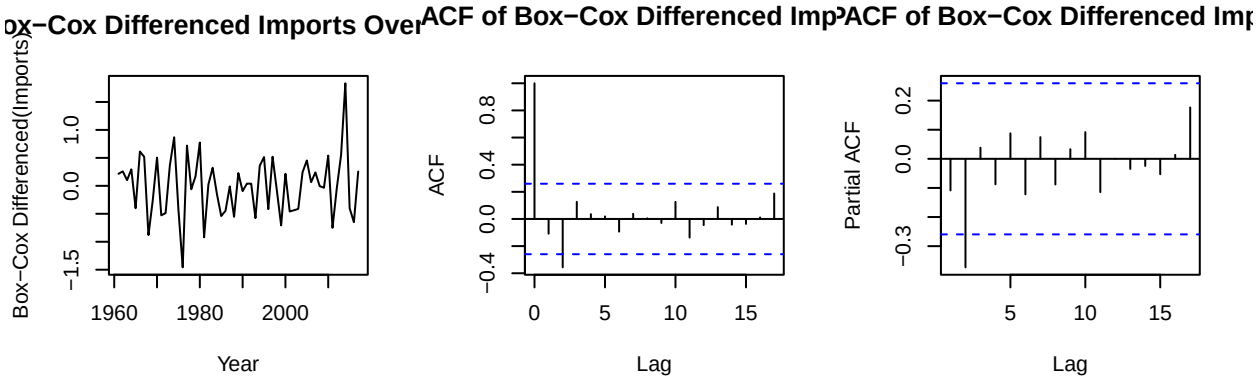




Box-cox Differencing Data

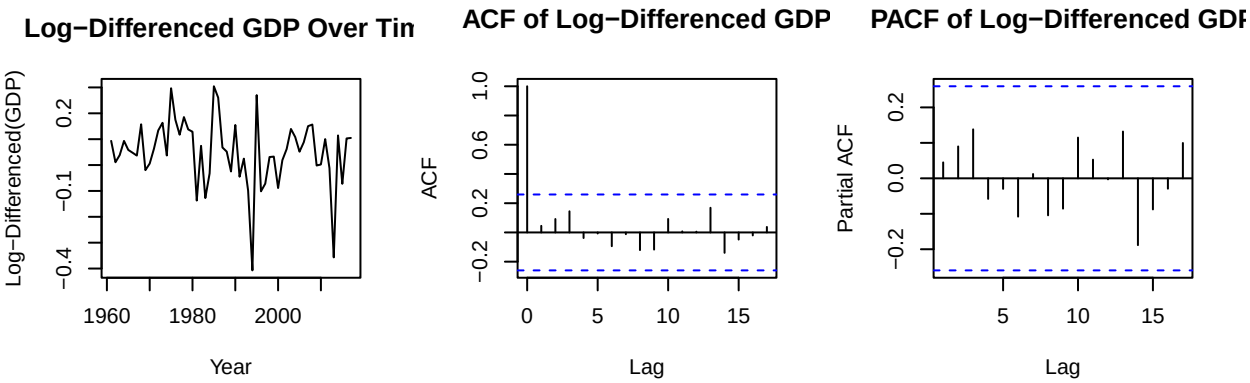
```r
# Box-Cox Differenced Data (first difference removes first year, so start = 1961)
boxcox.diff.gdp.ts <- ts(diff(boxcox.gdp.ts), start = 1961, frequency = 1)
boxcox.diff.import.ts <- ts(diff(boxcox.import.ts), start = 1961, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(boxcox.diff.gdp.ts, main = "Box-Cox Differenced GDP Over Time", ylab = "Box-Cox Differenced(GDP)",
acf(boxcox.diff.gdp.ts, main = "ACF of Box-Cox Differenced GDP")
pacf(boxcox.diff.gdp.ts, main = "PACF of Box-Cox Differenced GDP")

# 2. Imports
plot(boxcox.diff.import.ts, main = "Box-Cox Differenced Imports Over Time", ylab = "Box-Cox Differenced
acf(boxcox.diff.import.ts, main = "ACF of Box-Cox Differenced Imports")
pacf(boxcox.diff.import.ts, main = "PACF of Box-Cox Differenced Imports")
```

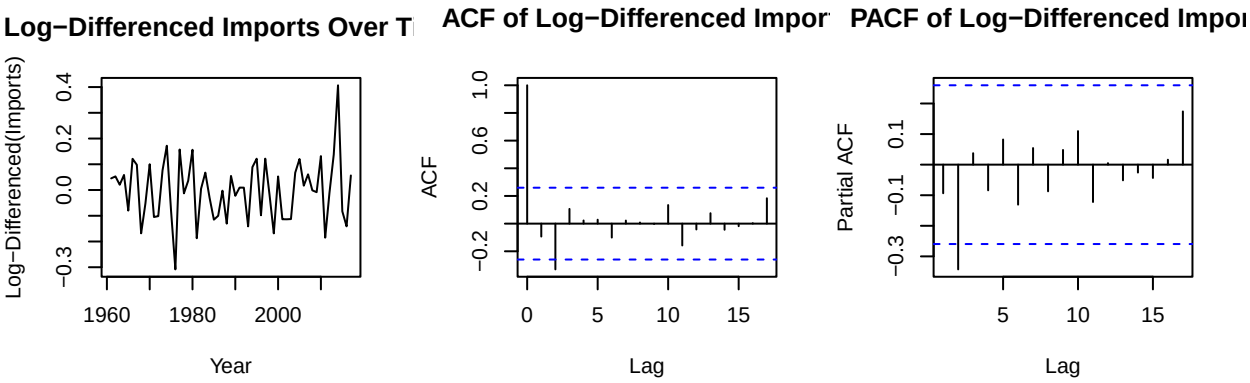




Log-Differencing Data (Final)

```r
# Log-Differenced Data (Final)
log.diff.gdp.ts <- ts(diff(log.gdp.ts), start = 1961, frequency = 1)
log.diff.import.ts <- ts(diff(log.import.ts), start = 1961, frequency = 1)

# Set up 2x3 plotting area
par(mfrow = c(2,3))

# 1. GDP
plot(log.diff.gdp.ts, main = "Log-Differenced GDP Over Time", ylab = "Log-Differenced(GDP)", xlab = "Yea
acf(log.diff.gdp.ts, main = "ACF of Log-Differenced GDP")
pacf(log.diff.gdp.ts, main = "PACF of Log-Differenced GDP")

# 2. Imports
plot(log.diff.import.ts, main = "Log-Differenced Imports Over Time", ylab = "Log-Differenced(Imports)",
acf(log.diff.import.ts, main = "ACF of Log-Differenced Imports")
pacf(log.diff.import.ts, main = "PACF of Log-Differenced Imports")
```





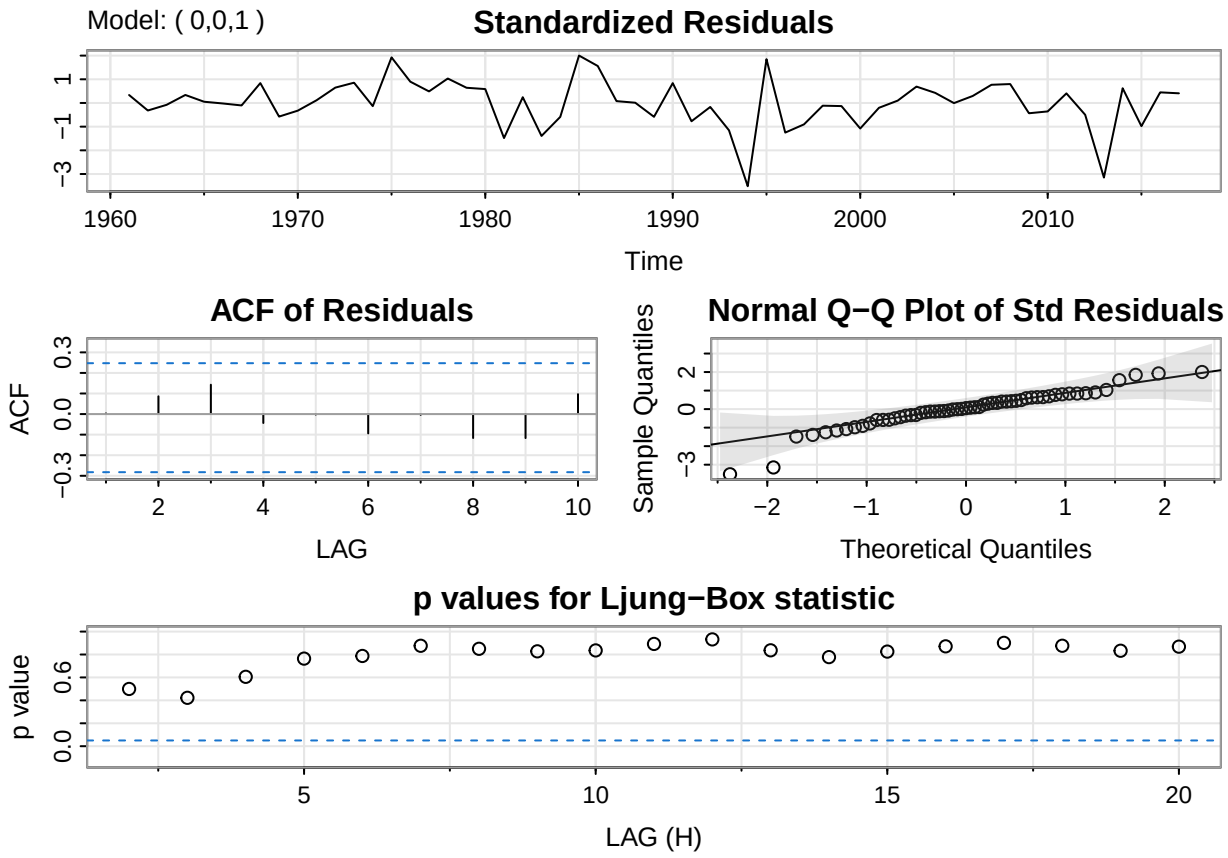
Model Selection

GDP

```
# Model Selection on Log-Differenced GDP (stationary)
# GDP

# Model 1: MA(1)
log.diff.gdp.ma1 <- arima(log.diff.gdp.ts, order = c(0,0,1))
log.diff.gdp.ma1.diagnostics <- sarima(log.diff.gdp.ts, 0, 0, 1)
```

```
## initial  value -2.051458
## iter   2 value -2.052302
## iter   3 value -2.052325
## iter   4 value -2.052325
## iter   4 value -2.052325
## iter   4 value -2.052325
## final  value -2.052325
## converged
## initial  value -2.052314
## iter   2 value -2.052314
## iter   2 value -2.052314
## iter   2 value -2.052314
## final  value -2.052314
## converged
## <><><><><><><><><><><><><><>
##
```

```
## Coefficients:
##       Estimate     SE t.value p.value
## ma1     0.0382 0.1228  0.3113  0.7568
## xmean   0.0502 0.0177  2.8416  0.0063
##
## sigma^2 estimated as 0.01649574 on 55 degrees of freedom
##
## AIC = -1.161487  AICc = -1.157589  BIC = -1.053958
##
```




```
log.diff.gdp.ma1
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(0, 0, 1))
##
## Coefficients:
##          ma1  intercept
##       0.0382     0.0502
## s.e.  0.1228     0.0177
##
## sigma^2 estimated as 0.0165:  log likelihood = 36.1,  aic = -66.2
```

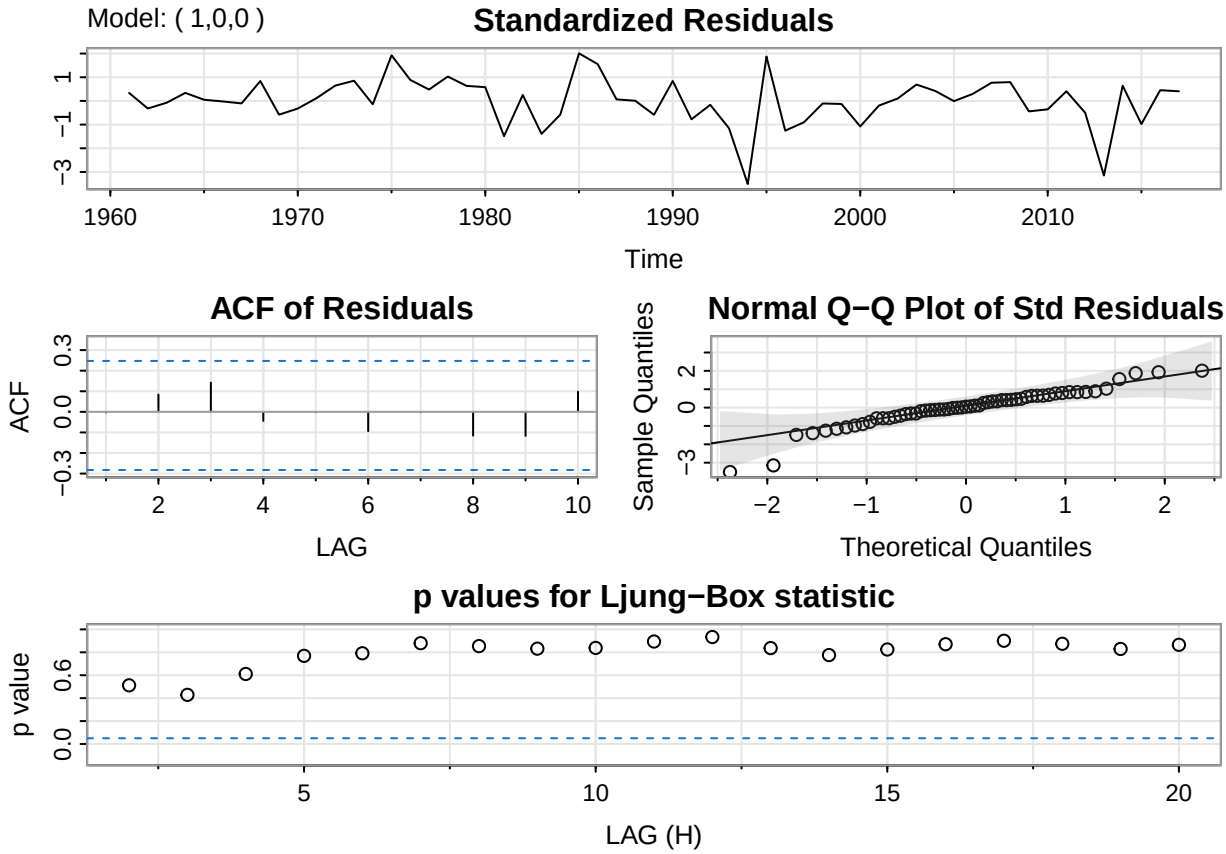
```
log.diff.gdp.ma1.diagnostics
```

```
## $fit
##
## Call:
```

```
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ma1   xmean
##       0.0382  0.0502
## s.e.  0.1228  0.0177
##
## sigma^2 estimated as 0.0165:  log likelihood = 36.1,  aic = -66.2
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ma1     0.0382 0.1228  0.3113  0.7568
## xmean   0.0502 0.0177  2.8416  0.0063
##
## $ICs
##       AIC      AICc       BIC
## -1.161487 -1.157589 -1.053958
```

```
# Model 2: AR(1)
log.diff.gdp.ar1 <- arima(log.diff.gdp.ts, order = c(1,0,0))
log.diff.gdp.ar1.diagnostics <- sarima(log.diff.gdp.ts, 1, 0, 0)
```

```
## initial  value -2.043604
## iter   2 value -2.044628
## iter   3 value -2.044635
## iter   4 value -2.044638
## iter   4 value -2.044638
## iter   4 value -2.044638
## final  value -2.044638
## converged
## initial  value -2.052438
## iter   2 value -2.052450
## iter   3 value -2.052458
## iter   3 value -2.052458
## iter   3 value -2.052458
## final  value -2.052458
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.0444 0.1315  0.3380  0.7367
## xmean   0.0502 0.0178  2.8207  0.0067
##
## sigma^2 estimated as 0.01649082 on 55 degrees of freedom
##
## AIC = -1.161776  AICc = -1.157878  BIC = -1.054247
##
```




```
log.diff.gdp.ar1
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(1, 0, 0))
##
## Coefficients:
##          ar1  intercept
##       0.0444     0.0502
## s.e.  0.1315     0.0178
##
## sigma^2 estimated as 0.01649:  log likelihood = 36.11,  aic = -66.22
```

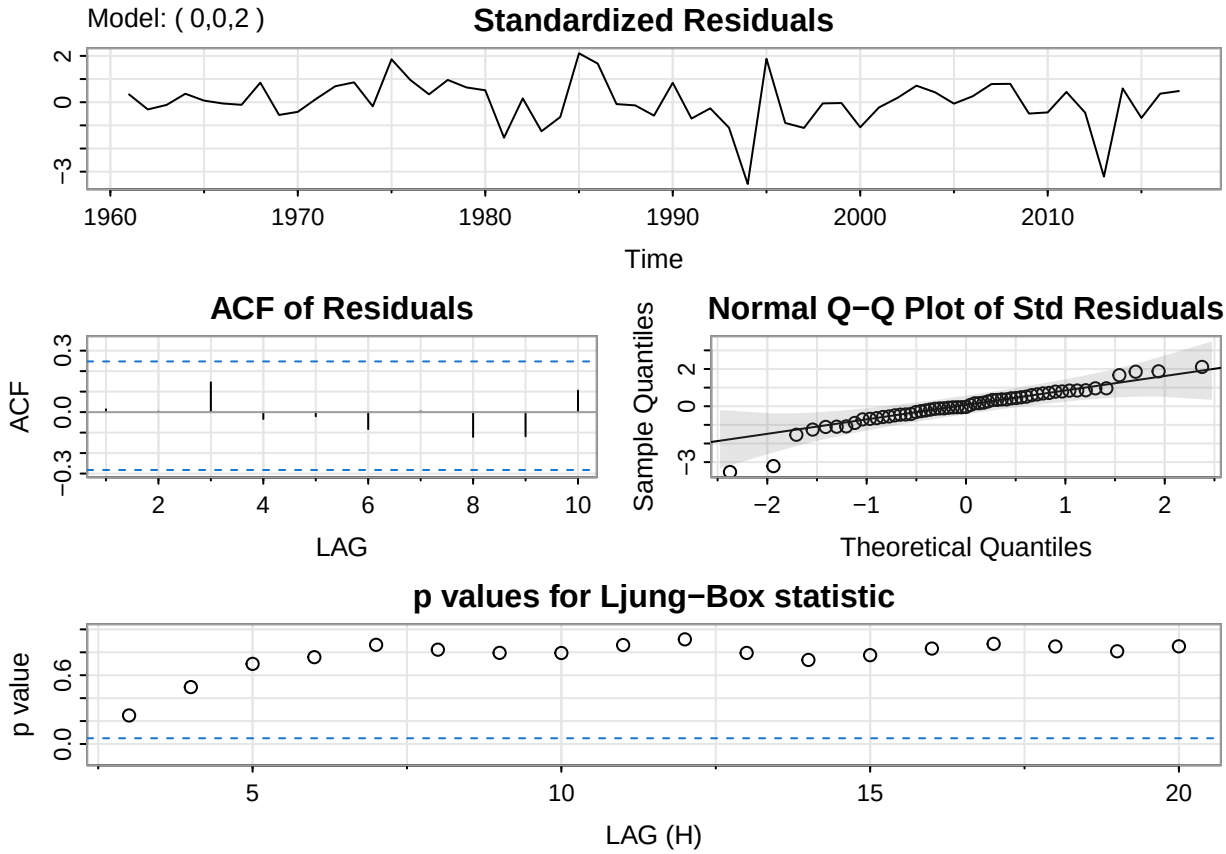
```
log.diff.gdp.ar1.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1   xmean
##       0.0444  0.0502
## s.e.  0.1315  0.0178
##
## sigma^2 estimated as 0.01649:  log likelihood = 36.11,  aic = -66.22
```

```
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.0444 0.1315  0.3380  0.7367
## xmean   0.0502 0.0178  2.8207  0.0067
##
## $ICs
##       AIC      AICc       BIC
## -1.161776 -1.157878 -1.054247
```

```
# Model 3: MA(2)
log.diff.gdp.ma2 <- arima(log.diff.gdp.ts, order = c(0,0,2))
log.diff.gdp.ma2.diagnostics <- sarima(log.diff.gdp.ts, 0, 0, 2)
```

```
## initial  value -2.051458
## iter   2 value -2.055718
## iter   3 value -2.056120
## iter   4 value -2.056142
## iter   5 value -2.056143
## iter   5 value -2.056143
## iter   5 value -2.056143
## final  value -2.056143
## converged
## initial  value -2.056002
## iter   2 value -2.056006
## iter   3 value -2.056006
## iter   3 value -2.056006
## iter   3 value -2.056006
## final  value -2.056006
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ma1     0.0159 0.1367  0.1162  0.9079
## ma2     0.0908 0.1388  0.6540  0.5159
## xmean   0.0503 0.0187  2.6901  0.0095
##
## sigma^2 estimated as 0.01636998 on 54 degrees of freedom
##
## AIC = -1.133784  AICc = -1.12584  BIC = -0.9904121
##
```




```
log.diff.gdp.ma2
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(0, 0, 2))
##
## Coefficients:
##          ma1     ma2  intercept
##       0.0159  0.0908     0.0503
## s.e.  0.1367  0.1388     0.0187
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.31,  aic = -64.63
```

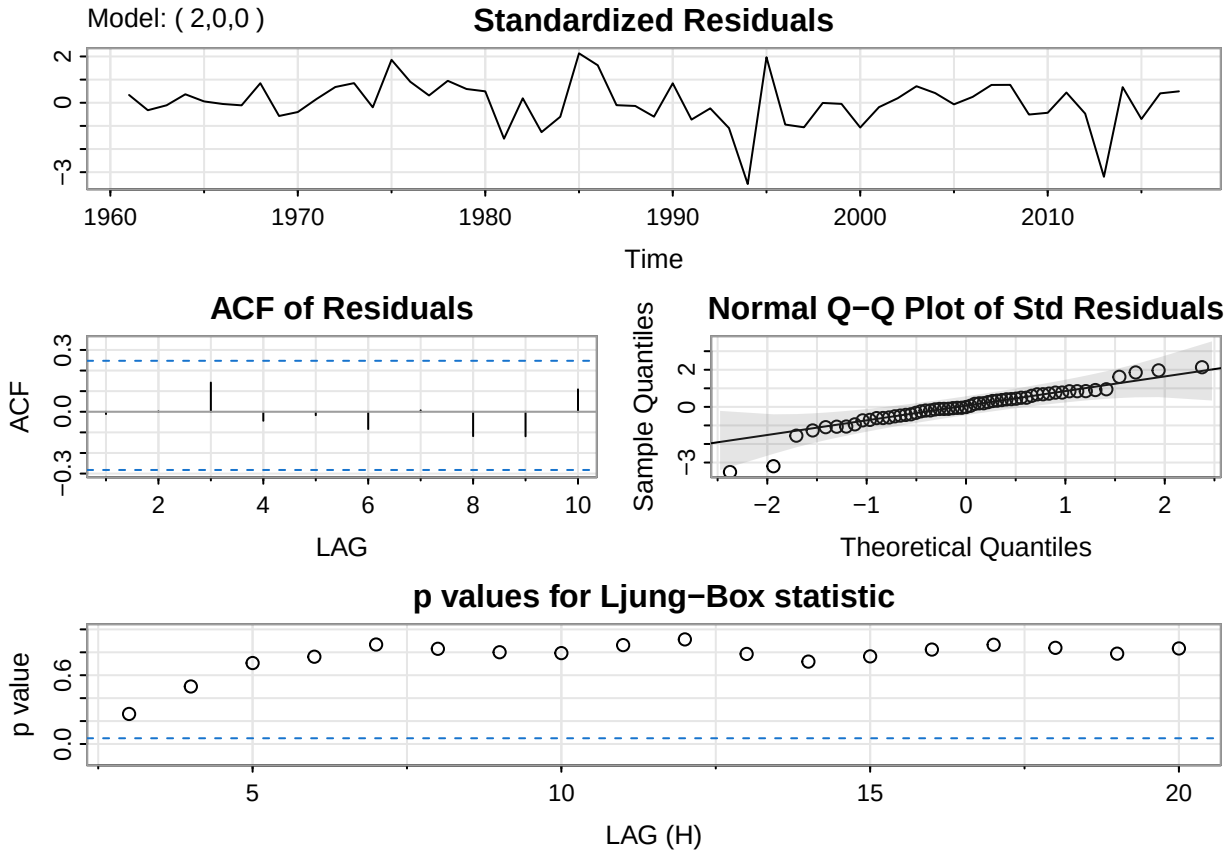
```
log.diff.gdp.ma2.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ma1     ma2   xmean
##       0.0159  0.0908  0.0503
## s.e.  0.1367  0.1388  0.0187
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.31,  aic = -64.63
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ma1     0.0159 0.1367  0.1162  0.9079
## ma2     0.0908 0.1388  0.6540  0.5159
## xmean   0.0503 0.0187  2.6901  0.0095
##
## $ICs
##        AIC       AICc        BIC
## -1.1337841 -1.1258397 -0.9904121
```

```
# Model 4: AR(2)
log.diff.gdp.ar2 <- arima(log.diff.gdp.ts, order = c(2,0,0))
log.diff.gdp.ar2.diagnostics <- sarima(log.diff.gdp.ts, 2, 0, 0)
```

```
## initial  value -2.035413
## iter   2 value -2.040611
## iter   3 value -2.040622
## iter   4 value -2.040622
## iter   4 value -2.040622
## iter   4 value -2.040622
## final  value -2.040622
## converged
## initial  value -2.056414
## iter   2 value -2.056422
## iter   3 value -2.056422
## iter   3 value -2.056422
## iter   3 value -2.056422
## final  value -2.056422
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.0405 0.1309  0.3097  0.7580
## ar2     0.0878 0.1302  0.6738  0.5033
## xmean   0.0504 0.0194  2.6026  0.0119
##
## sigma^2 estimated as 0.01635617 on 54 degrees of freedom
##
## AIC = -1.134616  AICc = -1.126672  BIC = -0.991244
##
```




```
log.diff.gdp.ar2
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(2, 0, 0))
##
## Coefficients:
##          ar1     ar2  intercept
##       0.0405  0.0878     0.0504
## s.e.  0.1309  0.1302     0.0194
##
## sigma^2 estimated as 0.01636:  log likelihood = 36.34,  aic = -64.67
```

```
log.diff.gdp.ar2.diagnostics
```

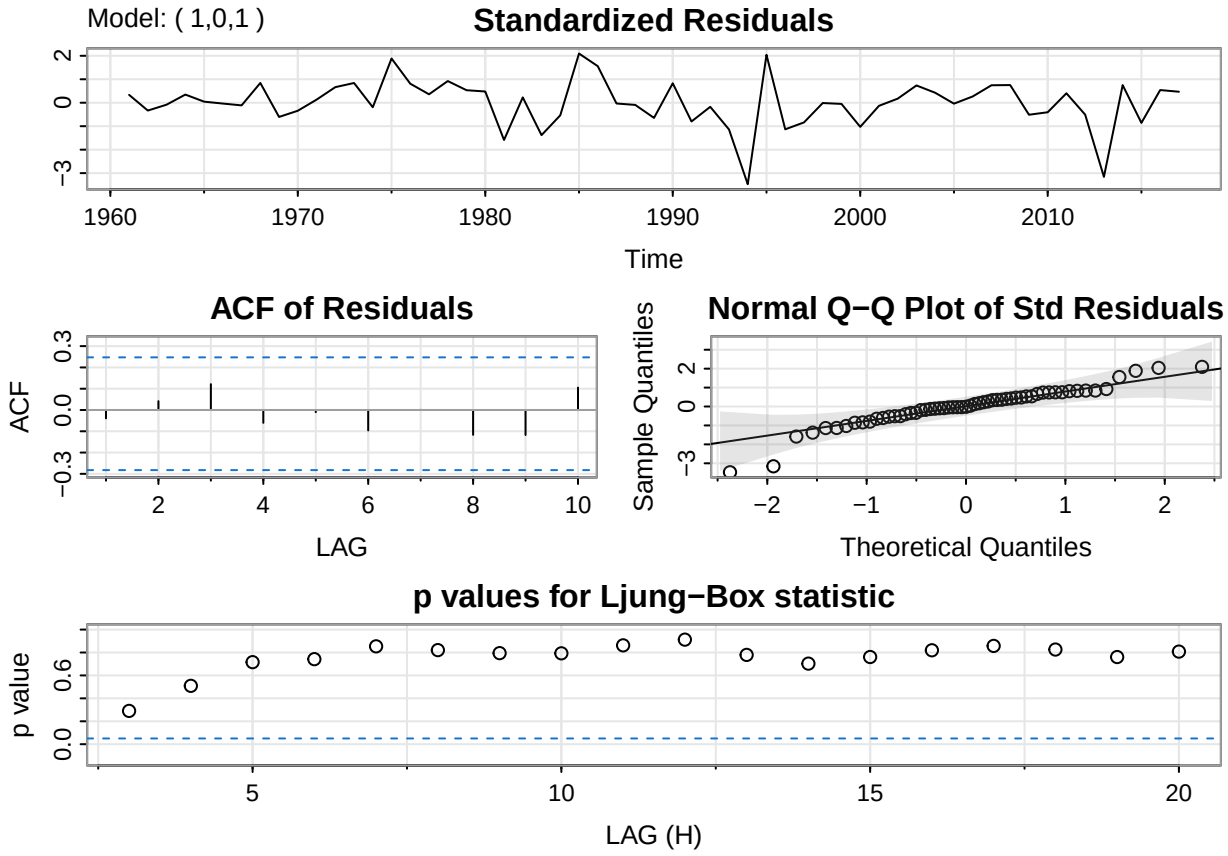
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1     ar2   xmean
##       0.0405  0.0878  0.0504
## s.e.  0.1309  0.1302  0.0194
##
## sigma^2 estimated as 0.01636:  log likelihood = 36.34,  aic = -64.67
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.0405 0.1309  0.3097  0.7580
## ar2     0.0878 0.1302  0.6738  0.5033
## xmean   0.0504 0.0194  2.6026  0.0119
##
## $ICs
##       AIC      AICc       BIC
## -1.134616 -1.126672 -0.991244
```

```r
# Model 5: ARMA(1,1)
log.diff.gdp.arma11 <- arima(log.diff.gdp.ts, order = c(1,0,1))
log.diff.gdp.arma11.diagnostics <- sarima(log.diff.gdp.ts, 1, 0, 1)
```

```
## initial  value -2.043604
## iter   2 value -2.044276
## iter   3 value -2.044596
## iter   4 value -2.044598
## iter   5 value -2.044625
## iter   6 value -2.044786
## iter   7 value -2.044823
## iter   8 value -2.044926
## iter   9 value -2.044934
## iter  10 value -2.044947
## iter  11 value -2.045576
## iter  12 value -2.046165
## iter  13 value -2.046254
## iter  14 value -2.047052
## iter  15 value -2.047177
## iter  16 value -2.047178
## iter  16 value -2.047178
## iter  16 value -2.047178
## final  value -2.047178
## converged
## initial  value -2.056080
## iter   2 value -2.056116
## iter   3 value -2.056124
## iter   4 value -2.056126
## iter   5 value -2.056128
## iter   6 value -2.056139
## iter   7 value -2.056145
## iter   8 value -2.056149
## iter   9 value -2.056149
## iter  10 value -2.056149
## iter  10 value -2.056149
## iter  10 value -2.056149
## final  value -2.056149
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
```

```
##       Estimate     SE t.value p.value
## ar1     0.6353 0.4481  1.4177  0.1620
## ma1    -0.5615 0.4727 -1.1879  0.2401
## xmean   0.0502 0.0202  2.4854  0.0161
##
## sigma^2 estimated as 0.0163663 on 54 degrees of freedom
##
## AIC = -1.134071  AICc = -1.126126  BIC = -0.9906985
##
```




```
log.diff.gdp.arma11
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(1, 0, 1))
##
## Coefficients:
##          ar1      ma1  intercept
##       0.6353  -0.5615     0.0502
## s.e.  0.4481   0.4727     0.0202
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.32,  aic = -64.64
```

```
log.diff.gdp.arma11.diagnostics
```

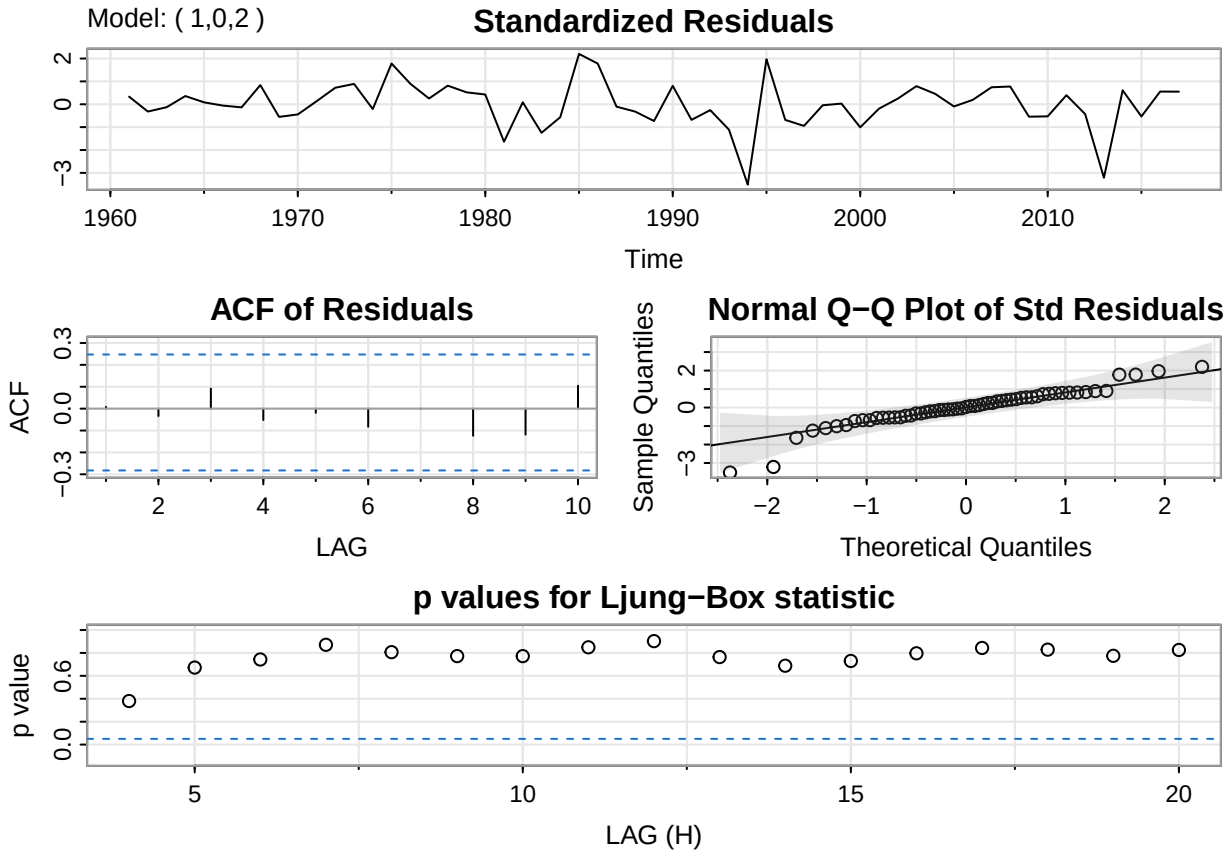
```
## $fit
##
## Call:
```

```
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1   xmean
##       0.6353  -0.5615  0.0502
## s.e.  0.4481   0.4727  0.0202
##
## sigma^2 estimated as 0.01637:  log likelihood = 36.32,  aic = -64.64
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.6353 0.4481  1.4177  0.1620
## ma1    -0.5615 0.4727 -1.1879  0.2401
## xmean   0.0502 0.0202  2.4854  0.0161
##
## $ICs
##        AIC       AICc        BIC
## -1.1340705 -1.1261261 -0.9906985
```

```
# Model 6: ARMA(1,2)
log.diff.gdp.arma12 <- arima(log.diff.gdp.ts, order = c(1,0,2))
log.diff.gdp.arma12.diagnostics <- sarima(log.diff.gdp.ts, 1, 0, 2)
```

```
## initial  value -2.043604
## iter   2 value -2.046177
## iter   3 value -2.048533
## iter   4 value -2.048624
## iter   5 value -2.048815
## iter   6 value -2.049700
## iter   7 value -2.051473
## iter   8 value -2.051869
## iter   9 value -2.052374
## iter  10 value -2.052542
## iter  11 value -2.052542
## iter  11 value -2.052543
## iter  11 value -2.052543
## final  value -2.052543
## converged
## initial  value -2.061088
## iter   2 value -2.061098
## iter   3 value -2.061118
## iter   4 value -2.061120
## iter   5 value -2.061122
## iter   6 value -2.061133
## iter   7 value -2.061138
## iter   8 value -2.061139
## iter   9 value -2.061139
## iter   9 value -2.061139
## iter   9 value -2.061139
## final  value -2.061139
```

```
## converged
## <><><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.4806 0.4350  1.1048  0.2742
## ma1    -0.4623 0.4343 -1.0644  0.2920
## ma2     0.1225 0.1510  0.8116  0.4206
## xmean   0.0505 0.0212  2.3799  0.0209
##
## sigma^2 estimated as 0.01619399 on 53 degrees of freedom
##
## AIC = -1.108962  AICc = -1.095467  BIC = -0.9297474
##
```




```
log.diff.gdp.arma12
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(1, 0, 2))
##
## Coefficients:
##          ar1      ma1     ma2  intercept
##       0.4806  -0.4623  0.1225     0.0505
## s.e.  0.4350   0.4343  0.1510     0.0212
##
## sigma^2 estimated as 0.01619:  log likelihood = 36.61,  aic = -63.21
```

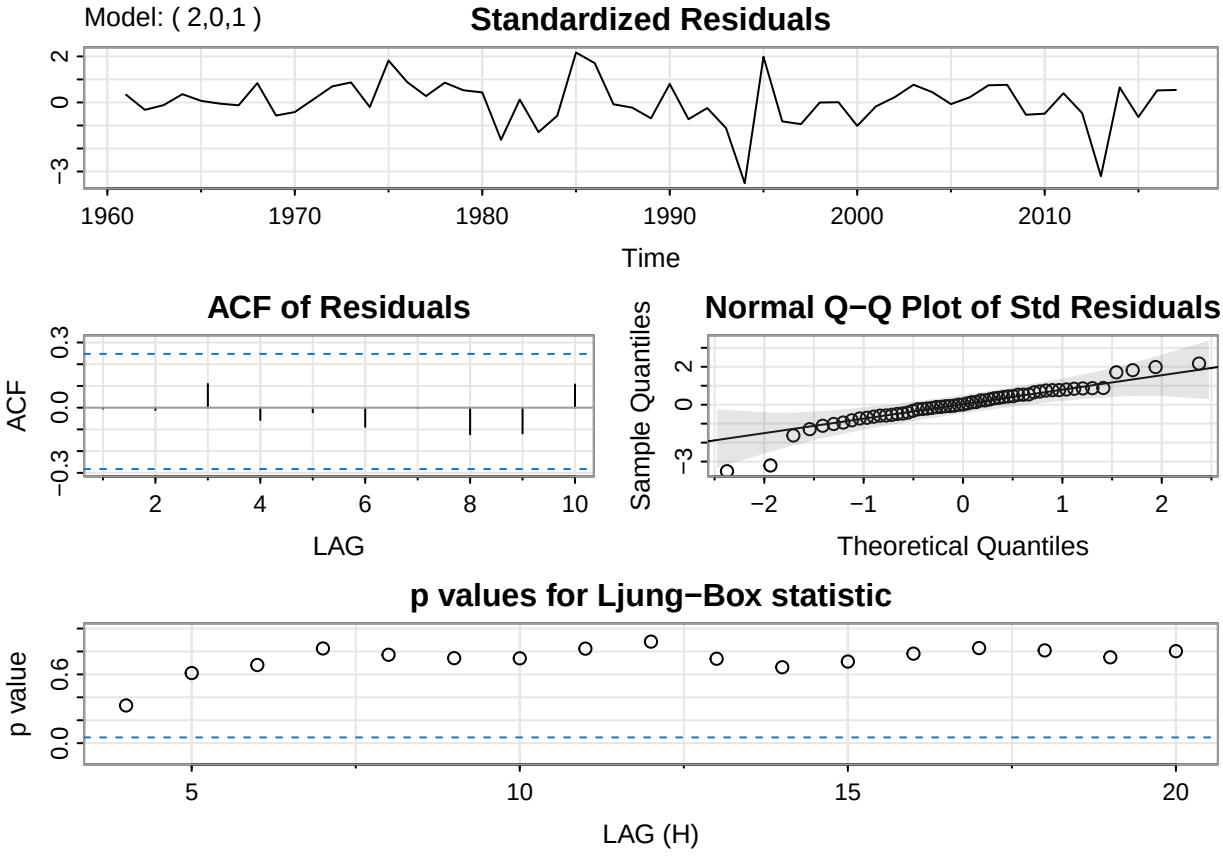
```
log.diff.gdp.arma12.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1     ma2   xmean
##       0.4806  -0.4623  0.1225  0.0505
## s.e.  0.4350   0.4343  0.1510  0.0212
##
## sigma^2 estimated as 0.01619:  log likelihood = 36.61,  aic = -63.21
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.4806 0.4350  1.1048  0.2742
## ma1    -0.4623 0.4343 -1.0644  0.2920
## ma2     0.1225 0.1510  0.8116  0.4206
## xmean   0.0505 0.0212  2.3799  0.0209
##
## $ICs
##        AIC       AICc        BIC
## -1.1089624 -1.0954671 -0.9297474
```

```
# Model 7: ARMA(2,1)
log.diff.gdp.arma21 <- arima(log.diff.gdp.ts, order = c(2,0,1))
log.diff.gdp.arma21.diagnostics <- sarima(log.diff.gdp.ts, 2, 0, 1)
```

```
## initial  value -2.035413
## iter   2 value -2.038759
## iter   3 value -2.040329
## iter   4 value -2.040504
## iter   5 value -2.040649
## iter   6 value -2.041805
## iter   7 value -2.042863
## iter   8 value -2.043277
## iter   9 value -2.043499
## iter  10 value -2.043624
## iter  11 value -2.043626
## iter  12 value -2.043627
## iter  13 value -2.043627
## iter  14 value -2.043629
## iter  15 value -2.043629
## iter  16 value -2.043630
## iter  16 value -2.043630
## iter  16 value -2.043630
## final  value -2.043630
## converged
```

```
## initial  value -2.059446
## iter   2 value -2.059459
## iter   3 value -2.059466
## iter   4 value -2.059467
## iter   5 value -2.059470
## iter   6 value -2.059475
## iter   7 value -2.059479
## iter   8 value -2.059480
## iter   9 value -2.059480
## iter   9 value -2.059480
## iter   9 value -2.059480
## final  value -2.059480
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1     0.4057 0.5161  0.7862  0.4353
## ar2     0.0918 0.1378  0.6661  0.5082
## ma1    -0.3718 0.5064 -0.7343  0.4660
## xmean   0.0504 0.0209  2.4092  0.0195
##
## sigma^2 estimated as 0.01625255 on 53 degrees of freedom
##
## AIC = -1.105644  AICc = -1.092148  BIC = -0.9264285
##
```



```
log.diff.gdp.arma21
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(2, 0, 1))
##
## Coefficients:
##          ar1     ar2      ma1  intercept
##       0.4057  0.0918  -0.3718     0.0504
## s.e.  0.5161  0.1378   0.5064     0.0209
##
## sigma^2 estimated as 0.01625:  log likelihood = 36.51,  aic = -63.02
```

```
log.diff.gdp.arma21.diagnostics
```

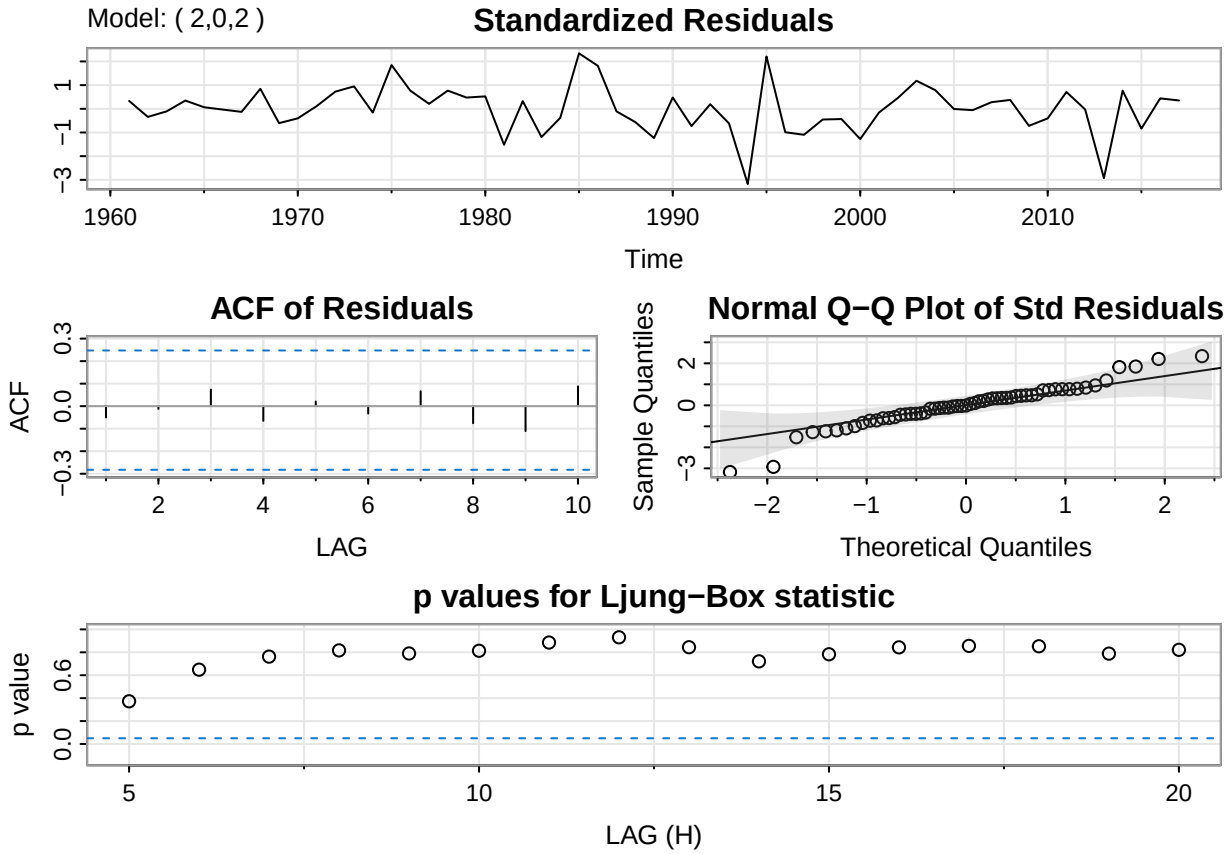
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1     ar2      ma1   xmean
##       0.4057  0.0918  -0.3718  0.0504
## s.e.  0.5161  0.1378   0.5064  0.0209
##
## sigma^2 estimated as 0.01625:  log likelihood = 36.51,  aic = -63.02
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1     0.4057 0.5161  0.7862  0.4353
## ar2     0.0918 0.1378  0.6661  0.5082
## ma1    -0.3718 0.5064 -0.7343  0.4660
## xmean   0.0504 0.0209  2.4092  0.0195
##
## $ICs
##        AIC       AICc        BIC
## -1.1056435 -1.0921482 -0.9264285
```

```
# Model 8: ARMA(2,2) (final)
log.diff.gdp.arma22 <- arima(log.diff.gdp.ts, order = c(2,0,2))
log.diff.gdp.arma22.diagnostics <- sarima(log.diff.gdp.ts, 2, 0, 2)
```

```
## initial  value -2.035413
## iter   2 value -2.038802
## iter   3 value -2.040478
## iter   4 value -2.040641
## iter   5 value -2.040791
## iter   6 value -2.042896
## iter   7 value -2.044919
## iter   8 value -2.044997
```

```
## iter   9 value -2.045682
## iter  10 value -2.045871
## iter  11 value -2.046747
## iter  12 value -2.047273
## iter  13 value -2.047581
## iter  14 value -2.047650
## iter  15 value -2.047702
## iter  16 value -2.047784
## iter  17 value -2.047792
## iter  18 value -2.047798
## iter  19 value -2.047815
## iter  20 value -2.047853
## iter  21 value -2.047961
## iter  22 value -2.048189
## iter  23 value -2.048265
## iter  24 value -2.048319
## iter  25 value -2.048351
## iter  26 value -2.048358
## iter  27 value -2.048491
## iter  28 value -2.048729
## iter  29 value -2.049412
## iter  30 value -2.050278
## iter  31 value -2.050624
## iter  32 value -2.050668
## iter  33 value -2.051024
## iter  34 value -2.051982
## iter  35 value -2.056774
## iter  36 value -2.059606
## iter  37 value -2.062725
## iter  38 value -2.067098
## iter  39 value -2.072222
## iter  40 value -2.088772
## iter  41 value -2.094336
## iter  42 value -2.097030
## iter  43 value -2.099760
## iter  44 value -2.099886
## iter  45 value -2.100899
## iter  46 value -2.101130
## iter  47 value -2.101203
## iter  48 value -2.101207
## iter  49 value -2.101215
## iter  50 value -2.101240
## iter  51 value -2.101241
## iter  51 value -2.101241
## iter  51 value -2.101241
## final  value -2.101241
## converged
## initial  value -2.050397
## iter   2 value -2.056026
## iter   3 value -2.081594
## iter   4 value -2.084062
## iter   5 value -2.085851
## iter   6 value -2.086765
## iter   7 value -2.087484
```

```
## iter   8 value -2.089790
## iter   9 value -2.091474
## iter  10 value -2.092000
## iter  11 value -2.093082
## iter  12 value -2.093234
## iter  13 value -2.093239
## iter  14 value -2.093261
## iter  15 value -2.093270
## iter  16 value -2.093279
## iter  17 value -2.093280
## iter  18 value -2.093280
## iter  19 value -2.093280
## iter  20 value -2.093280
## iter  21 value -2.093280
## iter  21 value -2.093280
## iter  21 value -2.093280
## final  value -2.093280
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE  t.value p.value
## ar1     1.5232 0.1236  12.3269  0.0000
## ar2    -0.9044 0.0924  -9.7861  0.0000
## ma1    -1.5297 0.1006 -15.2047  0.0000
## ma2     1.0000 0.1000  10.0004  0.0000
## xmean   0.0508 0.0196   2.5979  0.0122
##
## sigma^2 estimated as 0.01443647 on 52 degrees of freedom
##
## AIC = -1.138156  AICc = -1.117517  BIC = -0.9230984
##
```




```
log.diff.gdp.arma22
```

```
##
## Call:
## arima(x = log.diff.gdp.ts, order = c(2, 0, 2))
##
## Coefficients:
##          ar1      ar2      ma1  ma2  intercept
##       1.5232  -0.9044  -1.5297  1.0     0.0508
## s.e.  0.1236   0.0924   0.1006  0.1     0.0196
##
## sigma^2 estimated as 0.01444:  log likelihood = 38.44,  aic = -64.87
```

```
log.diff.gdp.arma22.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ar2      ma1  ma2   xmean
##       1.5232  -0.9044  -1.5297  1.0  0.0508
## s.e.  0.1236   0.0924   0.1006  0.1  0.0196
##
## sigma^2 estimated as 0.01444:  log likelihood = 38.44,  aic = -64.87
```

```
##
## $degrees_of_freedom
## [1] 52
##
## $ttable
##       Estimate     SE  t.value p.value
## ar1     1.5232 0.1236  12.3269  0.0000
## ar2    -0.9044 0.0924  -9.7861  0.0000
## ma1    -1.5297 0.1006 -15.2047  0.0000
## ma2     1.0000 0.1000  10.0004  0.0000
## xmean   0.0508 0.0196   2.5979  0.0122
##
## $ICs
##        AIC       AICc        BIC
## -1.1381564 -1.1175166 -0.9230984
```

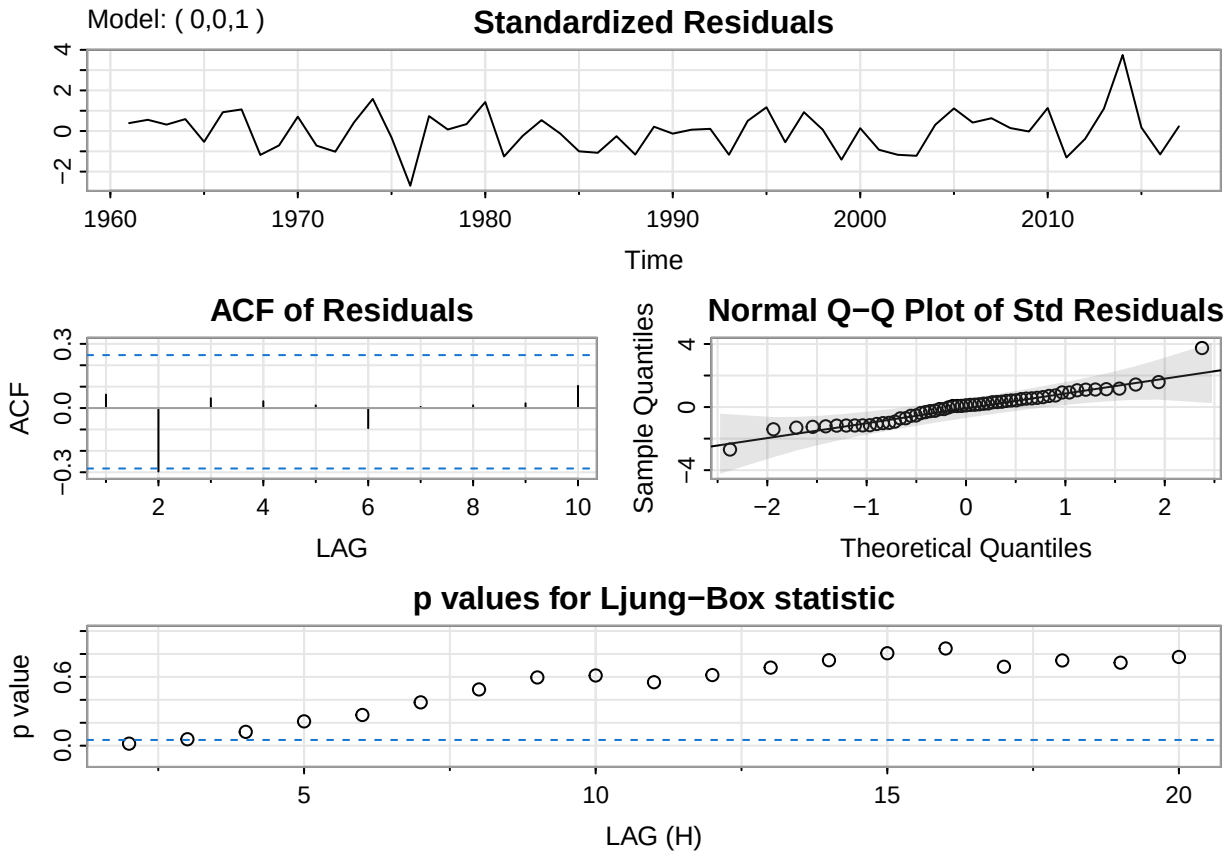
GDP Final Model: ARMA(2,2)

Imports

```r
# Model Selection on Log-Differenced Imports (stationary)
# Imports

# Model 1: MA(1)
log.diff.import.ma1 <- arima(log.diff.import.ts, order = c(0,0,1))
log.diff.import.ma1.diagnostics <- sarima(log.diff.import.ts, 0, 0, 1)
```

```
## initial  value -2.132354
## iter   2 value -2.139659
## iter   3 value -2.143882
## iter   4 value -2.144278
## iter   5 value -2.144320
## iter   6 value -2.144360
## iter   7 value -2.144361
## iter   8 value -2.144361
## iter   9 value -2.144361
## iter   9 value -2.144361
## iter   9 value -2.144361
## final  value -2.144361
## converged
## initial  value -2.143999
## iter   2 value -2.144005
## iter   3 value -2.144005
## iter   4 value -2.144007
## iter   4 value -2.144007
## iter   4 value -2.144007
## final  value -2.144007
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ma1    -0.2314 0.1836 -1.2604  0.2128
## xmean  -0.0016 0.0120 -0.1367  0.8917
```

```
##
## sigma^2 estimated as 0.01371892 on 55 degrees of freedom
##
## AIC = -1.344874  AICc = -1.340975  BIC = -1.237345
##
```




```
log.diff.import.ma1
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(0, 0, 1))
##
## Coefficients:
##           ma1  intercept
##       -0.2314    -0.0016
## s.e.   0.1836     0.0120
##
## sigma^2 estimated as 0.01372:  log likelihood = 41.33,  aic = -76.66
```

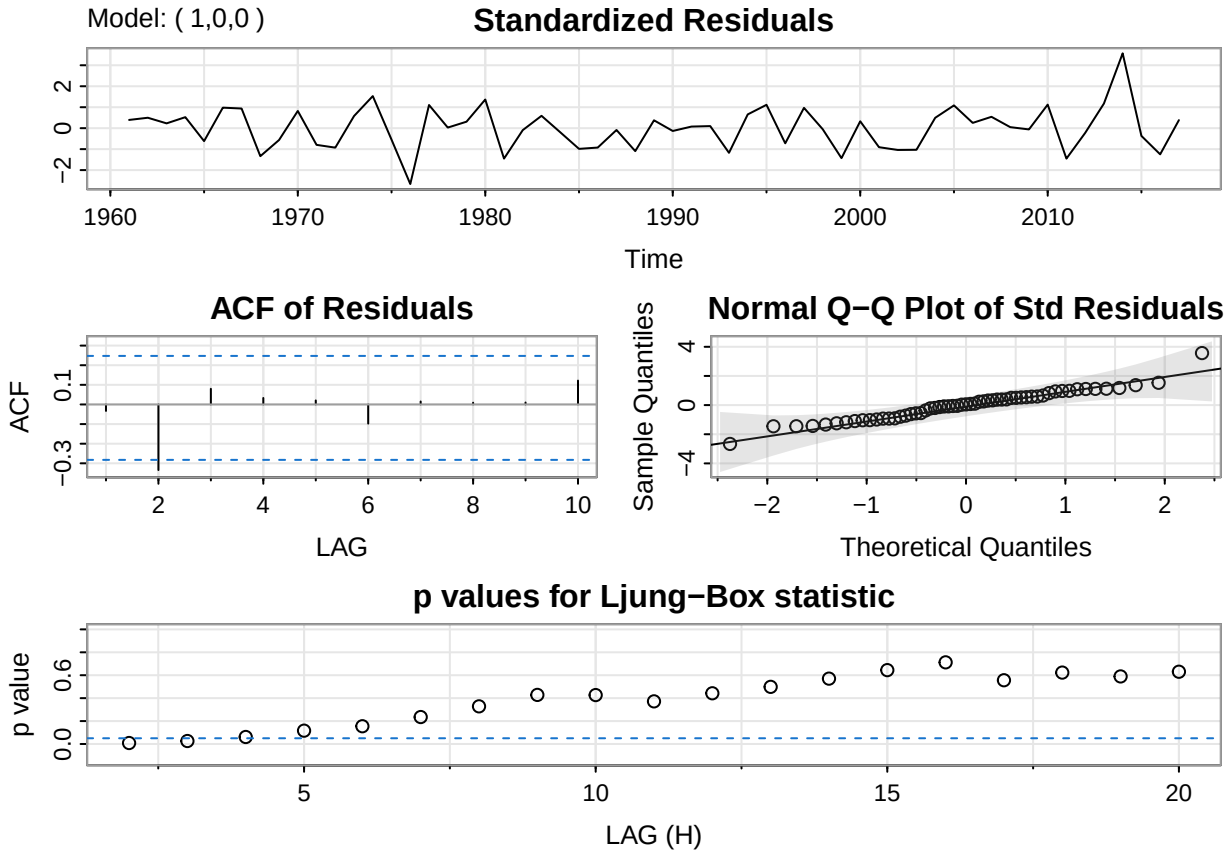
```
log.diff.import.ma1.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
```

```
## Coefficients:
##           ma1    xmean
##       -0.2314  -0.0016
## s.e.   0.1836   0.0120
##
## sigma^2 estimated as 0.01372:  log likelihood = 41.33,  aic = -76.66
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ma1    -0.2314 0.1836 -1.2604  0.2128
## xmean  -0.0016 0.0120 -0.1367  0.8917
##
## $ICs
##       AIC      AICc       BIC
## -1.344874 -1.340975 -1.237345
```

```
# Model 2: AR(1)
log.diff.import.ar1 <- arima(log.diff.import.ts, order = c(1,0,0))
log.diff.import.ar1.diagnostics <- sarima(log.diff.import.ts, 1, 0, 0)
```

```
## initial  value -2.124853
## iter   2 value -2.129291
## iter   3 value -2.129301
## iter   4 value -2.129306
## iter   4 value -2.129306
## iter   4 value -2.129306
## final  value -2.129306
## converged
## initial  value -2.136680
## iter   2 value -2.136694
## iter   3 value -2.136701
## iter   3 value -2.136701
## iter   3 value -2.136701
## final  value -2.136701
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.0926 0.1312 -0.7056  0.4834
## xmean  -0.0014 0.0143 -0.1002  0.9205
##
## sigma^2 estimated as 0.01393219 on 55 degrees of freedom
##
## AIC = -1.330262  AICc = -1.326364  BIC = -1.222733
##
```




```
log.diff.import.ar1
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(1, 0, 0))
##
## Coefficients:
##           ar1  intercept
##       -0.0926    -0.0014
## s.e.   0.1312     0.0143
##
## sigma^2 estimated as 0.01393:  log likelihood = 40.91,  aic = -75.82
```

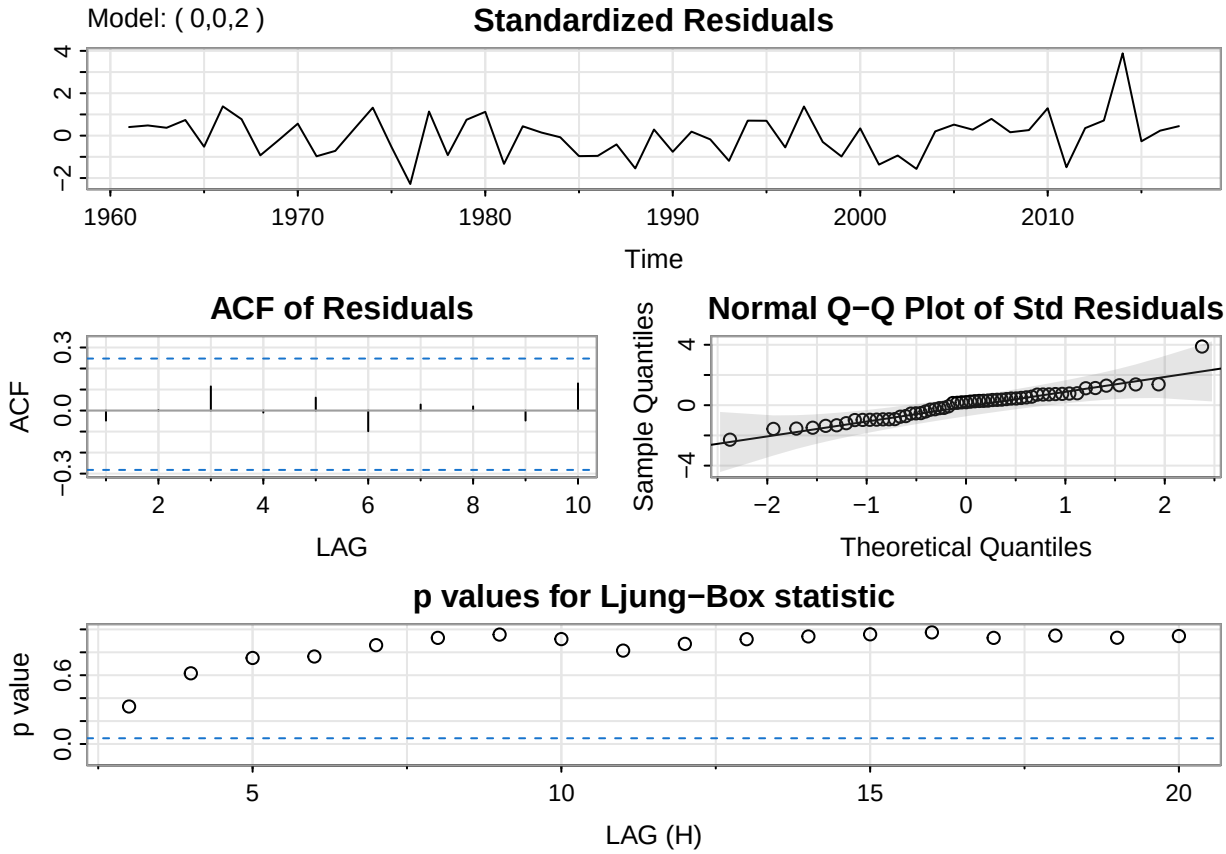
```
log.diff.import.ar1.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1    xmean
##       -0.0926  -0.0014
## s.e.   0.1312   0.0143
##
## sigma^2 estimated as 0.01393:  log likelihood = 40.91,  aic = -75.82
```

```
##
## $degrees_of_freedom
## [1] 55
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.0926 0.1312 -0.7056  0.4834
## xmean  -0.0014 0.0143 -0.1002  0.9205
##
## $ICs
##       AIC      AICc       BIC
## -1.330262 -1.326364 -1.222733
```

```
# Model 3: MA(2)
log.diff.import.ma2 <- arima(log.diff.import.ts, order = c(0,0,2))
log.diff.import.ma2.diagnostics <- sarima(log.diff.import.ts, 0, 0, 2)
```

```
## initial  value -2.132354
## iter   2 value -2.199743
## iter   3 value -2.200444
## iter   4 value -2.201470
## iter   5 value -2.202071
## iter   6 value -2.202083
## iter   7 value -2.202083
## iter   7 value -2.202083
## iter   7 value -2.202083
## final  value -2.202083
## converged
## initial  value -2.200774
## iter   2 value -2.200797
## iter   3 value -2.200822
## iter   4 value -2.200824
## iter   5 value -2.200824
## iter   5 value -2.200824
## iter   5 value -2.200824
## final  value -2.200824
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ma1    -0.0465 0.1315 -0.3538  0.7249
## ma2    -0.3865 0.1414 -2.7324  0.0085
## xmean  -0.0029 0.0086 -0.3335  0.7401
##
## sigma^2 estimated as 0.01218653 on 54 degrees of freedom
##
## AIC = -1.423419  AICc = -1.415475  BIC = -1.280047
##
```




```
log.diff.import.ma2
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(0, 0, 2))
##
## Coefficients:
##           ma1      ma2  intercept
##       -0.0465  -0.3865    -0.0029
## s.e.   0.1315   0.1414     0.0086
##
## sigma^2 estimated as 0.01219:  log likelihood = 44.57,  aic = -81.13
```

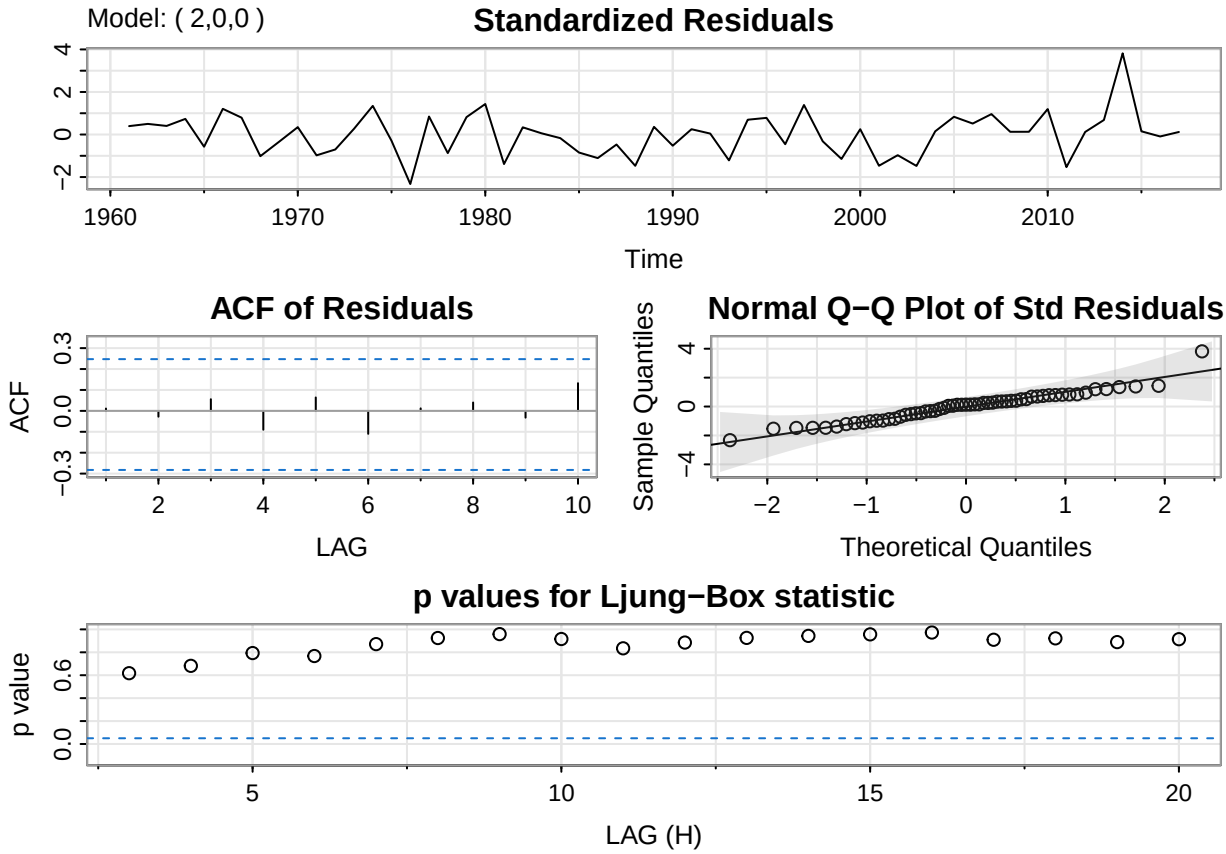
```
log.diff.import.ma2.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ma1      ma2    xmean
##       -0.0465  -0.3865  -0.0029
## s.e.   0.1315   0.1414   0.0086
##
## sigma^2 estimated as 0.01219:  log likelihood = 44.57,  aic = -81.13
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ma1    -0.0465 0.1315 -0.3538  0.7249
## ma2    -0.3865 0.1414 -2.7324  0.0085
## xmean  -0.0029 0.0086 -0.3335  0.7401
##
## $ICs
##       AIC      AICc       BIC
## -1.423419 -1.415475 -1.280047
```

```
# Model 4: AR(2)
log.diff.import.ar2 <- arima(log.diff.import.ts, order = c(2,0,0))
log.diff.import.ar2.diagnostics <- sarima(log.diff.import.ts, 2, 0, 0)
```

```
## initial  value -2.117685
## iter   2 value -2.186025
## iter   3 value -2.186679
## iter   4 value -2.186969
## iter   5 value -2.187031
## iter   6 value -2.187032
## iter   6 value -2.187032
## iter   6 value -2.187032
## final  value -2.187032
## converged
## initial  value -2.198846
## iter   2 value -2.198923
## iter   3 value -2.198958
## iter   4 value -2.198960
## iter   5 value -2.198962
## iter   5 value -2.198962
## iter   5 value -2.198962
## final  value -2.198962
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.1229 0.1240 -0.9912  0.3260
## ar2    -0.3417 0.1236 -2.7637  0.0078
## xmean  -0.0015 0.0101 -0.1499  0.8814
##
## sigma^2 estimated as 0.01224758 on 54 degrees of freedom
##
## AIC = -1.419696  AICc = -1.411751  BIC = -1.276324
##
```




```
log.diff.import.ar2
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(2, 0, 0))
##
## Coefficients:
##           ar1      ar2  intercept
##       -0.1229  -0.3417    -0.0015
## s.e.   0.1240   0.1236     0.0101
##
## sigma^2 estimated as 0.01225:  log likelihood = 44.46,  aic = -80.92
```

```
log.diff.import.ar2.diagnostics
```

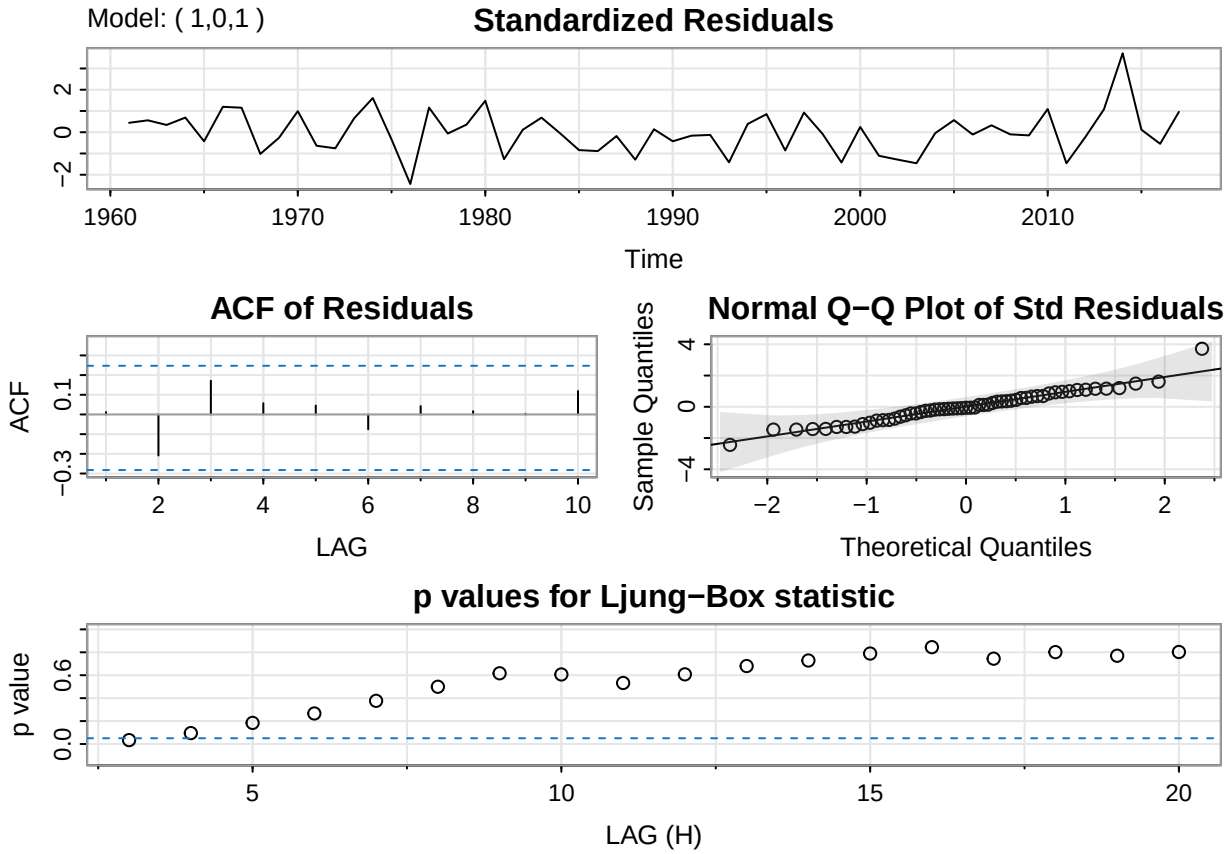
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ar2    xmean
##       -0.1229  -0.3417  -0.0015
## s.e.   0.1240   0.1236   0.0101
##
## sigma^2 estimated as 0.01225:  log likelihood = 44.46,  aic = -80.92
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.1229 0.1240 -0.9912  0.3260
## ar2    -0.3417 0.1236 -2.7637  0.0078
## xmean  -0.0015 0.0101 -0.1499  0.8814
##
## $ICs
##       AIC      AICc       BIC
## -1.419696 -1.411751 -1.276324
```

```
# Model 5: ARMA(1,1) (final)
log.diff.import.arma11 <- arima(log.diff.import.ts, order = c(1,0,1))
log.diff.import.arma11.diagnostics <- sarima(log.diff.import.ts, 1, 0, 1)
```

```
## initial  value -2.124853
## iter   2 value -2.131198
## iter   3 value -2.133231
## iter   4 value -2.137431
## iter   5 value -2.149287
## iter   6 value -2.160505
## iter   7 value -2.163489
## iter   8 value -2.164322
## iter   9 value -2.164496
## iter  10 value -2.164541
## iter  11 value -2.164592
## iter  12 value -2.164651
## iter  13 value -2.164733
## iter  14 value -2.164745
## iter  15 value -2.164927
## iter  16 value -2.165021
## iter  17 value -2.165204
## iter  18 value -2.166863
## iter  19 value -2.167436
## iter  20 value -2.176018
## iter  21 value -2.178987
## iter  22 value -2.192394
## iter  23 value -2.194201
## iter  24 value -2.200608
## iter  25 value -2.203958
## iter  26 value -2.210785
## iter  27 value -2.211433
## iter  28 value -2.219517
## iter  29 value -2.220264
## iter  30 value -2.226721
## iter  31 value -2.239600
## iter  32 value -2.253113
## iter  33 value -2.260094
## iter  34 value -2.260647
## iter  35 value -2.263222
## iter  36 value -2.264100
## iter  36 value -2.264100
```

```
## iter  37 value -2.264274
## iter  37 value -2.264274
## iter  38 value -2.264308
## iter  38 value -2.264308
## iter  39 value -2.264316
## iter  39 value -2.264316
## iter  40 value -2.264317
## iter  40 value -2.264317
## iter  41 value -2.264318
## iter  41 value -2.264318
## iter  42 value -2.264318
## iter  42 value -2.264318
## iter  42 value -2.264318
## final  value -2.264318
## converged
## initial  value -2.151965
## iter   2 value -2.159581
## iter   3 value -2.165801
## iter   4 value -2.167893
## iter   5 value -2.168446
## iter   6 value -2.168480
## iter   7 value -2.168526
## iter   8 value -2.168538
## iter   9 value -2.168538
## iter  10 value -2.168539
## iter  11 value -2.168542
## iter  12 value -2.168542
## iter  12 value -2.168542
## iter  12 value -2.168542
## final  value -2.168542
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE  t.value p.value
## ar1     0.7838 0.0991   7.9080   0.000
## ma1    -1.0000 0.0583 -17.1426   0.000
## xmean  -0.0075 0.0036  -2.0729   0.043
##
## sigma^2 estimated as 0.0126088 on 54 degrees of freedom
##
## AIC = -1.358857  AICc = -1.350913  BIC = -1.215485
##
```




```
log.diff.import.arma11
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(1, 0, 1))
##
## Coefficients:
##          ar1      ma1  intercept
##       0.7838  -1.0000    -0.0075
## s.e.  0.0991   0.0583     0.0036
##
## sigma^2 estimated as 0.01261:  log likelihood = 42.73,  aic = -77.45
```

```
log.diff.import.arma11.diagnostics
```

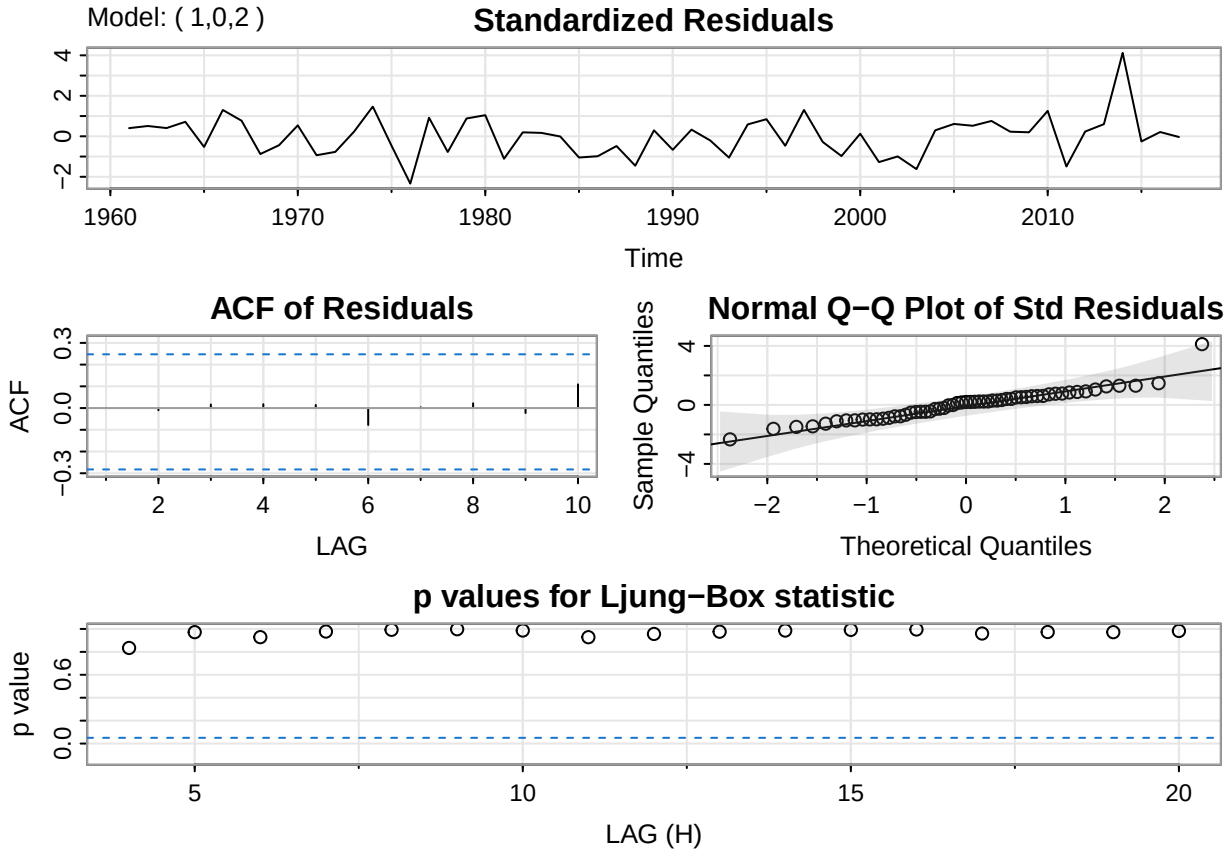
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1    xmean
##       0.7838  -1.0000  -0.0075
## s.e.  0.0991   0.0583   0.0036
##
## sigma^2 estimated as 0.01261:  log likelihood = 42.73,  aic = -77.45
```

```
##
## $degrees_of_freedom
## [1] 54
##
## $ttable
##       Estimate     SE  t.value p.value
## ar1     0.7838 0.0991   7.9080   0.000
## ma1    -1.0000 0.0583 -17.1426   0.000
## xmean  -0.0075 0.0036  -2.0729   0.043
##
## $ICs
##       AIC      AICc       BIC
## -1.358857 -1.350913 -1.215485
```

```
# Model 6: ARMA(1,2)
log.diff.import.arma12 <- arima(log.diff.import.ts, order = c(1,0,2))
log.diff.import.arma12.diagnostics <- sarima(log.diff.import.ts, 1, 0, 2)
```

```
## initial  value -2.124853
## iter   2 value -2.190526
## iter   3 value -2.196281
## iter   4 value -2.197046
## iter   5 value -2.198253
## iter   6 value -2.199359
## iter   7 value -2.201100
## iter   8 value -2.202005
## iter   9 value -2.202359
## iter  10 value -2.202363
## iter  11 value -2.202364
## iter  12 value -2.202364
## iter  13 value -2.202364
## iter  14 value -2.202364
## iter  15 value -2.202364
## iter  15 value -2.202364
## iter  15 value -2.202364
## final  value -2.202364
## converged
## initial  value -2.209843
## iter   2 value -2.209920
## iter   3 value -2.210058
## iter   4 value -2.210177
## iter   5 value -2.210240
## iter   6 value -2.210268
## iter   7 value -2.210269
## iter   7 value -2.210269
## iter   7 value -2.210269
## final  value -2.210269
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.3312 0.2902 -1.1412  0.2589
## ma1     0.2433 0.2720  0.8944  0.3751
## ma2    -0.4033 0.1267 -3.1820  0.0024
```

```
## xmean  -0.0025 0.0093 -0.2631  0.7935
##
## sigma^2 estimated as 0.01194652 on 53 degrees of freedom
##
## AIC = -1.407221  AICc = -1.393726  BIC = -1.228006
##
```




```
log.diff.import.arma12
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(1, 0, 2))
##
## Coefficients:
##          ar1     ma1      ma2  intercept
##       -0.3312  0.2433  -0.4033    -0.0025
## s.e.   0.2902  0.2720   0.1267     0.0093
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -80.21
```

```
log.diff.import.arma12.diagnostics
```

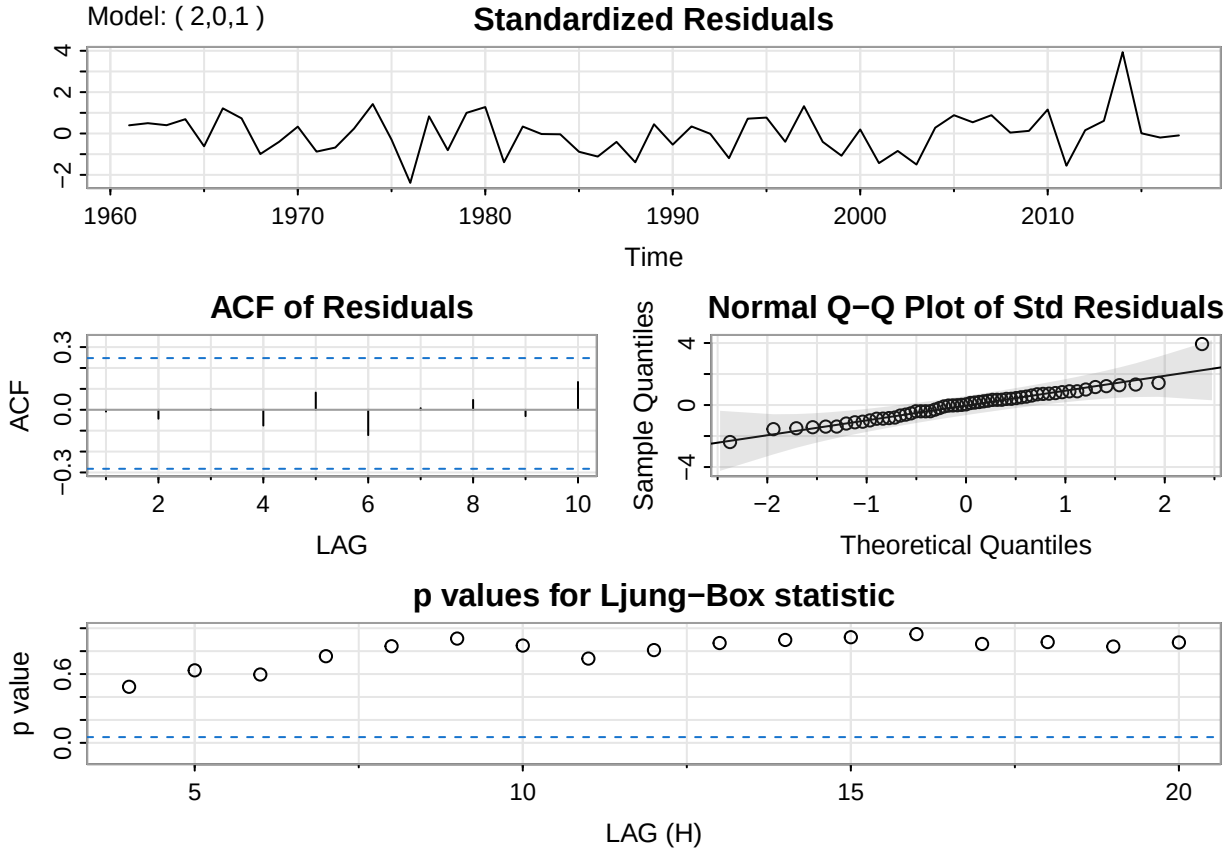
```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
```

```
##
## Coefficients:
##           ar1     ma1      ma2    xmean
##       -0.3312  0.2433  -0.4033  -0.0025
## s.e.   0.2902  0.2720   0.1267   0.0093
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -80.21
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.3312 0.2902 -1.1412  0.2589
## ma1     0.2433 0.2720  0.8944  0.3751
## ma2    -0.4033 0.1267 -3.1820  0.0024
## xmean  -0.0025 0.0093 -0.2631  0.7935
##
## $ICs
##       AIC      AICc       BIC
## -1.407221 -1.393726 -1.228006
```

```
# Model 7: ARMA(2,1)
log.diff.import.arma21 <- arima(log.diff.import.ts, order = c(2,0,1))
log.diff.import.arma21.diagnostics <- sarima(log.diff.import.ts, 2, 0, 1)
```

```
## initial  value -2.117685
## iter   2 value -2.183588
## iter   3 value -2.185259
## iter   4 value -2.185930
## iter   5 value -2.186463
## iter   6 value -2.186556
## iter   7 value -2.187485
## iter   8 value -2.187673
## iter   9 value -2.187720
## iter  10 value -2.187726
## iter  11 value -2.187727
## iter  11 value -2.187727
## iter  11 value -2.187727
## final  value -2.187727
## converged
## initial  value -2.199962
## iter   2 value -2.200019
## iter   3 value -2.200062
## iter   4 value -2.200062
## iter   5 value -2.200063
## iter   6 value -2.200064
## iter   7 value -2.200069
## iter   8 value -2.200072
## iter   9 value -2.200074
## iter  10 value -2.200074
## iter  10 value -2.200074
## final  value -2.200074
## converged
## <><><><><><><><><><><><><><>
```

```
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.3207 0.7160 -0.4479  0.6561
## ar2    -0.3531 0.1222 -2.8890  0.0056
## ma1     0.2253 0.8057  0.2797  0.7808
## xmean  -0.0016 0.0108 -0.1486  0.8824
##
## sigma^2 estimated as 0.01221868 on 53 degrees of freedom
##
## AIC = -1.386832  AICc = -1.373337  BIC = -1.207617
##
```




```
log.diff.import.arma21
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(2, 0, 1))
##
## Coefficients:
##           ar1      ar2     ma1  intercept
##       -0.3207  -0.3531  0.2253    -0.0016
## s.e.   0.7160   0.1222  0.8057     0.0108
##
## sigma^2 estimated as 0.01222:  log likelihood = 44.52,  aic = -79.05
```

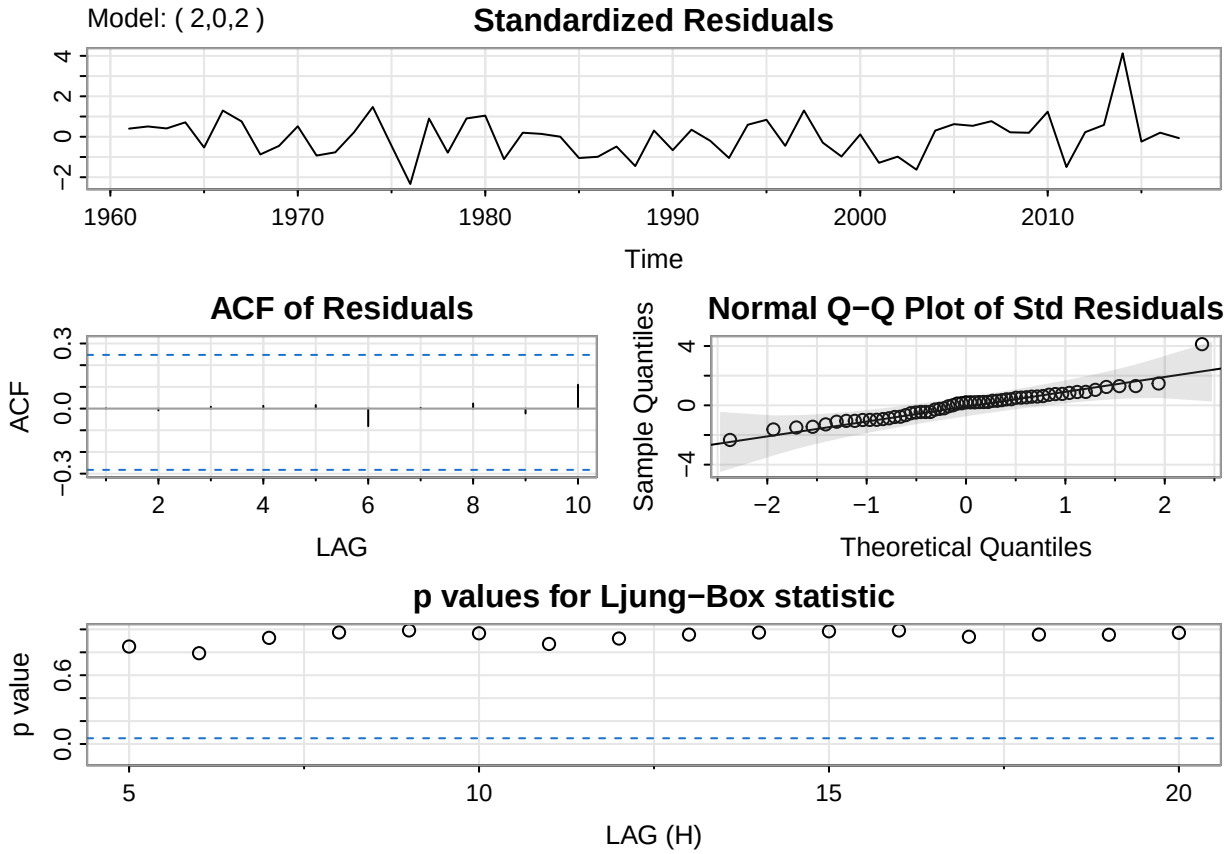
```
log.diff.import.arma21.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ar2     ma1    xmean
##       -0.3207  -0.3531  0.2253  -0.0016
## s.e.   0.7160   0.1222  0.8057   0.0108
##
## sigma^2 estimated as 0.01222:  log likelihood = 44.52,  aic = -79.05
##
## $degrees_of_freedom
## [1] 53
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.3207 0.7160 -0.4479  0.6561
## ar2    -0.3531 0.1222 -2.8890  0.0056
## ma1     0.2253 0.8057  0.2797  0.7808
## xmean  -0.0016 0.0108 -0.1486  0.8824
##
## $ICs
##       AIC      AICc       BIC
## -1.386832 -1.373337 -1.207617
```

```
# Model 8: ARMA(2,2)
log.diff.import.arma22 <- arima(log.diff.import.ts, order = c(2,0,2))
log.diff.import.arma22.diagnostics <- sarima(log.diff.import.ts, 2, 0, 2)
```

```
## initial  value -2.117685
## iter   2 value -2.146070
## iter   3 value -2.189558
## iter   4 value -2.191196
## iter   5 value -2.191658
## iter   6 value -2.194157
## iter   7 value -2.196115
## iter   8 value -2.199075
## iter   9 value -2.200842
## iter  10 value -2.202596
## iter  11 value -2.203318
## iter  12 value -2.203457
## iter  13 value -2.203461
## iter  14 value -2.203464
## iter  15 value -2.203467
## iter  16 value -2.203469
## iter  17 value -2.203470
## iter  18 value -2.203472
## iter  19 value -2.203476
## iter  20 value -2.203477
```

```
## iter  21 value -2.203477
## iter  22 value -2.203478
## iter  23 value -2.203478
## iter  23 value -2.203478
## final  value -2.203478
## converged
## initial  value -2.209480
## iter   2 value -2.209834
## iter   3 value -2.209908
## iter   4 value -2.210012
## iter   5 value -2.210118
## iter   6 value -2.210246
## iter   7 value -2.210304
## iter   8 value -2.210312
## iter   9 value -2.210329
## iter  10 value -2.210339
## iter  11 value -2.210345
## iter  12 value -2.210345
## iter  12 value -2.210345
## final  value -2.210345
## converged
## <><><><><><><><><><><><><><>
##
## Coefficients:
##       Estimate     SE t.value p.value
## ar1    -0.3480 0.3419 -1.0178  0.3135
## ar2    -0.0325 0.3434 -0.0947  0.9249
## ma1     0.2586 0.3225  0.8019  0.4263
## ma2    -0.3743 0.3343 -1.1197  0.2680
## xmean  -0.0024 0.0095 -0.2494  0.8040
##
## sigma^2 estimated as 0.0119451 on 52 degrees of freedom
##
## AIC = -1.372287  AICc = -1.351647  BIC = -1.157229
##
```




```
log.diff.import.arma22
```

```
##
## Call:
## arima(x = log.diff.import.ts, order = c(2, 0, 2))
##
## Coefficients:
##           ar1      ar2     ma1      ma2  intercept
##       -0.3480  -0.0325  0.2586  -0.3743    -0.0024
## s.e.   0.3419   0.3434  0.3225   0.3343     0.0095
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -78.22
```

```
log.diff.import.arma22.diagnostics
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = xmean, include.mean = FALSE, transform.pars = trans, fixed = fixed,
##     optim.control = list(trace = trc, REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ar2     ma1      ma2    xmean
##       -0.3480  -0.0325  0.2586  -0.3743  -0.0024
## s.e.   0.3419   0.3434  0.3225   0.3343   0.0095
##
## sigma^2 estimated as 0.01195:  log likelihood = 45.11,  aic = -78.22
```

```
##
## $degrees_of_freedom
## [1] 52
##
## $ttable
##       Estimate     SE t.value p.value
## ar1    -0.3480 0.3419 -1.0178  0.3135
## ar2    -0.0325 0.3434 -0.0947  0.9249
## ma1     0.2586 0.3225  0.8019  0.4263
## ma2    -0.3743 0.3343 -1.1197  0.2680
## xmean  -0.0024 0.0095 -0.2494  0.8040
##
## $ICs
##       AIC      AICc       BIC
## -1.372287 -1.351647 -1.157229
```

Imports Final Model: ARMA(1,1)

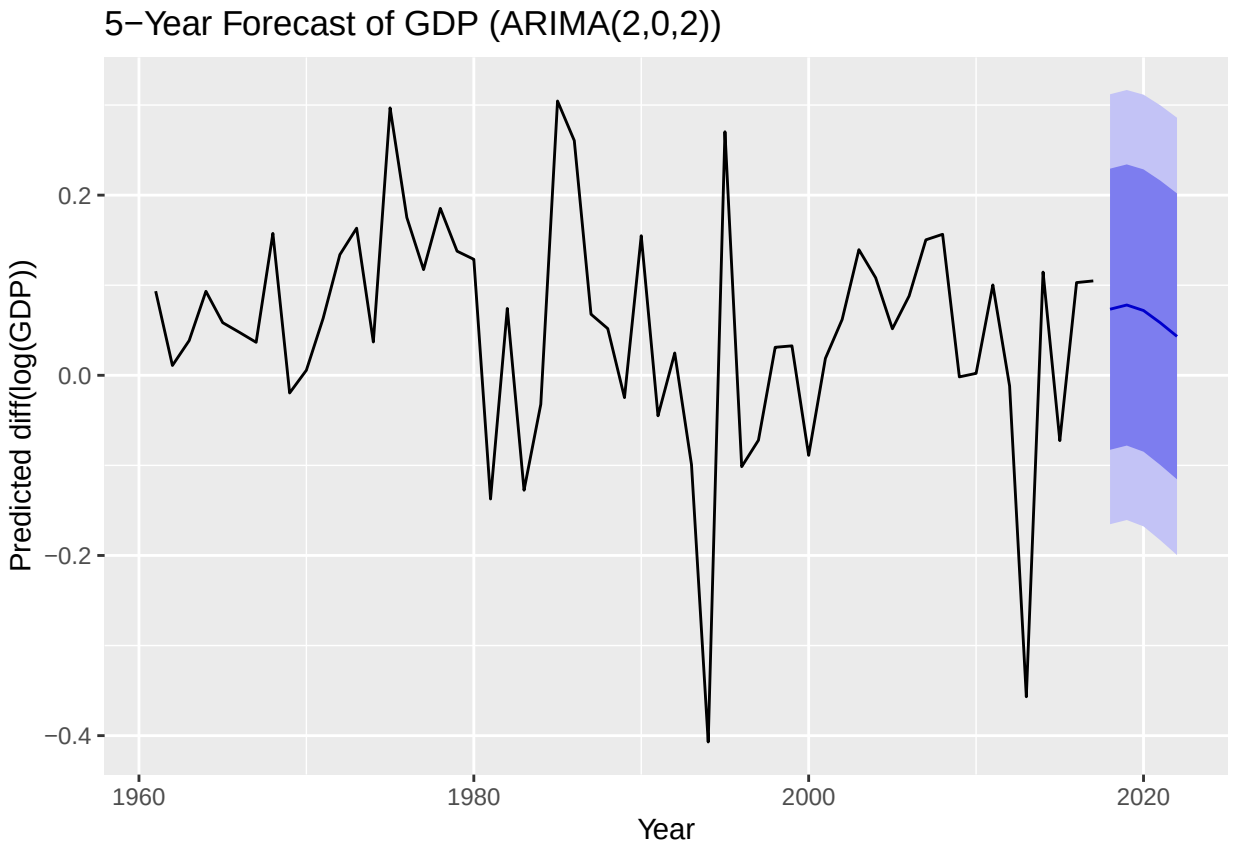
Forecast

GDP

```
# Forecast
# GDP
gdp_forecast <- forecast(log.diff.gdp.arma22, h = 5)
gdp_forecast
```

```
##      Point Forecast       Lo 80     Hi 80      Lo 95     Hi 95
## 2018     0.07335514 -0.08268150 0.2293918 -0.1652823 0.3119926
## 2019     0.07804164 -0.07800850 0.2340918 -0.1606165 0.3166997
## 2020     0.07190733 -0.08476349 0.2285781 -0.1677000 0.3115147
## 2021     0.05832534 -0.09951227 0.2161630 -0.1830665 0.2997171
## 2022     0.04318533 -0.11551828 0.2018889 -0.1995309 0.2859015
```

```
autoplot(gdp_forecast) +
  ggtitle("5-Year Forecast of GDP (ARIMA(2,0,2))") +
  xlab("Year") +
  ylab("Predicted diff(log(GDP))")
```

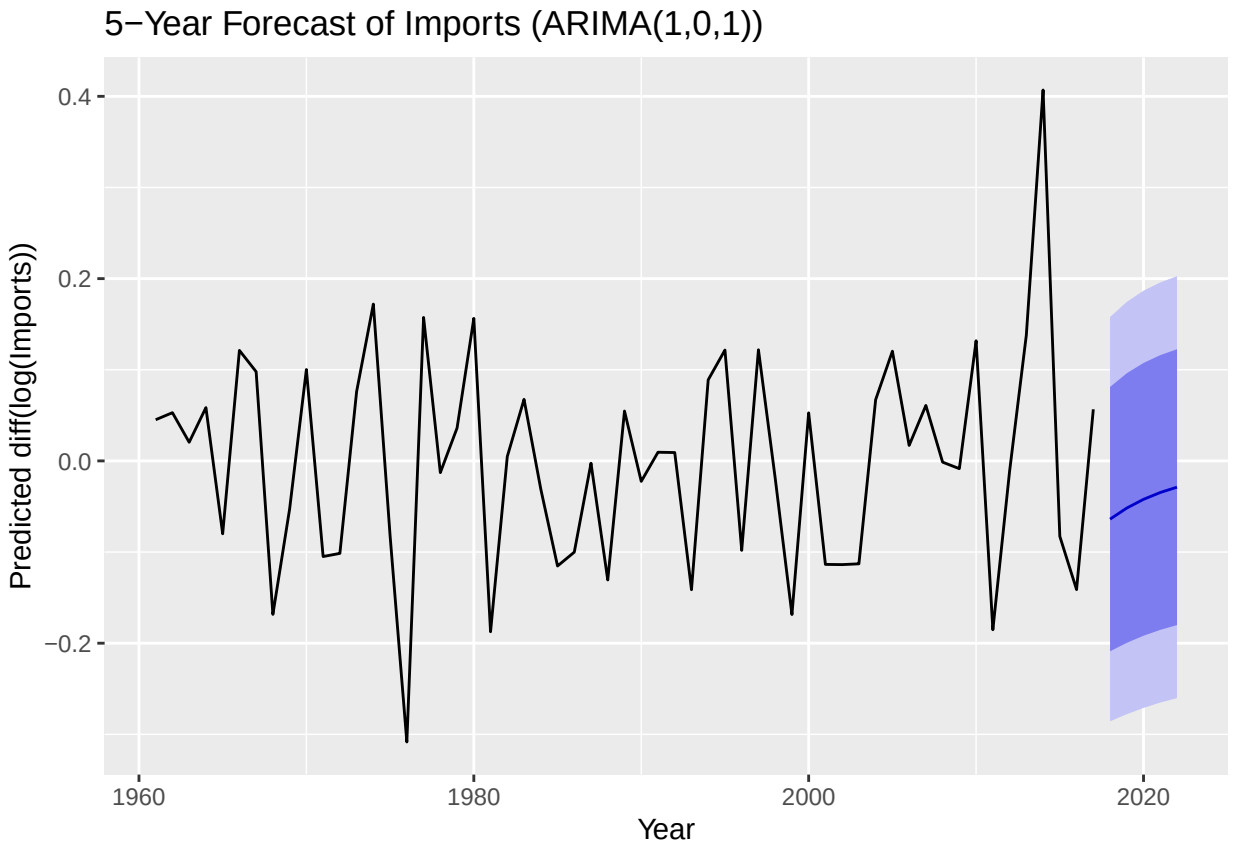


Imports

```
# Imports
import_forecast <- forecast(log.diff.import.arma11, h = 5)
import_forecast
```

```
##      Point Forecast      Lo 80      Hi 80      Lo 95     Hi 95
## 2018    -0.06390127 -0.2089035 0.08110099 -0.2856631 0.1578605
## 2019    -0.05171036 -0.1995994 0.09617871 -0.2778872 0.1744665
## 2020    -0.04215508 -0.1917900 0.10747989 -0.2710020 0.1866919
## 2021    -0.03466560 -0.1853631 0.11603193 -0.2651376 0.1958064
## 2022    -0.02879531 -0.1801419 0.12255130 -0.2602600 0.2026694
```

```
autoplot(import_forecast) +
  ggtitle("5-Year Forecast of Imports (ARIMA(1,0,1))") +
  xlab("Year") +
  ylab("Predicted diff(log(Imports))")
```


5−Year Forecast of Imports (ARIMA(1,0,1))
Predicted diff(log(Imports))
0.4
0.2
0.0
−0.2
1960
1980
2000
2020
Year